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ABSTRACT

The purpose of this paper was to refine an existing cognitive framework designed to characterize middle school students' statistical thinking. A case- study analysis was used to focus on two sub-processes of statistical thinking that were not adequately represented in the framework: students' use of multiplicative reasoning in analyzing data, and categorizing and grouping data. Twelve students, 4 from each of grades 6-8, were interviewed using a protocol comprised of 4 tasks designed to assess students' thinking across 4 levels: idiosyncratic, transitional, quantitative and analytical. Based on an analysis of the interview data, descriptors were developed for each of the 4 levels of statistical thinking for both of the sub-processes. These sub-processes will be merged with Mooney's framework and the refined framework will be validated in a future study. (Author)

REFINING A FRAMEWORK ON MIDDLE SCHOOL STUDENTS' STATISTICAL THINKING

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Abstract: This purpose of this study was to refine an existing cognitive framework (Mooney, in press) designed to characterize middle school students' statistical thinking. A case-study analysis was used to focus on two subprocesses of statistical thinking that were not adequately represented in the framework: students' use of multiplicative reasoning in analyzing data, and categorizing and grouping data. Twelve students, 4 from each of grades 6 - 8, were interviewed using a protocol comprised of 4 tasks designed to assess students' thinking across 4 levels: idiosyncratic, transitional, quantitative, and analytical. Based on an analysis of the interview data, descriptors were developed for each of the 4 levels of statistical thinking for both of the sub-processes. These sub-processes will be merged with Mooney's framework and the refined framework will be validated in a future study.

It is widely acknowledged that proficiency in statistical skills enables people to become productive, participating citizens in an information society and to develop scientific and social inquiry skills (National Council for the Social Studies, 1994; National Council of Teachers of Mathematics [NCTM], 2000; Secretary's Commission on Achieving Necessary Skills [SCANS], 1991). Moreover, in preparation for the workforce, SCANS (1991) recommended that benchmarks be established to inform education at the secondary level. Thus, calls for reform in mathematics education have advocated a more pervasive approach to statistics instruction at all levels (NCTM, 2000). This focus on the development and implementation of statistical application highlights the need to examine the development of students' statistical understanding, especially at the middle school level.

Researchers like Cobb et al., (1991) and Resnick (1983) have identified the need for cognitive models of students' thinking to guide the planning and development of mathematics curriculum and instruction. There is evidence that research-based knowledge of students' thinking can assist teachers in providing meaningful instruction (Fennema & Franke, 1992). With this in mind, Mooney (in press) developed and validated the Middle School Students Statistical Thinking (M3ST) framework based on a synthesis of the literature and observations and analyses of students' thinking in interview settings. The M3ST framework incorporated four statistical processes: describing data, organizing and reducing data, representing data, and analyzing and interpreting data. For each of these processes, the framework included descriptors that characterize four levels of students' statistical thinking ranging from idiosyncratic to analytical reasoning (Biggs & Collis, 1991).

In validating the framework, Mooney (in press) found that two key aspects of students' statistical thinking were not adequately assessed by the interview tasks used in his study. For the process organizing and reducing data, tasks did not elicit students' thinking about ways to categorize and group data. And for the process analyzing and interpreting data, no task was effective in evaluating students' use of multiplicative reasoning; that is, reasoning about parts of the data set as proportions of the whole to describe the distribution of data or to compare data sets. These two subprocesses of statistical reasoning are considered important to the overall development of students' statistical thinking (Cobb, 1999; Curcio, 1987; NCTM, 2000). Addressing these gaps in the framework is especially important if the framework is to be used by teachers to inform statistics instruction.

Theoretical Perspective

In this research, statistical thinking is taken to mean the cognitive actions that students engage in during the data-handling processes of describing, organizing and reducing, representing, and analyzing and interpreting data (Reber, 1995; Shaughnessy, Garfield & Greer, 1996). Descriptions of these cognitive actions are based on the general developmental model of Biggs and Collis (1991). Their model incorporates five modes of functioning: sensormotor (from birth), ikonic (from around 18 months), concrete symbolic (from around 6 years), formal (from around 14 years), and post formal (from around 20 years). Within each mode, three cognitive levels (unistructural, multistructural, and relational) recycle and represent shifts in the complexity of students' reasoning. According to Biggs and Collis, each of the five modes of functioning emerges and develops in a way that incorporates the continuing development of earlier modes. Thus, they also recognize two other cognitive levels: the prestructural which is related to the previous mode and the extended abstract which is related to the next mode. We consider the ikonic, concrete symbolic, and formal modes to be most applicable to middle school students.

Following the Biggs and Collis (1991) model, Mooney (in press) hypothesized that students could exhibit five levels of statistical thinking: *idiosyncratic*, (associated with the prestuctural level and representing thinking in the ikonic mode), *transitional*, *quantitative* and *analytical* (associated respectively with the unistructural, multistructural and relational levels; representing thinking in the concrete symbolic mode) and *extended analytical* (associated with the extended abstract level; representing thinking in the formal mode). However, students in his study only demonstrated the first four levels of statistical thinking and, thus, his M3ST framework characterized students' thinking for the idiosyncratic through analytical levels. Since the purpose of this study was to refine the M3ST framework, four levels of descriptors were developed to describe students' thinking when using multiplicative reasoning and in categorizing and grouping data.

Method

Participants

Students in grades six through eight at a Midwestern school formed the population for this study. Twelve students, four from each grade level, were selected for case-study analysis based on levels of performance in mathematics: one high, two middle, and one low.

Procedure

Based on the M3ST framework (Mooney, in press) and drawing from the research literature (e.g., Cobb, 1999; Bright & Friel, 1998; Lehrer & Schauble, 2000), four levels of descriptors were initially developed for the subprocesses, categorizing and grouping data and multiplicative reasoning. An interview protocol was designed to assess the middle school students' thinking within these subprocesses. Using the protocol, each student was individually interviewed during a 60-minute session. All interviews were audio taped and all student-generated work was collected. The interviews were transcribed for analysis.

Instrument

The researcher-developed interview protocol comprised four tasks (see Figure 1); each with a series of follow-up questions designed to assess students' thinking across the four levels, idiosyncratic through analytical. Questions were designed so students could respond orally or by generating tables or data displays. Although aspects of various statistical processes may be involved in completing each of the tasks, tasks 1 and 2 were designed to focus on the ways students grouped and categorized data. Tasks 3 and 4 focused on students' use of multiplicative reasoning.

Data Sources and Analysis

Data sources consisted of the transcribed interviews, students' written work and data displays, researcher field notes, and summaries generated during the analysis. Following the methodology used by Mooney (in press) to generate the M3ST framework, a double-coding procedure (Miles & Huberman, 1994) was used to analyze students' responses. The first two authors independently coded each student's response for each of the four tasks. Responses were coded by levels based on: (a) the initial descriptors for each level of the two statistical subprocesses and (b) descriptors generated from the data analysis that characterized students' responses, yet were not present in the initial descriptors. This occurred in the following manner. After all students'

Task 1: Shoe Sizes

50 eighth-grade students were surveyed about their shoe size. This list shows the data collected. The same information is on these cards. Your job is to arrange the data to be presented in the school newspaper.

Task 2: Pet Store

The teachers at your school were asked what kinds pets they have at home. In all, the teachers had 39 pets. A list of these pets is shown on this page. The same information is on these cards. Your job is to arrange the data to be presented in the school newspaper.

Task 3: Oscar Winners

This table shows the ages of the last 30 winners for the Best Actor and Best Actress in a movie. The same information is on these cards. The editor of the school newspaper wants you to arrange the data to be presented in the school newspaper and to determine which of these 3 headlines should go with the data.

Task 4: Study Habits

Mrs. Jones talked to the students in her mathematics classes one day about an article she read. It said that children who listened to classical music while studying performed better on tests than children who did not listen to classical music while studying. Some of her students planned to listen to classical music while studying for the next math exam. The results of the 80-point test are listed on this table. The students who listened to classical music have an "X" marked next to their name. The cards have the same information as the table.

Your job is to arrange the data to see if students who listened to classical music while studying performed better on the math test than students who did not listen to classical music while studying. The editor of the school newspaper wants you to present the data along with a headline about the comparison.

Figure 1. Interview protocol tasks.

responses to a question were read, the first two authors compared the responses to the corresponding descriptors to describe the levels of students' statistical thinking. If descriptors did not adequately characterize students' responses, the responses were examined as a whole to discern patterns of thinking. These patterns were used to revise the corresponding descriptors and the students' responses were then recoded using the revised descriptors to characterize students' levels of statistical thinking. If few or no students demonstrated thinking at a particular level of a subprocess, we interpolated the descriptor for that level based on students' thinking at other levels. Throughout this process, differences in coding were discussed and agreement was negotiated.

Results

Based on students' responses to the four interview tasks, we were able to revise descriptors for the subprocesses, categorizing and grouping data and multiplicative reasoning (see Figures 2 and 3). Overall, minor revisions were made in the wording of descriptors for purposes of clarification. A discussion about the construction of initial descriptors, patterns of thinking discerned from students' responses, and the revisions of descriptors based on the patterns of thinking for each subprocess are presented in the following paragraphs.

Categorizing and Grouping

In the M3ST framework, the statistical process *Organizing and Reducing Data* involves arranging, categorizing, or consolidating data into a summative form. The components of this process are considered critical for analyzing and interpreting data. Mooney (in press) identified three subprocesses: (a) categorizing and grouping data, (b) describing data using measures of center, and (c) describing the spread of the data. In his study, students were reluctant to organize data by arranging them into categories and they frequently displayed data in no particular order or arrangement. Thus, descriptors for this subprocess warranted further examination.

The initial descriptors for the categorizing and grouping subprocess (see Figure 2) were a modification of the descriptors in the M3ST framework and focused on students' ability to group or order data in a summative form. We hypothesized that students displaying idiosyncratic thinking would make no attempt to group or order the data in a summative form while students displaying analytical thinking would be able to group or order the data in a representative, summative form including a non-summative characteristic of the data such as the mean or a percentage. The grouping or ordering would be considered representative if information obtained in the arrangement coincided with information obtained from the original data source.

In analyzing students' responses to tasks 1 and 2, two patterns emerged. First, some students created arrangements (e.g., lists, tables, or graphs) by simply shifting data values around. In these arrangements, no new information was presented. For example, the student response at the transitional level shown in Figure 2 displays a listing of shoe sizes in numerical order. Nowhere in this listing is the frequency of each shoe size presented, even though this information could be determined from the arrangement. The second pattern pertained to students' creation of new categories to organize or consolidate the data. As shown by the student response at the analytical level in Figure 2, the categories of "fish" and "birds" were created to display the number of teacher's pets. In the data presented in the task, the particular types of pets were listed (e.g., goldfish, canary, cocker spaniel, garter snake), but no categories such as birds or fish were mentioned.

Initial descriptors were revised (see Figure 2) to reflect these patterns. In interpreting the first pattern, we recognized that students could arrange data in a non-summative form yet in a way that was reasonable or meaningful. Therefore, the descriptor at the idiosyncratic level was changed to characterize students who do not attempt to group or order the data. The descriptor at the transitional level maintained that students group or order data but not in a summative form. The second pattern in students' responses indicated that the development of new categories, rather than the inclusion of non-summative information, distinguished thinking at the quantitative and analytical levels. The descriptor at the analytical level was modified to reflect this distinction. In examining students' responses, we realized that it was not necessary for an arrangement to be representative of the data to assess a students' thinking about grouping and categorizing data. Given that the issue of representativeness is addressed in the descriptors of other subprocesses in the M3ST framework, it seemed appropriate to delete it from the descriptors for categorizing and grouping data.

Multiplicative Reasoning

The *Analyzing and Interpreting Data* process in the M3ST framework does not include descriptors pertaining to multiplicative reasoning because Mooney (in press) concluded that the tasks used in his study did not adequately distinguish levels of students' thinking for this subprocess. Based on the data handling processes that typically occur at the middle school level, we operationally defined multiplicative reasoning as reasoning about parts of the data set as proportions of the whole to describe the distribution of data or to compare data sets. We also considered multiplicative reasoning in terms of relative thinking as opposed to additive thinking. Thus, our initial descriptors (see Figure 3) focused on how students used relative thinking to describe or compare data. We hypothesized that a student exhibiting idiosyncratic thinking would not use relative thinking to describe or compare data while a student exhibiting analytical thinking would describe or compare data quantitatively and draw or justify conclusions using relative thinking.

Two patterns emerged in the analysis of students' responses to tasks 3 and 4. First, all students drew conclusions about the data regardless of how they used relative thinking to describe or compare data. Second, when comparing data, some students used relative thinking with only part of the data. For example, in task 3, students were asked to compare the ages of actresses and actors to determine whether there was evidence

Categorizing and Grouping

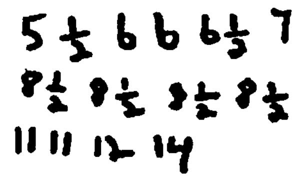
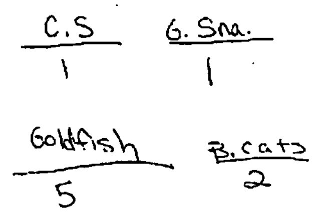
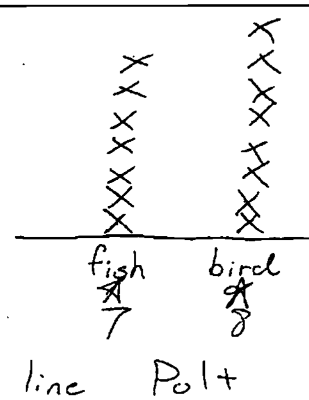
Levels	Descriptors	Sample Responses (Part of student work shown.)
Idiosyncratic	*Initial* Does not attempt to group or order the data in a summative form. *Revised* Does not attempt to group or order the data.	No responses at this level.
Transitional	*Initial* Groups or orders data but not in a summative form nor representative of the data. *Revised* Groups or orders data but not in a summative form.	
Quantitative	*Initial* Groups or orders data in a summative form, representative of the data. *Revised* Groups or orders data in a summative form.	
Analytical	*Initial* Groups or orders data in a representative summative from that includes at least one non-summative characteristic of the data. *Revised* Groups or orders data in a summative form by creating new categories or including categories not represented by data points.	

Figure 2. Categorizing and grouping descriptors (initial and revised) and sample responses.

of an age bias in Oscar winners. The student response in Figure 3 is indicative of thinking at the quantitative level. Here the student used multiplicative reasoning to conclude that for one-third of the years the actresses are older than the actors but made no explicit reference to the age comparisons in the other two-thirds of the years. In contrast, the student response for the analytical level indicates consideration all of the data when making the comparison. This student determined how many of the actresses were older than 40 (10/30) and how many of the actors were older than 40 (19/30) and concluded that the number of actors was greater than the number of actresses.

Our interpretation of these patterns led to changes in the descriptors for the quantitative and analytical levels. We concluded that it was students' use of only part of the data to make comparisons that distinguished thinking at these levels. Changes to the descriptor at the quantitative level were made to indicate that students at this level use relative thinking to make comparisons using only part of the data. At the analytical level, the descriptor was changed to indicate students' consideration of all of the data when making comparisons. Also, the reference to drawing and justifying conclusions was removed, based on the finding that all students made conclusions about the data.

Discussion

We sought to refine the Middle School Student Statistical Thinking framework (Mooney, in press) by developing descriptors for two subprocesses of statistical thinking that were not adequately addressed in the framework — categorizing and grouping data and multiplicative reasoning. The process of developing descriptors involved (a) creating initial descriptors, based on the M3ST framework and drawing from related research; and (b) refining these descriptors through the analysis of students' responses to an interview protocol designed to assess students' thinking across four levels — idiosyncratic, transitional, quantitative and analytical. For categorizing and grouping data, descriptors across levels characterized the summative forms of students' data arrangements and their use of categories to group data. However, further refinements to the descriptors for this subprocess might be necessary given that the tasks designed to assess thinking for this subprocess used only categorical data. The use of numerical data might reveal different patterns of thinking with regard to categorizing and grouping data. For multiplicative reasoning, descriptors characterized students' use of relative thinking and their consideration of the data set as a whole.

This study was one component of an extended research program that includes merging the descriptors developed in this study with the M3ST framework; validating the revised M3ST framework; and using the resulting framework with middle school teachers to guide instruction in statistics. As with Mooney's (in press) study, the descriptors we developed were based on the responses of only 12 students. Therefore, there is a need to validate the revised framework with a larger sample of students using a more comprehensive interview protocol. It is anticipated that the validation process will result in further refinement of the M3ST framework before it is used with teachers.

Categorizing and Grouping

Levels	Descriptors	Sample Responses
Idiosyncratic	*Initial* Does not use relative thinking to describe data or make comparisons. *Revised* Does not use relative thinking in situations that warrant it.	No responses at this level.
Transitional	*Initial* Uses relative thinking but does not use it to make quantitative descriptions of the data or comparisons. *Revised* Uses relative thinking but does not numerically describe data or make comparisons.	"Most of the male actors are middle age and most of the female actors are still young."
Quantitative	*Initial* Uses numerical relative thinking to describe or compare an event within data sets, but not across data sets. *Revised* Uses relative thinking to describe data or make comparisons, but considers only part of the data set.	"About in one-third of the years, the best actress is older than the best actor."
Analytical	*Initial* Describes or compares data quantitatively and draws or justifies conclusions using relative thinking. *Revised* Uses relative thinking, with the data set as a whole to describe data or make comparisons.	"Make a fraction out of it . . . 10/30 or 1/3 of the actresses are older [than 40] and 19/30 [of the actors] is . . . more than 1/3, that's older than 40."

Figure 3. Multiplicative reasoning descriptors (initial and revised) and student responses.

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