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AUTHOR	Verschaffel, Lieven
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ABSTRACT

This paper briefly reviews research grounded in cultural and socio-constructivist theories carried out to investigate the phenomenon of suspension of sense-making when doing school arithmetic word problems. It begins with ascertaining studies documenting and marking out the phenomenon, then provides an explanation of the observed effects that are found in the culture of the mathematics classroom. This explanation is followed by a review of some design experiments wherein researchers have tried to develop a new instructional approach aimed at the development in pupils of more appropriate conceptions about and strategies for doing word problems based on the modeling perspective. The paper discusses a number of educational implications of the research done so far as well as some challenges for the future of teaching mathematical applications and modeling. (KHR)

TAKING THE MODELING PERSPECTIVE SERIOUSLY AT THE ELEMENTARY SCHOOL LEVEL: PROMISES AND PITFALLS[1]

Lieven Verschaffel

University of Leuven, Belgium

WORD PROBLEMS AS MODELING EXERCISES IN HISTORY AND TODAY

Mathematics provides a set of tools for describing, analyzing and predicting the behaviour of systems in different domains of the real world (Burkhardt, 1994). This practical usefulness of mathematics for understanding the world around us, for coping with everyday problems, and for future professions, has always provided, and still provides, a major justification for the important role of mathematics in the (elementary) school curriculum (Blum & Niss, 1991). In particular, the inclusion of application and modeling problems was mainly intended to develop in students the skills of knowing when and how to apply their mathematics effectively in various kinds of problem situations encountered in everyday life and at work.

The application of mathematics to solve problem situations in the real world, otherwise termed mathematical modeling, can be usefully thought of as a complex process involving a number of phases. There are many different descriptions of this model (e.g., Burkhardt, 1994; Mason, 2001; Blum & Niss, 1991; Verschaffel, Greer & De Corte, 2000), but, in essence, they all involve basically the following components: understanding and defining the problem situation leading to a situational model; constructing a mathematical model of (or: mathematizing) the relevant elements, relations and conditions embedded in the situation; working through the mathematical model using disciplinary methods to derive some mathematical results(s); interpreting the outcome of the computational work in relation to the original problem situation; validating or evaluating the model(ing process) by checking if the interpreted mathematical outcome is appropriate and reasonable for his or her purpose; and, stating and communicating the obtained solution of the original real-world problem.

Evidently, this modeling process can not be described as a linear activity; it has to be considered cyclic (Blum & Niss, 1991; Burkhardt, 1994; Mason, 1997). Moreover, modeling is not a straightforward activity. In modeling, not all aspects of the reality can, nor should, be modeled. The modeler tries to capture the essentials of the situation in the model, but what is considered as essential, and, thus, taken into account in the mathematical model, is not fixed, but relative to the modeler and to the context wherein (s)he is confronted with the modeling task at hand. Ikeda and

Stephens (2001) point to the pivotal task for the modeler of seeking a proper balance between over-complexity and over-simplification, taking into account the goals of the modeling task, certain personal and contextual constraints, etc. Finally, while modeling is often performed in order to answer one or more well-defined questions, this is not necessarily always the case. Modeling also occurs in situations where no well-defined question(s) has been, or will be, posed, but where the goal is to grasp, understand, make sense of, represent, explain, predict, and so forth, a situation or a phenomenon (Niss, personal communication).

Historically, one major way of teaching modeling process is through word problems, i.e. verbal descriptions of problem situations, typically presented in a school context, wherein a question is raised the answer to which can be found by performing (a) mathematical operation(s) on the numbers in the problem (Verschaffel et al., 2000).

An analysis of the very long and worldwide history of word problems reveals that word problems have been included, and are still being used, with the ostensible aim of accomplishing several goals (Kilpatrick, 1985; Niss & Blum, 1991; Verschaffel et al., 2000). I will focus on their oldest, and probably most important goal, namely to offer practice for the situations of everyday life in which mathematics learners will need what they have learned in school. The (implicit) idea behind this goal is to bring reality into the mathematics classroom, to create occasions for learning and practicing the different aspects of applied problem solving, without the practical (organisational, financial...) inconveniencies of direct contact with the real-world situation evoked by the problem statement. By means of such "best alternatives" for the real-world situations outside the classroom, students become prepared for the mathematical requirements they would face in their (future) everyday lives.

For a very long time, word problems have played this application function without much reflection and critical concern. Of course, there have always been individuals showing (some) awareness of the bridging problem between reality and mathematics, and the risks involved (Lewis Carroll is a marvelous example), but many teachers, textbook writers and researchers, have been using, and still use nowadays, word problems as if there was no bridging problem at all.

During the last 10-15 years, it has been argued by many scholars from different disciplines, such as mathematics education, psychology, linguistics, and anthropology, that the current practice of word problems in school mathematics does not at all foster in students a genuine disposition towards mathematical modeling. First, in-depth linguistic analyses of word problems as a text genre have led to serious questionings of the "unproblematic acceptance of concepts of separable mathematical and real worlds and of word problems as a transparent bridge between the two" (Gerovsky, 1997, p. 22). A second relevant line of research comprises studies on "everyday cognition" revealing remarkable discrepancies and difficulties of transfer between applied mathematics in contexts in and out of school (Carraher, Carraher & Schliemann, 1985). Third, empirical studies, mostly grounded in sociocultural and socio-constructivist theories, have shown that after several years of schooling many students have constructed an approach to mathematical application problems whereby this activity is reduced to the execution of one or more arithmetic operations with the numbers in the problem, without any serious consideration of possible constraints of the realities of the problem context that may jeopardize the appropriateness of their standard models and solutions (Boaler, 1994; Lave, 1992; Nesher, 1980; Reusser & Stebler, 1997; Schoenfeld, 1991; Verschaffel et al., 2000). Altogether, these three related lines of research have led to a scepticism about word problems as a vehicle for promoting the development of students' disposition towards authentic mathematical modeling.

I will continue with a brief review and discussion of this third line of research carried out to investigate this phenomenon of "suspension of sense-making" (Schoenfeld, 1991) when doing school arithmetic word problems, beginning with ascertaining studies documenting and marking out the phenomenon, and then wending my way to an explanation of the observed effects, which will be found in the culture of the mathematics classroom. This explanation will be followed by a sketchy review of some design experiments wherein researchers have tried to develop a new instructional approach aimed at the development in pupils of more appropriate conceptions about, and strategies for, doing word problems, based on the modeling perspective. Finally, I will discuss a number of educational implications of the research done so far as well as some challenges for the future of teaching mathematical applications and modeling.

EVIDENCE OF LACK OF SENSE-MAKING AMONG STUDENTS

The most spectacular, and probably also the most quoted, case of "suspension of sense-making" is that of the French and German researchers (Institut de Recherche sur l'Enseignement des Mathématiques de Grenoble, 1980; Radatz, 1983) who posed non-sensical problems such as "There are 26 sheep and 10 goats on a ship. How old is the captain?" and found that many elementary school children supplied answers produced by arithmetical operations on the numbers in the text without expressing any concern about the appropriateness or meaningfulness of their computation-based answers on these absurd problems.

Another famous example is the buses item "An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?", that was used for the first time in the Third National Assessment of Educational Progress in the US (Carpenter, Lindquist, Matthews, & Silver, 1983) with 13-years-old students, and that elicited a remarkably large number of non-whole number answers ("31.3 buses") and answers wherein the outcome of the division was rounded to its nearest whole-number predecessor (rather than its contextually much more appropriate successor).

Inspired by these striking examples of suspension of sense-making in school mathematics (as well as some other examples), Greer (1993) and Verschaffel, De Corte and Lasure (1994) carried out pencil-and-paper studies with upper primary and lower secondary school students, using a set of problems including those listed below:

- Steve has bought 4 planks each 2.5 meters long. How many planks 1 meter long can he saw from these planks?
- A man wants to have a rope long enough to stretch between two poles 12 meters apart, but he only has pieces of rope 1.5 meters long. How many of these would he need to tie together to stretch between the poles?
- John's best time to run 100 meters is 17 seconds. How long will it take him to run 1 kilometer?

They termed each of these items "problematic" (P) in the sense that they require (from our point of view) the application of judgment based on real-world knowledge and assumptions rather than the routine application of one or more simple arithmetical operations. Each such P-item was paired with an S-item (for "standard") in which the "obvious" calculation is (we would argue) appropriate. For each item, the students, as well as recording an answer, were invited to comment on the problem and their response. A response to a P-item was classified as a "realistic reaction" if *either* the answer given indicated that realistic considerations had been taken into account *or* if a comment indicated that the student was aware that the problem was not straightforward. For example, a classification "realistic reaction" for the planks P-item would be given to a student who gave the answer "8" or who made a comment such as "Steve would have a hard time putting together the remaining pieces of 0.5 meters". In both studies, students demonstrated a very strong overall tendency to exclude real-world knowledge and realistic considerations when confronted with the problematic versions of the problem pairs. For instance, in Verschaffel et al.'s (1994) study, only 17% of all reactions to the P-items could be considered as realistic.

These initial studies were replicated in several other countries (e.g., Chili, China, Germany, Japan, Switzerland, The Netherlands, Venezuela), using similar methodologies and, to a considerable extent, the same items. The findings were strikingly consistent across many countries (see Verschaffel et al., 2000). Using the same criteria as in the two initial studies, none of the P-items was answered in a realistic fashion by more than a small percentage of students (except for two problems; see below), sometimes to the great surprise and disappointment of these other researcher(s) who had anticipated that the "disastrous" picture of the Irish and Flemish pupils would not apply to their students.

While the results were remarkably consistent across nationalities, there is increasing evidence, both from these replication studies and from several other studies with a somewhat different scope and methodology, that the tendency to respond to school arithmetic word problems in a stereotyped and non-realistic way is related to various kinds of task, subject and context characteristics.

With respect to *task variables*, P-items about the interpretation of a division with a remainder (i.e., the buses item and a similar item about sharing balloons) elicited considerably more realistic answers than the other kinds of P-items in the problem set in every study mentioned above.

With respect to *subject variables*, research evidence suggests that students' tendency to ignore plausibly relevant and familiar aspects of reality in answering word problems, is associated with age, gender, and social class. Children with less years of experience with (traditional) schooling (Radatz, 1983; Yeping & Silver, 2000), girls (Boaler, 1994) and working-class children (Cooper & Dunne, 1998) seem more likely to remain within an "everyday" frame of reference when doing application problems in a school context, leading to less appropriate answers if scored from a traditional point of view.

Finally, several follow-up studies have tested the effectiveness of variations in the *experimental setting*. A first group of studies assessed the effectiveness of making students more alert to consider aspects of reality and to legitimize alternative forms of answer produce. For instance, Yoshida, Verschaffel and De Corte (1997) gave half of the pupils an explicit warning at the top of their test sheet (a translation of Verschaffel et al.'s (1994) test) that some of the problems in the test were problematic, and invited them explicitly to write down and explain these unclarities or complexities. This manipulation did not result in a significant increase in the number of realistic reactions to the P-items in the test. In other studies too, this kind of manipulation provided, at best, only very weak effects (see Verschaffel et al., 2000). In a second set of studies, one or more categories of P-items were presented in a more authentic, performance-based setting. For instance, DeFranco and Curcio (1997) confronted sixth-graders with a version of the buses item in the context of a typical paper-and-pencil math test and, afterwards, asked them to make a telephone call to order minivans to take sixth-graders to a class party. The more authentic setting elicited a much greater percentage of appropriate responses than the restrictive school setting. In contrast to the studies about the effect of alerting pupils mentioned earlier, increasing the authenticity of the experimental setting, as done in the study of DeFranco and Curcio (1997) but also in other related studies (see Verschaffel et al., 2000), yielded much greater improvements in students' inclination to include the real-world knowledge they were so reluctant to activate and use under the previous, more restricted, testing conditions.

WHAT STUDENTS SAY AND THINK ABOUT WORD PROBLEMS

Besides deriving how students think about word problems from their responses to word problems in a paper-and-pencil test, it is, of course, possible to ask them directly.

Interviews carried out by Caldwell (1995) and Hidalgo (1997) suggest that, while unfamiliarity with the contexts involved in the problems and lack of appropriate heuristic and metacognitive skills may provide contributory explanations, (mis)beliefs about school arithmetic word problems constitute the major reason why so many students solve the P-items in a non-realistic way. For instance, a 10-year-old interviewed by Caldwell (1995, p. 39) commented as follows in response to the interviewer's question as to why she had answered a P-item in a non-realistic way: "I know all these things, but I would never think to include them in a math problem. Math isn't about things like that. It's about getting sums right and you don't need to know outside things to get sums right".

In a recent pilot study by Inoue (2001), upper elementary school children failed to give realistic answers to problems similar to the P-items from Verschaffel et al.'s (1994) test, but in the later clinical interview these answers were found not to be so unrealistic if they were judged based on the children's idiosyncratic interpretations of the problem situation. For instance, a student who had ignored the fatigue factor in solving a problem about finding how long it takes to finish a data-entry job based on the given rate (a problem very similar to the running item mentioned above), said that she ignored that fatigue factor since the calculational answer gives us the baseline information to judge how long it takes "theoretically". Her point was that such information is often useful for managing people calculating salary etc. in real life practices. Almost half of non-realistic answers given during the paper-and-pencil test were found to be of such type. This kind of "sensible unrealistic answers" (Inoue, 2001, p. 31) reminds also of Selter's (1994) findings with the infamous how-old-is-the-captain problem, wherein some students came up with highly ingenious explanations for their answers. For example, having responded to "There are 20 sheep and 5 goats on a boat. How old is the captain?" with the answer 25, one child suggested that may-be the captain's parents gave him an animal on each birthday so he would always know how old he is. Unfortunately, it is extremely difficult to distinguish solutions based on idiosyncratic interpretations that took place *during* problem solving from *post-hoc* rationalizations in defense of that response.

LOOKING FOR AN EXPLANATION: GOING BEYOND THE COGNITIVE

The results from the above-mentioned studies suggest that it is not a cognitive deficit as such that causes students' general and strong abstention from sense-making when doing arithmetic word problems in a typical school setting, but rather that they are acting in accordance with the "rules of the game" of the interactive ritual in which they are involved, or, as others would call it, in accordance with the "didactical contract" (Brousseau, 1997), or the "socio-mathematical norms" (Yackel & Cobb, 1996).

Several authors (e.g. De Corte & Verschaffel, 1985; Gerofsky, 1996; Kilpatrick, 1985; Lave, 1992; Reusser & Stebler, 1997) have carried out analyses of the hidden rules that seem to be used (implicitly, tacitly) by elementary school pupils (and by their teachers!) to make the "game of word problems" function efficiently, and have come up with rules such as the following:

- Any problem presented by the teacher or in a textbook is solvable and makes sense.
- There is a single, correct, and precise numerical answer.
- This single answer must be obtained by performing one or more mathematical operations with the numbers embedded in the text.
- The task can be achieved by applying familiar mathematical procedures.
- The text contains all the information needed and no extraneous information may be sought.
- Violations of your knowledge about the everyday world should be ignored.

De Corte and Verschaffel (1985) introduced the term "word problem schema" to refer to the system of beliefs about word problems shared by students and teachers, involving the perceived intent of word problems, interpretation of stereotyped semantic structure, and a complex network of implicit rules and expectations that govern playing of the "word problem game".

The above interpretation of students' non-realistic responses to word problems like the how-old-is-the-captain problem or the P-items from the test of Greer (1993) and Verschaffel et al. (1994), is in line with Schoenfeld's (1991, p. 340) suggestion that the children who produced such bizarre responses were not irrational but were engaged in sense-making of a different kind: "Taking the stance of the Western Rationalist in mathematics, I characterized student behavior... as a violation of sense-making. As I have admonished, however, such behavior is sense making of the deepest kind. In the context of schooling, such behavior represents the construction of a set of behaviors that result in praise for good performance, minimal conflict, fitting in socially, etc. What could be more sensible than that? The problem, then, is that the same behavior that is sensible in one context (schooling as an institution) may violate the protocols of sense-making in another (the culture of mathematics and mathematicians)". At a more general level, these children's seemingly meaningless behavior can also be interpreted in terms of Vinner's (1997, p. 97) particularly rich and relevant notion of "pseudo-analytical thought processes" in mathematics learning, wherein the person is looking for a satisfactory reaction to a certain stimulus and the thought process is guided by uncontrolled associations and superficial similarities, without any serious cognitive involvement.

THE CLASSROOM CULTURE AS EXPLANATORY FACTOR

This brings us to the question: How do these superficial strategies for and beliefs about the solution of school arithmetic word problem develop? As with most other "pseudo-analytical thought processes" (Vinner, 1997), the development of students' tactics for and conceptions about word problem solving is assumed to occur implicitly, gradually, and tacitly through being immersed in the culture of the mathematics classroom in which they engage. Putting it another way, students' strategies and beliefs develop from their perceptions and interpretations of the didactical contract (Brousseau, 1997) or the socio-mathematical norms (Yackel & Cobb, 1996) that determine(s) (explicitly to some extent, but mainly implicitly) how to behave in a mathematics class, how to think, how to communicate with the teacher, and so on. More specifically, this enculturation seems to be mainly caused by two aspects of current instructional practice, namely (1) the nature of the problems given and (2) the way in which these problems are conceived and treated by teachers.

Let's first have a look at the first of these two explanatory factors. In an attempt to summarize the characteristics of traditional word problems that appear in classrooms and textbooks and which lie at the basis of students' beliefs about and strategies for solving word problems as discussed above, Reusser and Stebler (1997, p. 323) wrote: "Only a few problems that are employed in classrooms and textbooks invite or challenge students to activate and use their everyday knowledge and experience. Most word problems used in mathematics instruction are phrased as semantically impoverished, verbal vignettes. Students not only know from their school mathematical experience that all problems are undoubtedly solvable, but also that everything numerical included in a problem is relevant to its solution, and everything that is relevant is included in the problem text. Following this authoring script, many problem statements degenerate to badly disguised equations." If the vast majority of the textbook and test problems have these characteristics, it should not be a surprise that many students develop gradually but inevitably strategies for and beliefs about word problem solving that are characterized by a lack of sense-making.

A second plausible explanatory factor for the development of the observed student beliefs about and tactics for word problem solving is the way in which these problems are conceived and actually treated by teachers in the mathematics lessons. A study that sheds some light on this second factor is an investigation by Verschaffel, De Corte and Borghart (1997), wherein (future) elementary school teachers were asked, first, to solve a set of P-items themselves, and, second, to evaluate four alternative answers from (imaginary) pupils to the same set of P-items as "absolutely correct answer, "partly correct and partly incorrect answer", or "completely incorrect answer". For each P-item, the four response alternatives always included the typical non-realistic answer and the most reasonable realistic answer. Only half of the student-teachers' own answers to the P-items in test 1 were scored as realistic, and, with respect to test 2, their evaluation of the non-realistic pupil answers to the P-items was considerably more positive than for the realistic answers based on realistic considerations!

In sum, the available evidence suggests that students' beliefs about and tactics for word problem solving do not develop as a result of direct teaching, but rather emerge from the nature of the textbook and test problems with which they are confronted and from the permanent interaction between teacher and students around these problems.

TAKING THE MODELING PERSPECTIVE SERIOUSLY

Based on the results of the above-mentioned theoretical and empirical work, several authors have put forward suggestions for improving the quality of the applied part of the mathematics education curriculum.

A minimal and rather easily achievable goal is to improve the quality of word problems as applications in numerous ways that have been suggested over many years such as:

- Break up the expectation that any word problem can be solved by adding, subtracting, multiplying or dividing, or by a simple combination thereof.
- Eliminate the flaws in textbooks that allow superficial solution strategies to be undeservedly successful.
- Vary problems so that it cannot be assumed that all the data included in the problem, and only those data, are required for solution.
- Weed out word problems in which the numbers do not correspond to real life.
- Accept forms of answer other than exact numerical answers.

However, on top of such a list of recommendations, which together constitute a minimal response to the identified flaws in traditional teaching of word problems, we propose in our book (Verschaffel et al., 2000) a more radical solution, namely to reconceptualize word problems *as genuine exercises in mathematical modeling*. In contrast to the truncated caricature of the multiphased and multidimensional model of mathematical modeling that underlies many traditional lessons in applied problem solving, at the basis of our approach is a much more elaborated version of the model of solving application problems, wherein all phases of the genuine modeling process are equally important and wherein

- knowledge about the phenomenon is not suppressed but considered as a valuable component in the initial stage of the solution process,

- the nature and the outcome of the mathematization act is influenced by the goals implicit in the situation, imposed by the teacher, or negotiated,
- the solver can make use of a rich variety of resources (including software modeling tools) in the stages of mathematical modeling and analysis,
- the interpretation and evaluation phase involve comparison and discussion of alternative models, and
- the task requirements may involve a communication phase that goes far beyond the bald reporting of the result of the calculation.

Starting from the above criticisms on the traditional practice surrounding word problems in schools and from the modeling perspective described above, researchers have set up design studies wherein they developed, implemented and evaluated experimental programs aimed at the enhancement of students' mathematical modeling and problems solving along the lines mentioned above. To mention just a few: several developmental research projects from the Freudenthal Institute in The Netherlands (see e.g., Gravemeijer, 1997), the Jasper studies of the Cognition and Technology Group at Vanderbilt (1997), wherein mathematical problem solving is anchored in realistic contexts using new information technologies, Lehrer and Schauble's (in press) experimental curriculum for mathematics and science teaching in young children built upon the modeling approach, and our own study about the design and evaluation of a learning environment for mathematical modeling and problem solving in upper elementary school children (Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts & Ratinckx, 1999).

Characteristics common to these experimental programs include:

- The use of more realistic and challenging tasks than traditional textbook problems, which do involve some, if not most, of the complexities of real modeling tasks (such as the necessity to formulate the problem, to seek and apply aspects of the real context to proceed, to select tools to be used, to discuss alternative hypotheses and rival models, to decide upon the level of precision, to interpret and evaluate the outcome, etc.).
- A variety of teaching methods and learner activities, including expert modeling of the strategic aspects of the modeling process, small-group work, and whole-class discussions; typically, the focus is not on presenting and rehearsing established mathematical models, but rather on demonstrating, experiencing, articulating, and discussing what modeling is all about (see also Mason, 2001).
- The creation of a classroom climate that is conducive to the development of the elaborated view of mathematical modeling and of the accompanying beliefs.

In most of these design experiments positive outcomes have been obtained in terms of performance, underlying processes, and motivational and affective aspects of learning. After reviewing the available research evidence, Niss (2001, p. 8) concludes that "application and modeling capability can be learnt, and according to the above-mentioned findings has to be learnt, but at a cost, in terms of effort, complexity of task, time consumption, and reduction of syllabus in the traditional sense".

To some extent, these characteristics of the modeling approach are beginning to be implemented in mathematical frameworks, and tests in many countries. However, according to Niss (2001), it is still the case, in general international terms, that genuine and extensive applications and modeling perspectives and activities continue to be scarce in the everyday practice of mathematical education.

PROMISES AND PITFALLS OF THE MODELING PERSPECTIVE

When putting the modeling perspective forward for serious consideration, it must be recognized that it is not without major difficulties and challenges.

A first critical issue is: how far can and should we go in our efforts to make the modeling tasks realistic? How much reference to the complexity of reality is possible and appropriate in the classroom context? I agree with Gravemeijer (1997) that there is, and always will be, an insurmountable difference between solving problems in the out-of-school reality and solving word problems in a mathematics lesson or test. But if we accept that there will always remain some gap between mathematical modeling in a school and an out-of-school context, what is the appropriate level of "reference to the real" that should be established in the mathematics classroom? And is encouraging students to use their real-world knowledge not opening a Pandora's box? I don't think that there is one appropriate level of realism nor that the non-existence of such a clearcut level forms an impassable didactical problem. It only suggests that the question about the model's degree of abstraction and precision may be regarded, not as a difficulty, but as a part of what we want students to learn to make deliberate judgments about, as one crucial aspect of a disposition towards realistic mathematical modeling. Such difficulties with respect to the level of realism and precision are most serious, I believe, when word problems are presented in a context that precludes discussion, such as a student working alone on a textbook problem, or sitting a written test. Within the context of discussion and collaboration, the degree of precision, the reasonableness of plausible assumptions, and so on, may be negotiated (Verschaffel et al., 2000).

Second: does the modeling perspective exclude traditional word problems? I would contend that it does not rule out mathematical teaching and learning activities around classical word problems wherein students learn to apply powerful schemes for identifying, understanding, and solving certain categories of problems. For example, the schemes of addition, multiplication, direct proportionality, etc. are very powerful, and applicable to a wide variety of situations. So, it is important that students (also) learn to master these schemes and to apply them in various contexts during their mathematics lessons. However, the problem arises when such a schema is automatically triggered by superfluous cues, i.e., when students are given no training in discriminating between those cases where it is appropriate and those in which it is inappropriate or, in the extreme sense, nonsensical (Hatano, 1997). So, there should be room for different kinds of word problems with distinct instructional goals. At one time they may be used mainly to create strong links between mathematical operations and prototypically "clean" model situations (with little room for timeless discussions about the situational complexities that might jeopardize this link), whereas at other times they may be used primarily as exercises in relating real-world situations to mathematical models and in reflecting upon that complex relationship between reality and mathematics[2]. Both roles are important in a balanced math curriculum, but they have quite different characteristics and priorities, which must be clear to the student as well as the teacher (Burkhardt, 1994). In this respect, Galbraith and Stillman (2001, p. 301) propose an interesting problem classification, consisting of four categories, with essential differences in terms of the kind of thinking processes they elicit in students, as well as in terms of their underlying assumptions about word problems and their relation to solving problems in the real world:

- injudicious problems, wherein realistic constraints are seriously violated;
- context-separable problems, wherein the context plays no real role in the solution and can be stripped away to expose a purely mathematical question;
- standard application problems, where the necessary mathematics is context-related and the situation is realistic, but where the procedure is (still) rather standard;
- genuine modeling problems, in which no mathematics as such appears in the problem statement, and where the demarcation and formulation of the problem, in mathematical terms, must be (at least partly) supplied by the modeler.

Whereas, I would agree with these authors, injudicious problems are to be avoided, because they strongly reinforce the belief that mathematics has nothing to do with the real world (expect if they are purposely used in the context of making students, or in-service or pre-service teachers, aware of this problematic relationship!), context-separable and standard application problems do have a meaningful role in mathematics education. However, when using them, one should be cautious for the risk that students will tend to overgeneralize the validity and relevance of familiar modeling notions (e.g. addition, multiplication, proportionality) and transfer these to settings to which they are neither relevant nor valid (Niss, 1991), as argued by Hatano (1997) and as amply evidenced, for instance, in a study of De Bock, Verschaffel and Janssens, (1998) for proportionality. Moreover, we should not labor with the latter two types of problems under the illusion that such problems will foster the ability to apply mathematics to solve realistic problems. Only in the fourth type of Galbraith and Stillman's (2001) categorization do we find the need to invoke assumptions integrating mathematical development with the real context as a key to progress.

A final important issue I want to address is whether teaching and learning mathematical applications according to the modeling perspective is important, and feasible for, all students. Over the past few years several authors, such as Keitel (1989) and Mukhopadhyay and Greer (2001), have made strong pleas for engaging *all* students in the modeling perspective, both for the empowerment of the individual and for the betterment of society. For Mukhopadhyay and Greer (2001), this "political aspect" can be considered as a third perspective from which mathematics education in general, and teaching and learning mathematical modeling in particular, should be analyzed critically, besides the (purely) cognitive and the social/cultural perspectives. In relation to this political aspect, the most important reason for introducing the modeling perspective to all students is to help as many people as possible "to become critical thinkers who can use mathematics as a tool for analyzing social and political issues, and can reflect on that tool use, including its limitations" (Mukhopadhyay & Greer, 2001, p. 310). Evidently, once mathematics educators start applying this modeling perspective on a larger scale and allow students to bring in their personal experience when trying to make sense of all kinds of technical, social and cultural issues and phenomena, they will be confronted quickly and inevitably with the diversity of these experiences in terms of gender, social class, and ethnic diversity (see also, Boaler, 1994; Cooper & Dunne, 1998; Tate, 1994). I endorse Mukhopadhyay and Greer's (2001) claim that engaging students in such modeling activities, with careful attention to the relevance of the problem contexts and all the diversity in views and approaches that they elicit, is the best way to prevent students from becoming alienated by mathematics and its authority, and to help them using mathematics as a powerful personal tool for the analysis of issues important in their personal lives and in society. Given the multiphased, multidimensional and non-straightforward nature of the modeling process, it is often viewed as an activity that is only within the reach of older and/or more capable students. The evidence from several recent design experiments reviewed in this paper suggests that it is not only important, but also feasible, to start applying the modeling perspective successfully in mathematics education of all students already from a (very) young age on and with a diversity of learners.

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[1] For this paper I relied partly on a book that I recently wrote together with B. Greer and E. De Corte on *Making sense of word problems*. Furthermore, I thank D. De Bock, B. Greer, M. Niss and W. Van Dooren for their valuable comments on an earlier draft of this paper.

[2] Besides these two major functions, word problems can play other roles too, like in "mathematical puzzles" that are used to train or test people's intelligence or mathematical ability (Verschaffel et al, 2000).

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