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ABSTRACT

This study describes a teacher preparation program in mathematics and science and explores what impact the reformed curricula and teaching methods instituted in the program have on prospective teachers' understanding of rational numbers and integers. The study pursues obtaining in-depth insights regarding prospective teachers' concept development. The pretest and the posttest consisted of tests for computational skills, number sense, and conceptual understanding. Results suggest a relationship between a reformed mathematical environment and enhanced student achievement. (KHR)

CURRICULUM REFORMS THAT INCREASE THE MATHEMATICAL UNDERSTANDING OF PROSPECTIVE ELEMENTARY TEACHERS

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As the mathematics curriculum in the United States is being transformed from a fact and algorithm based curriculum to a concept based curriculum (NCTM, 1989, 2000), universities and colleges have been taking a closer look at their teacher preparation programs (Arizona Board of Regents, 1995). It has been recommended that the experiences that preservice teachers have should conform to the standards developed by professional organizations, that the quality of the mathematics content courses should be strengthened, and that preservice teachers should experience the type of classroom environment they will be expected to develop.

Recently, a group of post-secondary institutions in a southwestern state were awarded a grant from the National Science Foundation to reform its preparation of teachers in Mathematics and Science. One of the courses targeted for improvement by the grant was a mathematics course that taken by all prospective elementary teachers. A team of instructors was assembled and current educational research was consulted in an effort to develop a rich curriculum designed to deepen preservice teachers' understanding of the fundamental concepts in mathematics. As a result, this required course has been dramatically modified to reflect the recommendations of NCTM (1991) and other national and local agencies. The course is now taught in what is considered a highly reformed manner (as opposed to a traditional lecture format). That is, the course is currently conceptually based, integrating tools and technology into the instructional delivery of the curriculum. These preservice teachers are now being asked to solve problems, write mathematically, think critically and construct concepts in the manner in which we expect them to provide for their students. For instance, these students used a variety of concrete models to study fractions. Activities with varied representations allowed them to see equivalent fractions, and to understand the motivation for the use of common denominators for fraction addition and subtraction. Array models used for whole numbers were extended to encompass fraction multiplication, and repeated subtraction was used to model and explain fraction division.

This study proposes to answer the following research question: What impact does the reformed curricula and teaching methods instituted in this course have on prospective teachers' understanding of rational numbers and integers. In order to obtain more in-depth insights regarding prospective teachers' concept development, these two fields were investigated rather than the entire curriculum. Research shows that

students, as well as teachers, traditionally have difficulty both understanding and conveying these concepts. Insights into students' understandings and misconceptions should shed light on the efficacy of the described reforms.

Framework

Understanding the concepts of signed numbers and rational numbers are fundamental for the acquisition of higher mathematical concepts. Understanding these sets of numbers involves more than knowing the operational algorithms; it also includes the ability to:

- Add, subtract, multiply and divide integers and rational numbers.
- Order integers and rational numbers.
- Predict how the basic operations affect size and order of integers and rational numbers.
- Explain how the basic operations affect size and order of integers and rational numbers.
- Solve problems using integers and rational numbers.
- Solve ratio and proportion problems.

These aspects of understanding provided the groundwork from which the assessment instruments were developed, and the methods in this study were designed to capture the presence or absence of these abilities.

Background

Most of the research conducted in this area either highlight areas where preservice teachers are mathematically deficient, or make recommendations for the mathematical development of preservice and in-service teachers. Studies show that both in-service and preservice teachers have been inadequately prepared in the past. Practicing elementary and middle school teachers in Arizona have self-reported that their preparation was fair at best and many had reservations about their ability to teach mathematics (Arizona Board of Regents, 1995, 1997). Post (1991) found that 25 percent of the middle school teachers studied could not successfully find the answers to problems that required basic computations with rational numbers. When asked to solve problems requiring conceptual knowledge, only half of them were successful. Ball (1990, 1990) studied the understandings that preservice elementary and secondary teachers have of division. She found that both groups studied had a shallow and fragmented view of division. While the prospective high school teachers had a better grasp of the rules, they were no better at explaining how the rules were derived or what they meant. Tirosh and Graeber (1990) found similar results when studying preservice teachers' understanding of division of rational numbers. They found that certain ways of thinking—for example the idea that "division makes numbers smaller"—while were true

for whole numbers became obstacles when reasoning about rational numbers. In a 1997 study, Behr, Khoury, Harel, Post and Lesh investigated the various ways preservice teachers approached a single problem involving rational numbers. In that study, the authors concluded that teachers must be aware of the many problem-solving strategies that a single problem can generate, and must provide activities that can develop a variety of cognitive structures. In a study that compared the conceptual understanding of a few Chinese elementary school teachers and American elementary school teachers, Ma (1999) found that even though those Chinese teachers had less formal education than their American counterparts, they demonstrated what she calls a profound understanding of fundamental mathematics, while teachers in the United States did not.

This collection of research studies suggests that teacher education programs in the United States need to better prepare prospective teachers by developing the conceptual underpinnings necessary to implement the new school curriculums driven by the NCTM Standards (1989). Many colleges and universities have responded to these finding by reforming their mathematics classes for prospective teachers. Yet there is little data available to suggest that these efforts are producing the desired results.

The Study

The subjects for this study were approximately 225 students enrolled in *Theory of Mathematics for Elementary School Teachers;* a one-semester course designed to promote deep understanding of the mathematics taught in elementary school. Also included in the study were the seven instructors assigned to teach the class. The research involved two groups -- a treatment group which involved four of the instructors and their students and a control group which involved the other three instructors and the balance of the students. Both groups were administered identical pretest and posttests. Participants from the treatment group were interviewed after both the pretest and the posttest. Interviewees were asked to solve selected problems, explain their reasoning, and express their general views about the class and their attitudes about mathematics. The interviews were tape-recorded and tapes were analyzed to look for insights into students understanding as well as misconceptions.

During the treatment phase of the study, the participants in the treatment group explored the topics in the context of activities that utilized concrete models and technology. Regular homework was assigned from a textbook and graded weekly, and an exam was administered at the end of the unit. In addition to tests, homework and class activities, students were asked to write about these topics on a regular basis in a math journal. While the control group did experience some "student-centered" curricula, they did not use the materials developed for the treatment group.

However, curriculum is only one facet of instruction. Implementation of the curriculum is critical to its success. For this reason, the instructional strategies used in the classrooms studied were also investigated. Since the treatment claimed to be "highly

reformed", it was important to not only substantiate that claim, but to also observe and compare the instructional strategies of both groups. The level of "reformed teaching" observed in these classrooms was measured using the Reformed Teacher Observation Protocol (RTOP). RTOP (Piburn et. al., 2000) was designed to quantify the level of inquiry or student-centered teaching that takes place in a classroom. Two researchers used the RTOP to independently evaluate the classrooms involved at least twice. The scores were averaged, and then correlated to the scores on the students' tests to determine if a relationship existed.

The pretest and the posttest consisted of three subtests: Computational Skills, Number Sense, and Conceptual Understanding. Many of the questions in this instrument have been used by other researchers studying the knowledge and misconceptions of prospective teachers. Detailed rubrics were developed for each of the conceptual questions during the pilot phase of the study. These questions were scored on a five-point scale, where 0 indicated that no attempt was made, and 5 indicated no flaws in work or reasoning. Questions from the other subtests were either correct or incorrect. Scores were established for each subtest as well as a total score for each participant.

Results

The pretest results show that a startling number of students enter the program under-prepared in mathematics. Even though the prerequisite for this course is College Algebra, 58% of those students tested were unable to execute simple computations involving rational numbers and integers without the use of a calculator. The poorest performance was reflected in those problems involving multiplication with mixed fractions, and division of two decimals. Interviews provided evidence for their strictly mechanical approach to these types of questions. Frequently students could not articulate their reasoning or their solution pathway, nor did they express confidence in their abilities.

Interviewer: *How did you arrive at this answer?*

Student: *I don't know – I just cross-multiplied.*

Interviewer: *Why did you do that?*

Student: *Well, I just remembered something about that with fractions you just cross over.*

Even when students arrived at the correct answer, their thinking and understanding of the problem was incorrect. When asked how she determined where to place the decimal point in the answer to $0.85 \div 0.2$, Carole replied, "*The decimal point was over 2 [places] so I figured it should be over two or three, so I put it over two.*"

When asked to estimate $72 \div 0.025$, most students responded that the answer would be less than 72. As Terry explained, "*When you divide something it gets smaller.*"

In response to the question, write a word problem that uses $1\frac{3}{4} \div \frac{1}{3}$, most students

wrote a problem that required division by 3.

Dana: *Joe and his 2 friends ordered two pizzas. They ate one pizza and three quarters of the other pizza altogether eating* $1\frac{3}{4}$ *of a pizza. How much did Joe eat, considering they each ate equal amounts?*

Interviewer: *You got as an answer* $3\frac{1}{2}$. *Does your answer make sense?*

Dana: *No—that doesn't make sense at all.*

Paired t-tests confirmed that while both groups made significant gains from the pretest to the posttest, the treatment group scored higher on the posttest than the control group. Not only did they perform better on the test as a whole, but also their sketches, drawings and explanations on the test papers revealed a deeper understanding of the content. The most dramatic and exciting increases were from the Number Sense and Conceptual Understanding subtests. Posttest interviews confirmed better understanding as well as more confidence in the subject matter.

Interviewer *How did you do this one, where you were to estimate* $\frac{12}{13}+\frac{7}{8}$?

Alice: *I figured* $\frac{12}{13}$ *is close to* $\frac{13}{13}$ *which is 1, and* $\frac{7}{8}$ *is close to* $\frac{8}{8}$ *which is 1, and so I figured* $1+1$ *is 2*

Many more of them were able to come up with appropriate word problems using division with fractions.

Dana: *I have* $1\frac{3}{4}$ *cantaloupes, and I want to give everyone* $\frac{1}{3}$ *of one. How many pieces can I get?*

Some of the interviewees were still confused division by $\frac{1}{3}$ with division by 3 when trying to create a problem. Yet, when those students were offered examples of both types of problems during interviews, they were able to select the correct one.

The interviewees also expressed enthusiasm for the format of the class. When asked if the activities they experienced helped them to understand fractions, decimals, and integers better, all but one stated that the treatment definitely helped their understanding of these concepts. They cited fraction manipulatives and base 10 blocks as well as paper folding as tools that enhanced their knowledge.

Alice: *I think a lot of the stuff helped me...it more. Like the reasoning behind it.*

Dana: *I think drawing it out, like say you have a certain fraction and you want to divide by another fraction...I wouldn't have thought before that I could divide each one into one third and then count them. I [also] like working in groups, I learn a lot more.*

Karen: *I'm like totally hands on. I've got to get into it.*

Ben was more reserved.

Ben: *A little bit. I'm still kind of confused. Like I said, I got very confused on how much exactly $\frac{4}{9}$ is. So a little bit. It seems like I do the Base 10 stuff and all in class, and then I try and apply it again and it gets more confusing. And then I go back and look at my activity book and it's not as clear for some reason as when I first did it.*

Comparing the Groups

There was a statistically significant gain from pretest to posttest for all subjects. In addition, the RTOP scores for the Treatment group were significantly higher then the RTOP scores for the Control Group. When RTOP scores were compared to the gains achieved from the pretest to the posttest, there was no correlation. (See Table 1) It would appear that the level of reform teaching had no impact on student achievement. However, an examination of the sub-tests revealed a different story. When RTOP scores were compared with the normalized gain (that is the gain divided by the potential gain) for the Computation Sub-test, there existed only a very weak correlation, indicating that a higher level of reform does not necessarily lead to better computational skills.

Table 1. RTOP Correlated with Selected Results

Group	RTOP	Overall Percent Increase	Overall Normalized Gain (Hake)(%)	Normalized Gain on Computational Subtest (%)
Treatment	83	11	26	59
Treatment	71	22	34	35
Treatment	73	32	50	72
Treatment	60	25	35	63
Control	61	25	31	55
Control	45	20	23	46
Control	61	18	24	64
Correlation		-0.21	0.37	0.22

Yet, when the RTOP scores are compared with the just post-test scores, the correlation jumps to 0.88. When considering only the Number Sense Subtest, RTOP correlates with both the posttest scores (r =0.76) as well as with the normalized gains attained (r =0.73). (See Figure 1.) A closer look at the Conceptual Understanding Subtest reveals a similar relationship. For a comparison of RTOP and the posttest scores of that subtest, r =0.89, and when compared with the normalized gains, r =0.73. (See Figure 2.)

Conclusions and Discussion

These results suggest a relationship between a "reformed" mathematical environment and enhanced student achievement. When prospective teachers experience a student-centered, conceptually based, Standards driven mathematics course, they benefit in at least two ways. They develop a deeper, richer understanding of mathematical concepts than they would in a traditionally taught, or even moderately reformed classroom setting. The treatment group displayed stronger "number sense" was better able to estimate reasonable solutions, to solve problems, and their computational skills were equal to those of the control group. They were also more articulate about their mathematical thinking. This is especially heartening; since number since and estimation are areas at which our students traditionally do not excel (TIMSS, 1996).

They also experience first hand the classroom setting that is advocated by various professional organizations. These students were able to feel the power of discovery, and the satisfaction of understanding. As a result, their attitude toward mathematics

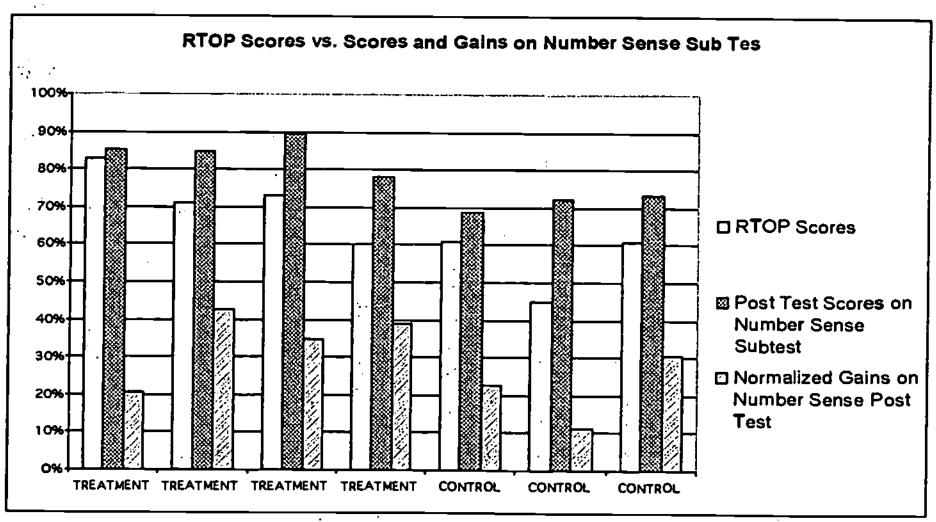

Figure 1.

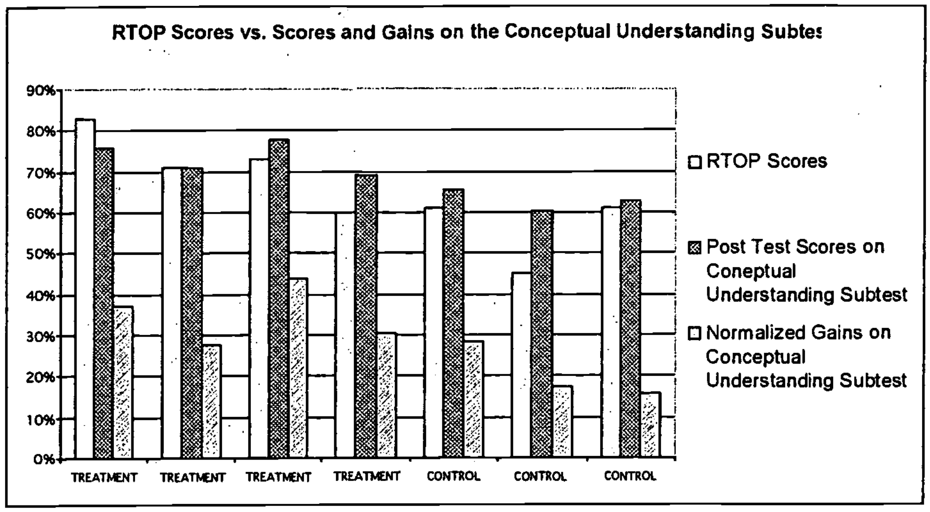

Figure 2.

learning and mathematics teaching changed. Most of the students interviewed liked the approach and reported that it enhanced their understanding of rational numbers and integers. They enjoyed working with the manipulatives and saw the value of using these methods with children. They benefited from the working in a group and having the opportunity to brainstorm and share with their peers. They also reported greater enjoyment of mathematics in general, more persistence, as well as more confidence in their own abilities.

These results also confirm the recommendations of many of the experts in this field, yet it also raises some issues that teacher preparation programs need to be aware of. Although all these students have a high school diploma and at least 3 years of high school math and the prerequisite of college algebra, they came to the program under-prepared. Not only did they not have the conceptual knowledge that one would expect of a teacher, they couldn't perform basic computations, and were unable to see that an answer did not make sense.

A short treatment like the one these students experienced may not be sufficient for all the students. One semester may not be enough to address the knowledge gaps that many prospective teachers bring with them. For instance, this treatment appeared to be deficient in proportional reasoning and decimal operations, as those are the areas of least improvement. However, even a short treatment can be valuable as demonstrated by the strong improvement noted in the estimation problems.

This is a positive start to developing a generation of teachers who understand the concepts, and who do not avoid mathematics. But this is only a start. One semester is clearly not enough. Many students still think in terms whole numbers only, and when unsure, tend to revert to mechanical methods. Students like Ben, need more exposure and reinforcement, or they will fall back into old habits. Methods curricula and professional development activities must be developed to continue this process of building conceptual knowledge, and to support them as they try to impart this conceptual knowledge in their classrooms.

Resources

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