DOCUMENT RESUME

ED 477 311 SE 067 984

AUTHOR	Singleton, Cynthia M.
TITLE	A Historical Overview That Led to the Fragmentation of College Mathematics Curriculum.
PUB DATE	2002-04-03
NOTE	54p.
PUB TYPE	Information Analyses (070) -- Reports - Descriptive (141)
EDRS PRICE	EDRS Price MF01/PC03 Plus Postage.
DESCRIPTORS	*Curriculum Design; *Educational History; Higher Education; *Mathematics Instruction; Postsecondary Education; Professional Development

ABSTRACT

This paper investigates how and why the fragmentations of college mathematics have reached the current form through looking at the history of mathematics education. (KHR)

A Historical Overview that led to the Fragmentation of College Mathematics Curriculum

Cynthia M. Singleton*

April 3, 2002

The major existing subdivisions of the present mathematics curriculum of post-secondary education in United States are: Arithmetic, Algebra, Geometry, Trigonometry, Calculus and Analytic Geometry, Statistics, and Complex Algebra. Recently many educators argue that the mathematics is taught in isolation and that one of the problems in the mathematics classroom is "the phenomenon that we can term the fragmentation of knowledge', due to the fact that mathematics subjects are taught separately from each other so that the result is islands in the learning process" (Furinghetti and Somaglia, 1998). Mathematics, which is difficult for many students, suffers from this situation and more than any other subject, is considered to be separated from the cultural context. As a result, the image of mathematics held by students is very poor: they think that mathematics is a very boring subject, without any imagination, detached from real life (Furinghetti and Somaglia, 1998).

To understand how and why the fragmentations of college mathematics have reached the current form one must look at the history of the Mathematics Education. Looking into the history the mathematics curriculum will clarify its boundaries, content, or methods and will increase understanding of the present nature and values of the discipline (Coxford and Jones, 1970, page 1)

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While there is evidence of the existence of genuine prehistoric mathematics in this hemisphere (Mayans of Yucatan in Mexico and Central America had a remarkably well developed numeration system) the history of the curriculum of college mathematics in America starts in1492. (Coxford and Jones, 1970, page 11). The effect of foreign influences upon the mathematics taught from the time of the earliest settlements to the middle of the nineteenth century was profound.

Other forces led to change and development, such as practical needs, church, religion, and intellectual curiosity (Coxford and Jones, 1970, page 13). The colleges founded prior to the Revolutionary war such as Harvard (1601), William and Mary (1693), Yale (1701), Princeton (1746), Pennsylvania and Philadelphia (1766), and Dartmouth (1770) did not have extensive requirements or offerings at first. Arithmetic was made an entrance requirement at Yale in 1745, at Princeton in 1760, and at Harvard in 1807. Harvard required the "the whole of arithmetic" in 1816 and later on in 1820 was the first university to require algebra. Geometry was not required for entrance until after the Civil War. As late as 1726 the only mathematics taught at Yale was a bit of arithmetic and surveying in the senior year. In 1748 Yale required some mathematics in the second and third years. Something of conics and fluxions (calculus) was taught as early as 1758, and by 1766 a program might have included arithmetic, algebra, trigonometry, and surveying. At Harvard the mathematical course began in the senior year and consisted of arithmetic and geometry during the first three quarters of the year and astronomy during the last quarter. It is interested to note that at that time each class concentrated for a whole day on one or two subjects. Mathematics or astronomy was studied on Monday and Tuesday (Coxford and Jones, 1970, page 19). The inclusion of

astronomy with mathematics was typical of the times and resulted from both practical forces and the intellectual climate. The practical need for astronomy in connection with navigation and surveying is obvious. At the same time, the European intellectual revolution stemming from the rise of science and the work of Copernicus, Galileo, Descartes, Newton, was communicated to the colonies quickly. By 1776 six of eight colonial colleges supported professorships of mathematics and natural philosophy (Coxford and Jones, 1970, page 19). In 1850 the Military Academy at West Point, the first technical institute in the United States introduced the study of analytical trigonometry (Coxford and Jones, 1970, page 29). Influenced by West Point the University of Michigan in 1837 adopted textbooks in Arithmetic, Algebra, Geometry, Conic Sections and Mathematics for the Practical Man.

During this time the introduction of the idea of graduate education had a profound effect on the development of the mathematics curriculum in the United States. Americans developed the tendency to do research in pure mathematics and in the general area of the foundation of mathematics rather than in applied mathematics, which became a real national handicap during the time of World War II. The idea of graduate studies and specialization associated with research and the creation of new mathematics is related to the founding of Johns Hopkins University in 1875, the founding of the *American Journal of Mathematics* and the founding of the American Mathematical Society established in 1888 (Coxford and Jones, 1970, page 30). The emphasis on the function concept and on interrelationships within mathematics introduced in 1893 had a profound effect in the development of college mathematics (Coxford and Jones, 1970, page 41). Psychological research and changing theories of learning were the major forces that

influenced the mathematics curriculum during this period. The works of Jean Peerage, Jerome Bruner had great influence on the on the mathematics curriculum (Herrera, 2001)

In the years that followed educational agencies such as NSF and Educational organizations played an important role in the changes of the mathematics curriculum that followed. The National council of Teachers of Mathematics founded in 1920 is responsible for many curriculum reforms. They are responsible for the "new math movement" of 1960-1970. During World War II, both educators and the public recognized that more technical and mathematical skills were needed to meet the needs of the developing technological age. The Commission on Postwar Plans appointed by NCTM made recommendations about mathematics curriculum. The goals were to establish the United States as a world leader and to continue the technological development that had began during the crises of war. The movement was further supported with the launch of Sputnik in 1957 because it crated the perception that the United States was "behind in the world scene" of technology and military power (Herrera, 2001).

In 1986, the Board of Directors of the National Council of Teachers of Mathematics (NCTM), recognizing the alarming need for reform in the way mathematics was taught, established Standards for the curricula of schools (Martinez, 1998).

The above historical overview of the development of mathematics curriculum in the United States suggests that some of the major factors that led to the fragmentation of mathematics and influenced the development of the mathematics curriculum are: practical needs, church, religion, and intellectual curiosity. Many educators now support the idea that the disciplines of the school curriculum should be viewed as a unitary body

of knowledge. A teacher who looks at the school curriculum with "interdisciplinary eyes" can discover a variety of situations to which mathematical methods can be efficiently transferred in a natural way (Furinghetti and Somaglia, 1998).

When looking at the school curriculum this way the role of mathematics changes. This subject really becomes the center of students thinking and hopefully they establish a richer image and understanding of the discipline (Furinghetti and Somaglia, 1998). Interdisciplinary (between mathematics and other disciplines and within mathematics) in the classroom would help students understand the power and beauty of mathematics and perceive it as interesting and useful subject.

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JN: Mathematics-in-School
SO: Mathematics in School v 27 no4 Sept 1998. p. 48-51
PY: 1998
PD: bibl
IS: 0305-7259
LA: English
AB: Part of a special issue on the history of mathematics. The writers describe how the history of mathematics illustrates the development of interdisciplinarity in schools. They provide examples in order to clarify the meaning of interdisciplinarity and the real benefits that mathematics in schools may derive from it. The projects they consider involve students aged from 13 years up. The research the writers provide illustrates that interdisciplinarity, even if differently approached, overcomes fragmentation of knowledge and renews the cultural dignity of mathematics, is characterized by strong interaction within a team of teachers involved in interdisciplinarity, and has the history of mathematics as a key element.
DE: Mathematics-History-Teaching; Mathematics-Correlation-with-other-subjects
DT: Feature-Article
TXI: Y
SZ: 24927
TX1: THE 'FRAGMENTATION OF KNOWLEDGE' AND THE IMAGE OF MATHEMATICS
One of the problems in classroom life is the phenomenon that we can term the 'fragmentation of knowledge', due to the fact that school subjects are taught separately from each other so that the result is islands in the learning process. In particular, mathematics, which is difficult for many pupils, suffers from this situation and, more than other subjects, is considered to be separated from the cultural context. As a result, the image of mathematics held by pupils is very poor: pupils think that mathematics is a very boring subject, without any imagination, detached from real life. This problem is very old. In the Association for the Improvement of Geometrical Teaching, there is the following verse quoted in the Daily News of December 16, 1892 as being often found written in a schoolboy's Euclid or Algebra:
'If there should be another flood, Hither for refuge fly, Were the whole world to be submerged This book would still be dry' (p. 23).
The author observes that 'there is strong reason to believe that these lines accurately set forth the opinion that is usually entertained' (Heppel, 1893 p. 24). He is also pessimistic about the possibility that the average pupil succeeds in seeing the beauty and the poetry in mathematics by himself. He rather thinks that 'the schoolboy's charge of dryness must be met in another way, by showing him how the progress of the Arithmetic, Geometry, Algebra, and Trigonometry that he is learning has gone on in answer to the needs that men have felt, and the desires they have formed' (Heppel, 1893, p. 24). In those times this opinion was widespread in the world of educators and of professional historians of mathematics(FN1).

Among historians the Italian, Gino Loria, who was also very interested in problems of mathematical instruction, strongly supported the introduction into schools of what he terms 'interdisciplinarity', i.e. a way of teaching which emphasizes the links between the different disciplines. In the paper (Loria, 1890) he points out the importance of history in promoting it. The basic idea in the two papers we have mentioned is that history of mathematics can help to see the genesis of ideas and the connections among the various subjects, thus making the culture transmitted in school a homogeneous body of knowledge.

ACHIEVING INTERDISCIPLINARITY IN THE CLASSROOM: AN OUTLINE OF POSSIBILITIES AND IMPLICATIONS

We can find the preceding ideas, which were enunciated in theory by the authors of the past, tried out in more recent times in different research projects. In the following, we briefly discuss some examples in order to make clear what 'interdisciplinarity' means and which real benefits mathematics in school may derive from it. The projects that we consider concern pupils aged from 13 years onwards. We describe how history can help to go across disciplines rather than focus on issues specific to the particular situations in which the projects took place.

In Grugnetti (1989), we find an example of an interdisciplinary activity involving pupils aged 14-16 years. Various historical periods are studied from different points of view: the point of view of the language used and the literature, the social and political situations of different countries, the technological development, and so on. According to the author, this initiative originates from the need to reveal a 'really humanizing aspect' of mathematics. Mathematics is not predominant over the other subjects and the teachers of all the subjects involved participate in the work. The conclusions of the paper do not show if the objective of emphasizing the 'humanizing' aspect of mathematics has been attained; we can only infer a more general result about 'humanizing' classroom life and communication among teachers of different subjects.

An analogous project in France with pupils aged 14-15 years is described in Guichard and Sicre (1988). As a starting point, a mathematician who was born in the region where the school is set was considered. They were very lucky since this mathematician was Francois Viete, a key character in the development of algebra. The authors state very clearly the objectives of their work in the classroom (in addition to the basic objective of making pupils aware of an important piece of local mathematical history):

-- to make pupils interested in an interdisciplinary approach;

-- to give a human dimension to sciences, presenting a part of mathematics (algebra) that is developed in the classroom within a historical perspective;

-- to show that mathematics (and its presence in school as a subject) is evolving all the time;

-- to work in mathematics classes on real problems on which mathematicians have worked, problems similar to those which generated the algebraic calculations.

The subjects involved in this project were architecture, sculpture, music, painting, ways of writing, history and geography of the 16th century. The authors point out two consequences of this experiment. On the cultural side, it appears that after the project there was a different attitude to the 16th century and to the life of the region. On the mathematical side, some basic algebraic concepts such as variables and parameters were discussed; thus it was possible to easily shift the focus of the algebraic teaching from pure manipulation to conceptual issues.

The problem of the image of mathematics held by pupils aged 17-18 years is the motivation of the experiment described by Rommevaux (1988). In this case the teachers of the mother language (French) and of mathematics planned and carried out together classroom work in which pupils analysed texts by Blaise Pascal and by Jean-Jacques Rousseau. The links between mathematics and literature, which in the usual way of looking at mathematics would be weak, reveal themselves as an interesting source of reflection on the nature of mathematics and its teaching, as it is shown in Furinghetti (1993).

There are two subjects, philosophy and art, that we often find associated with mathematics in history. A typical case of work involving the mathematics and the philosophy teacher concerns discussion of the concept of infinitesimals and infinity centred on the Zeno paradoxes. In Guichard and Sicre (1995), there is a description of an experiment involving high school pupils. As for mathematics associated with art, there are many experiments which link these two subjects. In Brin et al. (1995), an activity with 12- and

16-17-year-old pupils is described centred on the study of the methods used by Renaissance artists for representing objects of three dimensional space(FN2) in the two-dimensional plane. In this way, pupils come to know how the mathematical theory of perspective was born from the practice of art. Moreover, they acquire a precious 'key' for looking at masterpieces of art. In the paper by Menghini (1989), the link between art and mathematics is illustrated, not only through the study of Italian artists of the Middle-Ages and the Renaissance but also through the study of the use of conics in Roman Baroque architecture (the experience was actually carried out in Rome). The topic 'conic sections' has evident connections with the orbits of the planets which were studied around the period in which the Baroque churches were built. Thus two kinds of sky--the metaphorical domes of the churches and the real astronomical sky--link mathematics, art and astronomy. In the paper by Menghini (1989), other aspects of the connections between mathematics and art are discussed through the work of the Dutch painter M. Cornelius Escher. The well-known painting inspired by the Poincare model of non-Euclidean geometry was used in the classroom to introduce pupils to problems of non-Euclidean geometries and to motivate a discussion on the nature of space as it developed in the 19th century.

All of the previous experiments have been carried out in the classroom and were supported by worksheets, notes and other materials. We can see from these examples that interdisciplinarity, even if differently approached, has some common characteristics:

-- The fragmentation of knowledge mentioned at the beginning is overcome and mathematics regains its cultural dignity. The role of mathematics is seen from another point of view: it is no longer viewed as a discipline developed to serve other disciplines but is now seen, together with the others, as a discipline for solving problems.

-- There is a strong interaction within the team of teachers (usually in the same school) involved in interdisciplinarity. This interaction is very deep, since it does not only mean working together, but it mainly means to fix common objectives and to find efficient agreement on the methods for pursuing them. Interdisciplinarity reveals itself as a real way to overcome the isolation of which teachers often complain in their work. This fact is one of the factors pointed out by the teachers themselves who have carried out the experiment.

-- History of mathematics is a key element in going across disciplines.

INTERDISCIPLINARITY AND TRANSFERABILITY OF THE MATHEMATICAL METHOD

To focus on the deep meaning of interdisciplinarity and on the role of history in promoting it we analyse an experiment carried out in an Italian high school, the Liceo Scientifico. In this type of school, mathematics has an important role: its programme includes Euclidean geometry, calculus, trigonometry, some elements of logic and computer science. Italian, latin, physics, art, a foreign language, chemistry, sciences and philosophy are the other subjects taught. One of us (Somaglia) is a mathematics teacher at this school. In addition to the considerable cultural demands of the official programmes she has to face the students' difficulties in doing mathematics; thus she has tried different ways of teaching to overcome these difficulties. The usual strategy she uses to introduce mathematical concepts or procedures may be schematized in the following diagram: (Graphic character omitted)

The problem is where and how to find a suitable context in which the pupils' 'intuitive ideas' may arise. The history of mathematics has revealed itself to be very suitable for this purpose, as may be shown in the following example in which the teacher has to introduce the concept of integral, which is very difficult for pupils. Before this introduction she discusses in the classroom the concepts of area and volume, which are more familiar to pupils. Afterwards she presents the principle stated by the Italian mathematician Bonaventura Cavalieri in his treatise Exercitationes Geometricae Sex (published in Bologna in 1647). This is Cavalieri's statement(FN3):

' Two figures either plane or solid are in the ratio of all their indivisibles compared collectively and (if among them you find some unique common relationship) one by one.'

These words convey efficiently the idea of obtaining the area of a plane figure by sectioning it into 'infinitesimal elements' which are summed up afterwards. Cavalier's idea is very appropriate for preparing pupils for subtle reasoning and the obstacles in constructing the concept of integral, since the strategy of

sectioning a figure, as Cavalieri suggests, has an intuitive character and has a very telling graphical interpretation (see Fig. 1).

The context in which to develop the intuitive stage of pupils' learning is not necessarily to be found within mathematics. The teacher has used other disciplines that she considered suitable for this purpose, as we shall see in the following cases.

The first case concerns proof. The teacher ascribes great educational value to activities connected with proof but, as many colleagues do, she has to face the difficulties encountered by her pupils in the different phases of proving (conjecturing, exploring, validating conjectures, giving formal proofs). In particular, she has noticed that, in learning to prove, pupils have difficulty in grasping the overall structure of the proof at the semantic level. This lack of meaning implies pupils' inability to build proofs on their own since, even if they are able to reproduce the various formal passages, they are not able to understand what they are doing. To help pupils see the meaning of a proof, the teacher again looked for a suitable context in which pupils can work informally to activities linked to proof. Having in mind the ancient roots of mathematics and philosophy, she asked the philosophy teacher to involve pupils in an activity of analysis with 'mathematical eyes' of passages taken from works of the Greek philosophers. The colleague agreed to participate in the experiment. Here are some examples of the tasks given to pupils:

1. Give the structure of the proof ab absurdo (using the Eleatic axiom 'it is not possible to talk about nor think of what does not exist') through which Parmenides concludes that 'being is without end'.

2. On what assumptions are Socrates' arguments based--the ones that you have read in Plato's dialogue 'Gorgias' to prove that 'good' and 'pleasure' are the same thing?

-- Outline the fundamental points of Socrates' arguments.

-- Describe and compare the positions of Socrates in Plato's dialogue 'Protagoras' and that of Callicles in Plato's dialogue 'Gorgias'.

The aim of this activity is to accustom pupils to see the mathematical structure (logical sequences, hypotheses and conjectures, ...) in domains different from that of formal proof. In this way, it is hoped that pupils learn to pay attention not only to the syntactic, but also to the semantic aspect of proof. The pupils' arguments show their efforts to apply mathematical methods to a new context and also to use mathematical-like notations and structures. The following is an example of the structure of the reasoning ab absurdo (to deny what is to be proved leads to denying the Eleatic axiom) made by a pupil in the task: (Graphic character omitted)

To work in different contexts (the mathematical and the philosophical) reinforces the pupils' ability to analyse a text and both the subjects in question (mathematics and philosophy) may benefit. Philosophy gains a scientific approach to its theories, mathematics acquires a field in which to exercise the ability to decode colloquial language and to grasp the semantic aspect of a proof. In working together the mathematics and the philosophy teachers help pupils to overcome the psychological barrier of the language. Thus we have a two-way transfer of methods from mathematics to philosophy and vice versa.

In other situations art has been the discipline offering the informal context in which to develop pupils' 'intuitive ideas' on the mathematical concept 'geometric transformations'. A film (by Michele Emmer) on the work of the painter Escher was shown to pupils. This film emphasizes the strong connections of Escher's work with mathematics and allows discussion of the geometric transformations arising from a stimulating situation. After having analysed the mathematical content of Escher's work, the teacher posed a question: why has this painter taken inspiration from mathematics? The teacher, who is familiar with the history of mathematics, has introduced pupils to the methods used by historians. Letters written by the painter and people he met on his travels were considered. It emerged that from the start he knew from his brother, who was a biologist, that the geometrical transformations which were the leitmotif of his art were studied in crystallography. Afterwards he became interested in the mathematical research related to his art; we know, for example, that he had read the studies in crystallography of George Polya. Similarly, in the mathematical milieu, Escher's work was studied by the scientists and new theoretical developments in crystallography were stimulated by Escher's paintings. In this case it was the historical method which linked the different subjects (mathematics and art) to the benefit of both subjects. History of art gained

knowledge of the genesis of a style, mathematics appeared as a subject which may be seen as truly a part of human culture.

Escher's painting was also the key to enter into another branch of human knowledge which has connections with mathematics, i.e. music. In his writings, especially in his letters, the painter points out the 'analogy' of Bach's canon and his way of filling the plane of the painting (using symmetry and antisymmetry). What makes Escher's attitude interesting is that, unlike Bach in music, the painter is fully conscious that he is working on concepts which are common to various disciplines (art, mathematics and music).

Starting from the autobiographical notes of the painter on the relation of his painting to music the teacher has gone further in interdisciplinary teaching and has explored the rational construction of the music, making pupils aware of the important fact that 'several composers used mathematical ideas to structure their compositions' (Fauvel, 1996, p.244). With this activity the teacher has emphasized that there is a concept on which they are working (symmetry) which is common to various disciplines (see Clavarino and Somaglia, to appear).

CONCLUSIONS

In Galileo's works there is a passage in which the author complains of the criticisms of certain scientists of his time, who claim that mathematical ideas have to be developed separately from other subjects such as physics or philosophy. Galileo writes that truth is unique and is part of every discipline(FN4). This view applies to the school environment too. The examples that we have presented in this paper are inspired by the strong conviction of the need to demolish the barriers between subjects (and between teachers in the same school, too). Our intention was to suggest to teachers and readers that they consider the disciplines of the school curriculum as a unitary body of knowledge. A teacher who looks at the school curriculum with 'interdisciplinary eyes' can discover a variety of situations to which mathematical methods can be efficiently transferred in a natural way. We can say that the teacher's attitude has to be similar to the attitude of Escher himself who wrote explicitly that he found inspiration for his creativity by abandoning himself to the emotions of Bach's music. The contribution of the history of mathematics to this process of going across disciplines is in the opportunity it offers to lay bare the underlying ideas of the disciplines. By this way of looking at the school curriculum, the role of mathematics changes. This subject really becomes the centre of pupils' thinking and, we hope, pupils establish a richer image of the discipline.

Added material

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ACKNOWLEDGEMENTS

The authors would like to thank Marcella Bacigalupi, the teacher of philosophy, who supported the project referred to in the text and also John Earle for the accuracy and care taken in the editing of this article.

The editor wishes to thank Helen Easton for assistance with translation and interpretation.

Fig. 1 The original illustration of Cavalieri's principle

FOOTNOTES

1 It is worth noting that in those times there was a great ferment of initiatives in the school world (the Mathematical Association was established in 1894) and history of mathematics was assuming its present status of an autonomous discipline with specific journals and specialized researchers.

2 An interesting discussion of Piero della Francesca's skills in mathematics is in the paper by Field (1993).

3 The translations of the various passages are made by us.

4 See Enriques (1938).

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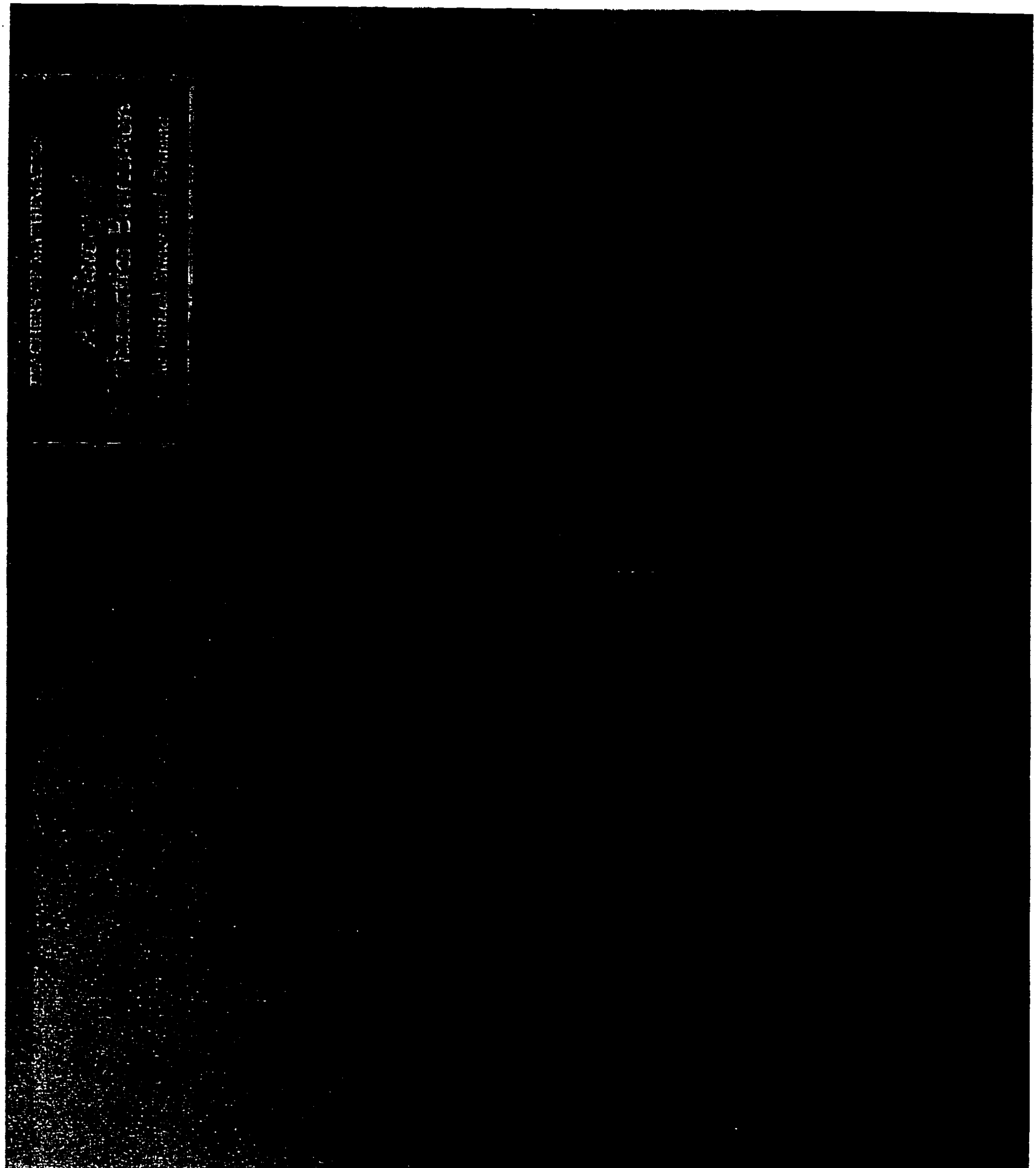

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PART TWO

Forces and Issues Related to Curriculum and Instruction, K–6

M. Vere DeVault and *J. Fred Weaver*

CHAPTER ONE

The Goals of History: Issues and Forces

1. *Our goals*

There are many kinds of history directed toward many goals. The chronology of events may serve for some as an end in itself, and for others as the basis of forecasts or of the search for causes and effects. Patriots or chauvinists may seek with history to revive feelings of nationalism, to justify past actions, to create or enhance a national image. The history of a special field may seek to clarify its boundaries, content, or methods and to increase understanding of the present nature and values of the discipline. Our chief goal in this history of mathematics education is to direct the thoughts of mathematicians, educators, and even the public at large to the issues of today, by showing that although the content and methods of mathematics education have changed significantly over the years, strong continuing forces imply that further change is imminent and important. The continuing development of new mathematics, new uses of mathematics, new pedagogical devices, and changing goals for a changing society all demonstrate the need for continued change in mathematics education.

However, a thoughtful view of history shows that many apparently new ideas are actually old and that they merit a revival only after the lessons of the past have been studied. The old approaches and materials should be examined in the light of both old experience and a

new situation. For example, Colburn's "inductive approach" to arithmetic and algebra has many elements in common with the "discovery teaching" of today; and attention to numeration systems and geometry in the elementry school has been urged on many occasions since the beginning of the nineteenth century. History suggests that past innovations have been adopted in response to changing needs and philosophies, although rarely with a speed that could be called revolutionary, and that they have often failed to be as magically effective as their proponents expected. It is hoped that steadier progress will result from a survey of the causes and effectiveness of past proposals and projects. For this reason our plan is to stress issues, and the forces leading to action on these issues, throughout this book.

»»»» 2. *Our themes: forces and issues*

We would like this book to be not a mere catalog of facts about mathematics education in North America, but a description of forces and issues related to mathematics education based upon such a catalog of facts. We regard *issues* as questions with reference to which there has been or may be some debate. Many issues exist in mathematics education today. Many of these are the same as, or slight modifications of, issues that have occurred frequently in our history. The emergence both of these issues and of proposed answers to them is due to a variety of *forces*—in mathematics itself, in educational theory and psychology, and in our changing culture. Our ultimate goal, then, is to provide a better understanding of the issues of today and the forces which gave rise to them, and to suggest answers for them. We believe that this may be achieved through a tracing over the past centuries of emerging issues and changing forces.

We believe that the major issues and forces relating to mathematics education can be considered under two headings: *curriculum* and *instruction*. The former, of course, includes both the content and the grade placement of materials taught. The latter is chiefly concerned with the method of teaching but also includes provisions for individual differences. This means that a complete separation of these two headings, curriculum and instruction, is not possible, since questions on individual differences are certain to involve questions of mental maturity which are related to the grade placement of materials in the curriculum.

Although we shall treat the evolution of mathematics education chiefly under these two headings, two other categories will frequently

appear as the sources of issues and forces. These are the changing views of *mathematics* itself and of the newly developing fields of *educational philosophy* and *psychology*. During the period of our story mathematics first developed in association with studies of the physical world, becoming a major tool in the scientific and technological revolution of the seventeenth, eighteenth, and early nineteenth centuries. Then, in the nineteenth and twentieth centuries, it became a collection of man-made, axiomatic structures rooted in the interests and intuitions of its builders but not in the physical world. Connections of mathematics with the physical world often developed later out of the work of persons other than mathematicians. As this view of mathematics developed, rigor, abstraction, and generalization increasingly characterized the work of the professional mathematician.

Educational psychology developed in somewhat the reverse manner. From the musings of educational philosophers it moved first into laboratories in which behavior was studied and then out to share in the redesigning of textbooks and the development of teaching procedures to embody the ever-changing views of the learning process and problem solving.

Further, although the authors believe in minimizing the distinctions between elementary and secondary education and in stressing continuity and articulation rather than differences in grade placement or course content, we have found it convenient to divide our discussion of forces and issues emerging through history in accord with the elementary and secondary levels of education. The extent and manner to which this distinction is being blurred today is a current issue, but we are writing history. We cannot deny that in the past one could differentiate rather sharply both the curriculum and instructional procedures according to grade level. For this reason we think that under today's circumstances our audience may be larger and more understanding if we heed this separation but try to point out both the issues involved in eliminating it and the forces tending to drive us in this direction. Along with this, we shall point out the issues and methods that are common to instruction at both levels.

These considerations have led us to design a book in six parts:

Part 1: Mathematics in the Evolving Schools
Part 2: Forces and Issues Related to Curriculum and Instruction, K–6
Part 3: Forces and Issues Related to Curriculum and Instruction, 7–12

Part 4: The Education of Teachers of Mathematics

Part 5: School Mathematics in Canada

Part 6: Epilogue: Present-Day Forces and Issues

3 *Progress by periods*

In Part One we will present the facts of the history of mathematics education, stopping occasionally to indicate themes that will be further developed in later parts. Our aim has been to clarify issues and identify forces which may both produce issues and help resolve them.

We have divided the story into five periods. Although time is a continuum which can be uniquely and sharply separated by specifying a single point, educational and cultural forces and issues are always overlapping and intermingling throughout all periods of time. However, it seemed sensible to us to begin with only a brief reference to the period prior to the first settlement in the United States and Canada (1607) and to extend the early history of mathematics education across the period of the beginnings of schools to the time of Warren Colburn, whose first book was published in 1821. We have selected Warren Colburn as representative of the beginning of a new concern for pedagogy. Although Colburn's arithmetic (125) and algebra (126)* written on "inductive principles" are neither the first mathematical books written or published in the Western Hemisphere nor the first to give thought to pedagogy, they represent a new and more general concern for pedagogical processes in this country.

For our second period we have chosen 1821 to 1894. The latter year dates the beginning of three new forces in mathematics education in North America. This was the year of the founding of the American Mathematical Society (AMS), which was derived from the New York Mathematical Society initiated in 1888 (244, pp. 105–6). The AMS itself has probably not been a significant factor in secondary and elementary education. However, certain early members of the society, notably Professor E. H. Moore, were much concerned with the problems of secondary school mathematics and gave time and attention to them, as well as giving this area of investigation the prestige that came from their interest in innovation in the teaching of mathematics. The year 1894 also saw the publication of the *Report of the Committee of Ten*

* Numerals in parentheses, as they appear here, refer in the Introduction and Parts One through Four to items in Bibliography A, for publications in the United States; in Part Five, to items in Bibliography B, for Canada.

on Secondary School Studies (72), the committee having been appointed two years earlier by the National Education Association (NEA). The appointment of this committee typifies two forces in American mathematics education: (1) the concern of persons with a major initial interest in education as a whole for the specialized subject-matter field of mathematics and (2) the influence of national committee reports as stimulators of reform.

The founding of the AMS and the role of the NEA in appointing the Committee of Ten represent another significant force that affected education during the latter part of the nineteenth century and the first part of the twentieth. This force was the rapid development of many professional organizations, both those associated with the organization and administration of the schools and those associated with scholarly research. The *Report of the Committee of Ten* will be discussed later, but it is interesting to note how many still-current issues received its attention: the introduction of more geometry of an intuitive sort into the elementary schools; the earlier beginning of algebra; the incorporation of solid geometry with plane; and the development of what later was called a "double track" program after the first year of algebra, with one syllabus for those planning to continue into college and another for those not so bent. This last provides an example of an issue and a related force. The issue is this: For what students and for what goals should the school program be defined? The force is the pressure on the secondary schools to plan curricula and guide students with college entrance requirements and examinations in mind.

We have chosen to terminate our third period with 1920, the date of the founding of the National Council of Teachers of Mathematics (NCTM) and of the issuing of the preliminary report of the National Committee on Mathematical Requirements, *The Reorganization of the First Courses in Secondary School Mathematics* (57). This committee had been appointed by the Mathematical Association of America (MAA) in 1916, prior to the entry of the United States into World War I. The major concern of the MAA, itself only recently founded (1915), was for the undergraduate mathematics curriculum in the colleges. Its sponsorship of this committee emphasizes the effect of college requirements and ideas on the secondary school curriculum and also the continuing concern of many mathematicians at the college level for improvement in the schools. A number of persons with close relations to the secondary schools—both persons training teachers and persons teaching in the schools—were sought out and included on the committee. The committee was spoken of with great approval by the

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first president of the NCTM, C. M. Austin, in his initial note to the members of that organization at the time it took over the *Mathematics Teacher* as its official journal in 1921 (289, p. 1). This period, 1894–1920, might then be labeled "The Growth of College and University Influence," or perhaps "First Steps toward Revision."

Our fourth period, 1920–45, introduces the effect of two strong forces outside the control of school or college personnel: economic depression and war. We have chosen 1945 as the terminal year of this period because it marked the end of World War II and the appearance of two of the reports of the Commission on Post-War Plans of the NCTM (16; 17). This committee was the first of a number of committees, commissions, and studies to be set up as a result of concern for the inadequacies in mathematics education that were brought out during the war.

The reports of the Commission on Post-War Plans may be regarded as one of the links between the issues of the prewar period and the reforms that began to get under way about 1952. Prior to the war mathematics educators had struggled to answer many criticisms. There were two major types: (1) The mathematical (arithmetical) competence of high school graduates was viewed as inadequate for many occupations such as business, elementary school teaching, and the armed services. (2) Much of secondary school mathematics, even extending to long division, was regarded as of questionable value for general education by many educational philosophers, psychologists, and guidance specialists. The Commission on Post-War Plans clearly exposed fallacies in this view. Other educators began to stress that the war had not only demanded mathematical competencies at many levels but had also led to the development of many new applications with postwar, peacetime significance. Both the applications of mathematics and much of the actual mathematics were new. This led to the view that to be prepared for unforeseen needs one must know and understand the structure of mathematics, not merely its facts and operations.

Two major issues of the prewar period persisted after the war: (1) Does school mathematics instruction serve the needs of individual students? (2) Does school mathematics instruction contribute effectively to the solution of the problems of our society? Before the war many people would have answered both questions with a no. The answer remained the same; but the reasons behind it, the remedies called for, and even the persons giving the answer changed during the war and the years immediately following it when proposals for extended experimentation and reform were developed.

This led to our definition of the years from 1945 to the present as our fifth and last period, a period of experimentation and reform which some termed a "revolution." Naturally, changes so extreme as to be called revolutionary brought forth some protests and reaction.

4. *Summary and preview*

In Part One, then, we will seek to provide the factual framework from which emerge the *issues* that have called for concern and change and the *forces* that have given rise to these issues and given direction to the changes.

In our introductory remarks we have passingly focused on such issues as these:

1. *What should be the goals of mathematics education?* Providing for personal and vocational practical needs? Providing "mental discipline"? Training in logic and problem solving? Preparing for further study?

2. *How can mathematical education in both content and instruction be adapted to the varied needs, capacities, and interests of students?* By "double tracks"? Self-instruction?

These are perhaps the *only* issues in the sense that perhaps all others are in some way subsidiary to them. However, we have also noted, implicitly at least, these additional issues:

3. *What mathematics should be taught?* "Facts" and operations—or structure?

4. *What students should take mathematics?* Everyone? All college preparatory students? Only would-be scientists and engineers?

Some issues which will arise in later chapters are:

5. *What persons or groups should direct mathematics education?* Professional mathematicians? If so, should they be the pure or the applied mathematicians? Educators? Psychologists? Mathematics educators? The general public?

6. *How can one provide for the experimentation needed to guarantee continued tested progress in both curriculum and instruction?*

7. *What levels of rigor are sound and desirable at different stages of a student's development?*

8. *What is the role of applications and mathematical models in motivating and clarifying instruction?*

). *How do we teach so that students perceive the excitement and* *iuty of mathematics as well as its facts and theorems?*

10. *Can we teach so that students will "discover" and be more crea-* *e?* If we can, should we?

Forces that have been mentioned in our introductory remarks in- ıde:

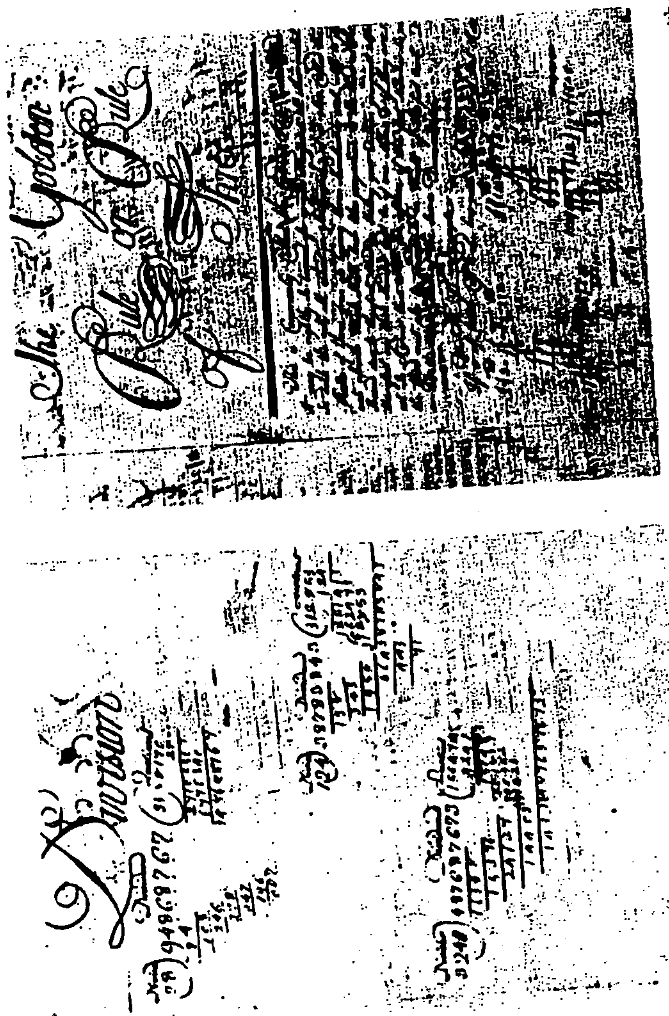
1. *Practical needs*—of explorers, soldiers, navigators.
2. *The research and beliefs of psychologists and philosophers*—with ·erence to the goals and methods of instruction.
3. *College entrance examinations and requirements.*
4. *The presence in the schools of increasing numbers of students* *·th increasingly varied interests and abilities, but all needing to be* *ained and educated.*

This list is incomplete, and some of these forces are subsidiary to hers in conception or importance. However, it is hoped that this :roductory list may make the reader more alert to perceive our eme and goal: the presentation of the history of mathematics educa- n as the story of changing issues and forces as they affect practices nich, in turn, provide a series of approximations to changing educa- nal goals. We hope that such perceptions may stimulate, prepare for, d help determine the best directions for future changes.

Mathematics in the Evolving Schools

PART ONE

Phillip S. Jones
Arthur F. Coxford, Jr.



A COPYBOOK IN THE ARCHIVES OF DARTMOUTH COLLEGE. These leaves illustrate the stress on rules and lengthy computation. The flowery description of the Rule of Three suggests verbatim copying of dictated material. (See also page xx.)

CHAPTER TWO

From Discovery to an Awakened Concern for Pedagogy 1492–1821

1. *The earliest American mathematics*

Few people realize that the Mayans of Yucatan in Mexico and Central America had a remarkably well-developed numeration system containing a zero symbol and utilizing place-value concepts but based on twenty rather than ten, long before the Spanish explorers came to the Western Hemisphere. In fact Cajori even suggests that the Mayan use of zero is the earliest use of such a symbol anywhere in the world (110, p. 11). This is a little uncertain because of the difficulties of dating Mayan hieroglyphics and also because it is debatable whether a special symbol used as a separatrix in the Seleucid period of Babylonian mathematics should be regarded as the use of a zero in that system. In any case, this was a rather remarkable prehistoric development in the Western Hemisphere, as was the somewhat similar development of a numeration system by the Aztecs.

There is also some evidence of the existence of genuine prehistoric mathematics in this hemisphere, or at least of number words in the languages studied by anthropologists who have worked with North American Indians. However, we shall pass quickly from these earliest periods to our major concern, the story of developments since 1492.

A little-known fact about the earliest mathematics in the Americas is that the first book of mathematical content printed in the Western Hemisphere was the *Summario Compendioso de las quentas de plata y ore*, published by Juan Diez Freyle in Mexico City in 1556. Although the major concern of this book was the conversion of gold ore into value equivalents in different types of coinages of the Old World, a problem requiring chiefly the use of ratio and proportion or what was called "the rule of three," it also contained a short chapter on algebra. In addition to this book, there were seven Mexican and four Peruvian books with substantial mathematical content published prior to 1700 (164, pp. 25–35). None of these books was devoted solely to mathematics. Their major concerns in fact were chiefly military matters such as the design and construction of camps and fortifications; surveying and navigation; and the calculation of the calendar, especially to determine various feast days and religious holidays. An educational sidelight on the times is suggested by the title of Pedro Paz's book, published in Mexico City in 1619, *Arte para aprender todo el menor del arithmetica sin maestro*.

Its theme of home study, or study "without a master," is typical of the time and of a land being newly exploited and settled. This is reflected also in the first book with mathematical content in the English language published in the Western Hemisphere. Published in Boston in 1703, this was a reprint of an English book by John Hill entitled *The Young Secretary's Assistant*. It included arithmetical ideas needed by the young businessman along with discussions of letter writing, bookkeeping, etc. This book represents several of the forces operating in a new land—the influence of foreign authors, of vocational needs, and of commercial demands—and again the role of home study and self-teaching in the educational system of newly settled territories.

The first English-language mathematical book both written and published in North America was entitled *Arithmetick, Vulgar and Decimal*. It was written by Isaac Greenwood and published in Boston in 1729. Greenwood, who served as the first Hollis Professor of Mathematics at Harvard from 1728 to 1738, had studied in England. Mathematics books in German, Dutch, and French appeared in 1774 (Germantown, Pa.), 1730 (New York), and 1775 (Quebec), respectively.

These facts further illustrate the continuing effect of foreign influences upon the mathematics taught from the time of the earliest settlements to the middle of the nineteenth century. Other forces led to change and development, and some of them continue up to the present:

1. *Practical needs*. The needs of exploration, navigation, and trade are dominant in a developing frontier community; later the needs of developing technology and science become important.
2. *Church and religion*. In the Spanish books cited above they influenced the content of the books; later they influenced the content and purpose of all education, especially higher education.
3. *Intellectual curiosity*. The existence and role of intellectual curiosity are suggested by the inclusion of unrelated work in algebra in Juan Diez Freyle's earliest book.

2. *The content and processes of colonial instruction*

At the outset, the earliest settlements had no schools at all. When schools did develop, their chief objective was the teaching of reading and writing. Reading and writing usually included the reading and writing of numerals and therefore counting. The famous hornbook of early American schools and homes included arithmetic only in the sense that on one side roman numerals were listed for memorization. Commercial arithmetic was often taught in special schools called reckoning or writing schools. Bronson Alcott's description of the schools in Massachusetts as late as the beginning of the nineteenth century included the note (121, p. 9):

> Until within a few years no studies have been permitted in the day school but spelling, reading and writing. Arithmetic was taught by a few instructors one or two evenings in a week. But in spite of the most determined opposition, arithmetic is now being permitted in the day school.

It was rare for a young student of the seventeenth or eighteenth century to possess an arithmetic book. As Robert Clason points out (441, p. 54), "Pupils were provided with books of blank pages, called ciphering books, given a rule and a problem, and set to work." A pupil of about 1810 described his experience thus (181, p. 45):

> Printed arithmetics were not used in the Boston schools till after the writer left them, and the custom was for the master to write a problem or two in the manuscript of the pupil every other day. No boy was allowed to cipher till he was 11 years old, and writing and ciphering were never performed on the same day. . . . Any boy could copy the work from the manuscript of any further advanced than himself, and

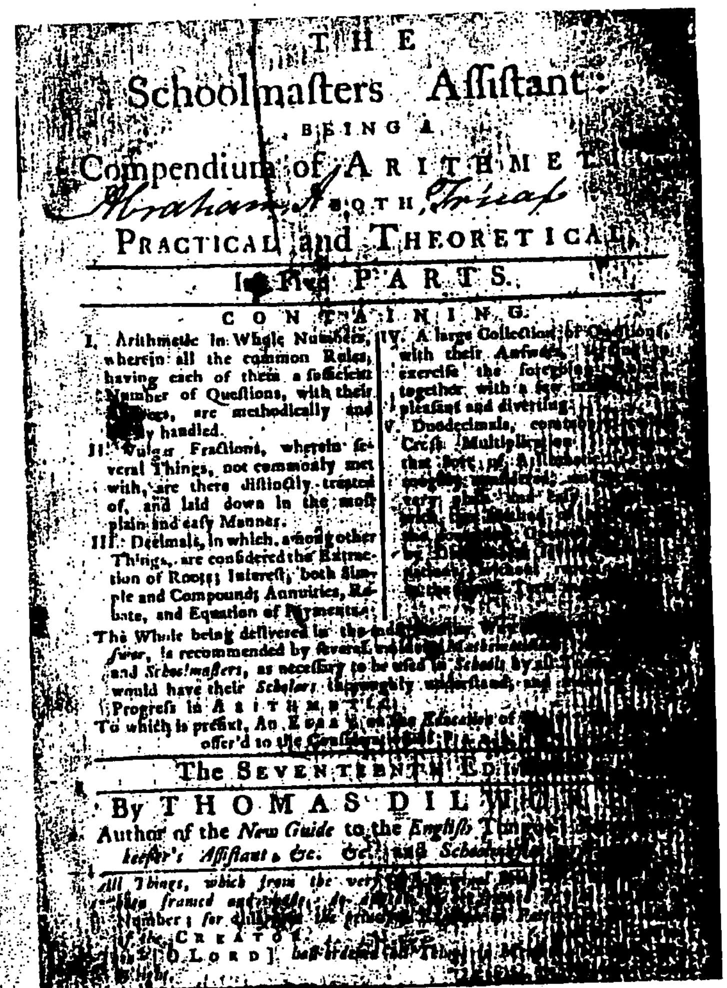
the writer never heard any explanation of any principle of arithmetic while he was at school.

Although students did not use printed texts in the ciphering-book approach to arithmetic, texts were available for teachers. These early arithmetic books were self-contained, complete, single volumes with no internal spiraling—each topic in its turn was treated completely and then dropped. The same book would be used over several years of study at different age levels, in or out of school. The most popular eighteenth-century arithmetic in America was Thomas Dilworth's *The Schoolmaster's Assistant: Being a Compendium of Arithmetic Both Practical and Theoretical*, a reprint of an English text. The first American edition (1773) was labelled the seventeenth edition. Its only concession to the new locale seems to be the addition in the preface of the book's fifty-first endorser, Nathaniel Wurteen, "Schoolmaster at Philadelphia." Karpinski (164, p. 73) lists fifty-eight American printings and editions through 1832. The topics listed in its table of contents were:

1. Arithmetic in whole numbers, . . .
2. Vulgar Fractions . . .
3. Decimals, in which, among other things, are considered the Extraction of Roots; Interest, both Simple and Compound; Annuities, Rebate, and Equation of Payments
4. A large collection of questions, with their answers, serving to exercise the forgoing rules; together with a few others, both pleasant and diverting
5. Duodecimals, commonly called Cross Multiplication

All this is encompassed in 192 small pages, and preceded by a "Preface Dedicatory" addressed to "Schoolmasters in Great Britain and Ireland" and an essay "On the Education of Youth" addressed to parents. Some extracts from the former will give some idea of the pedagogical problems and concepts of the day:

> I believe it is confessed by All, that it is a Task too hard for *Children* to be made compleat *Masters of Arithmetic*; and therefore the best Way of instructing them in it is, most certainly, first to give them a general Notion of it, in the easiest Manner, and next to enlarge upon it afterward, if there be Time; otherwise it must be done by themselves, as their increase in *Years* and Growth in Understanding will permit. . . .
>
> In all Places where it could be done conveniently, I have given Directions for varying the *Examples* by Way of *Proof*; because it not


THE Schoolmasters Assistant: BEING A Compendium of ARITHMETIC BOTH PRACTICAL and THEORETICAL. In Five PARTS. CONTAINING

I. Arithmetic in Whole Numbers, wherein all the common Rules, having each of them a sufficient Number of Questions, with their Answers, are methodically and briefly handled.
II. Vulgar Fractions, wherein several Things, not commonly met with, are there distinctly treated of, and laid down in the most plain and easy Manner.
III. Decimals, in which, among other Things, are considered the Extraction of Roots; Interest, both Simple and Compound; Annuities, Rebate, and Equation of Payments.
IV. A large Collection of Questions, with their Answers, ...
V. Duodecimals, ...

By THOMAS DIL[WORTH]
Author of the *New Guide* to the *English Tongue* ...

FIRST ARITHMETIC PRINTED IN THE UNITED STATES. From the University of Michigan's collections.

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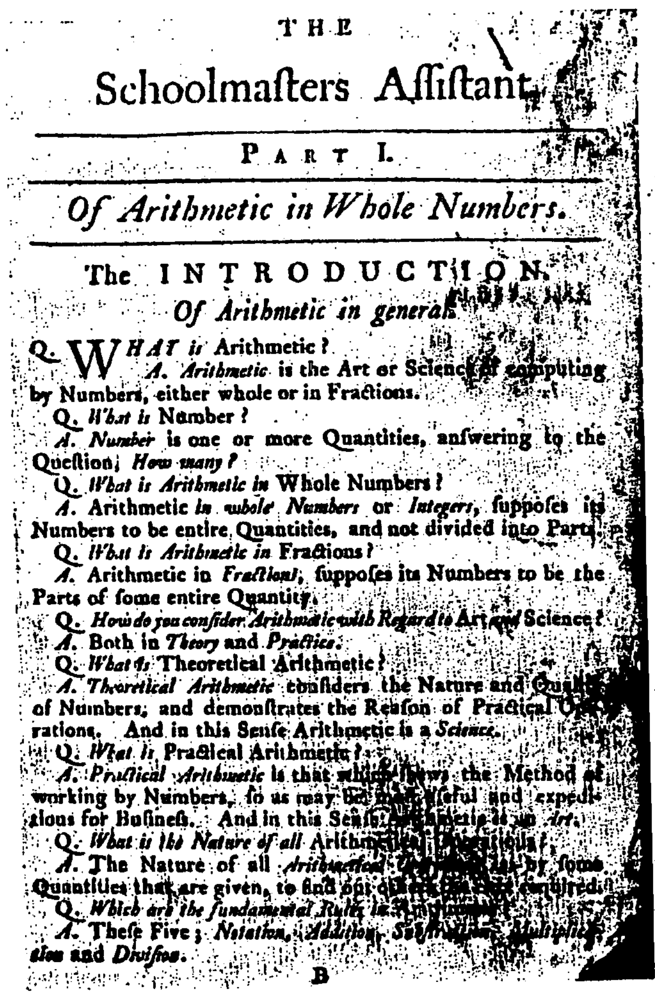

THE

Schoolmasters Assistant.

PART I.

Of Arithmetic in Whole Numbers.

The INTRODUCTION.

Of Arithmetic in general.

Q. WHAT *is* Arithmetic?

A. *Arithmetic* is the Art or Science of computing by Numbers, either whole or in Fractions.

Q. *What is* Number?

A. *Number* is one or more Quantities, answering to the Question; *How many?*

Q. *What is Arithmetic in* Whole Numbers?

A. Arithmetic *in whole Numbers* or *Integers*, supposes its Numbers to be entire Quantities, and not divided into Parts.

Q. *What is Arithmetic in* Fractions?

A. Arithmetic in *Fractions*, supposes its Numbers to be the Parts of some entire Quantity.

Q. *How do you consider Arithmetic with Regard to* Art *and* Science?

A. Both in *Theory* and *Practice*.

Q. *What is* Theoretical Arithmetic?

A. *Theoretical Arithmetic* considers the Nature and Quality of Numbers, and demonstrates the Reason of Practical Operations. And in this Sense Arithmetic is a *Science*.

Q. *What is* Practical Arithmetic?

A. *Practical Arithmetic* is that which shews the Method of working by Numbers, so as may be most useful and expeditious for Business. And in this Sense Arithmetic is an *Art*.

Q. *What is the Nature of all* Arithmetical [illegible]?

A. The Nature of all *Arithmetical* [illegible] by some Quantities that are given, to find out [illegible] required.

Q. *Which are the fundamental Rules in* [illegible]?

A. These Five; *Notation*, *Addition*, [illegible] *Multiplication* and *Division*.

B

LWORTH'S *SCHOOLMASTERS ASSISTANT*. English
ithmetics in dialogue and catechetical form date
ck to Robert Recorde, c. 1540.

only discovers the *Reason* of the operation, but at the same time both produces a new *Question*, and proves the *old One*. . . .

I have thrown the subject of the following pages into a *Catechetical Form*, that they may be the more instructive; for *Children* can better judge the Force of an *Answer*, than follow *Reason* through a Chain of *Consequences*. Hence also it proves a very good *examining Book*; for at any time in what Place soever the *Scholar* appears to be defective, he can immediately be put back to that *Place* again.

The note to parents is also fascinating in its reflection of modern pedagogical problems such as tardiness and homework. In particular, parents are urged to "never let their own Commands run counter to the Master's"; and a substantial paragraph is devoted to lamenting that "the fair Sex" are either not sent to school before they are eighteen or twenty years of age or that they do not stay at their "Writing Schools" long enough (more than a year).

The most popular arithmetic in America prior to 1850 was *Daboll's Schoolmaster's Assistant, Being a Plain Practical System of Arithmetic; Adapted to the United States*, first printed in New London, Connecticut, in 1800. This English text by Nathan Daboll and a revision by David A. Daboll had appeared in eighty editions by 1841. Daboll's book is much like his countryman Dilworth's: short (240 small pages with a six-page appendix on bookkeeping by Samuel Green in the edition examined), it is limited to arithmetic and stresses commercial topics and problems. Many of its problems involve large numbers and extended computation. Although dollars and cents appear, more problems involve pounds and shillings. Extended computations with varied units of measure and their "reduction" are included. It does not use a catechetical method but in each section gives a rule, a worked example, and problems.

The most popular American-written arithmetic text of this early period was Nicholas Pike's *A New and Complete System of Arithmetic Composed for the Use of the Citizens of the United States*, published in Newburyport, Massachusetts, in 1788. Together with an abridgment made in 1792 it appeared in nineteen printings and editions by 1822. The first edition was endorsed by B. Woodward, professor of mathematics at Dartmouth University; S. Williams, Hollis Professor of Mathematics and Natural Philosophy at Harvard University; and by the presidents of those two schools and Yale College. It was of a different character from Dilworth's book, much more extensive both in length (512 pages) and in subject matter, and clearly aimed at more

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mature students. Something of its range may be revealed by citing a few topics and summarizing its major categories:

Rules for Reducing all the Coins, from Canada to Georgia (pp. 111–23)
Tare and Trett [*sic*] (pp. 192–94)
Extraction of the Biquadrate root (p. 210)
Annuities (pp. 264–66, 300–313)
Circulating Decimals (pp. 323–28)
Permutations and Combinations (pp. 339–45)

A great deal of space was given to mensuration, the calendar, and elated astronomical problems, with less space devoted to such topics rom mechanics as falling bodies, the pendulum, lever, and screw. A ummary of Pike's second edition (1797) shows the following distribuion of materials: arithmetic, 396 pages; geometry, 4 pages; trigonomecy, 11 pages; mensuration (including the area and volume of an icosaiedron [*sic*] and the volume of a mash tub), 46 pages; algebra, 33 ages; conics, 10 pages.

This tabulation of contents makes it quite clear that Pike's book was ot a textbook for elementary school students. It was, in fact, adopted nmediately as a textbook by Harvard, Yale, and Dartmouth (240, p. 1). Pike's book has also been listed as the third book published in the merican colonies to include algebra. The first was Freyle's, mentioned the first section of this chapter. The second was *Arithmetica of yffer-Konst . . . Als Mede Een kort ontwerp van de Algebra*, by ter Venema, printed by J. Peter Zenger in 1730 in New York (240, 58).

In geometry, of the fourteen books published in America prior to 20, only three could be classified as dealing with demonstrative geomy. The remaining were works dealing with mensuration and surveyg, such as *Hawney's Complete Measurer* (1801), the first book aling with geometry printed in America. This, like the three deonstrative geometries, was written abroad. The geometries included o English books, Simson's and Playfair's; and one translation from ench, A. M. Legendre's *Elements of Geometry*. This latter book's st appearance in 1819 was followed by forty-two later editions and nslations, the last in 1890. It was used as a text at Yale University late as 1885. The popularity of Legendre's book and an analysis of unique features (such as its departure from Euclid's axioms and uence of theorems, its inclusion of some formulas, and its use of ebraic symbols and processes in proofs) show the strength of the nch influence on early American mathematics.

The introductions to these books also document the goals judged proper for mathematics education at the time. Legendre said the value of the "method of the ancients" is "to accustom the student to great strictness in reasoning," and also cited its value in mathematical research. Simson asserted that geometry was "of great use in the arts of peace and war" (358, p. 5).

In summary, even though a few books delayed or deleted the presentation of formal definitions and rules, especially at the elementary level, and a few used a catechetical question-and-answer format, the net effect was a heavy reliance on set exercises and texts with the result that the teaching process was "State a rule, give an example, and set exercises to be followed (or, in geometry, give proofs to be memorized)."

The content of mathematical education laid a heavy stress on practical applications—especially those related to exploration, commerce, and settlement (mensuration, bookkeeping, navigation, and surveying).

Instruction in mathematics was often minimal. It was irregular (or at least not daily), and it was often given by a private tutor outside of a formal day school.

However, the seventeenth century was notable for the early emergence of one of the strongest continuing forces in American education: the belief that education is necessary for the welfare of society. For example, a Massachusetts law of 1642 required the instruction of children in reading and catechism and also their apprenticeship in a trade. There was no mention of mathematics in this. In 1647 Massachusetts law required every town of fifty families to provide an elementary school teacher and every town of a hundred families to provide a Latin grammar school. The chief function of the Latin grammar schools was preparation for college; the college in turn taught Latin as a preparation for the ministry, law, and the teaching of Latin. These occupations and college attendance being limited, there was only limited popular support for the schools. The grammar schools had their effect on mathematics education somewhat negatively, since reading but not mathematics was required for entrance into the Latin grammar school.

The belief in the important of education for a society, its religion, and its government has continued to be a strong force up to the present moment. But as the early neglect of mathematics suggests, it necessarily leads to an important issue: *What* is the education that is necessary to the welfare of society?

»»» 3. *The Latin grammar school and the academy*

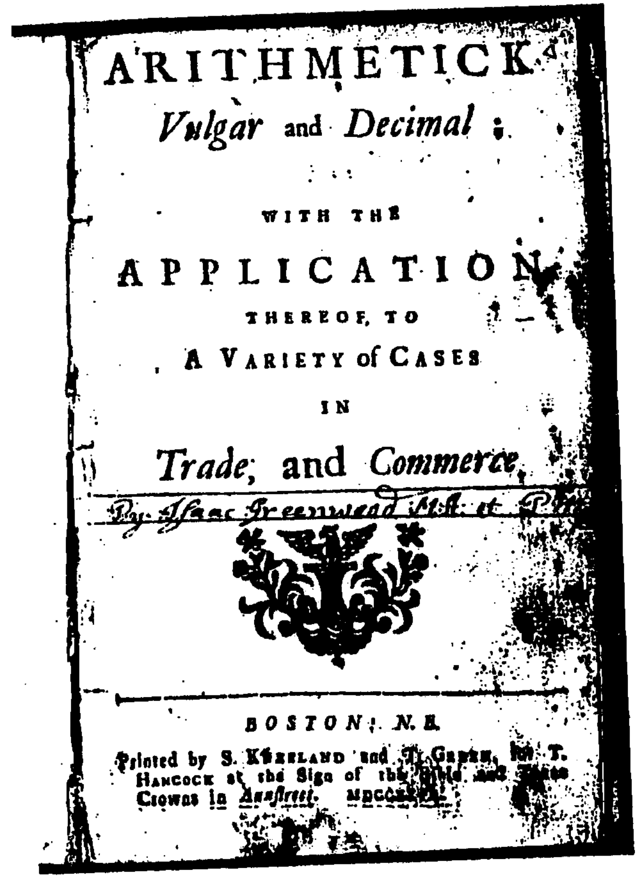
Although the Latin grammar schools had some public support, they ended to be modeled after the English grammar school, to be atended by children from wealthy families, and to direct their instrucion toward preparation for college—which in turn was directed oward the preparation of gentlemen, ministers, lawyers, and doctors. The Boston Latin Grammar School was founded in 1635. By 1700 bout thirty New England towns had such schools. Their curriculum vas chiefly Latin and literature, with some attention to writing. Later ome arithmetic was introduced, in response to public demand. But he cost and aristocratic nature of the grammar schools, together with need for a practical education for a growing group of merchants, tisans, navigators, and surveyors, led to a decline in the Latin gram-ar school. This was accelerated by opposition to taxation and by e American Revolution.

Academies developed in response to the forces listed above. The st academies, beginning with the one founded by Benjamin Frank-in 1749–51, were privately supported. The extensive development d spread of academies did not get under way until the nineteenth ntury.

The academy aimed at preparation both for life and for college. broader curriculum included mathematics for its practical utility d also for its mental discipline. The broadened curriculum and owing number of academies ultimately made necessary the institu-n of college entrance requirements.

»» 4. *The colleges and the mathematics community*

The colleges founded prior to the Revolutionary War—Harvard 101), William and Mary (1693), Yale (1701), Princeton (as the Col-e of New Jersey, 1746), Pennsylvania (as Philadelphia, 1755), Co-mbia (as Kings, 1754), Brown (1764), Rutgers (1766), Dartmouth 70)—differed somewhat in their origins and goals, but in mathe-ics none had extensive requirements or offerings at first. Arithmetic made an entrance requirement at Yale in 1745, at Princeton in o, and at Harvard in 1807. Harvard, though starting late, moved d rapidly, requiring "the whole of arithmetic" in 1816 (only the operations, "reduction," and the "rule of three" were required in) and then moving on to become in 1820 the first to require


ARITHMETICK
Vulgar and Decimal:
WITH THE
APPLICATION
THEREOF, TO
A VARIETY of CASES
IN
Trade, and Commerce.

BOSTON: N. E.
Printed by S. KNEELAND and T. GREEN, for T. HANCOCK at the Sign of the Bible and Three Crowns in Annstreet. MDCCXXIX.

FIRST ARITHMETIC WRITTEN AND PRINTED IN THE UNITED STATES. From the Plimpton Collection of Columbia University. A second copy there bears the autograph of Samuel Adams. (See also illustration on following page.)

algebra. Geometry was not required for entrance until after the Civil War.

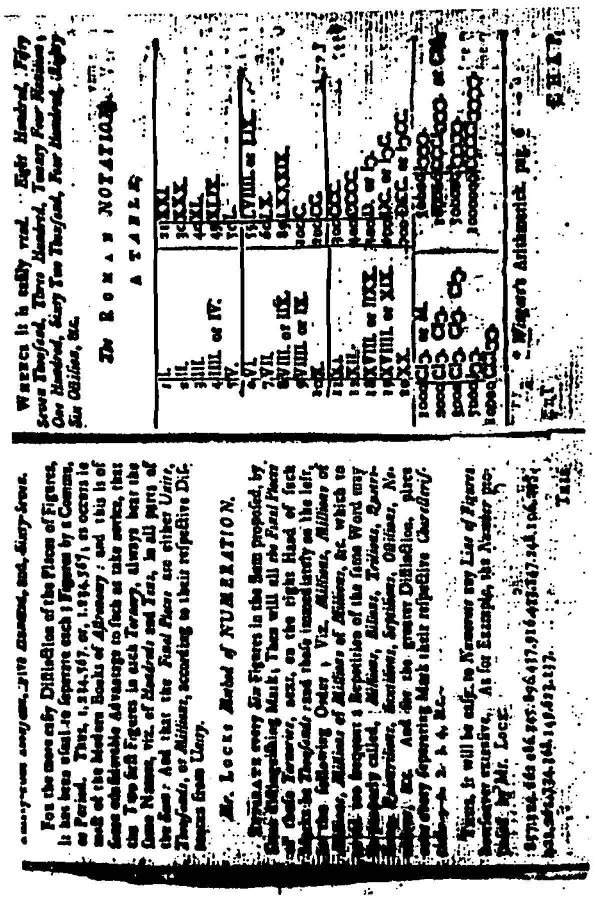
The early college mathematics curriculum was pretty scanty. As late as 1726 the only mathematics taught at Yale was a smattering of arithmetic and surveying in the senior year. In 1748 Yale required some mathematics in the second and third years. Something of conics and fluxions was taught as early as 1758, and by 1766 a program might have included arithmetic, algebra, trigonometry, and surveying.

At Harvard, whose program originally extended through only three years, the mathematical course began in the senior year and consisted of arithmetic and geometry during the first three quarters of the year and astronomy during the last quarter (121, p. 19). At that time each class concentrated for a whole day on one or two subjects. Mathematics or astronomy was studied on Monday and Tuesday.

Smith and Ginsburg (244, p. 32) quote a report to King George III, given after England's acquisition of Canada at the end of the French and Indian War in 1763 to the effect that there was "a pitiful contrast between the intellectual culture in the newly acquired Canada and the uncultured backwardness of the older English colonies." These historians then remark, "Whether or not the comparison was just, it is certain that the work in the new republic, judged by their curricula, was of a low grade."

Lecture notes show that Isaac Greenwood taught a formal sort of algebra based on the work of John Wallis during his period as first Hollis Professor of Mathematics and Natural Philosophy (1728–38). Greenwood's successor, John Winthrop, also taught calculus (fluxions), but his major interests and contributions were in astronomy (244, pp. 20, 52). The inclusion of astronomy with mathematics—and as a major part of mathematics—was typical of the times and resulted from both practical forces and the intellectual climate. The practical need for astronomy in connection with navigation and surveying is obvious. At the same time, the European intellectual revolution stemming from the rise of science and the work of Copernicus, Galileo, Descartes, Newton, and many others was communicated to the colonies rather rapidly. Witness to this is borne by the term "natural philosophy" as it recurs in the titles of courses and professional chairs at the colleges of the eighteenth century. By 1776 six of eight colonial colleges supported professorships of mathematics and natural philosophy (225, p. 29).

Mathematics had become a tool of science as well as of commerce, and science required more advanced mathematics than did commerce.



Pages from Greenwood's *Arithmetic*, showing his use of "billion" for a million million, and also the older forms of Roman numerals. (See also illustration on previous page.)

is force, the rise of science, was displayed in new curricula set up ly in the nineteenth century in such older colleges as Pennsylvania l Princeton and in the plans for such post-Revolutionary War col-es as West Point (1802), Virginia (1818–24), and Rensselaer Poly-hnic (1824).

Reference to the University of Pennsylvania and the University of rginia brings to mind two prominent early American intellectual l political leaders who did much for American education: Benjamin inklin (1706–1790) and Thomas Jefferson (1743–1826). Franklin, hough he did no mathematics himself, wrote "On the Usefulness of athematics," which appeared in the *Pennsylvania Gazette*, 20 October 35; and he included a school of mathematics along with schools English and Latin in the design of his academy, influencing the ign of the University of Philadelphia, which became the Univer-y of Pennsylvania. Thomas Jefferson published several mathematical icles about surveying, almanacs, astronomy, and Napier's theories spherical triangles (244, pp. 58, 61). He was a major influence in ting up the University of Virginia with an orientation that was ssical and humanistic but included such popular and practical new ojects as mathematics (225, p. 125).

»» 5. *Self-instruction*

The "mathematical community" of this early period included not y a few professors and sponsors but also some remarkable self-ght mathematicians whose major activities were outside the aca-nic world.

We have noted that in the earliest days, lack of formal mathe-tical instruction made self-instruction a necessity for all. Economic, nilial, and cultural pressures often denied to eager and able students at formal instruction was available. Three outstanding examples of h students are Benjamin Banneker (1731–1806), David Rittenhouse 32–1796), and Nathaniel Bowditch (1773–1838).

Benjamin Banneker, a Negro, was sent to an obscure school where learned to read and do arithmetic "as far as double fractions"; the school was open only during the winter, and he was allowed attend only until he became big enough to help his father on the m (383, pp. 32–37). His work, like that of the other two self-ght men, was largely in astronomical computations and the prep-tion of almanacs.

David Rittenhouse progressed from watchmaker to instrument

maker, surveyor, and astronomer. He served as vice-provost of the University of Pennsylvania, and George Washington appointed him director of the mint (244, p. 56; 218, pp. 21–28).

Nathaniel Bowditch is best known today as the original author of *The American Practical Navigator*, still published by the United States government. At the age of ten he was apprenticed to a ship's chandler, but he was allowed to read in the private Salem Philosophical Library. While sailing for many years he progressed from supercargo to captain; after his retirement from the sea he ultimately became president of an insurance company. His translation of Laplace's *Mécanique Céleste* (1829–39) contained many added explanations and commentaries and was the first book of substantial or advanced mathematics published in this hemisphere (356, pp. 333–38).

»»» 6. *The teachers and pedagogy*

There was a wide range in background and competence among teachers, especially in the earliest days. It was an exceptional teacher who possessed a fair knowledge of "fractions" and "the rule of three," according to Cajori (121, p. 9). The best-trained ones were very young college graduates and ministers who often engaged in teaching for only a short time early in their careers—and did that in the Latin grammar schools. For example, John Adams, later to become president of the United States, taught at the grammar school at Worcester for a time after graduating from Harvard and before beginning the study of law (p. 9). On the whole, the status of grammar school teachers was not high—it was about that of skilled laborers (118, p. 253).

The college teachers had somewhat stronger backgrounds. Both Isaac Greenwood and John Winthrop, who succeeded Greenwood at the age of twenty-four, had studied in England. Walter Minto (1753–1796) was appointed to Princeton in 1788 on the recommendation of the Earl of Buchan. He had studied at the University of Pisa, had translated the *Cours de Mathématique* of Bossut, and had published some original work (244, p. 30).

The dominant pedagogy of this period, as mentioned earlier, was to state a rule, give examples, and provide problems. However, the development of new educational views and procedures abroad soon had an effect in this country. In particular, a debt to the ideas of J. H. Pestalozzi (1746–1827) was explicitly recognized in the title of the first edition (1821) of Warren Colburn's *An Arithmetic on the*

Plan of Pestalozzi with Some Improvements. (The acknowledgement was later dropped.) Daniel Adams, whose first arithmetic appeared in 1801, tried to reconcile older procedures with Pestalozzi's concept of mental discipline as a goal for instruction and with his teaching methods. These methods placed a heavy emphasis upon concrete experiences as the basis of an induction from many examples to an understanding of rules. Adams's attempt is reflected in the title of his 1827 text, *Arithmetic in which the Principles of Operating by Numbers are Analytically Explained, and Synthetically applied; thus combining the advantages to be derived both from the Inductive and the Synthetic Mode of Instruction.*

7. *Summary*

Issues

In this chapter we have seen the following issues arise:

1. Should a free education be provided for all children?
2. Is the same amount of education appropriate for boys and for girls? For the children of the wealthy and for the children of tradesmen and workers?
3. Is the proper character and content of public education the humanities? The practical arts? The sciences? Is the goal of instruction mental discipline? Practical skills? Knowledge?
4. Does arithmetic (mathematics) have a place in the public school program?
5. Is rule-example-practice the best pedagogical procedure, or should inductive processes be used?
6. How can teaching processes be embodied in texts? Does a catechetical text embody teaching processes?

Forces

We have also seen the following forces at work:

1. The practical needs of exploration, pioneering, settlement, war, and a developing commerce
2. The church's need for trained ministers and missionaries
3. The belief that education is necessary to the welfare of society
4. The institution of college entrance requirements, including mathematics

5. The rise of "natural philosophy"—science, especially physics and astronomy
6. Individual initiative and self-instruction
7. Foreign texts and foreign-trained teachers
8. Intellectual curiosity and the scholarly interests of political leaders

Chronology

1556	Juan Diez Freyle's *Summario Compendioso* published in Mexico City
1629	Pedro Paz's *Arte . . . arithmetica sin maestro* published in Mexico City
1635	Elementary schools established in Quebec
1635	Boston Latin Grammar School founded
1642	Harvard College founded
1647	Massachusetts law requires towns to have schools
1655	A Jesuit college for secondary education established for Quebec
1703	John Hill's *The Young Secretary's Assistant* printed in Boston
1728	Isaac Greenwood becomes Hollis Professor of Mathematics and Natural Philosophy at Harvard
1729	Greenwood's *Arithmetick, Vulgar and Decimal* published in Boston
1745	Yale requires arithmetic for admission
1749	Franklin's academy founded
1788	Nicholas Pike's *A New and Complete System of Arithmetic* published in Newburyport (Mass.)
1801	*Hawney's Complete Measurer* printed in Philadelphia
1802	U.S. Military Academy founded at West Point
1803	Robert Simson's *The Elements of Euclid* printed in Philadelphia
1807	District Public Schools Act passed in Canada
1816	Common Schools Act passed in Canada
1820	Harvard requires algebra for admission
1821	English High School founded in Boston
1821	Warren Colburn's *An Arithmetic on the Plan of Pestalozzi* published in Boston

CHAPTER THREE

From Colburn to the Rise of the Universities: 1821–94

1. Forces at work

The period from 1821 to 1894 was one in which formal instruction t all levels developed and spread but was subjected to a number of onflicting forces. Practical, theoretical or philosophical, and scientific orces all influenced mathematics education. To the practical needs ted earlier, one might add the demands of a rapidly developing tech-ology, engineering, and westward expansion. Manufacturing that ied interchangeable parts, the telegraph, steamships, canals, railroads -all made some use of measurement and simple mathematical ideas. hese forces led to a recognition of the need for useful knowledge id the development of practical and engineering programs even in e colleges.

At the same time the rapid rise of science, especially mechanics and tronomy, called not only for *more* mathematics but for *more ad-nced* mathematics.

We have already noted the early effect of Pestalozzi's educational eas. His work was preceded by the philosophical writings of Locke

and Rousseau. Pestalozzi's stress on adapting instruction to the individual, beginning with motor skills and object lessons, and motivation by appeal to natural interests and instincts, led to new and more modern teaching methods. He was followed, chronologically, by J. F. Herbart (1776–1841) and Friedrich Froebel (1782–1852). Their methods coexisted with a belief in mental discipline as a goal and, later, an associated faculty psychology.

Two foreign wars had led to a growth of French influence in the United States, especially in mathematics.

Jacksonian democracy and waves of immigration led to newly franchised lower- and middle-class voters who favored government support for roads, railroads, and westward expansion. These forces stimulated both a contempt for purely intellectual activities and a demand, especially in the North, for free public education in "the three R's." In the South, wealthy planters wanted education for their children, with the result that until 1850 the South was ahead of the North in providing private academies and colleges (182, pp. 502, 530).

The Civil War further stimulated interest in engineering, manufacturing, and westward expansion, while at the same time retarding the growth of education in the South.

This was a period of general ferment and concern for such social reforms as abolition, women's rights, child labor, property qualifications for voting, and penal reform. Near the close of this period, several very strong forces for reform in education began to be felt. These included psychological experiments and theories discrediting the mental discipline goal; rapid growth of the school population; and the beginnings of advanced study, learned societies, and professional journals.

2. The elementary schools

During this period arithmetic largely passed from the secondary school to the elementary school. Colburn's first book was intended for five- and six-year-olds. His primary method was to use carefully sequenced series of questions, leading a child to discover his own rules or principles. Colburn's avoidance of formalized rules was attributed by one writer to his "natural aversion to every kind of rule" (362, p. 163); another criticized the method for leaving "the child to work out his own salvation" (399, p. 270).

Colburn's method was emulated in the "mental" or "intellectual" arithmetics that were commonly attached to arithmetic series as a supplement. These were problem books but not mere drill books. They

proposed problems that were to be reasoned out, often orally, rather than solved by direct application of rules. The stress on reasoning rather than rule is illustrated by the use of "analysis" or "unitary analysis." To illustrate:

> What is $\frac{3}{4}$ of $\frac{2}{5}$?
>
> *Answer:* We first get $\frac{1}{4}$ of $\frac{2}{5}$. Now $\frac{1}{4}$ of $\frac{1}{5}$ = $\frac{1}{20}$; $\frac{1}{4}$ of $\frac{2}{5}$ is, then, twice as much, or $\frac{2}{20}$, which equals $\frac{1}{10}$. Therefore $\frac{3}{4}$ of $\frac{2}{5}$, which is 3 times $\frac{1}{4}$ of $\frac{2}{5}$, is $\frac{3}{10}$.

This example is from the 1884 revision of *Colburn's First Lessons.* Such exercises were seen as an important means of exercising the mind. As a practical technique of problem solving they outlived the era of mental discipline.

The inductive discovery approach of Colburn was overshadowed in the middle of the nineteenth century by the introduction of a deductive structure. Whereas pre-Colburn texts had given rules for solving specific problems, the texts of this period stated general definitions and principles which were developed into a deductive system that made arithmetic the "science of number." In this system, numbers were defined and classified as concrete or abstract, like or unlike, simple or compound, and even as natural or artificial. The processes and applications of arithmetic were fitted to the system. For example, in multiplication, principles were enunciated which allowed the multiplicand to be either concrete or abstract but demanded that the multiplier always be abstract and the product be "like" the multiplicand. Thus 3 times 4 inches was 12 inches. Such rationalizations contrast with the earlier rule-example-exercise approach in which a student "never heard any explanation of any principle" (181, p. 45). They also contrast with Colburn's disinclination to formalize. Although Colburn was praised by historians and authors of texts, the "science of number" approach came to dominate the major texts, and sections of definitions and principles were often inserted even into the supplementary mental arithmetics. Edward Brooks, whose texts were widely used in the second half of the nineteenth century, likened arithmetic to synthetic geometry, maintaining that "in arithmetic we have the same basis, and proceed by the same laws of logical evolution" (106, pp. 165–66).

Both the deductions of the "science of number" and the reasoning of the mental arithmetics were viewed as exercising specific psychological faculties, a purpose that justified the inclusion of large amounts

of arithmetic in the elementary curriculum. Brooks stated the mental discipline thesis as follows (104, p. 37):

> *The mind is cultivated by the activities of its faculties.* . . . Mental exercise is thus the law of mental development. As a muscle grows strong by use, so any faculty of the mind is developed by its use and exercise. An inactive mind, like an unused muscle, becomes weak and unskillful. Hang the arm in a sling and the muscle becomes flabby and loses its vigor and skill; let the mind remain inactive, and it acquires a mental flabbiness, that unfits it for any severe or prolonged activity. An idle mind loses its tone and strength, like an unused muscle; the mental powers go to rust through idleness and inaction.

»»»» 3. *The secondary schools*

During this period grammar schools continued to decline. Academies rose in prominence from 1820 to 1845 and began a decline after 1850. A new type of secondary school, the high school, developed. The English High School was founded in Boston in 1821. Seven years later Girls High School got under way. There was a period of rapid expansion of high schools from 1840 to 1860, and by 1875 they were quite well established, at least in the North.

Morison (182, p. 532) states that by 1850 three basic principles not only had been formulated but were fairly well established: free elementary and secondary education should be available for all; attendance at school should be required; and teachers should have some professional training. However, only in New England were primary schools free and open to all. In other regions plans and theories gave way to practical necessities. In Ohio, for example, free school for grades K–12 was planned as early as 1850, but it was not possible to have it actually available everywhere in the state.

Arithmetic moved from the academies and high schools to become an elementary school subject by the end of the nineteenth century.

Algebra was on its way to becoming a mainstay of the high school curriculum when Harvard required it for admission in 1820, with Yale and Princeton following in 1847 and 1848 respectively.

Geometry moved down from the college level a little later. Yale made geometry an entrance requirement in 1865. Princeton, Michigan, and Cornell required it in 1868, and Harvard required geometry *and* logarithms for entrance in 1870.

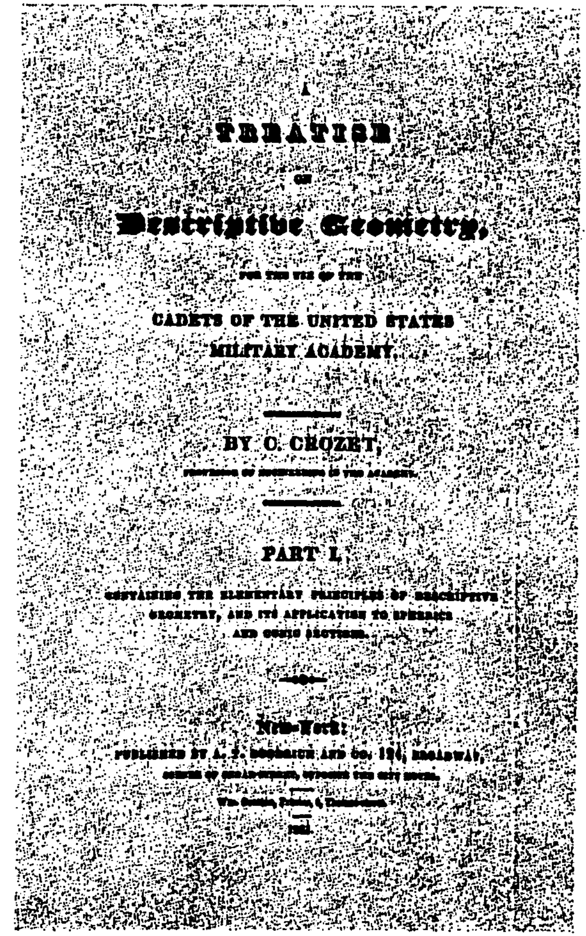
Algebra and geometry had the good fortune to be espoused by both of two groups whose programs sometimes were in conflict. A faculty

ychology and the need to prepare for further study combined with belief in the mental disciplinary value of languages and mathematics make mathematics required of the college-bound students. On the her hand, the practical utility of mathematics in the newly develop-g science and technology as well as in the older fields of surveying d navigation made persons planning non-college-preparatory pro-ams speak out for algebra, geometry, trigonometry, and mensuration. Although Harvard set up the first elective courses after an under-aduate rebellion in 1823, an extended elective system did not take ıld there until the presidency of Charles William Eliot, beginning in 169. He was motivated by a belief in individual differences and the fficulties being experienced with student motivation (225, pp. 118,)0–95). A little later this change came to academies and high schools the result of additional forces such as the decline of religious and ımanistic goals in favor of informational, social-civic, vocational, and ractical aims (118, p. 497). By the end of the nineteenth century the gh school curriculum was becoming more varied, and elective courses ıd programs were set up as a result of the changing and growing popu-tion of high schools, which motivated the changes in goals. Voca-onal, commercial, and manual training programs began to be estab-shed, sometimes in separate schools, during the latter part of the nine-enth century. These led to varied demands for mathematics and even the development of special courses.

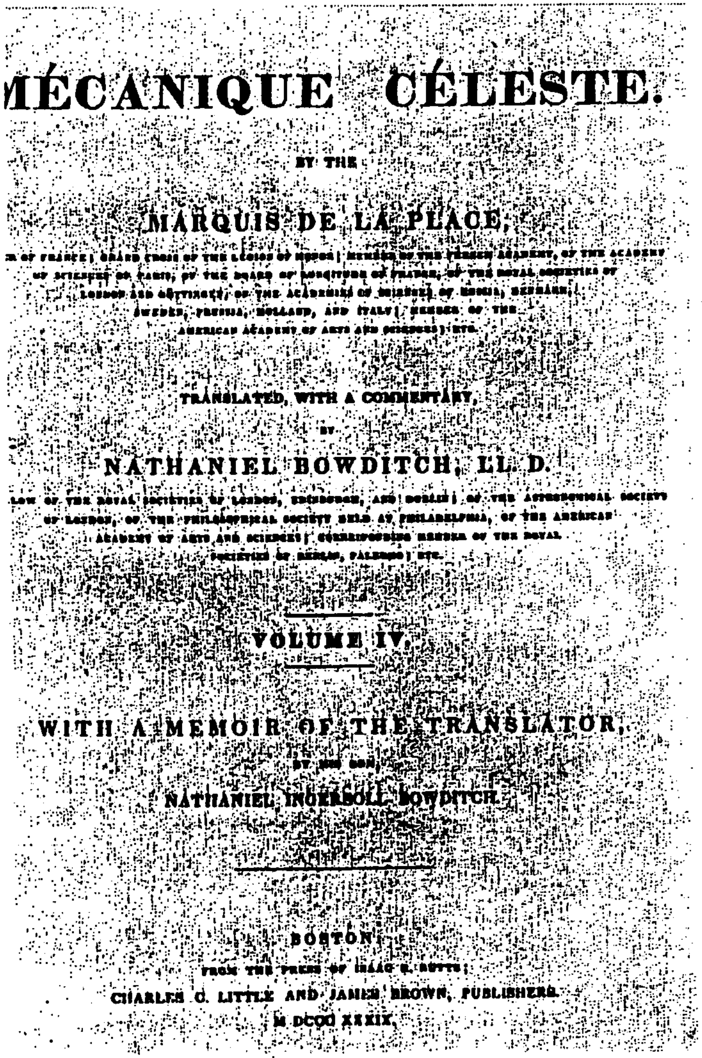
4. *The colleges and the mathematical community*

As mentioned in our introduction, this was a period when technical stitutes and midwestern and western colleges were being founded. hey included such "new" subjects as mathematics, science, English, ıd history in their programs; and even the older schools were de-ating the desirability of introducing these subjects. A famous Yale port of 1828 restated the case for the classical curriculum and the isciplinary values of mathematics. However, Harvard granted a achelor of science degree in 1851, and in 1852 Yale introduced a achelor of philosophy degree that originally had lower admission andards and required only three instead of four years, but admitted ıese new subjects as part of a degree program!

The first technical institute in the United States, the Military Acad-my at West Point, was founded in 1802. In its early days it was re-arded as a center for mathematics and innovative ideas. It brought achers, books, and ideas from France. These included descriptive


A
TREATISE
ON
Descriptive Geometry,
CADETS OF THE UNITED STATES
MILITARY ACADEMY.
BY C. CROZET,
PART I.
New-York:

FIRST ENGLISH WORK ON DESCRIPTIVE GEOMETRY. Claude Crozet, one of Napoleon's engineers, was brought over to teach at the U.S. Military Academy. He later served as a consultant in railroad engineering.



MÉCANIQUE CÉLESTE.

BY THE

MARQUIS DE LA PLACE,

TRANSLATED, WITH A COMMENTARY,

BY

NATHANIEL BOWDITCH, LL. D.

VOLUME IV.

WITH A MEMOIR OF THE TRANSLATOR,

BY HIS SON,

NATHANIEL INGERSOLL BOWDITCH.

BOSTON:

CHARLES C. LITTLE AND JAMES BROWN, PUBLISHERS.

M DCCC XXXIX.

FIRST ADVANCED MATHEMATICAL WORK PUBLISHED IN THE UNITED STATES. The first volume appeared in 1829.

geometry as a subject and the blackboard as a teaching device. Charles Davies, chairman of its mathematics department, translated many French texts for use there and in other American schools. In 1850 Davies published the first American methods book, *The Logic and Utility of Mathematics with the Best Methods of Instruction Explained and Illustrated* (133). The academy introduced the study of analytical trigonometry and encouraged learning and research through the founding of the Military and Philosophical Society, which collected materials and books and at whose meetings papers were presented.

The French influence in American schools and colleges is dealt with in other works (244, pp. 76–81). The role of West Point and of Davies's ideas and work in spreading this influence is apparent in the list of mathematics texts adopted at the University of Michigan in 1837 for its seven students: Davies, *Arithmetic;* Davies, *Bourdon's Algebra* (translated from the French); Davies, *Legendre's Geometry* (translated from the French); Davies, *Surveying;* Davies, *Descriptive Geometry;* Bridges, *Conic Sections;* Gregory, *Mathematics for the Practical Man.*

The scientific and technological trends of the period are also typified by the vigorous and nontraditional development of Rensselaer Polytechnic Institute (founded in 1824) under Amos Eaton, a Yale-trained botanist and geologist who was its first "senior professor." Among his contributions were an interesting practical textbook on trigonometry and surveying titled *Art without Science* (357, p. 421) and the setting up on a canal boat of a summer laboratory course in surveying.

The new colleges, especially those west of New England, had both peculiar advantages and peculiar problems. They did not have the classical humanistic tradition to impede progress, but neither could they count on their students having had adequate secondary school preparation. Many of them set up associated academies in which they enrolled students who, due to the lack of or inadequacy of secondary schools near their homes, were unprepared to enter college. To ameliorate these conditions, the acting president of the University of Michigan set up a system for the accreditation of high schools in 1870, and in 1871 President James Burrill Angell appointed a commission to inspect schools. This led to the founding of various regional accrediting associations, of which the first was the New England Association of Colleges and Secondary Schools, established in 1885 (118, p. 503). However, the increase in elective courses and programs in both high schools and colleges was creating entrance and articulation problems requiring more than regional associations for their solution.

ard the end of this period there was a further collegiate devel- that, eventually, had several significant effects on school mathe- This was the introduction in this country of the idea of mod- duate education. Master's degrees had been awarded by many :arly colleges, but the concept of extended advanced study and zation associated with research and the creation of new mathe- can largely be dated from three related events: the founding Johns Hopkins University in 1875 and the tenure of J. J. Syl- as professor of mathematics there (1877–83); the founding of *nerican Journal of Mathematics* there by Sylvester and W. E. and the founding of the American Mathematical Society 1 in 1894. Its forerunner, the New York Mathematical Society, en founded in 1888 and had begun to publish its *Bulletin* in 1891 pp. 105–6, 121). The Chicago section of the AMS was estab- n 1896 (244, p. 107).

he nineteenth century passed, it became increasingly true that ery few exceptions those Americans who had had advanced natical training or who were doing research in mathematics had l abroad and were largely employed as college teachers. How- the U.S. Coast and Geodetic Survey and Nautical Almanac supported the endeavors of such largely self-taught mathema- as Benjamin Peirce (1809–80) and Simon Newcomb (1835– Peirce spent most of his life as a professor at Harvard after studied there as an undergraduate and having assisted Nathaniel tch in the preparation for the press of his translation of Laplace's *ique Céleste*. His most famous work was *Linear Associative a*. This pure mathematical work was developed from lectures at the National Academy of Sciences and was issued in 1870 in l edition through the "labors of love" of persons in the U.S. and Geodetic Survey (86, p. 2). The tendency for American natical research to be very pure, and in the general area of the itions of mathematics rather than in applied mathematics, con- into the twentieth century, becoming a real national handicap time of World War II.

5. *Teacher training and pedagogy*

s period saw the rise of preservice and in-service teacher training f a group of professional educators. The first public normal was founded in Massachusetts in 1839 under the leadership of e Mann. Formal teacher training was instituted in universities as

early as 1832 at New York University, 1850 at Brown University, and 1860 at the University of Michigan. Teachers College, Columbia University, was founded in 1888 (118, p. 468). Courses in methods of teaching were offered in 1893 at the University of Michigan and at Michigan State Normal School. Louis Agassiz held a summer session at Harvard as early as 1873. In 1891 methods courses were also to be found in Kansas, California, Indiana, and Minnesota.

Horace Mann, who became chairman of the new Massachusetts Board of Education in 1837, had studied German educational ideas and translated Victor Cousin's report on Prussian education. The state of Ohio in 1836 sent Calvin E. Stowe to study European education systems (182, pp. 530–32). Such men as these not only spread word of the work of foreign educators including Pestalozzi, Herbart, and Froebel, but were active in initiating changes at home. The educational interests and theories of such European philosophers as John Locke (1632–1704), Immanuel Kant (1724–1804), and Herbert Spencer (1832–1903) were known and discussed in the books and periodicals on pedagogy which began to appear late in this period.

As noted above, Davies's methods book appeared in 1850. It was the first book in the United States for secondary school teachers of mathematics. Today it might be regarded as a mixture of foundations of mathematics and professionalized subject matter.

Nineteenth-century arithmetic texts, including those written for use in the normal schools, had many notes to teachers. However, the first book that can be viewed as a "methods" text for the elementary schools was Edward Brooks's *The Philosophy of Arithmetic* (1880). It seems to have been thoughtful, extensive, and based on considerable familiarity with pedagogical and mathematical literature. At that time, as today, there was considerable thought given to the nature of number by mathematicians, philosophers, and educators. Brooks discusses Hamilton's conception of algebra, and thereby arithmetic, as the science of time.

This interest in the nature of mathematics and its fundamental concepts, mentioned earlier in connection with American research interests, is further evidenced by the publication of several books. Albert Taylor Bledsoe, professor of mathematics at the University of Virginia, wrote *The Philosophy of Mathematics*, published at Philadelphia in 1868. It was largely concerned with the basic concepts of analytic geometry and calculus, discussing the work of Cavalieri, Descartes, Leibnitz, and Newton. In 1871 W. M. Gillespie of Union College published *The Philosophy of Mathematics, Translated from the Cours de*

'hilosophie Positive of Auguste Comte. Again, although this was far rom a methods book, many of its section headings have both a modern nd a pedagogical ring: "The Object of Mathematics," "The True Defiuition of Mathematics," "Concrete Mathematics," "Abstract Mathenatics," "The True Idea of an Equation," "Two Methods of Introducng Angles."

The rise of psychology as an offshoot from philosophy is perhaps ypified by the writings of Herbert Spencer (1820–1903). His father vas a schoolmaster who had written an inductive, laboratory-type, econdary-school text in geometry. Spencer wrote an introduction for ts American edition (247), telling how fascinated the girls in his ather's school had been with his inductive approach to geometry, which was a carefully sequenced series of questions and instructions. Today it would be termed a "discovery" or "heuristic" approach.

The inductive method of Colburn had neither the stress on objects dvocated by Pestalozzi, whom he professed to follow, nor as well leveloped a sequence of questions and activities as in Spencer's book. However, Colburn did reduce formalism and definitions and strive to ead the student to understanding by means of questions and explanaions. Colburn extended this approach to algebra in 1825 with the ublication of *An Introduction to Algebra on the Inductive Method of Instruction*. He said his object was

> to make the transition from arithmetic to algebra as gradual as possible [by] beginning with practical questions in simple equations, such as the learner might readily solve without the aid of algebra. . . . The most simple combinations are given first. . . . The learner is expected to derive most of his knowledge by solving the examples himself. . . . This method, besides giving the learner confidence, by making him rely on his own powers, is much more interesting to him, because he seems to be constantly making new discoveries.

In summary, then, this period saw the rise of teaching procedures timulated by changing philosophical-psychological ideas of the goals nd processes of education. Mental discipline as a viable goal of eduation, and drill as a procedure, were retained along with other newer oals and processes for more than thirty years after 1894, but the threetep process of "state a rule, give an example, practice" was yielding o inductive, reasoning, and discovery-teaching processes. Both goals nd processes were being questioned, and new psychological bases for earning and teaching were being sought.

6. *The climate of reform*

The many forces enumerated in the previous sections include the problem of high school–college articulation and the problem of "psychologizing" both the content and the methods of the secondary school curriculum to improve learning for all student groups but especially to adapt to the changing high school population with its many non-college-bound students.

These two related problems were studied by two committees appointed at the conclusion of this period and the beginning of the next. The appointment of these committees typifies several significant forces at work at this time.

The first of these was the Committee on Secondary School Studies. This group, often referred to as the Committee of Ten, was appointed in 1892, and it published reports in 1893 and 1894 (71, 72). The fact that it was appointed by the National Education Association (NEA) demonstrates the growing influence of professional organizations at this time. The NEA had been formed in 1870 out of the National Teachers Association, which had been founded in 1857. The subcommittee on mathematics of the Committee of Ten was the first national group to consider the goals and curriculum for mathematics education (306). This subcommittee was composed of leading mathematicians. Its recommendations were quite radical and forward-looking, even though it was largely oriented toward the goals of college preparation and mental discipline. Some of its recommendations, particularly the recommendation that the mathematics program be unified or integrated rather than compartmentalized, are still to be put into effect. However, the recommendation that introductory work in algebra and geometry come earlier and be more psychologically designed preceded the introduction of informal and intuitive geometry into the junior high program of the early twentieth century. This began a trend that is being revived today with the inclusion of more geometry in the elementary school and the movement of algebra into the eighth grade, together with even earlier introduction of preliminary algebraic concepts.

The other committee referred to is the Committee on College Entrance Requirements of the NEA, appointed in 1899, with some of the members named at the suggestion of the AMS. Its report (74) affected the period covered by our next chapter, but the problem and the forces and organizations associated with it had their roots in the 1821–94 period we have just discussed.

CHAPTER FOUR

First Steps toward Revision 1894–1920

1. *Forces at work*

The issues in mathematics education tend to endure, to change slowly if at all. However, the prominence of various issues tends to change with the shifting of the nature and direction of the forces exerted to change the educational program. The major force during the period of 1894 to 1920, resulting from the growth in school enrollments, was the pressure to provide an education for all children. Added to this were forces generated by psychological research and changing theories of learning. Mental discipline as an achievable goal of instruction was being questioned. Both educators and mathematicians were calling for an integration of content and a shift to newer methods using concrete, developmental, and intuitive approaches to this content.

This period (1894–1920) is characterized by a somewhat intensified consideration and refinement of the issues formulated in the previous period. One should also note the similarity of the problems and proposals of this period to those of the following period. For example, World War I diverted concern and energies from education while at the same time leading to some support for the continued education of returning veterans. Later, World War II gave a far greater push to mathematics and its applications as well as providing greatly expanded higher educational facilities for veterans. The pressure to enroll in

college in this later period paralleled the earlier pressure to complete a high school program.

In the period covered by this chapter much of the teacher training, especially for elementary school teachers, was done in normal schools. As a result, colleges and universities had much less concern for or contact with elementary and secondary education than they have today. In 1894–1920 the chief concerns common to the secondary schools and the colleges were the setting and administering of standards for the admission of high school graduates to college. Both the traditional college requirements and the then-new elective undergraduate college programs, along with developing graduate programs, exerted pressures for college preparation on the high schools. However, these pressures were not uniform. They varied with the colleges and with new college curricula. Further, these pressures often opposed the forces implicit in a growing high school population, much of which was non-college-bound and had immediate vocational-personal needs. The prominent issues became these: What are the goals of high school mathematics education? How can the school adapt to the varied needs, backgrounds, interests, and abilities of its students?

2. *The elementary schools*

The elementary schools underwent a terrific expansion, from 16 million to 24 million pupils in the interval from 1900 to 1930. The dominant organizational pattern of schools was beginning to change toward the end of this period. The "8-4" pattern, eight years of elementary school followed by four years of high school, was changing to "6-3-3," with a three-year junior high school inserted between a six-year elementary school and a three-year high school.

Arithmetic, once a college and secondary school subject, had moved to the elementary school. Now, in the elementary school, separate texts for early arithmetic work were disappearing. It was proposed that primary number ideas be treated incidentally and informally. The NEA's Committee of Fifteen on Elementary Education, which reported in 1895 (68), suggested that grades 1 through 6 complete basic arithmetic and that number work be continued into grades 7 and 8 where some algebra would be introduced, along with a reduction in the amount of time per day devoted to mathematics. This introduction of some algebra was to serve as a "transitional step to algebra proper" (p. 58), an idea in keeping with the purposes of the newly developing junior high school.

The arithmetic pedagogy of the latter half of the nineteenth century had tended to stress what Clason (441, pp. 72, 115–16) calls the "science of number," after the definition of arithmetic often given in the texts of this period. In this approach, as was noted in the previous chapter, considerable discussion was devoted to distinguishing different types of numbers such as "concrete," "abstract," and "denominate." These definitions associated number with units and collections of objects for the purpose of rationalizing the operations with numbers.

The early part of the period 1894–1920 saw substantial innovations in the approach to numbers. Herbert Spencer's dictum that number is *relation* led William Speer to stress number as the directly perceived ratio of quantities, lengths, areas, and volumes rather than as an abstract concept or a collection of discrete objects (441, pp. 135, 210, 302). Spencer's emphasis on the spiral nature of learning affected both the presentation and the organization of arithmetic texts, which in the earlier period had presented each new topic in its turn, once and for all (441, p. 62).

Later John Dewey (1859–1952) viewed number as originating through measurement. The text for teachers of arithmetic written by him and James A. McLellan, and the school books based on it, therefore stressed measurement activities, problems, and units (441, pp. 129, 205–10).

Following these innovative approaches there was a reaction typified by the eclecticism and avoidance of extremes to be found in the writings and texts of David Eugene Smith, according to Clason (441, p. 63).

Clason suggests that the next period in elementary school mathematics spanned the years 1917–35 and should be termed the period of connectionism. This term largely refers to the stimulus-response psychology so strongly presented by Edward L. Thorndike after the psychologists of the day thought they had thoroughly discredited the concepts of mental discipline and transfer of training. *The Thorndike Arithmetics* appeared in 1917. In *The Psychology of Arithmetic* (1922) and *The Psychology of Algebra* (1923) Thorndike stressed the importance of establishing many "bonds" by means of much practice (265; 266). This psychological approach led to the fragmentation of arithmetic into many small facts and skills to be taught and tested separately. This theory even led to the avoidance of teaching closely related facts close in time to one another for fear of establishing incorrect bonds. Thus Clason says, "For good or ill, it was Thorndike who dealt the final blow to the 'science of arithmetic' " (441, p. 64).

»»»» 3. *The secondary schools*

There were several approaches to "psychologizing" secondary school mathematics during this period. Perhaps the "Perry movement" in England should be mentioned first, since it was well publicized in this country by the efforts of Professor E. H. Moore, one of the leading mathematicians of the United States. Moore took the unusual step of devoting a major portion of his 1902 address on retiring as president of the AMS to a discussion of needed improvements in American schools (381). He urged that schools abolish the "watertight compartments" in which algebra, geometry, and physics were taught. He also suggested a stress on the interrelationships of mathematics with science. This latter was one of the three major proposals vigorously advocated by John Perry in addresses given in 1900–1902. Perry also urged that mathematics be made concrete by use of laboratory-teaching techniques and applications and by bringing into lower grade levels many simple ideas usually postponed to later grades. Moore endorsed all of this and further stressed the desirability of developing in the United States junior colleges, junior high schools, and improved subject-matter training for teachers in these schools and in the high schools. He also noted a forthcoming article in which "John Dewey, calling attention to the evolutionary character of the education of the individual, insists that there should be no abrupt transition from the introductory, intuitional geometry to the systematic, deductional geometry" (p. 48).

The National Committee of Fifteen on the Geometry Syllabus was appointed in 1908–9 under the joint auspices of the American Federation of Teachers of the Mathematical and Natural Sciences and the NEA. Its "Provisional Report" (66) appeared in 1911 in *School Science and Mathematics*, journal of the Central Association of Science and Mathematics Teachers. This report showed some effect of Moore-Perry views as well as a reaction to rising failure rates in algebra and geometry, which were still required courses in many high schools. This report called for the use in geometry of some informal proofs and for the exclusion of materials dealing with limits and incommensurables—with, however, continued stress on logical structure. It called for more use of concrete examples and graded "originals."

Many of these same pedagogical emphases were discussed in the *Report of the American Commissioners of the International Commission on the Teaching of Mathematics* (49) issued in 1912. The International Commission was set up at the Fourth International Congress

of Mathematicians held in Rome in 1908. It reported at the 1912 congress held in Cambridge, England. The final report of the American commissioners was based upon the reports of many subcommittees (39–48). As a continuation of this project and based on some later reports as well as those given at the congress, I. L. Kandel and R. C. Archibald prepared comparative studies of the training of teachers here and abroad. These appeared in 1915 and 1918 (163; 5).

The American commissioners in 1912 noted tendencies in this country to omit difficult or obvious geometric proofs, to postpone difficult topics, to reduce the stress on manipulation in algebra while increasing the stress on solution of equations and applications.

Archibald stated in 1918 (5, p. 4):

> At the present time superintendents . . . [in] the United States have been forced by public opinion to consider numerous radical changes in methods of secondary school education. If a high minimum standard of preparation were required on the part of each teacher, and the position of the teacher were made such as to attract in sufficient numbers the best talent in the country, other difficulties would disappear. Most countries considered in this bulletin have far higher standards than we with respect to teachers of mathematics in the secondary schools.

The most influential committee dealing with the teaching of mathematics appointed in the United States prior to the recent (1956–59) Commission on Mathematics was the National Committee on Mathematical Requirements, whose final report, *The Reorganization of Mathematics in Secondary Education* (59), was published in 1923. A preliminary report appeared in 1920 and a summary in 1922 (57; 58).

The committee was appointed by the MAA in 1916, before the United States entered World War I. The committee contained several persons with special interest in and experience with the secondary schools, in addition to the mathematicians who represented the association's major concern with the undergraduate college program. Its preliminary report was circulated through the U.S. Bureau of Education with a call for criticisms and comments to be submitted for consideration before preparation of the final report. The committee was also remarkable for having some financial support from an outside source for its staff and activities.

Its final report stressed many of the ideas mentioned above, such as the reduction of elaborate manipulations in algebra and of the memorization of theorems and proofs in geometry by decreasing the number "required" theorems and increasing the number of "originals." It also advocated a general mathematics program for grades 7–9 which would include topics from arithmetic, algebra, intuitive geometry, numerical trigonometry, graphs, and descriptive statistics. An entire chapter was devoted to the importance and unifying nature of the function concept.

This period, 1894–1920, can be partially typified by a concern for goals and curriculum revision. Several regional educational organizations and state education departments also prepared reports and syllabi.

4. *The colleges and the mathematical community*

An emphasis on the function concept and on interrelationships within mathematics had been strongly advocated since the middle of the nineteenth century by Germany's famous mathematician Felix Klein. He and a number of other foreign mathematicians attended the international conference held in conjunction with the Chicago World's Fair of 1893. Klein stayed on after the fair to participate in a colloquium at Northwestern University. This colloquium had a very stimulating effect upon mathematical research in the United States. Klein's summer lectures to German mathematics teachers were translated by E. R. Hedrick and C. A. Noble and published in this country in 1932 under the title *Elementary Mathematics from an Advanced Standpoint.*

A number of translations of important foreign treatises were published in this country around the turn of the century. G. B. Halsted translated source materials related to non-Euclidean geometry by Bolyai and Lobachevsky (translated in 1891 but published in 1914), and Saccheri (1920). E. J. Townsend's translation of Hilbert's *Foundations of Geometry* was published in 1902. Halsted, feeling that the high school geometry of his day was out-of-date and too full of errors, wrote *Rational Geometry* as a high school text based on Hilbert's axioms (151).

Axiomization became a major American research interest. E. V. Huntington and Oswald Veblen published a new set of axioms for geometry in 1904. Huntington axiomatized elementary algebra in 1905. Later continuing this American interest, G. D. Birkhoff devised a new set of axioms for geometry in 1933. This system, involving "ruler and protractor postulates," was later incorporated into a high school text, *Basic Geometry*, by Birkhoff and his mathematics-educator colleague at Harvard, Ralph Beatley (97).

Several books were written or compiled to convey the then-"modern" ideas of mathematical foundations to teachers. J. W. Young's

Lectures on Fundamental Concepts of Algebra and Geometry appeared in 1911 (284), and J. W. A. Young edited *Monographs on Topics in Modern Mathematics* (1911), which included articles by Veblen, Huntington, Birkhoff, and David Eugene Smith (286). J. W. Young of Dartmouth College is probably better known among mathematicians for his extensive axiomatic treatise on projective geometry written with Oswald Veblen. J. W. A. Young, after writing a dissertation in group theory at Clark University in 1892, was listed as "Associate Professor of the Pedagogy of Mathematics at the University of Chicago" in the 1920 edition of his book *The Teaching of Mathematics in the Elementary and Secondary School* (285). This book was first published in 1906. It was the second American book to deal extensively with the teaching of secondary school mathematics. Although Davies's earlier book purported to be for teachers, David Eugene Smith's *The Teaching of Elementary Mathematics* (241), which appeared in 1900, was the first book to resemble a modern methods text.

Another strong collegiate influence upon secondary school mathematics developed during this period. The College Entrance Examination Board (CEEB) was established in 1911. It included mathematics in the set of tests it administered for its member colleges, and it published syllabi defining the high school mathematics content upon which its tests would be based. The development of the board came after long concern for the problems of articulating the high school and college curricula and selecting high school graduates who were capable of succeeding in college. The Committee on College Entrance Requirements reported to the NEA in 1899 (69). This committee also advocated concrete geometry and introductory algebra in the seventh grade.

»»» 5. *Pedagogy and teacher training*

David Eugene Smith and J. W. A. Young may be regarded as representing an interesting development in this period—the appearance of "mathematics educators." Smith was professor of mathematics at Teachers College, Columbia University, from 1901 until his retirement in 1922. He played important roles in the AMS (86), MAA, and the NCTM. He served on many important committees, including the National Committee on Mathematical Requirements. He was one of the American commissioners on the International Commission on the Teaching of Mathematics. He was also a founder of the History of Science Society, a prolific writer, and the director of many studies in the teaching and history of mathematics.

A number of persons who were primarily mathematicians also maintained an interest in school programs. For example, the National Committee on Mathematical Requirements was appointed by E. R. Hedrick, president of the MAA. The committee included Smith and mathematicians J. W. Young, A. R. Crathorne, C. N. Moore, and H. W. Tyler. Vevia Blair of the Horace Mann School and Raleigh Schorling of the Lincoln School, New York City, were from experimental or "laboratory" schools. Both the existence of these schools and the appointment of persons from them to the committee were manifestations of the development of professional mathematics educators. Miss Blair also represented the Association of Teachers of Mathematics in the Middle States and Maryland. W. F. Downey was added to the committee later as a representative of the Association of Teachers of Mathematics in New England, and J. A. Foberg represented the Central Association of Science and Mathematics Teachers. The remaining members of this committee were A. C. Olney, commissioner of secondary education for California, and P. H. Underwood and Eula A. Weeks, mathematics teachers from Texas and Missouri.

Local and then national organizations concerned with the teaching of mathematics began to be formed along with the general growth of scholarly and educational societies. Three of these were cited in the previous paragraph. The MAA was founded in 1915. The NCTM was incorporated in 1920. It was largely stimulated by the interest and planning of the Chicago Men's Mathematics Club. The Council's journal, the *Mathematics Teacher*, was taken over in 1921 from the Association of the Teachers of Mathematics in the Middle States and Maryland.

Research pertaining to mathematics education was also carried on by general educators, especially by persons in the rapidly growing field of tests and measurements. We have already cited, for example, the work and influence of E. L. Thorndike.

»»» 6. *Summary*

ISSUES

In addition to a continuing stress on the issues cited in previous chapters, three new issues began to evolve. The continuing older issues were these:

1. What are the goals of mathematical instruction?
2. How can the mathematics curriculum and teaching methods be

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Title: The 'New New Math'?: Two Reform Movements in Mathematics Education.

Subject(s): MATHEMATICS -- Study & teaching -- United States; NATIONAL Council of Teachers of Mathematics

Source: Theory Into Practice, Spring2001, Vol. 40 Issue 2, p84, 9p

Author(s): Herrera, Terese A.; Owens, Douglas T.

Abstract: Compares the origins, curricular and pedagogical content and the impact of the new mathematics and the standards-based reform movements in the United States. Discussion on the mathematics reforms; Influence of psychological theories on mathematics; Factors to consider in mathematics curriculum development; Response from the National Council of Teachers of Mathematics.

AN: 4472840

ISSN: 0040-5841

Database: Academic Search Elite

Text Available:

Full Page Image

THE 'NEW NEW MATH'?: TWO REFORM MOVEMENTS IN MATHEMATICS EDUCATION

MATHEMATICS EDUCATION IS RIFE with the battle cries of so-called "math wars." On the one side are proponents of standards-based reform, a movement launched in 1989 with the publication of the Curriculum and Evaluation Standards for School Mathematics by the National Council of Teachers of Mathematics (NCTM). Opponents to the reform proclaim it a reincarnation of the mathematics of the 1960s, popularly known as the "new math" and popularly remembered as a pedagogical failure. Hence, these opponents term the current reform the "new new math," yet we have never seen an instance where they have actually compared the two reform movements.

In this article we compare the origins, curricular and pedagogical content, and impact of the new math and the standards-based reform movements. The first part of the article describes the events leading up to the new math era and the characteristics of representative curriculums. To conclude the first section, we consider the demise of the movement and events leading to the most recent reform.

The second part of the article describes the NCTM standards-based reform, an educational movement that shares some similarities with and yet differs substantially from the new math reform. We conclude with insights into the reaction to this current movement and its impact on school mathematics.

The New Mathematics Reform

X 2

The coalescence of concerns coming out of World War II, other events on the international scene, and discontent with high school mathematical preparation on the part of mathematicians fomented change--and the new math was born.

Sense of national crisis

The "new math movement" of 1960-1970 was 15 years in the making. During World War II, both educators and the public recognized that more technical and mathematical skills were needed to push forward the developing technological age. NCTM appointed the Commission on Postwar Plans to make recommendations about mathematics curriculum. The goals were to establish the United States as a world leader and to continue the technological development that had begun during the crisis of war. Although they documented problems with the preparation of youth in high school mathematics, the reports of the commission in 1944, 1945, and 1947 had little lasting effect. Osborne and Crosswhite (1970) characterize the reports as "basically regressive" (p. 246).

To add to the concern about the inadequacy of the mathematics curriculum for the emerging technology, the Soviet Union launched the first satellite into space in October 1957. This has commonly been cited as the event that marked the beginning of the new math revolution. The launch of Sputnik in 1957 created the perception that the United States was behind in the world scene of technology and military power. The launch served to loosen the public purse strings and catapult a movement that had already begun.

Curricular concerns

Prior to the launch of Sputnik, the University of Illinois Committee on School Mathematics (UICSM), representing the perspective of university mathematicians, was formed in 1951 "to investigate problems concerning the content and teaching of high school mathematics" (Osborne & Crosswhite, 1970, p. 251). Following a survey, it published a pamphlet, Mathematical Needs of Prospective Students in the College of Engineering of the University of Illinois (Osborne & Crosswhite, 1970, p. 251).

In 1955, the College Entrance Examination Board (CEEB) took a new kind of step to directly influence the school curriculum. CEEB appointed a Commission on Mathematics to consider how their examinations should reflect the changes in the field of mathematics that had taken place in the previous 50 years. To its credit, the commission represented university mathematicians, high school teachers, and college and university mathematics educators, three groups most directly concerned with secondary school mathematics curriculum (Osborne & Crosswhite, 1970, p. 260).

The commission's report, finalized in 1959, was characteristically restricted to college-bound students. The commission's nine-point program called for preparation in concepts and skills to prepare for calculus and analytic geometry at college entry. The commission wanted appreciation of mathematical structure in properties (e.g., commutative property, roles of 0 or 1) of natural, rational, real, and complex numbers. Use of unifying ideas such as sets, variables, functions, and relations was recommended. Treatment of inequalities along with equations was requested. For Grade 12, a half-year of elementary

functions was recommended along with alternative units in probability and statistics or modern algebra (CEEB, pp. 33-34).

Influence of psychological theories

The Swiss developmental psychologist, Jean Piaget, had been writing about cognitive development since the 1920s and 1930s and had begun to catch the interest of the mathematics education community. In 1952, the beginning of the new math era, The Child's Conception of Number was translated into English (Piaget, 1952). Although the ages at which the stages occur have been questioned, Piaget's stages of cognitive development and his concepts of conservation and reversibility, for example, had a significant impact on mathematics education, especially at the elementary school level. Attention to Piaget's theory had the effect of emphasizing the importance of concrete examples and physical manipulatives. However, unanswered questions about growth of mathematical ideas in young children gave rise to some teachers becoming overly cautious in offering challenges to children, albeit with concrete/manipulative means.

Jerome Bruner influenced mathematics curriculum and practice in the new mathematics era through his promotion of the discovery of mathematical ideas. Using well-chosen problems, he said students can do investigation and discovery rather than being told the relevant concepts and expected to practice skills. Through an oft quoted statement, Bruner (1960) made the point that readiness for a topic or concept does not depend altogether on the child's maturation. He said, "We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33). His point is that readiness is primarily a function of curriculum writers and teachers finding a suitable context or way of expressing the principle or concept to make it accessible to the learners.

Bruner (1966) also devised three stages of representation of mathematical ideas: enactive, iconic, and symbolic. The enactive mode is essentially a hands-on, concrete representation of an idea. For example base-ten materials are a manipulative representation of base-ten numbers. Diagrams and pictures of base-ten materials would be an iconic representation. Using ordinary numerals would be an abstract representation.

High school curriculum development

Max Beberman and his associates on the UICSM operated on the principle that improvement in mathematics curriculum would have to include classroom-tested instructional materials. The UICSM produced materials as a joint effort of the colleges of education, engineering, and liberal arts and sciences. The staff produced a high school program that was taught at the University High School. By 1958 a program was in place for the four high school years. When the program was more widely distributed, teachers were required to complete in-service education in the philosophy, program, and methodology, and were to be supervised by staff from the project. The UICSM program focused on significant changes in curriculum to identify unifying mathematical themes and use precise language and symbols. Features included:

> the move toward an integrated mathematics curriculum in which algebra is found throughout the four-year program; . . . the introduction of some modern concepts such as set terminology; . . . the relocation of several topics, such as . . . inequalities, to lower levels in the curriculum. . . . Care was taken to ensure precision of expression, even to the extent of inventing new terms and symbols (e.g., "pronumeral"). (Osborne & Crosswhite, 1970, p. 254)

Moreover, "discovery teaching and learning became the hallmark of the UICSM program" (Osborne & Crosswhite, 1970, p. 254).

Strongly influenced by college mathematicians, the largest curriculum project of the era was the product of the School Mathematics Study Group (SMSG). SMSG epitomizes much of the development of the era. Funded by the National Science Foundation (NSF), the writing sessions in the summer of 1958 and subsequent summers involved hundreds of mathematics teachers and mathematicians. SMSG operated by a process of summer writing sessions, classroom trials during the subsequent school year, rewriting, and publishing for national distribution (Osborne & Crosswhite, 1970, p. 274).

Over 60 texts were written as well as a variety of supplemental materials and reports. SMSG texts were to provide a model for commercial writers. Distribution was to continue with demand and eventually die a natural death (Nichols, 1968). Initially SMSG texts were written for high school programs to carry out the recommendations of the CEEB Commission on Mathematics, which included the following:

- treatment of inequalities along with equations
- structure and proof in algebra
- integrated plane and solid geometry with coordinate methods
- integrated algebra and trigonometry
- a twelfth grade course in elementary functions (NACOME, 1975, p. 5)

Junior high school projects

While there were many projects at the junior high school level, three represent the thrusts of the time. Unusual (at the time) chapter titles for seventh grade of the University of Maryland Project included symbols, properties of natural numbers, factoring and primes, the numbers one and zero, mathematical systems, and logic and number systems. Topics for the SMSG seventh and eighth grades included ideas of structure of arithmetic from an algebraic viewpoint, the number system as a progressive development, geometry with applications, measurement, and elementary statistics. "Careful attention is paid to the appreciation of abstract concepts, the role of definition, . . . precise vocabulary and thought, [and] . . . creation and discovery, rather than just utility" (Brown, 1961, p. 18).

Junior high school texts and curriculum guides developed throughout the 1960s indicate that students encountered

the concepts and language of sets, algebraic properties of number systems, non-standard numeration systems, informal properties of number systems, and number theory. [These topics are] rich preparation for high school study and a striking contrast to pre-1960 texts for grades 7-8. (NACOME, 1975, p. 9)

However, the National Advisory Committee on Mathematical Education (NACOME) was unable to determine the impact on the curriculum in general. For indirect evidence they turned to the National Assessment of Educational Progress (NAEP) tests to examine what was being tested.

Of approximately 200 items used to sample mathematical attainments of 13 year olds, NAEP included very few related to sets (6), probability (6), inequalities (2), and non-decimal numeration systems (2). . . . There were many geometry items, though stress was on measurement aspects that were not particularly novel. On balance the assessment pool shows some signs of a changing curriculum, but not dramatic upheaval. (NACOME, 1975, p. 10)

A third program, the Madison Project, was intended to prepare supplementary activities to stimulate children to be creative and develop enthusiasm for mathematics. Topics at grades 4 to 8 included axiomatic algebra, coordinate geometry, and applications to science. At the grades 6 to 9 level, topics were statistics, logic, matrix algebra, and applications to physics. Above all, the approaches to the topics were the mathematical laboratory and discovery methods (Nichols, 1968, p. 20).

Elementary school curricular change

In an attempt to reflect current changes, the elementary school textbooks of the era introduced the language of set theory, and the algebraic properties: commutative, associative, and distributive. Another topic characteristically added was arithmetic in bases other than ten (NACOME, pp. 15-16). The intent was to generalize the concepts of base and place value and thereby solidify the understanding of base ten.

Elementary school curricular change was slower and more difficult to implement than the high school programs. Few elementary school teachers were mathematics specialists, and little inservice training was offered to help them understand and implement the new curriculum ideas and materials. However, judging from popular textbook series or curriculum guides, some changes prevailed.

The label "arithmetic" has appropriately given way to "mathematics" as curricula incorporate varying amounts of geometry, probability and statistics, functions, graphs, equations, inequalities, and algebraic properties of number systems. (NACOME, 1975, p. 11)

A survey of teachers indicated that elementary school teachers gave more time to geometry, but other new topics such as graphs, statistics, and probability received less time and attention (NACOME, 1975, pp. 11-12). Standardized tests of the era point to a shift toward comprehension goals rather than focusing only on computation. However, the tests fell short of curriculum developers' hopes of being able to teach and measure students' understanding.

Reaction to the new math reform

Teachers responded enthusiastically by taking advantage of NSF funded summer-long institutes and training offered by the innovative programs. Parents reacted to the new mathematics curriculum with consternation due to their inability to help their children with mathematics. A few mathematicians were vocal in opposition to the new mathematics, notably Morris Kline who wrote the book, Why Johnny Can't Add (Kline, 1973).

The decade of the 1970s was characterized as a "back to the basics" era. Whether or not justified, the widespread sentiment was that the new math had failed, and that a return to the basics was needed. In particular, critics cited the College Board's report of a 10-year decline in Scholastic Aptitude Test scores (Usiskin, 1985).

The precise, structuralist language of sets, logic, and algebraic structures, hallmarks of the new math curriculum, were abandoned in favor of more emphasis on computation and algebraic manipulation. Socratic dialogue and pedagogical approaches of discovery were relinquished for those backed by principles of behavioral psychology. Lesson objectives were stated in terms of observable, measurable behaviors. Texts and teachers were to guide students through a prescriptive hierarchical curriculum (NACOME, 1975). Teaching practice during the back-to-basics era was described in National Science Foundation case studies:

In all math classes that I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked by the teacher or the students at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and the problems assigned for the next day. The remainder of the class was devoted to working on homework while the teacher moved around the room answering questions. The most noticeable thing about math classes was the repetition of this routine. (Welch, 1978, quoted in NCTM, 1991, p. 1)

It was in reaction to this curricular focus that the present reform in mathematics education emerged.

The NCTM Standards-Based Reform

This most recent reform was launched in 1989 with the publication of NCTM's Curriculum and Evaluation Standards for School Mathematics, a vision of teaching and learning that differs radically from the back-to-basics curriculum that preceded it. In this section, we consider its genesis, its conception of curriculum and pedagogy, and reaction to the movement. Our intent is to highlight for the reader the similarities as well as the differences in the two reform movements.

Curricular concerns

Reaction to the back-to-basics movement grew steadily within the mathematics education community during the decade of the '70s. The dominant place of computation in the elementary curriculum was questioned, as was the low priority given to problem solving. Also, dissatisfaction deepened as areas in the field of mathematics gained importance in a

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changing society but were not reflected in the school mathematics program (Usiskin, 1985).

Emerging concerns were voiced through conferences and reports. Among the most prominent was the 1975 report of the NACOME, which "called for a repudiation of a curriculum dominated by manipulative skills" (McLeod, in press) and recommended more work with technology and applications. In the same year, the National Institute of Education (NIE, 1977) sponsored a conference in Euclid, Ohio, that outlined 10 goals of mathematics, problem solving being the chief among them. Soon after, the National Council of Supervisors of Mathematics (NCSM), a body composed of leaders at university and district levels, published its Position Paper on Basic Mathematical Skills (1977), which defined "basic skills" as including not only computation but also estimation, geometry, problem solving, and computer literacy. In a summary of the concerns noted in various reports and conferences sponsored by the National Science Foundation in the 1970s, Fey and Graeber (in press) list the following:

- The public views mathematics as a set of arithmetic skills.

- Applications should be a more prominent part of mathematics curriculum.

- Problem solving should play a more central role.

- The role of technology needs to be considered in relation to computational skills and the handling of the "daily bombardment of statistics."

These reports prompted NCTM to appoint a committee to develop recommendations for school mathematics for the coming decade of the '80s. The product, An Agenda for Action (1980), was one of the earliest position statements from NCTM and a definitive step toward reform. It set problem solving as the curricular focus, recommended that the definition of "basic skills" be broadened to include such mathematical skills as estimation and logical reasoning, and promoted the use of calculators and computers in the classroom at all grade levels.

Although this publication articulated a direction for curricular change, its impact was insufficient to quell the rising sense of urgency among mathematics educators. Usiskin (1985), for example, in a strong argument for updating mathematics curriculum, concluded with this call for change:

Current student needs in mathematics cannot be met without modifying the very goals and nature of secondary school mathematics. Recent reports confirm that the current curriculum needs overhauling rather than adjustment, revolution rather than evolution. (p. 17)

Sense of national crisis

There was no Sputnik launch to ignite this reform, but rather a perceived falling behind in worldwide technological and economic standings. Concern with U.S. student performance had become linked in the public mind to the country's capacity to fill its technical needs (see Mathematical Sciences Education Board, 1989).

Results on international studies, such as the Second International Mathematics Study (SIMS) and the International Assessment of Educational Progress (IAEP), showed the U.S. as weak in several areas (Robitaille & Travers, 1992). Furthermore, despite the very focused emphasis of the back-to-basics period on procedural skills, "national tests showed that student performance in basic skills declined or stayed the same" (Kenney & Silver, 1997, p. 66).

While these voices grew louder within the halls of government and academia, an event occurred that awakened the general public to the sense of crisis: the publication of A Nation at Risk (1983). Commissioned by the National Commission for Excellence in Education (NCEE), this report struck a note of urgency: "If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war" (NCEE, 1983, p. 5). Its rhetoric linked education to a national crisis, as in the days preceding the new math movement.

The response from NCTM

The National Council of Teachers of Mathematics, which identifies itself as "the largest non-profit association of mathematics educators in the world," aims to represent "the broad community of stakeholders in the fields of mathematics and mathematics education" (NCTM, see "NCTM at a Glance": http://www.nctm.org/about/intro.htm). In response to the reports of the '70s and to the back-to-basics curriculum, NCTM adopted an activist stance. As Shirley Hill, NCTM president from 1978 to 1980, stated, "A major obligation of a professional organization such as ours is to present our best knowledgeable advice on what goals and objectives of mathematics education ought to be" (quoted in McLeod, Stake, Schappelle, Mellissinos, & Gieri, 1996, p. 24). With this motivation, NCTM stepped forth to produce a set of standards for school mathematics.

Using only organization funds, NCTM produced and distributed Curriculum and Evaluation Standards for School Mathematics (1989), a document intended to be a multivoiced statement reflecting the varied composition of the organization and to produce a consensus broadly acceptable to the mathematics community. To represent that community, the writing group was composed of mathematics educators ranging from elementary teachers to college faculty (including mathematicians), researchers to state and district supervisors, and people with expertise in technology, teacher education, and other areas.

The standards-based movement also produced Professional Standards for Teaching Mathematics (NCTM, 1991), which addressed explicitly the changes in teaching that were implicit in the first document, and Assessment Standards for School Mathematics (NCTM, 1995). These documents, collectively referred to here as the Standards, rounded out the NCTM vision of mathematics in K-12 classrooms.

Conception of pedagogy and curriculum

A reform, by definition, challenges current practice. As understood from the NCTM Standards, this reform in mathematics education advocates changes in content and pedagogy. A significant difference from the new math movement is NCTM's emphasis

on mathematics content and instruction suitable to all students, not only the college bound. This principle underlies its vision in both of these essential areas.

Content. The Standards do not list specific topics to be covered by the end of each grade. Instead, the documents present guidelines and provide examples to clarify its unique conception of school mathematics content. Some noted changes were:

• The opening of secondary mathematics to include discrete mathematics, statistics, and mathematical modeling, with increased attention overall to applications.

• Across the grades, stress on connections of mathematics to the real world.

• More integration of mathematics topics.

• Emphasis on higher-order thinking and on "making sense" of mathematics through problem solving, communication, connections, and reasoning.

• A change at the elementary level from almost total concentration on arithmetic to inclusion of such topics as geometry, patterns, and statistics.

Underlying these proposed changes in content is a central focus on the conceptual versus the merely procedural. For example, the Standards call for decreased attention at the K-4 level to long division and to "complex paper-and-pencil computation," but increased attention to mental computation, the meaning of operations, and "thinking strategies for basic facts" (NCTM, 1989, pp. 20-21).

Pedagogy. This reform views the learner as actively participating in the construction of knowledge, a view that leads to changes in teaching practice, such as:

• Active student involvement in discovering and constructing mathematical relationships, rather than merely memorizing procedures and following them by rote.

• The use of concrete materials, calculator graphics, tables, or other representations as a means to help students grasp abstract concepts.

• Group work, including students sharing and justifying their ideas.

• Student writing (including drawings, diagrams, charts) to encourage reflection on mathematical ideas, and oral presentation to promote communication of those ideas.

• The use of context, whether imaginary or real world, as a way to capture student interest in problems and as "a framework or structure upon which to secure concepts and study them" (Robinson & Robinson, 1999, p. iii).

• Teacher as orchestrator of classroom discourse and facilitator of learning experiences.

Reaction to the reform

Initial reaction was markedly positive, seemingly "an overwhelming national consensus on directions for change in mathematics education," but the reform movement is now "facing passionate resistance from dissenting mathematicians, teachers, and concerned

citizens. Wide dissemination of the criticisms . . . has shaken public confidence in the reform process" (Fey, 1999).

On the positive side, the reform has had an impact on school mathematics to the extent that most states have rewritten their frameworks to align with the Standards in language, grade level demarcations, and goals. Other evidence of support was the awarding of grants by the National Science Foundation to several projects to produce curriculum series aligned with the Standards. Schools now have available instructional materials that incorporate the goals envisioned in the reform (see K-12 Mathematics Curriculum Center, http:// www.edc.org/mcc). Despite this support for the NCTM vision, a backlash has developed.

One significant barrier to implementation of the Standards is the change required of teachers in their practice. They are asked to alter their role from the accepted position as transmitter of knowledge to a new, therefore uncomfortable, position as facilitator--one who engages the class in mathematical investigation, orchestrates classroom discourse, and creates a learning environment that is mathematically empowering (NCTM, 1991). McLeod et al. (1996), noting the expenditure of energy and time by teachers, questions, "How much of a burden is reasonable? How much of a burden is reform based on the NCTM Standards, a reform effort that is characterized by complexity and a lack of specificity?" (p. 115).

At the same time, the issue of accountability has come to the fore. High stakes testing is often not aligned with the Standards, which further raises frustration as parents and teachers grow increasingly concerned about student performance on these tests.

Further insights into public reaction reveal oft-quoted misinterpretations of the Standards, with statements such as, "Children don't need to know the multiplication tables, and geometry no longer requires proofs" (McLeod, in press). In addition, concerned citizen groups object to NCTM's support for calculator use in the primary grades, argue that the Standards are vague on the importance of basic computational skills, and feel that mathematical applications are overemphasized throughout. These groups work toward return to a traditional mathematics program with a well-defined set of grade level objectives. The following quote from one leading group, Mathematically Correct, conveys the flavor of its opposition:

Across the country, the way mathematics is taught in the classroom and in textbooks has been changing notably. Classrooms are often organized in small groups where students ask each other questions and the teacher is discouraged from providing information. . . . The use of blocks and other "manipulative" objects has extended well beyond kindergarten and can now be found in many algebra classes. Meanwhile, the students practice their fundamentals less and less. . . . Calculator use is growing and taking away expectations for student learning. Textbooks, if the students have them at all, are full of color pictures and stories, but not full of mathematics. The books often don't even give explicit definitions or procedures. That would be "telling" and the new idea is for students to discover all of mathematics for themselves. Many of these programs don't even teach the standard algorithms for the operations of arithmetic. Long division is a

devil that is to be beaten into extinction--and if they manage that, multiplication will be next. ("What Has Happened," 2000)

NCTM bore in mind these misinterpretations as a new writing committee worked to update, refine, and clarify the Standards documents. The recently published document, Principles and Standards for School Mathematics (NCTM, 2000), simplifies the reform message by presenting only five content standards that extend across all grade bands and clarifies them through examples of problems, student work, and classroom dialogue. The document as a whole attempts to respond to the concerns voiced by the many stakeholders in mathematics education, to clarify NCTM's positions, and to define its vision.

Conclusion

The standards-based reform is not a reincarnation of the new math movement, although there are similarities. Both grew out of discontent with student performance and the incompatibility of traditional content offerings with advances in mathematics. Therefore, each added new content to the K-12 curriculum, reflecting the field of mathematics current at the time. New math, however, emphasized deductive reasoning, set theory, rigorous proof, and abstraction, while the Standards emphasize applications in real world context, especially experimentation and data analysis. This difference determines what problems are investigated and how they are solved, and what is counted as evidence.

Another difference lies in the pedagogy embedded in each reform movement. Standards-based pedagogy is based on constructivism and, therefore, instructional practices focus strongly on process--communicating, reasoning, problem solving, making connections, and representations.

Public acceptance of both reforms was general at first, and both encountered strong countermovements toward traditional instruction. New math ended in educational oblivion. What will become of the current movement?

While state curriculum frameworks and textbook publications show decided change directly connected to the reform movement, at the classroom level only minimal change has taken place in important areas that affect students--how mathematics is conceptualized and how it is taught. McLeod (in press), summarizing the research on the impact of the NCTM documents on classroom teaching, states that on questionnaires teachers reported making several changes in both curriculum and pedagogy. But studies that used classroom observations and in-depth interviews with teachers concluded that "the changes suggested by the Standards remained a vision of the future, too difficult to implement fully in the context of schooling in the 1990s" (McLeod, in press; see also Ferrini-Mundy & Schram, 1997; McLeod et al., 1996).

Mathematics educational reform--whether state frameworks or instructional materials--can occur on paper, as it did in the new math era and is now in the current standards-based movement, without impacting deeply the teaching/learning process at the classroom level. Change is slow, change is complex. As noted by Fey and Graeber (in press), it is difficult to deliver a reform message throughout the nation's diverse K-12

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system, but it is even more difficult to sustain the momentum and allow time for the message to play out in the classroom.

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Source: Theory Into Practice, Spring2001, Vol. 40 Issue 2, p84, 9p.
Item Number: 4472840
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