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ABSTRACT

The increasing availability of computer algebra systems (CAS) for the high school classroom is expected to have a significant effect on how algebra is taught, when it is taught, and the expected learning types. This paper reports on a computer algebra approach to the teaching of algebra to 13-16 year old students in a high school in Melbourne, Australia. With the aid of the TI-92 graphing calculator the students were led through an exploratory approach to algebra that also included all the traditional algebra experiences that students of that age would normally encounter in their mathematics course. Generalized arithmetic and a functional approach were used to introduce algebraic concepts to the students. Throughout the classroom investigation, students demonstrated a willingness to use the TI-92 and the ability to quickly adapt to this new learning tool. The use of the TI-92 within the structured learning environment in which the students were involved appeared to provide them with positive learning experiences in algebra. (KHR)

Computer Algebra Systems in the Junior High School.

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Introduction

The increasing availability of computer algebra systems for the high school classroom will have a significant effect on how algebra is taught, when it is taught and the type of learning that is expected. Approaches to the introduction of algebra may be categorized according to Bednarz (1996) under four headings.

1. generalized arithmetic
2. problem solving
3. mathematical modelling
4. functional approach

Heid, Choate, Sheets, & Zbiek, (1995) point out that "Technology allows students to study algebra as the meaningful and related representations of functions, variables and relations rather than as the acquisition of skills in manipulating symbolic representations stripped of their meaning" (p.1). A Computer Algebra System (CAS), such as that available on the Texas TI 92, can enhance and compliment the approaches listed by Bednarz, by allowing the students to focus more on variables and functions and less on manipulation of variables. While the portability of the TI 92 mathematical computer allows students immediate access to a powerful algebraic manipulator where ever they may be.

During 1997, a computer algebra approach to the teaching of algebra formed the basis of work completed with 13 to 16 year old students in a high school in Melbourne, Australia. With the aid of the TI 92 the students were led through an exploratory approach to algebra which also included all the traditional algebra experiences that students of this age would normally meet in their mathematics course; in particular factorization, expansion and graphical representation involving linear, quadratic and cubic functions.

Classroom examples

The approaches of generalised arithmetic, a functional approach were used to introduce algebraic concepts to the student. The algebraic concepts were supported by students' access to the TI92 at all times during the class.

Algebra as generalized arithmetic.

Students were asked to investigate the results of the sequence of consecutive differences of reciprocals (after Ruopp, Cuoco, Rasala, & Kelemanik, 1997) and to record their results. (see Figure 1.)

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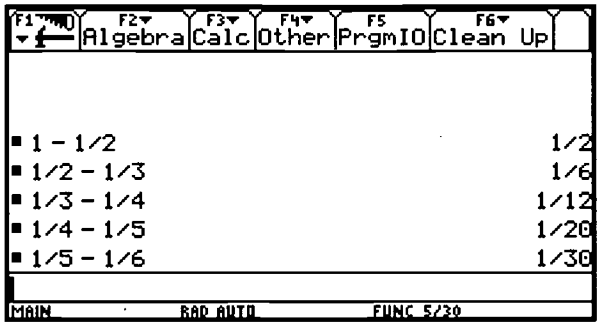

Figure 1. TI92 screen showing a series of differences of consecutive integer reciprocals.

Once the students had determined a pattern for the results, they were then expected to develop a rule that would satisfy all questions of this type. Students' written responses to the task included

> "You will receive your answer if you multiply the two denominators together"
> "The two denominators when multiplied together equal the denominator in the final result"
> "If you times the two numbers together you get the answer. With one on the top."

In each of these case there is recognition by the students that the solution was a result of the multiplication of the denominators. The key issue though is whether students could then develop a rule; using the standard mathematical operations of fractions.

The TI 92 or any CAS lends itself well to the generalisation of such problems, and enables the students to use variables to produce a rule that holds for all integer values. Equally importantly, the students were also expected to justify their result by defining the range of values their rule holds. In the beginning the students could see that they needed to represent each of the denominators by a variable, often their first choice was to use *1/a* and *1/b* to represent the reciprocals; that is,

$$\frac{1}{a} - \frac{1}{b} = \frac{b-a}{ab}$$

This method was clearly not helpful, because the students saw that the solution did not provide them with a generalised form of their numerical solution. When considering their numerical solution, many of the students then recognised that by using only one variable they would be able to develop a generalised solution. Their comments included;

> "Let x = be variable so the next consecutive variable is $x + 1$"
> "You need to get a single variable "

Figure 2 outlines a strategy that the students used to show that the denominator of the solution was equal to the product of the differences (most used x as the variable to represent the denominator).

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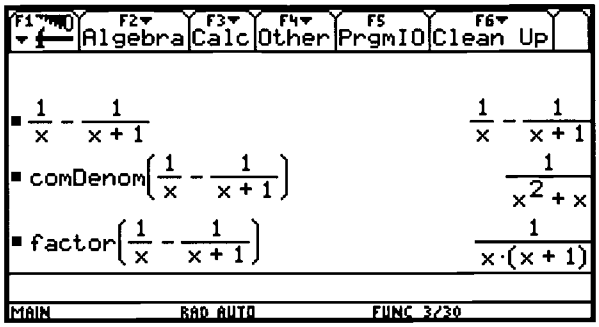

Figure 2. A rule that indicates the difference of consecutive integer reciprocals is equal to the product of the two consecutive integer reciprocals.

Important steps in the students' development of their rules included
a) recognising that the consecutive denominators could be written in terms of one variable.
b) using their rules for operations on fractions to algebraic fractions as well as using the factor command to determine the factors of an expression.

The significance of students being able to represent the equations in terms of variables should not be underestimated. Brenner et al., (1997) suggest that "The lack of expertise in problem representation creates difficulties as students attempt to make the transition from arithmetic to algebra" (p.664). That is the students will need to develop skills in selecting appropriate algebraic representations if they are to be successful solving problems requiring a generalised approach. The use of questions such as the difference of consecutive reciprocals can begin to support this development of the students understanding of variables and the correct selection of such variables.

Factorisation, x intercepts and sketching graphs.
A considerable amount of our time in mathematics classes has been spent on factorising and then solving to sketch a particular function. However with the availability of a CAS, the algebra can become the "language of representation" Heid et al., (1995), that is, the proficiency of manipulation of algebraic symbols is no longer the focus of school algebra. But as (Heid et al., 1995) point out:

> The study of algebra in schools must focus on helping students describe and explain the world around them rather than on developing and refining their execution of by hand symbolic manipulation procedures that are better accomplished through the use of computing tools. (p.143).

In this investigation the group of students were taken through a process of investigating factorisation with the support of the TI 92. The focus was on comparing the factorised form with the standard quadratic trinomial and their graphical representation with the intention of supporting the students' conceptual development in the area of factorisation and curve sketching. Because these students still needed to conform to the traditional assessment criteria for the school as well as state assessment policy, the first activity that students undertook was to complete a series of factorisations using the Factor command and then explain their results.(see Figure 3).

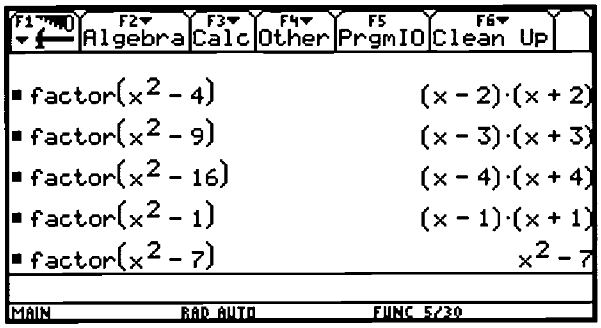

Figure 3. A series of factorisations that the students were expected to complete and explain.

Student's comments and explanations included:

> "To get the answer multiply the two numbers in the brackets"
> "The square root of the last number of the expression is equal to the x intercepts in the factorisation"

The final comment is an extremely important one particularly in the light of the final example given. That is when using Factor(x^2 - 7), the TI 92 will not provide a solution except in Approximate mode. The exact solution of $(x-\sqrt{7})(x+\sqrt{7})$ is not easy to obtain but is an important part of the solution and sketching of such graphs.

Students were then expected to investigate and draw conclusion from the sketching of graphs such as $y = x^2 - 4$ and $y = x^2 - 8x + 7$ (see Figures 4 & 5).

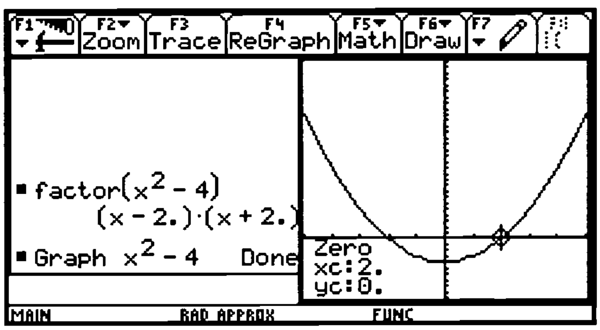

Figure 4: Investigating a simple difference of squares problem.

Students' explanation when trying to connect the previous representations included:

> "When you graph the factorisation, the line goes through the square root of the last number of the factorisation"
> "In the factorisation the answer contains x - or x + the points being the square root of the number in the question"

These types of response are important when considering assessment issues, with a CAS, where there is a need for an understanding of mathematical concepts as opposed to the familiarity with the use of a piece of technology. For example when looking at

the difference of perfect square a student could be asked to list the intercepts of the equation $y=(x^2-b)$. The ability to recognise that the x intercepts are given as the $\pm\sqrt{b}$ are important in determining the students understanding of factorisation and x intercepts versus their ability to use a piece of technology correctly.

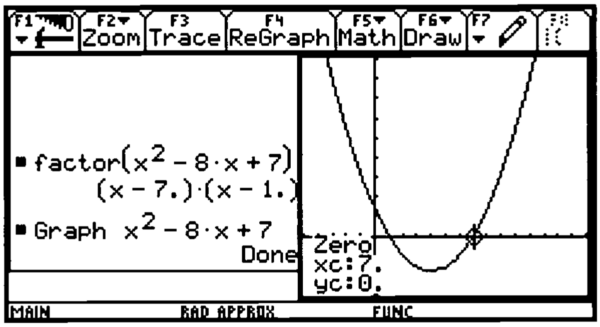

Figure 5. Extending the factorisation to other quadratic trinomials.

The students continued their investigations of the factorising quadratics with the TI 92 as shown in Figure 5, they were asked to explain the connections between the factorised form, the original equation and the graph. Comments such as the following helped to demonstrate the students understanding of the connections between the both symbolic representations and their corresponding graphical representation.

> "If you get a factor of (x -7)(x - 1) the line (curve) will pass through positive 7 and positive 1 on the x axis. The opposite (sign) applies for a factor of (x + 7)(x + 1)."
> "The x intercepts are the inverse of the factorised equation"

By approaching the calculation of intercepts from a visual and symbolic algebra perspective, students are able to construct an understanding of the null factor law, which can then be formalised for future use. By allowing the students to investigate a variety of algebraic representations with the TI 92, we are encouraging them to develop these ideas which in the past have simply been transmitted to them by the teacher.

Turning Point form

The students were then led through a series of exercises looking at turning point form of a quadratic; that is; $y=(x+b)^2+c$. Firstly they were provided with an example to investigate, similar to those shown in Figure 6.

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Figure 6. Investigating turning points when they are given in the form $y=(x+b)^2+c$.

Students' comments to the activity included;

> "The number inside the brackets is inversed to get the x axis location and the number outside the brackets stays the same to get the y axis"

While looking at the form $y=(x+b)^2+c$ one student stated;

> "When graphing b is inversed (change of sign) and c becomes the y turning point"

This is a clear recognition by the student of the connection between the turning point form and its graphical representation. The students have been able to take an overall perspective when looking at the form of the quadratic and key features of its graph. By being able to recognise the effect of the constants on the shape of the graph the students have moved towards a situation where they have been able to develop a "relational understanding" (Skemp, 1976) of the quadratic. The development of this relational understanding helps students to bring together all the aspects of quadratic functions and by doing so they gain a better understanding of the quadratic function. (Brown, 1996)

Assessing understanding.

(Skemp, 1986) states that "To understand something means to assimilate it into an appropriate schema." (p.43). At the conclusion of the series of lessons looking at factorisation, generalised arithmetic and sketching of graphs, the students were given a test. They were allowed to use their TI 92, but the questions were intended to test their understanding of the mathematical concepts, in contrast to their ability to utilise the functions of the TI 92.

One such question was
Give all possible integer values for [ ] that will allow $x^2+[\ ]\,x\ +24$ *to be factorised.*

This question was completed very well with 67% of all the students providing all 4 correct responses, the remainder of the students provided at least one correct solution.

The next question was designed to determine their ability to write an equation in factorised form given the graph of a quadratic. The question was written as follows;

The following (Figure 6) *is a graph of a quadratic. Express the quadratic in factorised form and expanded form.*

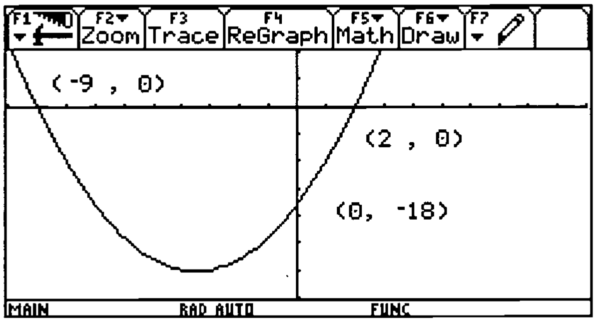

Figure 6. The graph provided for the students to analyse.

The students responses are given in the Table 1 below. The results support the suggestion that this method of introducing factorising and sketching of graphs does not inhibit the students understanding of the connection, between the x intercept and the factorised form. It can be seen that a majority, (62%), of the students were able to use the x intercepts provided on the graph to determine the correct factorised form for the graph.

Table 1. A summary of the students responses to the question in Figure 6.

Response	**Number of Students**
Correct response (x + 9)(x - 2)	13 (62%)
Symbols copied directly from points of intersection (x - 9)(x + 2)	4 (20%)
No Response	3 (14%)

A further question was then asked to determine the students' ability to write the equation of the quadratic when only the turning point is given.

The turning point on the graph is at (1, – 4), *write an equation for the graph.*

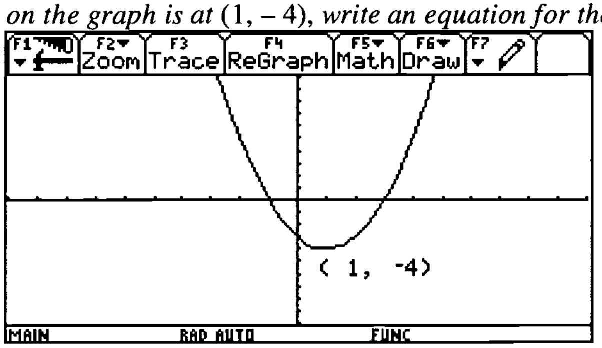

Figure 7. The graph with the turning point indicated that was given to the students.

The students were asked to write the equation for the graph and then to write this equation in expanded form. Most students used the scale to write the equation in the correct form while two students chose to use the turning point to find the equation. These two students were then able to expand the turning point form to give the equation. Overall a similar result was obtained to the previous question, indicating that

students had been able to interpret the x intercepts as the factors of the quadratic and then expand the equation.

Discussion and Conclusion

Throughout the classroom investigation students demonstrated a willingness to use the TI-92 and the ability to quickly adapt to this new learning tool. The use of the TI-92 within the structured learning environment in which the students were involved, appeared to provide them with positive learning experiences in an algebra subject that is often seen as dull and boring. The effectiveness of the TI-92 as a learning tool appears to have been supported by the students performance in their assessment completed at the conclusion of the activities. In this assessment most students were able to demonstrate the ability to determine the factorised form of a quadratic given its graph. The students written comments throughout their class work also indicated an understanding of the algebraic concepts that had been introduced.

A CAS often has its own peculiar syntax and this syntax can cause confusion for students. Ideally it would be preferable to allow students to generate their own symbolic representations and use the CAS to handle these representations, this can be done with the TI 92 using the define function. As Kaput (1996), says representations need to mean something to the students and we all too often go straight to using x and y while they are still struggling with coping with the meaning of these terms. Though this issue was not touched on in this project it forms part of a new study into algebra and mathematical modelling.

Students' were able to work in a structured investigative atmosphere and they were provided with sufficient time to investigate a variety of algebraic problems. Sierpinska (1994) recognises the importance of an investigative approach, when she talks about attention and intention in understanding, that is involvement in the problem. That is for the students to gain mathematical knowledge, they require time to interact with the problem and to attend to it. By providing the students with sufficient time to attend to the problem as well as structuring the investigation to cater for a range of abilities then there is an increased likelihood that students will gain an understanding of the concepts under consideration.

The CAS can provide students with a wholistic perspective on the principles governing functions and variables and, by so doing, it can help the students to develop a better sense of the effect of coefficients and constants on functions. For students' to obtain a better understanding of the effect of variables and what they represent they will need to gain a better symbol sense. Students' would appear to gain a better symbol sense by a greater use of mathematical modelling of real world functions. This work is now being undertaken with students of a similar age group.

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