DOCUMENT RESUME

ED 476 640 SE 067 914

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TITLE	The Effects of Multiple Linked Representations on Student Learning in Mathematics.
PUB DATE	2001-00-00
NOTE	9p.; In: Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (23rd, Snowbird, Utah, October 18-21, 2001). p997-1004 For full proceedings, see SE 065 231.
AVAILABLE FROM	ERIC/CSMEE Publications, 1929 Kenny Road, Columbus, OH 43210-1080. Tel: 800-276-0462 (Toll Free); Tel: 614-292-5680.
PUB TYPE	Reports - Descriptive (141) -- Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS	*Algebra; Classroom Environment; *Computer Software; Computer Uses in Education; Concept Formation; Curriculum Development; Elementary Education; *Graphs; Mathematics Education; *Student Attitudes; Teaching Methods

ABSTRACT

This study investigated the effects on student understanding of linear relationships using the linked representation software VideoPoint as compared to using semi-linked representation software. It investigated students' attitudes towards and preferences for mathematical representations--equations, tables, or graphs. An Algebra I class was divided into three groups of students: linked, semi-linked, and control. Findings regarding word problems, interpreting/constructing and reading graphs, solving and constructing equations, reading and constructing tables, and misconceptions were compared. Most of the students indicated that they found the software helpful in learning mathematics. (KHR)

THE EFFECTS OF MULTIPLE LINKED REPRESENTATIONS ON STUDENT LEARNING IN MATHEMATICS

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The utilization of technology in multiple representations has become one of the significant topics in mathematics education in the last decade. Here, multiple representations are defined as external mathematical embodiments of ideas and concepts to provide the same information in more than one form.

One example of this type of environment is educational software with linked multiple representations. Linked multiple representations are a group of representations in which, upon altering a given representation, every other representation is automatically updated to reflect the same change (Rich, 1995/1996). We define semi-linked representations as those for which the corresponding update of changes within the representations are available only upon request but are not automatic. It is the premise of this study that semi-linked representations are as effective as linked representations and that there is a role for each in different situations, at different levels, and with different mathematical concepts. The focus of this study is comparing three groups of students: one group using linked representation software, the second group using similar software but with semi-linked representations, and the control group. Briefly, the research questions of this study were:

1. What are the effects on students' understanding of linear relationships using linked representation software compared to using semi-linked representation software?
2. What are students' attitudes towards and preferences for mathematical representations—equations, tables, or graphs?

Theoretical Framework

Although there are a number of theories emphasizing multiple representations in the history of mathematics education, with Dienes' "multiple embodiment principle" this issue gained a significant prominence. The multi-embodiment principle suggests that conceptual learning of students is enhanced when students are exposed to a concept through a variety of embodiments (Dienes, 1960).

Constructivism suggests that students construct their knowledge by themselves actively in *their experiential world*. Through communication and interaction with other people, learners test how like (consistent) their constructs are with others' (Confrey, 1990; Goldin, 1990). Because of differences in experience, we cannot expect that everyone will understand a concept the same way from one representation or that one representation will be equally meaningful for everyone.

Integrating theoretical components from a number of mathematics educators, the theory of understanding relative to multiple representations is as follows:

- Students should be able to identify a given mathematical idea across different representations;
- Students should be able to manipulate the idea within a variety of representations;
- Students should be able to translate the idea from one representation to another;
- Students should be able to construct connections between internal representations;
- Students should be able to decide the appropriate representation to use in a mathematics problem;
- Students should be able to identify the strengths, weaknesses, differences, and similarities of various representations of a concept. (Dufour-Janvier, Bednarz, & Belanger, 1987; Hiebert & Carpenter, 1992; Lesh, Post, and Behr, 1987; Schwarz and Dreyfus, 1993).

The question is how understanding across multiple representations can be improved with educational technology. Kaput (1992) advocates the use of linked representations as follows:

> All aspects of a complex idea cannot be adequately represented within a single notation system, and hence require multiple systems for their full expression, meaning that multiple, linked representations will grow in importance as an application of the new, dynamic, interactive media (p.530).

According to Piaget's theory, cognitive development is driven by a series of equilibrium-disequilibrium states. If everything is in equilibrium, we do not need to change anything in our cognitive structures. Linked representational software gives students immediate feedback on the consequences of their actions with machine accuracy, but it may not engender the disequilibrium necessary for learning. Semi-linked software, by not showing the corresponding changes in other representations, by giving time to reflect or asking questions about what kind of changes will result from a change in any representation, forces students to resolve the dissonance in their cognitive structures. If their organization of knowledge is well established, they can deal with the question. However, if not, then they will need accommodations in their cognitive structures. Thus, semi linked representational environment puts students in a more active role as learners.

Data Collection Methods

Subjects of this study were ninth-grade Algebra I students. The class was divided into three groups of students—two experimental groups and a control group. Two experimental groups used the same software but with different linking properties—linked and semi-linked. VideoPoint is a software package that allows one to collect position and time data from QuickTime movies of two cars driving in the same direction with different constant speeds or two fish swimming towards each other. These data can be combined to form calculations such as distances between points and can be presented using different representations such as tables, graphs, and equations. Although VideoPoint was designed as linked representational software, the linkage for the table representation was not two-way. So, the software developer made changes, at the request of the investigator, to create the fully linked and semi-linked versions of the VideoPoint for this study.

The differences between the linkages in the linked and the semi-linked representations are summarized in Figure 1. As one can observe, the graph, table, and movie representations are linked two-way in the linked version. This means that when the user clicks on a point in those representations, the corresponding data points in all other two representations are highlighted. Moreover, when the linked version user clicks to see the algebraic form (the equation of best fit) of the phenomena, the line of best fit is also graphed in the graph window automatically. On the other hand, the user of the semi-linked version is not able to see any updates when s/he clicks on one representation. The only linkage that is available in the semi-linked version is between the graph and equation form. When the user estimates the coefficients in the algebraic form, s/he has an option to see the graph of the predicted equation.

Data collection methods included mathematics pre-and posttests, follow-up interviews with all students after the mathematics posttest, clinical interviews at the end of the treatment with 5 students from each experimental group, and classroom and com-

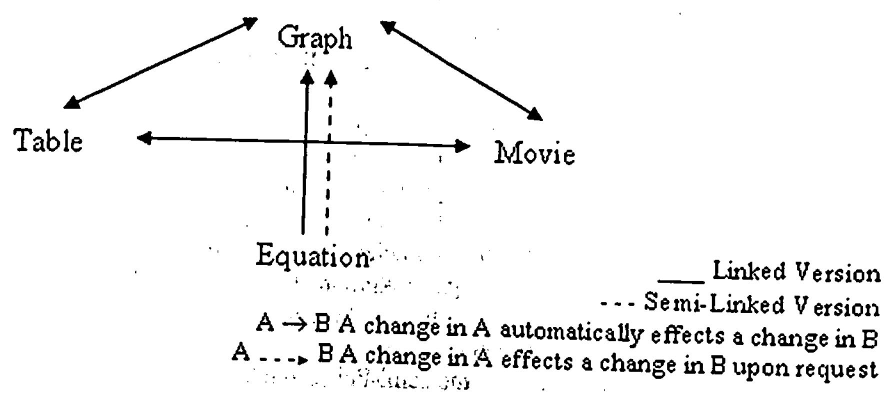

Figure 1. The linkages among linked and semi-linked representations in VideoPoint.

puter lab observations. A survey was conducted at the end of the study in order to see students' opinions about mathematics, representations in general, and the computer environment. Table 1 summarizes the research design with data collection and data analysis methods.

Results

Instead of studying each question separately, questions in all written tests used in this study were clustered into categories, and those categories were compared across the three groups—linked, semi-linked, and control. The categories were: Word Problems, Interpreting/Constructing and Reading Graphs, Solving and Constructing Equations, Reading and Constructing Tables, and Misconceptions (Height/Slope, Point/ Interval, Graph as Picture). These categories were compared using a nonparametric test—Kruskal-Wallis (a test for several independent samples)—to identify differences between the linked, semi-linked, and control groups. The results of this test showed that there were no differences in achievement between the groups in any category of problems on either the pretest or posttest at either the .05 or .1 confidence level. A nonparametric test—the Wilcoxon Test (a test for dependent samples)—was used to identify the improvement or decline from pretest to posttest within groups in each category (see Table 2). Some of the improvements were significant at the .05 level, such as experimental groups in the categories of interpreting graphs and constructing equations, the semi-linked group for the height/slope misconception category, and the linked group for the graph as picture category. Other improvements were significant at the .1 level, such as the linked group for the height/slope category.

In order to study the mathematical learning within the computerized environment, clinical interviews were conducted. It was found that in the linked software environment, when a question was asked, students either used the linkage directly to answer the question or they assimilated this new information and drew upon their previous knowledge to answer the question. When they used the linkage, their explanation for their answer was based more on the software; especially the movie. However, their answers were more based on the mathematical aspects of the question, when they did not use the linkage. When students provided an inappropriate answer to a question and they saw that they were wrong according to the linkage or computer feedback, disequilibrium occurred. Then they needed to go back and interpret this new information with their existing knowledge. If they could not interpret the new information, they needed to accommodate their preexisting knowledge in order to reach equilibrium. Sometimes students did not have the enough background to interpret this new information with their existing knowledge. Some students did not use the linkage at all, when they trusted their knowledge and answers..

When a question was proposed in the semi-linked environment, students relied mainly on their own existing knowledge with the help of the software. They assimilated new information and drew upon their existing knowledge to answer the ques-

Table 1. Data Sources.

Research Questions	Data Collection Methods	Description of the Data Collection Methods	Criteria or Indicators for Analysis
1. What are the effects on students' understanding of linear relationships using linked representation software compared to using semi-linked representation software?	Clinical and Follow-up Interviews	Five students from each experimental group were interviewed while using the computer software. Follow-up interviews after the pre- and posttests provided information about their reasoning in answering the questions.	Codes, patterns and themes were searched throughout the data.
	Mathematical Pre- and Posttest	Students' paper and pencil performance were analyzed.	Descriptive analysis Nonparametric tests for group differences and for achievement differences between pre- and posttest
	Teacher Interviews	To see the teacher's views about students' growth mathematically and their preferences	Codes, patterns and themes were searched throughout the data.
	Observations	Everyday classroom and computer lab sessions observations	
2. What are students' attitudes towards and preferences for mathematical representations: equations, tables, or graphs	Survey	Students' attitudes towards mathematics, mathematical representations and their rationales for their preferences towards representations were studied with Likert scale and open-ended questions.	Descriptive analysis Nonparametric tests for group differences Qualitative analysis for open-ended questions

Table 2. *Improvement Significance Scores*

		Improvement Significance Scores		
		Control	**Linked**	**Semi-Linked**
Word Problems (Verbal)		0.083*	0.058*	0.014**
Graphs	Interpreting/Constructing Graphs	0.059*	0.026**	0.007**
	Reading Graphs	0.785	0.038**	0.729
Equations	Solving Equations	0.157	0.317	0.180
	Constructing Equations	0.066*	0.042**	0.042**
Tables	Reading Tables	0.317	0.02**	0.102
	Constructing Tables	1	1	0.317
Height/Slope Misconception		0.684	0.061*	0.024**
Point/Interval Misconception		0.102	0.317	0.317
Graph as Picture Misconception		0.317	0.014**	0.157

*.1 significant **.05 significant

tion. Although the semi-linked environment did not provide such rich feedback as in the linked environment, a ready-made graph or table presented powerful visual information/feedback for students to use while answering the questions. Lack of linkage forced more mathematically-based explanations instead of movie-based explanations and empowered students to trust their answers and convince themselves and construct the linkages between representations by themselves. Some students needed the linkage in some situations in order to construct more empowering mathematical concepts.

The researcher tried to follow up the teacher's regular class sessions in the computer labs with the aim of giving opportunities to students to apply knowledge learned in class. Moreover, it was hoped that students would also carry their learning from the computer labs to the regular class sessions, which mainly consisted of paper and pencil tasks. There were a couple of incidents that showed students were carrying ideas back and forth from the class to the computer lab and vice versa.

Finally, the results from the survey revealed students' attitudes towards mathematics and their preferences for particular representations in paper and pencil and computer environments. All students exhibited somewhat positive attitudes towards mathematics. Students in each group all had similar attitudes towards the use of representations in mathematics. Most students agreed that mathematics problems can be solved in various ways by using different representations. Although they reported that they liked using more than one representation in solving mathematics problems, they also agreed that they found it easier to focus on one representation. They agreed that using different representations does not lead to totally different answers. Students reported that they preferred tables and equations to graphs. They indicated that they usually start solving mathematics problems with tables or equations. Previous experience/knowledge with a representation and knowing how to manage it was a common reason for students to choose a particular representation. There were specific reasons for choosing a particular representation, such as being able to find an exact answer with an equation, the visual advantages of graphs, or the organized information provided by tables.

Most of the students indicated that they found VideoPoint helpful in learning mathematics. Easy access to all representations at once was a common theme mentioned by students as a reason for finding VideoPoint helpful. Students reported that tables and graphs were the types of representation they liked the most while using VideoPoint. Graphs came to be the preferred representations due to the easy access to them with VideoPoint. Some students also mentioned how VideoPoint helped in constructing relationships among representations. They reported that they liked being able to see different kinds of representations all at once since it gave them a choice to work with one that they were more comfortable with or showed them there were various forms available. Several students also pointed out that VideoPoint was helpful in comparing different representations or checking their answers.

References

Confrey, J. (1990). What constructivism implies for teaching. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 107-122). Reston, VA: National Council of Teachers of Mathematics.

Dienes, Z. P. (1960). *Building up mathematics*. Great Britain: Anchor Press, Hutchinson Educational.

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 109-122). Hillsdale, NJ: Erlbaum.

Goldin, G. A. (1990). Epistemology, constructivism, and discovery learning mathematics. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 31-47). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Kaput, J. J. (1992). Technology and mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: Macmillan.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics leaming and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Erlbaum.

Rich, K. A. (1996). *The effect of dynamic linked multiple representations on students' conceptions of and communication of functions and derivatives*. (Doctoral dissertation, University of New York at Buffalo, 1995). Dissertation Abstracts International, 57 (1), Z142.

Schwarz, B., & Dreyfus, T. (1993). Measuring integration of information in multirepresentational software. *Interactive Learning Environments, 3* (3), 177-198.

VideoPoint 2.1.2 (Modified by the developer for this study) [Computer Software]. (2000). Lenox, MA: Lenox Softworks.

Von Glaserfeld, E. (1996). Aspects of radical constructivism and its educational recommendations. In L. Steffe, et. al., (Eds.), *Theories of mathematical learning* (pp. 307-314). Hillsdale, NJ: Erlbaum.