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ABSTRACT

No longer just a user of commercial software, the 21st century teacher is a designer of interactive software based on theories of learning. This software, a comprehensive study of straightline equations, enhances conceptual understanding, sketching, graphic interpretive and word problem solving skills as well as making connections to real-life and scientific phenomena. Developed using Maple and Hyperstudio, this software of 40 questions actively captures students' visual intelligence and evokes thinking to provide generative responses. Other features include corrective feedback, hide-and-show, scoring, timing and a student output file containing a student's 1st and 2nd responses and revealing any misconceptions held and difficulties encountered. A table shows the correspondence between the features of this software and Kuittinen's (1998) four criteria or demands for a good CAI (computer assisted instruction). A brief appendix provides programming information. (Contains 25 references.) (Author)

Teacher–Designed Software for Interactive Linear Equations: Concepts, Interpretive Skills, Applications & Word-Problem Solving

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Abstract: No longer just a user of commercial software, the 21st century teacher is a designer of interactive software based on theories of learning. This software, a comprehensive study of straight-line equations, enhances conceptual understanding, sketching, graphic interpretive and word-problem solving skills as well as making connections to real-life and scientific phenomena. Developed using Maple and Hyperstudio, this software of 40 questions actively captures students' visual intelligence and evokes thinking to provide generative responses. Other features include corrective feedback, hide-and-show, scoring, timing and a student output file containing a student's 1st and 2nd responses and revealing any misconceptions held and difficulties encountered.

Introduction

A straight-line equation, the most basic kind of mathematical function, is one of the most unifying ideas in mathematics. It has extensive applications in the world of science, especially physics, business and social science. From my experience as a high school mathematics and science teacher for about 20 years, I have seen how the lack of a conceptual understanding of straight-line equations has hindered students' understanding of science, especially physics. Some difficulties encountered by students include the inability to interpret a graph, to determine its algebraic representation, to verbalize the relationship between the two variables, to sketch a simple line without going through a table of values, or to see the graphical representation associated with an algebraic expression.

Dreyfus & Eisenberg (1982) found that students of low ability had difficulty with the graphical concept of a function. In another study in 1991 they contend that the chief source of difficulty encountered by beginning calculus students is their inability to exploit the visual representations associated with the concept of functions. Vinner and Dreyfus (1989) surveyed 307 college students on their concepts of a function and only 8 % of the students made reference to its graphical representation. In fact many studies investigated students' understanding of graphs of function and concurred that students lack the skill in interpreting graphs and fail to see the connection between the algebra and geometry. (Knuth, 2000).

In this computer age when a large quantity of data can be easily represented in different graphical forms, it is vital that students be proficient in the interpretation and extrapolation of graphs to deduce important information and relationships between the variables. Computer technology has freed students from the laborious task of graph plotting with pencil and paper but very often a student is not even aware that he/she has made a mistake in keying in information, resulting in the display of a different graph or function. Thus it is important that students have the conceptual, intuitive and visual understanding of straight-line equations in terms of positive and negative slopes, and y intercepts before embarking on the use of graphing software.

As cited in the Before It's Too Late report (2000), computer technology has not only changed the way we live and the way business is conducted, it has changed the demands of our work force, which calls for students to be equipped with the ability to think and solve problems. Educational reforms all over the world are bringing changes to meet such demands. NCTM (National Council of Teachers in Mathematics Standards) (2000) calls for a shift in emphasis from a curriculum dominated by the memorization of isolated facts to one that emphasizes conceptual understanding, problem-solving, functional relationships, multiple representations and connections, in particular between algebra and geometry, as well as to the real-world and other disciplines. My study of straight-line equations is in response to NCTM's multi-faceted frames of learning.

In Ontario, the New Curriculum, which consists of 4 years of high school compressed from 5, was introduced in 1999. Many topics, including straight-line equations, have been moved from higher to lower grades causing difficulties for many students. A recent report confirms a higher failure rate of our grade 9 and 10 students. ("Poor Math", 2001). Teachers with an increased workload of 6 courses from 5 are also faced with larger class sizes and new-immigrant students of very diverse ethnic backgrounds, languages and learning abilities. Professional developments for teachers and the use of computer technologies in the form of software or the Internet are high on the agenda to support student learning.

Computer Technology and Learning

Douglas Noble (1988, P. 241) has given one of the clearest discussions of the three rationales for the introduction of computer into schools.

"The first focuses on... technological society [which] requires new skills, including computer literacy...second rationale...technology of education: computer-based instruction offers new, effective and efficient ways to present material, to individualize instruction...third rationale focuses on a technology of mind: interactions with computers enhance cognitive skills while offering the possibility for intellectual mindstorms".

According to cognitive theories, computer technologies are cognitive learning tools, which expand human abilities such as memory and processing. Roschelle (1994) regarded technology as a form of inquiry with three functions derived from Dewey for the following:

a. to provide a stable, long-term access to a problematic situation in which the learner can repeatedly replay and reflect.
b. to provide focus and context to identify new features and relationship.
c. to augment ways of acting so that their meaning is more readily available to others.

Many studies have shown that calculators and computer-based materials have enhanced learning. Schwartz (1999) confirms the 5 aspects of mathematical activities, namely, conjecturing & exploring, evaluating & analyzing data, modeling, conceptually grounding manipulative skills, and the deepening of understanding, which can be enhanced by the use of computer technology. In mathematics, before students can create ideas or apply their ideas, they need to have a good grounding of basics, facts and conceptual understanding. Despite all the merits of computer technology, a teacher must use it wisely to ensure effective learning and must give clear instructions and guidance. I am in total agreement with Bland (1996, p.2) who says, "Technology should be used to expand possibilities for students, and to permit them to explore otherwise inaccessible problems. However, it should only be introduced after they understand the mathematical concepts involved, and are able to manually use them in simple problems".

Exploring and conjecturing are stimulating and can even evoke critical thinking for the average and above average students but for the less mathematically inclined, who often fail to discover patterns or conjectures, they feel very lost and frustrated. Mayes's (1992) study concluded that the average students performed better with the use of mathematical software with exploratory and programmable functions, but not the weak students. Recently, constructivism has been met with opposition by parents and educators as indicated in the "Math Wars" brewing in California which called for the adoption of the "three phase approach", namely the direct instruction in skills, the help phase and the self-regulated drill and practice (Becker & Jacob, 1998). Hirsch (1997, p.6) cites, "... only through intelligently directed and repeated practice, leading to fast, automatic recall of math facts, and facility in computation and manipulation can one do well at real-world problem solving".

Brady (1991, p.149) once said, "the best teaching aid is often those designed by the teachers themselves".
Thus, I have embarked on this project of designing a CAI software (mainly to serve the mathematically less inclined) with 40 questions that provides lots of practice, at the same time incorporating a multi-faceted frame of learning, delving in depth and width into all the topics related to straight-line equations. The research also examines students' conception or misconceptions of straight-line equations as well as ways of thinking and difficulties encountered in word-problem solving, interpretation of graphs, and scientific applications.

Software Design & Learning Theories

Piaget's and Skinner's Theories. Mellon & Sass (1981) suggest Piaget's developmental theory as a theoretical foundation for CAI. Piaget emphasized the movement of students through sequential stages of cognitive development; each stage builds on the previous one to develop effective learning. Skinner (1938) listed four important tenets about learning, which include a short learning process, reinforcement, immediate feedback and provision of "stimulus discriminations' for path to success, most of which are incorporated in the design.

Jonassen's Activity Theory. Interactivity has been identified as an important characteristic of the computer environment to the acquisition of deep learning. According to Jonassen and Rohrer-Murphy (1999), Activity Theory posits that conscious learning emerges from activity or performance. This software is designed with a high degree of interactivity, constantly asking the student what to do next and leading her through a continual process of interpreting, judging, constructing, applying concepts, translating modes of representation, and reinforcing algorithmic skills.

Real-Life & Interdisciplinary Learning. Students have often asked me "What good are the x and y in the real word? How are they helping me to find a job?" In the teaching of mathematics, it is imperative to bring in applications be they science, business or real-life problems, to foster conceptual understanding as well as convince students of the power and usefulness of mathematics.

Beane says that students are to "... integrate learning experiences into their schemes of meaning...And knowledge is called forth in the context of problems, interests and concerns at hand." (1995, p. 616). Carifio (1994) identifies key contextual features as familiarity, imageability and variable type (discreet or continuous quantities). The questions in the software are carefully prepared to incorporate such features i.e. real-life and scientific phenomenon that students are familiar with or can relate to.

Problem-based Learning. Problem solving is a form of higher-order thinking and depends on a student's resources, heuristics, control, and worldviews (Schoenfeld, 1985). My software aims at developing better heuristics and control in students as well as fostering the ability to identify relevant information, to transform words into symbolic equations, to assign suitable variables and notation for immediate recall of significant information, to discern the reversal error (Rosnick, 1981). and to apply the optimum algebraic skills. Through working with the word problems, students will learn to plan and sequence their tasks, justify their answers, and test their final answers by integrating algebraic and geometric methods.

Howard Gardner's Visual Intelligence. Gardner (1983) notes that many students have preferred ways of learning depending on their "intelligence" which could be kinesthetic, spatial or visual, musical, verbal, intrapersonal, interpersonal or naturalistic. This software captures the visual intelligence of the student through the many vivid representations of graphs created by Maple (Char et al., 1991) and fosters logical reasoning in the interpretation of graphs. According to Mones-Hattal & Mandes, (1995, p.891) visual literacy is "a cognitive process that includes mental rehearsal, introspection and visualization .....whereby each exposure to the visual image permits the observer to become a keener interpreter of the visual display".

My Interactive Straight-Line Equation Software

This software consists of 4 modules, namely, A: Slope by visualization or calculation, B: Graphical to algebraic form, C: Applications and D: Word-problem solving. The graphs were drawn with Maple. First the grid was drawn with tiny circles each representing integer coordinates or some multiples of integers. A set of horizontal points was generated using the Maple "seq" function and the corresponding points were drawn with the "plot" function. These two functions were placed inside a loop to draw subsequent lines of points forming a grid. The line $y = mx + b$ was then drawn and the whole graph was exported to Hyperstudio. (See Appendix I for Maple Program). Hyperlogo in Hyperstudio is used in achieving the more sophisticated interactive features such as keeping track of the number of times a student has answered an item or analyzing if the response is correct and furnishing a corrective feedback. The following table shows the correspondence between the features of this software and Kuittinen's (1998) four criteria or demands for a good CAI.

Demands (Kuittinen)	**Characteristics** (This software)
Domain - Dependent	-Questions are relevant to instructional aims and curriculum related -Questions are interdisciplinary or related to real-life phenomena
Instructional	-Each module carries at least one worked example -Generative responses as well as multiple-choice questions. -Immediate feedback; correct answer is displayed after 2 incorrect inputs. -A high level of interactivity to elicit continuous student participation; step by step guidance. -Ample questions for practice. -Questions are contextual in nature. -Student performance is assessed with an output file, which has a record of the 1st and 2nd responses. - Student's achievement score is recorded giving an indication of the student's progress. –Timing feature keeps a record of the time the student spends on a question giving an indication of perceived difficulty.
	-Powerful graphics with clear and precise graphs on a dotted grid to capture student's visual intelligence and to facilitate the skill in interpreting graphs. -Hide-

User Interface	and-show feature to capture student's attention and concentration on one particular idea at a time. -Drag-and-drop feature to help students tack on x and y coordinates to an ordered pair of a point. -Learner's choice of method in word-problem solving. Only when the student fails to achieve the answer does he have to follow certain prescribed steps, those steps taken by experts! Hopefully this trains students to emulate experts. -Accommodation of upper and lower case input. -Navigation is easy; each card is linked to the first card, which has the exit icon, as well as automatically to the next card or the menu selection card. – In some modules, questions are identified as easy, moderate and difficult. -Texts of different colours and highlighting for easy reading. -Students can work at his own speed.
Pragmatic	Hyperstudio will have to be installed in the computer to run the software. I am now preparing a compact disk with Hyperstudio embedded in so it can run independently. Thus the software is portable and does not require any technical support for installation. The modules could easily be posted on the Internet.

Table of Characteristics of Software

Nature of Questions & Links

Module C: Applications

1. Ohm's Law	6. Speed of sound
2. Density	7. Motion
3. Friction	8. Snell's law (light)
4. Electric charge	9. Heat
5. Electrical energy	10. Cooling of Water

Module D: Word-Problem Solving

1. Dimensions	6. Distance, speed
2. Mixtures	7. Distance, speed. (different starting times)
3. Money (coins)	8. Tickets, money
4. Age	9. Car rentals
5. Concentrations	10. Numbers

Visit http://fcis.oise.utoronto.ca/~vlawrence/stlines.html to view sample computer screens.

If you have **Quick Time 5** on your machine, you may try out a sample of the interactive software at http://fcis.oise.utoronto.ca/~vlawrence/hypertrl.html

Proposed Study

A class of grade 10 students who have covered the content of straight line equations will be given a pre-test of 8 questions , 1 on slope by inspection, 1 by calculation, 1 on transformation from graphical to algebraic form, 1 from algebraic to graphic form, 1 on formulation of equation given relationships expressed in words, 1 on the interpretations of a graph representing a scientific phenomenon, and 2 on word-problems. From the results of the pre-test, weak and average students (selected few) will be solicited for interviews. The whole class is then exposed to the interactive software but think-aloud protocols or audio-video taping may be employed to the selected few who work individually or in pairs. Students are then given a questionnaire about their learning experience with the software, followed by a post-test and a final interview (selected few), the latter to probe further into their misconceptions captured in computer files, audio-video taping and post-test.

Pilot Tests & Findings

In the first pilot study, 57 grade 9 students (from 3 classes) wrote a pre-test followed by treatments of modules B (graphic to algebraic) and C(applications). Students were then given a questionnaire in my absence, followed by a post-test. A quantitative analysis of the pre and post-test using the comparison t test for two means from dependent (correlated) samples pointed to a significant difference in the scores or gains. Feedback from questionnaires revealed that 91% of the students affirmed that the software had enhanced their learning.

In the second pilot test, three grade 10 students were interviewed individually followed by a pre-test. Each then interacted individually with module D (problem-solving) followed by a questionnaire and a post -test. Both feedback from the questionnaire and results of the post-test indicated significant gains in learning. Analysis of "captured files" also revealed certain misconceptions held and difficulties encountered by students, some of which were the inability to reduce (9A + 6S = 7500), relating to time frames (to add or subtract when leaving a point at a later time), handling special cases (solving C=40+0.2D and C=30+0.25D, did not see coefficient of 1 in C provides direct subtraction or equating), false commutation (length is 2 cm less than width as L = 2-w), faulty conversion between decimal and percent etc.

In general the students agreed that the software had helped them to attain a better conceptual understanding of straight-line equations, had improved their graphing, graphic interpretive, algebraic and problem-solving skills, in particular, the transformation of relationship in words to symbols. They also indicated that they view mathematics as more meaningful and have less fear of word-problem solving.

Conclusions

In view of positive pilot study results, this interactive software will not only enhance the learning of straight-line equations related to graphical interpretive and problem-solving skills but also foster a better understanding of scientific principles and usefulness of mathematics in the real world. Misconceptions and students' ways of thinking can shed light to how teachers should teach and serve as a resource for curriculum designers and planners. To accommodate students' learning in this computer technology era, software must not only cover all aspects of a topic; it must link them, actively and interactively engaging the user as a builder of his own knowledge. These principles of software design can be applied to other topics in the mathematics curriculum, in particular the extension to quadratic, exponential, logarithmic and trigonometric functions.

Appendix I

This MAPLE program generates the graph consisting of the grid points, coordinate axes and the straight line y = 10x + 20

```
with(plots):hlines:={};
                hlines := {}
for j from 1 to 6
do
  yval:= 10*j:
  yrlist:=[seq([i, yval],i=1..4)]:
  hlines:=hlines union {plot(yrlist, style=point,symbol=circle,colour=blue)}:
od:
display(hlines);
e:=plot(10*x +20, x=0..4):
 display({e} union hlines);
```

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