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ABSTRACT

This volume contains the proceedings of the 7th international conference on Adults Learning Mathematics--A Research Forum held in July, 2000 in Massachusetts. It includes posters and short oral reports under these section headings: (1) Research into Practice; (2) Large- Scale Issues: Frameworks, Standards, and Assessment; (3) Theoretical Frameworks; (4) Understandings; (5) Socio-Cultural Contexts; (6) Mathematics and the Parenting Role; (7) Instructional Approaches; (8) Technical and Vocational Education; and (9) Teacher Knowledge. Plenary papers include: (1) "Eavesdropping on the Conversation" (Roseanne Benn); (2) "A Teacher's Transformation into Teacher-Researcher" (Pamela Meader); (3) "Transitions between School and Work: Some New Understandings and Questions about Adult Mathematics" (King Beach); and (4) "Challenging the Researcher/Practitioner Dichotomy: A Voice from the South" (Gelsa Knijnik). (MM)

ALM

ADULTS
A Conversation Between
LEARNING
Researchers and Practitioners
MATHEMATICS - 7

Proceedings of ALM-7
the Seventh International Conference
of Adults Learning Mathematics-
A Research Forum

July 6-8, 2000

TUFTS UNIVERSITY

MASSACHUSETTS, USA

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the Seventh International Conference of
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July 6-8, 2000
Tufts University
Medford, Massachusetts, USA

Compiled by
Mary Jane Schmitt and Katherine Safford-Ramus

Sponsored by

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The National Center for the Study of Adult Learning and Literacy
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ALM
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About ALM

Adults Learning Mathematics – A Research Forum (ALM) was formally established at the Inaugural Conference (ALM – 1) in July 1994 as an international research forum with the following aim:

> *to promote the learning of mathematics by adults through an international forum which brings together those engaged and interested in research and developments in the field of adult mathematics learning and teaching.*

Within ALM we understand the term mathematics to include numeracy.

ALM and Charitable Status

In July 1999, the membership formally agreed at the Annual General Meeting (AGM) that ALM should become a UK-based Company Limited by Guarantee, and that the trustees appointed should seek charitable status. This has now happened so that ALM is now formally:

- a Registered Charity – Charity Number 1079462
- a Company Limited by Guarantee – Company Number 3901346

Board of Trustees

The affairs of ALM are managed by a Board of Trustees, formerly a Steering Group, elected by the members at the AGM, held in conjunction with the annual conference.

The Board of Trustees for the period 1999-2000 comprises:

Objects of ALM

The Charity's objects are the advancement of education by the establishment and development of an international research forum in the life-long learning of mathematics and numeracy by adults by:

- encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit;
- promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels, for the public benefit.

How We Try to Achieve These Objects

Specifically, our aim is to improve the learning of mathematics by adults. Several recent reports show that many adults have difficulties with the basic mathematical skills needed as parents, citizens and workers. For example, the Moser Report (UK, Department for Education and Employment, 1999) suggests that in Britain, 40% of adults have some numeracy problems and 20% have very low attainment in numeracy. The results of the International Adult Literacy Survey (Organization for Economic Co-operation and Development, 1997) indicate that there are similar situations in many other countries.

ALM's membership is concerned about this situation and works in a variety of educational environments to improve the learning of mathematics by adults. Currently many of our members are involved in the provision of adult basic education programs. Other members are involved in teaching mathematics at all levels in further and higher education, including initial teacher training and ongoing professional updating of teachers. Many of their students are adults.

The ALM forum provides opportunities for teachers to bring successful practice from their own classrooms to a wider audience. This happens at the annual conferences and throughout the year through the communication and networking encouraged by the forum. By reflecting on our own practice and the practice of others and by fostering international links between teachers, we are able to encourage the transference of good practice in curriculum design, and in teaching and learning materials and methods, which have evolved in different countries. In this way, innovative ideas are enhanced through critical appraisal by fellow participants and pass into the public store of educational material through the publication of the Proceedings of our annual conferences. Thus the teaching of mathematics to adults is improved and adult students benefit.

However, there is not enough information available about what mathematics is required by adults in their daily and working lives, how adults learn mathematics, and what the most effective androgogical practices are. Our aim is to connect research with practice, by bringing the experience of practitioners and students to bear on the formulation of research questions and the conduct of research, and by making academic research accessible to teachers and therefore benefit their students.

The organization does not support a single theoretical framework, or commission or conduct research. The presentation of papers at our annual conferences provides an opportunity for discussion on research methods and findings, which constitutes an active and participative public peer review process and quality enhancement mechanism.

People working in the field who are not members of the organization and/or who are not able to attend the annual conferences, are also able to make use of the activities of participants and therefore benefit their students by access to the published Proceedings of the conferences. The dissemination of the results of our work increases the sum of communicable knowledge about the mathematical education of adults. We believe that these collective actions are of direct benefit to the public.

ALM Discussion List

ALM encourages practitioners and researchers to discuss topics on adults learning mathematics. For this you may want to join the ALM discussion list.

To join the list, please send an email to: mailbase@mailbase.ac.uk

Write in the message space: join learning-maths 'firstname lastname'

After you have sent your message you will receive a welcome message through your email account. From then you can post your messages to

learning-maths@mailbase.ac.uk

ALM Website: www.alm-online.org

How to Join ALM

Anyone who is interested in the objects of ALM can apply for membership. The membership rates for 1 August 2000 – 31 July 2001 are:

Individual Member	£15
Students/unwaged/low-waged	£3 (minimum)
Institutional Member	£30

Please contact Sue Elliott (ALM Membership Secretary) for a membership form and payment details. If you live in any of the countries listed below you may pay in your own currency through your local ALM agent at one of the addresses below.

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Contents

Technical and Vocational Education

Preface

ALM's seventh annual conference was held in North America for the first time on July 6-8, 2000. During those three warm summer days, over 120 researchers and practitioners from 12 countries gathered on the Tufts University campus in Massachusetts. They came together to consider issues significant to adults learning mathematics in a variety of settings: formal ones such as adult basic education, technical and vocational school, college, and university, and informal ones such as the workplace, home, the community, and the marketplace. The conference title, "A Conversation Between Researchers and Practitioners," reflected both the aim of the organization and the nature of the discussion that took place throughout the conference. A key strength of ALM conferences has always been the dialogue that flows among the attendees, informing and improving the work of each when they return to their institutions. A second strength is the international flavor of that dialogue, which extends the knowledge base of participants through discussions of challenges and situations faced by colleagues in other countries. The ALM-7 conference was no exception.

This book is a compendium of 52 articles consisting of plenary speeches, research papers, workshop presentations, reports of symposia, and poster sessions shared during the conference. The proceedings begin with the commentary given by Roseanne Benn at the close of the conference, the result of a careful listening to the various conversations, and continue with the plenary speeches addressing the interplay of research and practice in interesting ways. Pamela Meader's narrative of her journey from GED mathematics teacher to a researcher in her own class and King Beach's perspective on the mathematics at school versus mathematics at work launched a dialogue about transitions. Gelsa Knijnik who, along with Marilyn Frankenstein, addressed the participants on the second day, depicted the principles of work conducted as a researcher-practitioner "learning in the process."

In an attempt to continue the spirit of that conversation, we have integrated the offerings by the themes that emerged during the conference, rather than by the type of paper. These themes were not pre-determined; rather, they emerged from the presenters' interests, and therefore provide a blueprint of that which stands out as important in this young field of endeavor: the development of adult mathematical thinking. *Research into Practice* highlights purposeful connections between researchers and practitioners; *Large-Scale Issues: Frameworks, Standards, and Assessment* focuses on local, national, and international governmental efforts; *Theoretical Frameworks* illuminates various theories of adults learning mathematics; *Understandings* deals with the ways in which adults think about mathematics; *Socio-Cultural Contexts* highlights circumstances in which individuals are imbedded; *Mathematics and the Parenting Role* deals with a variety of concerns about the mathematical learning of both adults and their children; *Instructional Approaches* provides a variety of promising practices across levels of instruction; *Technical and Vocational Education* focuses on preparation for and in the workplace; and *Teacher Knowledge* contains examples of practitioners' content and pedagogy development.

Of course, within each theme, a wide range of perspectives occurs and myriad questions are raised, and the reader will discover that many of the offerings cross themes. In every respect, this collection, along with the six volumes that precede it, serves as a vehicle that frames the emergent knowledge base and the current agenda of theory, practice, research, and policy in the field of adults learning mathematics.

Acknowledgments
The ALM-7 Conference owes its creation and success to many organizations and individuals. The conference was hosted by the National Center for the Study of Adult Learning and Literacy (NCSALL) at the Harvard University Graduate School of Education in conjunction with the Tufts University Department of Education and the NCTM-affiliated Adult Numeracy Network. The Office of Vocational and Adult Education of the U.S. Department of Education generously supported the work of several presenters at the conference. ALM is truly indebted to these U.S. organizations for helping to further the development of the international field.

ALM is also thankful to the local organizing committee who contributed so much of their time and expertise to the conceptualization and organization of the conference: Lynda Ginsburg, from the National Center on Adult

Literacy; Esther Leonelli, Cambridge Community Learning Center and ANN; and Analúcia Schliemann, Tufts University. Faith Harvey and Katherine Snead supported the conference preparation and Faith contributed greatly to the production of the proceedings. Finally, we sincerely thank John Comings, Director of NCSALL, and Ron Pugsley and Mary Lovell from the U.S. Department of Education for showing that leadership in U.S. adult education understands the importance of furthering work in mathematics and numeracy.

Mary Jane Schmitt and Katherine Safford-Ramus
Conference Co-chairs

PLENARY SESSIONS

Eavesdropping on the Conversation

Roseanne Benn
University of Exeter, UK

First I should like to thank Mary Jane Schmitt and the other organisers of ALM7 for asking me to give the concluding plenary session. When asked to suggest a title, I felt pleased to come up with the one above which I considered quite clever. However, I was chastened when several of the delegates who spoke fluent English even though it was not their first language confessed to me that that they did not understand the word "eavesdropping." A salutary lesson, echoed throughout the conference, that at an international event it is crucial to ensure not just understanding but common understanding of terminology used. We spent much of the symposium on *Developing a Theoretical Framework* and Tine Wedege's session on *Epistemological Questions about Research and Practice* doing just that. Much of the discussion was between those from the U.K. and those from the U.S. proving Winston Churchill's point that we are two nations divided by a common language!

I should also like to thank Lynda Ginsburg and the session facilitators who by taking notes of their sessions "lent me their ears" (to misquote William Shakespeare in Julius Caesar) and contributed to this paper.

I was interested at the beginning of the plenary in the response to my question as to who considered themselves first and foremost a practitioner, a researcher or for whom were the roles indistinguishable. A massive majority clearly saw themselves as practitioners with a scattering in the other two categories. This may be due to the large number of American conference attendees but I think that the result would have been the same at previous ALM conferences. This is a point we shall revisit later in the paper.

The Conference this year was entitled *A Conversation between Researcher and Practitioner,* seemingly a very appropriate but uncontentious title. After all ALM is an international research forum that brings together researchers and practitioners in order to promote the learning of mathematics of adults. This definition heads all the literature of ALM and is the banner under which many of us have met regularly for up to seven years. But the link between researcher and practitioner is not uncontentious, as heated discussions at earlier conferences have shown, so the topic of this year's Conference was timely and welcome. It allowed us, if we chose, to explore the relationship between researcher and practitioner. Is it symbiotic, a permanent union with each depending for its health on the other, with, for example, researchers drawing on the experiences and questions of practitioners to root their enquiries whilst practitioners gain by the utilisation of research results for their everyday work? By working together, strengthening and empowering the experiences of adult learners? Or is it parasitic, with one side dependant on and taking from but not giving to the other? One group living off, flourishing through, sapping the energy, strength, and knowledge of the other? Or is it some other form of relationship? Interesting questions which deserve reflective consideration. What did we hear?

The plenaries set the tone of the Conference. In the first plenary, Pamela Meader presented an impressive example of practitioner research. Very accessible and informative, she caught the hearts and minds of the audience. She showed that the outcome of her work was twofold. It both shone a spotlight on a patch of our field of work and study as good research should but also clearly led to more effective practice by developing reflexivity in both the teachers and the learners. However, the questioning at the end of the session illustrated some of the limitations of practitioner research or, perhaps more accurately, action research. Were the findings generalisable? What are the mechanics for sharing the results with others? Who gets to build on this work and how do we prevent the wheel being reinvented by others? All pertinent questions. We then heard a different kind of story from King Beach. From him we listened to "researcher" research, also presented in a very accessible way and interestingly illuminated by his "practitioner" research in Nepal. Other and different questions were raised by his presentation. Does this form of research draw on a knowledge base that is not shared by all? Is it expressed in a language (or perhaps more precisely a discourse) that we are not all familiar with (Beth Marr's session later in the Conference discussed this very issue)? When findings are disseminated in academic journals, who reads them? Is much of "researcher" research available only to other researchers? Is it more powerful because it is more global, generalisable, and built on the literature base? Is the researchers' side of the conversation more influential on the body of researchers and practitioners in the field, the institutions, and the policy makers? And if yes, should this be so?

What emerged from these two presentations was some of the undercurrents of the conversation. But also a clear message that the "whole" of the two kinds of research was greater than the sum the parts. An excellent start to the conversation.

In the second plenary, Gelsa Knijnik and Marilyn Frankenstein gave us two more stimulating sessions. What we heard here was a different tone to the conversation. Here the strong motivational factor for both was political empowerment for adults; the driving force, social justice. With Marilyn, it seemed impossible to separate the practitioner and researcher. She embodied both. This illustrated the false dichotomy of either/or. It can be both/and. We also heard the powerful message that researchers need not conform to academic norms and strive to be "objective" and distanced from their research. They can (and perhaps she felt should) be passionately involved. From Gelsa, we heard the unusual story of how a researcher had become a practitioner in the process of her research. She completed this circle by encouraging her learners from the Landless Movement to help empower their movement by becoming researchers in their turn. We heard how both speakers passed on to their students some of the power of the researcher by making the researched subjects not objects, giving them experience of interpreting policy and analysing how it impacted on their lives and using the culture and principles of the researched not the researcher to underpin the research. From both we heard how researchers can help the researched and/or students to understand and take control of their lives through active involvement in their learning and in their societies.

Let us now look more closely at this vexed question of the relationship between practitioner and researcher. As I noted earlier, discussions at previous ALM conferences have indicated that this relationship is neither easy nor automatic and have at times been heated. However, we have not usually had the time and space to unpack the issues. What follows is my attempt to do this.

In our post-modern world, we all freely acknowledge our and other's multiple roles and identities so I must be careful of positioning people. But the show of hands at the beginning of this plenary suggested that here in ALM many researchers either have been or still are practitioners. This is arguably not true for most researchers in the academy but true for many adult education researchers. Many of us just "happened" into our work. We are not typical researchers and academics but often have very diverse backgrounds grounded in practice. As many of the researchers here are/were practitioners, they have a self-image/identity of practitioner/researcher and so perhaps do not see a divide. Kathy Safford-Ramus suggested in her kind introduction to my talk that I, like many other researchers in ALM, conduct a researcher/practitioner conversation every time I talk to myself in a mirror! If this is the case, which I suspect it is, then the problems in the conversation are probably not seen as problems by this group.

But is the same true for the practitioners in ALM? Practitioners sometimes seem irritated, frustrated, and alienated by researchers and we need to delve deeper into why this is so. Many practitioners who help adults learn mathematics have constructed their own knowledge base which is often very substantial but which has been developed from experience and reflection rather than formal accredited learning. In our hierarchical and credentialist society, this may lead to feelings of vulnerability and insecurity. And it is arguable that these feelings are well founded. I can only speak for the U.K. but there maths adult educators are often (usually?) one or more of the following: badly paid; overworked; on part-time and/or short-term contracts; their work is seen as peripheral to the institution's mainstream; often located on the physical periphery (church basements, old buildings, the hut outside); they have access to limited resources and to cap it all, the institutions that often treat them so badly are themselves of low status in the pecking order. This may result in practitioners seeing researchers as privileged individuals coming from high status higher education institutions with low teaching loads. People who come in from "outside" with no sense of context. "What do they know about my course, my situation, my learners, their situation?" People who take and gain from the research process but who give little that is useful back to the practitioner or her students.

So has this conversation helped? I overheard one participant comment on how rare it is to have such a mix of researchers and practitioners at a conference. This has been a great opportunity for each group to eavesdrop on the other's conversations, talk to each other, influence the other, and raise concerns. It has been an opportunity to meet informally, develop relationships, and learn to respect and admire each other. But something even more important. One of the echoes throughout this conference was "fun." I think for example of Dave Tout's *Having some fun with math.* In his and other sessions we have laughed and made fools of ourselves together. We have had fun.

So is the relationship between practitioners and researchers parasitic or symbiotic? We know that in our societies researchers have more power and status. Their voices are heard (more) clearly. The "silent murmurings" (Foucault 1986, p. 28) of practitioners can pass unnoticed. But is this inevitable and/or immutable? Can we in ALM continue to develop a symbiotic relationship which recognises the different knowledges of the two groups and the different ways of knowing which together could lead to a greater empowerment of adult learners?

I leave you and ALM to discuss if this is possible and how we can ensure that the conversation continues in a balanced and fruitful manner.

Reference

Foucault, M. (1986). *Power/Knowledge.* Brighton: The Harvester Press.

A Teacher's Transformation Into Teacher-Researcher

Pamela Meader
Portland Adult Education, USA

For many practitioners, the word "research" is not welcomed with open arms. Many practitioners feel inadequate and not a part of the research world. Many feel that the research has no connection to what they do in the classroom and that researchers care little about practitioners' feedback. I felt quite a "disconnection" to the research world. While I would read abstracts by researchers in the K-12 arena, trying to keep abreast of new developments and then attempting to match these ideas with the adult model, I was not aware of research being done in my field of adult education or, more importantly, adult education and mathematics.

That began to change five years ago when I was asked to serve as a state representative in a new venture created by NCSALL (National Center for the Study of Adult Learning and Literacy) called the PDRN (Practitioner Dissemination and Research Network). The job description was somewhat vague and I assumed that my primary involvement would be to disseminate research ideas to my colleagues. Never did it cross my mind that I would be a researcher myself.

My first real "connection" to the research world came during a national meeting at Harvard University in Cambridge, Massachusetts, during the summer of 1996. As a PDRN representative for the state of Maine, I joined other PDRN representatives from the New England and Southeast regions to meet the researchers of the various NCSALL projects. All of us felt intimidated and nervous as we walked into a glorious room of giant mahogany tables. We were strategically placed around this room to get "closer" to the researchers and share in the process. It wasn't until I heard researchers John Comings, Rima Rudd, and Victoria Purcell-Gates speak that I began to relax and feel welcomed in this research arena. Matching faces to the research helped me to better "connect" to the ten research projects being conducted by the NCSALL researchers.

During the second year as a PDRN representative, I learned that my job description was changing. Each PDRN representative was asked to conduct their own practitioner research based on John Comings's research on learner persistence (Comings, Parrella, & Soricone, 1999). I was apprehensive at first, not knowing what "practitioner research" was. I soon learned that practitioner research was much like scientific inquiry where one posed a question or hypothesis, collected data, and then analyzed the findings. Because I am a math teacher, I found this process exciting and relevant.

The support and training I received from the NCSALL staff provided me with the much needed tools for practitioner research. We met three times during the year for support in our journey into research. Our first meeting helped us to develop and refine the questions, which many of us found to be the hardest task of all. Once our data were gathered, we met a second time to talk about ways to analyze data and present our findings. At our final meeting, we actually presented our projects and received wonderful support and accolades for our work.

In their research on persistence, Comings et al. found four supports to persistence: awareness and management of the positive and negative forces that help or hinder persistence; student self-efficacy; establishing a goal by the student; and progress toward reaching a goal. The idea of "persistence" certainly resonated with me. It had been a constant struggle to see students complete the variety of math courses we offered. In analyzing past persistence performances, I found that our persistence rates fluctuated between 40% and 60%. The idea of exploring force field analysis and incorporating goal setting into my math classes intrigued me. After much thinking and revisions, I decided to base my research on what effect continuous goal setting in a math classroom had on persistence rates.

I decided to begin collecting data with the first class meeting. I explained to the students about John Comings's findings on learner persistence and goal setting. I asked each student to fill out a goal setting questionnaire which incorporated force field analysis. That is, the survey asked students to consider what barriers would prohibit them from completing their goals and what positive forces would enable them to reach their goals. They were also asked to list daily action steps that they would follow to help reach their goals. Four weeks later we revisited their goals, then again at the halfway mark of the semester, and then toward the end.

What I learned from this research was far more than I had imagined. While Comings's research found transportation, childcare, and work as barriers, the greatest barrier for math students was math difficulties. Students listed lack of understanding, fear of failure, fear of math, frustration with math, math anxiety, and motivation as barriers to fulfilling their math goals. Clearly psychological and academic barriers were at work here, not situational barriers such as transportation.

As far as the effect of goal setting on persistence was concerned, the results were equally surprising. My higher level mathematics courses of Algebra showed no significant change while lower level courses showed a positive effect on persistence and goal setting. The class that had the greatest improvement in persistence was a Math Concepts (pre-Algebra) class of 21 students. They were compared to a non-goal setting group of similar size and content. The non-goal setting group finished with only a 40% completion rate while the goal setting group demonstrated a 75% persistence rate.

The transformation from teacher to researcher has made an everlasting impact on myself and my students. I continue to use goal setting in my classroom and feel it has made a difference in persistence. More importantly, I now have introduced the research process to my algebra students. As a performance assessment task, my students must develop a research question, collect data, graph and analyze the findings, and present their research to the class.

On another level, this research project has impacted my math department. Discovering that math difficulties were a clear barrier, I had my three math teachers conduct force field analysis in their classrooms. The results were identical to my findings. We then surveyed students who completed a required math class to see if their math attitudes changed. The majority of students went from hating and fearing math to liking math. Clearly a change in math attitude can have a direct effect on persistence.

To further substantiate the fact that changing math attitudes can affect persistence, I went back and looked at students' reflections on barriers when the students were about 6 weeks into the class. Many shared relief about presumed math fears. One student stated, "I'm not so embarrassed any more, there are tons of people out there with the same weaknesses." Another shares, "My step to overcome my fear is doing the math homework on time and that helped me a lot and decreased my fear of math."

As you can see, practitioner research has had a direct change on myself, my students, and my staff. I feel practitioner research is a viable procedure to help effect change. I have found that the practitioner research model and present research methods have similarities and differences. A practitioner's sample space may be small and not held accountable to statistical norms that a researcher must adhere to. However, effective practice change can be immediate in the practitioner's classroom while disseminating a researcher's findings and implementing them in the classroom may not be so immediate. Practitioner research makes research an "active" process while present research is disseminated to be read, a more "passive" process. In order for practitioner research to be a successful model, practitioners need to be supported, both administratively and financially. With the supports in place, practitioner research can cause immediate changes in the classroom and beyond.

Reference

Comings, J., Parrella, A., & Soricone, L. (1999). *Persistence among adult basic education students in pre-GED classes* (NCSALL Reports #12). Cambridge, MA: National Center for the Study of Adult Learning and Literacy.

Transitions Between School and Work: Some New Understandings and Questions About Adult Mathematics

King Beach
Transitions Research Group, Michigan State University, USA

Like many of you, I began my career in education as a mathematics teacher. A year of teaching in the United States and three years teaching in a school in rural Nepal moved me to wonder about the relation between what my students were learning in the classroom and what they did and learned in their lives outside of school. My experiences in Nepal in particular led me to examine the now often talked about distinction between learning in school and out.

Nepal did not have a national system of education prior to the 1950s. In fact, schooling had been outlawed in most parts of the kingdom for over a century. During the 1950s and 1960s teams of educators and development specialists from the United States were invited to help create a national system of education from the ground, drawing heavily on American and British systems of instruction and curricula. Thus schools as an entirely new form of social organization were rapidly introduced into Nepali communities. The dissonance between my students' lives in the subsistence-level agrarian community and their participation in school raised concerns for me as a teacher. Here are a few examples of what raised those concerns.

- Primary students attempting to mimic the elaborate finger calculation strategies of their older family members are reprimanded by classroom teachers. Such reprimands are supported by students' families because children are sent to school to learn "real math" that cannot be learned from family members.
- Village farmers describe their mental and finger calculation strategies as "guestimation" rather than *hisaab*, or math. When asked to explain why, they say that what they do was once considered to be *hisaab*. With the introduction of schooling to the village, though, *hisaab* has become exclusively that which you do with pencil and paper.

Learning transfer was one of the few concepts that seemed to have anything useful to say about learning across different contexts such as schools, homes, and workplaces. However, the concept of transfer did not help me much in puzzling through the complicated relations between culture, history, and mathematics that intertwined school with other institutions, and that seemed to be at the heart of my students' and their parents' struggles with *hisaab*. These sorts of struggles are perhaps more transparent in rural Nepal than they are elsewhere, but they exist in some version everywhere that people move between school and participation in workplaces, communities, and families. I suspect that many of you who are practitioners have spent far more time than I struggling with how to best support mathematics learning that is relevant to adult lives beyond the classroom—particularly in the workplace.

I would like to begin by talking about some shortcomings of transfer for understanding relations between mathematical reasoning in the classroom and the workplace. This will set the stage for the bulk of my talk where I will describe an alternative to transfer—that of consequential transition—and will illustrate it with three studies from my research group at Michigan State University. These are studies of school-work transitions in rural Nepal, the introduction of computerized machining into American manufacturing, and high school students' part-time work in fast food restaurants. I will conclude with a brief discussion of what this new conceptual tool of consequential transition might buy us in understanding and facilitating mathematics learning during transitions between school and work. As with any new conceptual tool, it generates new unresolved questions, and I will raise a few of these too.

Shortcomings of the Transfer Metaphor for Understanding What Happens Between School and Work

Our everyday use of the term "transfer" has a powerful metaphorical bearing on how we, as educators and researchers who also happen to lead everyday lives, think about learning transfer. In our everyday usage of the term, transfer involves the movement of a person, a transaction, or the shifting of an object from one place and time to another. As a construct in educational psychology, it refers to the appearance of a person carrying the product of learning from one task, problem, situation, or institution to another. It is here that the metaphor begins to break down.

- Commonsense suggests that generalization happens regularly on a moment-to-moment basis in our lives. Yet when we seek to study or facilitate it as transfer, we are rarely successful. This suggests that though the underlying phenomena are quite real, the transfer concept is inadequate for understanding them.
- Transfer either defines an extremely narrow and isolated aspect of learning (that learned on task/situation A that is applied on task/situation B) or is no different from "just plain learning," i.e., all learning involves transfer. Both make the concept relatively useless.
- Transfer environments are assumed to be static and pre-given. This excludes the creation of environments as part of the transfer process itself.
- Nothing new can be created in the process of transfer. Transfer assumes a model of person-environment relation that seals a person's initial learning off from being transformed in the new problem or situation.
- Transfer involves single processes such as recognizing isomorphisms or abstracting general representations. The actual generalization of mathematical reasoning from school to work is complex and cannot be reduced to single process explanations.

Despite these problems with the transfer metaphor, the important educational issues and challenges that underlie what we have called transfer remain central and important. I and the other members of our group at Michigan State University believe that the difficulties are significant enough that the transfer metaphor should be left behind in favor of a metaphor and a set of concepts that accept both changing persons and changing social contexts as central to understanding generalization between the classroom and the workplace. A sociocultural stance affords us this possibility.

To paraphrase Mike Cole in his 1996 book, *Cultural Psychology,* our distinctiveness as humans lies in our ability to modify our world through the construction of cultural artifacts in texts, technologies, symbols, and signs, along with our corresponding ability to reconstruct the modifications in subsequent generations through our schools, families, communities, and work. We thus transform our own learning and development. It is this recursive relation between changing individuals and a changing world that is central to sociocultural work, and to our conceptualization of consequential transition.

The Concept of Consequential Transition as an Alternative to Transfer

Experiences such as learning algebra after years of studying arithmetic, becoming a machinist, founding a community organization, teaching your first-born to walk, an elementary school class writing a letter to a local newspaper, collaborating with NASA scientists on a classroom project via the Internet, making the transition from student to teacher, and learning to do manufacturing quality control in your first job out of high school are all potential examples of the sort of things we are concerned with. Clearly the forms of generalization involved go far beyond learning transfer, but cover an educational terrain that has been reduced, metaphorically, to the carrying and application of knowledge across tasks. Each of these experiences shares a set of common features as consequential transitions.

- Transitions involve the reconstruction of new knowledge, skills, and artifacts, or transformation, across time and through multiple social contexts, rather than the reproduction of something that has been acquired elsewhere. Transitions therefore involve a notion of progress for the learner and are best understood as a developmental process.
- Consequential transitions involve a change in identity: a sense of self, social position, or a becoming someone new. Therefore individuals and institutions are often highly conscious of the development that is taking place, and have particular, sometimes publicly debated, agendas for how and why it should happen. Identity is what makes these transitions consequential.

- Consequential transitions are not changes in the individual or in the social context, per se, but rather are changes in their relationship. Both person and social context contribute to a consequential transition and are recursively linked to each other.

Illustrations of the Concept of Consequential Transition: Students and Shopkeepers in Rural Nepal

The first illustration of the concept comes from a study of Nepali high school students becoming shopkeepers and adult shopkeepers attending school for the first time (Beach, 1995a, 1995b). High school students near graduation were apprenticed to adult shopkeepers for a period of several months. Similarly, adult shopkeepers who had never attended school were enrolled in an adult literacy/numeracy class for several months. Changes in arithmetic reasoning were tracked during this period of time.

The high school students engaged in a lateral transition from school to work. Many students in rural Nepal go on to become shopkeepers, and therefore the transition was unidirectional toward their future career. The shopkeepers engaged in a collateral transition between the shop and the classroom. They participated in both activities with near simultaneity. They planned to remain shopkeepers. Their transition was not preparation for participation in a new activity. Rather, it was for the improvement of their existing activity—shopkeeping. Thus changes in both the students' and shopkeepers' sense of self and social position were engaged as a part of the consequential transition.

The high school students transformed their arithmetic reasoning as a part of the transition. Students retained a written form of arithmetic notation, but the notation was changed to represent modified forms of mental and finger calculation strategies used by the shopkeepers. By transforming their arithmetic reasoning the students retained the status associated with written arithmetic while acquiring the efficiency of the shopkeepers' strategies. The transition involved a transformation of the students' knowledge of arithmetic. Unlike students, shopkeepers added some aspects of paper and pencil calculation algorithms to their existing repertoire of calculation strategies and rejected others as not useful for shopkeeping, such as the writing-out of operation signs. They expanded and reorganized their existing knowledge of arithmetical calculation but did not construct a new form for representing calculations.

Illustrations of the Concept of Consequential Transition: The "Computerization" of Traditional Machining

The second illustration comes from a study of an industrial machine shop where machinists trained on mechanical machines were learning to use computer numerical control (CNC) machines (Hungwe, 1999; Hungwe & Beach, 1995). Mechanical machines are controlled with a series of dials, levers, and gauges that the machinist manipulates in real time to make parts. In contrast to this, program code that is written at a location distant from the machines before producing parts controls the CNC machines. The social and technological organization of the shop changed with the introduction of the computerized machines. Many of the machinists experienced an encompassing transition, a form of consequential transition occurring within the boundaries of a single social organization that is itself changing.

Machinists with decades of experience running mechanical machines mapped the CNC programming codes onto their prior knowledge of tool movement through Cartesian space and trigonometric calculations, albeit with some adjustment. However, machinists without those many years of experience with mechanical machines relied more directly on the structure of the programming code to think about tool movement and organize calculations in learning CNC machining. It can be seen from this example that it is the particular intersection of the history of the individual with the history of the social organization that determines the nature of knowledge developed during encompassing transitions.

The introduction of CNC machining supported the division of machining into machine operation and machine programming. Some machinists in the shop opted for overseeing the operation of the machines, whereas others began to program the machines. Several of the more accomplished machinists experienced a loss of craftsman identity as a part of the transition to CNC machines. They were no longer individually responsible for creating parts from start to finish. Despite having mastered the intricacies of CNC machining, these machinists returned to mechanical machines where they were fully responsible for the making of parts. Sense of self and social position, or identity, rather than knowledge and skill, drove the reversal of their earlier transition.

Illustrations of the Concept of Consequential Transition: High School Students at Work in the Fast Food Restaurant Industry

The final illustration comes from a study of high school students at work in fast food restaurants for the first time (Beach & Vyas, 1998). An exclusive focus on school subjects like math, science, and literacy gives the appearance that nothing new was gained during collateral transitions between high school and work in fast food restaurants. It fact, the situation appears to be very much one of classic transfer. Students' subject knowledge from school is applied to work in the restaurant. New understandings of math, science, and literacy are not constructed during the transition when these categories of knowledge are looked at in isolation.

A closer analysis indicates that the high school students do develop during the transition. They are learning how to learn in a production activity for the first time, in contrast to learning within a social organization that has learning as an explicit part of its agenda. Uses of language, math, and science on the job are reconstructed "on the fly," so to speak, while production is maintained. The students develop ways to learn how to avoid inefficient arithmetic calculations, call out orders that communicate without distracting, and avoid food spoilage, all without specific time and support for learning these things. Students do not see these as instances of math, literacy, or science. They are right in one sense. Math, literacy, and science each involve multiple concepts that reference each other within their respective domains, e.g., the concept of ratio is related to fractions, decimals, and division. Math-, science-, and literacy-like concepts in the fast food restaurant are referenced to aspects of production, and not to other mathematical, scientific, or communicative concepts.

Development can be found during collateral transitions when we move away from using the epistemological assumptions of one social organization—the school—to understand participation in the new organization, in this case the fast food restaurant. In doing so we are also putting aside ideological assumptions that value knowledge organized in the form of subject matter over knowledge organized is other ways, such as for production.

Some New Understandings and Questions

What might this new conceptual tool of consequential transition "buy" us in understanding and facilitating mathematics learning during transitions between school and work, and what new unresolved questions does it raise? Here are a few described in brief. A fuller explication of the concept of consequential transition can be found in a recent volume of the annual *Review of Research in Education* (Beach, 1999).

- Attempts to get mathematical reasoning to generalize by making the learning of mathematics in classrooms more like math at work, or by teaching core concepts "in the abstract," are misguided and not particularly effective.
- It is more productive to think about differences in school and work as presenting opportunities for mathematical learning and development, rather than boundaries to be overcome or transferred across. This suggests that efforts should not be directed at making school and work similar to each other, nor should seamless transitions between the two be promoted as a goal. Rather, we need to think about ways to directly support consequential transitions themselves as important pedagogical opportunities.
- Learning mathematics in classrooms engages adult learner identities in ways that are quite different from that of younger students. The sense of becoming someone new, or of not being someone, e.g., not being "educated," should be considered legitimate topics for discussion among adult learners of mathematics.
- How do we engage workplaces as environments for learning mathematics when learning and production often present competing and contradictory agendas?
- How do we maintain relations between that which the adult learner experiences as math and that which she does not experience mathematically, although we can understand both experiences as mathematical from our vantage point as teacher or researcher?

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Challenging the Researcher/Practitioner Dichotomy: A Voice From the South

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Introduction

When I sat down in front of the computer to begin writing this paper, I started to think about my first experience in Youth and Adult Education of the Landless Movement, at the beginning of 1991. At that time, I was in the small town of Braga, working at a Landless Movement Teacher Training Course. Late one afternoon, Father Sergio Görgen informed me of the ideas involved in the first Literacy Project for Adults in the Movement that was to begin soon in the state. I was being invited to participate in it, as an advisor in Mathematics Education. After hearing him out, I felt enthusiastic about the "new" aspect of this challenge, but, at the same time, aware of my inexperience with this type of work, I reasoned that I did not feel prepared to work on the project. He answered emphatically: "We intend to carry out this first national project. If you want to participate, and learn in the process, fine. If you don't, we'll do it without you." Choosing to "learn in the process" was one of the many lessons I learned and am still learning in the work I have been performing as researcher in the field of Mathematics Education in a social movement, a movement now internationally acknowledged for its struggle for land reform in the country with the highest concentration of land ownership in the world, a land reform that will enable a more equitable distribution of wealth and will promote social justice. The Landless Movement has also been internationally acknowledged for prioritizing Education as one of the central dimensions of its struggle, in particular the need of children, youths, and adults for Mathematical Education. This fact is even more relevant when the official data on schooling in Brazil are considered. Out of a population of 160 million, 20 million say that they do not even know how to write a note (IBGE, 1990). An enormous number of youths and adults are out of the educational system, victims of social exclusion. The principle of "learning in the process" was one of the foundations that supported that first project, involving, for 2 years, approximately 100 monitors and 2,000 students. Now, after 9 years, the principle still lives on in the work of the approximately 1,200 monitors and 20 thousand learners who participate in the Youth and Adult Education projects of the Landless Movement.

Students, monitors, and advisors must "learn in the process." It is necessary that we learn in the process, because the working conditions with which we deal are precarious from many standpoints: there is not a great tradition in Brazil regarding Youth and Adult Education projects in rural areas; even less experience has been accumulated in work organized *by* a peasant social movement (and not *for* it); and mainly because the level of schooling of the monitors, in many parts of the country, does not go beyond the initial grades of elementary school.

Prepared to, inevitably, "learn in the process," however, we had, since the beginning of that first project, a clear idea of the guiding principles. We were aware that Youth and Adult Education should be attuned to the Landless Movement Pedagogy, in particular with its educational principles. These principles are now expressed in the following points: Relationship between practice and theory; Methodological combination between teaching processes and training; Reality as a base of the production of knowledge; Socially useful formative contents; Education for work and by work; Organic link between educational and political processes; Organic link between educational and economic processes; Organic link between education and culture; Democratic administration; Self-organization of the students; Implementation of pedagogical collectives and continued teacher education; Research attitude and skill; and Combination of collective and individual pedagogical processes (MST, 1996).

These are the principles that have guided the Landless Movement work in Education, and they constitute points of reference for the implementation of Youth and Adult Education projects in the Brazilian camps and settlements. This implementation has occurred rather unequally, considering the heterogeneous levels of organization of the Sector of Education in each state, and the possibility of advice from universities and NGOs (non-governmental organizations).

If, on the one hand, from the beginning we had identified the theoretical perspective of Ethnomathematics as being the one most directly connected with the educational principles of the Landless Movement Pedagogy, on the other hand we realized at the time (what we still realize today) that "applying" this perspective in the Youth and Adult Education of a peasant social movement with very specific characteristics, as is the case of the Landless Movement, is not a simple operation: there is what is "new" produced socially by the Movement, also in the dimension of Mathematics Education, requiring other pedagogical answers; there is the "new" brought by Ethnomathematics, a relatively recent area, whose formulations are still, in a sense, incipient; and there is the "new" of implementing an attitude and research skill, one of the founding principles of the Landless Movement Pedagogy. The implementation of this attitude and research skill has challenged us to break up the Researcher/Practitioner Dichotomy. In dealing with this challenge, I have learned a few lessons. They are the core of the second part of my talk.

Lessons From Research and Practice

The work that I have been developing with the Landless Movement can be summarized in what I have called the Ethnomathematics Approach. This consists of: the investigation of the traditions, practices, and mathematics concepts of a social group and the pedagogical work which is developed in order for the group to be able to interpret and decode its knowledge, to acquire the knowledge produced by academic Mathematics, and to establish comparisons between its knowledge and academic knowledge, thus being able to analyze the power relations involved in the use of both these kinds of knowledge.

Guiding an educational process in this approach means, necessarily, articulating research and practice. The pedagogical practice here is linked with research in two dimensions. The first relates to the process of investigation of the traditions, practices, and mathematical concepts of a social group. This investigative process implies the need to carry out fieldwork, in which ethnographic techniques, such as participant observations, audio recordings, field diaries, and interviews are used. But this is not anthropological work or ethnography in the strictest sense of the word. However, in using elements of ethnographic techniques, inspired by anthropological knowledge, I have been watchful about questions that have been asked contemporarily by Anthropology, an area strongly marked by its links with the colonial area, with the description of the "Other." However, as clearly shown by Henrietta Lidchi (1997):

> Asserting that ethnographic texts are not accurate descriptions made of one culture by another but by the *writing of one culture by another* would, today, be a starting point in an analysis of ethnographic work, rather than a radical statement. (p. 200)

Thus, in the description of practices, traditions, and mathematics concepts of a social group, I do not delude myself that I am "discovering" what "is there." The act of *writing of one culture by another* implies in fact "constructing one culture for another. What is being produced therefore is not a reflection of the 'truth' of other cultures but a representation of them" (Lidchi, 1997, p. 200).

In this process of representation, I consider that my ethical and political position makes me take into account, as stated by Clifford Geertz (1983), that:

> To us, science, art, ideology, law, religion, technology, mathematics, even nowadays ethics and epistemology, seem genuine enough genres of cultural expression to lead us to ask (and ask, and ask) to what degree other peoples possess them, and to the degree that they do possess them what form do they take, and given the form they take what light has that to shed on our own versions of them. (p. 92)

This is an ethically relevant question, which has led me to try to understand the practices, traditions, and mathematics concepts in the social world of which they are part. According to Rabinow (1977):

> Culture is interpretation. The "facts" of anthropology, the material which the anthropologist has gone to the field to find, are already themselves interpretation. The baseline data is already culturally mediated by the people whose culture we, as anthropologists, have come to explore. Facts are made—the word comes from the Latin *factum,* "made"—and the facts we interpret are made and remade. Therefore, they cannot be collected as if they were rocks, picked up and put into cartons and shipped home to be analyzed in the laboratory. (p. 150)

This care to not treat the material collected in the fieldwork like "rocks" has allowed it to provide information for the pedagogical process, giving it permanent feedback. As I mentioned previously, the pedagogical work is developed in order for the group to be able to interpret and decode its knowledge; to acquire the knowledge produced by academic Mathematics; and to establish comparisons between its knowledge and academic knowledge, thus making it able to analyze the power relations involved in the use of both these kinds of knowledge. For practitioners to be able to implement this complex process, which articulates the relativistic and legitimistic perspectives of culture (Knijnik, 1997), they must necessarily also be researchers.

In a second dimension, the Ethnomathematics Approach also connects practice and research. This dimension concerns the follow-up of the pedagogical process. Here I, as a practitioner, am dedicated to writing and examining my own practice, establishing an interlocution with the theorizations in the field of Ethnomathematics, thus seeking to discuss the limitations and potentialities of the work I am doing. Here, the practitioner is challenged to break up the dichotomy between research and practice, since it is her/his own practice that is taken as a subject for analysis. This research perspective which, in a more conservative view, could be criticized as lacking neutrality and objectivity, now, as we live in a time when these myths have been undone, can produce other meanings for the act of researching, enabling a crossing of borders and a breaking of the dichotomies between "insider" and "outsider," between the language of research and the language of practice. This crossing of borders, which had previously been outlined with such accuracy, involves the act of writing about one's own research and presenting it at conferences, which means definitely to take on, oneself, the act of self-representation.

From this perspective one can understand even more radically the position of the Landless Movement in establishing partnerships with Brazilian universities to create college-level Education Courses that prepare their participants in the field of Education. As Marli, a student in one of these courses, said: "We, better than anyone else, know what we must know about ourselves to be able to go on in our discussions." As a specific intellectual, in the sense given to it by Foucault, I have participated as a researcher and practitioner in these discussions, careful to permanently problematize the asymmetrical power relations involved in this interaction with the Landless Movement.

In the kernel of this interaction is the Ethnomathematics Approach I have been developing inside the Landless Pedagogy, which problematizes scientificity, the apparent neutrality of academic Mathematics, and brings to the scene "other" Mathematics, usually not mentioned at school, as a cultural production of non-hegemonic groups (Knijnik, 1998).

Our role in these processes of including or excluding knowledges in the school curriculum is above all and mainly political. These processes, defining which groups will be represented and which will not be heard from in schooling, are, at the same time, products of power relations and producers of these relations: a product of power relations because it is the dominant groups that have the cultural capital to define what knowledge is legitimate to become part of the school curriculum. These processes are also producers of power relations, because they produce very specific subjectivities, positioning people in given places in the social sphere and not in others. These places have not been defined once and for all. The field of Adult Mathematics Education is also a place of resistance, of protest, of struggle.

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RESEARCH INTO PRACTICE

Issues in Research and Practice for Adults Learning Mathematics

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Abstract
In this workshop we drew on our experience in ALM and of editing the book, Perspectives on Adults Learning Mathematics: Research and Practice *(Coben, O'Donoghue, & FitzSimons, 2000) and invited participants to explore current issues and develop future agendas for research and practice in the field of adults learning mathematics. We are keen to facilitate new research collaborations, especially of an international and comparative nature.*

Introduction
We based our introductory discussion in our book, *Perspectives on Adults Learning Mathematics: Research and Practice* (Coben, O'Donoghue & FitzSimons, 2000), on issues we had to consider as editors that could be said to mirror issues in the field of adults learning mathematics as a whole. This book is about adults learning mathematics wherever and in whatever circumstances they do so. It starts with an international review of research in this field, by Gail E. FitzSimons and Gail L. Godden, both of whom are based in Australia. While the review is as comprehensive as possible, the authors highlight areas where further research is needed and echo the conclusion of Tine Wedege, Roseanne Benn, and Jürgen Maaß (1999) that adults learning mathematics is an under-theorised domain which needs to draw upon as many relevant disciplines as possible in order to develop. The main content is then divided into four sections: Section I aims to give insights into some current research directions; Section II takes a broad look at adults, mathematics, culture, and society; Section III focusses on adults, mathematics, and work; and Section IV focusses on teaching mathematics to adults. Our aim in editing the book was not to attempt to shape the field, but rather to convey a picture of a rapidly developing and still very open field, showing something of its development to date, at least amongst those writing in English (although not always as their first language). Accordingly, rather than concluding with a chapter attempting to sum up developments in the field, the book ends with a Postscript on the legacy of the late Brazilian educationalist, Paulo Freire, for adults learning mathematics.

Most of the book's contributing authors are present or past members of ALM and many of them were attending the ALM7 conference; some attended this workshop. We were proud to reflect that through our contributions to the book, as well as through ALM, we had contributed to developing and raising the profile of the field. The book amply demonstrates the international nature of work in the field, with chapters by, for example, Gelsa Knijnik, reporting on her work in mathematics education with the Landless People's Movement (Movimento dos Sem Terras, MST) in Brazil, Gail FitzSimons, who looks at mathematics in the Australian vocational education and training system, and Katherine Safford, whose innovative approach to teaching algebra in the United States is revealed through the voices of her adult students; other contributors discuss work in Denmark, Israel, Argentina, the United Kingdom, New Zealand, Ireland, and Australia.

In planning the book we were mindful of the fact that research in the field is characterized by multiple perspectives and research paradigms. Given this diversity, it was important to decide what constitutes research in the field and for the purposes of the workshop we adopted Bishop's (1992, p. 711) view that a study must have three components:

- Enquiry...research must be intentional enquiry.
- Evidence...evidence samples the reality on which the theorizing is focused.

- Theory...theory is the essential product of the research activity and theorizing is, therefore, its essential goal.

Contributions to the book by, for example, Adrian Simpson and Janet Duffin on trying to understand adults' thinking about mathematics, Juan Carlos Llorente's enquiry into adults' mathematical processes in jam-making in Argentina, and Diana Coben's work on mathematics and common sense in relation to adults' mathematics life histories, could all be seen as examples of intentional enquiry, involving the sampling of evidence and theorizing.

We also felt that there was a need to differentiate between learning and education, where appropriate. We tried to situate adults' mathematics *learning* in terms of time, place, social/cultural context, prior education, and possibly other factors, in line with Bishop, Hart, Lerman, and Nunes (1993, p. 1), who identify four groups of influences which appear to be of crucial importance for learners of mathematics. These influences seem to hold true for adult learners as well as children. They are:

- the demands, constraints, and influences from the society in which the mathematics learning takes place;
- the knowledge, skills, and understanding that the learner develops outside school and which have significance for their learning in school;
- aids to learning such as teaching materials, texts and computers;
- the tutor or teacher.

We also attempted to situate adult mathematics *education* in the wider field of lifelong learning and lifelong education and within mathematics education as a whole. We needed "tools to think with" and found Bishop's (1993) distinction between Formal Mathematics Education (FME), Non-formal Mathematics Education (NFME), and Informal Mathematics Education (IFME) helpful as far as it went. Bishop developed his categorisation on the basis of Coombs' (1985) distinction between different types of education. Coombs describes Formal Education (FE) as full-time, sequential, extending over years, intended for all young people in society. Non-formal Education (NFE) is any organized, systematic, educational activity carried on outside the formal system to provide selected types of learning to special subgroups, for example, adults; it is often part-time. Informal Education (IFE) is a lifelong process through which people acquire knowledge, skills, attitudes, and insights from experience. It is unorganised, unsystematic, and sometimes unintentional, but accounts for the great bulk of a person's total lifetime learning.

The last two categories are seen by many adult mathematics educators as the most relevant to their work and there was lively discussion in the workshop of the extent to which any *education* (as opposed to *learning)* can ever be unintentional. The prevailing view was that education cannot be unintentional, hence the categorisation was flawed. However, given the prevalence of the FE/IFE/NFE formulation in the literature on adult learning/adult education it was difficult to avoid. Furthermore, it has become commonly accepted for a very good reason: the need to differentiate the education of adults in a multiplicity of contexts outside formal schooling from the stereotypical view of the school classroom that prevails even today, perhaps especially in the popular view of mathematics education.

We also considered Paul Ernest's (1998) categorisation of research in mathematics education. His framework encompasses research into:

- the teaching and learning of mathematics at all levels in school and college;
- out of school learning (and teaching) of mathematics;
- the design, writing and construction of texts and mathematics learning materials;
- the study of mathematics education in pre-service teacher education;
- the graduate study of mathematics education texts and results;
- research in mathematics education at all levels.

This framework was felt to achieve the right balance between specificity and generalisation and offered a way of locating adults' mathematics education within a broader perspective on research in mathematics education for learners of all ages. Ellerton and Clements' (1998, p. 154) view that mathematics education research "is an enterprise conducting careful studies that are informative, in the sense that they generate share-able knowledge that is simultaneously non-trivial, applicable, and not obvious" is also applicable.

One participant, Dorothea Steinke, made the interesting suggestion that we should look at the "field" of adults learning mathematics and research on it in a completely different way: as a sea. She pointed out that the ancient Greeks and Romans had conceptualised the world as centring on the Mediterranean, while for the Vikings the sea was the important thing, a route to new and unknown (to them) lands, including, possibly, North America. She argued that we were in danger of focussing too hard on the "land" (the parts of adult mathematics education which we already know) and not seeing the possibilities opened up by a more "fluid" conception.

Another view represented in the workshop by Juan Carlos Llorente, amongst others, and in the book, was that adult mathematics education, like any other curriculum area within the education of adults, should be considered in relation to a broader conception of education *per se*. In other words, the fact that we know that in many instances when people say "education" they are thinking of the education of children in school (FE, in Coombs' terms, or FME for Bishop) should not prevent us from arguing for a more inclusive conception, encompassing education at any stage of life, along the lines proposed by Malcolm Tight (1996). He states that lifelong education: builds on and affects all existing educational providers, including schools and Higher Education Institutions; extends beyond the formal educational providers to encompass all agencies, groups, and individuals involved in any kind of learning activity; and rests on the belief that individuals are, or can become, self-directing, and that they will see the value in engaging in lifelong education (Tight, 1996, p. 36).

While we cannot pretend to have come to any definitive conclusions, the workshop certainly continued the work of contributors to the book, and participants at other sessions at the ALM7 conference, in exploring issues in research and practice for adults learning mathematics and defining a new research domain through our individual and concerted efforts.

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Whose Thinking Is It Anyway? Role-Pair Blends in the ALM Community

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Evidence of tension between teachers and researchers regarding the work that researchers undertake in classrooms surfaces from time to time. The title of this conference—"A Conversation between Researchers and Practitioners"—testifies to the importance of interaction between the users and providers of research. Indeed, in this paper, we will argue that the traditional role-pair interactions, which may be the cause of the tension, can be replaced with what we will call a "blended role-pair." This blend, we suggest, merges research and action, connects the researcher with the researched, and combines the ownership of the knowledge produced from the research.

Further, the unique nature of working with *adults* learning mathematics allows a similar role-pair blending to take place between practitioners and students. The symmetry of the problems associated with traditional teacher-pupil role-pairs and those associated with traditional researcher-practitioner role-pairs allows us to consider a symmetrical response. This blended role-pair, we suggest, merges presentation and construction of mathematical knowledge, connects the teacher and the learner, and combines the ownership of the mathematics—all concepts in the best tradition of the ALM community. Further, we will speculate that the triangle of role-pairs can be completed by considering the researcher-student pairing and whether a similar blending can be usefully developed.

Traditional Pairings

Skemp (1979) introduces the notion of the role-pair. He defines the concept of a "role" as a schema that is brought to bear in a particular situation, delineating the forms of interaction the person can have with others in this situation. Other researchers have called this a script (Shank and Abelson, 1977) or a frame (Davis, 1982; Minsky, 1975). In each case, the role is a mental structure (evoked by a given situation) that brings with it certain expectations, certain degrees of freedom, and certain degrees of restriction. A doctor, for example, has the freedom to ask their patient to submit to investigations that would not be permitted in other roles and, with those freedoms, come restrictions about what the doctor may *not* do. Of course, this example amply demonstrates that the role is not absolutely fixed, nor is it made completely explicit (even where some culture imposes quite rigorous legislation).

Skemp notes that most roles in fact form half of a role-pair. The doctor's role is matched by a role played by the patient. Each half of the role-pair has a "script" of freedoms and restrictions. The lack of an entirely deterministic script on either side inevitably leads to some possible conflict in even very well defined role-pairs, but generally the two scripts are roughly compatible.

Of course, the role-pairs of most interest here are the teacher-student and teacher-researcher pairings (and, at the end, we will briefly consider the research-student pairing). In the teacher-pupil role-pair, which Skemp examines in the traditional context of the compulsory schooling of children, each side of the role-pair has a script which indicates, to some extent, what the other side should be doing.

For example, in the traditional caricature of teaching, the teacher's script contains expectations that pupils will attempt to do the tasks which have been set, while the pupil's script contains expectations that the teacher will set tasks from which they will learn. Within this traditional caricature, the pupil's script may contain the expectation that the teacher has a complete command of the subject matter while the teacher's may contain the expectation that their expression of their knowledge is to be accepted as fact. Brousseau and Otte (1991) call the set of expectations and restrictions in a teacher-learner situation the "didactic contract." In situations in which the behaviour of either the teacher or the pupil moves outside that set by the corresponding script, a conflict can be caused (Moshovitz-Hadar, 1993).

Of course, in different classrooms quite different didactic contracts are negotiated and different forms of role-pair are developed. Later, we will suggest that the opportunity exists in the ALM classroom to form a "blended" role-pair in which the ownership of knowledge is shared, the traditional sharp teacher-pupil distinction blurred, and the construction and presentation of knowledge are shared.

We have argued elsewhere (Duffin & Simpson, 2000) that blending can similarly address some of the perceived problems of traditional researcher-practitioner role-pair. Saul (1995) writes of his concern for the value of mathematics education research to him, "having searched the standard journals in vain for insights" into his classroom. This concern tells of his perception of the separation between himself and those who publish in standard journals. Some of those, having moved from school teaching to university lecturing, recall this sense of separation: Day (1995), for example, "never knowingly read a research paper" as a teacher.

There is little doubt that if teachers can get access to the results of educational research, can relate it to their classroom practice, and can modify that practice in ways they feel is warranted by the work, it can be of great benefit. Saul and Day highlight the problem that this access is far from simple. It may well be the case that research is not easily available to practitioners, because it is published in journals housed in university libraries that not all can access rather than in the professional journals read by practising teachers. Moreover, the language of academic journals tends to be that developed for the sharing of information between researchers and their peers rather than being in the linguistic register of the practising teacher. Indeed, even within the research domain, some work can be inaccessible to other researchers where very specific language has been developed by a particular group within the whole research community so it is not surprising if teachers find research difficult to read.

These concerns seem to fit with the caricature of traditional research paradigms. Grundy (1994) highlights his concerns about this type of research. Along with other criticisms, he notes that much research:

- separates research from action;
- separates researcher from the researched; and
- gives the ownership of knowledge to the researcher and not the practitioner.

Wright (1997) notes similar concerns about this traditional pairing: the researcher "represents the 'other'"—through their writings they speak with their voice about another's culture.

Working from the ideas of Daly (1970) we have elsewhere noted that traditional forms of research render some legitimate questions about the teaching and learning process invisible. For example, in accepting only quantitative evidence as significant (Tooley, 1998), whole areas of concern including those which may be of direct relevance to classroom practitioners may be dismissed.

Day notes a further concern that he sees particularly in newly appointed education lecturers. Traditionally recruited directly from successful school teaching, they are required to satisfy the research requirements of their new job. However, as Saul's and Day's comments indicate, the caricature role-pair script they bring with them from their positions as teachers may contain the very concerns raised by Grundy as expectations for research!

We are not suggesting that Grundy's concerns apply to all educational research. Indeed, the field of action research, particularly where the researcher and the practitioner are the same person, clearly avoids Grundy's points completely. Similarly there are cases such as Jaworski (1994) and Nolder (1992) who moved from being successful classroom practitioners to roles as university researchers and, in doing so, brought with them a sensitivity to the views and concerns of teachers which became an integral part of their research.

Our Way of Working

While Day provides one possible answer to Grundy's concerns through the adoption of an action research framework, we will suggest an alternative strategy. We offer this model as a generic example of collaborative working which could be adapted by others to their situation.

The development of our way of working came from a shared interest—initially in just one piece of work. Our interest in the work of an 8 year old girl in developing a general method for squaring numbers led us to share our views (Duffin & Simpson, 1991). In doing so, we found that the root of our interest in the work was profoundly different. One of us focussed on the formal generalisability and efficiency of the mathematical algorithm. The other focussed on the way in which the pupil had come to develop the method for herself based on her experience of a distinctive teaching style (within the CAN project described by Shuard, Walsh, Goodwin, and Worcester, 1991, and Duffin, 1996).

The fact of these wildly contrasting perspectives became almost of as much interest as the girl's work and we began to focus on how our own backgrounds had influenced our reaction to the pupil's work (in the same way that the pupil's background within the distinctive teaching style had influenced her methods).

At this time, it was not clear to us that we were doing research. However we were prompted through our conversations with others to reflect on the ways in which we worked to develop our ideas.

In doing so, we saw three characteristics to our way of working:

- Introspection: the consideration of our personal reactions to any episodes or issues we investigate.
- Co-spection: the sharing of those personal reactions so as to arrive at a composite view of the experience we are investigating.
- "As if from inside": the attempt to see the situation from the viewpoint of the learner.

These three characteristics allow us to *blend* our roles. Introspection allows us to examine the way in which our own background and examinable mental schemas influence our reactions to situations. In doing so we have some sense of direct access to the ways in which internal mental structures manifest themselves as external behaviour. Clearly the problems associated with this mirror those highlighted by Watson (1913) in the early development of scientific approaches to psychology. However, these are countered by combining introspection with the other two characteristics. With "co-spection" we begin by sharing our personal reactions and the explanations from our introspection. In doing this we gain access to another (usually quite different) perspective on learning and behaviour. However, co-spection goes beyond this to attempt to develop a "fused" perspective in which both explanations join together to give a richer interpretation of the incident under discussion.

With the final characteristic, "as if from inside," the three different perspectives (two introspective and one fused co-spective) on the ways in which internal structures manifest themselves as external behaviour allow us to ask a question of the learning incident we see. What internal mental structures might this learner have which would explain the external behaviour? In doing this, we can observe a learner "as if from inside."

Blending the Researcher-Practitioner Role-Pair

These characteristics, developed over the years of the research partnership, result in a role-pair which no longer has the sense of distinct elements (with different obligations, rewards, and behaviours) in Day's description of traditional, separate researcher-researched role-pairs.

Any particular implementation of this model relies on the individuals within the role-pair. Clearly the distinct forms of explanation generated through introspection are, of necessity, unique. We do not believe, however, that this prevents the general form of the model being adapted to blending the user/provider role-pair in other situations.

Within the ALM community, the researcher-practitioner role-pair provides excellent nourishment for the growth of this model. The personal explanations generated by introspection will probably come from quite different perspectives. The sharing of the explanations, through co-spection, will rely upon both the intimate involvement with the classroom culture of the teacher and the research culture of the researcher. This fused perspective could give a much clearer insight into the ways in which internal mental structures manifest themselves, in the classroom context, as behaviour.

Clearly the blended role-pair produced is based on the different perspectives, formed through introspection and shared through co-spection. Thus each role-pair formed brings a set of lenses through which to view incidents "as if from inside."

Facing the Concerns

In its ideal form, the model proposed addresses the three concerns set by Grundy.

Because our way of working results in a shared perception of incidents and learning episodes encountered, practitioner and researcher would be able to perceive classroom episodes with a shared perspective. This would mean that the teacher's knowledge of their classroom would now be shared more fully with the researcher who would therefore no longer be separated from the researched, for the researcher would now be in a position to see the issues which concern the practitioner through that shared perspective. This does not mean that either lose their personal perspective but that, in the fusion of perspective that comes from the collaboration, each would have a new enhanced perspective that includes both their original perspectives. Thus the blended role-pair is as much a part of the teacher's classroom as the teacher is and the separation of researcher from researched is removed. Further, the blending of the role-pair also addresses Wright's concerns about "representing the 'other'"—the practitioner is no longer the "other" in the role-pair and, as Wright suggests in her work, can develop the skills of being a researcher themselves.

By taking part, with the teacher, in what goes on in the classroom, and with the shared perception that they would have developed, the research undertaken would become part of the teacher's action, now shared with, and by, the researcher. So the traditional problem of the researcher coming into the classroom to find answers to their own research questions which had little connection with the teacher's classroom concerns will be subsumed in their now common purpose and interest. It would make the agenda a commonly perceived one rather than being that of either the researcher or the teacher, as separate people. Thus the actions of the blended role-pair are the unified actions of teaching and researching.

Similarly, because knowledge is constantly and repeatedly being generated and shared between researcher and teacher so that it becomes the possession of both, both will own the research, both will be dedicated to the pursuance of the questions to which they both want answers, and both will own the knowledge they gain from the research. Indeed, rather than seeing the knowledge as equally shared (which implies a fair *division*), the *whole* knowledge and its situation within the classroom and research cultures are owned by the blended role-pair.

Adapting to *Adults* Learning Mathematics: Blending the Practitioner-Student Role-Pair

The opportunities afforded by constructing blended role-pairs in this way have particular significance for the ALM community. The role-pair described by Skemp (1979) in a traditional school setting is, in his terms, an "imposed" role-pair. The pupil is required by law to attend the school and the didactic contract and the scripts developed for both sides of this role-pair are carefully maintained by the disciplinary structures of the school culture.

By contrast, the role-pair in the ALM community is "elective." While the learners may have a wide variety of reasons for wishing to study mathematics, some of which may be seen by the learner as an imposition, the teacher is probably not a part of structure which imposes. Inevitably, in the absence of an alternative, part of the script which has been developed in school may be used initially to try to understand the system when the adult learner returns to learning mathematics. Without negotiating a new didactic contract, the learner may still expect that the teacher should have complete command of the subject and that, as learners, they are expected not to question the teacher. Of course, other experiences from school may colour the initial script with which they return to learning mathematics: every question has one, and only one, correct answer (Copes, 1982); mathematics is hard (Buerk, 1982), and learning means precisely recalling the correct procedures to implement in the appropriate contexts.

Thus, the model of collaboration we have developed to blend the researcher-practitioner role-pair may be adapted to helping blend the teacher-learner role-pair and assist in the negotiation of a new didactic contract. In the researcher-practitioner blended role-pair described above, introspection involves the participants in the careful examination of their own ways of thinking. This has just as useful a function to play in constructing a blended practitioner-student role-pair. The fact that the students are adults means that they are much more likely to be able to reflect carefully upon their learning and try to make sense of it.

The sense made by both students and teacher in looking at their own ways of thinking about a particular piece of mathematics can then be shared through a practitioner-student version of co-spection. As the learners recount their own ways of thinking and listen to others, including the teacher, they may begin to question the traditional "mathematics lesson" expectation. In the elective role-pair of adult learners and practitioners, the sharing of thoughts and feelings can be value-free. This co-spection may enable the learner to blur the boundary between the teacher as having complete command of their subject content and themselves as having poor command.

The final component of the blended researcher-practitioner pairing is "as if from inside." In the practitioner-student pairing, the sharing of ways of thinking about a piece of mathematics through co-spection certainly may help the practitioner construct models of the students' ways of thinking. However, the adult students can also construct models of the ways of thinking of their peers, including the teacher. In doing so, they are constructing a wide variety of mathematical ways of thinking which they may choose to adopt.

The Calculator Aware Number (CAN) project in the 1980s and 1990s in the UK implicitly adopted some of the constructions we have suggested here (Shuard, Walsh, Goodwin, & Worcestor, 1991). In providing young pupils with calculators and in instructing teachers to avoid teaching standard arithmetic algorithms, the project encouraged a non-directive, listening, and observing style of teaching. Pupils were involved in developing their own mental and written methods with the teacher as a guide and partner in those inventions. The children felt they owned their mathematics as much as the teacher did.

Possibilities: Blending the Researcher-Student Role-Pair

The nature of the learners as adults in ALM classrooms provides one more opportunity for a blended role-pair. Although it is a concern of standard methodological texts (Cohen & Manion, 1994), traditional research on learners in school almost obscures the issue of pupils' rights to the ownership of research. While permission is sought from the children (and their parents) and the results of the research may nominally be made available, there is an even stronger power imbalance in the traditional researcher-pupil role-pair than in the traditional researcher-teacher pair.

However, the ability of adult students to reflect on their thinking as outlined above gives the opportunity to blend this last role-pair in the triangle. In Aspinwall, Shaw, and Presmeg (1997), the mature student Tom is highly articulate about his thinking, is able to share some of the deepest aspects of his teacher and, to some extent, look back upon himself with some detachment (in a version of "as if from inside"). Ultimately, one could imagine opportunities for researchers and adult learners of mathematics to move through versions of introspection, co-spection, and "as if from inside" and produce research that is the genuinely co-owned product of a blended researcher-student role-pair.

The most obvious way in which we can see this happening is where the teacher takes on the role of researcher of their own classroom. In developing a partnership with her students, Tomlin (1998) encourages students to investigate their own thinking. Similarly, O'Hagan (1994) demonstrates an openness with her students and a sense of learning together which begins to blend their roles. Thus, even in the last role-pair under consideration, we may be able to see value from blending.

The triangle of researcher, practitioner, and student role-pairs, in its traditional form, separates knowledge and divides ownership. Blending role-pairs through the adoption of new ways of introspecting, co-specting, and examining others (and, paradoxically, ourselves) "as if from inside" allows knowledge to be connected and ownership of that knowledge to be combined.

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Two Dilemmas in Communicating Mathematics in Adult Basic Courses: "How to Meet Pre-Knowledge of Adult Learners" and "How to Support Democratic Classroom Decisions"

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Conversation Between Researcher and Practitioner

It just seemed so obvious. As the theme of ALM-7 was announced as "A Conversation Between Researchers and Practitioners," it came naturally for us to prepare a joint presentation: Lena Lindenskov as a researcher in mathematics education and Eigil Peter Hansen as an adult teacher in mathematics. Our common aim of the presentation at ALM-7 was to present and discuss experiences from a one-year mathematics course in adult basic education, conducted by Eigil and observed by Lena. This course was given in a suburb of Copenhagen, the capital of Denmark.

We acknowledge the need to develop, explore, and document different ways of connecting theory and practice in adult mathematics education—different ways of cooperation between research and teaching. Our cooperation in this project is one way among several possible ways, and we might choose another way in future cooperative projects. Our cooperation this time can be characterized by the following components:

- The teacher has absolute power over mathematical content and pedagogical methods, and the researcher has no influence.
- The students have the right to a veto on participating in research conversations and interviews. In the first lesson the teacher asked the class if they would allow research visits, and each student was, at any time, free to break off research conversations and interviews.
- The researcher has absolute power over research questions, methods, and analyses.

The purpose of our cooperation is research and production of theoretical knowledge. Teaching practices and learning practices play two roles. Understanding for improving teaching practices and learning practices is an *ultimate goal*, as for all theorizing in adult mathematics educational research. But teaching practices and learning practices are also *means*: instead of setting up any experimental design, ongoing teaching practices and learning practices in natural settings provide grounds for theoretical analyses.

Lena started the presentation with her relatively optimistic view on research-teaching cooperation, strongly supported by personal experiences of taking different roles as a teacher and a researcher. Lena sticks to the obligation of ensuring research being not-teaching. In research you can observe and interview individual students and groups, immerse yourself in the extensive literature in mathematics education, and visit and observe the diversity of math-containing practices in different educational institutions and different settings outside education. These are activities which are not included in normal teacher practices, and that's why it is important that researchers make the most of it.

Research Questions

The project we presented at ALM-7 provides knowledge of different agents' understanding of everyday mathematics inside and outside education. The project explores how adults' perspectives, intentions, blockages, resistance, and fascinations are being reconstructed during a mathematics course in adult basic education. The project compares adult mathematics education in different institutional settings: adult basic education; adult vocational training; and informal, general education (the last is called day-folk-high schools).

In the model below, which consists of four areas, the core questions are how the fourth area influences the third area. The fourth area contains elements that are specific to the profile of the particular educational institution. It includes elements such as the profile of the teachers, curriculum, tasks, materials, fitting up of the classroom, and the establishing of relations between the ongoing teaching-learning practice and other practices.

Research Model

F o u r t h

First Area	Second Area	Third Area
The adults' experiences from inside and outside schooling	The adults' Math beliefs Math attitudes Math feelings Math concep-tions and abilities	The adults' Perspectives Intensions Blockages Resistance Fascinations

A r e a

The specific research questions were:

1. How do elements in the fourth area facilitate the adults' intentions (of what and how to learn and use methods and wisdom in mathematics) in being expressed, lived, and developed?
2. How do elements in the fourth area facilitate the adults' blockages and resistance in being expressed and interpreted by others as blockages and resistance?
3. How do elements in the fourth area facilitate the adults' fascination in being expressed, lived, and developed?

The idea for Lena's observing and documenting Eigil's teaching and the adults' learning was to explore, to learn, to be surprised and let new doubts arise. Lena has learned a lot. She has started thinking of teaching as being confronted with dilemmas without best solutions, and thinking of dilemmas as core issues to reflect upon. Concepts of dilemmas might become important building blocks for theory development.

The Climate for Learning in the Class
Starting the class it was important for Eigil to support a good climate for learning. The class spent time on mattering: beliefs people have, whether right or wrong, that matter to someone else, that are the object of someone else's attention, and that others care about and appreciate. Eigil sees four dimensions of mattering:

- Attention
  The feeling that another person notices you or is interested in you.
- Importance
  Others seem to care about what you want, think, and do.

- Dependence
  You feel that you are a contributing member and others are counting on your participation.
- Ego-extension
  You believe that others are interested in your successes and disappointments and actively follow your progress.

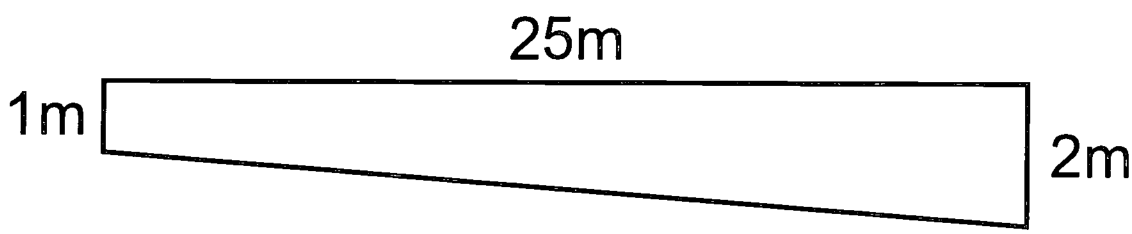
The First Dilemma
We define a dilemma as a right-right choice situation. In the presentation we talked about two dilemmas which we consider as general in teaching adult basic mathematics.

The first dilemma, faced by educational planners and teachers in adult mathematics, is how to meet the adult learners' already established conceptions and procedures:

> How are learners' established conceptions and procedures actually met by the teaching materials and how do teachers meet them in classroom conversations? And how could/should they be met?

It is often said that best practice is helping the adults become aware if they have methods to calculate, approach, and solve problems. It is important to give the adults time to remember and discuss their methods. It is important to arrange learning situations where the adults can build upon their methods. But observing and interviewing the adults in Eigil's class challenged these ideas and showed the relevance of introducing light and shadow into these ideas.

Among the adults in Eigil's class we saw three groups. The first group did not have any methods themselves. They liked to be introduced to and to engage in developing new methods. They might feel lost and spend time for no purpose in classroom work and discussions on learners' methods. One adult articulated it this way: "Oh, but I have nothing in my own head, so of course I want to get methods from the teacher and use those methods." A second group had their own methods and actually did not care about the teacher's or other learners' methods. The second group did not try to make new methods usable. In the presentation we showed examples from working with area and volume where adults stuck to their own old methods.



Different ways to calculate the area:

$$1 \times 25 + \tfrac{1}{2} \times 1 \times 25$$
$$2 \times 25 - \tfrac{1}{2} \times 1 \times 25$$
$$\tfrac{1}{2} \times 25 \times (1+2)$$

A third group had some methods already, but improved them or replaced them during the course. We gave examples in the presentation from calculating percentages, where several different methods existed among the adults. The different methods were discussed in the class.

Examples of different methods of calculating problems with percentages:

150 increases by 15%: 150 + 15% of 150 (150 + 0,15 x 150) or 115% of 150 (1,15 x 150)
150 decreases by 15%: 150 – 15% of 150 (150 – 0,15 x 150) or 85% of 150 (0,85 x 150)

150 increases to 180 - How many percent?
180 – 150 = 30 (30/150) x 100 = 20% or 180/150 = 1,20 (= 120%) = an increase of 20 %

150 falls to 120 - How many percent?
150 – 120 = 30 (30/150) x 100 = 20% or 120/150 = 0,80 (= 80%) = a fall of 20%

As we see the question of how to meet existence and non-existence of pre-knowledge as a dilemma, we cannot describe just one right answer to give planners and teachers. We see many right answers as to how to meet pre-knowledge. Which answers are better than other answers depends on complex elements, e.g., the institution's and the adults' intentions with the course. And it might be highly influenced by individual characteristics.

A year after the course ended, we invited the adult learners to an informal meeting. Only three came. A remark from one of them illuminated the dilemma, when she told that one day at her job she told about the new percentage method from the course. She had been both proud of the method and worried that it was far from everyday use. Then one of the more educated colleagues responded that it was a well-known method that was often used in the company. After that day she always used the new method. For us it is obvious that the social reaction to new methods from education is overwhelmingly influential, e.g., as to whether the adults will remember, appreciate, and use new methods and knowledge or not.

The Second Dilemma
The second dilemma concerns the power of the learners. To give you some idea of the learners' intentions and feelings, let us tell you about Anett. She was born in 1966. In her thirties it became her intention to get an education. Mathematics was her worst subject in school as a child. She told us that still as an adult she can feel shocked when a mathematics teacher approaches her, but expected that as an adult it MUST be possible for her to understand mathematics. She said she finds it highly demanding to express out loud when there is something she does not understand. She knew she herself was responsible for getting it expressed, but still it was felt both difficult and tough. The best moments for her were the warm feeling of understanding. Then she felt like crying out loud: "YES YES YES, I understand." After those moments she went home thinking it was worth the struggle.

According to Danish legal provision, the learners participate in the ongoing planning of the course. In Eigil's class two themes were democratically chosen: *The first was art, the second was food.* Most of the adults imagined "food and mathematics" more relevant to everyday life than "art and mathematics," e.g., Anett. Anett spoke highly in favor of food and mathematics in the democratic decision process, but she did not mention food and mathematics at all at the end of the course. Instead she wrote that working with mathematics in art had been "cool." It was fun and exciting, combined with creativity (cutting and pasting), measuring, calculating, hanging up the pictures, and getting the photos of themselves. At the end of the year the adults as a group considered "art and mathematics" as being mostly relevant. This was confirmed a year later at the meeting. The adults said that mathematics and art had changed their way of looking about their surroundings and had given inspiration for holiday activities with their sons and daughters. So the second dilemma is how to provide the learners with adequate information to base their decisions upon. Again it is not to be expected to find the one and only right answer, but to be able to reflect upon several good answers.

We have given some documentation on how some Danish adults choose between proposals, and we feel the need of two concepts on motivation in order to understand the dilemma:

- We need a concept of what might be called "Motivation A." Motivation A drives you to choose between proposals.
- We also need a concept for what might be called "Motivation B." Motivation B drives you through the ongoing work, the detailed mathematical ideas, the detailed tasks, the detailed calculations, etc.

What kind of material should we then provide the learners with? We doubt that a discussion of the relevance of the theme (art, food) is suitable. That only serves the purpose of involving motivation A, but not motivation B. The examples given show that when a theme such as "mathematics in art" has the potential of expanding the horizon of the learners, but is not directly applicable in everyday life, then motivation A might be low, and motivation B might be high.

Why "Motivation A" might be high for mathematics and food
The students deal with different items such as:

- cooking, recipes, temperatures;
- new and old units of measurements;
- shopping;
- calories, weight, exercise.

Why "Motivation B" might be high for mathematics and art
The students deal with different items such as:

- basic rules and tools for rectangle, circle, and triangle;
- permutation;
- the divine fraction and the golden rectangle;
- area measuring;
- the Pythagorean theorem.

And at the same time they learn and train rules about mathematic models, equations, etc.

Mathematics Anxiety and the Adult Student: An Analysis of Successful Coping Strategies

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A number of strategies designed to alleviate mathematics anxiety have been proposed (Hembree, 1990; Higbee, 1989; Robertson, 1991; Tobias, 1987). A partial list includes attending a mathematics tutoring clinic, keeping a "math autobiography," engaging in relaxation training, assertiveness training, stress inoculation, or systematic desensitization, learning to improve time management skills and study habits, and even adopting a "Math Anxiety Bill of Rights" (Davidson & Levitov, 1993; Hackworth, 1982; Tobias, 1991).

It has been suggested that just as mathematical problem solving has both a cognitive as well as an affective component (Bessant, 1995; Fennema, 1989; McLeod, 1988), so do intervention strategies designed to alleviate anxiety (Robertson, 1991; Williams, 1988). Cognitive (or math-dominated) interventions focus on the learning of mathematics content, assuming that the more mathematics students understand, the less anxious they will be, while affective interventions focus on psychological techniques such as peer support groups, counseling, and relaxation training that help students reduce their anxiety level.

The above list of coping strategies designed to help students alleviate mathematics anxiety has been presented in many formats, such as self-help manuals, diagnostic clinics, videotapes, pamphlets, handbooks, and seminars (Arem, 1993; Davidson & Levitov, 1993; Hackworth, 1982; Kogelman & Warren, 1978; Robertson, 1991; Sembrera & Hovis, 1993; Zaslavsky, 1994). Nonetheless, there is a lack of empirical research on student and faculty assessment of their comparative effectiveness. The present study examined this issue.

The primary purpose of this study was to evaluate the relationship between college students' level of mathematics anxiety and the strategies they employ to cope with it. Additionally, the study considered the effects of course enrollment, in either a remedial algebra course or a nonremedial precalculus course, and gender upon students' assessment of coping strategies. A secondary goal of the study was to examine both counseling and mathematics faculty's ratings of the same coping strategies that the student subjects evaluated. Aside from analyzing each group of faculty assessments individually, the study also compared these groups to the student ratings of the coping strategies.

Two-hundred seventy-nine community college students, enrolled in either a remedial algebra or a nonremedial precalculus course, completed the Composite Math Anxiety Scale (Tobias, 1993). This psychometric instrument, composed of twenty Likert type items, was adapted from the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1986) in order to provide an overall mathematics anxiety score. Afterwards, the students were asked to rate ten Likert type mathematics anxiety coping strategies, designed by the investigator, with regard to frequency of use and helpfulness. This set of coping strategies is presented in the Appendix. In addition to the student subjects, a total of fifty faculty members from the Mathematics and Student Development (Counseling) Departments at the same community college rated the set of coping strategies, but only in terms of faculty perception of helpfulness to students.

A multivariate analysis of variance (MANOVA) was performed on the student data. The three independent variables were Mathematics Anxiety level (high or low), Gender (male or female), and Course Enrollment (remedial or nonremedial). The dependent variables were the ten coping strategies, each of which was rated for frequency of use and helpfulness. In addition, the mean scores of the coping strategies (in terms of the helpfulness factor) were rank ordered and compared among the student, mathematics faculty, and counseling faculty groups.

A major finding of the study was that students with low mathematics anxiety both utilize and value a wider variety of coping strategies than their high anxiety counterparts. In fact, seven of the strategies (RELAXATION, DISCUSS STUDENTS, ASK QUESTIONS, HOMEWORK, REMIND YOU'RE GOOD, EXTRA STUDY TIME, and INSTRUCTOR KNOW), were preferred more by students with low mathematics anxiety than by those with high mathematics anxiety. Perhaps, because anxiety itself has a disabling effect on students, a low anxiety level may place students in an enhanced coping mode, thus empowering them to participate in the majority of coping strategies considered in the study.

It is also important to consider the types of coping strategies in which low vs. high mathematics anxiety students engaged. Namely, high mathematics anxiety students used tutoring services (TUTOR) and had discussions with their counselors (DISCUSS COUNSELOR) significantly more than low anxiety students. Both of these behaviors, which were the only two engaged in more frequently by high mathematics anxiety students, were considered among the least helpful by all of the students.

There were significant gender differences for three of the coping strategies. Practicing systematic relaxation, physical activities, or exercise (RELAXATION), an "avoidance" strategy which males utilized more than females, was considered one of the least helpful coping strategies by all students as well as by both groups of faculty. Alternatively, completing homework assignments on time so that you don't fall behind (HOMEWORK) and letting your instructor know if you don't understand the course material (INSTRUCTOR KNOW), two "approach" strategies which females found more helpful than did males, were both regarded as among the very most helpful coping strategies by all students as well as by both groups of faculty. A few significant differences were also found for course enrollment. Algebra students utilized certain strategies more than did precalculus students.

The three groups of subjects that participated in the study (mathematics students, mathematics faculty, and counseling faculty) essentially agreed on the helpfulness of the coping strategies, placing HOMEWORK, EXTRA STUDY TIME, ASK QUESTIONS, and INSTRUCTOR KNOW, all of which are "approach strategies" (since the individual directly confronts the stressor), in the top half of the list. This ranking is consistent with the work of Holahan & Moos (1987) who found such strategies to generally be the most successful category of coping behaviors as compared to "avoidance strategies" where the individual temporarily leaves the stressful situation in order to reduce anxiety.

Although the three groups of subjects fundamentally concurred in their ratings, some differences are noteworthy. First of all, both the counselors and mathematics instructors rated using a tutor (TUTOR) as more helpful but rated practicing systematic relaxation, physical activities, or exercise (RELAXATION) as far less helpful than did the students. The first difference possibly may be due to the fact that both groups of faculty can only perceive the tutoring experience from an outsider's point of view. The students themselves, who actually partake in the tutoring process, may not regard it as being valuable since they are more aware of both its advantages as well as its shortcomings. One possible example, although not specifically assessed in this study, may be that some tutors use different methods to explain course material than those used in class by the mathematics instructors. This dichotomous approach can be potentially disadvantageous.

In sum, as previous research suggests (Holahan & Moos, 1987), this study has demonstrated that approach strategies are regarded as the most helpful group of coping behaviors and that low mathematics anxiety students tend to utilize and value coping strategies more than do high mathematics anxiety students. A smaller number of coping strategies yielded significant differences with respect to gender and course enrollment, with females showing a tendency to utilize more coping strategies than males and algebra students more than precalculus students. In addition, mathematics students, mathematics faculty, and counseling faculty all essentially agreed when rating the coping strategies in terms of their helpfulness. All three groups regarded approach strategies as the most helpful. In particular, asking your instructor questions in class, completing homework assignments on time so that you don't fall behind, setting aside extra study time for review before class exams, and letting your instructor know if you don't understand the course material were valued the highest.

A number of pedagogic recommendations can be made based upon these findings. These include having mathematics faculty partake in training workshops that specifically focus on approach strategies as well as having peer tutors work together with instructors in the classroom so that students will ideally regard both of them as a "coordinated team." The author has successfully implemented both of these recommendations and will report the results in future work.

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APPENDIX

COPING STRATEGIES SURVEY FOR STUDENTS

MATH COURSE (CIRCLE ONE): ALGEBRA PRECALCULUS

SEX (CIRCLE ONE): MALE FEMALE

Directions: The following is a list of strategies that students may use in order to learn mathematics effectively and do well in their mathematics courses. Please respond to <u>both</u> questions listed below <u>each</u> of the following behaviors by circling **any** number from 1 to 5 where:

1 = not at all 3 = somewhat 5 = very much.

Once again, all responses will be kept confidential and used for research purposes only.

1. Using the school's tutoring center or a private tutor.

   a. How often have you tried this? **1 2 3 4 5**

   b. How helpful has it been OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

2. Practicing systematic relaxation, physical activities, or exercise.

   a. How often have you tried this? **1 2 3 4 5**

   b. How helpful has it been OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

3. Discussing experiences or difficulties related to your mathematics course with other students in your class.

   a. How often have you tried this? **1 2 3 4 5**

   b. How helpful has it been OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

4. Discussing experiences or difficulties related to your mathematics course with your school counselor.

   a. How often have you tried this? **1 2 3 4 5**

   b. How helpful has it been OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

5. Using additional textbooks or review books other than the required text.

   a. How often have you tried this? 1 2 3 4 5

   b. How helpful has it been
   OR how helpful do you think
   it would be if you tried it? 1 2 3 4 5

6. Asking your instructor mathematics questions in class.

   a. How often have you tried this? 1 2 3 4 5

   b. How helpful has it been
   OR how helpful do you think
   it would be if you tried it? 1 2 3 4 5

7. Completing homework assignments on time so that you don't fall behind.

   a. How often have you tried this? 1 2 3 4 5

   b. How helpful has it been
   OR how helpful do you think
   it would be if you tried it? 1 2 3 4 5

8. Reminding yourself that you are a good student if you start to feel incompetent.

   a. How often have you tried this? 1 2 3 4 5

   b. How helpful has it been
   OR how helpful do you think
   it would be if you tried it? 1 2 3 4 5

9. Setting aside extra study time for review before class exams.

   a. How often have you tried this? 1 2 3 4 5

   b. How helpful has it been
   OR how helpful do you think
   it would be if you tried it? 1 2 3 4 5

10. Letting your instructor know if you don't understand the course material.

   a. How often have you tried this? 1 2 3 4 5

   b. How helpful has it been
   OR how helpful do you think
   it would be if you tried it? 1 2 3 4 5

A Review and Summary of Research on Adult Mathematics Education in North America (1980-2000)

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Introduction

Since its inception as a field of study at the turn of the last century, the discipline of mathematics education has matured and become accepted by the North American education community as a credible research area (Kilpatrick, 1992). Over the past twenty years, substantial research has been conducted at all levels of mathematics education, kindergarten through graduate school, and reported in dissertations and journals. The identification of the subset of that research that examines mathematics education for adult or non-traditional students is the subject of this study. In this presentation I will report on the dissertations in mathematics education that are concerned with that population.

Methodology

The databases of Dissertation Abstracts International were searched using the Boolean argument "Adult AND Mathematics AND Education" on the "Subject" key. Two-hundred-and-five dissertations were found, 195 dating from 1980. The abstract for each dissertation was downloaded, printed, and reviewed for applicability. Eighty-two were culled for a variety of reasons. Some did not report research conducted in North America, others, although typified as adult, did not address the teaching of mathematics to adult students. For example, the students tested in one study were junior high school students. Eight abstracts were removed because they addressed interventions with teachers of mathematics and were deemed by this researcher to be concerned more with teacher in-service rather than direct teaching of mathematics to adults. Admittedly, this was a subjective decision and another researcher might choose to include them.

The journalist's questions of "Who, what, when, where, why, and how?" served as the basis of the analysis of the abstracts. The exact questions are indicated later in this report, but one example would be "Who is funding or supporting the research?" A coding template was then superimposed on each abstract printout and the data coded. Again, subjective decisions were made at times and the researcher invites others to duplicate this process in order to validate her findings. Results were then totaled and recorded in a spreadsheet, the contents of which served as input to the creation of the overhead transparencies shown during this presentation.

Who

From its inception, Adults Learning Maths (ALM) has been a forum that promoted dialogue between researchers and practitioners. An integral part of that dialogue is the encouragement for practitioners to undertake research projects in their work and an invitation to researchers to apply theory to practice by working in learning situations. The first "Who?" question, therefore, is "Who is reporting research?" The abstracts did not always provide clear answers to this question. In some cases, personal knowledge allowed me to assign the designation. Otherwise, unless the abstract clearly identified the author as a practitioner, the individual was counted as a researcher. As a result, an overwhelming majority of the dissertations, 88%, were determined to have been written by candidates whose primary role was researcher.

The second question under this heading was "Who funded or supported the research?" Support in many cases was manifest as "permission to conduct" rather than financial support. This was determined to be the case when an abstract indicated that the research was conducted in a particular section or class within an institutional setting. Sixty-three percent (71) of the dissertations fell into this category. Support for 23% (26) could not be determined from the abstract. The remaining 16 dissertations were funded by government agencies (11%) or industry (4%).

What

In recent years, qualitative research has continued to gain acceptance within the mathematics education research community. Abstracts were therefore examined to determine if the researcher had incorporated one or more qualitative methodologies into the study. Only 17 (15%) dissertations were determined to be purely qualitative studies. An additional 14 (12%) combined qualitative and quantitative methods. A full 82 (73%) were conducted using quantitative methods. The following table shows a breakdown by year of studies that employed each method:

Table 1

Type	*80-81*	*82-83*	*84-85*	*86-87*	*88-89*	*90-91*	*92-93*	*94-95*	*96-97*	*98-99*
Qual	1	1			1	2	3	2	6	1
Quan	8	7	13	8	10	7	7	10	9	3
Both	1		1	4	1	1	3		2	1
Total	10	8	14	12	12	10	13	12	17	5

It would appear that qualitative methods have gained a foothold, but still represent the method of choice for a minority of doctoral candidates.

The program for ALM-7 was constructed around nine themes that emerged from the proposals submitted. The investigation of "What?" asked the question: "What ALM-7 themes, if any, are represented by the dissertations?" While a few abstracts could not be characterized by the themes, most could. A list of the dissertation authors sorted by ALM-7 themes is contained in the Appendix. While, as stated earlier, it was rarely clear that the researcher was a practitioner, the number of dissertations that could be characterized as "Research into Practice" or "Instructional Approaches" indicates a strong link between researcher and practitioner in adult mathematics research. The following table shows the breakdown by ALM theme:

Table 2

Theme	*Number*	*Percent*
Assessment/Frameworks/Standards (AFS)	14	12
Contexts	3	3
Instructional Approaches	31	27
Parents	2	2
Research into Practice	36	32
Teacher Knowledge	4	4
Theory	2	18
Understandings	6	5
Workplace/Vocational	7	6

Other non-ALM themes emerged during the abstract review process. Twenty-six (23%) of the dissertations investigated tests or personal factors that can be used to predict the success of an individual in the adult mathematics classroom. Another 11 (10%) reported findings about classroom methods or teaching styles that contribute to student achievement. Interventions with math-anxious students were discussed in 11 (10%) of the abstracts. Finally, the application of technology to adult learning situations was reported in ten entries, nine of which concerned computer-aided instruction (CAI) and one of which involved television.

When

Adult mathematics instruction occurs at levels comparable to all traditional student settings. Adult basic education starts with the simplest of mathematical concepts. Adult secondary education and General Educational Development (GED) preparation provides the equivalent of high school mathematics content. In 1996, 3,000,000 adults enrolled in ABE classes while over 900,000 pursued ASE instruction. Some 263,000 individuals over the age of 24 took the GED test in 1998. While not all these students engaged in mathematics classes in those years, it is safe to assume that a substantial number did at some point in their studies. Post-secondary institutions offer mathematics instruction in both remedial and college-credit courses. In 1997, 35% of the undergraduate students at degree-granting institutions were at least 25 years old. If the threshold is lowered to 22 years of age (an additional 15.6% of the undergraduate population), then adult students were in the majority, 6.3 million out of 12.45 million. Now assume that students take mathematics during three of their eight undergraduate semesters. That assumption translates to 2.36,000,000 adult students taking undergraduate mathematics each year. It is probably more likely that the returning student enrolls for two semesters of remedial mathematics followed by two semesters of credit courses, so a less conservative figure would be 3.15 adult students per million per year.

The first "When?" question therefore asked the level of instruction addressed by the research in the dissertation abstracts. The distribution is quite different from the distribution of the students and, in all probability, reflects the academic position of the researcher. Fifty-eight of the dissertations (51%) were conducted at undergraduate institutions. Twenty-one (19%) were ABE level, while 13 (12%) were either ASE or GED. In some ways, the 51% figure is lower than might be expected. In the United States it is becoming increasingly difficult to obtain or maintain a position, get tenure, or achieve promotion at a tertiary institution without a doctoral degree. Candidates for a doctoral degree, therefore, would be likely to come from that educational level and use their classrooms as laboratories. On the other hand, instructors with elementary or secondary certification often supply ABE mathematics instruction. They are frequently part-time faculty and have little financial or career incentive to pursue a doctoral degree. In that light, the 19% rate is surprisingly high, even though the ABE population accounts for a substantial portion of the adults in formal instruction.

The second "When?" question asked, "In what year was the research conducted?" The reader can refer to Table 1 and see that, with the exception of a spurt of dissertations between 1996 and 1997, the rate has been fairly constant. It is interesting to note that the adult-centered degrees issued in that period accounted for 15% of the mathematics education doctorates awarded. It is heartening to note that researchers in adult mathematics education are persisting despite a paucity of research money compared to elementary and secondary funding sources.

Where

"Where" questions had less definitive answers than previous types. Abstracts were often vague about the geographic location where the research was conducted or reported as well as the institutional setting in which it was accomplished. Twenty-four dissertations (21%) gave no clue about the type of institution where the research took place. Sixty-one were situated at a tertiary institution: 31 (27%) at junior or community colleges, 30 (27%) at universities. Adult learning centers harbored 15 researchers (13%), with the remaining split between industry (6%) and government (5%). There was no apparent geographic clustering of either investigative or reporting locale.

How

The quantitative nature of most of the dissertations depended on easily quantified research instruments. Fifty-eight percent of the abstracts identified tests, predominantly normed tests, as the vehicle used to conduct the study. Forty-two percent used questionnaires or surveys. Qualitative components relied heavily on interviews (23%) with analysis of student records of transcripts (10%) and observations (9%) accounting for all but one of the studies. One dissertation was based on a case study. Keep in mind that dissertations often used more than one method, so the percentages reported total more than 100%.

Why

"Why?" was the question with the least clear answer. Obviously, the researchers undertook their task to complete doctoral degree requirements. Some reasons for pursuing the degree have already been suggested. Securing a present post or creating the potential for advancement are likely explanations. The selection of a topic for the project leaves more room for conjecture. It was disappointing to read abstracts that seemed designed as a quick and dirty approach to fulfilling a degree requirement. Far more appealing were those studies whose authors conveyed a passion for testing a theoretical framework or novel methodology as the heart of their studies. Their work provides a basis upon which later studies can be built.

Discussion

The number of dissertations that investigated adults learning mathematics was larger than I had anticipated when I undertook this project. They seem to be clustered around a few major topics: prediction of success, methods that may contribute to success, and math anxiety that may inhibit success if not neutralized or, at the very least, decreased. At first I found the interest in predicting success disappointing, almost a condemnation of teacher effort. Viewed in a positive light, however, the existing research provides a teacher with tools to identify "at risk" students. Their needs can then be addressed with interventions rooted in the research base on good practice and math anxiety programs.

What was discouraging was the paucity of studies that investigate the applicability of the K-12 National Council of Teachers of Mathematics (NCTM) standards for adult populations (NCTM, 1989, 2000). Only two of the abstracts indicated the standards and the reform movement as their focus. Mathematics as problem solving is a key theme of the new elementary and secondary curricula, yet only three abstracts identified that idea as a keystone. Other aspects of mathematics study, such as communication through writing or cooperative activities, were also neglected.

Only one of the dissertations addressed distance learning and the use of the Internet as a focal point, despite the fact that both of these ideas are hot topics in educational circles. It takes several years for dissertation research to reach fruition, so it is possible that these methodologies are currently being investigated and findings will be reported in the next few years. Many adults in the United States are not native speakers of English, yet none of the dissertations investigated the impact of English as a Second Language (ESL) or the implications of childhood mathematics learning outside North America on the study of mathematics in adulthood.

Conclusion

The body of doctoral research in adult mathematics education is small but cohesive. Much is known about the symptoms of student problems and work now needs to be continued or begun to devise and test "treatment plans" to help adult mathematics students gain confidence and to become successful in their studies of mathematics at all levels of the education system. Learning theories and teaching methodologies from traditional system research need to be analyzed and adapted for adult populations and then tested via doctoral studies.

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National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Appendix: Dissertations Ordered by ALM-7 Theme

Assessment/Frameworks/Standards
1981 Carabin, Robert Jerome
1983 Robinson, Donna Regina
1985 Moss, Lester Lavahn
1986 Puchon, Charles Anthony, Jr.
1989 Banner, Doris Vance
Leitsch, Patricia Kearns
Meeks, Kay Irene
1990 Carey, Karen Waite
Wheeler, Elizabeth (B. J.) Johanna
1992 Oliver, Richard Thomas
Refsland, Lucie Tuckwiller
1994 Blair, Judith Dee
Kestner, Sandra Fey Shelley
1996 Smith, Joseph Garratt

Parent Education
1990 Craig, Elizabeth L.
1991 Doering, William George

Instructional Approaches
1980 Willing, Delight Carter
1981 Jain, Barbara Jean
Stark, Jean Peterson
1982 Beachner, Lynne Anne
1984 Munyofu, Paul Malima
1985 Barnett, Thelma L.
Burnham, Paul Thomas
Friedman, Susan O.
Gould, Lillian Joyce Venable
Ungson-Devito, Maria Teresa
Wilson, Shirley Anne
1986 Massey, Frances Ann
Reid, Margie N. Barron
Robichaud, Kathleen Kienzle
1987 Farr, Charlotte Webb
Wilding, Marcella G.
1988 Basinski, Ida Rockwood
Grout, Don Hall
1990 Pace, John Patrick
1992 Ellman, June Christine Scholten
1993 Hsieh, Feng-Jui
1994 Wilder, Margaret Ramsey
1995 Burton, Beatrice Spencer
Greenwood, William Franklyn
1996 Berry, Andrew Jonathan
MacLeod, Susan H.
Newman, Glenn Austin Robert
1997 Gunasekera,Thilak Wijenayaka
Ramus, Katherine Safford
Wardlaw, Roosevelt

Instructional Approaches (continued)
1998 Martelly, Diana I.

Contexts
1982 Owings, Maria Facchina
1992 Masingila, Joanna O.
1997 Millette-McGuire, Beverly

Research into Practice
1980 Arnold, Carol Palmer
Boysen, Vicki Allen
Czarnecki, Karen Gordon
Mullinix, Patricia M.
1982 Ross, Kenneth Scott
Ehring, Howard A.
1983 Grady, Donna Katherine
Miller, Kathleen Noble
1984 Huntimer, Linda Carol
Jones, Martha Jane Everman
1985 Dugan, Patricia Ann
Lehmann, Christine Elyse Heinecke
McCarthy, William Francis
1986 Hoffer, Sharon Marie
Porter, Albert H.
Stewart, Barbara Martin
1987 Altiere, Gaetan
1988 Borakove, Larry Steven
Romero, John Edward
1989 Tobing, Asmara Raphy Uli Lumban
1990 Cunningham, Donna Davidson
Mayta, Fabian Esteban
1991 Russakoff, Marilyn
1992 Marsh, Joan Czaja
1993 Bartlett, Lucy
Fazzari, Alan Joseph
Skane, Marie Elizabeth
Sneller, Lowell Lee
1995 Galloway, Linda Jean Lowrey
Harper, Linda C.
Richardson, Samuel
1996 Johnson, Rayneld Rolak
1997 Cook, Roberta Parrino
Dias, Ana Lucia Braz
Szanto, Gabriella
1999 Steig, Mary Jo

Teacher Knowledge
1980 Richardson,Mikel Freeman
1993 Arriola, Leslie K.
1995 Nesbit, Tom
1998 Brown, Angela Denise Humphrey

Theory
1990 Hartman-Abramson, Ilene
1994 Barker, John Lewis

Understandings
1988 Ulrich, Mary Beth
1990 Munn, Anne Hodgkins
1992 Bockbrader, Barbara Ellen Berger
1995 Khooury, Hani Qustandi
1996 Beauford, Judith Elaine Mitchell
1997 Le, Xuan

Workplace/Vocational
1983 Holloway, G. Yvonne
1984 Hollobaugh, Patricia Susan
1986 Immergut, Brita
1988 Vanis, Mary Irene
Wilson, Odell D.
1994 Stephens, Geralyn Esther
1998 Moussavi, Massoud

No ALM-7 Theme Appropriate
1980 Travis, Thomas Riley
1987 Hopkins, Janet Claire
1989 Walsh, Velma Joy
1992 Trutna, Kevin Wayne
1997 Jost,Oliver R.
Parker, Sheila Latralle Blackston
1998 Bryant, Debra Deon
1999 Zachai, Judith

How the Brain Learns: Research, Theory, and Classroom Application

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Teachers are better able to help their students succeed when they understand how the brain learns and how to apply this understanding in the classroom.

Our brain is a physical organ in our body. Like all our other organs, the brain has evolved to perform specific functions; and, like the other organs, it performs its functions according to natural, innate processes. The brain has many functions; among them, and perhaps of most interest to educators, are those of empowering oneself, thinking, learning, and remembering.

The human brain has evolved as a survival organ impelled to learn how the world works and how to make it work—in order to survive and thrive. The brain has the ability to perform these functions because it is, from birth, a natural pattern seeker. In fact, it is impelled (as the lungs are impelled to breath and the heart to beat) to see, find, and make sense of patterns (conceptual structures) in the world (Freeman, 1995; Jensen 1999; Mehler & Dupous, 1994). This innate pattern-seeking ability is indispensable in mathematics.

> Mathematics experts are also able to quickly recognize patterns of information such as particular problem types that involve specific classes of mathematical solutions....The expert knowledge that underlies the ability to recognize problem types has been characterized as involving the development of organized conceptual structures...that guide how problems are represented and understood. (Bransford, Brown, & Cocking, 1999, p. 21)

Research shows that the brain's pattern-seeking drive is indeed innate and natural. For example, 5- to 12-week-old infants are "capable of perceiving, knowing, and remembering [and] begin to grasp the complexities of their world" (Bransford, Brown, & Cocking, 1999, p. 72). Wynn (1992) finds that infants are even capable of doing mathematics. Healy (1994) and others (e.g., Jensen, 1999), in reviewing the research, report the same phenomenon: human beings are natural and apt pattern-seekers, thinkers, and learners from birth—and the brain uses the same natural processes throughout life.

When teachers understand the brain's natural thinking, learning, remembering, and self-empowering processes and how to develop and implement brain-based curriculum in their classrooms, they are better able to help their students be the motivated, successful learners they were born to be. Brain research gives us a clear picture of what happens in our brain when we are learning. However, because the brain and its processes are inordinately complex, we will look here only at the least we need to know in order to be able "to put the brain into the classroom."

Something happens in our brain when we live through an experience (emotional, social, intellectual, cultural, physical); are confronted with a new phenomenon; or are introduced to a new concept, skill, or body of information. What happens is that as—and *because*—we experience, explore, interact with, practice, become familiar with, try to make sense of, think critically about, use for our own purposes the new object of interest, some of our approximately one hundred billion brain nerve cells (neurons) grow fibers (axons and dendrites). These fibers reach out and make connections (synapses) with other neurons and their fibers. This growing and connecting of fibers form increasingly more complex neural networks. The growing and connecting of these physiological structures *is* learning, and the new neural networks that have been thus constructed *are* our understanding and knowledge of any and every experience, phenomenon, concept, skill, or body of information. Moreover, endorphins, the "pleasure chemicals," are produced in the brain during learning. The brain is not only naturally impelled to learn, it also has a natural, internal motivation for learning: when we are learning we feel good. Nature, in fact, has provided us with both a need and a desire to learn.

Thus, if we want our students to learn, we must help them grow and connect their neural fibers and construct ever-more-complex neural networks about each object of learning. The brain does all this physiological work on its own. We do not yet fully understand how the brain knows where and how to grow dendrites, create synapses, and construct neural networks. Fortunately, however, we do know what learners have to do to make their brain structures grow and connect. Knowing that, we can give our students opportunities in the classroom to use their brain's natural learning process—as a result, their brain structures will grow, and they will be motivated and successful learners.

Field research (Smilkstein, 1989), conducted over a fifteen-year period, finds that people learn by a natural human learning process (NHLP). The subjects were approximately 5,000 students and teachers; these were multicultural students in middle and high school, community college, university, and graduate school classes across the curriculum and teachers in all disciplines in a variety of faculty development venues. In classes and groups of various sizes, subjects were asked how they learned outside school to be good at something. Each group reported an almost identical four-to-six-stage process of learning: 1) preparing to learn/using current knowledge/making a connection with something they already knew; 2) experimental practice; 3) skillful practice; 4) knowing in more detail/taking classes/reading/refinement/more practice; 5) gaining fluency/being creative; and 6) wider application/continued improvement/mastery/teaching.

This field research shows there is a progressive, incremental, cumulative natural human learning process that exists across age, gender, culture, and educational level. This research converges with the brain research, which also shows a connective, constructive process of learning and explains that learning is occurring—the learner's skill, understanding, and knowledge are increasing—*because* the learner's neuronal fibers and synapses are growing and connecting, forming complex neural networks. This convergence suggests a theory of learning: the learning activities and experiences in the four-to-six stages of the natural human learning process cause the brain's fibers, synapses, and neural networks to grow, connect, and form—i.e., cause learning to occur. Thus, if we create curriculum that provides opportunities for students to learn according to the NHLP stages, our students' brain structures will naturally grow and our students will be motivated, successful learners.

There is currently a mathematics reform movement underway, as promulgated by the National Council of Teachers of Mathematics in their document *Principles and Standards for School Mathematics: Discussion Draft* (1998), known as Standards 2000. Successful mathematics teachers have seen from their own experience, as well as in the research, that some curricular and instructional methods are conducive to learning while others are counterproductive. The research and theory presented here explain *why* certain methods are successful and also how to improve methods that are less successful. Some of the methods recommended in this reform document and movement, and discovered through experience by successful instructors, are identical to key curricular and instructional methods in the research-based, brain-compatible NHLP approach. These convergent methods include allowing "for opportunities for students to construct their own knowledge," providing "students with efficient, appropriate, and meaningful [authentic] learning situations[, and]...reducing drill and practice [on worksheets and in workbooks]" (Vasquez, 2000, p. 2).

Other recommended, and self-discovered, effective methods of mathematics instruction that also converge with research-based NHLP methods include having student-centered, rather than teacher-centered, approaches to instruction (Holmes, 1985) because students learn when—and what—they practice, i.e., what they grow and connect brain cells for. Moreover, "encouraging classroom talk represents a departure from past practice...as math used to be the time when there was silence in the classroom" (Willis, 1999, p. 5). "Students have expressed disapproval when teachers work problems and students have to watch" (Morrison & Payne, 2000, p. 2). As the NHLP research shows, and Morrison and Payne assert (2000, p. 2), students are motivated, successful learners when they "are able to participate in the instruction through active, hands-on experiences or problem-solving in groups." However, the instructional system is "still predominantly lecture and [has] changed little over the past 50 years" (Miles, 2000, p. 20). Uri Treisman, in an interview with May Garland (1993, p. 15) reported that his research found that community building and collaborative work are central features of successful mathematics instruction.

Talking and working together in a student-centered classroom converges with brain-compatible, natural-learning instruction, which uses a three-step cycle as its instructional strategy: 1) students work individually on a task so each one can use his or her own brain (neural networks); 2) students share and discuss in small groups for interaction, collaboration, and feedback; and 3) students reconvene for debriefing (teacher asks what they came up with and writes their contributions on the board *verbatim*) and general discussion for interaction, community building, collaboration, and feedback.

Following is one piece of NHLP curriculum, including the three-step NHLP instructional strategy. It was created by Joanne Rawley (1999), a St. Petersburg (Florida) Junior College basic-level mathematics teacher whose students had a difficult time understanding and using fractions. She designed this NHLP lesson plan to be the introduction to, and foundation for, a unit on fractions.

<table>
<tr><th></th><th>Stage 1
Using Current Knowledge</th><th>Stage 2
Experimental Practice</th><th>Stage 3
Skillful Practice</th></tr>
<tr><td>Indiv-idual</td><td>Teacher gives each student four 8x11 sheets of paper, each one a different color, e.g., white, blue, red, green: “Tear the blue sheet into two equal pieces and place them on the whole white sheet. Write down how you would tell or explain to someone how many of the blue pieces one of the blue pieces is.”</td><td>“Now tear the red sheet into four equal pieces and place them on the white sheet. Write down how you would tell someone how many of the red pieces one of the red pieces is, then how many two of them are.”</td><td>If the self-evaluation was 1-3: “Write down how you would tell someone how many of the red pieces two of the red pieces are, then how many three pieces are.”

Modification: If the evaluation at the end of Stage 2 was 4-6, the teacher modifies this activity as follows: “Write a fraction for what one of the red pieces is, then for two pieces, then three, then four.”</td></tr>
<tr><td>Small Groups</td><td>“Tell each other what you wrote down. Discuss what you were thinking when you were trying to figure out what to write.’ “</td><td>As before.</td><td>As before.</td></tr>
<tr><td>Whole Group</td><td>“What did you write down? (Teacher writes all answers on the board.) What were you thinking when you were trying to figure out what to write?” (General discussion.)

Note: This is a complete cycle: Individual, Small Groups, Whole Group (I, SG, WG).</td><td colspan="2">After the Small Group discussion at Stage 2, teacher writes 1----6 on the board and points to 1: “How many feel very confused about what we’re doing?” Then points to 6: “How many feel you fully understand what we’re doing?” Repeats with 3-2, 4-5. If most are at 3 or lower, teacher goes on to Stage 3, ending the Stage 3 Whole Group activity with the material at the end of the paragraph below.

However, if most are at 4-6, the teacher concludes Stage 2 this way: After the discussion and self-evaluation above, if no one has come up with ½ or ¼, the teacher says, “We say each piece is a fraction of the whole. This is how we write a fraction (puts ½ and ¼ on the board), and we say ‘one-half or one-quarter.’” Then the teacher does the modified Stage 3 (boldfaced). Again, if students were 1-3, teacher says this at the end of Stage 3 instead.</td></tr>
</table>

	Stage 4 *Knowing in More Detail*	Stage 5 *Gaining Fluency*	Stage 6 *Wider Application/Mastery*
Indiv-idual	If students were 1-3: "Write a fraction for what one of the red pieces is, then for two pieces, then three, then four." However, if students were 4-6: "Go back to the white and blue pieces and write a fraction for each piece."	"Now tear the green sheet into eight equal pieces and place them on the white sheet. Write the fraction for one green piece. Then write a fraction for two green pieces, then for three pieces, then four, five, six, seven, and eight green pieces."	"Write six fractions for any different numbers of blue, red, and/or green pieces. You decide what color pieces you will use and how many pieces you will have in each of your fractions."
Small Groups	As before.	As before.	"Show your fractions to each other. Discuss what all your fractions are saying about the number the pieces are of each whole."
Whole Group	As before. If students were 1-3, do another Stage 4 cycle (I, SG, WG) with the activity above, i.e., writing fractions for the white and blue pieces.	As before. Do another Stage 5 cycle (I, SG, WG). Then do a *third* Stage 5 cycle. Students are to write in response to the question, "What do you know about fractions now?" (I, SG, WG).	As before. Do another Stage 6 cycle (I, SG, WG): "Write fractions for other numbers of pieces of different colors." Do a *third* Stage 6 cycle: "What do you know about fractions now?" (I, SG, WG).

This introductory set of NHLP activities might take more than one class session to complete. But this "front loading" must be done so students without neural networks for (with little or no understanding of) a new concept or skill can construct a firm foundation for it, in this case, for fractions. Learners need time and practice to grow and connect the new dendrites, synapses, and neural networks that *are* their new concept or skill. With such a foundation, students will have a concept and skill (neural networks) to which higher level knowledge and skill (neural networks) can connect. This is how knowledge and skill increase. And students are motivated to learn—and are successful—when they have the opportunity to learn the way their brain is made to learn.

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Affective Research and the Mathematics Curriculum for Distance and Online Education

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Over the last 30 years educationalists have explored factors related to achievement in learning. For the purposes of this paper the focus is on mathematics learning. These investigations have taken place from a number of perspectives influenced by the humanist and cognitive scientist views and have included the seminal works of Bloom, Mesia, and Krathwohl (1964), Knowles (1980), Ausbel (1968), and Tinto (1993) (Figure 1). In the area of mathematics learning, research into the affective domain has featured heavily, with numerous variables identified from the literature including academic and mathematical self-concept, mathematics self-efficacy, mathematics anxiety, perceptions of the usefulness and value of mathematics, motivation, self-esteem and locus of control (Pajares, 1996; Higbee and Thomas, 1999).

As the nature of tertiary education in Australia changes and diverse groups of students are encouraged to participate, Australia has seen a proliferation of courses designed to facilitate access to and retention in tertiary study (Atkins, 1994). At The University of Southern Queensland, a regional university in Australia, such programs deliver learning packages to large numbers of distance education students. Currently, USQ is operating in 4th generation of distance education technology (Taylor, 1995). Instruction in mathematics is seen as an essential component of such courses and integrates the principles of instructional design developed by Gagne, Briggs, and Wagner (1988) and principles of good practice for numeracy teachers developed by Marr and Helme (1991).

However, such principles for good practice for numeracy teachers have been developed with face-to-face instruction in mind, and the transfer to instruction for students studying at a distance or online, where often no face-to-face contact is ever possible, is challenging, especially in view of the fact that such packages are often prepared months ahead of being delivered to the student. Arnold, Shiu, and Ellerton (1996, p. 725) in their discussion of curriculum design for distance education of mathematics indicate that text, software, audio visual materials, and student support are all important components of the distance education package for mathematics instruction. In particular, they warn that "when text is itself the learning medium the challenge is to initiate active learning instead of passive learning."

It is within this context of practice that the present distance education package for development mathematics evolved. This paper aims to describe how components of the affective domain are addressed within learning packages designed for distance or online education. The students for which these packages are designed have been described previously by Taylor (1995), Fogarty and Taylor (1997), Mohr (1998), Bedford (1998), and Clarke and Bull (1998).

Print Development

The printed version of the course is characterized by the inclusion of the following strategies which were included at different stages of the development:

- An introductory module which contains discussions of past students' experiences of mathematics and mathematics learning with suggestions about study strategies.
- An introductory assessment which asks students to detail their past mathematics learning. This is completed in their first week of enrolment which allows for students to make contact with their tutor as early as possible.
- The presence of learning diaries which allow students to reflect on their learning experience associated with each module and to make personal contact with their tutor.

- The presence of the Mathematics Learning Essay which asks students to reflect on their whole mathematical learning experience viewing any changes in attitudes and gathering together study strategies that worked for future reference.
- Text Materials are written in informal language with little mathematical jargon to make them more accessible to novice students. Formal mathematical language is introduced at a slow pace.
- Content is presented in context and applied to events that many students may be expected to have encountered previously. The section "A taste of things to come" includes applications modified from their future university studies.
- Content and activities are grouped in small bursts to allow students to experience early success and to build confidence, knowledge, and skills gradually.
- All activities have fully worked solutions with problem solving prompts to guide students through the steps.

A formal evaluation of the core curriculum of the TPP was conducted in 1997 (Bedford, 1998), with the mathematics component reported on in Mohr (1998) and with Taylor and Mohr (in press) reporting on the development of the process over a number of years.

Online Developments

A video and related CD-ROM (with capability for online delivery) extensions of the course demonstrated in this workshop introduces you to Mary-Anne, Lenny, Cindy, David and Susan: five students from five different backgrounds (played by a series of actors). We follow them through the highs and lows of studying as they use their own experiences to overcome their fears of mathematics and prepare themselves for success in tertiary studies such as nursing, teaching, psychology, and business. Details are included in Taylor, Spielman, Ross, Galligan, and Mohr (1998) and Taylor, Spielman, and Ross (1999). These components are designed to address all of the principles of teaching and strategies to overcome maths anxiety detailed in Marr and Helme (1991). It achieves this by integrating the following learning strategies into the story of the five students. Students will be able to:

- see video clips of students solving maths problems in context;
- see video clips of students involved in group problem solving sessions;
- view and hear more formal instructional sessions which will include animations;
- participate in interactive examples;
- test themselves using Self Test (Taylor 1998); and
- create and print a diary of personal notes and comments.

Evaluations of these interactive multimedia have been completed but are yet to be published.

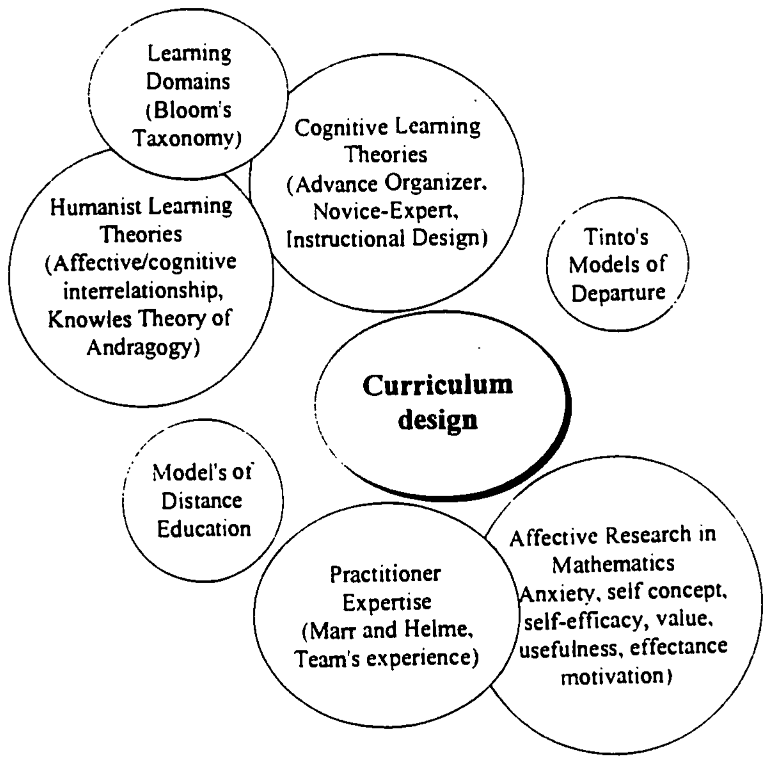
Workshop Components

Workshop participants were challenged to identify the affective components necessary to teach mathematics to a diverse group of students studying at a distance and to discuss whether distance education was as effective as face-to-face teaching in this domain.

Acknowledgments

The course described in this paper has developed over a number of years initiated by the work of Pamela Surman, Peter MacNamee, Linda Galligan and Joan Mohr and continued by Robyn Pigozzo. Multimedia developments were designed by Elizabeth Spielman, Linda Galligan, Joan Mohr, and David Ross and myself.

Figure 1: Research Influences on the Curriculum Design of a Mathematics Course Offered at a Distance



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Arkansas GED Mathematics Instruction: History Repeating Itself

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The advancement of technology throughout the past 15 to 20 years has transformed the workplace. The workplace is now more complex and multifaceted with changing expectations about work. Because of the changed expectations, an educated and proficient work force with a different skill set is now required.

Different mathematical skills are a component of the different skill set. Consequently, mathematical expectations in today's workplace have expanded from paper-and-pencil computation to mathematical reasoning, logic, problem solving, and connection to real-world use and application (NCTM, 1989, 2000). Carnevale (1991) maintains that the focus of mathematics skills in the workplace should be on problem solving rather than computation.

Even with the changed expectations and skills, the typical General Education Development (GED) participant is still performing pencil and paper, drill and practice of isolated computation problems that have no connection to real-world use or application (Gal, 1993). Adults who enter educational programs perceive mathematics as a barrier to upgrading skills for job advancement or for personal gratification (Gal, 1993). This perception can be a result of past classroom experiences and/or learning disabilities. However, adult educators who are given the task of helping the adult learner overcome a difficult learning history are in need of training/staff development to upgrade their own math skills as well as to change their attitudes toward mathematics (Gal, 1993). Instructors, who are often part-time, are not always prepared to teach mathematics. Furthermore, little research exists concerning how mathematics is taught to adults in GED programs and how prepared the instructors are to meet the numeracy needs of adults.

This research was adapted from a study by Mullinix (1995), *Exploring What Counts: Mathematics Instruction in Adult Basic Education*. Mullinix examined mathematics instruction in Adult Basic Education (ABE) in Massachusetts, using both quantitative and qualitative methods to obtain a comprehensive picture of the ABE mathematics learning environment.

Purpose of the Study

The primary purpose of this study was to determine the current status of the delivery of mathematics instruction in GED programs in Arkansas by investigating the experience/background, teaching techniques/methods, training/staff development, and professional involvement of GED instructors. The secondary purpose was to determine if there was a relationship between the instructor's experience/background and their instructional practices, support of mathematics reform, and professional involvement.

This study was conducted in Arkansas where 57 adult education centers house the GED programs. GED programs are housed in a variety of settings, including technical schools, community colleges, and public schools. Each setting has one or more instructors who can be at different sites within the program.

Methodology

Since a list of GED instructors was not available from the Adult Education Section, Arkansas Department of Workforce Education, a two-step process was necessary to identify the instructors. A mailing list of 57 adult education program directors was obtained from the Adult Education Section. Because two of the program directors indicated their program did not offer GED instruction, instructors from 55 programs were included in this study. Secondly, a list of all instructors teaching mathematics at each facility was obtained along with a work address for each instructor. A work address was requested so the questionnaire could be sent directly to the instructor.

A list of 257 GED instructors was obtained from the 55 program directors. Several program directors gave their instructors an option of participating, and three program directors did not provide names of their instructors. Therefore, the sample of GED instructors used for this study is not necessarily the total number of GED instructors in Arkansas. Thirty additional questionnaires were sent to the three program directors that did not provide names or numbers of instructors. Therefore, 287 questionnaires were distributed to GED instructors. One hundred sixty nine questionnaires (59%) were returned. Of those returned, two were disqualified because the instructors did not teach mathematics. Consequently, data from 167 (58%) questionnaires were analyzed.

The demographic data obtained from the Mathematics Instruction Questionnaire included the position; teaching experience; post secondary degrees; major and minor subjects; highest level of mathematics taken in high school and college; whether mathematics is the only subject taught; the number of hours of mathematics taught per week. Additional data addressed the curriculum, instructional methods, training, and professional involvement.

Findings

Typical GED Instructor

The results of this study provided data for the following description of the typical Arkansas GED instructor: The instructor is employed full-time (56%), teaches all subjects (96%) included in the GED curriculum, and teaches mathematics between 1 and 10 hours per week (71%). The instructor has a Bachelor's degree (64%) in Elementary Education (25%) with a minor in Social Studies (15%). He or she has taught in grades 4 through 9 (87%) but has held more than one teaching job and has taught a variety of levels. The typical instructor has an average of 4 to 6 years experience (19%) teaching GED mathematics. The highest level of mathematics taken in high school and college was typically Algebra II (37%) and College Algebra (54%), respectively.

The curriculum chosen by the typical GED instructor (72%) is most often Steck-Vaughn (85%). Individual instruction (95%) and repeated practice (99%) are the instructional format and method of choice with repeated practice considered the most effective (89%) instructional method. The typical GED instructor (64%) did not participate in training related to teaching mathematics to adults in the two years previous to this study.

Knowledge and Support of Mathematics Reform Topics

Data gathered concerning the instructor's knowledge of the mathematics reform topics is presented in Table 1. As indicated, the highest percentage reported for considerable knowledge of a reform topic was 50% for problem solving. The remaining reform topics in the considerable column exhibit percentages of less than 50%. In fact, the topics that require high order thinking skills, reasoning, logic, and application fall well below the 50% level. The percentages regarding some knowledge of the reform topics are higher than those presented in the considerable column.

Table 1: Knowledge of Mathematics Reform Topics

	None	Some	Considerable
Problem Solving	6%	44%	50%
Teaching in Context	9%	48%	43%
Reasoning	8%	50%	42%
Use of Calculators	8%	58%	34%
Communication	17%	52%	31%
Open-End Problem Solving	18%	61%	21%
Computer Spreadsheets	47%	38%	15%

As indicated in Table 2, the range of the instructor's support of the mathematics reform topics was from 84% to 34%. Instructors indicated overwhelming support for the reform topics except for computer spreadsheets. Over one third of the participants indicated no opinion on open-ended problem solving and use of calculators and 57% have no opinion concerning computer spreadsheets. Those who do not indicate support of the mathematics reform topics indicate that they have no opinion rather than that they do not support their use.

Table 2: Support of Mathematics Reform Topics

	Do Not Support	No Opinion	Support
Problem Solving	1%	15%	84%
Reasoning	2%	22%	76%
Teaching in Context	2%	24%	74%
Communication	3%	22%	63%
Open-Ended Problem Solving	6%	36%	58%
Use of Calculators	14%	34%	58%
Computer Spreadsheets	9%	57%	34%

Professional Involvement

The majority (50% to 88%) of the participants were not familiar with the journals and organizations that focus on the mathematics reform movement and the teaching of mathematics. In fact, several indicated they had never heard of the organizations and journals listed on the questionnaire. Only six participants indicated membership in a professional organization related to teaching mathematics.

Data Analysis

Data concerning curriculum, instructional methods, instructor training, and professional involvement were analyzed using the chi-square test of association and an independent t-test. Results of data analysis using the chi-square test of association revealed empty cells in the member/subscribe category of professional involvement. In order to obtain more reliable results, the member/subscribe category was recoded to be included in the familiar with category. Therefore, two categories instead of three categories were included in the chi-square analysis. The categories familiar with and not familiar with, for the purpose of this study, were the descriptors used to indicate professional involvement. In addition, an independent t-test was performed comparing the mean of the subject's number of years teaching mathematics in K-12 and their professional involvement.

A chi-square test of association was computed to compare the number of hours of training received by instructors and their professional involvement. Results indicated a relationship between the training received by GED instructors and their familiarity with professional organizations and journals that focus on teaching mathematics and the mathematics reform movement. The ratio of the instructors with no training who were not familiar with to those instructors who were familiar with professional organizations and journals was about 7:5. As the hours of training increased, familiarity with professional organizations and journals increased. The ratio of those instructors with four or more hours of training who were not familiar with to those instructors with four or more hours of training who were familiar with appropriate professional organizations and journals was about 1:4.

A chi-square test of association was calculated comparing the instructor's support of reform mathematics and their professional involvement. Results indicated the ratio of those instructors who did not support to those who did support the themes of mathematics reform and were in the not familiar with category was about 3:70. The ratio of those instructors who did not support to those who did support the themes of mathematics reform and were in the familiar with category was less than 1:90.

An independent t-test compared the mean of the participant's number of years teaching mathematics in K-12 and their professional involvement. Results indicated a relationship between the number of years the instructors taught mathematics in K-12 and their professional involvement with organizations and journals that focused on teaching mathematics and the mathematics reform movement. As the years of experience teaching K-12 mathematics increased, the instructors were more likely to be familiar with the professional organizations and journals.

Discussion
The results of this study indicated that the current status of the delivery of mathematics instruction in GED programs in Arkansas was inadequate for the GED student of the present and the future. Typically, the instructors were using textbook/workbook instructional methods (repeated practice, 99%) and an instructional format (individual instruction, 95%) that promotes a passive learning environment (Nesbitt, 1996). In addition, almost all of the instructors (96%) teach all subjects in the GED curriculum. Typically, the instructors were using a curriculum based on algorithms and were showing their students how to perform mathematics operations rather than teaching them to understand mathematical concepts (TIMSS, 1996).

Training related to teaching mathematics to adults is a major concern. Over 63% of the respondents had not received any training related to mathematics instruction in the two years previous to this study. Furthermore, the participants in a mathematics manipulative workshop provided in the spring of 1998 indicated that they had never received any training related to teaching mathematics and that training was needed (Ward, 1998).

Although the instructors indicated knowledge and support of the mathematics reform topics, those topics were not applied in the classroom setting. The use of a textbook/workbook curriculum (85%) with an individual instructional format (95%) and paper and pencil repeated practice (99%) indicated that the reform topics were not used in GED mathematics instruction. A curriculum based on these methods sets a standard of minimum expectations and deprives the student of a curriculum rich in critical thinking and problem solving.

Since this study did not have a qualitative component, it is impossible to determine the instructor's definition and understanding of the reform topics. However, the results indicated that the instructor's definition and understanding of the reform topics are defined and limited by the textbook/workbook.

The instructors were not professionally involved with organizations and journals that focus on the teaching and learning of mathematics. While the results of this study indicated a relationship between training and professional involvement, and support of mathematics reform and professional involvement, the relationship appears weak. Although the instructors indicated familiarity with professional organizations and journals that focus on the teaching and learning of mathematics and the reform movement, only six of the participants indicated membership in any of the organizations listed in the questionnaire. There is considerable difference between familiarity with an organization and supporting that organization with membership.

Recommendations
The lack of staff development and training for teaching mathematics to adults is a major concern. The instructional methods and the curriculum used by the instructors does not promote high order thinking skills, problem solving, reasoning, logic, and teaching mathematics in context. The development of an extensive staff development program that will provide instructors with new ways of teaching and thinking about mathematics is recommended.

Since there is little research concerning mathematics and adults, more should be done in all areas of adult education and mathematics instruction. Children are the focus of most of the research concerning the teaching and learning of mathematics. As a result, research based upon children is being generalized to the adult population. We need to know more about how adults learn mathematics, ways to teach adults mathematics, and if, in fact, the research for children can be generalized to adults.

This study can be used as the foundation for determining whether extensive staff development and training in alternative mathematics instruction methods impacts the delivery of mathematics instruction. Additionally, the new version of the GED test, which will be initiated in 2002, introduces the use of a calculator for part of the math test. This study can be used as a foundation for determining if the introduction of the calculator impacts mathematics instruction.

Conclusion
This study revealed the following: the typical GED student was performing paper-and-pencil, textbook/work-

book repeated practice; the focus of mathematics was on teaching how to manipulate numbers rather than on problem solving, critical thinking, and mathematical reasoning; the typical GED instructor needs staff development to upgrade her/his own math skills. This profile of a typical Arkansas GED mathematics instructor and the way mathematics is taught fits the description provided by the literature.

Furthermore, different mathematical skills and expectations are required in today's workplace. These skills and expectations encompass a broader range of knowledge and application than textbook/workbook repeated practice provides. The need for different mathematical skills points to the need for change in the way mathematics is taught.

The history of mathematics instruction is textbook/workbook repeated practice and manipulation of numbers. Mathematics instruction in GED programs in Arkansas represents history repeating itself.

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LARGE-SCALE ISSUES: FRAMEWORKS, STANDARDS, AND ASSESSMENT

Learning Outcomes: Skills or Function?

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Abstract

Skills or function? That's the question. How should learning outcomes in adult numeracy reflect this dichotomy? In the workshop, two different approaches to describing learning outcomes were used to promote a discussion among participants around the role of skills and function within an adult numeracy curriculum. In the first part of the workshop, participants worked in small groups to analyse some different real life stimulus materials to develop some teaching ideas, then to try to see how these would fit against two models representing the above two approaches for describing learning outcomes. In the second part, participants discussed their experiences and looked at the differences and the merits of the two approaches.

Background

In the past, adult literacy and basic education has mainly been an informal, student focused form of education with no formal accreditation process or system wide curriculum, with students learning reading, writing, mathematical, oral communication, and general education skills. But in the 1990s the pressure of competency based education and training has meant that the Adult Literacy and Basic Education field too has been required to develop accredited curricula based on competency based learning outcomes.

It has been the challenge therefore to develop accredited curricula that try to espouse and maintain the principles established for a student-focused and relevant form of education. It seems that there have been two different approaches to describing learning outcomes: base them on teaching mathematical skills, often replicating school-based views on teaching mathematics, or base them on functional mathematics, on using and applying mathematics in real life.

Two Different Approaches

In Ontario, Canada, learning outcomes for numeracy are being implemented based on traditional school math strands (number, measurement, space and shape, data, and algebra). They are essentially skill-based outcomes. These are called the Literacy and Basic Skills (LBS). The LBS matrix features the following learning outcomes for numeracy:

- Perform Basic Operations with Numbers
- Use Measurement for Various Purposes
- Solve Geometric Problems
- Manage Data and Probability
- Use Patterning and Algebra

The LBS learning outcomes for numeracy come directly from the Common Curriculum now being used in the elementary schools in Ontario and as such are very hierarchical and structured and don't take into account how adults learn.

For details of one of the LBS Learning Outcomes see Appendix 1.

In Australia, numeracy learning outcomes have been developed which focus on the social purpose and use of mathematics within meaningful contexts. The outcomes include skills and knowledge in an organisational structure based on function where mathematics is seen as the knowledge and skills to be applied and used for a range of purposes and in a variety of contexts. These are called the Certificates in General Education for Adults (CGEA).

The Learning Outcomes are organised into four different categories or domains, according to different purposes and functions of using mathematics.

- **Numeracy for Practical Purposes** addresses aspects of the physical world to do with designing, making, and measuring. There are two learning outcomes: **Numeracy for Practical Purposes - Design** and **Numeracy for Practical Purposes - Measuring.**
- **Numeracy for Interpreting Society** relates to interpreting and reflecting on numerical and graphical information of relevance to self, work, or community. The two learning outcomes are: **Numeracy for Interpreting Society - Data** and **Numeracy for Interpreting Society - Numerical Information.**
- **Numeracy for Personal Organisation** focuses on the numeracy requirements for personal organisational matters involving money, time, and travel. There are two learning outcomes, one dealing with money and time, the other to do with location and direction.
- **Numeracy for Knowledge** is only introduced at level 3 and deals with mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings and/or assumptions. There are learning outcomes to do with problem solving, algebraic, and graphical techniques.

For details of one of the CGEA Learning Outcomes see Appendix 2.

Developing Teaching Ideas
Sets of three different functional materials or stimulus materials were distributed to small groups of participants. (They were: a tourist map of Boston; a menu from the Near East Cafe; and a newspaper article: "American League baseball standings.")

The question that was posed was:

What would you teach with these materials if they were brought to your classroom by your students?

A wide range of teaching ideas and activities were developed by the different groups. These ideas were shared by the whole group. This illustrated how much math was embedded within such common materials.

Using the Learning Outcomes
A brief explanation of the two Learning Outcome schemes (CGEA and LBS) was given and excerpts from the two frameworks were distributed.

The question that was posed here was:

How would you fit the learning activity that you just discussed into each of these frameworks? In other words, "map" the activity to the learning outcome.

This was done to also illustrate and model to participants that no matter what the standards or curriculum frameworks that are introduced or imposed upon teachers, it is possible to start with what students want or are interested in, and then afterwards to map the activity to the learning outcome rather than try to start with the standards or learning outcomes.

The Discussion
Two questions were posed here:

- *What are the differences between these two frameworks?*
- *What are the advantages and disadvantages of the two frameworks?*

Here it was almost unanimously agreed that the main difference was that the Ontario LBS outcomes are based on "Math" while the Australian CGEA outcomes are based on "Context." However, it was agreed that both can get to the same end—it depends on learning styles, on different ways of looking and viewing the world of math and numeracy.

It appeared that those teachers who were more inclined to teaching a traditional math curriculum found the Ontario LBS scheme much more comfortable and user friendly, whilst those who taught in a more holistic and integrated way, often where literacy and numeracy were taught together, found the Australian CGEA scheme more attractive. One argument that was raised was that for teachers who are not math trained, the Australian CGEA Learning Outcomes were much more understandable and would be easier to work to.

Conclusion
Alan Mortiboys (1984) warned against the extremes of teaching mathematical skills without a context or of adopting a purely functional approach with myriads of timetables, menus, and advertisements. Terry Riley (1984) concluded: "We need to adopt a balanced approach: one in which mathematical rules are understood and practised, and where appropriate, used in situations deemed to be relevant to the student by the student." Although these references seem old, this balanced approach, where function is integral, is based on principles of adult learning that are rooted in context and relevance to every day life.

One of the dangers of following a school-based approach such as in the LBS outcomes and in the absence of curriculum guidelines and appropriate training, is that literacy instructors who have little training in numeracy will use these learning outcomes as the prescribed course of study. One literacy tutor has already expressed her difficulty in understanding the schema, for example with the success marker which states: "models numbers grouped in 10s and 1s and uses zero as a place holder." This terminolgy is too abstract for a literacy practitioner who has no formal training in mathemetics. Furthermore, new assessment tools based on the LBS learning outcomes look like the school tests that failed literacy learners in the past.

We believe that numeracy provision needs to be a balance between function and skill development. The CGEA scheme says that mathematics skills are an important and vital part of the Learning Outcomes, but they are not the up front focus—the function or purpose is the organising structure. In that sense we believe therefore that the Australian Certificates in General Education for Adults Numeracy and Mathematics Learning Outcomes better represent the aims and ideals of adult basic education. As such they have better met the challenge to develop accredited curricula that try to espouse and maintain the principles established for a student-focused and relevant form of education.

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Appendix 1 **Ontario Literacy and Basic Skills Program (LBS)**

Sample LBS Learning Outcome

Learning Outcome: Manage Data and Probability – Concluding and Reporting

Level 1 Success Markers	Level 2 Success Markers	Level 3 Success Markers
❑ relates objects to numbers on a graph with one-to-one correspondence [1] ❑ records data on charts or grids given by the instructor [1] ❑ organises materials on concrete graphs and pictographs using one-to-one correspondence [1] ❑ reads and discusses data from graphs made with concrete materials and demonstrates understanding in a variety of ways (for example: use informal language to discuss) [1]	❑ relates objects to number on a graph with many one-to-one correspondence (for example: 1 Canadian flag represents 100 Canadian citizens) [3] ❑ organises data in Venn diagrams and charts using several criteria [3] ❑ constructs bar graphs (with discrete classes on one axis and number on the other) and pictographs using scales with multiples of 2, 5, and 10 [3] ❑ interprets data from graphs (for example: bar graphs, pictographs, and circle graphs) [3]	❑ recognises that graphs, tables, and charts can present data with objectivity or bias [5] ❑ constructs labelled graphs both by hand and by using computer applications [5] ❑ evaluates data presented on tables, charts, and graphs and uses the information in discussion (for example: discusses patterns in the data presented in the cells of a table that is part of a report on a scientific experiment) [5]
	Transition Markers	**Transition Markers**
	❑ identifies the parts of a graph: labels, scales, title, data [2] ❑ organises data using graphic organisers (for example: diagrams, charts, graphs, webs) and various recording methods (for example: placing stickers, drawing graphs) [2] ❑ constructs and labels simple concrete graphs, bar graphs, and pictographs using one-to-one correspondence [2] ❑ interprets displays of numerical information and expresses understanding in a variety of ways (for example: use informal language to discuss) [2]	❑ recognises the purposes of different parts of a graph: title, labels, axes [4] ❑ constructs labelled graphs (for example: labelled with titles, horizontal and vertical axes, intervals, and data points) both by hand and by using computer applications, and creates intervals suited to the range and distribution of the data gathered (for example: a graph with a range of 100 years is better divided into intervals of 10 years than 1 year) [4] ❑ reads and interprets data presented on tables, charts, and graphs (for example: circle graphs) and discusses the important features [4]

Note: *(1) This learning outcome "Concluding and Reporting" has two more levels: 4 and 5.*
(2) The number at the end of each Success or Transition Marker, e.g., [4], denotes the corresponding grade level in the Ontario Elementary School Common Curriculum.

Appendix 2 **The Certificates in General Education for Adults (CGEA)**

Sample CGEA Learning Outcome

Learning Outcome 2.5: Numeracy for Interpreting Society - Data

Can use and create everyday graphs and charts to represent and interpret public information which is of interest or relevance.

Assessment criteria

Not all assessment criteria need to be met in the one assessment task or activity

Mathematical Knowledge & Techniques

(a) interpret the key features, conventions and vocabulary of everyday graphs or charts, including the concept of scale
(b) use whole numbers, percentages, decimals and simple ratios found on charts and graphs
(c) collect, sort and record data in a table using simple techniques
(d) interpret and discuss meaning of text that incorporates graphs or charts
(e) mark scales and axes appropriately
(f) represent data in simple bar or line graphs

Language

(a) use the descriptive language of graphs and charts such as maximum, minimum, increasing, decreasing, going up, constant, changing, slope, etc.

Interpretation

(b) relate meaning/information of graph or chart in terms of personal implications and/or social consequences
(c) decide on the fairness or bias of the data in response to teacher prompting.

Performance range

- The types of graphs or charts could include simple pie charts, bar graphs, line graphs, pictograms, etc. of the kind found in newspapers, on household bills, information leaflets, etc.
- Scales created should count in 1's, 2's, 5's or 10's.
- Scales interpreted from public information not limited to the above simple scales - can interpret from more complex scales available on public information.

Assessment of Math Skills in ABE: A Challenge

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1. Introduction

Testing adults in Adult Basic Education (ABE) on their math skills is not easy, especially when it involves non-native speakers, but it is a challenge to try to discover what adults really know and can do when they enter ABE and what this means for math education in ABE.

In 1998 qualitative research was done into the mathematical knowledge and skills of a group of thirty-two non-native adults in ABE in the Netherlands. These adults came from twelve different countries. All of them received primary school education in their own countries and some of them also participated in low vocational courses. They did an intensive language training for about half a year before they started the math course. Part of this research was content analysis of their results on a placement test for ABE, developed in the Netherlands. Information based on this analysis should provide insight into their knowledge and skills and into appropriate ways of assessing non-native adults in order to improve math education in ABE. In this paper a few criteria for placement tests for testing adults in ABE are discussed and supported by a few students' examples.

2. Some Background Information

In the early 1970s Realistic Mathematics Education (RME) started as a new way of mathematics education in the Netherlands. RME was developed by Professor Hans Freudenthal and his coworkers at the IOWO institute (Institute for the Development of Mathematics Education), currently the Freudenthal Institute, in Utrecht. It starts with Freudenthal's assumption that mathematics has its roots in real life and constructs and reconstructs reality. This is a never-ending process. RME is a learning theory on its own and has five starting points: learning is a constructive activity; learning moves through various levels of abstraction with the provision of models, schemes and symbols; learning takes place by reflection; learning is a social activity; and learning mathematics leads to a structured and interwoven entity of knowledge and skills (Freudenthal, 1973, 1991; Gravemeijer, 1994; Treffers, 1991)

RME was enthusiastically embraced in ABE in the Netherlands when the new adult education system started in 1987. Based on the RME principles the first experimental materials for adults were developed in the years 1987-1992. RME in ABE started from scratch. ABE learning centers needed everything: a curriculum, student books, but also a way to determine the starting level of adults when they enter ABE.

3. Testing Adults in ABE

At the start of ABE in 1987 many teachers came from open school centers and from literacy projects. They were used to working based on Paulo Freire's theory: learning from experiences. Most of them did not want to use placement tests because that would be too "school-like" and might remind adults of negative school experiences. However, in the new system we had to grow to a way of systematic learning and teaching, including tests. That forced us to think about a way in which we could collect relevant information but not in a school-like way and more than only right/wrong answers, as was common for paper-and-pencil school tests at that time. Such tests might be sufficient to determine a placement level, but our concerns were multiple:

1. Native adults in ABE are often blocked by math anxiety, due to past negative school experiences. They may feel uneasy in a test situation, especially at the start in ABE.
2. Weak readers and non-native adults may encounter language problems in a paper-and-pencil test that could affect their actual knowledge and skills on math.
3. Tasks that only ask for applying mathematical procedures, like doing algorithms, may give an incorrect impression of the actual mathematical knowledge and skills of adults, especially when it involves second language learners.

4. A placement test based on only collecting right/wrong scores does not provide qualitative insight into the mathematical procedures of adults.

From these negative experiences with paper-and-pencil tests we wanted to come to a way of testing that would be friendly for adults and include both quantitative and qualitative information. Also, in this test the starting points of RME should be visible. That means, in our interpretation of RME for ABE, that items should be related to real life situations and that people should be able to show their own knowledge and skills in a constructive way. That brought us to the next set of criteria for testing adults in ABE:

1. Adult students should be enabled to show the best they can.
When adults come back to school they often have developed a mix of school knowledge and real life knowledge. They may have forgotten lots of formal school math procedures and developed more own-invented informal procedures instead. On the other hand, many adults with little school education show insight into mathematical problems and may know more than what they ever learned in school, based on real life experiences. Therefore, tasks on ABE tests should not only ask for supposed school knowledge and skills, but should enable adults to show what they actually know and can do. *For that, a math test for adults should provide mathematical problems in contexts derived from real life situations that can be solved in different ways.*

2. Language in a placement test should not hamper the student from doing the math test.
Math contexts do not need to consist of only text. In real life situations contexts occur as "problem-situations" that can be embedded in or supported by visual representations. Such visualizations should be the essence of math contexts in the test. Text in the context itself should be restricted to a minimum. Only essential information should be given. Instruction text in the item questions should be worded in a consistent style, indicating particular tasks or types of questions always in the same style and with the same or similar words. Instructions should be written in short, simple, not complex sentences. In this way the language barrier can be kept as low as possible. During the actual test situation people should still feel free to ask for help if there is any language problem.

3. Adults, in particular second language learners, should have a chance to apply their own mathematical procedures and the algorithms they are used to.
In an international setting cultural differences should be taken into account. Such differences become clear in, for example, different notation systems, not only in the students' work, but also in the offered mathematical contexts. It is very difficult to create items for placement tests that offer cross-cultural possibilities. For example, text in contexts should be unambiguous, e.g., 1/5 can be pronounced as "one-fifth" or "one-over-five" or "one out of five." In an context that says "One-fifth of the students in High School are too heavy, what percent is that?" it would be better to use the fraction notation "1/5" instead of words, or make it visible in a graphic. This may prevent misunderstanding. Also, people from different countries may apply different algorithms. Therefore, in a placement test we should not ask students to do algorithms to conform to the host country's style. We should offer contexts in which students can apply their own algorithms.

4. The test should yield qualitative information about the adults' mathematical skills in order to enable teachers and program developers to set up adequate and well-tuned programs for ABE students.
If the students are asked to write down their computations in their copies of the placement test, this may provide a lot of qualitative information to teachers and program developers. What kinds of computations does the student show in the test? Is the student used to formal algorithms or does he apply informal computations? Is it based on insight? Does the student profit by graphics? Does the student use schemes or mental models? Does

the student understand the text in the context, in the item question? etc. Information acquired here should be the starting point in the math classes.[1]

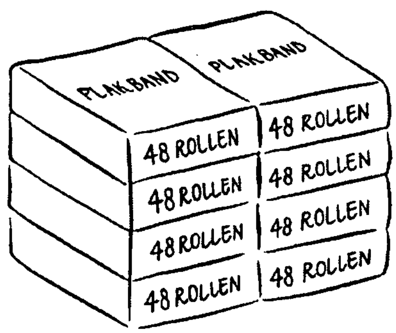
In 1992 a first experimental framework with accompanying placement test was published for ABE: the *Supermarket Strategy* (van Groenestijn et al., 1992, 2000). This test consists of 64 contexts, accompanied by a leaflet. The content is based on mathematical activities in the context of a supermarket, spread over different mathematical strands and levels. The test is based on oral interviews. The color leaflet and the contexts enable the interviewer to build up the interview in an informal way of a friendly talk with the interviewee about math topics related to their real life experiences. Guided by a session-flow procedure and a structured interview scheme, the interviewer is able to set a profile and determine the placement level in about 45 minutes to one hour. The interview yields a lot of qualitative information about the mathematical skills of the adult. Also, non-native speakers can be helped when they encounter language problems. This test showed teachers in ABE that testing adults could be done in a non-school-like way, which was an important step.

However, at the follow-up of this placement test, the elaboration of the *In Balance* student units, we were strongly advised to develop a new placement test, a paper-and-pencil test, due to time constraints in ABE institutions (van Groenestijn et al., 1994-2000). In this test the same principles were used as described above, but now the challenge was to develop items that could yield as much information on paper as in an oral interview. The text in the item contexts and item questions should be minimal, unambiguous, and consistent for similar items at different levels. A precondition for a paper-and-pencil test is that it looks attractive and challenging to adults to lessen math anxiety and blocks during the test session. People should be introduced to the test in the actual test situation, to become familiar with the setup and the layout.

4. Two Examples

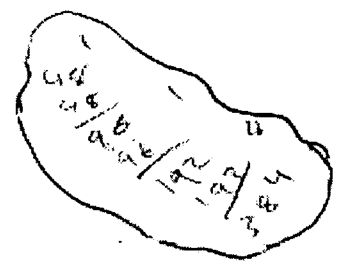
To give a brief impression of the type of items based on the criteria above, two examples are presented here. These items were part of the research described in the introduction. The first task is a multiplication context about tape rolls in boxes. The item question is:

How many tape rolls in total?



The graphic shows 8 boxes with the Dutch word "tape" on it. The text "48 rolls" indicates the content of each box. The text in the context and in the item question is minimal but essential. This context offers possibilities for formal and informal computations based on, for example, repeated addition, doubling, multiplication, smart computations by rounding up to 50, combining two boxes to 100-4, etc. A few students' computations show the following results:

[1] Note: Here we come to the question whether the student should be allowed to use a calculator at the placement test or not. The use of a calculator is a study in its own and not part of this paper. I am all in favor of the use of calculators in ABE, though under certain conditions. However, regarding the placement test I advise not using the calculator in order to get clear what the student actually can do by himself. After the placement is done, we could ask the student to do one or more extra tasks, allowing him to use a calculator, or spend some time during the first group session for a few tasks with a calculator in order to see how the students deal with a calculator. Personally, I prefer the latter.

	200 - 8 = 192 200 - 8 - 192 8 1014 400 - 16 = 384	48 · 8 ——— 384
Student 1	Student 2	Student 3

Student 1 shows computations based on repeated doubling. By this action she shows her insight and her way of thinking. She can apply the addition algorithm, but probably not the multiplication algorithm. It is even unsure whether she knows how to multiply. Other items should give additional information.

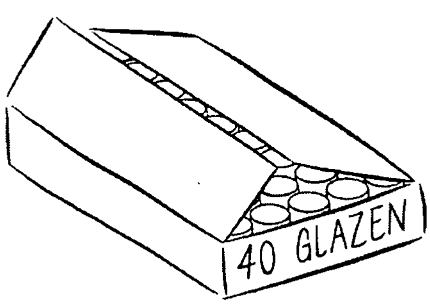
Student 2 starts in a smart way by using the number 200 for 4 boxes, minus 8. This student shows insight in the context, but then he applies a wrong addition algorithm. He adds 2 and 2, making 4. Then he adds 9 and 9, but he writes down 1 and keeps 8 instead of the other way. Finally he adds 1 and 1, plus 8 makes 10. (Total 1014) However, in the final notation below, probably done mentally, the student shows the correct answer (384).

Student 3 shows a correct multiplication algorithm, although it does not conform to the Dutch style.

For the record: only 53% of this student group got the correct answer to this item.

The second context shows a multiplication/division problem. The text says:

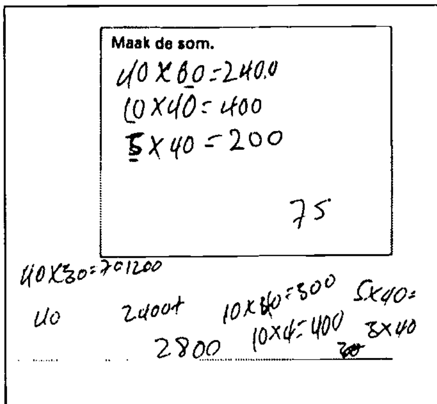
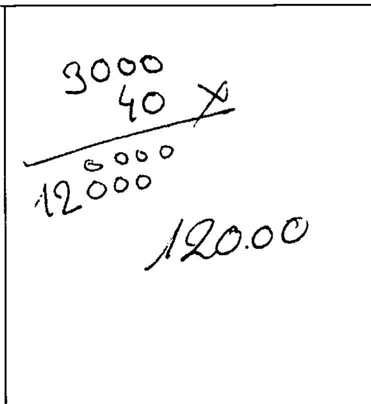
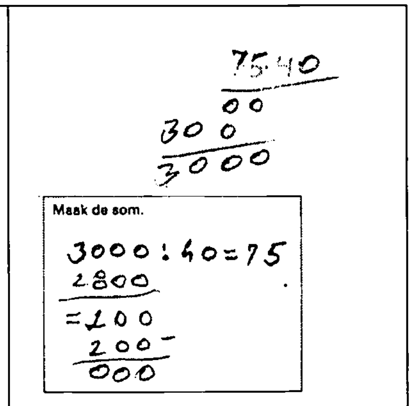
How many boxes?



A restaurant owner buys 3000 glasses.
The glasses are packed in boxes.
How many boxes should he buy?

The graphic only shows one box with the total "40 glasses" on it. It also shows the structure 8x5 in it. The instruction text is simple. The text on the box is minimal but essential to solve the problem.

This context offers many possibilities for solving the problem. A few examples:

		
Student 2	Student 4	Student 5

Student 2, the same student as in the previous example, starts by doing an estimate. Below the computation we see 40 x 30 = 1200 and below that, the number 2400. In the computation box we see 40 x 60 = 2400. Then we see in his scribbling 10 x 30 = 300 and 10 x 40 = 400 plus the number 2800. He adds in the computation box: 10 x 40 = 400. In the final step he adds another 5 x 40 = 200 and writes 75.

This student shows insight into the context problem and solves it by estimation and doing multiplication mentally, given his notations. The student probably does not have mastery of algorithms.

Student 4 does not understand the actual problem. He could read the text, as we checked afterwards. He shows a multiplication algorithm that does not fit in the context and with an incorrect answer. This is an example of "blind ciphering." This student showed more of such meaningless algorithms in his work. It may mean that he can do formal computations but is probably not used to solving context problems. Also, the point in the answer may indicate that he confuses this number with money or does not know exactly how to deal with commas and points that separate thousands or decimals in a number. In the Netherlands we use points for separating thousands, millions, etc. For the decimal point we use the comma, though the use of calculators may confuse people a lot.

Student 5 shows insight into the context problem. He can solve the problem by applying a correct long division and checks his answer by applying a multiplication. Again, this way of doing algorithms is not common in the Netherlands.

For the record: only 44% of this ABE student group got the right answer to this item.

5. Developing Placement Tests for Adults in ABE: A Challenge

The sample responses to the two items discussed above indicate that it is possible to construct a paper-and-pencil test in which ABE students can indeed show in a constructive way whether they understand a mathematical context and what kind of mathematical knowledge and skills they have acquired. The challenge with these items is that test developers cannot plan in advance that an item is meant for, e.g., "only" addition or multiplication or division. They can only indicate that it is meant for "basic operations." Such items enable ABE students to show their capacities on math, in these examples the four basic operations, but also their formal or informal strategies, or mental math. Test developers hence need to create a sufficiently broad set of items that can cover the grid of different skills according to curricula for ABE, taking into account the cross-linked nature of items as discussed in this paper. This way of testing may also provide additional information about people's ways of problem solving.

Information acquired from placement tests should provide directions for learning and teaching in ABE. Interesting conclusions can be drawn about people's ways of thinking and doing computations. For example, it may show that it is not necessary at all to learn (or to teach) algorithms in ABE when students are able to solve problems in more informal, but effective, alternative ways. Informal procedures, in combination with the sensible use of a calculator, may have as good results as formal procedures. Given the goals of ABE, at least in the Netherlands, this could have an important effect on math curricula in ABE.

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Developing a National Framework for Adults' Mathematics Education in Ireland: A Pilot Study

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Introduction
The authors have been involved in the development of a national framework for adult mathematics education within the remit of the National Council for Vocational Awards (NCVA). This work has been reported at previous ALM conferences, most recently at ALM-6 (O'onoghue, in press), where the proposal for a national framework was presented and discussed in a workshop. This work has continued under the aegis of the NCVA. One level of the national framework for adult mathematics education, namely, Mathematics, Level 2, was piloted during the academic year 1999/2000.

This paper reports on issues related to the pilot study that was directed by the authors from October 1999 to May 2000. It is intended to make Mathematics Level 2 available on a national basis from September 2000. The role of practitioners in this process is highlighted as it is novel in the context of national curriculum developments in Ireland.

Educational Context
Ireland has a highly centralised education system (see O'Donoghue, 1995, and Colleran & O'Donoghue, 1998, for a more detailed treatment of the system). There are now four distinct levels, Primary (4-12+); Second level (12-17+); a newly constituted Further Education sector; and Higher Education. Central in the emergence of the Further education sector as a distinct level within the Irish education system has been the work of the National Council for Vocational Awards (see O'Donoghue, in press).

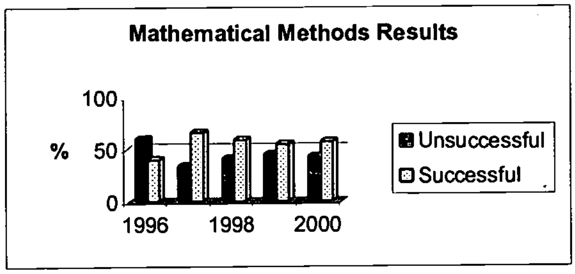
The process of mathematics curriculum development has evolved in Ireland from being determined centrally by the Department of Education and Science to the current partnership arrangement where the partners in the process are consulted before new syllabuses are introduced. The evolution from curriculum by decree to a negotiated curriculum and on to the current partnership arrangement is well documented (Oldham, 1992, 1993).

Background and Work to Date
O'Donoghue (1995, p. 404) posed the question "*What is the appropriate mathematics for the emerging FE sector?*" The author suggested as a solution an appropriately graded numeracy programme which includes functional mathematics for individual needs, employment-oriented mathematics, basic mathematics on which to build further studies, and entry standard mathematics for higher education.

In 1996 NCVA introduced Mathematical Methods Level 2 on a pilot basis. The standard set was targeted at a level that subsequently proved too high. This was evident from the high failure rates for the module (Figure 1) and supported by feedback from practitioners at centres. This situation was problematic in itself but assumed added importance because:

- mathematics acts as a gatekeeper to entry to this type of programme as many Irish Institutes of Technology specifically require successful completion of mathematics at NCVA level 2 or equivalent; and
- more candidates (~250 in 1996 increasing to ~900 in 2000) were completing the module to gain entry to technological-type programmes in Higher education, mainly Institutes of Technology.

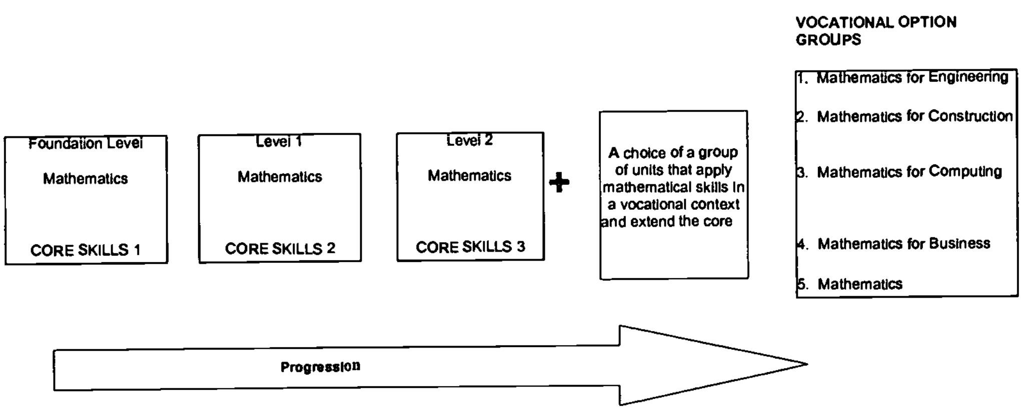
Figure 1: Results for Level 2, Mathematics 1996-2000



Subsequently the NCVA initiated a review of mathematics provision. The review group instituted a twin-track approach to deal with the situation.

- A short-term strategy, that involved increasing candidate question choice and time available to complete the examination was put in place to deal with learners currently enrolled in Mathematical Methods.
- A longer-term strategy to develop a new Level 2 Mathematics module, the proposal for a national framework for adults' mathematics education that articulates three levels of mathematics, Foundation, Level 1 and Level 2 was developed by the NCVA (O'Donoghue, in press). This framework is comprised of a progressive core (essentially concerned with number, geometry, algebra, data handling/chance) leading to vocational units related to specific vocational areas (Figure 2).

Figure 2: Proposed National Framework for Mathematics



Foundation Level Mathematics - This module develops core skills that relate to elementary number concepts.

Level 1 Mathematics - This module extends core skills 1 (numeracy, geometry, algebra, statistics) and provides learners with an opportunity to create and consolidate the base from which they can further develop their mathematical knowledge and skills.

Level 2 Mathematics - The mandatory core of mathematical skills extends the learner's foundation in essential mathematics and provides the basis for progression from Foundation Level to Level 1 to Level 2 and from Level 2 to higher education. Optional module units that extend these mathematical skills in an applied context provides the vocational focus to cater for the needs of a range of occupations e.g. Business, Computing, Construction, Engineering and Mathematics.

Dialogue with Practitioners

The group set up to review mathematics provision resolved from the outset to involve practitioners in every aspect of the process and measures were taken to ensure that this happened.

Practitioner Questionnaire and Feedback

In Summer 1998 practitioners[1] were surveyed and given the opportunity to input into the review process at an early stage. This survey revealed that the existing level 2 module was delivered by centres as part of technology-based courses (computing, engineering, electronics, etc.). Forty percent of the candidates who successfully completed these courses progressed to Higher Education. Sixty percent of the candidates are over 18 years of age. Eighty percent of candidates achieved a grade C or less in mathematics at Leaving Certificate ordinary level.

Practitioners were asked to rank the factors they thought influenced performance in mathematics at level 2 (Figure 3).

Figure 3: Factors Influencing Performance in the Mathematical Methods Module (as indicated by practitioners)

1	The calibre of the candidate completing the module	Most Likely
2	How the module is written and the standard that it sets	↓
3	The numbers of teaching hours available	
4	The way the module is delivered and the resources in the centre	
5	The way the module is assessed.	Least Likely

Gathering practitioner views is an integral part of the NCVA development process. The detailed information obtained through the use of the questionnaire gave valuable information on all aspects of the existing module and provided a springboard from which the review group could begin to revise the standard for mathematics at level 2.

The initial work of the review group focused on the following:

1. the revised module should have a vocational thrust.
2. the level of the content should be adjusted
   - there was too much or inappropriate assessment.
   - the new modules must be different from the leaving certificate so that candidates are not given *more of the same.*
   - the standard should be the catalyst for changing delivery in the classroom.

By September 1999, a drafts of three new modules were completed: Mathematics, Mathematics for Computing, Mathematics for Engineering. The NCVA approved the modules to run on a pilot basis during 1999/2000.

The review group recommended that practitioners should be given the opportunity for further input at this stage in the development process. All practitioners were invited to attend a workshop to consider the new modules in mathematics.

Workshop for Practitioners

The format for the workshop involved a series of brief presentations covering the vocational framework, mathematics teaching, innovative approaches to assessment, and an overview of the rationale behind each of

[1] Number of practitioners surveyed = 30, number of respondents = 15.

the units in the modules.[2] Practitioners were encouraged to adopt new approaches to delivery and assessment.

Support notes with sample assignments and questions drafted by the review group were circulated giving a clear indication of the standard. There was an opportunity for practitioners to discuss elements of the modules with a vocational specialist from the review group. Changes were subsequently made to the modules as a result of these discussions.

In all, 37 practitioners attended (about 90% of those delivering the module). The feedback on the proposal for the mathematics framework was very positive. Practitioners were interested in the approach to assessment by assignment, project, etc, rather than the traditional examination. However, many expressed concern about the availability of resources to support the new modules and whether, even with the new approach, their learners would successfully complete the modules.

As part of this process practitioners were invited to volunteer to use these modules as part of the pilot for 1999/2000. Those who agreed were required to provide feedback to the review group on their approach to delivery and assessment.

About half of those at the workshop expressed an interest in becoming part of the pilot. The others felt that:

- since the workshop was held in early October by which time learners had already started their programme it was too late to change from the existing module.
- opportunities for candidates to progress to Higher Education might be compromised since the new modules had not been formally approved for the Higher Education Links Scheme (a formal articulation agreement between Further and Higher Education).

Pilot Phase
The pilot was limited to six centres. Four of the centres were dedicated Colleges of Further Education; the remaining two also delivered at second level.

Five centres piloted Mathematics for Computing and one centre piloted Mathematics for Engineering. About a quarter of all candidates for mathematics at level 2 in 1999/2000 were included in the pilot group.

The practitioners met again in March 2000. By then all had completed the core units and were working on delivering the vocational units. Comments on the new modules were positive. The group shared the approach that they had taken to assignments and agreed sample examination questions for the vocational units.

The main concern was with the content of the vocational units in Mathematics for Computing. Practitioners themselves were having great difficulty coming to grips with teaching mathematics in a vocational context.

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[2] Core units: (All Modules)
Unit 1: Modelling using Mathematics; Unit 2: Graphs, Functions and Rates; Unit 3: Geometry and Trigonometry:; Unit 4: Statistics and Chance;

Vocational Units:
Mathematics *Unit 5: Further Calculus; Unit 6 : Complex Numbers and Trigonometry;*
Mathematics for Computing *Unit 5: Numeral Systems and Boolean Algebra;*
Unit 6: Algorithms and Computations;
Mathematics for Engineering *Unit 5: Engineering Magnitudes and Scales;*
Unit 6: Motion and Vectors;

Peer support on the day helped to solve a number of the problems highlighted. However, it was clear that the standard set in the vocational units in computing was too high. To address this problem the pilot practitioners recommended specific changes. In addition the practitioners also recommended that the final standard set for Mathematics at Level 2 should be deliverable by teachers of mathematics as well as vocational specialists. Revisions to the modules were made by the review group with immediate effect, and circulated to those involved in the pilot.

In June 2000, following year-end assessment, the pilot practitioners met again and gave the following feedback:

- the new modules were better than the existing module at level 2;
- learners liked the fact that the module was different from Leaving Certificate mathematics;
- regardless of background, at least some aspects of the module were new for all learners;
- learners liked the assignments which were radically different from an examination;
- the vocational approach made the subject more interesting to learners;
- there were still some concerns about aspects of the vocational computing units in Mathematics for Computing.

In terms of candidate achievement the 2000 results for both Mathematical Methods and Mathematics for Computing[3] are outlined below (Table 1).

Table 1: Results for NCVA Mathematical Methods and Mathematics for Computing June 2000

Module	**Mathematical Methods**	**Mathematics for Computing**
Grade	% of Candidates	% of Candidates
Distinction	12%	11%
Merit	16%	29%
Pass	27%	30%
Unsuccessful	45%	30%
Sample Total	100% (n=570)	100% (n=247)

Discussion

Creating a purposeful dialogue between practitioners and module designers is a challenging enterprise even when there is a willingness to engage. The benefits in this case are obvious to the authors and other members of the review group. There are still problems, as indicated by the unsuccessful rate that have to be addressed. Nevertheless significant progress has been made as indicated by the drop from 45% in the existing module to 30% in the proposed new module. These figures are greatly influenced by the prior mathematics/achievement of learners that in this case is dominated by low grades in the school Leaving Certificate (Mathematics). Indeed, it may well be that there is little scope for improvement here, however other outcomes of the pilot study are more encouraging.

A significant measure of agreement has been achieved between practitioners and the review group on the rationale and framework for adults' mathematics education under the aegis of NCVA, its content, pedagogy, and assessment. The new Level 2 Mathematics module has been endorsed by practitioners as being both teachable and learnable. Practitioners have contributed to the development of a support system including support notes and an informal practitioner network of peer support.

[3] One centre completed Mathematics for Engineering with a very small number of candidates; these results have been excluded from this data.
Number of centres completing Mathematics for Computing = 5.

In short, there is strong evidence that practitioners have now taken ownership of the new module and this, together with the new supports that have been fully endorsed by the review group, leads to optimism concerning the future success of mathematics education in the vocational context and a substantial reduction in the numbers of candidates that are unsuccessful.

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What Makes One Numeracy Task More Difficult Than Another?

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Abstract

This paper reports on the development to date of the numeracy conceptual framework for the Adult Literacy and Lifeskills (ALL) Project. The inclusion of a numeracy scale in the ALL survey offered a significant opportunity to develop a new conceptual framework for adult numeracy. This paper discusses the meaning of numeracy and the notion of numerate behavior, presents a framework of five facets, and looks in some detail at a complexity-rating scheme used to guide construction of the assessment tasks that looks at the question of what makes one numeracy task more difficult than another.

The ALL Project

An international survey of the numeracy abilities of adults is to be part of the Adult Literacy and Lifeskills (ALL) Project, formerly called the International Life Skills Survey (ILSS), planned for the year 2002. This comparative survey is being jointly developed by Statistics Canada and by the United States' National Center for Education Statistics (NCES), in cooperation with the Organization for Economic Cooperation and Development (OECD). The ALL project is a follow-up to the International Adult Literacy Survey (IALS), the world's first large-scale comparative assessment of adult literacy.

Tasks will assess performance in several skill domains, including Numeracy as well as Document and Prose Literacy, and Problem Solving, while other variables will be assessed via a background questionnaire.

To this date, items and background questions developed for the survey have undergone a feasibility study in the Netherlands and the U.S. After translation of the materials, a pilot study will be administered in the participating countries in 2001. The first round of the actual survey is scheduled for 2002.

Why Include Numeracy?

Numeracy is becoming a growing concern for diverse sectors. Countries are increasingly attending to topics such as improving workplace efficiency and quality processes, to resulting lifelong learning needs, and to civic participation. It is seen as vital that nations have information about their citizens' numeracy, among other skills, if they want to plan effective education and lifelong learning opportunities.

The concept of numeracy is also specifically related to the dialogue about the goals and especially outcomes and impact of school mathematics education. More educators now encourage links between knowledge gained in the mathematics classroom and students' ability to handle real-life situations that require mathematical or statistical knowledge and skills. However, while numeracy may be a key skill area, its conceptual boundaries, cognitive underpinnings, and assessment have not received much prior scholarly attention.

What Is Adult Numeracy?

One of the scales of the International Adult Literacy Survey (IALS), the Quantitative Literacy Scale, was a measurement of the respondent's ability to apply arithmetic operations to numbers embedded in diverse texts. While this scale produced useful data, survey developers recognized that it was limited in scope. The Numeracy scale of ALL is designed to go above and beyond the QL Scale, while avoiding reliance on formal, curriculum-based knowledge of mathematics.

IALS, following on a framework established in previous studies in the U.S. and Canada, made use of three literacy scales, Prose Literacy, Document Literacy, and Quantitative Literacy, to make operational its conception of literacy. The general definition of Literacy used in IALS was:

> Using printed and written information to function in society, to achieve one's goals, and to develop one's knowledge and potential.

The definition for Document Literacy (DL) was:

> The knowledge and skills required to locate and use information contained in various formats, including job applications, payroll forms, transportation schedules, maps, tables, and graphics.

The definition for Quantitative Literacy (QL) in IALS was:

> The knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials.

The working definition we have for numeracy in the ALL is:

> The knowledge and skills required to effectively manage the mathematical demands of diverse situations.

Facets of Numeracy

We have sought a view of numeracy that acknowledges the diverse purposes served by adults' mathematical (and statistical) knowledge, that encompasses the different suggestions regarding the skills adults need to effectively function in home, work, community, and other contexts, and that takes into account the cognitive, metacognitive, and dispositional processes that support or affect adults' numeracy.

Overall, numeracy is a multifaceted and sometimes slippery construct. Our basic premise is that numeracy is the bridge that links mathematical knowledge, whether acquired via formal or informal learning, with functional and information-processing demands encountered in the real world. An evaluation of a person's numeracy is far from being a trivial matter, as it has to take into account task and situational demands, type of mathematical information available, the way in which that information is represented, prior practices, individual dispositions, cultural norms, and more.

A full assessment of all elements of such a broad conception is beyond the scope of ALL. We have thus chosen to focus on numerate behavior, which is revealed in the response to mathematical information that may be represented in a range of ways and forms. The nature of a person's responses to mathematical situations depends on the activation of various enabling knowledge bases, practices, and processes.

Table 1 presents our elaboration of numerate behavior. It has been used to guide development of items for a Numeracy Scale for the ALL. The statement in Table 1 distinguishes five facets, each with several components.

Table 1: Numerate Behavior and Its Facets

Numerate behavior involves:

Managing a situation or solving a problem in a real context
everyday life
work
societal
further learning

by responding
identifying or locating
acting upon
- order/sort
- count
- estimate
- compute
- measure
- model

interpreting
communicating about

to information about mathematical ideas
quantity & number
dimension & shape
pattern and relationships
data & chance
change

that is represented in a range of ways
objects & pictures
numbers & symbols
formulae
diagrams & maps
graphs
tables
texts

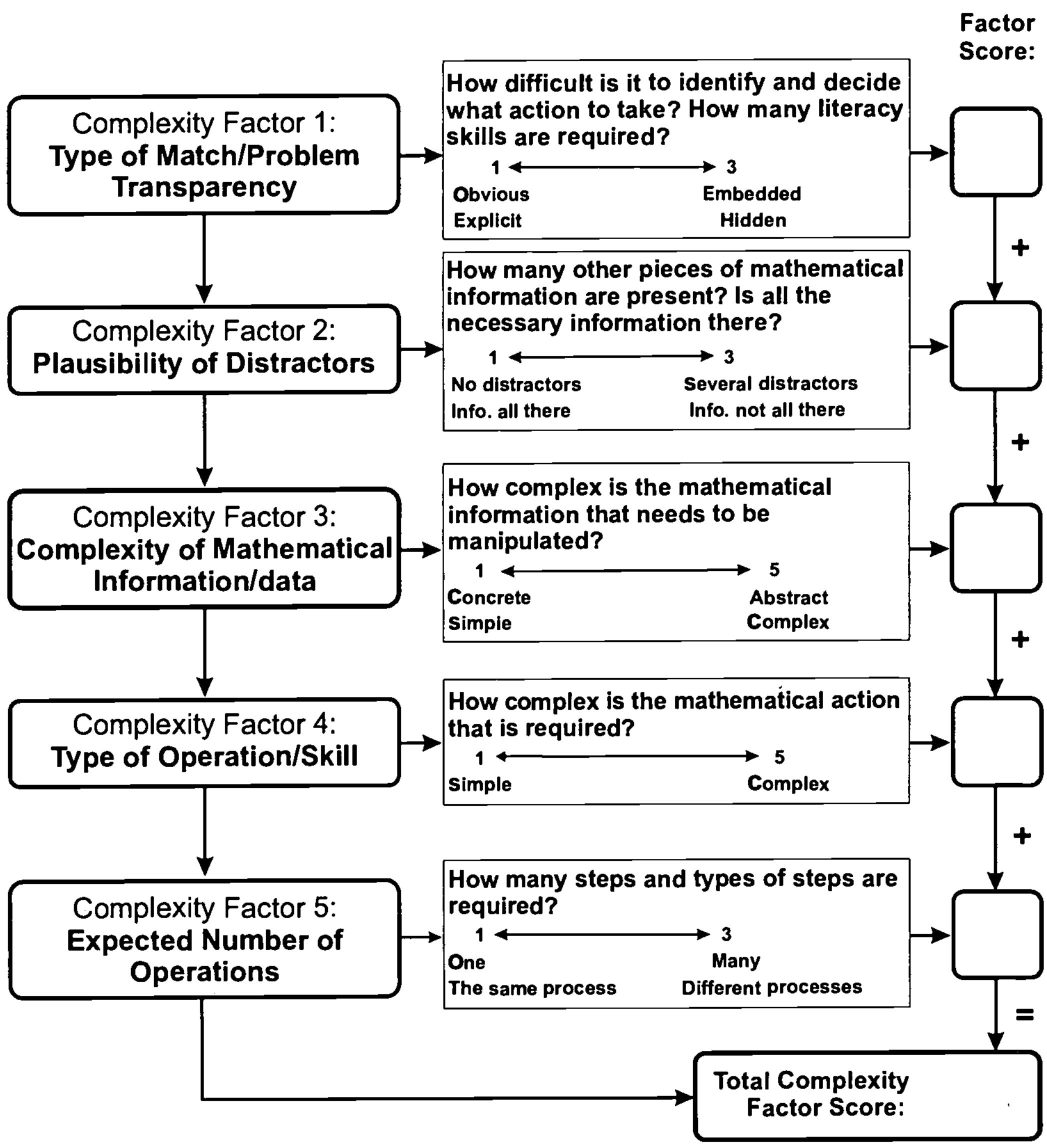
and requires activation of a range of enabling knowledge, behaviors, and processes
mathematical knowledge and understanding
mathematical problem-solving skills
literacy skills
beliefs and attitudes.

A comparison of these facets of numeracy with what was included in the previous concept and definition of quantitative literacy of the IALS demonstrates how far the ALL concept of numeracy goes above and beyond the QL Scale. While the IALS QL scale depended on stimuli that were text-based and often required considerable literacy skills, the numeracy scale includes stimuli that span the range from minimum text dependency to high. The IALS QL scale only assessed the application of arithmetic operations while the numeracy scale includes the assessment of a broad range of skills and knowledge.

Complexity Factors
A scheme of five factors was developed to account for the difficulty of different tasks, enabling an explanation of observed performance in terms of underlying cognitive factors.

These five factors are: (1) Type of match/problem transparency; (2) Plausibility of distractors (including in text); (3) Complexity of mathematical information/data; (4) Type of operation/skill; (5) Expected number of operations. These factors have been used to attempt to estimate, separately and in interaction, the difficulty level of the numeracy tasks. Each numeracy task or item that was developed for the test was given a rating between 5 and 19, the sum of the numbers assigned for each of the factors as outlined in the flow chart.

Diagram 1: Complexity Flow Chart



Each of these five factors has been developed in detail in the following tables.

Scoring for Each of the Complexity Factors

Complexity Factor 1. Type of match/Problem transparency How difficult is it to identify and decide what action to take? How many literacy skills are required?		
Score 1	**Score 2**	**Score 3**
In the question and the stimulus, the information, activity or operation required: - is clearly apparent and explicit—and all required information is provided - is specified in little or no text, using familiar objects and/or photographs or other clear, simple visualizations - is about locating obvious information or relationships only - closed question—not open-ended	In the question and the stimulus, the information, activity or operation required: - is given using clear, simple sentences and/or visualizations where some translation or interpretation is required - is located within a number of sources within the text/activity. - fairly closed question	In the question and the stimulus, the information, activity or operation required: - is embedded in text where considerable translation or interpretation is required - and/or - may need to be derived or estimated from a number of sources within or outside the text/activity - and/or - the information or action required is not explicit or specified - more complex, open-ended task

Complexity Factor 2. Plausibility of distractors How many other pieces of mathematical information are present? Is all the necessary information there?		
Score 1	**Score 2**	**Score 3**
- no other mathematical information is present apart from that requested—no distractors	- there is some other mathematical information in the task that could be a distractor - the mathematical information given or requested can occur in more than one place - may need to bring to the problem simple information or knowledge from outside the problem.	- other irrelevant mathematical information appears - mathematical information given or requested appears in several places. - necessary information or knowledge is missing, so outside information or knowledge needs to be brought in

Complexity Factor 5. Expected number of operations How many steps and types of steps are required?		
Score 1	**Score 2**	**Score 3**
one operation, action or process	application of two or three steps, the same or similar operation, action or process	integration of several steps covering more than one different operation, action or process

Complexity Factor 3. Complexity of mathematical information/answer required
How complex is the mathematical information that needs to be manipulated?

Score 1	Score 2	Score 3	Score 4	Score 5
Context Based on very concrete, real life activities, familiar to most in daily life.	Based on common, real life activities.	Based on real life activities, but less often encountered.	Based on real life activities but unfamiliar to most	Based on abstract ideas or unfamiliar activity in a context new to most.
Quantity Whole numbers to 1,000 Fractions, decimals, percents - benchmark fractions ($^1/_2$, $^1/_4$, $^3/_4$) - decimal fraction for a half only (0.5) and equivalent as a percentage (50%)	- large whole numbers including millions - other benchmark fractions, like $^1/_3$ and $^1/_{10}$ - common decimals, like 0.1, 0.25 to 2 decimal places - common whole number percents, like 25% and 10%.	- large whole numbers including billions - other fractions - decimals to 3 decimal places (other than money) - all whole number percents	- negative integers - all remaining fractions, decimals and percentages	- all remaining types of rational (and some irrational) numbers including directed numbers
Pattern and relationship - very simple whole number relations and patterns	- simple whole number rates and ratios - whole number relations and patterns	- rates and ratios - relations and patterns including written everyday generalizations	- complex ratios, relations, patterns - simple formula	- formal mathematical information such as more complex formulae, knowledge of relationships between dimensions or variables, etc
Measures/ Dimension/Space - standard monetary values - common everyday measures for length (whole units) - time (dates, hours, minutes) - simple, common 2D shapes - simple localised maps or plans (no scales)	- everyday standard measures for length, weight, volume , including common fraction and decimal units - common 3D shapes and their representation via diagrams or photos - common types of maps or plans with visual scale indicators	- other everyday measures (area included) including fraction and decimal values - more complex 2D and 3D shapes, or a combination of 2 shapes - area and volume formulae - common types of maps or plans with ratio type scales	- all kinds of measurement scales - complex shapes or combinations of shapes	
Chance/Data - simple graphs, tables, charts with few parameters and whole number values - simple whole number data or statistical information in text	- graphs, tables, charts with common data including whole number percents—whole number scales in 1s, 2s, 5s or 10s - data or statistical information including whole number percents	- graphs, tables, charts with more complex data (not grouped data) - more complex data or statistical information including common average, chance and probability values - scales - more complex whole number, fractional or decimal scales	- complex graphs, tables or charts including grouped data - complex data or statistical information including probabilities, measures of central tendency and spread	

Complexity Factor 4. Complexity of Type of operation/skill
How complex is the mathematical action that is required?

Score 1	Score 2	Score 3	Score 4	Score 5
Communicate no explanation - a single simple response required (orally, or in writing)	- no explanation - a simple response required (orally, or in writing	- simple explanation of a (level 1 or 2) mathematical process required (orally, or in writing) -	- explanation of a (level 3) mathematical process required (orally, or in writing)	- complex, abstract and generative reasoning or explanation required
Compute - a simple arithmetical operation (+, -, x, ÷) with whole numbers or money	- calculating common fraction, decimal fraction and percentages of values - using common rates (e.g. $/lb.); time calculations; etc - changing between common equivalent fraction, decimal and percent values, including for measurements e.g. $^1/_4$ kg = 0.250kg	- more complex applications of the normal arithmetical operations such as calculating with fractions and more complex rates, ratios, decimals, percentages, or variables - simple probability calculations	- applications of other mathematical operations such as squares, square roots, etc	- more advanced mathematical techniques and skills e.g. trigonometry
Estimate	- estimating and rounding off (when requested) to whole number values or monetary units	- estimating and rounding off to requested number of decimal places	- making a contextual judgement re whether a found answer is realistic or not and changing the answer to the appropriate correct rounded (but not necessarily mathematically correct) answer.	
Use formula/ model	- evaluating a given formula involving common operations (+, -, x, ÷)		- developing/creating and using straight forward formulae - using strategies such as working backwards or backtracking (e.g. 15% of ? = $255)	- generative reasoning - using and interpreting standard algebraic and graphical conventions and techniques
Measure - knowing common straight forward measures - naming, counting, comparing or sorting values or shapes	- visualizing and describing shapes, objects or geometric patterns or relationships - making and interpreting standard measurements using common measuring instruments	- using angle properties and symmetry to describe shapes or objects - estimating, making and interpreting measurements including interpolating values between gradations on scales - converting between standard measurement units within the same system	- calculating measures of central tendency and spread for non-grouped data - converting between non-standard measurement units within the same system - counting permutations or combinations	- converting between measurements across different systems
Interpret - locating/identifying data in texts, graphs and tables - orientating oneself to maps and directions such as right, left, etc	- reading and interpreting data from texts, graphs and tables - following or giving straight forward directions	- interpolating data on graphs - calculating distances from scales on maps	- generating, organising, graphing non-grouped data - extrapolating data - reading and interpreting trends and patterns in data on graphs, including slope/gradient	- generating, organising, graphing grouped data - calculating measures of central tendency and spread for grouped data

The development of a scale that attempts to predict the complexity/difficulty of numeracy tasks was one of the more exciting, and challenging, aspects of the project.

Results and Next Steps

Based on the conceptual framework, over 100 items were developed and tested in a feasibility study in the U.S. and the Netherlands. A pool of 80 items remains that includes tasks at diverse levels of difficulty and that covers key facets of the conceptual framework for numeracy. For those 80 items, we found that the theoretical factors that were predicted to account for task difficulty were strongly correlated with the observed difficulty of items. A scatter plot of the data shows a significant negative linear correlation with an r of -0.859. Thus, about 74% of the variation in performance on the 80 items is accounted for by the complexity factors.

Preliminary results therefore provide initial support for the content validity and the construct validity of the numeracy scale. More reliable data are expected from the pilot study and the main survey.

The next steps are to revise the complexity scheme in the light of feedback received to date, and to ask other interested teachers or researchers to rate items using the complexity scheme. Initially this could be to test the inter-rater reliability of the scheme on the 80 pilot study items. A further validation stage could be to test the scheme on new sets of items, and compare the theoretical predictions of difficulty levels with the actual performance of the items with sample respondents.

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The Numeracy Team consists of: Yvan Clermont, Statistics Canada; Iddo Gal, University of Haifa, Israel; Mieke van Groenestijn, Hogeschool van Utrecht and the Freudenthal Institute, Netherlands; Myrna Manly, El Camino College, California; Mary Jane Schmitt, National Center for the Study of Adult Learning and Literacy, Harvard University, USA; and Dave Tout, Language Australia.

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From Research to Reality: Progress Toward Integrating Research and Practice in a Large School District

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Many adults are looking for a second chance to prove they can be successful in academic course work required for a traditional high school diploma from an accredited school. For a sizeable percentage of these people it is important to have the actual diploma rather than an equivalency certificate. The Los Angeles Unified School District, second largest in the United States, offers this choice to thousands of adult learners each year through the Diploma Plus Program. This program is a personalized, competency-based, adult high school diploma program designed to foster learning in the academic areas that enhance job skills and life skills in today's world.

In 1995 a plan was put in place to revise and upgrade the math curriculum for Diploma Plus as a need had been identified to improve the overall effectiveness in the way math was being taught. The plan reflects the 1989 NCTM Standards document and other research in adult learning practices. Even before the state or local district had produced comprehensive student learning standards it was clear that student learning standards must be incorporated into the new curriculum. The goal of this plan: Mathematically literate people who can read, understand, appreciate, share, and apply the mathematics found in the many roles played in life.

The new math program was researched and developed by a group of several credentialed, field-based, adult education math teachers. The practical result of this work is seven math courses, each comprising a 60-hour semester block, equivalent to the traditional high school course structure in Los Angeles Unified School District. Since adult learners do not always come to Diploma Plus with requisite math skills equal to a ninth grade math education, the first three of the seven semester courses are remedial in content. The final four semester courses are integrated with algebra and geometry concepts in a spiraling model that builds on skills from previous courses.

The foundation of each course is a course outline and a companion instructional document known as a student learning contract. These contracts are instructional road maps that contain all of the assignments to be completed for credit. The student contract assignments are drawn from a wide variety of carefully selected materials that include manipulatives and computer software to enhance student retention and enrich math literacy. Student learning contracts can be used in diverse ways to guide instruction in small groups, or as guides for larger group activities. The primary value of this modality is, however, the accountability it provides to individualized instruction, the core of the Diploma Plus program.

To strengthen instructional effectiveness of this model, teachers can access continuous training opportunities through the adult education division of the school district. Many teachers in Diploma Plus do not have a strong background in math education. These staff development sessions provide support and guidance for the less-than-confident adult education math teacher.

The task of integrating research with classroom instruction in adult education in the Los Angeles Unified School District is an ongoing process. The Integrated Math package of courses in Diploma Plus has proven to be an outstanding opportunity for adult learners as well as at-risk youth to challenge the low expectation mentality that many have for these learners. Our informal research demonstrates that these learners can and do succeed in more rigorous learning situations. Diploma Plus will continue to review and respond to the challenges of new research in adult learning as resources allow.

Adult Numeracy Teaching – An Australian Focus on Social Contexts

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Abstract

There are three interconnected aspects to this presentation:

- *Teaching numeracy in social context is well embedded within both Australian numeracy frameworks and teaching practices. Some of the guiding documents and their origins are referred to. The purposes of accreditation are questioned. Testing (regardless of which calculator you use) is compared unfavorably with ongoing assessment integrated into students' learning activities.*

- *A schema is proposed of what we desire students to achieve. In addition to* ***functional performance*** *(such as catching public transport, banking, household measuring) and* ***transferable understanding and skills****, an additional category of* ***social knowledge or understanding*** *is described.*

  It is questioned whether there is particular socially important knowledge, based on understanding of mathematically encoded information, which numeracy teachers ought to teach explicitly. Examples will be given such as, in the U.S. context, the importance of teachers supplying numerical facts about the correlation between gun availability and gun death.

- *Various teaching activities are offered to contribute to the direct practical usefulness of the Conference. They feature the Olympics, fitness, kitchen design, gambling, measurement sampling using alternative units (but there is only space for one here), and some Australian Internet sites are indicated.*

This presentation introduces approaches to numeracy teaching in Australia, particularly emphasising the social contexts of what is learnt, and indicating my own position on some pertinent issues.

Contents

This presentation refers to:

- A schema of what numeracy teaching involves.
- A common statement of numeracy knowledge domains.
- The Australian attempt to portray numeracy in genres similar to those used for literacy.
- Some reflection on assessment, accreditation, and course outcomes.
- Particular socially important knowledge based on understanding of mathematically encoded information which numeracy teachers ought to teach explicitly (which is the key point).
- Professional development and teaching materials created by the leading group of Australian numeracy developers.

What Should Students Achieve?

To know how we ought to teach we need to answer the question "What should students achieve?" Plato thought it was *arete* (virtues/excellence), then wondered what constituted *arete*. We have more limited aims here.

A number of precepts will be taken for granted; the gamut of adult learning principles which render the whole enterprise learner-focused. Within that, my schema of numeracy achievement, and hence numeracy teaching, involves:

- Transferable understanding & skills.
- Functional performance.
- Social knowledge.

The idea of **transferable understanding & skills** is well understood. As the U.S. National Council of Teachers of Mathematics (NCTM, 2000, p. 5) puts it in their Principles and Standards: "When students understand Mathematics, they are able to use their knowledge flexibly." In a social context an example would be for a student to understand the meaning of a percentage when they come across it in a newspaper article.

Functional performance is similarly familiar to the adult numeracy field. Relevant social examples of functional performance include being able to take medicines, follow recipes, check change when shopping, catch public transport, and use automatic banking machines.

These first two categories have been well discussed, most recently by Ciancone and Tout at ALM7, but the third, **social knowledge**, though widespread in the practice of low-level numeracy teachers, has not been articulated, recognised, and developed as it could be. This formal overlooking hitherto has consequences for the governing frameworks describing what we want students to *know* in addition to what they are able to *do*.

Social Knowledge

This category of knowledge is important information, which numerically skilled people, such as numeracy teachers, know about the society and world they live in. It includes the good advice teachers typically give their students. A state of partial understanding, of knowing without totally knowing, is exemplified when we may understand some of the precepts and consequences of theories of relativity or quantum mechanics without ever having followed the mathematics. Many professionals who regularly apply statistical tests in their line of work do so without ever having comprehended the mathematical derivation of those tests. There is much about the world, both natural and social, which is based on mathematical understandings, and important to know regardless of whether the mathematics is itself understood. Numeracy teachers regularly teach their students such facts, and it is worth recognising this as part of the adult numeracy curriculum.

Before further developing this idea, we will examine briefly other conceptions of numeracy teaching, quickly surveying the development and strength of the "social" in numeracy teaching in Australia, and some issues in reporting student achievement.

As an example of how well established the "social" approach is in Australia, these quotes are taken from an end-of-course report by a teacher about her class:

> These skills are required to interpret and make sense of the world, and so should be taught not in isolation, but as part of other contexts and applications.
>
> Mathematical literacy is essential for full participation in Australian society. Adults need to be able to interpret graphs, tables and leaflets. The ability to talk about and describe different shapes, to follow and give directions and to measure are everyday skills which require maths to be seen as part of our language and culture. (CARWP, 1994)

In Australia socially-contextualised numeracy teaching practices have been developed through the numeracy stream of the Certificate of General Education for Adults (CGEA). The first version of the CGEA divided mathematics into five strands: number, space and shape, data, measurement, and algebra. Such arrays of mathematics topics are standard worldwide. We find similar categorisations used in Canada (Ciancone, 2000) and in school mathematics in the U.S. (NCTM, 2000). Such lists of topics are useful for checking that your teaching is covering a breadth of mathematics fields. In Australia a rather different schema is now being tried.

Five ways to make meaning in mathematics were put forward by Betty Johnson (1994, p. 32, quoted in Tout, 1999). These are through:

- **ritual**, where meaning is acquired by rote learning of atomised content;
- **conceptual engagement**, where mathematical meaning is constructed through problem-solving, process, and cognitive dissonance;
- **use**, where meaning is developed through use in everyday contexts;
- **historical and cultural understanding**, where meaning is enhanced by an understanding of the genesis and cultural use of specific mathematics; and
- **critical engagement**, where meaning is generated by asking "in whose interest?" and also questions about the appropriateness and limits of the maths model in the real situation.

Adult literacy teaching in Australia has been strongly influenced by systemic functional linguistics and the genre theorists. A schema for the practical teaching of reading and writing has been provided in the work of Rob McCormack et al. (Bradshaw, 1993, p. 137). They used genres of literacy as procedure, as public debate, as self-expression, and as knowledge to describe the literate repertoire. Stressing the **purposes** of everyday mathematics Beth Marr et al. have tried to mirror this categorisation in numeracy, yielding Numeracy for Practical Purposes, for Interpreting Society, for Personal Organisation, and for Knowledge.

- **Practical Purposes** addresses aspects of the physical world to do with designing, making, and measuring.
- **Interpreting Society** relates to interpreting and reflecting on numerical and graphical information of relevance to self, work, or community.
- **Personal Organisation** focuses on the numeracy requirements for personal organizational matters involving money, time, and travel.
- **Knowledge** deals with mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings and/or assumptions. There are learning outcomes to do with problem solving, and algebraic and graphical techniques.

This was a bold vision. In hindsight it seems neither entirely successful, nor necessary. The rubric associated with description of performance becomes overly specific, and potentially bureaucratic in its application to teaching and assessment. In practice, these genres simply do not fit as well in numeracy as they do for describing writing.

Measuring Learner Achievement Testing? Or Performance of Holistic Tasks?
In Australia, and increasingly in some other countries (Ireland, for instance), assessment is based on a portfolio of student work in which the student evidences competence with the numeracy in question. The new NCTM Principles and Standards for School Mathematics (2000, p. 5) hold that "Assessment should be an integral part of instruction that guides teachers and enhances student learning. Teachers should be continually gathering information about their students."

What distortion does the GED test in the U.S. bring to good numeracy teaching?
Indeed, assessment is an integral part of teaching processes (though not of learning processes). But the rationale of assessment for certification ought to be carefully considered. What is the accreditation for? The learner, the teacher, the teaching institution, third parties? Who actually uses the Certificate? What difference does the Certificate actually make? The answers differ markedly from one country to another.

In the U.S. a high school certificate or its equivalent, the General Educational Development Certificate (GED), plays a large role in adult access to further study, acting as a hurdle to be overcome. Lacking one has consequences. Yet most research shows negligible economic consequences of GED success (NCSALL Research Briefs).

In Australia, with the notable exception of entry to the armed forces, the CGEA is almost entirely without consequence, except psychologically to the learner, and to the bean-counters in funding bodies who want to be assured that courses achieve outcomes. Adults (over 23) can usually gain access to further study without specifically obtaining high school equivalence.

The CGEA was created to give strategic legitimacy to an education field in danger of being overlooked, because it was not part of the school and vocational mainstream.

The U.S. National Center for the Study of Adult Learning and Literacy (NCSALL) has begun good work identifying what it is that students gain from our courses. Bingman (2000, p. 14) informs us that learners reported

> changes in their lives that were varied, contextual, and inter-related. Measuring changes in educational levels with standardized tests will not give programs and policy makers information about these outcomes. The tests are neither broad enough nor sensitive enough to capture the changes that matter to learners or to measure the performance of programs in supporting these changes.

Bingman et al. (1999, p. 1) had previously pointed out that "the other positive changes occurring in students' lives, especially those outside the classroom, generally are not assessed, perhaps because they are difficult to track and to measure."

Whom do we teach? Mostly legally competent adults, who themselves are the individuals best placed to decide whether a course is beneficial to them. Anecdotal reporting by learners can be a valid measure of course success.

Social Knowledge: Actual "Mathematical" Information about the Student's World

The excellent new adult numeracy TV series being produced in New York, TV411, has a segment in which rent-to-buy (hire-purchase) is shown to be much more expensive than straight-out purchase. This is an example of the sort of knowledge we are advocating be taught directly to our students **whether or not they understand the mathematics behind it.** Numeracy teachers are in a position to provide introductions to more complex mathematical ideas of particular relevance, without teaching the mathematics in full.

The teacher has a responsibility to pass on the most significant **mathematical** social (and natural) understandings of the world that they have. Examples of this are:

- If you gamble in a net loss game, it is increasingly likely over time that you will lose, and lose more (see teaching example below).
- In Australia, each withdrawal from a cheque account is taxed. So if you use a cheque account as your every day account, say, making frequent small withdrawals from automated teller machines, you pay a lot of tax.

In the U.S. context appropriate social facts to teach may be that

- The rate of gun death in the U.S. is four times that of Australia, and more than 20 times that of Britain. The table below illustrates this, but a bar graph would be much more effective, though setting appropriate scales would be difficult for students, with the U.S. disappearing off the page and Japan hardly visible (CSGV, 2000).
- With new gun restrictions in Victoria, Australia, domestic gun murders of women fell (Halloran, 1993).

Such numerical facts do not replace debates about freedom and gun availability, but they do inform them significantly.

DEATHS DUE TO FIREARMS
(rates are per 100,000 people)

	Total Firearm Deaths		Firearm Homicides		Suicides		Fatal Accidents	
	Rate	Number	Rate	Number	Rate	Number	Rate	Number
United States (1995)	13.7	35,957	6.0	15,835	7.0	18,503	0.5	1,225
United Kingdom (1994)	0.57	277	0.13	72	0.33	193	0.02	12
Australia (1994)	3.05	536	0.56	96	2.38	420	0.11	20
Canada (1994)	4.08	1,189	0.6	176	3.35	975	0.13	38
Germany (1995)	1.47	1,197	0.21	168	1.23	1,004	0.03	25
Japan (1995)	0.07	93	0.03	34	0.04	49	0.01	10

A Class Activity with Sampling/Statistical Content:
Principles of Gambling, or the Immorality of Unfair Games
All commercial gambling activities (that are not completely fraudulent) run on a basis similar to this game.

Take **three** coins. Bet one dollar. You toss the coins.
If you get two heads exactly (tails if you prefer), I'll give you back two dollars, i.e., you would win a dollar.
Do you want to play?

How about this one then? Take **two** coins. Bet one dollar. Throw two heads, and I'll give you back 3 dollars!
Roll up! Roll up!

Now we play until most of you are convinced that you are losers at this game, and in (almost) any case we can see that there are more people who have lost more money than a lucky winner may have won.

This may be thought to be a scheme to supplement casual teachers' paltry wages, but the main aim is to have you understand how gambling games work. They are set up as net loss games for the players.

Using a decision tree we can construct a plan of all possibilities and then compare the numbers of winning outcomes with losing outcomes to work out how the payout differs from what would be paid out if these were fair games.

Can you construct a simple game (simple enough for us to later demonstrate the mathematics of it) that looks attractive to players on first glance, but fleeces them nevertheless?

Australian Resources
In Australia the triumvirate of Beth Marr, Betty Johnson, and Dave Tout have played towering roles in the creation of the field of adult numeracy teaching. Their teaching resources and professional development materials are highly regarded for their specific application in teaching adults.

Finally, some numeracy teaching sites which may be beneficial:

http://www.staff.vu.edu.au/mcaonline
This site, created by Syed Javed and colleagues at VUT, provides interactive assistance with mathematics learning at a wide range of levels, but lacks the social contextualisation distinctive of the Australian contribution in this field.

http://sunsite.anu.edu.au/language-australia/numeracy
This site is managed by Dave Tout. It includes a bank of numeracy lessons written by teachers from around Australia.

http://www.vicnet.net.au/~acenet/newspaper.html
This site is managed by Dale Pobega. It has a range of CGEA online numeracy courses.

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THEORETICAL FRAMEWORKS

Developing a Theoretical Framework for Adults Learning Mathematics: ALM-7 Topic Group

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Research in "adults learning mathematics" is a new domain between research in adult and mathematics education. Some of the questions are sociological, some psychological; other questions are mathematical, educational, philosophical, or anthropological. "The field itself is ill-defined—or wide open, depending on one's point of view" (Coben, 2000, p. 47). In order to catch the specificity of the domain we have introduced different metaphors—area, field, moorland, borderland, etc.—at the last four ALM conferences. In workshops and topic groups, a growing group of participants has engaged in an on-going exploration of the theoretical frameworks for adults learning mathematics. This session at the conference has always proved contentious, demanding, and highly stimulating.

At the ALM-4 conference in 1997, a group started the discussion on the paper by Tine Wedege entitled "Could there be a specific *problematique* for research in adult mathematics education?" i.e., a systematically linked problem field in which questions and answers about the subject field are formulated on the basis of a certain theoretical and/or methodological approach (Wedege, 1997; see also Wedege in this volume). A central question in the debate concerned the situation of our research and practice in the scientific landscape. At the ALM-5 conference in 1998, we continued the debate. This time the perspective was "Adults Learning Mathematics as a community of practice and research." We tried to characterise the ALM practice and research as situated in the border area between adult education and mathematics education. It was a point that there are no clear boundaries in our domain (Wedege, Benn, & Maasz, 1998). During the ALM-6 conference in 1999, the reflective discussion was continued in the form of a topic group "Developing a theoretical framework." Three themes structured the debate: (1) The value of multiple theories and perspectives; (2) Should our emphasis be on the A, the L, or the M (Adults, Learning, or Mathematics) or on the relationship of the three? and (3) The value of ALM is that there is a dual emphasis on research and practice. The combination of researchers who practice and practitioners who conduct research is what makes ALM unique (Benn, Maasz, & Safford, 2000).

During the ALM-7 conference in Boston, the reflective discussion was continued in the topic group "Developing a Theoretical Framework for Adults Learning Mathematics." It was stated that different theories offer a larger perspective in practice and research and that interdisciplinarity is a must. Thus, we began to make explicit some of the possible theoretical and methodological choices (e.g., What do we mean by mathematics education?) and how these choices do affect our practice.

The debate will continue at ALM-8 in June 2001, Roskilde University, Denmark. Join us there!

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Tracking Ways of Coming to Know With the Shifting Role of Teachers and Peers: An Adult Mathematics Classroom

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Introduction

Epistemological perspectives are the ways students interpret or make meaning of their educational experiences as a result of their assumptions about the nature, limits, and certainty of knowledge (Perry, 1970). Collectively these perspectives form ways of knowing (Kitchener, 1983). Recent research initiatives in the area of women and mathematics stress the need to develop strategies to counter traditional mathematics pedagogy and epistemology which is proposed to have alienated many girls and women by not appreciating or validating their ways of coming to know (Becker, 1996; Burton, 1995, 1996; Jungwirth, 1993; Roger & Kaiser, 1995). How students come to know mathematics, as well as the nature of mathematics itself, is particularly pertinent for students who are traditionally under-represented in mathematics.

The perception that mathematics is absolute is an epistemological perspective and is common among women returning to study mathematics where it is associated with a perceived need for procedural learning or rule following (Beesey, 1995). In this context the teaching philosophy proposed to provide the opportunity for success incorporates the women's common-sense knowledge, draws upon and values their own experiences, focuses on practical applicability, and promotes a collaborative classroom environment (Feil, 1995; Helme, 1995; Isaacson, 1990). Research on whether the women's epistemological assumptions about mathematical knowledge change over the duration of such courses to promote more complex ways of knowing is in its infancy (see Taylor, 1995). Becker (1996) suggested the theories of Belenky et al. (1986) and Baxter Magolda (1992) as possible guides in future research on gender and mathematics.

Theoretical Framework

Belenky et al. (1986) and Baxter Magolda (1992) re-examined the cognitive developmental theory of Perry (1970), considered by Foley (2000) to be one of the most interesting theories for adult educators. Perry (1970) proposed a schema of how male[1] university students shift from seeing knowledge as something that is handed down to them from authorities to seeing knowledge as relative (everyone has the right to their own opinions) to seeing that knowledge is constructed by people in particular social contexts in accordance with particular values. The models of Belenky et al. (based on women's experiences from a broad range of socio-economic backgrounds) and Baxter Magolda (based on both men and women's experiences in a university context) broadly overlap with the schema proposed by Perry (1970), but significant divergence was noted. Belenky et al found that many women experienced greater obstacles in their intellectual development, that they had more distant relationship to authority, and along with Baxter Magolda (1992), also noted how women are more likely to come to know through inter-personal relationships.

Using a grounded theory approach Belenky et al. (1986) proposed five perspectives from which women view reality and draw conclusions about truth, knowledge, and authority: *Silence; Receiver; Subjective; Procedural;* and *Constructed* (see Table 1). The "Silent" perspective is particularly pertinent for women returning to study mathematics due to the anxiety many have with the learning of mathematics as a consequence of poor experiences at school. The "Receiver" perspective most closely aligns with the emphasis on rule based learning. A "Subjective" perspective, a focus on relying on personal experiences to create meaning, may also represent a further hurdle to overcome to allow for the acceptance of mathematical knowledge derived by external authorities to be viewed with legitimacy. The term "Procedural" knower is unfortunate in the context of learning mathematics as it is usually associated with rule following rather than the systematic reasoning that Belenky et al. convey. Two voices emerged for the Procedural knower, Connected and Separate knowing. These voices appear to be the precursors to Baxter Magolda's gender-related reasoning patterns discussed

[1] Perry (1970) did interview some women but his schema was based entirely on men's experiences.

below and in Table 1. Belenky et al. were reticent to propose a hierarchical schema and suggested future research would shed light on this aspect of their work.

Baxter Magolda (1992) identified four qualitatively different ways of coming to know that she proposed were stage-like: *Absolute; Transitional*; *Independent;* and *Contextual* (see Table 1 for a description). While Transitional knowers are struggling to move away from an Absolute orientation, to come to terms with knowledge being inherently uncertain and socially constructed, thinking for oneself is the core element of the Independent knower. Contextual knowers extend this position "to thinking for oneself within the context of knowledge generated by others" (p.168). While all four ways of knowing were evident for both men and women, gender-related reasoning patterns were identified (see Table 1). Women are proposed to more likely develop intellectually via the receiving, interpersonal, and inter-individual reasoning patterns while men are proposed to more likely develop intellectually through the mastering, impersonal, and individual reasoning patterns. The integration of the models proposed by Belenky et al. and Baxter Magolda are provided in Table 1 but also see Brew (1999).

The positioning of the subjective knower described by Belenky et al. with respect to Baxter Magolda is difficult to integrate having elements of absolute and transitional knowing but different all the same because of the reliance on relying on personal knowledge rather than knowledge derived from authorities to determine what is truth. As over half of the women interviewed by Belenky et al. conveyed this orientation and a shift into this perspective often preceded women making the decision to return to study, it remains an important perspective in the context of adult further education.

Baxter Magolda's model was chosen as the starting framework for two main reasons. First because she described how students shift in their epistemological perspectives with respect to five domains of learning: the role of the learner, peers, the teacher, assessment, and perception of knowledge. These domains provide a useful structure for a formal learning setting. Second, it was important to be open to hearing diverse voices, the gender-related, not gender-dictated reasoning patterns. The five perspectives of Belenky et al. were then integrated as feasibly as possible.

Here I focus on two key learning domains of Baxter Magolda, the role of the teacher and peers, to explore whether the integrated framework provides a useful lens to consider women's ways of coming to know mathematics. Ultimately the aim is to provide a framework to guide the teaching of mathematics where the absoluteness of mathematical knowledge can be challenged while at the same time holding on to the objectivity, the precision, and the accuracy that such knowledge can provide.

Methodology

Data were collected from students enrolled in a full-time women's only Technical and Further Education (TAFE) science course where mathematics was a significant component. The course was developed for early school-leavers to provide them with the option of entering into non-traditional areas of further study. Here there was a commitment to collaborative group work as the teacher believed there was immense value for learners to verbalise their own understanding and hear others clarify their reasoning.

Initial interviews with participants occurred at the end of their second term and this was after three weeks of attendance at classes. Follow-up interviews occurred at the end of the course and many of these were by telephone. Observation of classes continued on a regular basis. Interviews explored the students' perspectives on mathematical learning with respect to the five domains of learning. Their reasons for returning to study along with past school experiences were also gathered. I focus on two case studies using interview and observational data to illustrate different epistemological perspectives with respect to the role of the teacher and peers.

Table 1: Summary of the Integrated Epistemological Models of Belenky et al. (1986) and Baxter Magolda (1992)

	Silence (Belenky et al.)	**Absolute Knower** (Belenky et al. & Baxter Magolda)	**Subjective Knower** (Belenky et al.)	**Transitional knower** (Baxter Magolda)	**Independent knower** (Baxter Magolda)	**Procedural knower** (Belenky et al.)	**Contextual** (Baxter Magolda) **& Constructed knower** (Belenky et al.)
General description	Sense of feeling dumb and stupid	Dualistic thinking. Replicate knowledge of authorities. Confused by ambiguity	The mathematics has to be personally owned or will be rejected	Some knowledge is uncertain where multiple perspectives are legitimate	Absolute perspective replaced by the valuing of diverse methods & explanations. Judgment as to which methods are more valid is rare.	Some truths are truer than others. Systematic analysis. Elements of Independent knowing	An authentic voice. Ambiguities & complexities in mathematical knowledge authentic. Indicative of the use of mathematics in social contexts
Role of the teacher	Passive, reactive and dependent on external authority for direction and truth	*Receiver reasoning pattern* Provide a relaxed environment. Minimal interaction expected.	To make the mathematics relevant.	*Inter-personal reasoning pattern* Encourage peer interaction, the verbalising of the mathematics	*Inter-individual reasoning pattern & Connected knowing* To provide an environment in which different approaches to solving mathematical problems are valued		The gender-related reasoning patterns of the Independent knower & the two Procedural voices proposed to converge
		Mastery reasoning pattern Strong identification. Expectation to be quizzed.		*Impersonal reasoning pattern* To challenge them to explain their mathematical reasoning	*Individual reasoning pattern & Separate knowing* To provide an environment in which their own and others' approaches to solving mathematical problems can be debated		
Role of peers	An inability to find meaning in the words of others or learn from others' experience	*Receiver reasoning pattern* Social. To create relaxed atmosphere and to ask questions to relieve pressure	To support making the mathematics real, relevant to their everyday lives	*Inter-personal reasoning pattern* Openness to peers' knowledge as this provides different ways of understanding the mathematics	*Inter-individual reasoning pattern* Greater valuing of peers' reasoning. Assists understanding their own.	*Connected knowing* Seek to understand how others develop different perspectives from their own	
		Mastery reasoning pattern Quiz peers to aid each others' mastery of knowledge		*Impersonal reasoning pattern* Peer interaction to establish credibility	*Individual reasoning pattern* Seek debate with peers to support their own mathematical understanding	*Separate knowing* Play devil's advocate, seek logical contradictions in other's arguments	

Case Study 1

Cheryl was 22, lived with the father of their small child, and wanted to be a nurse. Cheryl completed grade 12 but described how she would feel physically sick before going to a maths class. In interview 1 Cheryl conveyed a subjectivist perspective in coming to know mathematics through her recognition that she discards knowledge that makes no personal sense. The anxiety that she struggles with also resonates with the silent perspective in terms of an inner voice telling her she is incapable of learning mathematics.

> *Unless something makes complete total sense to me I refuse to acknowledge it, my brain just discards it. ... I flash back, "see you're stupid, you don't get it, you were never meant to do maths."*

Regarding the role of the teacher, her reflections also conveyed a mastery reasoning pattern through enjoying being visible in class and being competitive with her peers. As this reasoning pattern was more evident among males in Baxter Magolda's sample it was perhaps significant that Cheryl wished there were males in the class.

Role of the Teacher

> *She put the sheet in front of us and I was thinking I won't say anything, no one else has said anything, they must just get it. ... I was the only one left in the room and I just cried, [laugh], and the teacher goes "what is wrong?" ... She said "just tell me next time" and went through it. Ever since then she always looks at me and gives me a nod, and I'm like, "yeah that's fine," or I will frown at her. And she will come over and go through it. ... fractions was my biggest block. ... She gave us a full circle, half, thirds, ... up to 16ths, and we sat for a week with these ... at the end I am the best in the class.*

Role of Peers

> *I am cooperative ... but only if you are cooperative with me. ... I find I am very loud, and I know I have pissed people off, but at the same time I let other people talk. ... Nancy and Kate ... they just race ahead of me, ... what I hate is when they don't get something and I do, they expect me to sit and explain it to them. ... Maybe if there were half a class of guys and half a class of women it would balance out.*

In interview 2 Cheryl spoke about becoming more focused on her own mathematical learning as if this was at the expense of the learning of her peers. As peers were still conveyed as playing an important role in her learning where their alternative ways of thinking were valued, this would suggest a perspective consistent with the impersonal reasoning pattern of the Transitional knower. Further evidence of the impersonal reasoning pattern emerged later in the interview in relation to the role of peers in terms of them providing the opportunity through engagement to establish her mathematical credibility.

> *In the end I was like I am doing this for myself, I am not doing this for you. ... I was still very reliant on Nancy and Kate, like "am I doing this right?" ... Nancy and I were very much in common because I would look at things one way and she would look at things in a completely different way, sometimes I was right, sometimes she was right.*
> I: You did get into a lot of explaining to Nancy.
> *Oh I loved it! [laugh].*
> I: Did that help you?
> *Yeah! It just reinforced that I do know this, and I could prove it because look here is the answer.*

When I followed up her idea of wanting men in the class, her response suggests that they could contribute to her learning in a different way from women, albeit in terms of their more impersonal approach to learning.

> *Most men I know tend to ... this is a generalisation ... this is how you've got to do it ... they say this is the rule, follow this and you are done. Whereas women need to know where did that rule come from?*
>
> I: You like that way of thinking?
>
> *I can accept it more now and understand it, now that I know these rules just don't come out of thin air and actually have meaning. ... Whereas before ... I would have looked at the rule and gone I don't know how to do that [laugh]. ... But now I know why they are the rule, ... like if we went to uni and they decided to teach that way I wouldn't have a problem with it.*

Case Study 2

Pauline was 39, had four adult children and left school in grade 11. She was different from Cheryl, being at a different stage of her life, and had also enjoyed mathematics at school. From observations she was very quick to solve work sets. In interview 1 Pauline conveyed that she appreciated the encouragement of student interaction and that the teacher did not present herself as the ultimate authority. Nancy also gained an appreciation of her peer's alternative methods. Overall her comments are consistent with the inter-personal reasoning pattern of the Transitional knower who is beginning to legitimize peers' knowledge and view the role of the teacher as a facilitator, rather than to impart knowledge of authorities.

Role of the Teacher

> *You really do feel you are on a level footing, ... if you don't understand something you can ask them to clarify it without making you feel stupid. ... The teachers aren't up there "we know it all and you know nothing." The teachers are prepared to consider what the students have to say.*
>
> I: Were you a bit nervous initially to speak out?
>
> *Yeah ... I sat back in the corner and slowly integrated.*

Role of Peers

> *We are not there to compete, you are there to help each other. ... Like with Fran, I was showing her the way I was doing it, and she was sort of not really understanding it. But then she did it another way and said "well is this way wrong?" I said "no, it is just different" ... She did it her way and still got the same answer. I was able to let her know, it is not always just one way to do something, there is more than one way, as long as you get the right result.*

In interview 2 by telephone, Pauline spoke again about this specific interaction with Fran. Here evidence emerged of her applying her new epistemological outlook on mathematical learning to her every-day life. That is, Pauline had begun to recognise the value of applying ideas learnt within a collaborative mathematics classroom to areas of her life that were quite distinct from the learning of mathematics.

> *I think I mentioned last time with Fran. ... I tried to explain the way I was doing it, and then she said "well this is the way I started to do it is it wrong?" And I looked at it and said "no, it is not wrong it is just different from my way."*
>
> I: Were there any other interactions like that?
>
> *Not that I can think of, that one was sort of an important one for me. I don't know why.*
>
> I: It was just a really important one?
>
> *Yeah .... I think it was good for me at that point of time because there were a few things going on ... that I wasn't getting things right.*
>
> I: What was that?
>
> *Oh personal stuff.*

I: So you related it to that. Did it help you?
It helped me sort of get on and put a couple of things behind me.
I: Thinking that two people could be right?
Yeah. So that is why it stuck in my mind.
I: So that was in terms of your personal relationships but you connected it to your experience of learning mathematics?
Yeah.
I: If you had seen maths in terms of there being only one right way, would that have meant you would have struggled with your personal situation?
Yeah. I AM SURE OF IT. [laugh]
I: You were sure you were right or you were wrong? [laughing]
In that instance I thought I was right, and that was it! [laughter]
I: You realised there was another perspective?
Yeah [giggling]

Discussion

Using the two case studies I have endeavored to show how different experiences of peers and the teacher in the same adult mathematics classroom might be viewed through an epistemological lens. It is worth conjecturing that if students' epistemological perspectives are to shift in such a course then they do so quite dramatically within the first few months as long as the learning environment is conducive. Baxter Magolda discusses how Absolute knowers "do not begin to view themselves as knowers until the learning environment implies or states directly that they have something of value to say" (p.273). In this classroom the teacher was vigilant in her efforts to validate the students' ideas and encouraged them to examine each other's methods and this would appear to have set the context. Perry (1970) noted that epistemological shifts occurred suddenly rather than gradually for his interviewees. Women who are early school-leavers, who have made the commitment to return to study, are already in a transition phase with respect to thinking about their own learning and are therefore probably quite open to accepting alternative modes of teaching and this was confirmed by the teacher.

Case study 1 is a student who shifted enormously in her attitude towards mathematics—who came to the course with what can be described as a silence perspective, a belief that she was incapable of learning mathematics. She also expressed a view of coming to know mathematics that could be linked to subjective knowing—a strong sense of needing to own and understand the knowledge personally, consistent with the notion of personal agency or authorship described by Burton (1999). Baxter Magolda's framework was then helpful in viewing subtle shifts in relation to the role of the teacher and peers in her mathematical learning for the notions of connected and separate knowing are too advanced epistemologically. Instead the mastering and impersonal reasoning patterns, precursors to separate knowing, better described Cheryl's predominate way of coming to develop her mathematical voice.

Case study 2 is a student who came into the course feeling very comfortable with mathematics. Her reflections suggest that she came into the course as an absolute knower, who perceived that mathematics was about being right and wrong. This notion was challenged by peer interaction and interestingly her apparent epistemological shift with respect to mathematics flowed into her personal life. Goldberger (1996) states that what determines a person's shift in epistemological perspective may be contextual rather than developmental. For Pauline, the notion of a developmental rather than a contextual shift is more evident.

The types of student perspectives conveyed here are not new and other lenses could be used to describe them. Through overlaying an epistemological model the notion of stages of mathematical development is proposed in the context of women returning to study in the further education sector. Such a model, I suggest, allows for the noting of subtle shifts in women's reasoning over time and an epistemological perspective on why difficulty can be experienced by students when asked to engage collaboratively with peers. A pathway towards encouraging more complex ways of knowing mathematics may also be made clearer.

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Adult Multiple Intelligences and Math

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Howard Gardner, author of the theory of Multiple Intelligences, defines intelligence as "...the psycho-biological potential to create or solve a problem or fashion a product that is valued in one or more community or cultural settings" (Kallenbach & Viens, 2000, p. 13).

In December 1996, ten teachers of adults from the northeastern region of the United States, myself included, initiated work on the Adult Multiple Intelligences (AMI) Study. This research project, a collaboration between Harvard Project Zero and the New England Literacy Resource Center (NELRC)/World Education under the auspices of the National Center for the Study of Adult Learning and Literacy (NCSALL) at the Harvard Graduate School of Education, lasted 18 months. During that time, we explored the ways that Howard Gardner's Multiple Intelligences (MI) theory could support instruction and assessment in various adult learning contexts.

In the book that summarizes the findings of our study, *MI Grows Up: Multiple Intelligences in Adult Education: A Sourcebook for Practitioners*, Julie Viens, co-director of the project, explains the eight intelligences identified by Gardner (Kallenbach & Viens, 2000, pp. 15-17):

Linguistic Intelligence
- involves perceiving or generating spoken or written language
- allows communication and sense-making through language
- includes sensitivity to subtle meanings in language
- encompasses descriptive, expressive, and poetic language abilities

Logical-Mathematical Intelligence
- enables individuals to use and appreciate abstract relations
- includes facility in the use of numbers and logical thinking

Spatial Intelligence
- involves perceiving and using visual or spatial information
- [involves] transforming this information into visual images
- [includes] recreating visual images from memory

Bodily-Kinesthetic Intelligence
- allows you to use all or part of your body to "create"
- refers to the ability to control all or isolated parts of one's body
- includes athletic, creative, fine, and gross motor movement

Musical Intelligence
- involves creating, communicating, and understanding meanings made out of sound (music composition, production, and perception)
- includes ability dealing with patterns of sound

Naturalist Intelligence
- involves the ability to understand the natural world
- includes the ability to work effectively in the natural world
- allows people to distinguish among, classify, and use features of the environment
- is also applied to general classifying and patterning abilities

Interpersonal Intelligence

- involves the capacity to recognize and make distinctions among the feelings, beliefs, and intentions of other people
- allows the use of this knowledge to work effectively in the world

Intrapersonal Intelligence

- enables individuals to understand themselves and to draw on that understanding to make decisions about viable courses of action
- includes the ability to distinguish one's feelings and to anticipate reactions to future courses of action

After attending a three-day institute and reading several recommended books (see bibliography), each AMI participant developed a research question based on her own teaching practice and interests related to MI theory. Over time, the research questions were clarified and modified as the participants' understanding of the theory evolved. The participants' various inquiries covered a range of applications, including the following questions:

- Will awareness of their own intelligence profiles help my students become more independent learners?
- Can MI-informed lessons help the progress and attendance of Learning Disabled and Attention Deficit Disorder students preparing for a GED (Tests of General Educational Development)?
- How can teacher and student, working collaboratively, a) identify the student's strongest intelligences through MI-based assessment and classroom activities? b) use the understanding of these intelligences to guide the learning process?
- What will happen when I use MI theory/instruction in teaching math?
- What kind of MI-based instruction and assessment can be developed that will help adult learners deal with math anxiety so they may reach their stated goals?

Participants in this qualitative research project collected their data in several ways. All teachers were required to keep a journal of their lessons and reflections. Other data collection strategies included interviews, analysis of student work and videotaped lessons, surveys, and dialogue journals. Two co-directors supported the teachers' research efforts through classroom visits. Over the course of the study, the participants met at quarterly institutes to discuss the progress of their research and communicated regularly both online and by phone. Upon the completion of the research project, each teacher wrote a final report detailing her findings. The co-directors analyzed all the data after it was collected and extracted common themes that emerged from the research. This information was published as a draft sourcebook that was then piloted by twelve teachers from Maryland, Texas, Ohio, and Washington.

MI Grows Up: Multiple Intelligences in Adult Education: A Sourcebook for Practitioners is a compilation of the information gathered as a result of our three-year AMI Study. The major themes addressed in the book fall under two main categories: MI Reflections and MI-Inspired Instruction. These themes and, most significantly, their connection to math instruction for adult learners will be outlined in this paper.

MI Reflections

The term "MI reflections" focuses on ways to teach about MI theory and, consequently, use it as a tool for student self-reflection and self-understanding. The teacher/researchers who participated in the study found that they had to make a conscious decision whether or not to explicitly discuss MI theory with their adult students. Those who chose to teach about the theory used a variety of ways to introduce it to their students, including presentations, handouts, activities, dialogue journals, and discussions. A number of factors influenced how much time teachers spent dealing with this topic, including student expectations, interests, and cultural considerations. In their cross study of the research findings of the ten participants in the AMI Study, co-directors Julie Viens and Silja Kallenbach found that there were three important reasons why teachers might want to spend time introducing MI theory to their students (Kallenbach & Viens, 2000, p. 27):

- "Learning about MI" to provide a rationale and explain "new," unfamiliar, nontraditional, MI-informed activities;
- "Learning about ourselves" to build student awareness of their own strengths, and to develop self-efficacy; and/or
- "Learning about our ways of learning" to help students find learning strategies that fit their strengths/interests.

Because I taught a higher functioning ASE class of students who were pursuing a GED or adult high school diploma, I found it useful to introduce the theory to my students by having them complete an informal survey that generated a discussion about the students' areas of strength (Kallenbach & Viens, 2000, pp. 74-80). The benefits of such an experience are best highlighted by the following description of how one of my students used this self-knowledge when trying to learn a new math skill.

One result of having the students acknowledge and appreciate their own intelligences was a marked change in their willingness to approach the learning process from a different perspective. This story of one student's work on math word problems provides a good example to illustrate this point. Often, when this student had to face a word problem, he would develop what I refer to as "math paralysis"; he would sit there staring at the problem, not knowing where to begin.

One evening, soon after we had created our AMI Profiles, we began working on problems that involved finding the area of a right triangle. The class had previously only worked on the calculating area of a rectangle. Without any additional instruction, I posed this problem in written form:

Consider rectangle ABCD. If side AB = 12 inches and side BC = 9 inches, what is the area of triangle ABC?

The student was stumped. I asked him what intelligences were evident in the problem. He noted linguistic and logical-mathematical. I then had him look at his AMI profile and asked him to recall his strongest areas. My notes from that evening indicate that I could actually see the student relax; his shoulders became less tense and he let out a sigh of relief when he realized that he should draw the figure before trying to compute the answer. Within a short period of time, he had the problem solved. From that point on, the student was willing to work with manipulatives and use drawing when solving math problems.

This was something I had been encouraging all my students to do for many months prior to this, with no results. It was almost as if they had not seen this as "real math" because this was not the way they had previously been taught to solve math problems in school. Their AMI Profiles became a touchstone, giving them permission to try new ways of learning, to experiment, to take risks.

MI-Inspired Instruction: The AMI Experience

The teacher/researchers in the AMI Study found MI theory to be helpful because it encourages teachers to analyze their instructional practice and, as a consequence, provide students with a range of learning opportunities based on student strengths and interests. In many cases, those involved in the study found that MI theory validated instructional practices they had already found successful when working with adults, including multi-modal, real-world based lessons and assignments. It is important to remember that MI theory, just like any other theory of education, is meant to inform, not prescribe. Each teacher/researcher involved in the AMI Study applied the theory as she thought best, based on teaching context and student population. The following information, taken from *MI Grows Up: Multiple Intelligences in Adult Education*, explains some of the key findings that emerged from the cross study of the AMI experience.

Using MI theory leads teachers to offer a greater variety of learning activities (Kallenbach & Viens, 2000, pp. 83-85).

The teacher/researchers' understanding of the plurality of intelligences led them to offer a greater variety of entry points or ways to engage the learner in his/her study of any given topic or skill. Teachers found themselves providing their students choices in how they went about learning the subject matter and a variety of ways by which the students might demonstrate their understanding of the work. An especially popular activity, known as "Choose 3," was developed by one of the AMI teacher/researchers, Martha Jean. It allowed students to select from a comprehensive list of activities designed with the eight intelligences in mind. One example of this format, "'Choose 3'—Angles," appears below (Kallenbach & Viens, 2000, pp. 167-168).

"Choose 3"—Angles

Materials: Paper, pencil, pens, rulers, protractors, paint, Play-Doh™

1. In 2-5 minutes list as many angles as you find—acute, right, obtuse and straight—inside the class or outside.
   a. Make a graph showing each type you found.
   b. Which angle is most common? Why?
2. Using your arm and elbow form five angles.
   a. Draw those angles and write approximate measures for each.
   b. Are there any kinds of angles that cannot be made with an elbow?
   Explain your answer.
3. Discuss with someone and write responses to these questions:
   a. What does someone mean when someone says, "What's your angle?"
   b. If you were on an icy road and did a "360," what happened to you?
   c. Why do you think this shape, ∟, is called a right angle?
4. Using Play-Doh™ and/or paper, show the angles 180°, 135°, 90° and 45°.
5. Find or make five triangles. Measure each angle and find the total number of degrees in each triangle by adding up the sum of the three angles.
6. Draw, make with Play-Doh™ or paint a place you know. Mark and measure the angles in your design.
7. Write a poem, song, chant, or rap using some of the following words about angles:
   - figure formed by two lines, intersection, elbow, notch, cusp, fork, flare, obtuse, acute.
   - point of view, perspective, viewpoint, outlook, slant, standpoint, position.
   - purpose, intention, plan, aim, objective, approach, method .

Increasingly, all AMI teachers found themselves using more open-ended assignments as part of their teaching repertoire. For instance, when studying perimeter, instead of merely asking the students to calculate the distance around the sides of a 4-inch by 8-inch rectangle, I would ask them how many different figures they could draw that had a 24-inch perimeter. One of my students, who especially enjoyed challenging problems like this, termed working on these assignments "intense."

The most engaging MI-based lessons use content and approaches that are meaningful to students (Kallenbach & Viens, 2000, pp. 85-86).
The AMI teacher/researchers found that an understanding of MI theory helped them develop lessons and interdisciplinary units that provided authentic learning experiences for their adult students. As a result, these authentic learning experiences were more meaningful to the students. In my classroom, we worked on team building activities that allowed the students to display their strengths through project work. Students were given open-ended assignments, including the following two exercises: *What can we do as a group to make our center a more comfortable place in which to work and learn? How can we, as a group, encourage more adults to attend classes at our center?* My students expressed interest in working on these real-life challenges, often saying that this was their favorite part of our program. One student made the following comment to me during an interview session:

> The project is very important to me because I'm learning more with every step we take. It's exciting to find out what's next and begin the project. The most exciting part is the finished project because we all worked together to complete it.

As the students worked on these projects, I had time to observe them in authentic settings as they solved problems and created products. Once the project was underway, I found ways to tie in various math skills, such as computing percentages, calculating area, and graphing, to the project. The students got to experience math in the real world while completing real projects that were meaningful to them.

Other Key Findings

In addition to the information detailed above, several other findings emerged from the AMI Study (Kallenbach & Viens, eds. 2000: 86-91):

- MI-based approaches advance learning goals.
- Implementing MI-informed practices involves teachers taking risks.
- Persistence pays off with MI-based instruction.
- MI-informed learning activities increase student initiative and control over the content or direction of the activities.
- Building trust and community in the classroom supports MI-based instruction.

Adult students often express a need to know why they are being asked to complete a certain assignment, especially if it does not resemble what the students might consider traditional class work. I found it very beneficial to explain frequently to my students the reasons behind the various lessons and activities I presented to them. I also included them in decision-making processes as frequently as possible. While I was sifting through my data and sketching out the themes that I saw emerging in relation to my own research question, I began to realize more and more how my students had become "co-researchers" with me on this project. For my December 1997 interview, I decided to ask them what they thought I should tell other teachers about our classes and what advice they would want to give teachers to help them plan effective lessons for adult learners. Their responses support the significance of emphasizing all the intelligences and, in particular, the personal intelligences when planning an effective ABE program.

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Epistemological Questions About Research and Practice in ALM

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Abstract

The subject area of the didactics of mathematics is "always-already" structured and delimited by the concrete forms of practice and knowledge that are currently regarded as mathematics teaching, learning, and knowing. The new research and practice area "adults and mathematics" is situated in the borderland between mathematics and adult education. At the fourth and fifth ALM conferences there was a debate about the characteristic features of the research area. Questions were formulated about "Adults Learning Mathematics" as a community of practice and research within the didactics of mathematics, a community where adults' learning and numeracy are placed at the centre and where the answer to the so-called justification problem (Why teach mathematics?) is empowerment for social and working life. Epistemological reconnaissances have given rise to the author's construction of the concept problematique for didactic activity in this field (adults and mathematics) where interdisciplinary studies (mathematics, sociology, pedagogy) are necessary to bridge mathematics and adult education. Reconstruction of "math-containing qualification" as a didactic concept is given as an example of interdisciplinarity.

The object of study in mathematics education research encompasses all phenomena and processes involved in, and around, the actual or potential teaching and learning of mathematics. The researchers', teachers' and learners' conceptions and definitions of mathematics are important. Thus, the subject area of this research field, also named the didactics of mathematics, is "always-already" structured and delimited by the concrete forms of practice and knowledge that are currently regarded as mathematics teaching, learning, and knowing.

By the *didactics of mathematics* I mean the scientific discipline related to research and development work in mathematics education. According to Mogens Niss there seems to be a preference in Europe for using the label "the didactics of mathematics," inspired by names such as "Didaktik der Mathematik" (German), "didactique des mathématiques" (French), "didáctica de la matemáticas" (Spanish), "matematikdidaktik" (Scandinavian languages), in spite of the slightly oblique connotations attached to the term "didactical" in English (Niss, 1999, pp. 1-2).

Three Interrelated Subject Areas

I describe the research subjects of the didactics of mathematics as stretching between three superordinate subject areas: teaching, learning, and knowing mathematics. The specific subject fields are structured and construed within or across these subjects (Wedege, 1997). On the basis of Niss' analytical description of the domain (Niss, 1999) and my own research (Wedege, 2000), I give the following short description of the three subject fields:

(1) Mathematics and math-containing teaching

Problem complexes, which can also be interdisciplinary, begin with: why, what, how, who, and where?
The focus is on communicating mathematics.
By *math-containing instruction* I mean organised communication of a single or interdisciplinary subject area where mathematics is an integrated but identifiable part. The instruction can be informal (for example, learning from one's colleagues at a place of work) or be part of a course or a study program.

(2) Learning mathematics

The problem complexes begin with: who, what, how and where?
The focus is on the learner.

(3) Mathematics knowledge, math-containing knowledge, and attitudes to mathematics

The problem complexes start with: what, where and who?

The focus is variously on human knowledge or mathematics, the context or the situation (teaching, working life, social life, culture, etc.).

The two subject areas, (2) learning mathematics and (3) mathematics knowledge, may only be separated analytically by distinguishing between process and product. The subjects complement each other in the sense that the one does not exist without the other.

If mathematics knowledge is not only academic mathematics and school mathematics but also ethnomathematics and numeracy then mathematics teaching and learning encompasses a lot of informal activities outside schools and educational institutions. We shall look at two constructions broadening the conception of mathematics and thus the research domain.

Two Constructions in the Subject Field

The construction and delimitation of mathematics is always, explicit or implicit, on the agenda in the didactics of mathematics. In order to illustrate that, I have chosen the two concepts "ethnomathematics" and "numeracy" which both have expanded the subject area and the problem field in mathematics education and visualised the need for interdisciplinary studies.

Ethnomathematics

Mathematics has traditionally been considered a culturally independent phenomenon, but from the end of the 1970s mathematical didacticians have been aware of the societal and cultural aspects of mathematics and mathematics teaching. Ubiratan D'Ambrosio presented his ethnomathematical programme at the fourth international congress of mathematics education (ICME4) where he contrasted *academic mathematics* (the mathematics taught and learned in schools) with *ethnomathematics,* which he describes as the mathematics practised in cultural groups and subgroups (D'Ambrosio, 1985). The point of departure for the ethnomathematical project is studies that have shown that teaching which conflicts with the hidden mathematical competencies of the participants, e.g., in their craft skills, can result in severe obstacles to learning. Ethnomathematics adopts a cultural-anthropological viewpoint thus expanding the problem field of mathematics education. (See also Gelsa Knijnik in this volume.) A problematique is constituted by means of the common value base in the research community of ethnomathematics.

Paulus Gerdes has analysed ethnomathematics as a research domain, as a subject area, and as a field of practice. He argues that it makes sense to speak of an "ethnomathematical movement" characterised for example as follows (Gerdes, 1996, pp. 917-918):

- Ethnomathematicians adopt a broad concept of mathematics, including counting, locating, measuring, designing, playing, and explaining.
- Ethnomathematicians emphasise and analyse the influences of socio-cultural factors on the teaching, learning, and development of mathematics.
- Ethnomathematicians argue that the techniques and truths of mathematics are a *cultural product* and stress that all people—every culture and subculture—develop their own particular forms of mathematics.
- Ethnomathematicians look for *cultural elements and activities* that may serve as a starting point for doing and elaborating mathematics in the classroom.
- Ethnomathematicians generally favour a socio-critical view and interpretation of mathematics education which enables students to reflect on the realities in which they live, and empowers them to develop and use mathematics in an emancipatory way.

The ethnomathematical approach is based on a thesis concerning the relationship between everyday knowledge and school knowledge, a thesis that also lies behind my interest in the everyday math-containing competencies of unskilled workers.

Numeracy

In the field of research, "adults learning mathematics," the construction and further development of a concept of "numeracy" is a task that many researchers relate to. Adult numeracy is an everyday competence bridging mathematics and everyday life. The term "numeracy" was introduced for the first time in the United Kingdom in the late 1950s as a parallel to the concept of "literacy." The need was felt for a concept for necessary, basic arithmetical proficiency corresponding to the concept for reading and writing skills.

In the last twenty years, there has been lively debate between educational planners and researchers in the English-speaking countries (the United Kingdom, the United States, Australia, etc.) about the content and meaning of the concept of "numeracy." The discussion has, inter alia, concerned questions such as: How broad is the competence? How deep? How general? How specific? Is it also a matter of democratic competence? (See, for example, FitzSimons, 1997; FitzSimons & Godden, 2000.)

The new research and practice area "adults and mathematics" is situated in the borderland between mathematics and adult education. At the fourth and fifth ALM conferences we had a debate about the characteristic features of the research area. Questions were formulated about "Adults Learning Mathematics" as a community of practice and research within the didactics of mathematics, a community where adults' learning and numeracy are placed at the centre and where the answer to the so-called justification problem (Why teach mathematics?) is empowerment for social and working life (Wedege, 1997; Wedege, Benn, & Maasz, 1998).

Two General Research Questions

My own research concerns adults' empowerment for working life including the capacity to understand and modify technology. The preliminary *didactic* question defining my problem field was:

- How could mathematics education contribute to adults' (further) developing technological competencies at the workplace? (1993-95)

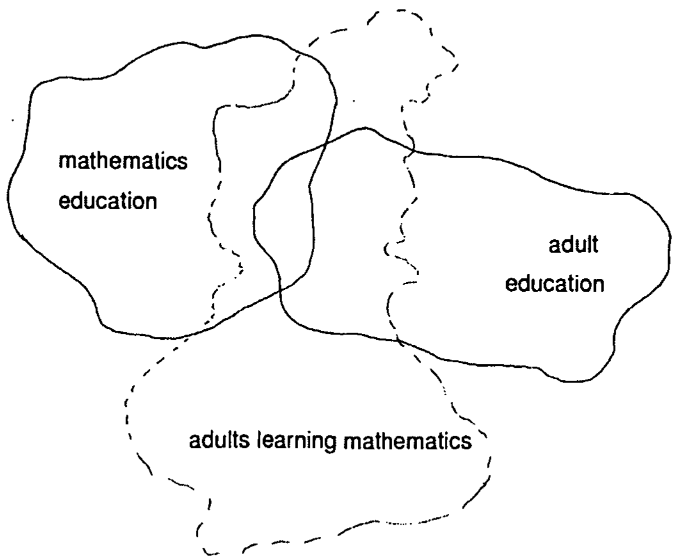
"But are you sure that mathematics is the answer to your question?" asked the German educational researcher Peter Alheit. Thus, the didactic question was reformulated and an *epistemological* question was added: What characterises a scholarly *problematique* which makes possible studies of the question:

- Is it possible that mathematics education contributes to adults' (further) developing technological competencies at the workplace? (1995-present)

In 1999, I finished and defended my doctoral dissertation with the following title: "Mathematics knowledge and technological competencies in adults with brief schooling—Reconnaissances and constructions in the borderland between the didactics of mathematics and adult education research" (Wedege, 2000). Adults Learning Mathematics is situated in the borderland between mathematics education and adult education. (See Figure 1.)

Roseanne Benn has illustrated research in adults learning mathematics in relation to the neighbouring disciplines (maths, adult education, maths education, sociology, psychology, etc.) by concentric rings (Benn in Wedege et al., 1998). "But are the 'neighbours' correctly positioned and is the picture complete?" asks Diana Coben (Coben, 1999, p. 49). Research in both adult education and mathematics education is multidisciplinary or interdisciplinary—drawing on psychology, sociology, philosophy, anthropology, etc.

Figure 1: "ALM" Situated in the Borderland Between Mathematics and Adult Education



Five Conclusions on ALM

The reconnaissances have resulted in five conclusions on the international research forum "Adults Learning Mathematics" (ALM).

(1) Preliminary place in the scientific landscape:
The ALM community of practice and research is accepted as a domain within the didactics of mathematics.

(2) Subject area:
The learner is the focus of the ALM studies, and her/his "numeracy" is understood as mathematics knowledge.

(3) Problem field:
Didactic questions are integrated with general adult education questions in ALM and the studies are interdisciplinary.

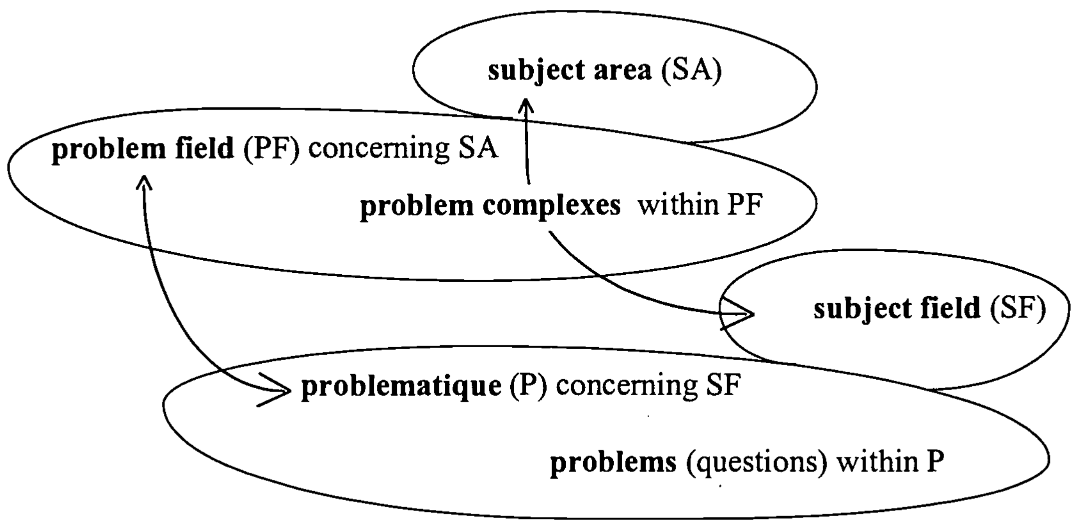
(4) Two perspectives:
The duality between the objective and subjective perspective is implicit, or explicit, in all ALM problematiques.

(5) Justification problem:
The general aim of ALM practice and research is "empowerment" of adults learning maths (Wedege in Wedege, Benn, Maasz, 1998; Wedege, 2000).

Two different lines of approach are possible and intertwined in the research: *the objective line of approach* (society's requirements with regard to adults' math-containing competencies) and *the subjective line of approach* (adults' need for math-containing competencies and their beliefs and attitudes to mathematics).

Problematique for Didactic Activity
Epistemological reconnaissances have given rise to my construction of the concept problematique for didactic activity in the field "adults and mathematics" where interdisciplinary studies (mathematics, sociology, pedagogy) are necessary to bridge mathematics and adult education. In the sections above, I have used the terms subject area, subject field, problem field, and problematique. (For a definition see Wedege, 1997.) In Figure 2, the conceptual framework is visualised.

Figure 2: An Epistemological Framework



A *problematique* in the didactics of mathematics may be characterised, inter alia, by a specific perception of mathematics, mathematics knowledge, and learning, and it can contain a value-based answer to the question: Why should society offer mathematics teaching? (the justification problem). As we have seen, ethnomathematics is an example of a new didactic problematique expanding the problem field of mathematics education.

In the didactics of mathematics, as in adult education research, multi- and interdisciplinary studies are necessary. Within the problem fields concerning ethnomathematics and numeracy, interdisciplinarity is necessary because the reasons for teaching and learning mathematics are located outside mathematics, and not within mathematics.

During the 1980s there was growing interest in practice-learning. In 1991, Lave and Wenger paved the way for a theory of learning as an integral part of social practice with the concept of learning as *legitimate peripheral participation*. However, the theory of the current community of practice as the only explanatory framework provides no possibility for understanding the inertia in adults' disposition to change their attitudes to mathematics. It is possible partly to explain how the adult learns but not why she does not learn.

Pierre Bourdieu's concept of *habitus* covers an incorporated system of tenacious dispositions as principles for readiness to act. I claim that habitus can provide a theoretical framework for the subjective conditions for adults' learning mathematics. By analysing an interview with a 75-year-old woman (Ruth) about mathematics in her life, I have illustrated and discussed by using the two analytical concepts their suitability for analysing adults' mathematics knowledge in different situation-contexts (school, work, the family, and leisure time).

On the basis of an epistemological discussion, I claimed that the concept of habitus, which was developed in a sociological problematique, could be imported into a didactic problematique about adults' learning mathematics together with the concept of learning as legitimate peripheral participation (Wedege, 1999, 2000). Habitus is a sociological concept about people's socialisation. Another sociological and pedagogical concept to be imported and reconstructed in an ALM problematique is "qualification."

"Math-Containing Qualification" as a Didactic Concept

Reconstruction of "math-containing qualification" as a didactic concept is given as an example of inter-disciplinarity.

Figure 3: The Didactic Subject Field Compared with the Sociological and Pedagogical

Sociology	Didactics of Mathematics	Pedagogy
Socialisation Qualification (of the working power)	Mathematics knowledge Learning mathematics	Knowledge Formation/Education Qualification Learning
The societal function of teaching	Teaching mathematics and its function	Teaching

Let us take a closer look at the concept of qualification as an important link between the social and pedagogical research fields where studies in the subject area of "adult education for mathematics in the workplace" are situated. As such the concept provides a framework for didactic reflection on the relation between adult education and work (Wedege, in press b).

I define *qualifications* as the knowledge, skills, and properties that are relevant to technique and work organisation as well as to their interaction in a work function. In my definition of the concept of qualifications, I speak of *relevant* knowledge, skills, and properties rather than of *necessary* knowledge, etc. This makes it possible to perceive qualifications from two different points of view: subjective and objective, i.e., from the point of view of individual workers as well as from the point of view of the labour market. I distinguish analytically between two types of qualification:

- *specific professional qualifications*: technical-professional knowledge and skills that are directly and visibly present when the individual work function is being carried out; and
- *general qualifications*: general and professional knowledge, abilities, and competencies such as literacy and numeracy that are (often indirectly) present when a wider range of work functions are being carried out.

A third type of qualification is introduced as a quality inherent in the two others:

- *social qualifications*: personal traits/attitudes that are present in the work process such as precision, solidarity, flexibility, and ability to co-operate (Wedege, 2000).

Empirically, these types of qualifications are interwoven in the single individual. A skill or understanding might be analysed as a specific professional qualification in a work context and as a general qualification in another context. For example, skill in reading diagrams and applying this knowledge is a specific qualification for the driver of a forklift truck, while skill in reading and understanding a chart of absence due to sickness is a general qualification for workers.

Human qualifications constitute a central element in technology where they are used, challenged, and developed at work in co-operation with and in contrast to technique and work organisation. On the basis of this conception of technology and technological development it is necessary to distinguish between necessary and relevant qualifications in analyses which are to be used for purposes of educational planning.

Conclusion

In the *International Handbook of Mathematics Education,* FitzSimons, Jungwirth, Maasz, and Schloeglmann (1996) characterised the field of adults and mathematics as having "great heterogeneity." Previously, I have argued that this is due to lack of a "grand narrative" concerning adults and mathematics and the great complexity of the subject area (Wedege et al., 1998). My preliminary reconnaissances on "Adults Learning Mathematics" as a subject area, as a research domain, and as a field of practice have led me to the conclusion that it makes sense to speak about ALM as "a community of practice and research" in spite of this heterogeneity. The reconnaissances have also given rise to construction of the terminology (subject field, problem field, problematique) which could be a means to make explicit some of the epistemological choices made by researchers and practitioners in the field (What do we mean by mathematics? What do we mean by knowledge? What do we mean by learning? etc.).

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UNDERSTANDINGS

Different Interpretations of Chance by Brazilian Adults

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Editor's Note: This paper reports on an important line of research that was very dear to the heart and mind of Dr. José Elias Damasceno. His questioning and plans for future research were suddenly stopped when, after a brief illness, he died on August 24, 2000, at the age of 39. At his burial service his scientific spirit was highly praised by his colleagues, who are honored to have known him and deeply miss him.

It is somewhat of a challenge that this paper will be written in English. Why? Because the English word *chance* has at least two very distinct meanings. And while if we were using another language we might have two different words available, one for each of the two different meanings we may wish to communicate, in English we will have to use the word *chance* indiscriminately.

This certainly is not a limitation, for we are sure that the reader is able to distinguish between the two meanings, even though he or she may be used to seeing them denoted by the same word *chance*, sometimes even in the same paragraph, as below:

> Chance opens the door to luck. When the chances are against success and it nevertheless occurs, you are lucky. Conversely, if the chances favor success and nevertheless it fails to ensue, you are unlucky. (Rescher, 1995, p. 42)

In the first sentence of the quote, the word "chance" is used to denote something that can be thought of as having real and objective status. In the sentences that follow, "the chances" are "the odds," or measurements of probability.

Chance in the title of this article is intended to have the first meaning mentioned above: not a measure of probability, as in "the chance that the flip of a coin yields tails is ½," but a representation of "some distinct creative or administrative agency" (Venn, 1888, p. 235), the same as *acaso* in Portuguese, or *hazard* in French.

But why bother with such a highly philosophical concept, one that touches at centuries-old questions such as those about free will versus predetermination or about the appropriateness of the assumptions we make about causality and indeterminism in science?

Our decision to pursue this investigation arose from the confluence of several of our research interests:

- We are convinced of the influence of students' cultural backgrounds in their learning of mathematics. People's beliefs and worldviews, especially in the case of adults, are the frames on which they are going to construct new mathematical concepts. And one aspect of culture is particularly important: the language in which students think and receive new information.
- We understand that the comprehension of the concept of chance, as well as the learning of probability, should be a goal for our students. Although chance and probability are conceptually independent concepts (von Plato, 1982), the concept of chance permeates probabilistic thought and therefore deserves the attention of educators.

Meanings of Chance

The words chance and fortune have been part of the philosophical vocabulary since Aristotle, and while the Greeks had a Goddess of Chance, the objection against the existence of chance is also very old (Venn, 1888; Gigerenzer et al., 1989). The question of whether an event should be attributed to chance or to causation is a philosophical enquiry that has attracted a lot of popular attention, too, and is often part of the questionings of non-academics as well.

Some of the meanings of chance identified in everyday language by Ayer (1965) are:

- An event is said to occur by chance when no one intended it to happen. In other words, it is an undesigned, but not necessarily uncaused, event.
- A chance event is due to the concurrence or coincidence in time or place of events belonging to causally independent series. One would cease to attribute the event to chance if the concurrences happen repeated times.
- A chance event is one that has an *a priori* probability of occurring. There is no implication that the event is uncaused.

Relationship Between Chance and Probability

Different literatures present contradictory views on the relationship between probability and chance. This has direct implications for education.

In Piaget's view, the idea of chance is a prerequisite for the development of the understanding of probability (Piaget, 1974). A historical analysis, however, indicates that a working knowledge of probabilities does not necessarily entail the embracing of the currently normative concept of chance.

At the birth of probability theory the aspect that became most evident in philosophical discussions of chance was its opposition to divine purpose. The very mathematicians who proposed a place for chance in the natural and moral sciences maintained that what we called chance was mere ignorance of the true causes of an event and that every event was governed by necessary causes, even if hidden or unknown (Hacking, 1990). Probability theory would then, for these theorists, be useful for us because of our ignorance and limitations. Eminent mathematicians such as Jakob Bernoulli and Laplace maintained that if we knew all necessary causes we would be able to predict every event and would have no need for probabilities. Needless to say, these mathematicians were experts on the calculus of probabilities, hinting at the independence between the philosophical stance and the mastery of the theory.

The Piagetian view is also opposed by Metz (1993), who examined kindergartners' and third graders' chance, probabilistic, and alternative interpretations in a spinner's task. She categorized children's manifestations of interpretations of the spinner's outcome into the categories *Probability without Chance, Chance without Probability*, and *Chance with Probability*, as well as into two other categories for spurious interpretations, finding results that violated Piaget's model: 23% of the kindergartners and 28% of the third graders exhibited Probability without Chance, whereas Chance without Probability was rare at both grade levels. Metz concluded that people have difficulty in conceptualizing the source of the variability of results of events, the bounds of the predictable, and the bounds of control.

This common difficulty is a challenge that we propose to face, by seeking first to understand what people think the sources of common events are, and how they interpret the role of chance in them, so that we can later move toward the incorporation of discussions on variability, control, predictability, causation, and chance into the curriculum. For this objective, it is imperative that we investigate the usage of the language related to chance among the populations we will be serving (Freire, 1968).

The Study

The conclusions reported in this paper refer to data collected as part of a research project involving students in Brazil, Canada (Quebec), and Hungary, the SIMULO project. A questionnaire, mostly based on Green's (1982) instrument for assessing the level of probability understanding, was administered to students in the three countries, with the objective of selecting students with various levels of probability thinking for a study on modeling and simulation. Of the 18 questions of the questionnaire, four concemed language use. In this paper we report our conclusions about data relative to one of these questions, answered by 63 students in the 18-26 age range.

In the activity in question, students were to write a sentence ending with "... is something that happens by chance." Since, in the larger context, one of the aims of the study was to investigate linguistic issues in probabilistic thinking, with this assignment we not only wanted students to give examples of events that they thought were ruled by chance, but also wanted to know how they used the expression "by chance" in everyday language. So instead of just asking students to give examples of chance events, we shaped some activities in the original study to assess language use.

Our first thought was to fit the data into four categories: deterministic, probabilistic, nonsensical, or blank, according to the nature of the students' answers. Immediately afterwards we realized those categories were not good for data arising from the question we posed to students. In simplistic terms, a deterministic stance is one in which causes are attributed to every event. Giving as an example of chance events one that clearly has a chain of causes related to it does not characterize a deterministic point of view but, rather, a personalistic perception of chance. Moreover, the classification above, besides being very hard to code reliably, would bring little if any clarification into the language issues we sought to investigate. We then decided to create new categories, according to what we thought were significantly similar interpretations of chance among some of the students' answers: chance as justification for unfortunate events and chance related to events of very small frequency.

Chance as Justification for Unfortunate Events.
A large number of answers dealt with unfortunate or undesirable events. Many of those events had evident causes that were often even explicit in the sentences, but chance was apparently used to console someone or oneself. Notice that we are not saying that students conceptualized chance as something that rules unfortunate events, or saying that only bad things happen by chance, but that they were *using* the term chance and the idea of chance as a justification for undesired outcomes, or even as a way to diminish bad feelings about the events. The examples below illustrate this:

> "To get a bad grade in the math test...(is something that happens by chance)."
> "To be barred at the door because I was not wearing the uniform..."
> "A pregnant woman wants a male son and a baby girl is born..."
> "Don't worry. Your father will come back. This is ..."
> "To be late to school ..."
> "Getting bad grades in subjects I study hard..."

It was for us so surprising to see this type of sentence be given in response to the request to build a phrase ending with "is something that happens by chance" that we had to look for a different interpretation of them. This was when we came up with this category, and suddenly a large number of answers started to make sense. With this interpretation, the first sentence would be saying "I got a bad grade in the math test. It is bad, but it is OK, these things happen (by chance)." The sentence "Don't worry, your father will come back" could thus be understood as "Something chancy may have delayed him, but he (and you) will be OK." Or, referring to the last example, "Getting bad grades in subjects I study hard is something bad, but it happens (by chance)." And maybe: "Thus I shouldn't feel so bad about it."

Chance Related to Events of Very Small Frequency
Another class of answers made no apparent sense as examples of chance events, but started to make sense once we realized they all had a common feature: they dealt with events of very small probability—or at least would make more sense if we assume the events are very infrequent. For example:

"Seeing my father in a good mood..."
"Getting a good grade in Portuguese..."
"An improbable happening..."
"A poor person become rich, a rich person share with the poor..."
"Finding the principal at the school..."
"A garbage collector becoming the manager of the Urban Garbage Collection Service..."
"Having an education and being well accepted in today's society..."
"Having a liberal principal..."
"Sleeping during the afternoon, for me..."
"It is very unlikely that this happens, because..."
"Yesterday I went out..."

In our interpretation, giving the sentence "Yesterday I went out" in response to the activity proposed could make sense if this happened very infrequently and the person uses the expression "is something that happens by chance" to mean "is something very infrequent." According to this interpretation, the first example would mean "my father is rarely in a good mood." Other answers suggest this category even more clearly, like the one: "It is very unlikely that this happens, because this is something that happens by chance." This sentence definitely does not make sense if we hold on to the usual concept of chance, for an event may be ruled by chance and still have high probability of happening, such as "getting a number different than one in the roll of a die," which has probability 5/6.

Discussion

The analysis above points out that students may have very unorthodox uses for the word chance. We believe that idiosyncratic language and the meanings it conveys must be known by us before we attempt to influence students toward the application of probability theory in everyday matters. We agree with Humphreys (1989) when he says that it would be unfortunate if a psychological prejudice in favor of fate or some unusual conception of chance prevented students from an objective evaluation of a probabilistic theory. Even if we cannot argue against philosophical or religious stances, one way to deal with this matter in class is, as Humphreys proposes to do in his discussion of indeterminism in *The Chances of Explanation*, to design a polemic to undermine prejudices.

Prior to designing such a polemic, however, we need to be well informed about what conceptions our students have of chance. Our research program parallels that of Cohen (1973) in his study of what he calls "psychological probabilities." His justification for his endeavor can then add meaning to what we are striving to do. Cohen emphasizes that the probability of which he speaks is not to be confused with the varieties of subjective probability in its axiomatic treatment. While the subjective interpretations of probability (those of different subjectivist schools) carry a prescriptive character, that is, describe how an ideal person would reason hoping that real people can apply those norms to their own reasoning, Cohen's psychological probabilities describe the thought of real people, who make mistakes and do not reason in the normative way. He argues that until we have a well-documented natural history of human error, such as the study of psychological probability can give us, the contribution that the subjectivist school can make toward the elimination of human error will be rather limited. We too think that until we bring into discussion what our actual students think chance is, we will have little chance of influencing them toward using probability theory.

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Math Is in the Eye of the Beholder

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For successful teaching to take place, there must be a teacher and student(s). The teacher must know and communicate the information to the student in a way that the student can understand, use, and appreciate. To do that, the teacher must be aware of how the student learns, how the student feels about learning, what goals the student has, and what needs the information fulfills for the student. The student must be ready to learn and must believe the teacher can teach him or her. The student must be able to translate the information into an understandable form, be able to connect the information with what was known before, and have a context in which to apply the information. Whether articulated or not, every good teacher understands this.

In order to teach more effectively, teachers engage in a lot of theoretical discussions. Researchers take select groups of students and test theories. Both groups reflect on their own successful learning. They listen to professors and read journal articles. A consensus is reached on the most effective teaching methods. Teachers distill the consensus with their own opinions and past experiences and then use the framework to conduct their classes.

Students who object to the format or goal of the class have three choices: argue convincingly for a different approach, trust the teacher and wait for results, or leave. Overwhelmingly in adult education classes, they leave.

Of course, attrition has many reasons. There are situational causes: lack of babysitters, transportation, new job schedules. There are emotional causes. Students in adult education classes have impulsively left commitments before, dropping out of high school, quitting jobs, leaving other adult education classes. There are institutional causes: changes in class schedules, costs or locations. Whatever the causes, it is common for programs to have 50-70% attrition rates (Quigley, 1995).

The perception that the class is not providing a forum in which they can learn is an important contributing factor in attrition. In a classroom research study conducted by Pamela Meader, a math teacher in Portland, Maine, "math difficulties were the barrier that students perceived as keeping them from finishing a class." Over 60% of the students mentioned math difficulties as a barrier to persistence compared with the next highest category of fewer than 10% for a child's illness (Meader, 2000, p. 9).

Students and teachers do not see math and math instruction in the same way. Most math teachers enjoy math, have been successful at it, and relish the challenge. They consciously use math in their everyday lives, perhaps especially chosen among their family and friends to figure the tip or keep the family finances. Students, on the other hand, have often been unsuccessful at math, avoiding it as much as possible. Math learned in school appears useless, a creation of teachers and textbook writers. "Most math is just a game. If you like it, you play it. I don't like it." (student, Prairie State College, Illinois, 1992).

> I don't care how many cookies Sally made. And I don't care how many were oatmeal and I don't care how many were chocolate chip and I could care less who ate them. You know, I'll never in my life forget that problem as long as I live. Who cares?! You cook 'em, you eat 'em. (Curry, Schmitt, & Walton 1996, p. 15)

It is not surprising then that students and teachers have very different perceptions of how a math class should be conducted.

These facts were brought home to me this year during an afternoon substituting in a GED math class at Harper College in Palatine, Illinois. One of the tasks for the six students in the class was to learn how to divide fractions. I approached it with manipulatives, discussion, and problem-solving. The students participated and seem to successfully grasp the concept. At the end of the class, I asked for their reactions. One student was enthusiastic both at her learning and the way the material had been presented. The other students responded more negatively. These are some of their comments:

"We don't need to understand why this works. In life you are just given a problem, you follow the rules and you do it."

"Understanding is too hard."

"Real math teachers in high school don't teach like that."

"You know, she hasn't had much math so this seems good to her. But we all remember a little how to do this and we want to start from where we remember."

"All I want is to know the rules and get my GED. I don't care about math."

Were these general attitudes with all students or just those almost ready for the GED? Was this, perhaps, just a reaction to a particular class and a difficult concept? I created a survey completed by students in adult education math classes at four widely separated community colleges in Illinois.

I. *Palatine area including Harper College and several workplace sites.* This college is in the far northwest suburban area of Chicago. Most students have jobs and are somewhere in the middle income range. There is a fairly even distribution of Hispanic, Caucasian, and African-American students along with a significant number of other ethnic and racial groups.

II. *Elgin Community College.* Elgin is a middle-sized older city about 45 miles northwest of Chicago. The students are similar to those at Harper with a large percentage of immigrants, especially from Mexico.

III. *Olive Harvey Community College* is located on the south side of Chicago. The students at Olive Harvey are primarily African-American although there are other ethnic and racial groups represented. The income level varies from low to middle class. Many students have jobs, but there are also many who are unemployed.

IV. *Prairie State Community College* is in Chicago Heights, a southern suburb of Chicago. The student population is similar but somewhat more diverse than at Olive Harvey.

The teachers surveyed came from two groups:

I. Those at the schools where the students were surveyed.

II. Teachers belonging to the Adult Numeracy Listserv.

Both groups were asked to complete the survey from the viewpoint of a student. The teachers were asked to imagine themselves as students in a math class.

MATH SURVEY

Under each number, mark **1** by the answer you most agree with and **2** by the second best answer.

1. **What should a math class teach?**
   A. The rules that tell how to do math problems.
   B. The understanding behind the rules.
   C. The ability to look at problems and develop your own rules.

2. **Which kind of problem would you most enjoy working on in your class? They will all teach you the math.**
   A. How many years of carpet would it take to carpet your living room?
   B. Could all the people in the world float in Lake Superior?
   C. Develop a budget.
   D. Do practice problems from the GED test.
   E. Figure out how much money in interest you pay on your credit card bill.

3. **Do you prefer:**
   A. Working on your own in a good workbook. The teacher helps you if you get stuck.
   B. Working with a partner or in small groups studying and doing problems together.
   C. Working as a whole class learning the same math strategies all together step-by-step.

4. **Do you think it is more productive to:**
   A. Do the word problems that are most common in math workbooks.
   B. Learn math by problem solving to find solutions to the real math problems in student's lives in the classroom.
   C. Spend most of the time practicing addition, subtraction, multiplication, division on whole numbers, decimals and fractions and not do too many word problems.

5. **Would you rather learn math by:**
   A. Listening to an explanation from the teacher.
   B. Practicing problem solving by using games and puzzles.
   C. Watching an example done on the board by students or teachers.
   D. Using rulers and other "hands-on" tools that help you to "see" the math.
   E. Discussing and working problems in pairs or small groups of other students.

6. **Why are you learning math?**
   A. For my job.
   B. To pass the GED
   C. Because I always felt I didn't know the math I should have learned in school.
   D. To help my own kids.
   E. Because it is interesting.
   F. To use in my everyday life.

The rationale behind the questions and choice of responses is obvious with the exception of Question Two. In Question Two, the following is the reasoning behind the responses:

1. What kinds of problems would you most enjoy working on in your class?

   A. How many yards of carpet would it take to carpet your living room.
   This is a false "real-life" question found in many math texts. Most people when they need carpeting either give dimensions to the carpet store or have the salesman/installer measure the room. Few people actually figure the area.

   B. Could all the people in the world float in Lake Superior?
   This is an intriguing question both for the information and the challenge of solving. It has no connection to any essential need.

   C. Develop a budget
   Many people have the need to do this and have tried before entering math class. It is necessary, but it is a process that people frequently feel they know already.

   D. Do practice problems from the GED
   A common belief is that if you practice similar type problems, you will do better on the test. Even for those not ready for the GED, it may be a goal.

   E. Figure out how much money in interest you pay on your credit card bill.
   This is a real-life, important challenge that many of us cannot do.

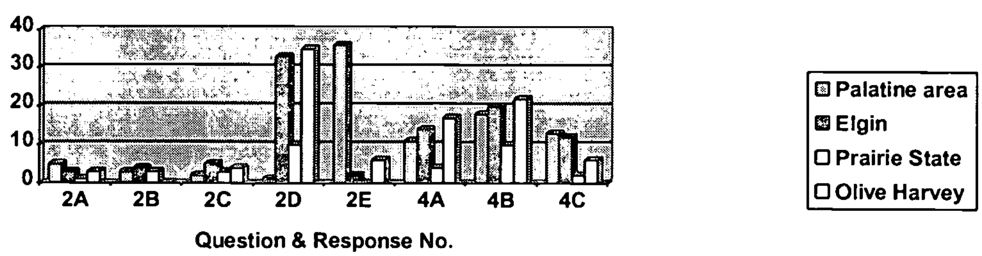
Students completing the survey: 161
Teachers completing the survey: 13

Palatine area students: 46
Prairie State students: 19
Elgin students: 47
Olive Harvey students: 49

Charts for total student responses and total teacher responses are on the next page.

There were some differences in the sites.

Comparison of First Choices: Students



2. Which kinds of problems would you most enjoy working on in your math class

All sites but Palatine chose **2D**: "Do practice problems from the GED test." Palatine preferred **2E**: "Figure out how much money in interest you pay on your credit card bill.

Students were not distinguished according to what type of adult education math class they were in. It is possible that those in Palatine were not in a GED program to the extent that the other groups were.

4. Do you think it is more productive to:

Three sites were quite close between choices **4A**: "Do the word problems that are most common in math workbooks" and 4B: "Learn math by problem-solving to find solutions to real math problems in student's lives in the classroom." However, Prairie State overwhelmingly chose **4B** and Elgin and Palatine gave a strong showing to **4C**: "Spend most of the time practicing addition, subtraction, multiplication, division on whole numbers, decimals and fractions and not do too many word problems."

Looking back at the Student Total chart, note that students were strongly unanimous on most choices. They want "The rules that tell how to do math problems." They would like to "do the practice problems on the GED test." They would prefer "working as a whole class" over working alone or in small groups. They want to find solutions to "...real math problems." Instruction should come directly from the teacher, and the emphatic goal is to pass the GED.

Teachers, on the other hand, want "the understanding behind the rules." They agree with students on doing "practice problems from the GED," "working as a whole class," and doing "real life math problems." However, teachers prefer to "work in pairs or small groups," and they see the reason to learn math "because it is interesting."

Teachers' choices are much more varied than student choices. For instance, 3 out of 13 teachers, approximately 23%, chose "Could all the people in the world float in Lake Superior" as their favorite type of problem to work on in class. Only 10 out of 161 students, or 6%, chose the same problem, intellectually interesting but neither "real world" nor directly applicable to the GED.

One of the most striking contrasts was in the attitude toward small group work, with 38% of the teachers favoring it and only 14% of the students in question 3 and less in question 5.

The results of the survey questions are consistent whether the numbers are for just first choice or the sum of the first and second choices.

What implications does this have for teaching?

The National Council of Teachers of Mathematics issued standards for math education in 1989. These were translated into standards for adult education in a 1996 research study funded by The National Institute for Literacy titled: *A Framework for Adult Numeracy Standards*. The similarity of selected standards is shown below:

NCTM Standards, grades 5-8
(selected standards with selected bullet points)

Standard 1: Mathematics as Problem-Solving

- *Use problem-solving approaches to investigate and understand mathematical content*
- *Formulate problems from situations within and outside mathematics;*
- *Develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;*

Standard 2: Mathematics as Communication

- *Model situations using oral, written, concrete, pictorial, graphical and algebraic methods;*
- *Reflect on and clarify their own thinking about mathematical ideas and situations;*
- *Develop common understandings of mathematical ideas, including the role of definitions;*
- *Discuss mathematical ideas and make conjectures and convincing arguments;*

Standard 3: Mathematics as Reasoning

- *Understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs;*
- *Make and evaluate mathematical conjectures and arguments;*

Standard 4: Mathematical Connections

- *Use a mathematical idea to further their understanding of other mathematical ideas;*
- *Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science and business;*

Standard 5: Number and Number Sense

- *Understand, represent and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and scientific notation) in real-world and mathematical problem situations;*
- *Develop number sense for whole numbers, fractions, decimals, integers, and rational numbers;*

A Framework for Adult Numeracy Standards

- **Relevance/Connections**
- **Problem-Solving, Reasoning/Decision-Making**
- **Communication**
- **Number and Number Sense**

There has been support for these standards both in the K-12 and the adult education community. (The NCTM Standards were amended in 2000, but they were not substantially altered.)

In 1995, District 2 in Manhattan, New York introduced a new standards curriculum.

"Before the district adopted TERC, a full-year pilot project showed comparable performance between the comparison group and the TERC group on traditional computation tasks. But the TERC group demonstrated greater flexibility in their choice of approach to problems and greater accuracy in solving them" (Hausman, 1998).

"In 1994, in Arlington, Virginia, 110 adult educators from 30 states met for a three-day working conference on adult mathematical literacy. Their recommendations included the following:

1. Class math activities should be collaborative, involve problem solving, and help learners develop reasoning skills. (Ciancone, 1996)."

"Educators interested in developing numeracy (rather than in merely teaching mathematics) are challenged not only to address learners' formal knowledge of mathematics or their ability to solve word problems, but also to attend to their abilities to transfer skills and effectively manage numeracy situations" (Gal & Stout, 1997-1998, p. 15).

This information presents a contradiction. The research indicates that learners need to have "the ability to look at problems and develop (their) own rules." They need to problem solve and communicate their thoughts to others moving away from knowledge centered in and directly transmitted from the teacher. Students need to experience math in a variety of ways including with physical objects, in conversation, and by symbolic abstraction.

But if this teaching makes students uncomfortable and challenges their trust in the efficacy and accessibility of the math class, they will drop out. If the student is not in class, it does not matter if the method of math instruction is the most optimal or not.

Perhaps if the teacher steps back a little with respect toward students' perceptions, both good teaching and retention can be accomplished.

- Start with whole class instruction and move toward small groups as the comfort level in the class increases. It is not bad for the teacher to be respected as a source of information. Discussion and thought can occur in a whole class as well as in a small group.
- Let students, who are frequently unsure and apprehensive, have the confidence of learning a procedure and applying it successfully. Give rules. Just because students learn a rule does not mean that they will never understand it.
- Recognize and applaud students who are seeking a GED. It may not be the only goal in adult mathematics education, but it is a worthy one. Let students see GED type problems. Practicing problem types reduces anxiety, which helps test taking.
- When students seem confident enough to move out on their own, give choices. Let them decide if and who they will work with. Some students learn better if they struggle with the problem on their own. Some work best with one friend. Some like the camaraderie and ideas generated in a group.
- Validate the bits and pieces of knowledge the student has accumulated. Begin a discussion by asking them what they already know. Stepping off from familiar ground is reassuring and makes it easier to explore the unknown.
- Encourage students to use "math words" such as "balance," "rate," "measure," "weigh," "share," etc. to develop problems that are relevant to their lives and past experience.
- As the class becomes more oriented toward problem solving and independence, explain not only the math but also the reasons for approaching it in this fashion.

In summary, treat the class with the same compassion and support as you would any friend who came to you for help even if you and that friend have some differences in perception as to how the help should be given.

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How Can They Belong If They Cannot Speak the Language? Enhancing Students' Language Use in the Adult Mathematics Classroom

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A sociocultural educational perspective sees learning as induction into "discourses" or "communities of practice" through interaction with more expert others. At the heart of any discourse are the language and symbols which carry its special meanings. To become a member of the discourse one must begin to learn its language. However, in traditional mathematics classrooms, the students, those who need to learn the language, usually have the least opportunity to speak it (Pimm, 1987).

A teaching experiment with an Australian adult numeracy class used audio and video tapes to record the talk of students as they participated in a range of mathematical learning activities. Included were group and pair tasks designed specifically to encourage meaning-making talk and to enhance students' use of mathematical language. The ongoing, grounded analysis revealed two distinct aspects of language use in the mathematics classroom. The first was the "opportunity to speak": the space, and invitation, given for students to articulate and modify their thoughts about mathematical tasks, through speech. The second identified aspect was the "means to speak": students' understanding of mathematical concepts, and their ability to use the mathematical terminology intrinsic to them. This paper will use qualitative data from the study to differentiate between these two aspects of language use and indicate advantages of attending to both within the context of adult mathematics learning.

Language From a Sociocultural Theoretical Perspective

A sociocultural, discourse oriented approach to the study of learning takes as a premise the social construction of knowledge (Bauersfeld, 1995; Lerman, 1996; Voigt, 1995; Wertsch, 1995). Fixed concepts do not exist in their own right but are invented and passed on within established conceptual frameworks within "communities of practice" or "discourses." Within these discourses are specialist language, meanings, symbols, and established forms of expression and argument of the discourse—its "register" (Pimm, 1987) and accepted ways of using that language within it—"communicative competence" (Cazden, 1986). The sociocultural perspective sees learning as induction into discourses through interaction with more expert others such as experienced players, workers, or teachers. Part of natural induction to any practice is modelling, scaffolding, and instruction of language within the practice. From this perspective, teachers of mathematics should introduce students to the language of mathematics as a natural part of their teaching.

> Having resource to language affords power to name and rename, to transform names, to use names and descriptions to conjure, communicate and control our images, our mental worlds. (Pimm, 1995, p. 1)

Conversely, what might it mean not to have the power of language? How can one reflect on things one cannot name?

Analysis of traditional school mathematics classrooms has shown that students have little opportunity to practice mathematical language. Rather, classroom discussion is usually teacher dominated. Using characteristic questioning patterns, teachers tend to elicit from students minor responses in tightly orchestrated step-by-step processes, so that those who need to learn the language get the least practice at using it (Barnes, 1976; Cazden, 1986; Edwards & Westgate, 1994; Voigt, 1995; Wood, 1994). "During frontal teaching, (teachers) elicit and control the classroom discourse step by step" (Pimm, 1987, p. 178).

Linguistic analysis by Veel (1999) revealed that tight control of this nature is more pronounced in traditional mathematics classes than in other school subjects, partly because of the dearth of explanatory written texts available to the students.

> Whereas most other subject areas rely on an extensive canon of written prose (to be found in textbooks, encyclopedias and school libraries) to provide the impression of stability and permanence to knowledge, this is noticeably absent in mathematics. Textbooks tend to be pastiches of repetitive activities and fragments of knowledge. (p. 187)

Consequently, within the traditions of mathematics teaching and learning, there is a heavy reliance on the teacher's verbal explanations to carry the knowledge and understanding of the subject. Reliance on the spoken mode begins to explain the "catechistic" type of interaction so prevalent in mathematics classrooms. A tight control must be kept on the dialogue if the teacher's knowledge is to become the students' knowledge and so power roles of teacher as "primary knower" and students as receivers of knowledge are firmly established within the traditional mathematics classroom culture (Veel, 1999).

Educators who believe that language is an important learning tool in mathematics use alternative activity structures such as group and pair investigations, with report back sessions to encourage students to find their own voice in the classroom (Adler, 1977; Cobb et al., 1995). However, detailed investigation of student utterances by Adler, Pimm, and Veel reveal: that in secondary school mathematics classes there are large gaps between students' language use and that of accepted mathematical discourse (Veel, 1999); that students seem unable to express their strategies clearly to an audience of their peers (Pimm, 1994); and that report back sessions do not reflect the richness of the small group investigations (Adler, 1997). Veel suggests that more explicit attention to mathematical discourse is necessary.

Arising from this concern are questions regarding how students can best be introduced to mathematical registers and what kind of frameworks are productive for analysing attempts to do so.

Adler (1999), describing language as a tool for learning, uses a window metaphor to illustrate two aspects of a tool: "useability" and "visibility." A window can be regarded from two different perspectives. It is usually something that people use to *look through*, a "useable" means of letting in light and views, enabling them to see. However architects and their clients, designing a building, are also concerned about the shape, size, and style of the windows. They look specifically *at the window* itself, making it "visible."

The window metaphor applied to language within mathematics learning sees students using language, first, as a means to "see," to access meaning, and to convey thinking, understanding, and strategies. For this useability of language to occur the students need to be given an opportunity to speak, to let in the light. For example, collaboration in groups or pairs, and writing or reporting to the class, might aim at encouraging talk, but not necessarily focus on language itself. However, at other times teachers might want students to look specifically at the language, to become aware of the mathematics register. They should understand its formal terms, modes of expression, and argument, such as "if...then" connections, and its forms of generalisation. They should be able to use this language themselves. This "visibility" requires teachers to specifically address the mathematical register (Pimm, 1987; Veel, 1999).

The Research Study

The study reported in this paper involved observation and analysis of the participants, culture, language, and communicative competence, encouraged by traditional worksheet methodology in an Australian adult mathematics classroom. A one semester teaching experiment, grounded in ongoing observations, then introduced a range of supplementary "Intervention Activities" and considered the changes in students' talk patterns and appropriation of language facilitated by the activities. The Intervention Activities incorporated group and pair structures, hands-on materials, specific language focused activities, and open-ended investigations. Using ethnographic, case-study techniques, data were collected through audio and video taping of student and teacher talk, interviews, field notes, and collection of artefacts (Bolster, 1983). The data were analysed weekly using a grounded approach, which enabled the teaching experiment to respond in an adaptive manner to the ongoing findings.

The Program
The study took place on the main inner-suburban campus of a large multi-campus Technical and Further Education (TAFE) Institute. The class was part of a full-time certificate program: The Certificate of General Education for Adults (CGEA), a qualification framework of four levels, 1 up to 4 (ACFE, 1996) that included Mathematics in the course package. These students were roughly Level 3/4 mathematics, but varied in literacy levels. Classes ran in three-hour blocks, consequently, single sessions could include a variety of activity formats and topics.

The Teacher
The teacher, from a progressive, primary teaching background, was one of the most popular in the program. His classes were particularly enjoyed by mathematically anxious women.

The Student Group
The adult learners exemplified a range of ages, cultures, educational backgrounds, and motivations for studying. From a language perspective, there were two major groups. Seven were primarily in the program learning English as a second language to improve employment prospects. Originally from Italy, Libya, Lebanon, and the Philippines, they had worked in Australia prior to attending this course, some for as many as thirty years. Although the classes were conducted in English, Arabic and Italian were sometimes used between students. The twelve Australian born students had either left school early or had interrupted schooling and saw the course as a means of gaining entry to further study, some for interest or self-improvement (usually women), others because education was essential for gaining employment to support themselves and their families.

Traditional Teaching Methodology
The usual teaching utilised calculation-based worksheets, either abstract skills practice type—derived from typical school textbook tasks, such as calculating the areas of a number of complex shapes—or "reality based" worksheets designed by the teacher around realistic applications which he assumed students would easily relate to. For example, one asked students to act as contractors to calculate fence material and grass planting costs for the local council.

In the regular classroom culture students were always given individual copies of the worksheets. Although they were encouraged to assist each other, no structuring of pair or group work was observed.

The Intervention Activities
Activities introduced into the classroom as part of the teaching experiment were intended to serve two major purposes. The first was the exploration of mathematical concepts through hands-on materials, reflection, discussion, and sometimes writing, in group and pair situations. For example, one task asked students to use string and grid paper to investigate statements related to areas of garden that could be made with a fixed length of fencing. In another, students in small groups estimated the volumes of household containers, then collaborated to write responses to "reflective prompts" (short questions designed to encourage students to think beyond specific answers and formulate generalisations) such as "Have any of your ideas about the volume of containers changed during this activity?"

The second purpose, grounded in early observation and analysis (see below), was to encourage students to engage with the vocabulary of the topics they were studying, strengthening their use and understanding of the mathematics register. For example, one task asked students to use the mathematical language of shape to "Write Three Sentences" describing a container. Some "cloze" language activities, based on English as a Second Language (ESL) teaching models in which students are required to select the correct words to fill gaps in sentences, were also created. A more open-ended activity asked pairs of students to list the *similar* and *different* properties of the two cylinders which could be fashioned using a sheet of A4 paper.

Framework for Analysing Classroom Talk

An organisational framework for reporting the facets of talk remained problematic throughout the progressive analysis of the data. Adler's window metaphor finally provided the inspiration for establishing two categorisations: "the opportunity to speak" and "the means to speak."

The opportunity to speak: This manner of looking at student talk refers to the space, and invitation, given for students to articulate and modify their thoughts about the tasks and engagement with the tasks, through speech. It implies an aspect of both speaking and being heard by responsive others—an "audience." It asks questions about when, and to whom, students talk as a means of exchanging meaning and feelings about the subject.

The means to speak—mathematical language: This second manner of examining student talk refers to students' access to mathematical meaning and the "mathematical register" (Pimm, 1987, 1995), including their understanding of mathematical concepts and their comprehension and use of mathematical terminology related to, and intrinsic within, the mathematical concepts. It asks questions relating to the quality of the talk, terms understood, and confident use of mathematical language.

Opportunity to Speak in the Traditional Culture

Lack of opportunity for students to speak in traditional whole class discussions

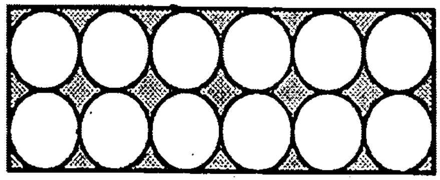
Classroom discourse patterns within this adult classroom confirm many features of school classrooms described in the literature. However, a particularly jocular and light-hearted atmosphere created by the teacher counteracted students' past anxieties about mathematics classes. Individual students felt free to contribute spontaneously throughout, because he refrained from nominating particular speakers or evaluating individual responses.

Using traditional worksheet routines of topic introduction, student practice, followed by teacher review, teacher-led segments occupied more time in total than the student practice. For example, there would be 27 minutes of teacher-led time compared to 25 minutes on student work and interaction. During these whole-class segments students customarily listened to teacher explanations and filled in answers in short manageable steps (Note: .. indicates a pause in speech; .... segments of speech omitted from transcript):

> *T:* *I just want to go over the way you work out some areas .. to see if you've forgotten them or not. So ..* [draws a figure on the board as he speaks] *Area of a rectangle, and this includes squares really, .. that's seven and that's four. How do you work it out?*
> ***Ss*** ***Seven times four/twenty-eight***
> *T* *You multiply it together ....*

Writing and drawing on the board, he continued to detail procedures for rectangles, triangles, and circles, leaving the occasional gap for brief student responses. Students' utterances were usually one or two words only, characterised by numbers or arithmetical processes. Typical dialogues of this kind, in which teachers use controlled "discussion" to provide information, severely limit students' *opportunity to speak* in the classroom. For example, in one representative 320-word transcript, the students contributed a mere 28 words: 11 in numbers; 5 in numerical processes, such as "divide it by three"; and 5 words in a flippant interjection.

Students were sometimes invited to explain their reasoning. However, a tendency to co-opt, or rephrase, student explanations (probably to clarify them for others), meant that the intended elicitation of students' ideas was not always reflected in the quality of the outcomes. The following example illustrates the teacher mediating a student's attempted explanation by slowing down the reasoning. However, in doing so, he took over the explanation rather than encouraging student expression. (Note: ../ signifies an interrupted utterance.)

d) 

10 cm diam
holes

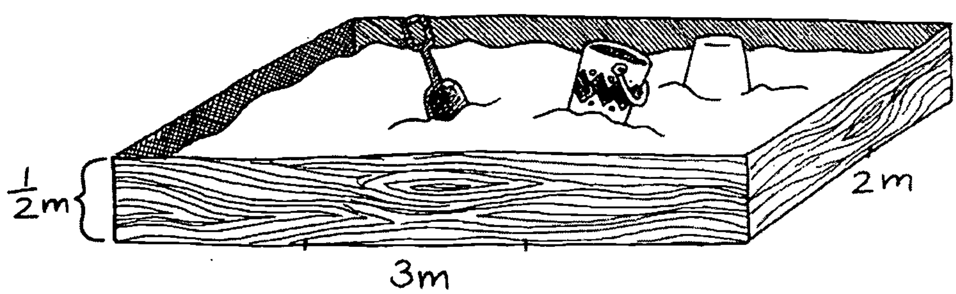
Ester	[enthusiastic to speak] *You find the area of those holes and you subtract that ../* [interrupted by teacher]
Teacher	*So you find the area of one hole ../*
Greta	*Ten ../*
Teacher	*....which is going to be .. if ten's the diameter five is going to be the radius .. so it's five times five times three point one four ....*

The mediation seems logical since the teacher's proximity to the board diagrams allows him to clarify students' explanations. However, the effect is that the majority of school mathematics teachers do most of the talking and mathematical naming. The interruption of students' speech limits their responses to brief numerical calculation strategies and numbers, at the expense of encouraging the use of a variety of language from the mathematical register. The students' *means to speak* is neither explored nor encouraged.

"Cries for help" during student practice create some "opportunity to speak"

Predictably, practice periods interrupted the asymmetry of power and control of dialogue illustrated above. They provided more opportunity for students' mathematical communication as they requested help to proceed with exercises. However both teacher-student and student-student communication during this time seemed to follow patterns of "repair work" (Bauersfeld, 1995) in which one person, the giver of knowledge, provides procedural or "how to" information to fill gaps for another less knowledgeable person, the receiver of knowledge. During these teacher-students interactions, again the teacher does most of the talking, with student responses limited to "yes" and "ah-ha" type utterances.

Similar interchanges occurred between students, although the student-student interactions were extremely procedurally focused with little discussion of why the processes were appropriate. For example, in the interchange below, Sophia clearly gave all responsibility for the process to the written rule on the board, and Mina seemed to follow her step-by-step instructions without indication of understanding. (Note the mixture of Italian and English in their private conversation.)



Mina	*Come e fatto? .. three times two .. e poi?* [What did you do .. three times two .. and then?] Translation ...
Sophia	*Oh you .. the way it's explained up there you multiply the height.* *You go .. three metres by two metres.*
Mina	*So three times two?*
Sophia	*Yeah*
Mina	*..and doppo what about the half?* [and then what about the half] Translation
Sophia	[inaudible]
Mina	*So you times?* *Times half? .. come half? ..* [inaudible word] *a half?* [how do you do the half?] Translation
Sophia	*Nought point five.*
Mina	*Point five ../*
Sophia	*For half. Then it comes out to three cubic metres.*
Mina	*Point five .. put the zero or no?*

Chains of assistance

Instructions were shared generously from one student to another. For example, after the above exchange Mina relayed the procedure to another student: *"Three times two equals six times .. five .. **point** five .. point five."* This phenomenon of sharing information, which I have called a "chain of assistance," was common during student work. However, the instructions usually became more procedural and stripped of explanatory information as they passed along the chain.

Although students received greater *opportunity to speak* during worksheet practice periods, their *means to speak*, the appropriation of the mathematical register, and understanding of concepts, was not necessarily increasing.

Opportunity to speak using the intervention activities

Structured group and pair tasks, centred around concrete materials, proved to be one means of changing traditional communication patterns. Since students were not expected to undertake calculations alone, there seemed less necessity for long teacher introductory dialogues, so the *opportunity to speak* became more immediate. The nature of student-student interactions shifted from givers and receivers of information to a more exploratory form of collaboration (Mercer, 1995). Rather than one student waiting until another had worked out an answer then requesting help, which came as fully thought out opinions or procedures, students explored more conceptual ideas together in a collaborative manner. (This was a process not only facilitated by the cooperative structures and concrete materials, but by the different nature of questions they allowed.) For example, students manipulating string on grid paper explored area shape and perimeter together:

Elaine	*What she's saying is she owns a rectangular garden and she wants to make another rectangular garden with the same amount of fencing and a larger area.* [Interpreting the question]
Greta	*Yeah. Yeah.*
Elaine	*It can be done.*
Sarah	*Can it?....* [briefly inaudible]
Robyn	*Instead of making the garden long and thin. That's what you are saying aren't you? But it's still a rectangle.*
Elaine	*Yeah.*
Sarah	*OK. Is that what you mean when .. you make it with the same amount of string?*

As well as becoming more collaborative in nature, the character of student-student interactions became less procedural. In many cases their conceptual knowledge and intrinsic terminology was clarified because understanding language, such as "rectangular" or "cube" was embedded in the tasks. For example, when exploring cubes made with small MAB blocks, Jackson suddenly realised how to determine the number of cubic centimetres in a cube and its connection with $x^2$and x^3 formulations. ***"That's it! That's it!*** *Don't go any further! The number has to be timesed three times to make a cube."* He then shared his discovery in an excited voice:

> *.. Cube the number mate. Equals four time four times four. .. That's, that's why we have square root - you know how we have "square" and "cubed." That's what cube stands for.*

Responses to the true or false statement "Stephan carried a cubic metre of soil in his wheelbarrow," typified similar exchanges about meaning:

Mina	*Quanti a cubic metre?*
	[How big is a cubic metre?] Translation
Bruna	*The larghetsa .. the altetsa .. egualli..*
	[The length and the height are equal] Translation
	Her explanation was then taken up in English:
Sophia	*Imagine a .. imagine a whole container... the same length and the same width and the same - a cubic metre*

"Rehearsing" student explanations to increase student voice

An explicit strategy for encouraging longer student explanation involved asking pairs of students to rehearse their reasoning for brief questions such as "Which is bigger 1600 cc or 1 1/2 litres?" This strategy significantly increased the length of student utterances to the whole class. For example:

> *.... we thought together that ... you convert the one cubic centimetre to millilitres .. millilitres you get um .. then it's equal the one and a half litres .. and so therefore the one thousand six hundred is greater than one thousand five hundred. So the one thousand six hundred ccs is the greater. I went around the world to say that ....*

Claire's final comment implies that the speech seemed very long to her. It indicates the expectations of student speech within the prevailing culture, where students were seldom encouraged to explain their thinking. However, in comparison to the teacher's usual contributions, Claire's was quite short.

During these invited student explanations other students paid close attention and acknowledged their understanding. *"See that's what she did. She converted it to liquid, and it makes it simple."*

Divergent tasks change teacher-student interactions

Students' divergent approaches to group investigation problems allowed a change in the quality of teacher-student interactions. Differing responses gave the teacher a real purpose to act as "audience" listening to students, rather than always answering their cries for help as before. For example, *"we counted up the squares, right? and then we get the area ...and then we did a triangular shaped fence."* Or when Jackson related to the teacher his strategies for finding all possible dimensions of box shapes with a volume of twenty four: *"Well I did it in equation form right? I thought of all the different equations that times to get twenty four. So have a look at this."*

Means to Speak During the Intervention Activities

Although the initial activities trialed provided more *opportunity to speak,* there were early indications that students' *means to speak* was problematic. Students did not use the mathematical register naturally; rather, they tended to rely on "spontaneous," or everyday, language (Boomer, 1986) which made it difficult to express their conceptualisations adequately. For instance, early in the teaching experiment students were asked to write group reflections after the volume/shape estimation task. It became clear that they did not naturally use the term "volume" but tried to describe the concept with *"carry the same amounts"* or the imprecise *"bigger."* It was these results that prompted the creation of tasks specifically designed to focus on mathematical language, an important aspect of the *means to speak.*

The introductory brainstorm for the first language task demonstrated that even English speaking students lacked confidence with the language of shape: *"Spherical? Is that a word?"; "Is a pyramid a triangle?"* However, their questions also indicated the students' willingness to "attend" to language and experiment with it. For instance, when Jackson experimented with the "-ical" ending the teacher had modelled for conical, and cylindrical: "*... Hexogonical .. hexoconical .. Hexagon? .. so it must be hexagonical .. I've never heard of such a word .. Can I borrow your dictionary?"* Similarly Mina's appropriation of the modelled language was apparent as she rehearsed her written response: *"the volume .. the cylindrical shape .. has more volume."*

Tasks which asked students to write and discuss conceptual ideas gave them permission to take the language of the subject seriously. In her interview Sarah, a keen mathematics student, commented on the effort involved in putting her mathematical thinking into words:

> *.... like myself, I fall over my words a lot .. not fall over my words .. but you know what you are saying but you can't explain.. You have to I don't know, be really articulate .. to come sort of out with .. or you know, really verbal .... but I suppose you get taught that.*

When she did manage to articulate her thoughts she was obviously pleased with her efforts *"It is a cylindrical shaped bottle with a round base - mm quite eloquent."* So although she found it difficult to express herself in mathematics, she seemed pleased that she was improving in this aspect and had become conscious of it as a part of the subject.

Benefits from specific language tasks appeared to flow into later activities as students' conversations became qualitatively different, using language appropriated from earlier tasks in later discussions. For example, when considering "How many litres are there in a cubic metre?" and "What would it weigh if it was filled with water?" a group measured the litre (large MAB block) along the edges of the constructed cubic metre. *"But how many of those go across?"; "It'll be ten"; "Well is that ten by ten by ten, to get the three dimensions? .. So a thousand litres."* "Dimension" was a word that had not been available to students several weeks earlier, but had been introduced to help them explain the difference between a circle and a cylinder.

When listing similar and different properties of A4 cylinders, students used many terms that had been rehearsed and clarified in the prior cloze exercise. For example, English learners Isabel and Bruna, usually diffident regarding language, contributed: *"It holds more .. it is different in volume," "circular base," "cylindrical,"* and *"capacity"* to the class discussion. This type of contribution, involving the naming of mathematical properties, contrasts with student utterances during the calculation-based worksheets, in which numerical answers would have been the only verbal responses. More specific studies would be necessary to see whether communication changes of this nature would be lasting, however the results from these few instances were sufficiently encouraging to indicate that further work in this area is worthwhile.

The results of this experiment indicate that curriculum planning in adult mathematics and numeracy classes should take account of increasing students' communicative competence. The study demonstrates that rehearsal for a larger audience, the need to express thoughts during collaborative tasks, and finally the need to contribute to written reflections all provide students with greater *opportunity to speak* than traditional mathematics pedagogy. These tasks also awaken in students an interest in acquiring language with which to express themselves, the *means to speak.* The results show that this aspect of language will benefit from supplementing the conceptual tasks with activities that focus on written and oral use of mathematical terminology.

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The Effects of Digital Measuring Equipment on the Concept of Number

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Abstract

Over the last twenty years the use of calculators and digital measuring equipment has to some extent replaced mathematical mental/written activity and also the use of analogue measuring equipment. This paper explores some aspects of number concept, reading the number line, and estimation from scales. The students being considered are mainly part of a foundation year in science/engineering/computing, many being mature students who are returning to education. The foundation year provides an alternative entry to degree courses for students without the conventional entry qualifications. The results obtained are compared with research undertaken in schools. The results of the research will be used to inform the teaching of future foundation cohorts.

Introduction

Electronic technology has revolutionised the ways in which we work with numbers. This paper attempts to address the question "Has the technical revolution effected a change in the understanding of number?"

Aids to calculation have existed for centuries, from the ancient Chinese abacus, through to logarithms, ready reckoners and mechanical adding machines, each offering its own set of shortcuts. The hand held technology of the last generation is simply an extension of this. However, digital technology has found an application in many areas where previously a graduated scale may have been used. The most obvious examples are a digital watch replacing an analogue watch, digital weighing and pricing scales in food stores replacing traditional scales. There are many more specialised uses of this technology, which abound in certain professional areas, e.g., health care, building surveying, etc. The advent of desk-top computers, with their easy to use spreadsheets capable of generating many types of graphs and charts, means far fewer graphs are produced using pencil and paper and with it the necessary exercise of deciding which scales to use and how to fit the graph on the page. The display from digital equipment is presented as a given number of decimal places. There is no need for the recorder to choose or have specified the degree of accuracy. So even at the most basic level of approximation, "rounding up" is removed. The evidence suggests that the tendency of technological devices is to bury the mathematics. These advances lessen the need to engage with numbers in a context where one may have regard for size or place value. After all 0.05 and 0.005 are both small numbers and they can both be easily given on a digital display. In some settings students may be able to differentiate between a factor of ten; however, when reading scales or digital displays, this may prove problematic as the context has been removed.

History of Digital Equipment/Calculators and Their Use in Education

As early as the 1970s electronic calculators were introduced into schools. Since this period there has been a debate concerning the appropriateness of calculating aids and their effect on the mathematics school curriculum. Within a decade changes could be seen to have been made in the design of the GCE A level syllabus, which now permits the use of programmable graphic calculators, through to the primary sector where young children are encouraged to play with calculators.

Opponents to the use of calculators argue that it can result in a deterioration in a pupil's ability to do basic computations. The recent introduction of the National Numeracy Strategy (NNS) discourages the early introduction of calculators, emphasising the development of computation in the early years. Students are taught about place value in terms of a number line. Numbers are "visualised" as existing along an infinite line. This understanding moves from the discrete counting numbers to a situation where other units of measure including rationals and decimals can be handled with increasing facility.

What counts as a mathematics curriculum? The use of calculators presents potential for more time to be available for understanding the structure of mathematics. However the use of calculating aids in the teaching of mathematics does need to continue to be re-evaluated.

Digital equipment may have started with the calculator, but the majority of adults use push button telephones and ATM machines on a regular basis. This usually necessitates the keying in of a PIN. Numbers are often given in the form seven, six, two, three rather than seven thousand, six hundred and twenty three. In other words, the number is being used as a token (Pimm, 1995, pp. 60-61) rather than as something that represents a particular quantity. Possibly this is the most frequent use of number outside the mathematics classroom. Inside the classroom students tap numbers into a calculator, inputting them from right to left as they move across the digital display. Even the keypad has no mathematical structure in its layout and is very similar to that of a telephone or ATM. The press of a single calculator button will result in the appearance of another number on the display. What meaning does the student attribute to this—are the numbers and the results still tokens or something that represents a specific size value? In order to make some attempt to answer this question a survey was undertaken.

Analysis of Questionnaire Results

Students surveyed were mainly foundation science and technology students at the University of North London. These students are insufficiently qualified for starting a degree programme and first undertake a one-year foundation programme, which normally includes two mathematics modules. The students are from very diverse backgrounds; about 40% are mature and may not have studied for some years and another 60% have just completed a pre-university course, e.g., A Level or GNVQ, but have not reached the required level for progression. Three questionnaires were used at different times during the academic year.

In discussing scales, reference will be made to major divisions, which are always labelled with a number and minor divisions, which are not labelled. Three activities related to scales are considered: (a) locating or plotting points on a labelled scale; (b) constructing and/or labelling a scale and then locating or plotting points; and (c) reading plotted points from a labelled scale.

The first questionnaire was undertaken during the first maths module at a time when students were starting to study graphs. The questionnaire had two parts: questions about digital equipment and questions relating to scale.

The digital equipment questions showed about half the cohort still wore watches with hands and that the use of measuring equipment mainly involved the use of graduated scales but mostly on an occasional basis. However a minority of students had some experience of using digital weighing scales and thermometers.

Activity (a) - Plotting Points on a Labelled Scale

Point to plot plus scale details		*Results*			
Point	Major Divisions (Minor Divisions)	*Accurate (%)*	*Close (%)*	*Not Attempted (%)*	*Incorrect values plotted in error (%) and *comment on close values*
2.5	0,1,2, (0 minor)	96	2	0	5.5 (2%)
7.8	0,1,2 (0 minor)	98	0	2	
37	0,50,100 (0 minor)	74	14	2	370 (10%) - factor of 10
0.63	0.4,0.5,0.6 (1 minor)	82	16*	2	*3/10 along 1st division
0.046	0,0.01,0.02 (1 minor)	84	1	2	0.0046 (2%), 0.043 (6%) ie 6/10 along 1st division, 0.011,0.021
0.0025	0,0.01,0.02 (1 minor)	44	16*	12	0.025 (20%) - factor of 10 0.001 (4%), 0.005 (2%), 0.009 (2%) *at around 0.002 or 0.003

The above table shows a high level of accuracy when estimating tenths of a division for 5.5 and 7.8. This accuracy drops in the case of 37, which is incorrectly placed at 370 in 10% of cases. Students appear to be reading the 350 and 400 as 35 and 40. This error again manifests itself when 0.0025 is incorrectly plotted at 0.025 in 20% of cases. Difficulties in understanding scale with one minor division are apparent in the plotting of both 0.63 and 0.046 as students estimate using tenths in the first section only, ignoring the existence of the second section.

Further scale questions required students to construct and label scales and then plot points, activity (b). Students were given blank scales with 10 major divisions each with 4 minor divisions. Students were expected to label using the largest suitable scale and then plot 4 or 5 points. Students who could correctly label the scale could generally plot the points correctly.

Expected choice of Major Division	10	0.1	0.01
Facility	75%	65%	55% (35% didn't attempt)

The facility decreases as the numbers become smaller (very large numbers were not tested).

These results informed the construction of two subsequent questionnaires, which were designed to test out some of the errors that were occurring in the first set of results. These questionnaires were completed by much smaller cohorts of students, towards the end of the academic year.

Both questionnaires asked respondents about what timepiece they visualised when they thought of reading the time and which they found easiest to use. Again about half thought of clocks or watches with hands and half thought of a digital timepiece. In terms of which was easier to use, 65% said digital timepieces were easier, 25% thought clocks with hands were easier, and 10% didn't mind. Reasons for choices given:

Digital: easy to read; it's done for you; I've grown up with this; on screen display; exact figures; gives the precise time; no working out where the hands are; etc.

Analogue: it's fixed in my memory; 24 hours is confusing; associate it with direction; can see the full 60 minutes, i.e., the time remaining and the time gone; I'm used to it; etc.

Both these lists indicate a sense of comfort with the familiar. However, at a deeper level, comments indicated the advantages of a digital display which gives time to the nearest minute and of a clock whose hands can show the proportions of time.

All the scales in the second questionnaire involved either 5 or 10 minor divisions, some representing numbers like 25 and 0.004. Hence, locating points involved students in too much division. When unable to interpret the scale, some students resorted to counting minor divisions in "ones." This brought a sharp focus back to the study of place value, which was perhaps becoming secondary, so this questionnaire was abandoned after completion by 15 students and the third one designed with simplified questions on scale.

On the third questionnaire, questions were designed to test a particular potential error, which had been identified earlier. This was completed by 43 students, 10 of whom were from a local further education college with most of these studying advanced level mathematics.

This questionnaire tested reading points from an existing scale which was the reverse of what was done in the first questionnaire where students had to plot or locate the points.

Activity (c) - Reading Points From a Labelled Scale

Point to read plus scale details		*Results*			
Point	Major Divisions (Minor Divisions)	*Accurate (%)*	*Close (%)*	*No Attempt (%)*	*Values read in error (%) and *comment on close values*
3.5	0,1,2, (0 minor)	85	15	0	
480	0,50,100 (0 minor)	40	40*	2	48 (2%) - factor of 10, 450.7,457 and similar (16%), *estimated as 475 or 470
0.05	0,0.1,0.2 (4 minor)	60	9	9	0.5 (7%) - factor of 10 0.045 (9%), 0.02 and 0.03 (6%)
0.78	0,0.2,0.4 (1 minor)	83	7	0	0.68 (5%), 0.73 and 0.7 (5%)
0.002	0,0.01,0.02 (1 minor)	45	26	12	0.02,0.025,0.25 (12%)- factor of 10 0.004 (5%) or 4/10 along 1st division
0.02	0,0.1,0.2 (9 minor)	56	2	26	0.2 (7%) - factor of 10, 0.01 (5%), 0.04 (2%), 00.8
0.23	0,0.1,0.2 (9 minor)	68	0	24	0.26 (5%), 0.28 (3%)
0.405	0,0.1,0.2 (9 minor)	42	27*	26	0.045 (5%) *0.4 or 0.41, inability to estimate in between value.

The accuracy rates are ranges from 40% to 85%. However the combined accuracy and closeness range is 58% to 90%. The majority of errors involve either place value or reading of minor divisions. The place value errors now exhibit a new way of including the estimated part of the division, i.e., 450.7 for 480 and also the scrambling of the digits in the 0.405 giving 0.045.

Activity (b), completing scales and locating points. Two scales each with the first two major divisions already labelled were provided. Students were expected to use these scales and locate three points. Not all students completed the last section of the questionnaire so these results refer to those that did.

Ability to label and interpret scale				*Comments on plotting points*	
Major Divisions (Minor Divisions)	*Correct*	*Incorrect*	*Change in sequence*	*Points to plot*	*Comments on value plotted in error*
0,0.2,0.4 (4 minor)	54%	29%	17%	0.004, 0.09, 0.105	0.004 plotted as 0.008, 0.002 0.105 plotted as 0.11 and 0.0105
0,0.1,0.2 (9 minor)	76%	14%	10%	0.058, 0.82, 1.12	1.12 plotted as 1.02

The scale involving counting in twos proved more difficult than counting in ones; some students changed sequence at 0.2, i.e., 0, 0.2, 0.3, 0.4, another changed at 0.6. Similarly with the second scale some changes of sequence took place after 0.9 and 1. Plotting errors, when the scale was interpreted correctly, mainly involved misinterpretation of minor divisions. There was less scope for place value error in these exercises.

On ability to use scales, 42% rated it as very important, 50% as quite important, and 8% as hardly important. Seventy-eight percent thought their ability to use scales affected their understanding of place value.

The foundation students plot graphs as part of their first maths module. The most frequent error that occurs is when a scale is inconsistent, for example, 0, 2, 4, 6, 8, 10, 20, 30, . . . or 0, 1, 2, 4, 6, 8, . . . This type of error was also observed in the questionnaire scales. Another common error is to use the heavy lines of the graph paper to represent the given data values without any regard for scale, so for example one could have 2.3, 4.6, 5.1, 6.7 all plotted at equal intervals. This often seems to be a direct result of the way a student has been taught, whereas the former is more to do with a student's perception of number.

School Studies

Although it is not possible to map the above to exactly the same studies in schools, there are elements of research undertaken that contain some comparable elements. The Concepts in Secondary Mathematics and

Science (CSMS) study and the Low Attainers in Mathematics Project both look at children's understanding of decimals and place value. In particular is the interest is a child's ability to move from an understanding of discrete counting to that where "measurement can be refined by smaller and smaller units of measure" (Dickson et al., 1984, p. 91). The CSMS study found the facility of 15 year olds to locate 5.8 at a minor division between 5 and 6 was 85% (a similar question to foundation students had a 96% facility) and to locate 2.74 at a minor division between 2.7 and 2.8 was 71%. This dropped to 61% when children were required to locate 14.65 on a tenths scale where an estimation was needed between unlabelled minor divisions at 14.6 and 14.7 (Hart, 1981, pp. 60-61). The facility of 15 year olds to write down a number between 0.41 and 0.42 was 71%. In a slightly different context this is similar to students who approximated 0.405 to 0.4 or 0.41 in order to plot it. As one might expect, foundation students seem to show competence at the simplest levels and score higher than their school counterparts. However, some students exhibit difficulty when interpreting numbers where the place value is not explicit and the number or scale are more complex.

Overview – Reflection on Findings

The findings from the questionnaires indicate some students have a problem with identifying place value. It often appears that when students make errors they are reading the digits but not reading the place value. In other words, they are looking for like digits and not like place or size. Another common error was the inability to accurately read the minor divisions. Sometimes a single minor division is ignored and estimation is undertaken in only one section. When four or nine minor divisions are used they can be attributed the wrong value, e.g., counted in twos instead of ones. The findings show a range of misconceptions but can only provide a limited insight into the students' mathematical thinking. The higher education students in the sample are studying mathematics in order to progress into their chosen field. They bring to their study a range of different experiences. Also, their views of mathematics in terms of its purpose may vary. For a majority of the students mathematics is a means to an end; it is viewed in terms of its potential usefulness in their field of study.

Conclusions and Suggestions for Changes to Teaching for Foundation Students at UNL

The challenge for the teacher is to use the technology not only as a computational device but as a pedagogic device to enhance a student's understanding of mathematical structure alongside any other necessary devices like the use of scales.

The knowledge adult learners bring may be fragmented and contain misconceptions. In terms of self directed learning we can recognise four main characteristics suggested by Alan Rogers (1986, p. 69):

- Episodic: the task is completed in a short burst of intense activity, usually followed by a period of no activity.
- Goal orientated: a means to an end, e.g., progression to chosen degree course. Learning is limited to task at hand with no desire to extend knowledge or draw on existing compartmentalised knowledge. Usually technique oriented rather than concept oriented.
- Use of a wide range of strategies: trial and error methods, learns by imitation, but takes longer to absorb than other learners, needs to understand fully the whole process and make a meaningful whole.
- There may be little interest in overall principles, hence what is stored is the "how" rather than the "why."

The teacher needs both to work with these characteristics and also to employ strategies to complement students' shortfalls. For example, provide short activities focussing on a specific misconception: what is wrong with estimating 550.7 when 570 was intended; what is a suitable measure; what level of accuracy is appropriate? The activities should develop the understanding of how place value and measurement are embedded in their particular area of science or computing. Students need to have things to imitate, starting from the simple to the more complex, enabling them to build up their own more complex "meaningful wholes." Students could be shown how numbers a factor of 10 apart can exist on the same scale, e.g., 0.02 and 0.2. This fluency should then enable active learning trial and error methods to test out what may be the best scale or measure appropriate to a given situation. This should include reading from a calculator with a view to "sensible size" for purpose.

Students should be encouraged to build up an understanding of size and place value, which shows the connectivity and simplicity of our number system.

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Does "Part-Whole Concept" Understanding Correlate With Success in Basic Math Classes?

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Introduction

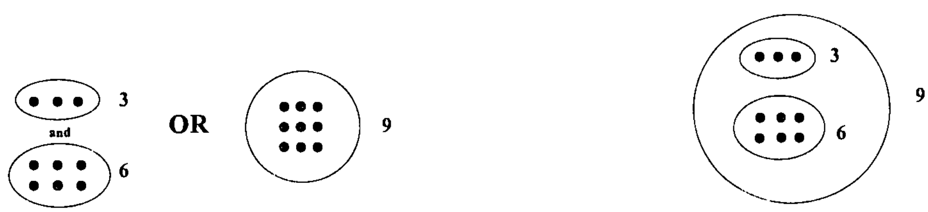
Adults with difficulty in math may lack skills in number manipulation or may fail to grasp fundamental concepts of number relationships or both. It is important to uncover students' weakness in conceptual understanding so that math "skills" can be applied appropriately.

One major transition concept from concrete to abstract thinking in math is the part-whole concept. Students who grasp this concept have the sense that the parts or partitions of a quantity and the whole quantity exist together at the same instant, rather than only the whole or only the parts existing at any given point in time (Figure 1).

Figure 1

NO PART-WHOLE
Either the parts or the whole exist

PART-WHOLE UNDERSTANDING
Both the parts and the whole exist at the same time



Pilot studies (Steinke, 1999) indicated that many adults lack the part-whole concept. A later study (Steinke, 2000) reported a high percentage of errors on problems requiring part-whole understanding on a standardized test (Test of Adult Basic Education Summary) among a large sample population already possessing a high school diploma or GED. The current report relates part-whole understanding on the same standardized test to final grade in a Basic Math or Algebra I course in portions of the large-sample population.

Background

In studies with children, Steffe and Cobb (1988) proposed a 3 Stage model of number understanding, the third stage being grasp of the part-whole concept. Using the physical signs and oral responses described by Steffe and Cobb as differentiating the 3 Stages, Steinke (1998) developed a short non-pencil-and-paper assessment for part-whole understanding. In three different pilot groups of adult volunteers, Steinke (1999) found unexpectedly high percentages of adults who lacked the part-whole concept: 4 of 11 students from a general population at a two-year college (36%); 8 of 12 students in a pre-GED class (67%); and 2 of 15 pre-service teachers in a "mathematics methods" class (13%). Even with visual cues (circles to represent the given part; the written numeral for the whole), these adults were unable correctly to answer, or showed lack of part-whole understanding in the way they answered, a question of the form: If I have 7 cookies on the plate and 23 cookies altogether, how many cookies are still in the box?

Attempting to support or refute these unsettling pilot results, a *post-hoc* data analysis was undertaken (Steinke, 2000) of a large sample of responses from the mathematics portion of the Tests of Adult Basic Education (TABE), Summary Form (CTB, 1990). The entire Summary test is used as a placement exam for entering students at an independent two-year post-secondary institution in New Mexico. A TABE "locator" test determines the appropriate reading level for a person, and the correspondingly difficult form of the TABE Summary is then administered (Form A, D, M, or E). Prior to data collection and analysis, the questions in the Math Application portion of the TABE were reviewed to determine which required part-whole understanding.

Single-operation math problems fall into two types:

1) given the parts, find the whole (P + P = ?) (7 + 9 = ? or ? – 9 = 7); or
2) given the whole and a part, find the missing part (P + ? = W) (9 + ? = 16 or 16 – 9 = ?),

The second type requires part-whole understanding to find the "missing part."

This segregation of problems into two basic types is supported by comments from the literature. Steffe and Cobb (1988) speak repeatedly of "part-whole operations." Kamii (Kamii with others: 1985, 1989, 1994) and Ross (1986, 1989) imply these two categories of problems in their writings. Carpenter et al. (1999) use the terms "part unknown" and "whole unknown" to describe word problems.

Analysis for Type 2 "missing part" problems in the TABE Summary found 5 problems in Form A, 7 in Form D, 4 in Form M, and 4 in Form E. Scores on the "missing part" problems in the large sample (2,909 individuals) showed a high level of errors (Steinke, 2000). For all such problems, the percent of wrong answers (% errors) on "missing part" problems were: 6.6% to 50.0% in Form A (424 tests), 12.3% to 65.1% in Form D (1,066 tests), 1.2% to 42.7% in Form M (1,118 tests), 7.6% to 48.5% in Form E (301 tests).

Procedure

Three of the Type 2 problems from each TABE Form, those most straightforward and most similar to each other across the Forms, were used as the basis for determining part-whole understanding in the present analysis. The % errors for these three problems were: Form A: 6.6%, 24.3%, 37.7%; Form D: 12.3%, 24.9%, 24.2%; Form M: 1.3%, 25.7%, 42.8%; Form E: 48.2%, 26.6%, 48.5%.

For the present analysis, the database was organized to show TABE scores, by-item TABE errors, math classes attempted, and final grades for students who took the TABE Summary Math Applications Section and enrolled in a math course from the Fall 1996 term through the Summer 1999 term. "Final Grade" is recorded as the standard A through F plus W for "withdraw." "W" students left the course between four weeks from the start of the term and four weeks before the end of the term with no penalty. In other words, many of those who are recorded as W may have earned a poor grade had they not withdrawn.

Fewer students took math classes than the total who took the TABE. Of particular interest were individuals who had taken either Math 100 ("basic" math) or Math 102 (high school Algebra I) as their initial math course. Also of interest were students who took Math 100 and continued to Math 102.

The data from all four TABE Forms were combined with all results sorted by math class and final grade (Table 1). This was done to provide more data for each grade/errors pairing. For example, all the errors made by students who earned an A grade in Math 100 were summed and divided by the number of such students (N = 159) to give All Problems Average Errors of 3.1447. To provide clearer independence of data, the Average Errors for the three "part-whole" problems (3 P/W) and for the remaining 12 problems (12 Prob.) excluding the three "part-whole" problems were also found by final grade in each course.

Table 1: Average Errors by Letter Grade in Math 100 and Math 102 for All Forms of the TABE Summary Combined

Math 100 (Basic Math) N = 500				Math 102 (Algebra I) N = 291			
Grade/N	All Prob.	12 Prob.	3 P/W	Grade/ N	All Prob.	12 Prob.	3 P/W
A 159	3.1447	2.6038	0.5409	A 56	2.6964	2.4107	0.2857
B 107	3.1121	2.3551	0.7570	B 54	2.6852	2.2407	0.4444
C 87	3.4367	2.6207	0.8161	C 59	3.2034	2.6271	0.5763
D 33	3.0303	2.1212	0.9091	D 23	3.2174	2.0870	0.6957
F 64	2.9688	2.2188	0.7500	F 25	3.1600	2.4800	0.6800
W 50	3.3600	2.6000	0.7600	W 74	3.3378	2.7887	0.6622

This combining of Forms was felt to be justified by the TABE publisher's assertion of the equivalence of the Forms when the Form on which a subject is tested is based on taking a "Locator" test for reading level, as was done for all these subjects. Furthermore, separate analysis of scores for Forms M and D showed a pattern of results within each Form similar to that of the scores for the combined forms.

Because of interest in success in Algebra I (102) of those who had taken Basic Math (100) compared to those whose initial math class was Algebra, another analysis of the sample was done (Table 2). In this smaller group, Forms A and E had too few samples to be fairly distributed across letter-grade categories. Only the Forms D and M pairings were used. Students' math registration was known from Fall 1996 forward through Summer 1999. The "Initial Math 102" sample was limited to those Math 102 students who took the TABE no earlier than July 1996 and whose first registered math course after that date was Math 102. There were 156 students in this group. Another 130 students took Math 100 followed by Math 102 in this time period.

Table 2: Math 102 Final Grade and 3 P/W Average Errors for Prior Math 100 Students and for Initial Math 102 Students (TABE Forms D & M)

	100 Prior	% Tot. N	3 P/W	**102 Initial**	% Tot. N	3 P/W
Grade	N (Tot. 130)		Av. Error	N (Tot. 156)		Av. Error
A	16	12.31	0.3750	36	23.08	0.2500
B	18	13.85	0.8333	28	17.95	0.2857
C	34	26.15	0.5588	37	23.72	0.4595
D	31	23.85	0.8065	11	7.05	0.4545
F	11	8.46	0.4545	15	9.62	0.5333
W	20	15.38	0.8500	29	18.59	0.3793

Analysis

What is striking about the letter grade/TABE All Problems and 12 Problems Average Errors relationships in Math 100 students (Table 1) is that the people who received D's or F's in the course did better on the TABE than those who got A's, B's, or C's. Also, those who withdrew (W) did better than the people who earned C's.

Students in Math 102 who earned D's and F's had fewer average errors on the 12 problems (those other than the three selected part-whole problems) than did students who earned A's. Granted there are other factors affecting classroom success, but at face value it seems that this version of the TABE does a poor job of predicting success in basic math and first year algebra for this sample.

A student's grasp of the part-whole concept (3 P/W average errors) prior to starting Basic Math (Math 100) does seem to relate to final grade in the course. The same trend appears for those who successfully complete Algebra I (Math 102) (see Chart 1). This group of Math 102 students probably includes some who took Math 100 prior to 1996; this may have the effect of showing slightly higher 3 P/W average errors in Math 102.
The reason for the 3 P/W average errors for F and W students in Math 100 being close to that for B and C students has any number of explanations. One factor in academic success is self-motivation. Perhaps the F and W students could earn a B or C but give up. One reason for giving up is that they are aware that they aren't "getting it." They are unaware that "it" is an understanding of the part-whole concept.

Chart 1:
3 Part/Whole Problems Average Errors on TABE and Students' Final Grades in Math 100 and Math 102

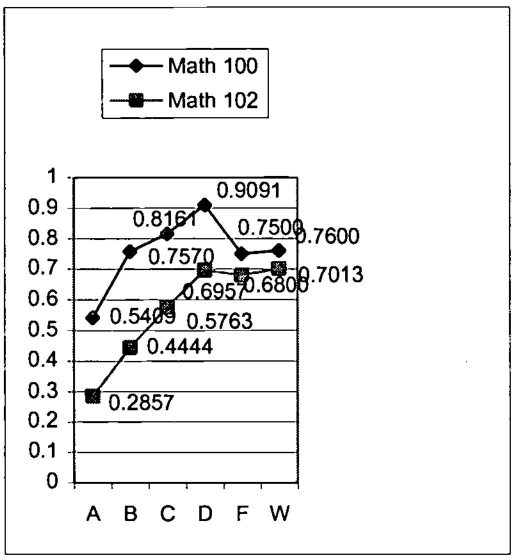
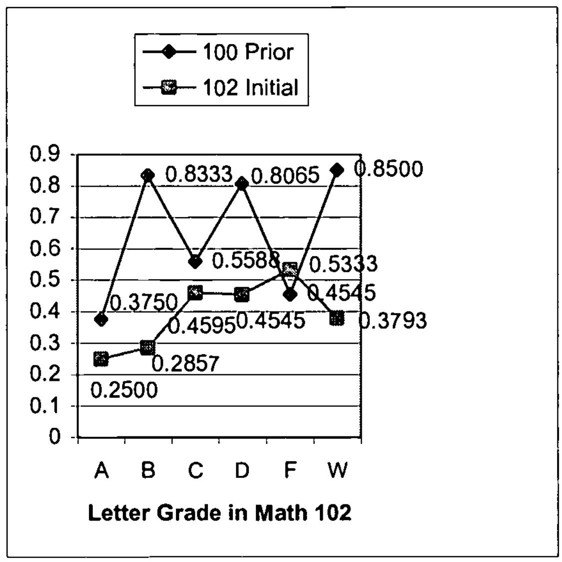


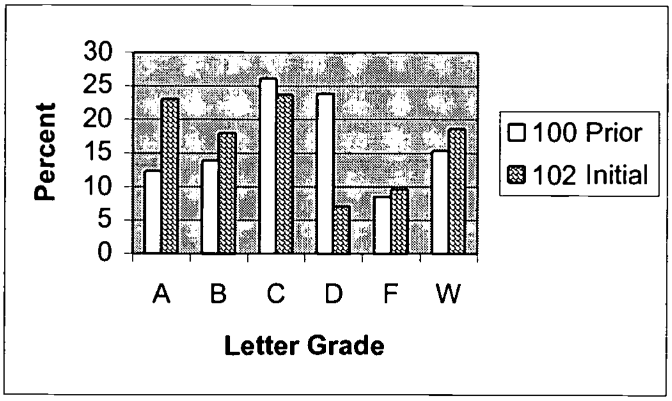
Chart 2:
Math 102 Final Grade and 3 P/W Problems Average Errors on TABE for Students Taking Math <u>100 PRIOR</u> to Math 102 or Taking Math <u>102</u> as <u>INITIAL</u> Math Course



The better question is: do B and C students in Basic Math (Math 100) get better grades by improving their part-whole understanding? Perhaps they are able to earn an acceptable grade by using rote skills on tests, turning in homework, and doing "extra credit" work. The success rate of Math 100 students going on to Math 102 provides further insight.

A comparison of 3 P/W average errors and final grade was made between students who took Math 102 after taking Math 100 (100 Prior) and students who it is certain took Math 102 as their first math course at the college (102 Initial). The comparison was limited to Forms D and M (for a fair distribution of Form/Grade pairings) and "Math 100 Prior" students with grades of A, B, or C in Math 100 (Table 2). The "102 Initial" group shows a 3 P/W-to-grade trend; the "100 Prior" group does not (Chart 2). Again the question is whether the "100 Prior" students improved their part-whole understanding in Math 100 or worked harder in Math 102.

Chart 3:
<u>Percentage</u> of Students at Each <u>Final Grade in Math 102</u>
With <u>Math 100</u> as <u>Prior</u> Course or <u>Math 102</u> as <u>Initial</u> Course



Another interesting comparison between the Math 100 Prior/Math 102 Initial groups is the percentage of students in each group at each final grade in Math 102 (Table 2; Chart 3). The percentage of F and W are close. In the Math 102 Initial group the remaining grades are skewed toward A. In the Math 100 Prior group the remaining grades are skewed toward D. If the importance of the part-whole concept to success in math were accepted, then it would seem that students in Math 100 (Basic Math) who lack the concept might not be picking it up from current instruction methods. If they were, the Math 100 Prior success rate, measured by percentage of students at each letter grade, should match that of the Math 102 Initial group.

Another source provides insight into the extent of the lack of the part-whole concept. After providing in-depth instruction on the fundamental principles of arithmetic to pre-service teachers, Bloom and Zimmerman (2000) reported a statistically significant increase in the number who could correctly answer these questions without calculating them:

1. Without calculating an exact answer, select the best estimate for $\frac{12}{13}+\frac{7}{8}$.

A. 1
B. 2
C. 19
D. 21
E. I don't know

2. John had two pizzas. He gave one-third of one pizza to his sister and one-half to his brother. How much pizza remains?

A. More than 1 pizza
B. Less than 1 pizza
C. Exactly 1 pizza
D. No pizza

However, even after a semester of instruction, 20% of the pre-service teachers in the course still were unable to answer these problems correctly. Solving these questions requires part-whole understanding, a sense of the whole coexisting with all the parts.

Concluding Remarks

Evidence for lack of part-whole concept understanding as a hidden cause of innumeracy is reported here. The evidence is based on "data mining" and as such is subject to hidden traps. Therefore no confidence values (p values) have been attached to the numbers reported. Still, the data do show a trend of a higher final grade corresponding to better part-whole understanding at the start of the course.

In light of this evidence, it would seem that direct instruction of the part-whole concept should be part of adult basic math and algebra classes. This is not the case with any of the many adult math texts reviewed by this author. Part-whole understanding is assumed. Therefore, many adult math students continue to struggle and continue to fail. They do not know that it is this concept, rather than skills, that they are lacking; neither do the curriculum developers.

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SOCIO-CULTURAL CONTEXTS

All for One and One for All: Citizenship and Maths Education

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What Is Active Citizenship?

The concept of citizenship has a long history but still remains problematic. Newton (1999, p. 4) argues that it can be viewed as a set of ordered relations between people that seek to avoid the Hobbesian "state of nature" where life is "solitary, poor, nasty, brutish and short." This perspective depicts a social contract aimed at promoting security and well being within the community and necessarily deals with how individuals ought to act to achieve such an end. It can then be viewed as effective, skilled, and knowledgeable public-spirited work to solve common problems (Merrifield 1997).

The concept of citizenship underpins that of democracy but, in British society at least, "citizenship" has until recently been an unfamiliar notion. The Commission on Citizenship (1990) found that the word was not in common use and, even when used, it had a diversity of meanings. Crewe and Searing (1996) supported some of the findings of the Commission and found that although the British understand the concept, they do not define themselves as citizens. When they do talk about citizenship, it is in terms of civic engagement, i.e., participation in the institutions of civil society. Rather than voting and other forms of electoral participation, the British see citizenship as, for example, working in local voluntary associations. So from a British perspective, citizenship is involvement in social networks, in the groups, organisations, and voluntary associations that connect citizens with the life of their communities. Motivations to engage in other aspects of citizenship, such as attention to political and public issues, are reinforced through participation in informal groups and voluntary organisations and engagement in civic and communal activities from good neighbouring to charity giving to more formal socio-political activity.

A reasonably representative definition of citizenship can be taken as how an individual activates him- or herself to be able to consciously influence their own situation and the situation of others in a democratic society (Bron, 1996). Crewe and Searing (1996) argue that the key components of citizenship are civic engagement and public discourse. This links very closely with Putnam's notion of "social capital" (1993) which has as a major component the social networks of individuals, groups, and organisations.

Active Citizenship Is on the Agenda Now

It is arguable that the fundamental aim of a democratic society is to enable all citizens to participate as fully as possible in cultural, economic, political, and social life, and the active engagement of citizens is part of the broader concept of citizenship of ensuring that people can take the project of shaping the future into their own hands. However, there is a growing concern that there is a democratic deficit and a fading of citizenship values and practices (Thorne, 1998). In the US, Putnam (1995a, 1995b) argues that there has been erosion, over the last thirty years, of the propensity of individuals to associate together on a regular basis, trust one another and engage in community affairs and a weakening of the civic engagement. However, despite the evidence that aggregate levels of social capital have not declined to an appreciable sense in Britain in recent years and that civic engagement also seems to remain relatively high (Hall, 1999) and similar positive results for other European countries (see van Deth, Maraffi, Newton, & Whiteley, 1999), there are still fears that there is a decline in social capital in Britain and elsewhere in Europe. This has led to calls for an increased emphasis on combating social exclusion and the encouragement of an active and engaged citizenry possessing the skills and confidence to contribute as fully as possible. The 1997 European Commission Report *Learning for Active Citizenship* focuses on learning for citizenship as one of the key challenges facing the Union in the years to come. The Report argues that having the right to participate is not equivalent to doing so in practice nor being equipped to do so on equal terms. It asserts that active citizenship is being empowered to handle the practice of participatory democracy and so calls for opportunities to learn and practice autonomy, responsibility, co-operation, and creativity and develop a sense of self worth and expertise in confronting and tolerating ambiguities and oppositions. There is, however, a strong caveat to any discussion of adult education's contribution to a more active citizenry. Many social factors such as poverty, ill health, gender, race, or age may disadvantage parts of the population and prevent their participation. Structural inequality impedes participation.

Issues Raised for Adult Education

But however desirable active citizenship is to leaders throughout Europe, many individuals lack relevant information, skills, and confidence as well as access to opportunities for participation and engagement. It is necessary, therefore, to identify and develop sites for affective and pragmatic as well as cognitive learning

However, the number of adults who willingly choose curricula in civics or even politics or economics is small (Field, 1995). An alternative approach is to locate sites for learning citizenship skills not in "civics" classes but intrinsically and extrinsically in the adult education curriculum. This would mean that whether adults came to learn history, politics, literature, or, as we are interested in here, maths then the curriculum would have as an explicit objective the acquisition of knowledge and skills which contribute to active citizenship. We will first look at some aspects of the content of the numeracy curriculum. We will then consider the skills of citizenship suggesting that these are fundamentally transferable. These skills can be learnt in any adult classroom then "transferred" into a more active participation in society.

Active Citizenship Involves Numeracy

To suggest the content of an adult numeracy course which would contribute to active citizenship requires an examination of the citizenship situations where adults need numerical skills. Some examples are given below but there are many more.

Thorstad (1992) identified school governors as a prime example of citizens who work responsibly and without pay on behalf of the community. Some people are deterred from standing for governorship in the first place due to lack of knowledge or confidence in financial matters, and even those who are elected may be making decisions on shaky ground due to similar inadequacies. The numerical skills identified as being of most use to a governor were the ability to: follow an argument that includes (especially large) numbers; do a quick estimation; check other people's calculation; and calculate accurately with speed and agility but using a calculator. This is a mismatch with numeracy practice encouraged in many formal classrooms. As a result, adults were insecure with mathematical skills half-remembered from school or informally learnt as an adult or a confusion of the two. The result was that some non-specialist governors, including parents, did not take an active part in crucial debates or were being asked to rubber-stamp financial decisions made by the financial subcommittee.

Another study into active citizenship investigated the numeracy issues raised by the introduction of the Council Tax (Hind, 1993b). Hind found that the resultant inability to interpret numerical information led to a lack of knowledge of new developments such as the Council Tax and a failure to understand its implementation. This meant that the citizen did not have the requisite information to make decisions about, for example, tax payments, the fairness or otherwise of the tax, or how to claim for benefits or discounts. This affects the ability of the citizen to operate effectively in a democratic society.

Voluntary bodies, pressure groups, and women's organisations may require citizens to produce or seek out data then analyse it and understand the context where it was produced. Hence certain mathematical skills are needed for critical citizenship and include: how to obtain information produced but not published; methods for the production of information at a small scale level in the community; and the interpretation of information from other sources or one's own research (Evans, 1990). An important skill required in the struggle for critical citizenship is access to and a grasp of official statistics from which we obtain most of our information about government spending, unemployment, poverty, and so on. This access takes place mainly through the media (one has only to think of television news) but in the context of the current political, social, and economic climate. These factors have an immense implication for the accuracy of information disseminated.

However, the numeracy curriculum is currently constructed around the immediate personal or work related curriculum of the individual learner or based on the school mathematics curriculum. It could be extended, as has been done with literacy, to integrate numeracy skills with issues of public concern such as school budgets or new tax proposals. Adults' expressed needs should not be ignored and any widening of the curriculum should not replace the instrumental goals and self-development requirements of the learner but enhance these. Adults can be encouraged to recognise and value the mathematics learning that takes place in all facets of their everyday life. The role of the adult as citizen, in addition to worker, can provide a wealth of suitable material for accessible everyday "really useful" knowledge.

The data for critical explorations of important social issues could come from newspapers, official Government statistics, or the newsletter of the Radical Statistics Group (RSG). The Winter 1995 edition of the RSG's Newsletter contains useful starting points for investigations such as "The unofficial guide to official health statistics," "The Department of the Environment's index of local conditions: don't touch it," and "Retiring into poverty." A statistics literacy course could aim to convey basic knowledge such as an understanding of terms, like average, percentages, etc., graphs, and the logic behind certain concepts such as why averages are used. Most importantly, it could aim to encourage learners to think statistically and appreciate that the application of statistics is valuable (Gal, 1996).

Abilities Required for Citizenship

But it is not sufficient to just consider the content of the curriculum. Other skills and abilities are required for active citizenship. The concept of citizenship is a complex and slippery one. Crewe and Searing (1996) suggests that good citizenship involves two factors: civic engagement and public discourse. "Civic activity," "being an active citizen," or "civic engagement" refers to participation in any significant way in community or social activities and/or involvement in community or social organisations. "Public discourse" refers to discussions in private and public settings ranging from casual conversations to serious deliberations on public affairs topics from community concerns to party political matters. To be a good citizen in the above terms requires an individual to possess certain abilities, confidences, and knowledge. Drawing on these and other definitions from the literature, this paper suggests that the following lists of attributes promote active citizenship—having:

1. the ability to

- negotiate and co-operate with others;
- deal with difference and conflict;
- listen constructively to others;
- obtain information (e.g., from libraries, the Web, authorities, public meetings etc.); and
- voice ideas and opinions.

2. the confidence to

- be proactive;
- have independent opinions;
- act independently if they think it is right;
- take responsibility; and
- assume that their voice will be heard and taken into account.

3. the following knowledge:
- how society is structured;
- how local government works;
- how national government works;
- the basic ideas of the main political parties; and
- political philosophies/ideologies.

A group of adult learners were asked to evaluate the skills and confidence that enhance active citizenship that they possessed (Benn, 2000). The results indicated a lack of perceived ability to deal with difference and in voicing ideas and opinions. The most actively demanding skills (dealing with difference and conflict; voicing ideas and opinions; negotiating and co-operating with others; obtaining information; and listening constructively to others) are those that fewer people felt they had. There is a corresponding higher confidence in the more passive abilities. When asked where they had acquired these abilities, adult education scored well on developing skills in listening constructively, finding information, and having and voicing ideas and opinions. It scored less well on the other perhaps more active abilities.

Lessons for Adult Maths Education

This paper has argued that citizenship needs to be learnt, that it is not only about rights but also about the everyday participation in our society, and that this participation is both a measure and a source of society's success. The challenge to our society is to create ways in which citizens can participate fully and effectively in conditions where all who wish can become actively involved, can understand and participate, can influence, persuade, campaign, and "whistleblow," and be involved in decision-making. The challenge for adult educators is to contribute to this vision (Benn, 1997).

Citizenship has to be learned like any other skill. Participatory democracy is learned through practice and therefore the adult education experience should itself be an experience of participatory democracy. In this way it can be an affective as well as cognitive learning experience that both citizenship and adult education are "for us" and not just "for other people." What does seem clear is that if maths adult educators have a serious commitment to developing an active citizenry, then they might do well to consider their own list of citizenship skills perhaps using the ones given here as a starting point. The curriculum, pedagogy, and approach to the programme could then be constructed with the aim of developing these skills. That is not to say that this should take precedence over the "subject." If adults come to learn maths or gain a particular qualification, then that should continue to be the prime outcome of the course. It is also important to note that in Britain funding is linked primarily to qualifications and, in an assessed course, the prescription of the syllabus almost inevitably brings limitations. Nevertheless, within these constraints many adult educators do still have a freedom denied to other parts of the education sector and within that freedom might lie the potential to contribute to a more democratic society.

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The Role of Mathematical Tools in Addressing Problems in Everyday Life: The Interplay Between Socio-Cultural Factors and the Individual's Feelings and Ways of Thinking

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Abstract

I asked a group of women to discuss activities in which they used mathematics and number in their everyday lives, encouraging them to describe the socio-cultural contexts in which these activities took place. In this paper I investigate one woman's accounts of her use of some of the numerical and mathematical tools which are an intrinsic part of Western culture today. She describes her proficiency with using some tools and difficulty with others. Her experiences engender strong feelings in her, both positive and negative, and her accounts reveal that aspects of her self-identity are bound up with her use of tools and feelings about them. I discuss how the interplay between the socio-cultural factors and her individual mode of thinking shapes the strategies she chooses to address problems that emerge in her everyday life.

Introduction

The problems in people's everyday lives and the way in which they are solved are inseparable from the socio-cultural contexts of the activities out of which they arise. These contexts are both the person's relationships with other people and with the environment in which they live. People interact with both through the use of tools and conventions (Saxe, 1991). The way these relationships are conducted and the tools and conventions are themselves products of the culture to which they belong. Tools and conventions are created by people to enable them to take part in activities which are part of the culture (Lave & Wenger, 1999).

In the Western post-industrial society of today, people have a range of sophisticated tools with which to engage with their environment and each other. Many of these tools are used to solve numerical and mathematical problems: calculators, computers, tills. Others use numbers and mathematics to solve other kinds of problems: telephones, video-recorders, cash machines.

Ruth was a participant in a focus group where a group of women discussed how they use mathematics and numbers in their everyday lives. The group met four times for a total of about seven hours. Eleven women participated in the group at different times: attendance at each session varied from two to ten people. The conversations in the group were tape-recorded and the tapes were transcribed. The transcriptions were analysed using Lincoln and Guber's method of developing grounded theory (1985). For a fuller description of the composition of the group and its organisation, see my previous paper (Colwell, 1998).

Ruth recounted several events in her life that involved the use of tools with a numerical or mathematical function. In this paper, I will show how her self-identity is inextricably linked to her use of tools; how they can provoke strong feelings in her, varying from nightmarish horror to intense enjoyment; and how most of these events happen in the context of social relationships.

Ruth also brings her own prior understandings (Saxe, 1991) to the solution of the problems she meets in her everyday life. Her competence varies dramatically, from professed inability to do mental calculations about money, to proficiency in reading maps and using computers.

The Use of Tools

Learning to Use a Calculator

Ruth described finding herself in a job where she was required to use a calculator to work out VAT (Value Added Tax) on building contracts. She wasn't given any formal training, just told, "So here you are ... here's the bill, here's the paper, sit down and put it on there. Here's a calculator."

But when she told the person who was showing her the job that she didn't know how to use a calculator, they still gave her very little instruction, "They just told me ..., 'Well, you know, you just multiply, here's the multiplication sign button, ... multiply it like this.'"

In spite of this cursory training, Ruth seems to have learnt how to use a calculator effectively. She had to calculate both the VAT and the total including VAT. She entered the sum, put it into Memory, then multiplied by .15 to calculate the 15% VAT (as it was then). Then she added the VAT into Memory and pressed the Memory Recall button to get the total price. This is a fairly sophisticated way of using a calculator.

This experience Ruth had of learning to use a calculator enabled her to go on to learn to use computers, "Well that was good then, because that led me on then to sort of using spreadsheets and stuff, once I'd used a calculator."

Handling Cash at Work

But Ruth had a great deal of trouble when she later got a job at a sports stadium, taking the money. "I just couldn't do it, because it was money, I think, particularly, I couldn't handle it really. ... I can't do mental arithmetic, full stop."

She gave an example of the kind of mental calculation she had to do:

> Well sometimes people would want to book the tennis courts and they would want to book them for so long. And one person had a pass, so therefore they were entitled to a reduction and the other person didn't, so therefore they weren't. Plus they wanted to hire a racket and ball.

Ruth used two common methods of giving change. She put the proffered notes on the till, using them as tools to remind her of what amount she was giving change for. Then she used the coins as tools to calculate the change, "if something came to 18p, take 2p and then 10p out (of the till)." But Ruth found that these methods did not really help her. "But then I would see the note that I put in front of me that they gave me and I'd think, 'Well they gave me ten and I am about to give them three back.' By that time I am in a big muddle, you know."

Finding Her Way

Ruth is very competent at map-reading: she knows exactly where she needs to go. When she was coming to the group for the first time, she looked at the A-Z map before she left home. She told herself which road to follow from the station and where to turn. She was able to use this information successfully to find her way. She found the task "quite easy really."

Feelings and Self-Identity

Learning to Use Calculators and Computers

When Ruth discovered that she was expected to use a calculator in her job, she said she "had to confess" that she didn't know how to use it. In other words she was embarrassed or ashamed, maybe apprehensive that she would find that she couldn't learn how to use it and would therefore not be able to keep her job.

She found that the person showing her how to do the job "thought this was highly amusing, fortunately," that she had never used a calculator. Although it must have been uncomfortable for Ruth to be laughed at, this attitude on the part of her colleague probably contributed to her ability to learn to use the calculator: it was a social situation where she was expected to be able to do it.

Learning to use the calculator made a great impression on Ruth. She thought, "Wow!, you know, this is a revelation to me." This experience gave Ruth the confidence to go on to learn to use computers. She said that using computers and programming video-recorders is so easy anyone could do it, if they didn't think it was difficult.

Handling Cash at Work
When Ruth was working on the till in the sports stadium, she was in a social situation where she was dealing with members of the public and she felt she had to calculate fast. "So I'd be struggling trying to do that (work out the discounts and total the prices), while they give me the money. And I'm a stage behind them and then trying to work out the change." She described the job as "an absolute nightmare." Buxton (1981) recognised these two factors, imposed speed and exposure to other people, as important factors in creating maths anxiety. Ruth described her feelings of panic, "I just know that I have got a bit of a problem with numbers generally, ... with money anyway. ... I feel a bit panicky if I have to think about it really."

Ruth was also in a social situation with her fellow workers. She was shown how to do the job by someone who was also having trouble with managing it and they supported each other. Ruth thought it was "fortunate" to be working with someone who had as much difficulty with the job as she had. She wasn't annoyed that she had not been shown the job properly. She was glad that someone else had the same difficulties she had: someone she could identity with.

Ruth also said that she didn't think that the workers in that job were really expected to be able to do the work effectively, "It was such a badly paid job that, you know, everybody who worked on the till had some kind of problems. ... It was like a Scale 2 council job, which is the lowest you could possibly go." Ruth did not give any indication of gradually improving her skills in this job and feeling more confident. It was as if the management's expectation of the workers, or Ruth's perception of them, and the other workers' inability to do the job effectively made Ruth unable to learn to do the job properly.

Finding Her Way
Ruth really enjoys looking at maps. When she is on holiday with her partner, he drives and she navigates. "I do like going to places I haven't been to before. ... I like looking at the map and then going there and finding my way around." She described herself in relation to this as "sort of anorakish, somehow" and then "sort of swotty, really." These are both mildly derogatory terms: it is as if she feels she has to defend her self-identity as a person who is good at and enjoys reading maps.

Categorising Ways of Thinking
Ruth is very competent with using calculators, computers, and maps and programming video-recorders, probably more competent than the average adult. But she panicked when she had to mentally calculate prices, take money, and give change.

Pask's model of different kinds of thinking distinguishes between serialist and holist thinking (1972). Ruth probably likes maps because they enable her to think about where she is in a holist way. "I like to ... have the map as I am sitting there and know where we are in relation to the whole of Spain or whatever." She put her household accounts on a spreadsheet which enabled her to see them holistically. She finds programming the video easy because she can see all the variables together on the screen. Both the spreadsheet and the video programme obviate the necessity of her doing any calculation, which would require what Pask defines as serialist thinking.

Whereas when Ruth was working on the till at the sports stadium, the thinking required to do the job is serialist: she had to take discounts off prices, total them up, and calculate the change from the proffered money. The way she said she coped with this situation was, "I kind of went into a trance-like state I think, when I was doing it." She may have been able to do the job by taking her focus of attention away from the detail of what she was doing, rather like being able to ride a bicycle without consciously focusing on balancing it (Polanyi, 1983), which is probably similar to thinking in a holist way. However, a calculator cannot be used in a holist way: the numbers have to be keyed in, then read off in a series. So Ruth can manage this kind of thinking: she just seems to feel more confident with tools which can be used in a holist way.

Ruth has constructed her life around avoiding situations where she might have to calculate. When I asked her how she would work out a 50% reduction, she said, "Phew! Just avoid it really, if possible." But she has an enthusiasm and curiosity for mathematical tools which I think is very unusual for someone who has problems with calculation.

Summary

The problems Ruth has to solve in her daily life have a socio-cultural context. They are embedded in her relationships with other people: her partner, and her customers and colleagues at work. They involve tools and conventions like calculators and money. Ruth's self-identity both influences and is formed by the situations she is in and she sometimes experiences strong feelings as a result. But Ruth also brings her individual understanding and modes of thought to the solution of problems. It is the interplay between her thinking and the culture which produces the results she describes: a facility with using maps and computers and an avoidance as far as possible of calculating money.

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MATHEMATICS AND THE PARENTING ROLE

Implications for Women and Children When Mothers Return to Study Mathematics

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Introduction

This paper is part of a larger research project that seeks to integrate two feminist epistemological frameworks: Belenky, Clinchy, Goldberger, and Tarule (1986) and Baxter Magolda (1992). The aim is to provide an epistemological lens through which to understand better the intellectual experiences of women returning to study in the further education sector, with a particular focus on the learning of mathematics. One of the motivating factors in the women's reasons for returning to study mathematics was to support their children academically and this motivation appears to support the women's own intellectual growth. This is an issue that has received little attention to date.

The theoretical background to the study is consistent with current writings in the "women and mathematics" area that call for strategies to counter traditional mathematics pedagogy and epistemology which has alienated many girls and women by not appreciating or validating their ways of coming to know (Becker, 1996, Burton, 1996). This direction is also consistent with current writings on mathematics education more generally that is focused on investigating the socially situated and cultural aspects of learning mathematics (Cobb & Bauersfeld, 1995; Mercer, 1995). Becker (1996) suggested that the frameworks of Belenky et al. (1986) and Baxter Magolda (1992) might prove fruitful in future research on gender and mathematics. A tentative integration of the models has been proposed in the context of the adult mathematics classroom and the reader is referred to Brew (1999; 2000) to provide background reading.

Baxter Magolda (1992) described the shifting role of significant others (peers, teachers) and the changing nature of students' perceptions of knowledge as shifts in their epistemological perspectives occurred. For example, Absolute knowers, those who hold the perspective that knowledge is certain and that it is derived from external authorities (e.g., the teacher), do not tend to view peers as holding legitimate knowledge. Rather, the role of peers is mainly social and to share what they have learned from the teacher. Transitional-Independent knowers, described as shifting their perspective to viewing knowledge as uncertain, are more open to hearing peers' views—alternative perspectives—and expect the teacher to facilitate peer interaction. In this paper the role of children is tentatively proposed as a further significant other for women with children returning to study, extending the framework of Baxter Magolda (1992).

In the study by Baxter Magolda (1992) on the pathways of intellectual development of college students, children were not discussed and Belenky et al. (1986) only did so briefly. In a more recent study, Belenky (1996) noted that programs set up by midwife teachers to provide self help networks had a positive impact on the relationship between the women and their children. These positive impacts were traced to changes in the women's ways of knowing.

> These women influenced their children's behaviour by engaging them in reflective dialogue, drawing out their problem solving abilities. By contrast, the women who did not see themselves as thinkers seemed much less aware of their children's thinking processes. They relied almost exclusively on authoritarian, power-oriented child rearing techniques....These programs might well lead to more democratic families and the ripple effect will be felt down through the generations. (Belenky, 1996, pp. 396-397)

The lower school achievements of the children of working class parents are generally described within the context of "social reproduction" theories (Burns & Scott, 1997). One perspective describes a deficit model in which parents are proposed to value education less highly and to have lower aspirations for their children. Another perspective put forward is that working class parents lack knowledge and confidence to be involved in their children's education due to limited or poor school experiences (Marjoribanks, 1995). Social reproduction theory does tend to assume that a family's class position is generally fixed by early adulthood based on occupation/education and the associated values acquired by the parents-to-be (Burns & Scott, 1997). These researchers ask, "... but what happens when social class becomes more fluid, and parents markedly raise their own educational status after their children are born? Do the children inherit their old level of cultural capital or the new level?" (Burns & Scott, 1997, p. 210).

According to Crane (1996) the relationship between home environment and mathematical skills is not well documented though Reynolds and Walberg (1992) found for grade 8 students it had the largest indirect effect. In the study by Crane (1996) the effect of the home environment was also large, particularly when the children were younger, though for older children it had significant effects as well. Crane (1996) suggested that further research was required to find ways to influence the home environment to enhance young people's mathematical competence, as the outcomes could be potentially large.

Mature age women who complete tertiary courses commonly report gaining new confidence, greater understanding, wider interests, and better conversation skills (Burns, Scott, & Cooney, 1993; Kelly, 1987). Burns and Scott (1997) also presented evidence that for women in the tertiary education context there is a substantive flow-on effect to children, particularly for older children. Apart from specific tutoring skills, most respondents emphasised more global factors such as their influence as a role model, their increased ability to understand the child's thinking, and the more intellectual climate in the household. This paper provides evidence of similar outcomes in the academic step prior to tertiary study—the further education sector—with a focus on the learning of mathematics.

Participants and Data Collection

Data were collected from 11 women enrolled in one of two adult mathematics classes at a Community Learning Centre, Melbourne, Australia. One class covered basic mathematics as a component of a full time Information Technology course; the other class was equivalent to secondary school grade 11 mathematics. Eleven women ranging in age from 33 to 50 years old participated and nine were early school leavers. These women had one to four children at home ranging in age from 2 to 17 years. After three weeks of class observations participants were interviewed about their perceptions of mathematics, the role of the learner, teacher, peers, and assessment in their learning, as well as their reasons for returning to study, their previous school experiences, particularly in mathematics, and the influence of parents on their schooling. Regular observations of classes continued to the end of the courses and final interviews were by telephone. The role of children emerged spontaneously as a strong theme in interview 1 despite not being an initial focus and this was followed up more systematically in interview 2. Interview 2 focused on any perceived shifts in the students' epistemological perspectives on the role of the teacher and peers in their mathematical learning and also on the role of children in their studies.

Findings

Apart from personal and financial motivations, seven of the 11 women stated that one reason for returning to study was to better support their children's academic studies. This was usually stated in terms of wanting to help them with their homework but more global issues also emerged such as wanting to be a good role model and wanting to break the cycle of learning difficulties that they themselves had suffered.

In either the first or second interview, six women reported that their goal to better support their children with their school work since undertaking their mathematics course did eventuate. Four of these women also said they had obtained similar assistance from their older children. The women's discussion of their interactions with their children suggested that the quality of their conversations was changing. From here I focus on two case studies, Linda and Samantha, to illustrate the nature of these changing interactions.

Case Study 1

Linda was in her early 40s, married with three children ranging in age from 6 to 14 years old, and was enrolled in the Informational Technology course. Linda described a difficult secondary school experience in a poor and rough working class suburb. In terms of the notion of social reproduction theory, her parents seemed to have had both low educational aspirations for their daughter and lacked the knowledge and confidence to be involved in her education. Linda's aim in returning to study was motivated, in part, to break this cycle.

> I: How did your parents influence you in your studies?
> *They didn't sort of worry, ... like school was supposed to take care of that. It wasn't to be brought home. Occasionally I had trouble and my homework wasn't done and Dad would sit down and try and help do it. But for Mum, well, wouldn't be able to. You were a girl and you were going to finish in fourth form (grade 11) anyway and go on and do office studies.*
> I: Is that what they used to talk to you about?
> *Yes. Dad would say, "well, you are not doing any good so, as soon as fourth form comes up you are out of there." Not to say sort of, "why aren't you doing well in maths."*
> *The business went broke so obviously I had to get out there and just move myself. A bit of reality sets in and the old pay packet gets a bit thinner (laugh)... and I just needed my brains to get working. ... the kids were coming home and talking and some days I was just switched off. And I thought this is hopeless, ... I am just stagnating. ... With my son he is 14 now and I have noticed his work is becoming a bit daunting, and I felt the need to sort of help him, whereas, my parents didn't do that.*

In the second interview Linda's comments resonate with the notion of the development of a genuine voice, and a moving away from *Silence*, epitomised by a sense of feeling dumb and stupid (Belenky et al., 1986). With this shift in perspective came greater honesty in Linda's relationship with her son. What is critical in the context of this paper is how her greater confidence with mathematics was integral to this process, though returning to study, in general, undoubtedly contributed. Linda also reported that her two daughters (one a pre-schooler and the other junior primary level), were also influenced in a positive way by her return to study as they enjoyed simulating her homework activities.

> *I feel more self-assured and speak out a bit more than I used to. Whereas I tended to hang back and think I better not say that because that might be stupid. ... Speaking to my son I speak out more about schooling than I did before, in the way that he should think about school.*
> I: In the first interview you said his work was becoming daunting. Are you able to help him now?
> *Yes. I am now because he is doing similar things to what we did last year. Working out the areas, perimeters. Actually we were only doing that yesterday, and I thought "Wow, I did this! I can do this with you." ... he is actually asking me questions (laugh).*
> I: So that wouldn't have happened before?
> *I probably would have looked at it and gone "Ask your father." Whereas now, ... I haven't had to say "wait till your father comes home" or "don't show me that."*
> I: Did you say that, don't show me that?
> *Oh yeah. ... A couple of times I have said "I haven't got time for that" only because (voice dropped) I didn't really know what he was talking about. ... But now I can read it, say "all right, lets have a look," and work it out.*

Evident in Linda's comments below is an apparent belief that culturally she should be thinking of her own personal academic goals as being separate from those of her children's. Yet for Linda her personal and academic goals were intimately connected to those of her children in her decision to return to study.

> I: You said in the first interview that the kids were coming home and talking and you were just switched off, you thought this is hopeless. It sounds like the kids kind of triggered you in some ways?
> *Yeah you do things for your family ... well I did it for myself I suppose ideally, but in the back of your mind ... like I had this mundane job, ... I had to get out there ... improve myself. FOR the kids.*

Having completed the course Linda applied successfully for a part time office worker position.

The family was saying oh they won't pick you, you are too old ... the first interview I came back and I said "I got that job!" (laugh). They just looked at me. I mean my husband was very supportive, but the rest of my family they were just astounded. ... they said, "Mum, I am so surprised, I really didn't think you would get anything." They were quite open about it. And it is so good for the children to see ... hey if you go back to school ... you put the time in, you do get something better.

Case Study 2

Samantha was in her early 30s and was married with three children ranging in age from 8 to 15 years old. Samantha was enrolled in the grade 11 mathematics and wanted to pursue a teaching career. Samantha attended a remote rural school and left in grade 9. She described an uncomfortable relationship with mathematics at school and is aware of, and concerned about, similar patterns of thinking emerging with her children with respect to low academic confidence. A somewhat Absolute orientation towards mathematical knowledge is also evident.

Maths is a lot of things that don't make sense, that somewhere in the end they do. ... You have to abide by the rules to get that solution, you can't go off and make up your own rules ... I remember sitting in the maths room and quaking in fear of all this big maths that we were going to learn, in the very early stages of year 7. I just remember walking to class from assembly and thinking, "Oh, I don't know if I can do this." That's funny, because I hear by kids saying it now and I say "yes you can, yes you can."

Samantha described how her motivation to return to study was intimately related to her children's education, particularly mathematics.

I realised they are requiring different things of me. They are not requiring to have their nappy changed, mashed food on the table, they are requiring a more intelligent person to be around them. I decided I had the life experience, probably I needed to brush up on the academic aspect. That has encouraged me more than anything and I think maths is definitely an area where I need to be knowing what they are doing.

At the time of the first interview Samantha had already experienced a shift in the types of conversations she was having with her oldest son. These interactions are somewhat consistent with the role of peers for the Absolute-receiver orientation, knowledge obtained from the teacher can be shared. The apparent focus on genuine engagement with understanding, and the verbalising of uncertain knowledge, also suggests the emergence of a Transitional inter-personal orientation.

I have interesting conversations with my oldest son now, where I can't understand something he'll say "have you seen this or have you done that?"... And yet there will be times when I say "can you help me with this?" ... And he will say "what is that?" And I can teach him where I am up to but I can't go the next step. We've shared. That is important to me. I want the kids to know they are not on their own.

As I became more aware from the initial interviews about the role of children in the women's study life I asked the participants at the next observational visit to tell me whenever they had conversations about mathematics with their children. The day I requested that input Samantha revealed that her 14 year old daughter (grade 8) had come home that week and announced that she had excelled in an Algebra test. Samantha said her daughter normally failed such tests. When this outcome was followed up in interview 2 her daughter had clearly moved ahead in her mathematical studies, yet Samantha did not believe that her daughter would consider that she had been influential.

> *I think it would be a refusal comment to say that mother helped (laugh). Mothers don't get much credit when they are this age. But ... she has become more open to realising that maths can be fun even if it is tough. ... to the point where she has gone from not understanding the concept of maths at all, to doing a lot of maths that was at my level. Her maths teacher said, "I want her in my advanced maths class next year." ... But I don't know if the credit would directly come back to me. I think the environment of seeing maths as being a regular thing in my house, my books over the table, my determination, the late nights, and the successes in the end have inadvertently led to an openness for the subject.*

In interview 2 Samantha described a shift with respect to the role of the teacher and this was linked to her peers' mathematical knowledge gaining greater legitimacy. Her reflections suggest an epistemological shift towards a Transitional inter-personal orientation.

> *The teacher was definitely the primary source of information, ... but when it came to making the maths real it was from talking.*
> I: Can you think of any examples in the class when you discussed maths together?
> *Yes I can. We were all trying to grasp something and we couldn't.... It was algebraic, ... like and unlike terms. It came down to Denise explaining it in terms of apples and oranges (laugh). Everyone all of a sudden understood it ... The teacher agreed she couldn't have explained it any better. ... it kept coming back ... that Denise was a very good teacher ... it was amazing, Wow! I know the teacher had been able to break through on a number of things ... but this was US working it out, US describing it to each other.*

Samantha also described a very satisfying application of her new mathematical knowledge in her everyday life. Evidence of perhaps a further shift into an Independent knowing perspective emerged, epitomised mathematically here by valuing alternative ways of solving a problem, while at the same time valuing her own method equally. What I think is also important to note is how mathematics was used by Samantha to demonstrate a new sense of her own intellectual space.

> *My husband was asked to build tables to accommodate some computers. Now it took a lot of maths to work this out ... you had a big pole in the middle and around it you had to build six units to come together neatly.... So the whole thing had to be done by way of angles. My husband ... credits himself with working it out, but he worked it out by his logic as a builder. And then I was able to work it out through using angles and the laws of circles to make sure what he had worked out was correct. ... It was really frustrating to start with, like what did I learn? But it all worked out, ... we were so accurate that the people who cut the timber could not get it right. ... That was a good example of putting something I had learnt into practice in a very practical way and seeing the end result was perfect.*

Discussion

What I hope to have conveyed is that for women returning to study mathematics, having older children at home can provide a fertile environment to encourage them to verbalise their mathematical knowledge and understanding. In the classroom, encouraging women to verbalise their mathematical thinking can be quite challenging for adult practitioners and this may provide a fruitful avenue to explore.

Also evident from the extracts provided is that dramatic changes in children's attitudes towards and achievement in mathematics can occur. These findings support the assertions of Crane (1996) and Reynolds and Walberg (1992) that when the home academic environment is improved the benefits for children's mathematical achievements are potentially quite large.

Finally, the experiences of the women suggest that their intellectual growth is intimately connected to being able to better support their own children's development. For the two case studies discussed, these women did create a new sense of independence and their own personal intellectual space. What is significant is the extent to which their new mathematical knowledge and their motivation to support their children played a crucial role in creating this for themselves.

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Parents as Learners and Teachers of Mathematics: Toward a Two-Way Dialogue

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In a regular classroom we would not have learned it like this. Here we learn thanks to the comradeship and "confianza" [trust].

In this paper I pursue the work I presented at ALM-5 and ALM-6 (Civil, 1998, 1999) on parents as adult learners of mathematics. My emphasis has been on establishing a two-way dialogue through which, while I engage the mothers in explorations about mathematics, we also collaboratively reflect on uses of mathematics in our everyday life and possible ways to establish connections between different forms of mathematics. I want to stress this collaboration aspect: we meet as a group to discuss mathematics. Granted, I come to the group as the "mathematics educator" but I am not there to "teach" but more to converse with them about mathematics. We are all learners and teachers in the group. Just recently, I have been reading about dialogic learning, in particular through the work of Flecha (2000). This work is grounded on the experiences of a group of adults in a working class neighborhood who participate in a literary circle that bears many similarities to "El Club de Literatura" in which we carry out our work in mathematics. I was struck by the similarity in orientation and will highlight some of the key ideas in Flecha's work as part of the theoretical framework.

In this paper I focus on the process of establishing a dialogue about mathematics by looking at our experiences learning geometry. The geometry theme started through typical reform-based school mathematics tasks that allowed us to explore area and perimeter and their relationship (e.g., fixed perimeter, varying area). We then moved to a geometric exploration around the craft of "papel picado" (punched paper). The mothers taught me how to do "papel picado" and as we conversed through the practice, we brought up different mathematical questions to explore. The activity of "papel picado" provided a setting in which we all had something to learn and something to teach.

Context

Our work involves a small group of women who have been participating regularly in what we call "talleres matemáticos" (Mathematical workshops). The group consists of three mothers who have children in the middle school where we meet, one young woman who is a relative of one of the mothers, the school librarian, my colleague in the project, and myself. Other mothers have also joined us at different times, but I will focus on the group of "regulars." These women are part of an ongoing project—"El Club de Literatura"—that started in 1995 and that centers around the reading and discussing of literature (some examples of authors read include Marcela Serrano, Angeles Mastretta, García Lorca, etc.).

When the opportunity came up to expand the group activities to incorporate mathematics, these women were eager to do that. A question that came to my mind right away was: will we be able to engage in mathematical discussions that parallel the kinds of discussions and conversations that characterize the literature circle? This is a key question in our work since many of the activities we work on could probably be characterized as "mathematics for the sake of mathematics," or "mathematicians' mathematics" (such as the area/perimeter exploration I will discuss later in this paper), and thus, they may be less "personal" and harder to relate to (in comparison to the activities in the literature circle).

Theoretical Framework

Our work relies on three main bodies of literature: a) research on parental involvement that critically examines issues of power and perceptions of parents (especially minority and working class parents) (Henry, 1996; Vincent, 1996); b) research on adult education, especially that grounded on critical pedagogy (Benn, 1997; Frankenstein & Powell, 1994; Harris, 1991; Knijnik, 1996); c) research based on a socio-cultural approach to education (Forman, 1996; Moll, 1992; Rogoff, 1994; van Oers, 1996). (See Civil, 1999; Civil, Andrade, &

Anhalt, 2000 for some discussion of this research basis.) More recently, the work of Flecha (2000) on dialogic learning has become very helpful in my thinking about our work.

> [dialogic learning] leads to the transformation of education centers into learning communities where all the people and groups involved enter into relationships with each other. In this way, the environment is transformed, creating new cognitive development and greater social and educational equality. (p. 24)

In particular, here is what he writes about one of the seven principles of dialogic learning—egalitarian dialogue:

> A dialogue is egalitarian when it takes different contributions into consideration according to the validity of their reasoning, instead of according to the positions of power held by those who make the contributions. (p. 2)

Flecha points out three kinds of barriers to dialogue: a) cultural ("most members of the population are dismissed as incapable of communicating with each other using dominant knowledge" [p. 9]); b) social ("many groups are excluded from the production and evaluation of valuable knowledge ... Experiences that do not fit the mold are excluded" [p. 9]); c) personal ("many people's personal histories [and how they report them] lead to self-exclusion from many formative practices" [p. 9]).

These barriers are particularly relevant for us as the group of women we work with belongs to a minority group that has encountered many of these cultural, social, and personal barriers as women, Mexican immigrants, and working-class individuals. Furthermore, their personal histories as they relate to their learning of mathematics share many commonalties with so many other people who have been excluded from the "in-group" as mathematical learners through years of schooling that led them to believe that mathematics was not for them.

Of particular interest to our work in mathematics education is Flecha's view that dialogical learning encompasses but goes beyond constructivism:

> [in dialogical learning] the meaning formation process does not depend solely on the intervention of education professionals, but also on all the people and contexts related to the student's learning. (p. 23)

As we hear the women in the group sharing their learning with each other and with their family members, we see the role that other people and contexts play in their learning of mathematics.

The Mathematics Workshops

During the academic year 1999-2000 we met every two to three weeks for about two hours per workshop (the group kept on meeting once to twice a week with my colleague as part of the literature circle). In September we started by working on a combinatorics problem that the teachers in the research project BRIDGE[1] were also going to try out in their classrooms (the work with the mothers is one of the components of project BRIDGE). The problem led to an investigation of patterns and of Pascal's triangle. From there, we continued with a few more sessions on counting problems and pattern problems. The group expressed an interest in learning algebra. Below is an excerpt from my journal as I reflected on this request:

[1] Project BRIDGE (Linking home and school: A bridge to the many faces of mathematics) is supported under the Educational Research and Development Centers Program, PR/Award Number R306A60001, as administered by the OERI (U.S. Department of Education). The views expressed here are those of the author and do not necessarily reflect the views of OERI.

> The mothers had been asking to learn about algebra. Their children bring algebra homework and they don't know how to help them. Algebra... what I can I do that would be both meaningful and truthful to our approach, yet would allow them to help their children?
> They want to learn so much and I feel we have so little time in a sense. As I introduced the notation with y's and x's (or n's), some of them brought up what their children are learning, they wanted to understand how to manipulate equations.
> Here is my dilemma, do I start going over the often meaningless symbol manipulation that characterizes some of the algebra teaching that takes place in our schools? Or do I keep them working on the problems the way we do, discussing approaches, constructing meanings together? Eventually, we'll reach the symbol manipulation, but hopefully in a meaningful way, but what about their immediate needs of wanting to help their children?

This excerpt captures one of my dilemmas as I work with the mothers. Do I put the emphasis on them as *adult learners,* or on them as *mothers* (Civil, 1999)? When we mention our work to others, quite often their reaction to having mathematics workshops for parents is "oh, so that they can help their children." While this is certainly one goal, and may be what brings parents in initially, it is not our only goal. We are constantly reflecting not only on what we discuss in these workshops (issue of content) but also on why we are discussing it. We are aware that these women's knowledge of mathematics is likely not to be recognized, considered marginal and of less value (Fasheh, 1991; Frankenstein & Powell, 1994; Harris, 1997). As Benn (1997) points out, we look for ways to connect learners' everyday discourses with the unfamiliar discourse of academic mathematics. The mothers have made it very clear to us that they want to learn school mathematics, that is, the mathematics that their children are learning in school. As one mother writes:

> I am so happy with all these mathematics workshops because I realize how to help my children understand mathematics in a different way, from a fun approach, all together as a family. ... And also for us, because one never knows when we may need /use it, and this way we move forward, and no one is going to mandate that is has to be the way they say, because we also think and solve problems.

Our Work in Geometry

Our work in geometry started with a series of activities on area and perimeter adapted from a middle school mathematics curriculum (Shroyer & Fitzgerald, 1986). Through the exploration of rectangles with constant perimeter and fixed area, as well as rectangles with constant area and varying perimeter, we continued our work in algebra through graphing, function writing, and some equation solving. The group became fascinated by the connections between the different topics and by how we went from hands-on explorations with tiles to getting a glimpse of parabolas and hyperbolas. Thus far, this was what I would describe as mathematics for the sake of mathematics. I personally enjoy these explorations and I enjoy even more seeing how others make sense out of the concepts. For example, I am interested in how they start conceptualizing what shape will have the largest or smallest area for a given perimeter and why. I am also interested in how they make sense of the graphical representation in particular in cases that I thought were rather "abstract" (for example, in looking at rectangles of area 12 tiles, we graphed one side as a function of the other; that is, since $L*W = 12$, we have $L = 12/W$; that graph gives us a branch of a hyperbola).

Throughout their work on this topic, I was clearly leading them. Because by then "confianza" [trust] had been established, these women were comfortable asking questions, challenging my and each other's explanations. "Confianza" is an important concept in this work and these women wrote about it in the newsletter they put together in the Spring 2000:

> When I integrated into the group de las Señoras, for me the most important foundation was the confianza that each one offered me.... I can say that all that I now know and have learned has been accomplished by means of the confianza (Newsletter, Spring 2000, pp. 2-3).

> At last, I also have someone that more than a teacher is a friend and most importantly inspires me: Confianza [confidence, trust] the confianza that I in particular never had with any other teacher of mathematics. ...Thanks to the confianza that exists in the group we can work without problems and pose any sort of question without fear (Newsletter, Spring 2000, pp. 2-3).

Thus, even though I was leading many of the activities, I feel comfortable that had this been not of interest to them, they would have let me know. The "curriculum" is negotiated constantly.

The next topic in our exploration of geometry had them leading the activities as they taught me how to do "papel picado." This is a traditional Mexican craft that is quite typical in our local context. Using colorful paper one creates intricate designs by cutting out shapes in a careful and pre-planned fashion. This practice involves lots of opportunities for the exploration of symmetry, and can be pushed in other directions such as finding areas and perimeters of the shapes being cut out. The task of learning to do "papel picado" was challenging for me (planning and visualizing what may come out is hard for me). As we sat around the table folding and cutting, the conversation would take different directions, but a main thread was to try to identify the mathematics in this practice. As a non-expert in "papel picado" and as someone who considers herself as rather limited visually, I was having a hard time connecting my years of academic mathematics to the practice of papel picado.

Since we had just been working on area and perimeter, one of the women in the group suggested cutting shapes for which it would be easy to find their area and then we could figure out how much of the sheet of paper had been cut out. What captured my interest in this activity is the fact that their much better understanding (than mine) of the whole process of "papel picado" made them the leaders of the activity. As I watch the videotape of that session, they sound very confident, they were in charge of the exploration and of teaching me how to make "papel picado." They right away realized that to make 2 by 2 squares, they would have to make a 2 by 1 cut (because of the fold; when opening it, the result would be 2 by 2 squares). They also came up with a strategy to find the area that had been cut, by using the fact that many of the shapes were congruent and thus one only needed to find a few areas: "we'll compute the area and multiply it by the number of shapes." They suggested using grid paper to measure (we did not have enough rulers that day!). Each square on the grid was 1/4 in on its side. Each square in the "papel picado" took 4 squares on the grid (2 by 2); there were 40 squares total, so in terms of grid paper, 160 squares on the grid. How many square inches is this? This question poses a challenge, as the tendency is to divide by 4 (because of the 4 squares on the inch). This was in fact their initial reaction. We analyzed the situation and they realized that it took 16 squares on the grid to cover 1 square inch.

Our "papel picado" had circles and squares. This led to an exploration on how to find the area of a circle. The younger woman in the group kept referring to "Pi" and for example for the area of the circle (they were of diameter 2 squares on the grid), she said "6.28 (pause), ah no, that's perimeter." The other women in the group wanted to understand, they didn't want to hear about all this "three point and whatever...." They did not want to use formulas that they did not know where they came from. In our next session we worked on "discovering Pi." Then, this whole notion of Pi took a life of its own for this group of women. When I arrived at the next workshop, they had started an exploration on "applications of π to our daily lives." They had collected data among their family members on the measurement around the head and their height and were working on finding a relationship among these measurements. They graphed the data, discussed it, looked for how it may relate to π. I contributed to the exploration (with my measurements; by bringing in graphing calculators and introducing the notion of regression), but they were leading the exploration. They explained to me (as they had been working on it outside the meetings of the "taller") that they wanted to know if the height was equal to "π times the circumference of the head." As I was trying to visualize what they were saying, the following dialogue took place:

> E: So, Marta, what I do, to make it real, is our height is the circumference.
> M: Ah, so what I'm calling circumference becomes
> E and others: the diameter

> M: the height, our height
> Several voices: becomes a circle
> E: the circumference of the head is the diameter [of that circle that represents the height]

What the dialogue does not capture is the ease and the confidence with which they explained to me how they were visualizing the situation. Developing this confidence in themselves as learners of mathematics may help address my dilemma of working with them as adult learners and as mothers, as the following excerpt from one of the mother's 15 year-old son shows:

> Now that she [his mother] is attending these talleres she is learning in a different way, understanding the why of the formulas and where they come from and how they can be applied in her life; she shares it with the entire family and we all get involved in a mathematical reunion that is fun. We are all teachers and students at the same time, there is no difference and that there be much respect and confianza is most important. (Spring 2000 newsletter)

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Putting Math Into Family Life: What's Possible for Working Parents[1]?

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Abstract

We report on parents' use of a set of parent-child math activities designed to help busy, working parents do math with their children as part of everyday situations such as cleaning up and making dinner. In our initial study with a small and diverse sample, parents' use of the activities did not appear related to their education, mathematical comfort and expertise, or occupation. Instead, parents drew upon their knowledge of their children, family interactions, and the situations they regularly face at home.

Eleven employees are gathered around a table in the lunch room at their workplace. Their occupations range from receptionist to project leader. Some have a high school education, others have finished college, and one has a doctorate. Some work closely together, others are barely acquainted. Their ages range from mid-20s to early 50s. As they open their lunch bags, they chat about what brings them together: children and math.

After a few minutes, Lina, a member of the company's human resources department, welcomes the group to this workshop on putting math into everyday family life. She gives everyone a booklet of math activities for families, and she explains that they'll start by doing one of them. She describes the activity: First, someone chooses a "Number of the Day," then everyone comes up with ways to make that number. If Number of the Day is 11, one solution is $8 + 2 + 1$.

Lina invites the group to come up with other solutions. Before long, everyone is engaged. Some work alone, others talk to a partner. Some challenge themselves to find unusual solutions; others try to solve the problem as they think their children would.

After a few minutes, Lina asks for volunteers to share their ways to make 11. These include $13 - 2$, $99 \div 9$, $22 \times 1/2$, and $3 + 4 + 5 + 6 - 7$. She records their ways on chart paper.

Next, Lina raises the question of *when* working parents can find time to do this activity with their children:

In my house, it can be a challenge to find time to do math with the kids. I pick them up on my way home from work, and when we get home, we're all tired, and I've got to make dinner and do the laundry. Sometimes, we do Number of the Day while we're doing evening chores. Are there some times you can think of when you might be able to do Number of the Day—maybe even tonight?

Everyone jumps in with ideas:

In the car, driving home. That's the only time we get to talk.

When I'm braiding my daughter's hair. It takes an hour, and she always gets bored.

Bed time. I tell him a story. We could do a little math, too.

When we're cleaning the house.

At the laundromat.

[1] We use parents to refer to those primarily responsible for children.

In the kitchen, when I'm making dinner. This could keep the kids busy so they don't get in my way.

At bath time. When she's in the tub, and I'm just sitting there to keep her company.

At this point, the group has been together for half an hour, and some employees need to return to work soon. Lina uses the remaining time to give an overview of the other math activities in the booklet. She concludes by encouraging everyone to set a lunch date with someone else at the workshop, to talk about what happened when they used the math activities with their children.

Bringing Math to Working Parents

In the U.S. today, there are widespread calls for parents to become more involved in supporting their children's math learning at home (e.g., NCTM, 2000). For some parents, involvement is a challenge. While many parents have ideas on how to help their children with literacy learning, they're not sure what to do when it comes to math (Hartog, Diamantis, & Brosnan, 1998). The often conflicting realities and demands of parents' lives pose another difficulty. Many working parents have little time for learning how to support their children's math learning, and little spare "family time" in which to do math with their children. U.S. parents face increasingly demanding work schedules, requiring many hours away from their children and homes (Hewlett & West, 1998). Mothers are spending an average of 85 hours per week on the combination of paid work and household work (Hochschild, 1989). A full 20% of parents work two jobs, and the average employee works 163 more hours per year now than twenty years ago (Schor, 1992).

If working parents are to do more math with their children, they need approaches that mesh with their time and schedules. We are investigating one possible approach: reaching parents at their workplaces, where they are already spending a great deal of time, and providing them with materials that show them how to put math into things they're already doing with their children, such as household chores, local transportation, and shopping.

In 1999, we began this work with a three-year grant from the U.S. National Science Foundation. In the first year, we developed a set of math activities for families with elementary grades children (aged approximately 5 to 11). We also developed a short workplace-based orientation to those activities, designed to be conducted by a working parent with no special expertise in math. Beginning in 2001, the materials will be disseminated by Ceridian Performance Partners, a company that supports businesses across the U.S. by providing a broad range of employee benefits, including educational materials. The individual businesses pay for these benefits, so they are free to employees.

Throughout the project, we will be researching how working parents use the math activities with their children. In this paper, we report on the first phase of this research and present preliminary findings based on a small, diverse sample of parents.

Subjects and Method

In the first year, our research was conducted as part of activity development. On each of several rounds of testing and revision, we distributed activity drafts to small groups of parents at their workplaces. We asked them to use as many of the activities as they wanted, as often as they liked. Two to four weeks later, we gathered their feedback through questionnaires and, in one workplace, through 30- to 45-minute individual interviews.

The seven parents interviewed include a broad range of occupations (from clerical assistant to senior research leader), educational levels (from high school to doctorate), ethnicity, and family structures (including families with one, two, and three children, a single parent, a single custodial grandparent, and two-parent families). Five of the parents are female and two are male. The children attend suburban and inner-city public school, parochial school, and private school. Most use math curricula that emphasized rote learning of facts and algorithms; a couple use programs that include invented algorithms and explaining thinking. (Although the parents who completed questionnaires reported experiences similar to those interviewed, the interview data is much richer, so we base our discussion on that data.)

The math activities (listed in Figure 1) were designed to give parents extensive support in doing math as part of regular family life. Each activity consists of a 2-page sheet, and includes basic steps, variations, and information on working with children, such as ideas for helping them if they get frustrated with the math and adapting activities for use with more than one child. Activity sheets also include information on children's mathematical thinking, such as ideas for encouraging children to talk about how they solved problems, to check their work by solving problems in more than one way, and to use mental math.

Figure 1: Math Activities

How Much Is on the Floor? Parents use math to make chores more interesting: children estimate how many things are scattered on the floor and then count the things as they put them away.

What's Fair? Parents involve children in the math of dividing up food fairly, so everyone gets the same amount.

Taking Turns Parents put math into turn-taking by giving children turns that are 5, 10, 12, or some other number of minutes long. Children figure out when their next turn begins.

When Should We Leave? Parents involve their children in the math of planning when to leave so they arrive places on time.

How Much Longer? Parents involve their children in calculations with time, so that they can figure out how long until an exciting event begins.

How Much Do We Save? As parents make shopping lists and plan food budgets, they get children involved in the math of grocery store coupon savings, and food prices.

Wish List Parents use advertisements and mail-order catalogs as a basis for math: children make a "wish list" of items they could buy for a pretend spending limit.

Which Holds More? Parents involve children in estimating volume. Children look at several household containers to find the one that holds the most water.

Junk Mail Families investigate paper use and paper waste, as they gather data on how much junk mail comes to the house in a week.

Number of the Day Parents engage children in a computation game that they can do just about anywhere.

Results

Although parents came from a very wide range of backgrounds, the ways in which they used and adapted the activities did not appear to relate to their education, mathematical comfort and expertise, or job. Instead, parents drew upon their knowledge of their children, family interactions, and the situations they regularly face at home.

Parents used (and repeated) the activities that fit best with their family life. For instance, their use of "How Much Is on the Floor?" related to how they approached cleaning. In some families, children didn't like cleaning their bedrooms, but the estimating and counting in the activity made it tolerable, so parents used it repeatedly. Some families were very neat and left nothing on the floor; parents adapted the activity so children were estimating and counting things as they took them out of the clothes dryer or dishwasher. In one family, the parent and child often argued about cleaning. Adding on another task—estimating and counting—only deepened the conflict, and the parent dropped the activity.

Parents' use of the written materials seemed related to family interaction styles. Some parents wove the activities into family life, and never told their children they were doing a math activity. These parents felt that their children would not be receptive to something they knew was math. Other parents took a more direct approach. They not only told children they were doing math activities, they read their children the activity sheets.

Parents also differed in how they prepared for working with their children. Some carefully read the activities in advance and planned out just how they would introduce them; others only skimmed the activities beforehand. Some parents explained that their use of the written materials was characteristic of their general approach to new things.

Parents valued the concrete information on working with children, even though they didn't always use it. Parents agreed that the clearly defined and detailed activity steps were essential. Most mentioned that they appreciated the ways to work with children (such as ways to help children feel successful when working on a difficult problem), although they often relied on their own strategies for this. Some parents commented that this information set up realistic expectations, giving a sense that the activities can be done in real families, where everyday goals are a high priority.

Parents occasionally added more "real-life" math skills to the activities. Several parents used the time-related activities to teach their children to use analog clocks. Digital clocks are suggested in these activities, but these parents believed that their children should be fluent with analog clocks and saw these activities as an opportunity for practice. None of the activities require calculators, but a few parents introduced them with the money-related activities because they themselves use calculators when working with money.

Parents thought their children learned useful skills. When asked what value they saw in the activities, most parents brought up addition, estimation, counting, and other number and computation skills. Many also mentioned applications of math, such as calculating the price of catalog orders, planning when to leave to get somewhere on time, developing a better sense of time, and finding out how much things cost. They also discussed non-mathematical benefits, such as helping children to keep their bedrooms neater or to get ready on time. Most did not mention children's growth in conceptual understanding or ability to explain their thinking. In fact, they typically reported that they did not take the time to follow the suggestions for supporting children's thinking, such as asking children to explain how they arrived at solutions.

Discussion

Our workplace project is one of a spectrum of approaches to parent-child math learning in the U.S. These range from intensive classes for parents (e.g., Morse & Wagner, 1998) or parents and children (such as Family Math, see Stenmark, Thompson, & Cossey, 1986) to public awareness campaigns, such as Figure This! (www.figurethis.org). Many of these approaches seek to engage parents at the outset in learning about children's mathematical thinking or re-learning math themselves in order to help their children.

By contrast, we start by giving parents step-by-step math activities grounded in the realities of everyday family life. We also provide parents with information on supporting children's mathematical thinking, to use if and when they are ready. So far, our approach seems promising: The seven parents were readily able to integrate the activities into the things they were already doing at home with their children. In using and adapting the activities, they drew upon their deep knowledge of their children, family interactions, and familiar situations, and they relied upon their strategies for handling a variety of family situations. Parents tended to use (and repeat) the math activities that helped them accomplish an everyday goal, such as cleaning up.

Parents felt that the activities helped children learn and practice with valuable math skills; they reported focusing more on children's answers than on the thinking behind those answers. In the coming two years, as we work with a larger sample of parents, we will investigate the extent to which this changes over time: Once parents are experienced integrating math into family life, do they focus more on discussing how children arrived at their answers? Do parents see such discussions as both valuable and feasible? Do the priorities inherent in the everyday situations on which the activities are based—cleaning up, getting the family ready to leave on time, and helping children to take turns—preclude time to discuss the math? To what extent does support from peers, such as co-workers using the same math activities with their children, contribute to the frequency and nature of parent-child math interactions? Do background factors such as parents' education and occupation, and the math curriculum children use at school, make a difference?

All of these questions bear on identifying just what engages and sustains parent involvement in children's math learning. To date, there has been very little U.S. research in this fundamental area, despite the proliferation of parent involvement projects in math. As these projects evolve, it will be important to investigate and share findings about impact over time, in order to extend our understanding of the limits and possibilities of different approaches for different populations–including parents who are able to attend classes or events on mathematics or children's mathematical thinking, as well as those who prefer to start out by putting math into the things that they already do with their children.

Acknowledgments

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Ellis Parents Active in Math

Alice Levine
Boston Excels Family Literacy Program, USA

Editor's Note: The following is a description of a poster session displayed throughout the ALM-7 Conference.

The Context

Boston Excels, a program of the Home for Little Wanderers, operates in five of the Boston Public Schools, providing a wide array of services to support children and their families. At the David Ellis Elementary School, a key component of Boston Excels is the **Family Literacy Program**. When the parent classes began in 1998, parents were asked what they wanted and needed to learn. Many parents expressed their frustration with their children's math homework: "Why does it look so different from when we were in school?" Thus, the Ellis Parent Math Class was born.

The Class and Its Impact

In the Parent Math Class, parents try out the kinds of problems that their children are expected to solve. They learn to *grapple* with open-ended problems (where there may be more than one right answer!) and to use math manipulatives or household objects to help solve problems. Parents work collaboratively, share their math strategies with each other, and discuss how the new teaching/learning approaches differ from those used when they were in school.

The Ellis parents come to the class initially because they want to be able to help their kids, but somewhere along the way, they get so excited about math "MAKING SENSE" to them for the first time, that they are clearly also learning for themselves. As they spend time working through problems with each other, the parents become excited and animated and their whole relationship to mathematics begins to change. They are then able to pass these new attitudes and feelings about math on to their kids.

We have also found that many parents in the class have become interested in helping other Ellis parents to understand and support their children's math education. The math class participants have become leaders in the school—leading math workshops for other parents, staffing math stations at our extremely successful Family Night, and working with K-5 teachers to find more effective ways of involving Ellis parents in their children's math education.

Massachusetts Parent Involvement Project

Janet Stein
Museum Institute for Teaching Science, USA

Editor's Note: The following is a description of a poster session displayed throughout the ALM-7 Conference.

About This Display
These activities have been developed by the Massachusetts Parent Involvement Project (MassPIP), a collaboration between the Massachusetts Department of Education and the Museum Institute for Teaching Science (MITS, Inc.). The project's mission is to increase parents' involvement in their children's mathematics, science, and technology education. The project focuses on parents and caretakers who avoid or are unable to go to school settings, or who experience language and cultural gaps with their children's teachers.

MassPIP Hands-On Kits for Public Sites are designed to bring hands-on, inquiry-based math and science explorations to families in non-traditional settings. The kits have been used in public housing complexes, clinic wait rooms, parks, after-school programs, supermarkets, and other places where families find themselves. The activities are organized by local community coalitions in MassPIP districts and are presented by coalition volunteers. After trying the activity, families receive a take-home bag with supplies and ideas for investigating at home.

MassPIP kits are available free to participating coalitions within the project. We also have Kit Books which contain copy-ready versions of all the activities and take-home sheets, as well as information on ordering supplies needed to assemble the kits. The Kit Books are available from MITS, Inc. at cost.

Activity by Mail is a series of four packets designed to provide ongoing opportunities for families to do math and science activities. The first kit is delivered in person, and subsequent packets are mailed directly to the home. Activity by Mail is still being field-tested, and we are exploring new ways to distribute the initial kit to families least likely to have access to these kinds of children's materials. Like the MassPIP kits, Activity by Mail has been developed by a team of museum educators from science and children's museums in Boston, Worcester, Holyoke, and Springfield.

Activities by and for Parents came out of a mini-grant program that enlisted Adult Basic Education programs to incorporate math and science for parents into their existing work. The samples on display are from the Operation Bootstrap Even Start Program in Lynn, the Immigrant Learning Center in Malden, and Read/Write/Now in Springfield. Literacy instructors worked with adult learners to select books and games which were then presented by the learners to other adults and children.

INSTRUCTIONAL APPROACHES

Teaching Adult Students Mathematical Investigations – 6

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Abstract

Teaching Adult Students Mathematical Investigation is based on the continuing research work carried out for the last ten years of teaching on the Foundation Course in Mathematics at Goldsmiths, University of London. Teaching Mathematical Investigation to adult students is a very challenging and often rewarding experience for adult educators as well as for the adult students.

The theme of the investigation is that adult students are asked to make Ice-cream Cones containing two scoops of two different flavours. First, they are asked to decide the flavours, then make as many different combinations of two scoops of Ice-cream Cones, using each flavour only once.

The paper concludes that many adults already have a wealth of experience. Teaching methods such as "mathematical investigation" and discussion with adults can provide a bridge to new learning initiative for adult students. Clearly, the evidence in this investigation suggests that adult students gain more when they know what work they are personally accountable for and what to do when they have finished, with the "Ah, Ha!" experience.

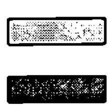
Introduction

> The idea of "mathematical investigation" is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to solve problems in very many fields. (The Cockcroft Report, 1982)

Mathematical Investigation (MI) is a source designed to provide effective practice for teachers of mathematics in some basic concepts in mathematical education and the development of adults' learning skills in a form that generates interest and enthusiasm. Many adults already have a wealth of experience, and teaching methods such as mathematical investigation and discussion with adults can provide a bridge to new learning initiatives for adult students.

In this investigation, adult students are asked to make Ice-Cream Cones containing two scoops of two different flavours. First, they are asked to decide on the flavours, then make as many different combinations of two scoops of Ice-Cream Cones, using each flavour only once. In their different groups, students are encouraged to discuss their investigation work with each other and use constructive criticism to develop ideas.

Two Flavours

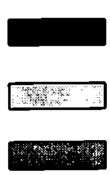


Strawberry

Chocolate

Only one combination is possible
1

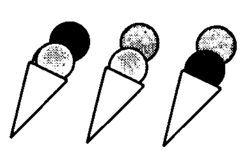
Three Flavours



Toffee

Strawberry

Chocolate



Three possible combinations
3

On observation, one group introduced the problem by assuming that an ice-cream seller has different flavours for making cones and that each cone must just contain two different flavours. The group raised the following question: How many different cones can the seller make with the following 9 flavours: lemon, vanilla, strawberry, toffee, chocolate, orange, apple, and peach?

The group then went on to identify the purpose of the investigation, which is to find the maximum number of choices they can have, given a number of flavours and the number of combinations. A combination consists of two scoops of two different flavours

They let the flavours be represented respectively by their first letter. In order to avoid confusion between Chocolate and Coffee, which have the same initial letter, Coffee was represented with a K.

L, V, S, T, C, K, O, A, P.

Method of Investigation and Analysis

We suggested starting the investigation with the two first flavours and then increasing one by one until exhaustion of the list. That is to say, we will work out from L and V, and then for L, V, and S, …, and so on. At the end of the list, we will collect the results in a table and will try to find the rule which allows us to work out the number of combinations for a given numbering of things if we know how to combine them.

Let us now start.

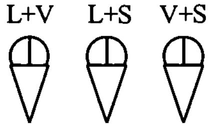
For 2 flavours L and V, how many ice-cream cones:

As we can use each flavour just once, we have

L,V → L+V  → 1 cone

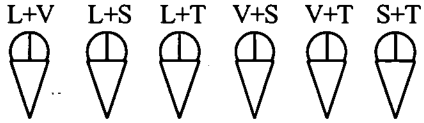
For 3 flavours

L,V,S → L+V L+S V+S → 3 cones



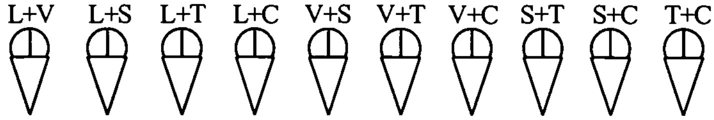
For 4 flavours

L,V,S,T → L+V L+S L+T V+S V+T S+T → 6 cones



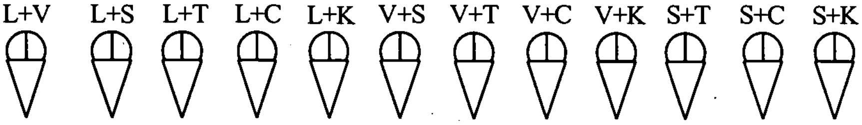
For 5 flavours

L,V,S,T,C → L+V L+S L+T L+C V+S V+T V+C S+T S+C T+C → 10 cones

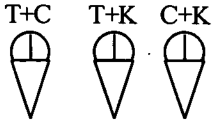


For 6 flavours

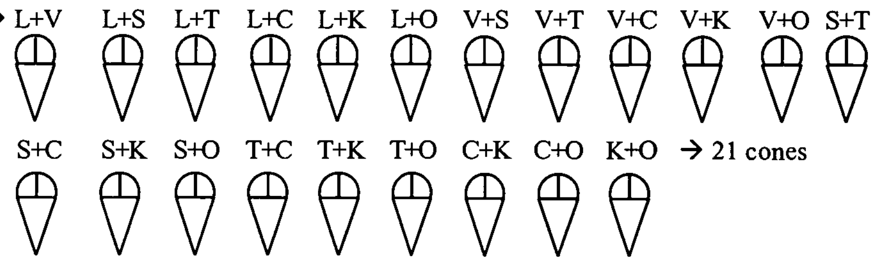
L,V,S,T,C,K → L+V L+S L+T L+C L+K V+S V+T V+C V+K S+T S+C S+K



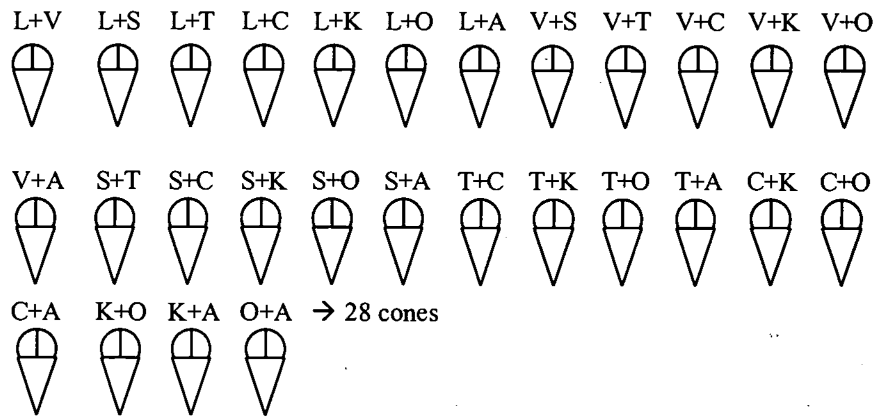
T+C T+K C+K → 15 cones



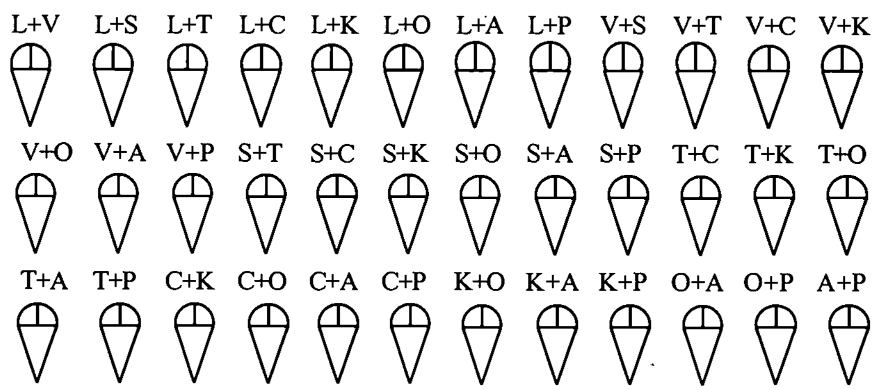
For 7 flavours
L,V,S,T,C,K,O →



For 8 flavours
L,V,S,T,C,K,O,A →



For 9 flavours
L,V,S,T,C,K,O,A,P →



→ 36 cones

Now let us collect the results altogether in a table; f stands for flavour and c for cone.

TABLE ROA/1

f	c
2 →	1
3 →	3
4 →	6
5 →	10
6 →	15
7 →	21
8 →	28
9 →	36

What do we learn by observing this table?

First Observations
The results in the table show that:

When f = 2, c = 1, and when f = 3, c = 3. Thus the difference between c-1 and c equals f-1, i.e., 2.

When f = 4, the difference between c-1 and c equals 3, which is equal to f-1, c = 6, c-1 = 3.

When f = 5, c = 10. Here also there is a difference of f-1 between c and c-1. f-1 = 4 → c-1 = 6.
So, c = (f-1) + (c-1)

When f = 6, c = 15. Again c is different from c-1 by f-1, i.e., 5.

When f = 9, c = 36. Once again c-(c-1) = f-1 = 8.

Second Observation
If c is always different from c-1 by f-1, what about the first row of the table, i.e., the case where f = 2 and c = 1? Is the first observation also applicable to this case? Let's try!

c must be different from c-1 by f-1.
c = 1, then c-1 = 0 and the difference is 1.
Now f = 2, then f-1 = 1, therefore c is different from c-1 by f-1.

Hence the first observation is applicable to the case where f = 2 and c = 1, the first row of the table.

Third Observation
c increased by arithmetical progression, which means by addition. For example, when

f = 2, c = 1
f = 3, c = 1 + 2 = 3
f = 4, c = 1+ 2 + 3 = 6
f = 5, c = 1 + 2 + 3 + 4 = 10
f = 6, c = 1 + 2 + 3 + 4 + 5 = 15
f = 7, c = 1 + 2 + 3 + 4 + 5 + 6 = 21
f = 8, c = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28
f = 9, c = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36

Fourth Observation
f = 2, c = f-1
f = 3, c = (f-1) + (f-2)
f = 4, c = (f-1) + (f-2) + (f-3)
f = 5, c = (f-1) + (f-2) + (f-3) + (f-4)
f = 6, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5)
f = 7, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5) + (f-6)
f = 8, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5) + (f-6) + (f-7)
f = 9, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5) + (f-6) + (f-7) + (f-8)

So if f = 1, c = f-1, which is nought; since each cone must necessarily have two different flavours, there is no combination and c = 0.

What does the fourth observation tell us?

It tells us that for any number of f, provided $f \geq 2$, c will be equal to the sum of f_i, f_{i+1}, $f_{i+2} \ldots$ until $f_{i+(n-1)}$ [where i = 1]. For example, if f = 3, then n = 2, f_i = f-1, and $f_{i+(n-1)}$ = f-2.

So $\sum_{i=1}^{n-1} f_i$ is one rule to work out the number of combinations

The Σ is called sigma and means sum

$\sum_{i=1}^{n-1} f_i$ is read sigma f_i from i = 1 to n - 1

Another way to work out the possible combinations is to apply the following combination formula.

$$^{m}Cp = \frac{m!}{p!\,(m-p)!}$$

Where m stands for number of elements to combine, C for combination, and p for number of elements in a combination.

This formula is read as combination of m elements taken p by p equals factorial m divided by factorial p, factor of factorial m minus p.

Let us open a parenthesis to show how the factorial works.

2 ! = 1 x 2
3 ! = 1 x 2 x 3
4 ! = 1 x 2 x 3 x 4
n ! = 1 x 2 x 3 x .... x n
1 ! = 1

And by convention, 0 ! = 1

To use this formula for our ice-cream investigation, we just have to substitute f for m to get

$$^{f}Cp = \frac{f!}{p!\,(f-p)!} = c$$

Combination of f flavour taken p by p equals factorial f divided by factorial p factor of factorial f minus p, equals c.

So, for f = 2 and p = 2, we have c = $\frac{2!}{2!\,(2-2)!} = \frac{2}{(2 \times 1)\,0!} = \frac{2}{2 \times 1} = \frac{2}{2} = 1$

for f = 3, p = 2 ċ = $\frac{3!}{2!\,(3-2)!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} = \frac{3 \times 2}{2 \times 1} = \frac{6}{2} = 3$

for f = 5, p = 2, C $\frac{5!}{2!\,(5-2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1} = \frac{5 \times 4}{2 \times 1} = \frac{20}{2} = 10$

for f = 9, p = 2, C $\frac{9!}{2!\,(9-2)!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{9 \times 8}{2 \times 1} = \frac{72}{2} = 36$

Conclusion

We have shown through this investigation two ways to work out the possible number of combinations of a set of elements without repetition, if we are given the number of elements in each combination. The first one is by arithmetical progression using the sigma

$$\sum_{i=1}^{n-1} f_i$$

and the second one is the combination formula using the factorial notation: the product of all the positive integers from n down to 1 is denoted by n!

$$^{f}Cp = \frac{f!}{p!\,(f-p)!}$$

In this investigation, both ways are equivalent. However, I think that the combination formula is easier to use than the first one for a large number of elements. Nevertheless, we can use the sigma formula for a smaller number.

From the pictorial representations and observations made, it can be observed that the combination columns follow that of the Blaise Pascal Triangle of numbers. The numbers in Pascal's triangle also occur when we consider the number of ways we can combine different flavours. There is need for more research into adults returning to study mathematics and numeracy, focusing on both cognitive and affective domains. These need to be complemented by a third, the ontological dimension.

I would argue that whilst teaching techniques in mathematics education of the past have shown that adult students can be trained to use their minds and not to think, teaching techniques in mathematics education today (e.g., mathematical investigation) should require adult students to think, as well as young people in the education system. This has wider educational implications for improving professional teaching standards and skills for adult educators, improving learning strategies, improving learning objectives, resources, learning activities, and outcomes. Implications for teaching adult students should be drawing attention to the professional development, both initial preparation and continuing education, of more practitioners in the field.

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Overcoming Algebraic and Graphic Difficulties

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Universidade Católica de Brasília, Brazil

How does one manage to teach 200 freshmen first-year calculus if they have difficulties with basic algebra and graphs? Teaching Business Majors at Universidade Católica de Brasília, a school that gets its students from the low socio-economic portion of the population of Brazil's Federal District, I have been trying to find this out, and will share with readers some of the difficulties I have found and some of my thoughts on the solutions.

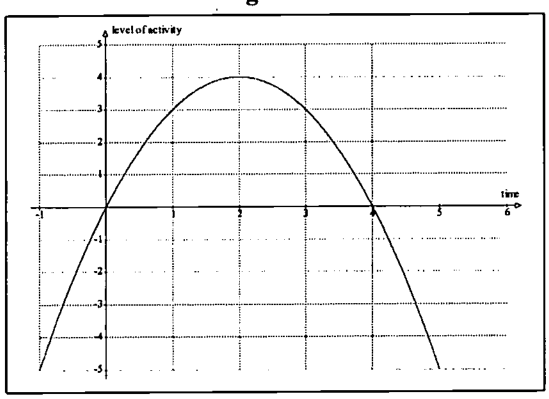
Difficulties with Algebra

Let F: $A \rightarrow B$, $x \in A$, $y \in B$. This is the kind of language usually found in the books. Students are expected to read or understand "let F be a function going from set A to set B, *x* a variable whose domain is set A and *y* a variable whose range is set B." Students, on the other hand, ask me, when faced with "f(4)" on a text: "What is this f in front of the 4? Can I just disregard it?"

Difficulties with Graphs

Graphically, given a graph of a function F like that in Figure 1, students cannot tell what the image of three by the function F is.

Figure 1



Students also usually do not have an understanding of the continuum. For example, even after being instructed about the difference between having graphs composed of discrete points and graphs which are lines, very few students could answer the following question in terms of intervals:

What values of the variable *time* have positive images? (See Figure 1.) Usual answers were "1, 2, and 3" or "0, 1, 2, 3, 4." When prompted about there existing numbers between 1 and 2 with images on the graph, a student answered: "Oh, yes, 1/2." I insisted "Only half? No other numbers?" She timidly jotted a 1/4 on the paper.

Lack of Meaning

The most popular textbooks used for this course bring problems that bring up situations such as: "The number of members in an association is given by $f(x) = 100(2x^3 - 45x^2 + 264x)$." The students question how the number of members in an association can be given by such a function. When a problem says "The daily production of a certain factory is $Q(L) = 20{,}000\ L^{1/2}$," the students ask whether they are going to be given such formulas when they start working. The reason for this lack of meaning on the majority of textbook word problems is that there is no effort to acquaint students with the modeling process. Learning how mathematics is really used through modeling is the only way students will be able to use it in their careers later on.

Problems with the natural language also exist, and they are of two kinds: inability to interpret word problems and confusions caused by the usual meaning of a word when its mathematical meaning is different. For example, the word used for slope in mathematics in Portuguese is *declive*. However, in everyday language this word means what in mathematics we would call a negative slope. So every time I said "slope" and pointed to a positive slope on a graph I heard whispers and confusion among the students. One of them finally said she was seeing no slope. She was seeing an *aclive* ("positive slope," in everyday language).

Approaches to Improve the Situation

Syncopated Algebra

To overcome the difficulties with algebraic notation, groundwork had to be done to let students understand what a variable is. They all had used variables procedurally since elementary school and basically think that "variables are letters." If the students do not know that variables may be other symbols and that not all letters are variables, and, more importantly, if they do not know when to create and use a variable, they will not do mathematics.

At our course at UCB's business school I have been emphasizing that variables are symbols chosen to represent any one generic element in a set, the domain of the variable. I have worked with the students using real data from surveys or research found on the Internet. I try to bring from real research variables that are not "a letter" but words or expressions, to avoid that habit of specially using x's and y's. So they work with variables such as SEX, DEGREE, WRKSTATUS, and can have ordered pairs such as (SEX, DEGREE) instead of the usual (x, y). I also have them collect their own data and create variables, specifying their "names"and their domains, and to form general sentences about the elements of their sets of data.

Using syncopated algebra is also helpful. As much as possible, I write, for example, "the image of 4 by the function G" instead of g(4) for a while until they are tired of it and accept and understand the g(4). Or

$$\frac{\text{resultant variation on the dependent variable}}{\text{variation occurred on the independent variable}} \text{ instead of } \frac{\Delta y}{\Delta x}$$

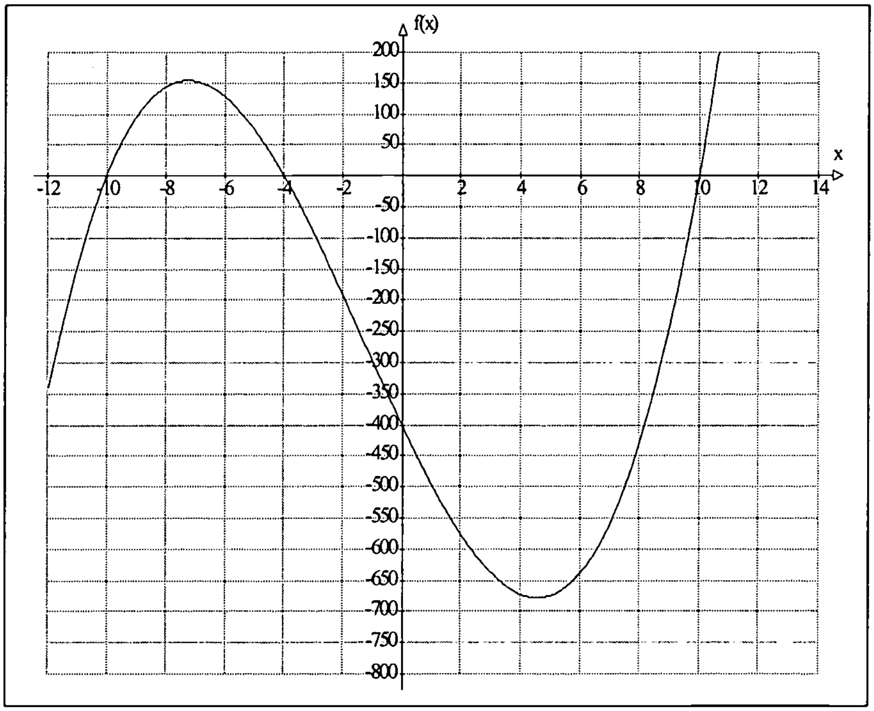
Lots of Work with Graphs

Facing the students' difficulties, initial work had to be done with graphs, starting by explaining the whole idea of algebrizing the plane and how any point on the plane can be determined by an ordered pair. This of course has to be preceded by an understanding of the "Real line." Students bring these ideas vaguely from high school, but, as said before, they lack conceptual understanding. They all think they can plot a point given the coordinates, and they do when the coordinates are (1,2), but give them the graph of a function F and ask them to point to (2, f(2)) and the difficulties start showing. As for the real numbers, as mentioned before they carry ideas such as that there is a finite number of numbers between two integers or that there is "a next real number after 1.1," for example.

Working on simple exercises such as the one in Figure 2 is a situation for a lot of learning.

Figure 2

Plot on the Cartesian plane below the following points:
(2, 50)
(4, -200)
(4, f(4))
(-8, f(-8))
(2, f(-8))
(6, - f(-8))



Students had a lot of difficulty in seeing that the distance from a point on the horizontal axis and its image on a graph can be the ordinate of another point, in another location of the horizontal axis. Transferring lengths and using them as coordinates of different points was a good exercise.

The students have to start viewing a coordinate as the measure of a segment. Understanding how the real line was constructed, based on a chosen unit, and that for each length of line segment a real number is associated (that is, for each point a real number is associated) is essential. Work like this may be too rigorous and formal, but how to even have the procedural knowledge the textbooks emphasize without conceptual knowledge?

This exercise showed students that their previous knowledge on points and coordinates was not as well established as they thought. Apparently they had learned "Go to the horizontal axis to mark the first coordinate and to the vertical axis to mark the second coordinate." However, when they had to mark a point such as (2, f(-8)), they had to:

1. Go to the -8 on the horizontal axis.
2. Measure the length f(-8) above (or below) it until the curve of the graph of F.
3. Transfer that length to the 2 on the horizontal axis.

Many of the students just could not agree with this. They said: "To mark the second coordinate I must go to the vertical axis, and here am I going to the horizontal axis!"

Students finally became accustomed to the idea of coordinates when we started writing them as (side distance, height). For example, a point (4, 5) is 4 units distant from the vertical axis to the right, and five units high from the horizontal axis. This also helped a lot with the sign of quantities, because sometimes they would not admit that f(-8) or that -f(8) could be positive. When we started talking about "positive height," "negative height," "units to the left" or "to the right of the vertical axis," and other strange things that for us made sense, things started to get clearer.

Modeling

In different semesters I have tried different approaches to deal with students' lack of knowledge of the modeling process. I have tried to *teach about* modeling, what it means and its steps, showing examples, and I have tried to have students model real situations. While the second alternative is more difficult to work with, it is much more engaging for students. While we naturally cannot expect students to create great models at this stage, they can at least experiment with each step of the process, and with the help of a curve fitter, make simplifications and approximations to real situations, such as forecasting of maxima and minima or future values using the functions chosen.

Understanding the difference between the Rationals and the Continuum brings in a good discussion on modeling and the difference between discrete and continuous variables. The students become much more critical of all the theory they are learning when they think of which of the measurable variables in the real world are continuous or when they can be assumed continuous.

Textbooks

This year I was particularly happy to find that one of Deborah Hughes-Hallett's calculus books (Hughes-Hallett et al., 1994) had been translated into Portuguese. This book addresses a lot of these same ideas, such as the intense work with the graphic aspect of every concept and the easiness with the algebraic notation. One can find there, for example, $\frac{\text{vertical increment}}{\text{horizontal increment}}$ (my re-translation into English), instead of $\frac{\Delta y}{\Delta x}$. I promptly adopted it as a textbook (although still doing original side work, especially in modeling and curve fitting with software), but found that it is still too complex for my students. To really study deeply the first chapter of the book I might need a whole semester, so I have to help my students go through the book, leaving a lot of good things behind.

To give the readers an idea, two of the simplest exercises in Hughes-Hallett's book, in Figures 3 and 4 (Hughes-Hallett et al., 1997, pp. 23-24, my re-translation), gave rise to the long, step by step worksheet I had to write for the students (Figure 5). They had trouble seeing the relationship between the curvature of a graph and the sign (positive or negative) of the variation of the dependent variable, especially the case of decreasing functions. Specifically, they would think that greater and greater "falls" on a decreasing graph (downward curvature) meant that the variation on the dependent variable was increasing (suffering a positive variation).

Figure 3

Each function in table 1.10 is increasing, but each increases in a different way. Which of the graphs of figure1.24 best fits each function?

t	g(t)	h(t)	k(t)
1	23	10	2.2
2	24	20	2.5
3	26	29	2.8
4	29	37	3.1
5	33	44	3.4
6	38	50	3.7

Table 1.10

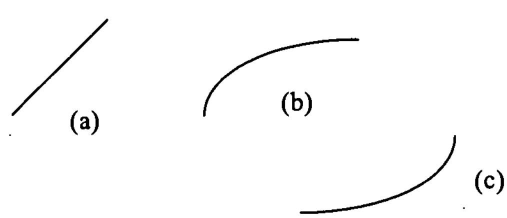


figure 1.24

Figure 4

Each function in table 1.11 is decreasing, but each decreases in a different way. Which of the graphs of the figure1.25 best fits each function?

x	f(x)	g(x)	h(x)
1	100	22.0	9.3
2	90	21.4	9.1
3	81	20.8	8.8
4	73	20.2	8.4
5	66	19.6	7.9
6	60	19.0	7.3

Table 1.11

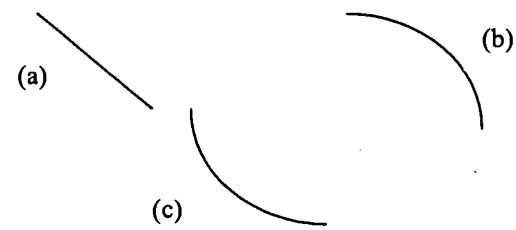
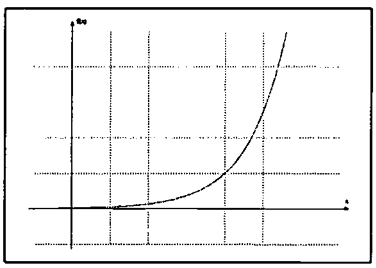


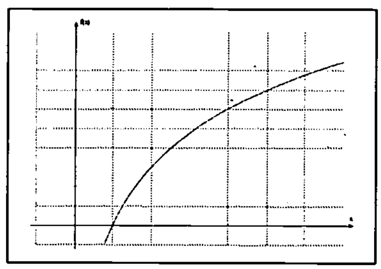
Figure 1.25

Figure 5
Worksheet

First, let us examine the graphs below:



Graph 1



Graph 2

In both graphs, the dependent variable increases as the independent variable grows.

As for the **variation on the dependent variable** for a given **variation on the independent variable** (use the grid to examine intervals of the same length on the horizontal axis), as the independent variable increases:

The variation of y ________________ on graph 1 (increases or decreases?)
The variation of y ________________ on graph 2. (increases or decreases?)

We see then that, when a graph is **concave downward**, as the independent variable increases, the variation on the dependent variable for a given variation on the independent variable, ____________ (increases or decreases)?
And when a graph is **concave upward**, as the independent variable increases, the variation on the dependent variable for a given variation on the independent variable, ____________ (increases or decreases)?

Save the conclusion you got above (let us call it conclusion (*)) because we will compare it with what you conclude later.

Let us examine now the tables in exercise 5:

The table below gives us the values of h(t) for the integer values of t from 1 to 6. In the third column, we started to calculate the variation that h(t) suffered when t varied by 1 unit, from the previous line to the line in question. Continue to fill out that last column:

t	h(t)	variation on h(t)	
1	10		
2	20	(from t=1 to t=2)	**10**
3	29	(from t=2 to t=3)	
4	37	(from t=3 to t=4)	
5	44	(from t=4 to t=5)	
6	50	(from t=5 to t=6)	

Do the same in the following table, with the values for g(t) (Calculate the variation of g(t) from one line to the next):

t	g(t)	variation on g(t)	
1	23		
2	24	(from t=1 to t=2)	**1**
3	26	(from t=2 to t=3)	
4	29	(from t=3 to t=4)	
5	33	(from t=4 to t=5)	
6	38	(from t=5 to t=6)	

Finally, do the same in the following table, with the values for k(t) (Calculate the variation of k(t) from one line to the next):

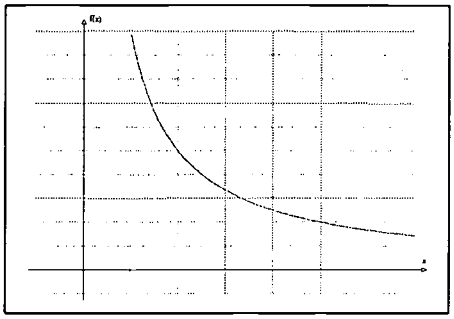
t	k(t)	variation on k(t)	
1	23		
2	24	(from t=1 to t=2)	**0.3**
3	26	(from t=2 to t=3)	
4	29	(from t=3 to t=4)	
5	33	(from t=4 to t=5)	
6	38	(from t=5 to t=6)	

We see that, as t increases, h(t), g(t), and k(t) increase. However, as t increases, h(t) **increases faster or slower** every time? _______________ So, how would its graph look? Concave upward or concave downward? _______________

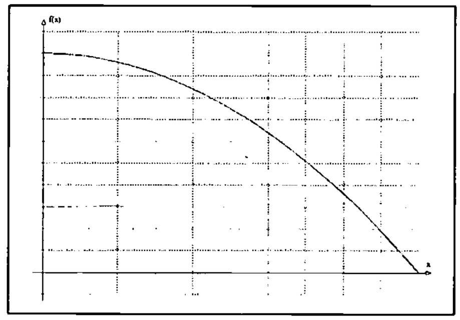
What about g(t), does it increase faster or slower every time? __________ So, how would its graph look? Concave upward or concave downward? _______________

And what do you say about k(t)? _________________________

Let us now examine decreasing functions:



Graph 3



Graph 4

In both graphs, as the independent variable increases, the dependent variable decreases. But does it decrease slower or faster every time?

In the graph that is concave upward, the dependent variable decreases _________ (slower or faster)?

In the graph that is concave downward, the dependent variable decreases _________ (slower or faster)?

Now let us examine the tables in exercise 6:

x	f(x)	variation on f(x)	
1	100		
2	90	(from t=1 to t=2)	**-10**
3	81	(from t=2 to t=3)	
4	73	(from t=3 to t=4)	
5	66	(from t=4 to t=5)	
6	60	(from t=5 to t=6)	

We see that, as x increases, f(x) decreases. But does it **decrease faster every time or slower every time**? ________________

So, how would its graph look? Decreasing and "falling" greater and greater distances or falling smaller and smaller distances? That is, would it be **concave downward or upward**?
____

Now, in terms of the **variation of f(x)**, as x increases, is the variation of f(x) increasing or decreasing? (See the values with which you completed the table and answer. But pay attention! We are dealing with negative numbers. -10 is greater or smaller than -6? So the **variation of f(x)** is increasing or decreasing?) _____________________

Repeating what you have just written: the graph of F is concave _______ (upward or downward?) and, as x increases, the variation in f(x) for a given variation in x _______ (increases or decreases?).

Now compare this conclusion with the conclusion (*) that you reached in the case of increasing functions. They should be the same (that is, (*) should still work).

Continue the exercise for the functions G and H: (...)

Conclusions

The goals in introductory mathematics courses for non-mathematical careers may vary from having students memorize some rules and procedures (such as rules of derivation and rules for the relationship between curvature of the graph of a function and the sign of the second derivative of the function) to helping them gain insight into the mathematical concepts they will be using. If we have the second goal, more time and effort than usual may be necessary if the students did not have adequate preparation in their previous schooling. The activities described in this workshop may seem condescending and too easy for college students. However, my experience has been that after the hard work students have thrilled at being able to understand the exercises and theoretical parts of the textbooks. They have often made positive manifestations about our work in class and the extra worksheets and asked for more of this kind of guidance.

Reference

Hughes-Hallett, D., et al. (1997). *Cálculo, vol. I.* Rio de Janeiro: Livros Técnicos e Científicos Editora.

A Structured Approach to Mathematical Enhancement

Donna K. Ellis
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Currently within the UK there is an increased need to support students throughout their Higher Education. This is due to the trend towards wider participation and employers increasing their demand for vocationally apt students.

Due to these trends and life long learning, students now enter the system with varied qualifications and experiences, hence the need for support.

The Student Learning Support Facility (SLSF), at the Robert Gordon University, was established to provide such support. The support is provided in terms of assistance with Writing and Communication Skills, IT applications, and Mathematics.

The workshop will concentrate on the Mathematics thread, giving an indication of how the SLSF operates and the learning strategies adopted to help students.

All students' entering first year undertake a mathematics diagnostic assessment. This allows us to identify students' strengths and weaknesses needing addressing at a very early stage. This provides valuable information for both students and staff. Furthermore, it allows the SLSF to provide a tailored learning programme and guidance on resources available to help them, thus increasing student retention.

At the facility we have:

- commonly used reference textbooks,
- worksheets (sold at cost),
- computer aided learning packages,
- personal or group tuition, and
- a study area that provides a quiet place for students to work, with a tutor being available at most times.

The Faculty of Science and Technology within the Robert Gordon University (RGU) identified mathematics as a subject that students have difficulties with. It was recognised that mathematics was an important skill required for their undergraduate courses. Hence in 1996 the Mathematics Learning Support Centre (MLSC) was founded. As the MLSC established itself, lecturers throughout the university began to identify other skills students had difficulties with. As a result the MLSC became the Student Learning Support Facility (SLSF) in 1998. The facility now provides a service to the entire University dedicated to enhancing students' Writing & Communication skills, use of Information & Communication Technology (ICT) applications, and Study Skills, as well as Mathematics and basic Statistics. The facility has both full-time and part-time tutors available.

Currently within the United Kingdom there has been an increased requirement to support students throughout their Higher Education. This is due to the trend towards wider participation and employers increasing demand for vocationally apt graduates. Due to these trends and life long learning, students now enter the system with more varied qualifications and experiences, hence the need for extra support. The main beneficiaries of the facility are school leavers and mature students, of which the latter seem to gain most through the use of the facility. We believe this is due to the school leaver's perception of additional learning support to be remedial classes for students with extreme learning difficulties. Hence the SLSF has changed its name to Study Support Centre (SSC).

One of the facility's main functions is the Mathematical Diagnostic assessment of each new intake of students within the first weeks of their arrival to the university. The assessment determines the proficiency of the student's mathematical skills, highlighting any strengths and/or weaknesses. The results of the assessment allow, firstly, course tutors to identify if a student is embarking on the correct course (e.g., transfer from a Higher National Diploma (HND) course to a Bachelor of Engineering (BEng) course). Secondly, it provides an indication of students who may have potential difficulties, allowing both tutors and the SSC to make appropriate provision. The SSC provides extra support, enabling students to overcome any weaknesses and reduce failure/drop out rates.

At present the facility uses a paper based diagnostic assessment. It has been developed from the software package "Diagnosys" designed by Newcastle University, and customised to the requirements of RGU. It assesses knowledge previously gained from school or college along with some basic skills taught in the first year mathematics units of RGU's undergraduate courses. This assessment is informal yet has a formative purpose, however it does not contribute towards a student's grades. Each question is associated with a basic mathematical skill. The answer to each question is recorded for identifying underlying weaknesses in individual skills. The assessment has the questions grouped into topics, such as Basic Algebra, Calculus, and Trigonometry, allowing us to identify areas of mathematical weakness. For the past four years all engineering students have been assessed with over 200 scripts to be hand marked each year. A vast range in abilities has been recorded with trigonometry and basic calculus being the main topics where students lack knowledge. On completion of the assessment a report is generated for each student and their course tutor. The report contains information about (i) the student, (ii) topic results, (iii) individual concepts that the student appears to lack knowledge of, and (iv) worksheets that may help the student to overcome their weaknesses. It is important to mark and return results to students with constructive feedback quickly. Discussing the results of the assessment with the student on a one to one basis would enhance this feedback. This allows the SSC to arrive at reasons why there are difficulties and how to eliminate them for a particular student, hence why we believe diagnostic testing is required at an early stage of undergraduate studying. Unfortunately this process is very time consuming, resulting in reports not always being produced within the correct time frame. Thus we are currently automating the diagnostic system.

Unfortunately the software "Diagnosys" which the paper assessment is based upon is incompatible with the computer system in RGU. Therefore we are searching for a more appropriate piece of software to automate the assessment and are considering the development of a new piece of software. Due to restrictions the assessment is at present being redeveloped in the software package "Question Mark" for the next intake of students.

The diagnostic assessment pass mark is 30%. If a student does not meet the grade a formal referral is **sent** to both the student and their tutor, suggesting that they visit the facility for additional study support. Unfortunately the majority of students who require support do not approach the SSC. However it appears that this is a national problem that requires addressing.

The SSC offers a variety of support mechanisms for students within the University. Personal and group tuition is given using course notes along with textbooks and worksheets available at the facility. The facility stores all commonly used mathematical textbooks available for reference only. Both mathematical and language worksheets can be obtained on a cost basis. The facility has a large, quiet, open plan study area for students to work with the benefit of a tutor being available. A limited number of Computer and Video Aided Learning packages are also available for students. The SSC attempts to create a friendly, informal learning environment ensuring students are comfortable and are able to openly learn and study. Personal/group tuition is the main form of support, conducted by full-time and part-time staff at the facility.

The SSC has slowly increased its resources over the past 4 years and has a number of ambitions for the future which include improving the usage of ICT applications, an ICT hotline, and increasing tutor availability.

The ICT hotline is an immediate development that the facility would like to provide for students. This would provide support in PC applications including the Internet. Providing this support would increase the ICT knowledge and understanding of graduates thus making them more appealing to employers. From experience we believe mature students would benefit most from this service.

Packages such as CALMAT (Glasgow Caledonian University) and MATHWISE (UK Mathematics Courseware Consortium) are readily available in the United Kingdom. These have been developed through the government funded SHEFC and TLTP Programs. The SSC aims to incorporate such packages into our students' learning programmes. This would ease the workload of the tutors and in turn allow them to provide a better service to the University and its students.

Additional tutors would provide greater support in all areas of student learning difficulties, however funding is a problem that all Universities have. Ideally both the facility and students would benefit from a full-time Study Skills tutor, Writing & Communication tutor, ICT tutor as well as Mathematics tutor.

We believe that Universities with similar support groups should be establishing greater links both nationally and internationally. The ALM-7 conference has enabled RGU to establish such links. Within RGU we are currently developing stronger bonds with our support services, including the Disability Unit and the Special Needs Unit. The current focus will be on helping students with dyslexia. RGU is actively involved in the research into Dyscalcula in conjunction with other Universities.

The SSC has two long-term goals. Firstly to re-design the paper distance learning course into an online course via the RGU virtual campus. This would allow students to discuss problems with tutors and online discussion groups. Secondly to transform the diagnostic assessment so that it is available on the RGU INTRANET. This development process would include instant online reporting and assistance where required. The report would also be sent to the SSC so that a learning programme could be developed following a one to one discussion with the student.

References

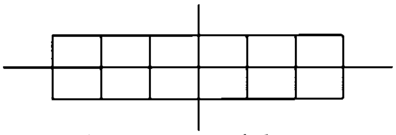
CALMAT v2. (2000). Glasgow, UK: Glasgow Caledonian University.
DIAGNOSYS v3.24. (2000). Newcastle Upon Tyle, UK: Newcastle University.
MATHWISE v2.1, UK Mathematics Courseware Consortium, UK.

Visual Mathematics

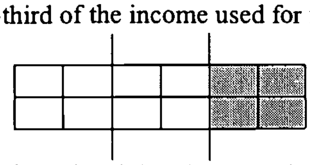
Diane E. Ferris
Portland Community College, USA

Introduction to Visual Mathematics

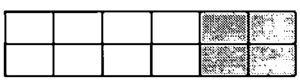
An adult education student typical of many preparing for the GED mathematics test, Margie was anxious as she pondered over a word problem: If a family spends one-fourth of its month's wages on rent, and one-third on food and utilities, what fraction of their income remains? Striving to make sense of the question, she reached for an egg carton and some short lengths of yarn and some colored markers. First, she used the yarn to separate the egg carton sections into four parts, each having the same number of sections. She chose blue markers to fill one-fourth of the carton. The illustration below represents the one-fourth of the income used for rent.



Margie then rearranged the yarn to separate the egg carton into thirds and used red markers to fill one-third of the carton, being careful not to overlap any of the blue sections. This red portion of the carton represents the one-third of the income used for food and utilities.



Margie realized that the unmarked sections were the answer to her math problem, but she still needed to identify the name of the fractional amount. Since the carton had a total of twelve sections, and five of the sections were left unmarked (uncolored), Margie decided that the fractional remainder of the family's income must be five-twelfths.



A quick check of the answer key to verify her conclusion prompted a beaming smile from Margie. Later, she reflected upon her long struggle to become a successful problem solver for everyday types of math situations. During her elementary school years she'd experienced learning challenges, especially in mathematics using the rote memorization and paper and pencil methods, which were the primary instructional methods at that time. She thought that her current success could be attributed to using a different learning approach. The new approach involved using math manipulatives and visual, hands-on approaches to problem solving. Margie began to realize that she was a very capable thinker when she could use realia to help her visualize the problem she was trying to solve.

Tools to Think With

In his book, *Mindstorms: Children, Computers, and Powerful Ideas*, Seymour Papert described being fascinated by the working machinery on the farm where he spent much time as a youth. He was especially interested in the myriad large and small gears with their cog teeth interacting at various rates, and he spent hours watching the interaction. He realized later that he was able to make sense of many concepts in mathematics and science by drawing upon his memories of how the gears worked. For Papert, the gears had become a tool to think with.

As an educator, Papert noticed that many learners seemed to lack "tools to think with." Papert developed the computer program called LOGO and identified other computer activities which help learners develop their thinking and reasoning skills by doing projects and by developing mental images and models to draw from.

Resources for Using Visual Approaches to Teach Mathematics to Adult Learners
Math and the Mind's Eye, a curriculum project targeted for middle-school students, was developed by The Math Learning Center in Portland, Oregon, beginning in 1984 with support from the National Science Foundation. The materials provide learners with sensory experiences to develop mental images that serve as an aid in understanding math concepts and relationships. The lessons are written so that teachers can adapt them to fit learners' backgrounds. The materials are appropriate for learners of all ages, and they are especially respectful of the adult learner. The lessons can be used individually as stand-alone activities or they may be used as part of a longer project.

Visual Mathematics is an evolutionary outgrowth of *Math and the Mind's Eye*. Each lesson has three components: Connector, Focus, and Follow-up activities. The Connector activity helps students recall facts, ideas, or relationships and connect learning with aspects of life outside of mathematics. The Focus activity guides the instructor in facilitating the exploratory activities for which students build on their previous knowledge and learn by doing. The student Follow-up activities reinforce and maintain students' learning.

Visit The Math Learning Center on the Internet at *www.mlc.pdx.edu/* for additional information about *Math and the Mind's Eye* modules and *Visual Mathematics* (which has a new name: *Math Alive*). Look in the curriculum section for sample lessons.

Introducing Adult Learners to Visual Mathematics
Most adult learners, especially those who have struggled to understand math concepts using traditional approaches, can benefit from using a visual approach to learning mathematics. As a first lesson, it is important to introduce students to aspects of the visual learning process and to goals for the math course so that, from the outset of class, learners can use these as a basis for monitoring the development of their knowledge and disposition about math.

Sample Lessons
Lesson One: The Introduction to Visual Mathematics Connector Activity is included here with permission by The Math Learning Center. (See Appendix.) Students find the activity an engaging one. They are asked to share what they see in their "mind's eye" when they hear the word "rose." Students are able to describe vivid images in great detail for words such as "rose," "tree," and "house." Students are then asked to share what they see in their mind's eye when they hear "thirty-two." For most students, images for "thirty-two" are much less vivid.

During the Focus activity, students are arranged in groups to discuss their own interpretations of the Visual Mathematics Philosophy (Foreman & Bennett, 1995, p. 5):

- There is a mathematician within each of us.
- Experiences with models for math concepts help us understand, invent, and remember important math ideas.
- Learning math is a social activity.
- Learning math is an ongoing process of knowledge construction.
- "Disequilibrium" is a sign of new learning.
- Mathematics is a fascinating world of its own.
- The world of mathematics connects to many other worlds.

With a limit of two to three minutes for "teacher talk," the Visual Mathematics Goals (Foreman & Bennett, 1995, p. 7) for the class are shared with students. The goals state that:

We are a community of mathematicians working together to develop our:
a) visual thinking,
b) concept understanding,
c) reasoning and problem solving,

d) ability to invent procedures and make generalizations,
e) mathematical communication,
f) openness to new ideas and varied approaches,
g) self-esteem and self-confidence, and
h) joy in learning and doing mathematics.

Lesson Thirteen: Folding Circles. This lesson is an excellent activity to introduce geometry vocabulary and recall or discover information about geometric shapes. The lesson is included here with permission from The Math Learning Center. (See Appendix.)

References

Foreman, L., & Bennett, A. (1995). *Visual mathematics*. Salem: The Math Learning Center.
Maier, E. (1990). Mathematical swindling. *Teaching today*. University of Idaho.
Maier, E., Nelson, T., Arcidiacono, M., & Shaughnessy, M. (1987). *Math and the mind's eye*. Salem: The Math Learning Center.
Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York: Basic Books, Inc.

Appendix

Connector Teacher Activity

OVERVIEW & PURPOSE

Students recognize and discuss ways in which they already use mental imagery to help them recall facts, ideas, or relationships in aspects of life outside of math. This discussion relates to the major purpose of this course—to engage students in sensory experiences that provide them with meaningful mental images for important mathematical ideas.

MATERIALS

✓ *Mathematics and Visual Thinking* and *Mathematical Swindling*, teacher background readings, see *Starting Points for Implementing Visual Mathematics*.

✓ *Math and the Mind's Eye* and *A Change of Course* (two optional videos), teacher background information, available from The Math Learning Center.

ACTIONS

1 Point out to students that one ol the authors of their math program lives in the city of Portland, Oregon, also known as the City of Roses. Ask them to share what they see in their mind's eye when you say the word "rose."

2 Repeat Action 1 for "tree" and then "thirty-two."

COMMENTS

1 Invite students to close their eyes if they wish. Allow a few moments for students to think privately and then ask volunteers to describe their mental images. Students generally have vivid images to report—a single long-stemmed rose, a favorite rose garden or bush, a stained-glass window depicting a rose, a favorite aunt named Rose, etc. Some report imagining the fragrance of a rose or the feel of touching it.

2 Students' images for "tree" are usually vivid. For students whose mathematical experiences have been primarily symbolic, images for "thirty-two" are often quite different. They are mostly composed of symbols—the numeral 32, or the numerals for four and eight with a multiplication sign between them, or perhaps a favorite ball player's jersey with the numeral 32 on it. If the quality of people's images of "rose" were of the same nature, their only image would be of the word itself. Thus, conceiving a design for an emblem for the Portland Rose Festival would be a daunting task!

To develop trust and promote sharing, it is important to acknowledge all images and to avoid suggesting one is better or more meaningful than the other. You may get clues, however, about the nature of some students' prior experiences in math. When asked to think of the number 32, students who have had a more hands-on, visual experience in math may report images such as 3 strips of 10 and 2 units or a 4 by 8 array of tiles. Such images carry meaning and can be useful in thinking abour mathematical situations. Providing experiences that will enable students to develop a range of such useful images is the basis of this course.

Connector Teacher Activity (cont.)

ACTIONS

3 Ask the students if they know of any examples of sports or other activities in which they or other people visualize or use their mind's eye to aid their performance, understanding, or recall.

4 Ask the students to mentally recall the method they use to tie a shoe and/or which way to turn a jar lid to loosen it. Discuss their methods for recalling this information.

5 Ask volunteers to share eaamples of how kinesthetic memories help them to semember.

6 Tell the students that Albert Einstein, the famous scientist, said he relied primarily on visual and kinesthetic thinking in his work. In fact, he told that his most important thought was accomplished with visual and kinesthetic images rather than with words and numbers. Point out that the intent of this course is to provide sensory experiences that will enable the students to develop a collection of meaningful mental and kinesthetic images that will help them (like Einstein!) understand, retain, recall, and invent mathematical ideas and methods.

COMMENTS

3 Students may remember watching athletes (swimmers, skiers, runners, weight lifters, gymnasts, etc.) on television or at sporting events as they practice visualization in preparation for competition. These athletes believe that in order to win, they must first visualize themselves successfully completing each stage of competition. For eaample, before he actually hits the ball, professional golfer Jack Nicklaus always imagines hitting it and, in his mind's eye, "watches" the ball through its flight until it lands.

Prior eaperiences with objects provide a basis for recalling many facts and relationships. Students may enjoy predicting the following by picturing the objects in their mind's eye: On the American flag, is the top stripe red or white? Which direction does Lincoln's head face on a penny? How many sides are on a wooden pencil? On a traffic light, is the top light red or green? In which hand is the Statue of Liberty's torch? etc.

4 Certain information is easier for some people to recall by combining a mental image with "kinesthetic memories." For example, people often remember which way to turn a jar lid to loosen it by re-creating the action in their mind's eye while twisting their hand in midair.

5 Some possibilities are: opening a combination lock, loosening a screw, shooting a basket, or shifting gears on a bicycle.

6 Students may enjoy investigating and reporting on other famous mathematicians, scientists, artists, or writers who "thought in images." A few examples are provided in the teacher background readings, *Mathematics and Visual Thinking* and *Mathematical Swindling*, which are included in *Starting Points*.

The video *Math and the Mind's Eye*, which is available from The Math Learning Center, provides additional teacher background information regarding the role of visual thinking in mathematics.

Connector Teacher Activity

OVERVIEW & PURPOSE

Students fold circular regions and examine and discuss the folded regions. This activity introduces some vocabulary and geometric relationships that the students will work with in the Focus activity.

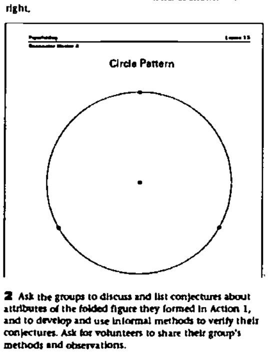
MATERIALS

✓ Connector Master A, 1-2 copies per student.

✓ Scissors, 1 pair per student.

ACTIONS

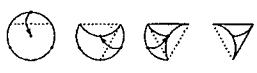
1 Arrange the students in small groups and distribute a copy of Connector Master A and a pair of scissors to each student. Ask each student to carefully cut out their circular region and then to fold it so the 3 points on the circle coincide with the center of the circle, as shown at the right.



2 Ask the groups to discuss and list conjectures about attributes of the folded figure they formed in Action 1, and to develop and use informal methods to verify their conjectures. Ask for volunteers to share their group's methods and observations.

COMMENTS

1 To save time, cut out circles prior to class. Groupmates can provide help to each other if the directions are not understood by everyone.



Note that the folded figure is technically a *triangular region* and its boundary is an equilateral triangle. However, it is common practice to refer to the shape as an equilateral triangle. Similarly, the remaining actions of this activity will produce regions, but we will discuss only the perimeters of the regions.

2 Acknowledge a variety of observations and methods of verification. Here are examples of some possible observations:

The folded region is triangular.

All angles are congruent. To verify equality of angle measures, corners can be traced or folded and superimposed on each other.

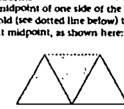
All sides are congruent. To verify equality of side lengths, mark a segment which is the length of a side of the

(Continued next page.)

Connector Teacher Activity (cont.)

ACTIONS

3 Ask the students to locate and mark (by creasing lightly) the midpoint of one side of the triangle. Then have them fold (see dotted line below) the opposite vertex to that midpoint, as shown here:



Ask the groups to discuss and write a list of the attributes of this new shape, and to develop and use informal methods to verify their observations. Acknowledge and discuss the groups' observations and methods.

COMMENTS

2 (continued.)
triangle on the edge of a sheet of paper and then compare that length to the other sides.

The triangle is equilateral. A triangle whose sides are equal in length and whose angles are equal in measure is called *equilateral.*

The triangle has 3 lines of symmetry. These lines also happen to be the *medians* of the triangle, i.e., the segments connecting each vertex to the midpoint of the side opposite that vertex. Line symmetry can be verified by folding congruent halves onto one another.

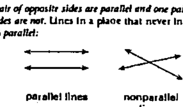
The triangle has 120° and 240° rotational symmetry, with the center of the circle as the center of the rotation. Rotational symmetry can be verified by tracing a frame around the triangle and showing that, in addition to a rotation of 360°, the triangle can be rotated 120° and 240° to fit back into its frame. Rotational and line symmetry were introduced in Lesson 16 of *Visual Mathematics, Course I.*

It is not necessary to make formal definitions or vocabulary lists at this time. It is intended that terminology becomes familiar through use over time. However, students may wish to add new terms to their journal glossaries (see *Starting Points*).

3 Here are examples of observations students have made:

The shape formed is a trapezoid. Note that a *trapezoid* is a *quadrilateral* (a 4-sided polygon) with 2 parallel sides (many textbooks define a trapezoid as a quadrilateral with exactly 2 parallel sides; such a definition disallows classification of a parallelogram as a trapezoid).

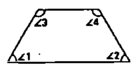
One pair of opposite sides are parallel and one pair of opposite sides are not. Lines in a plane that never intersect are called *parallel:*



One way to verify that two of the sides are nonparallel is by opening the figure back to the original triangle formed in Action 1 to show the sides intersect at a vertex of the original triangle.

Connector Teacher Activity (cont.)

ACTIONS	COMMENTS
	The base angles are equal in measure, i.e., measure of angle 1 = measure of angle 2 and measure of angle 3 = measure of angle 4.

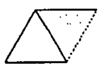


Three sides are equal in length. Note that the trapezoid formed is called *isosceles* because its 2 nonparallel sides are the same length.

The longest side is twice the length of the other 3 sides.

This trapezoid also has 1 line of symmetry and no rotational symmetry.

4 The trapezoid formed in Action 3 consists of 3 triangles. Have the students fold one of the end triangles over the middle one as shown below.



Ask the groups to list, verify, and discuss the new figure's attributes. Discuss their results.

4 Some properties that may come up are: opposite sides are the same length; opposite angles have the same measure; and opposite sides are parallel. This quadrilateral is called a *parallelogram* because both pairs of opposite sides are parallel. It is also called a *rhombus* because all 4 sides are congruent.

5 Ask students to fold the 2 remaining triangles over each other and discuss the results.

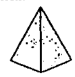
5 The resulting shape (shown below) is a smaller equilateral triangle, whose attributes can be verified as in Action 1.

Note that this new equilateral triangle is said to be *similar* to the triangle in Action 1, because it has the same shape (i.e., corresponding sides of the 2 triangles are proportional and corresponding angles are congruent). Similarity is explored in Lesson 30.

Connector Teacher Activity (cont.)

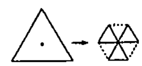
ACTIONS	COMMENTS

6 Allow the 3 triangles to open up and come to a point, as shown below. Discuss.



6 This 3-dimensional figure is a *tetrahedron*, a 3-dimensional figure whose sides are 4 triangles.

7 Now have students open the figure back to the large equilateral triangle from Action 1 and fold the 3 vertices to the center point, as shown below. Ask the groups to discuss, list, and informally verify the attributes of the new figure. Discuss their methods for verifying their conjectures about side lengths and angle measure.



7 This figure is called a *regular hexagon* and the following are some of its characteristics: it has 6 sides that are congruent; all angles are congruent; and it is formed by 6 equilateral triangles. Any *regular* polygon has congruent sides and angles.

Each *diagonal* of the hexagon is a line of symmetry, as are the lines connecting the midpoints of opposite sides. The hexagon has 60°, 120°, 180°, 240°, 300°, and 360° rotational symmetry.

8 Ask students to lift the points folded over in Action 7 and to carefully push toward the center to form the 3-dimensional figure shown here. Ask them to list the attributes of this figure.

8 This 3-dimensional figure is called a *truncated tetrahedron*. It has 3 sides that are isosceles trapezoids and upper and lower faces that are equilateral triangles.

9 (Optional) Ask students to unfold the circles and to identify the polygons formed by the creases.

9 There are several sizes of triangles and trapezoids. There are also parallelograms.

10 (Optional) Give the students a new circle pattern and ask them to find and explain methods for folding the circle to form other shapes.

10 An extension/alternative activity would be to give each student 1-2 additional copies of Focus Master A and have them show an adult at home several shapes (ones explored in class and others) that can be formed by folding the circle, demonstrate to the adult the properties of those shapes, and then write a summary of the experience.

Steps a)-e) on the next page illustrate one method of forming a square and a regular octagon (8 equal sides).

a) Using a new circular region, fold the region over onto itself and crease. This forms a diameter of the circle.

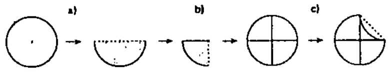
Connector Teacher Activity (cont.)

ACTIONS	COMMENTS

b) Fold the new region over so that the crease lies on itself. Crease again, forming 2 radii that are perpendicular (at right angles) to each other.

c) Unfold to expose creases that form 2 perpendicular diameters. Using an endpoint of each diameter, fold to form a side.

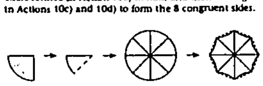
a) b) c)



d) Repeat Action 10c) to form the remaining three sides, as illustrated here:

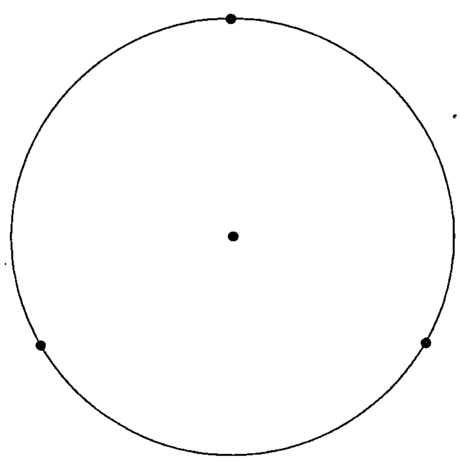
Since the resulting quadrilateral has 4 congruent sides and 4 congruent angles, it is called a *square*.

e) A *regular octagon* can be formed by folding the quarter circle formed in Action 10b) in half, and then folding as in Actions 10c) and 10d) to form the 8 congruent sides.



Connector Master A

Circle Pattern



Warming Up With Slope-Intercept

Judith M. Graves
Butte Community College, USA

Editor's Note: The following is a description of a poster session displayed throughout the ALM-7 Conference.

The ability to interpret and graph linear equations in two variables on a rectangular coordinate system is crucial to the success of beginning algebra students. For adults enrolled in an introductory algebra class for the first time or who never fully understood the graphing in quadrants they were taught as youths, such mastery may be critical to attaining an AA (two year) degree for entry into particular careers or as prerequisite material for transferring to a university to pursue a BA (four year) degree. But how can mathematics instructors teach such concepts with instructional methods that relate to the "real world" and are therefore more comprehensible and more readily retained in their students' long-term memories?

One alternative uses Slope-Intercept form with ordered pairs that relate to temperature conversion formulas (Centigrade to Fahrenheit in this example). Students are divided into groups to discover the equation of a line from as little information as two given points, i.e., (15, 59) and (0, 32). Their collaborative work demonstrates that cooperative problem solving is a powerful tool for interpreting graphs, converting data into equations, understanding independent vs. dependent variables, and explaining thinking visually, verbally, and in writing.

A factor in this activity's success is each group's familiarity with these equations. The numbers are not as basic as those mathematics texts use to demonstrate finding the equation of a line from two points, but students have experience with these fractional slopes, having used them in earlier lessons. Finding the y-intercept becomes a proof that $b = 32$, a fact members observe after drawing the lines on their graphs. If a group recognizes the familiar pattern of the equation, it may discover the real world context because 0 degrees Celsius is equivalent to 32 degrees Fahrenheit. By switching variables, students can see the equation of the line $y = 9/5\ x + 32$ relates to the temperature conversion formula: $F = 9/5\ C + 32$.

This lesson was shared at the ALM-7 conference via a poster session. Samples of student work were displayed, including prerequisite skill development using the temperature conversion formulas and overhead graphs drawn by groups for in-class presentations to peers. Handouts included lesson plans with extension ideas, samples of related assignment sheets, and examples of possible assessment problems.

During the discussion session from 1-2 p.m. on Friday, July 7, conference participants were invited to form groups for solving the sample problem, "It only takes two points." After distributing assignment sheets, including graphs with identified ordered pairs, prerequisite concepts were reviewed. "Students" spent at least 3 minutes on individual thinking before sharing ideas that led to group solutions. Researchers and practitioners experienced why students might receive this lesson more warmly (whether they measured temperature in Celsius or Fahrenheit) than they would one utilizing abstract points with no relation to their cultures. This presentation concluded with an informal conversation regarding the California community college programs, student demographics, goals, and success/transfer rate.

Mathematics for Everybody

Frank Haacke and Mieke de Laat
School for Adult Education ROC Eindhoven, The Netherlands

"Mathematics for everybody" is a cooperative project of Mieke de Laat, Dr. Frank Haacke, and Dr. Harrie Sormani, CINOP. The goal of the project is to give students a change of attitude towards mathematics and confidence in their mathematical abilities. We aim to change the cry of despair: "All those incomprehensible symbols and figures, I always hated them," into a cry of amazement: "It is more fun than I thought. I didn't know mathematics could be like this."

Fortunately, we have had a lot of experience in this field through our practice with adult education. As teachers of basic mathematics we have had to deal with the problem of attitude with nearly all our students.

Our solution, as we are typical Dutch people, has always been "Realistic Mathematics," a movement which was started by Freudenthal, who is also Dutch. However, we experienced how unmanageable reality is. The middle-class culture of our teachers differs considerably from the culture of our students. To design tasks that fit the reality of the students is difficult if you only have the teachers' cultural view.

Action-research gave new insights into the culture of the students. We found that research in itself functions as an educational process for our students. Key skills, especially problem-solving and communicative skills, can be trained when research is an integrated part of education.

The research our students did into their culture was done with the help of two instruments, of which "The Integrated Mathematical Activity" is one and "Hands On Activities," particularly Origami, the other. We have chosen Origami as the basis for our "Hands On Activities" after having met Prof. Eliana Guedes from the University of Taubaté, Brazil, at ALM-4 in Ireland. She visited the Netherlands three times and taught workshops. Frank Haacke saw her work in Brazil last year and is still impressed.

Now, two years later, the math teachers in Eindhoven are still enthusiastic and experiment and research together with their students as a result.

Structure
Van Hiele sees visual structures as the most fitting for learning mathematics. Therefore, knowledge of elementary geometry is essential for our students. He states: "...one should understand the word 'visual' in a broader meaning: every structure is related to our senses....Perhaps it would have been better to speak of sensory structures...." (Van Hiele, 1997, p. 32).

In mathematics the experiences with sensory perceptions and feelings are translated into visual structures. "Hands On Activities" makes the sensory perceptions and feelings conscious so that non-verbal thinking, intuitive thinking, is activated. Van Hiele sees **"nonverbal thinking"** as the first level of learning. "During the study of each subject one bears in mind that at first the student always thinks on a visual level and is totally unapproachable for the words of the **describing level**" (Van Hiele, 1997, p. 181). For adult students this isn't always true!

> The transition from the visual to the describing level demands a conversion in thinking: one must be willing to change the seeing of things into discussing them. This level in the learning process is called "making explicit." In "making explicit relations," relations which were known implicitly in the learning process, but had not yet been given a name, become explicit by discussion. (Van Hiele, 1997, p. 181)

The command of mathematical language makes it possible for students to do research, analyze, and discuss. The students get acquainted with the use of mathematical language during the visual level when they discuss the structures they are observing with others and in doing so support their own thinking process, for now the structure can be visualized with the name of the structure.

A student works on the **theoretical level** when he doesn't need either a visual structure or a description of a structure to solve a mathematical problem. He can then work with formulas and, if necessary, he will know how and when to use them in the real world.

Hands On Activities, Origami

Origami provides concrete structures by folding paper in three-dimensional structures. Unfolding the paper produces two-dimensional structures. By folding the student experiences the structure as a part of himself and activates his motor memory.

Nonverbal Level or Visual Level

During the folding the teacher uses formal mathematical words. The students will experience these words as new and thrilling, but they will also understand them immediately because they can feel and see the meaning of those words. For example: "Fold the bottom-right angle of the paper to the center of the square." The names of the different geometrical structures which can be recognized in the folded paper or unfolded paper are given by the teacher or fellow-students.

Students who are on this level mostly point to the structure with their fingers and outline it to explain what they mean.

Describing Level

On this level discussion takes place and students train to use the formal mathematical words by making explicit the structures they see. It is the teacher's task to activate the students by asking open questions so that *all* students have the chance to discuss and research them. The students observe and analyze a structure and the relationship between structures.

In an adult-group there are different levels. Observing is an essential task for the teacher during the discussion. Amazingly it has been found that dropouts from high school sometimes are on the visual level.

Theoretical Level

The results of the analysis of the structures can be transformed into mathematics; this could be a formula, graphs, or mathematical tables and such. The teacher gives an example and activates the students to do research. Open questions help a lot.

After working with origami we make the students do research into how the formulas and graphs they have learned about are used in their personal and professional lives. Then we can see on what level they are operating and if they didn't just learn a trick.

Repetition appears necessary.

The Importance of Working in a Group

Discussion is necessary to learn new mathematical words. "Language is mainly learned in the course of strongly interactive communication..." (Freudenthal, 1991, p. 180). Without language, thinking is impossible.

During the hands on activities the teacher instructs with formal mathematical language. The students experience this language as a new phenomenon. They favor the formal language for two reasons: first, formal language upgrades their status. They learn like everybody else.

Second, they can express themselves and aren't stuck anymore at the nonverbal level.

> Mathematics can boast a language, so specific that it looks like mathematics itself is a language (which is often, though wrongly, identified with mathematics as such). Language is mainly learned in the course of strongly interactive communication; ...they are steered by reflection on one's own mental activities, which is stimulated by observing oneself by means of observing others, and reinforced by the distant levels of the participants in the learning process. Cooperative learning foreshadows cooperation in adult life and professions. (Freudenthal, 1991, p. 180)

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Numeric Literacy in Two Hours: A Language-Symbol Approach to Teaching Reading and Writing of Larger Numbers

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Valley Opportunity Council, USA

On numerous occasions in teaching Special Education, ESL, GED preparation, and Remedial Math, the author of this article has found learners lacking the skills of reading and writing larger numbers. Many older children and adult learners withdrew from, or otherwise opposed, instruction or remediation that involved manipulables. Also, place-value explanations at times seemed to compound misconceptions. Instruction in place value is often used to introduce reading and writing of larger numbers. However, place value concepts are not essential to reading and writing numbers, and difficulties with those concepts need not prevent acquisition of the practical vocational skills of reading and writing larger numbers in the hundreds of thousands, millions, and billions. The author has found that removing the issue from the domain of conceptual mathematics and treating it as a language-symbol problem can yield quick and lasting results. Moreover, the indications are that anyone who correctly reads and writes all of the numbers 0 to 999 can quickly and easily build on those skills to master numbers into billions and trillions.

Two Ovid computer searches of ERIC provided the background research for this article. Both were keyed to the terms "number$" and "place value." One search covered 1965 to 1984. The other covered 1985 to March 2000. The searches yielded a total of 270 citations, all of which were reviewed. Of those, six were selected for consultation of the full documents; four documents actually treated with the reading and/or writing of larger numbers. Morgenstern and Pincus (1972) maintained a "lack of any organizing principle" in errors of elementary school students. Closer examination, however, might have revealed that each student's errors were consistent (and therefore systematic), though idiosyncratic. Finkelstein (1980), in a text purportedly developed for use by slow learners and students with learning disabilities in high school, instructed learners to read the comma in a larger number as "thousand," but made no mention of millions and billions, nor did he address zeroes, either singly or in clusters. Cooper (1994) did not deal with zeroes either, though he did observe the repeating pattern in three-digit clusters. He did designate commas as "trillion," "billion," "million," and "thousand" without saying so explicitly. Shea and Capleton (1984) designated the three-digit clusters between commas as "periods," and instructed that "when we get to the comma, we say the period name." Once again, however, zeroes went unaddressed, and instruction could be much more expeditious without introducing the concept of periods within a number. For many learners, adding this element to instruction could lead to more bewilderment than clarification. The teacher must always be mindful of the age-appropriateness of any instructional material or technique, along with pragmatic considerations of availability, cost, and ease of implementation. These four factors make chalkboard instruction, with paper-and-pencil practice, particularly appealing, as long as the technique is efficient and effective. The technique described below adheres to these four contingencies while assuring that no gaps are left by leaving information to the learners' inference.

This technique presupposes mastery of the skills of reading and writing the numbers zero to 999. It also treats the reading of zeroes and clusters of zeroes as discrete skills. Consequently, instruction on zeroes will be explained separately below, so only the digits 1 through 9 are used in instruction until learners achieve mastery of all numbers up to 999,999,999,999 without zeroes.

When mastery of numbers up to 999 has been established, write a four-digit number on the board, and separate the hundred's place from the thousand's place by a comma. Teach the students that the comma represents the word "thousand," and as they read the number, they are to say the word "thousand" when they arrive at the comma. An easy way to illustrate and cue this is by writing the word "thousand" under the comma:

4,735
t
h
o
u
s
a
n
d

As the students develop this as an automatic response, introduce 5- and 6-digit numbers. In order to focus learner attention on the digits in the thousand's cluster, cover the comma and the digits to the right of it. After they have read the digits in the thousand's period, uncover the comma, at which they are to say "thousand," then uncover the final 3-digit cluster for them to read. The skill of writing the numbers up to 999,999 should be introduced as soon as reading them is mastered. This is accomplished by instructing the students to write a comma whenever they hear the word "thousand" as numbers are read aloud. After mastery to 999,999, introduce a 7-digit number with a comma to the right of the million's place. Teach that this new comma represents the word "million." Once again, illustration and cueing can be accomplished by writing vertically:

6,231,759
m
i
l
l
i
o
n

If necessary, cover all of the digits and commas that you do not want the learners to attend to, gradually revealing the commas and 3-digit clusters as they read through the elements of the number. At this point, teach learners to write " , _ _ _ , _ _ _ " as soon as they hear the word "million." These blank spaces provide a visible physical structure on which to fasten the subsequent auditory in-put. (This will also greatly facilitate later instruction in how to respond to clusters of 3 zeroes between commas.) When reading mastery is established in numbers to 9,999,999, introduce tens of millions, then follow with hundreds of millions. After reading is internalized, follow with instruction in writing skills. On mastery of numbers up to 999,999,999, extend the same technique to teaching billions, tens of billions, and hundreds of billions. On mastery of hundreds of billions, introduce zeroes. (In the author's experience, the above learning sequence progresses much more rapidly than that which follows.) Point out that since zero means "nothing," we do not say anything when we encounter zeroes. For instance, in the number 7,023 the zero means that we do not say the word "hundred" in the cluster where it occurs.

In a separate lesson, teach that the occurrence of three zeroes between two commas means that the name of the comma to the right is silent. For example, 23,000,417 is read "twenty-three million four hundred seventeen." For some learners, it might be necessary to augment the auditory in-put with visual by covering the mouth with a hand. For some learners it will be entirely appropriate to include instruction on how to write numbers that have two or more clusters of zeroes, depending upon vocational goals.

As the successive reading and writing skills are mastered, the teacher must integrate them into math, reading, writing, and content-area instruction. The author (teaching adults in a state that offers major lottery jackpots) has found that learners can compare jackpot amounts and also read and write the amount awarded to each member of a group purchasing a winning ticket. Government expenditures and budget deficits are also ready and current discussion topics into which to integrate these numeric skills. In the course of this instructional sequence, learners are exposed to new language phenomena. The author has found that instruction and explanations are most effective when delivered in the learners' own terms. Therefore, it is advisable to let the learners name these phenomena themselves out of their own experiences and vocabularies if they choose to do so. If learners are not forthcoming with these names, then alternatively the instructor can offer names that relate to some shared experience of the group.

The author has used the above-described technique with learners of a wide range of ages and cognitive levels and in a variety of settings. It is demonstrably an efficient and effective instructional device for meeting short-term practical learner-centered goals. However, research could reveal it to be a mechanism for acquiring skills that could lead to successful conceptualization of place value. Perhaps conventional practice places the cart before the horse in teaching place value as an entry behavior for numeric literacy when, in fact, learners should be using these practical reading and writing skills as a foundation on which to build concepts of place value. This would conform with the basic pedagogical principle of movement from the concrete to the abstract.

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Connecting Students, Sense, and Symbols: A Workshop of Practical Activities From Personal Experience, and Informed by Research

Beth Marr
RMIT University, Australia

Connecting Students
A great deal has been said about the benefits of group work in mathematics learning. Indeed, I have found it a wonderful way to conduct major sections of all my adult classes, both in basic education and pre-service teacher education. In my own classrooms, and in those I have observed through research, I have found many positive benefits from students working together on a variety of structured group problems. The comments which follow are drawn from recent research (Marr, 2000, see elsewhere in this volume) in which I observed the differences in student interactions when engaged in traditional worksheet tasks, in contrast to small group activities.

Collaborative Structures Change Quality of Student Interactions
I found that explicit group and pair structures tended to change the quality of students' interaction and collaboration from that which occurred when using worksheets. In traditional interactions resulting from worksheets, one student tended to wait until another had worked out an answer alone before requesting help, and then was likely to receive more fully thought out opinions or procedures, "explanatory speech" (Barnes, 1976). It seemed that collaborative structures encouraged students to share thoughts whilst they were still in the formative stages, "exploratory speech" (Barnes, 1976). This exploratory interaction style put the students on a more equal footing in the early stages of engagement with the task, and tended to change the power dynamics. Evidence of "intersubjective" meaning making (Cobb, 1995) or jointly constructed knowledge (Mercer, 1995) was found in excerpts of students' dialogue. For example, interrogative phrases like "that's what you are saying aren't you?" and "is that what you mean?" indicated that the students were clarifying ideas together and building on each other's explanations. Thus it seemed that the asymmetry of power and knowledge was less defined in group and pair tasks than it was in worksheet situations.

Group Collaboration Expands the Range of Interactive Roles
Group collaboration also tended to expand the range of interactive roles students could take on, thereby increasing their means of actively contributing to discussion and signaling their participation. Some took on social roles within the group: explaining the task to latecomers, keeping the group on track, and asking clarifying questions. Others contributed differing types of knowledge. For example, some contributed knowledge drawn from their use of measurement at home, social discourses (Lee, 1994), exemplified by contributions about cups and cubic metres of soil. Others contributed from their knowledge of the mathematical discourses such as fractions and areas. These roles were not assigned, or even discussed, in this classroom, but occurred spontaneously during the group work. Their existence was noted only after analysing tapes and videos of the group interaction. The emergence of these different roles within the task interaction allowed more students to contribute positively to the group and to learn from others whilst doing so.

The notion of contributing and learning as a peripheral participant (Rogoff, 1995) was illustrated by one student who took on a social facilitation role in the small group. This role allowed her a way to participate actively although she had little confidence in her knowledge of the topic. In doing so she signalled her involvement and learned from the discussion of the other group members. These multiple roles in the small groups appeared to allow for diversity of participation, in contrast to the situation of "primary knowers" and those who asked for their help (Veel, 1999), which predominated in worksheet situations.

Catering for Individuals Versus Social Needs
However, adult classes are often made up of students with a wide range of needs, and levels of knowledge. This tends to mean that educators are swayed towards providing individual work that satisfies the expressed needs of each student, and deciding that working in groups will not be appropriate. Although I agree that catering for

each individual student is essential in most adult situations, I also think it is important to consider making connections within the groups as well.

I believe that for many adult students there is an unspoken social need that is integral to their return to study. Leaving the isolation of the home and mixing with others seems an important part of the new learning experience. In my experience the need for this aspect of returning to study is especially true of adult women students. If their classes are organised to include some time in which students mix with each other in structured group and/or pair activities then they have opportunities to create the necessary social links. Mixing purposefully with other students allows for the creation of personal connections through sharing information about one another. Beach (1992), referring to this type of interaction between students as "self disclosure about their lives and experiences" (p. 99), asserts that in some circumstances the social bonds created make students more willing to listen to each other's subject related ideas. So not only does such conversation create possibilities of social communication and friendship but it also creates links which facilitate students helping each other with the subject.

My own teaching experiences with adults indicate that group activities, which foster discussion through hands-on activity, can change students' attitude to the subject and make the classroom atmosphere lively, cooperative, and enjoyable. In the classroom research I found that not only did sharing the materials bring students together to solve problems, the materials were often a trigger for other forms of communication. For example, sharing memories of childhood games was provoked by the string used for area and perimeter investigations, and the plumbing pipes fitted together to model a cubic metre facilitated recollections of building cubby-houses and many jokes about children. These connections with childhood pursuits could be seen as negative distractions if they continued for too long. However, this was not usually the case, and it is also possible to view these off-task moments from the perspective suggested by Baynham (1996). Baynham refers to "identity work": interchanges between the adult class members and the teacher that enabled them to see themselves in equal adult roles outside of the classroom situation. Baynham sees this identity work as an important aspect of students maintaining their self-respect and adult status in the potentially disempowering classroom situation. Similarly, Benn (1999) refers to establishing "cultural touchstones," a range of common interests between teacher and students, which are referred to in classroom communication and act to bridge cultural and status gaps. Thus sharing stories of common childhood experiences or "positioning" the students in the parent role (Walkerdine, 1988) can be seen as a valuable dimension of the class. In this case it was facilitated by hands-on materials.

Household containers and bottles used in several activities also seemed to provoke a sharing that "positioned" the students in the adult world. For example, alcohol bottles, ice cream, and yoghurt containers encouraged exchanges about drinking habits and food preferences. These off task exchanges, triggered by the introduction of real world artefacts into the classroom, allowed for the personal disclosure, identity work, and creation of cultural touchstones described above.

Connecting Sense and Symbols

The examples discussed illustrate valuable connections between people, the teacher and students, that can be facilitated by group and pair work using reality based situations and artefacts. If, in addition, the group and pair activities are selected to encourage students to make meaningful connections between their lives, mental images, and the symbols and formulae of mathematics, then the time is doubly well spent.

In the research classroom real-life materials such as the domestic containers seemed to contribute positively to the mathematical dimensions of the tasks for these adults. The data revealed the domestic artefacts acting as "footholds" from familiar discourses into the mathematical activity (Boomer, 1986). Or, put another way, the students could "position" themselves in a field where their prior knowledge or conceptual frameworks were connected with the current mathematical meanings. For example, their practical knowledge of familiar containers assisted estimating unfamiliar volumes. The deceptive visual impact of tall thin cylinders was "noticed" because it resonated with students' adult consumer awareness. This not only allowed for discussion of packaging strategies, but also for formulating hypotheses about shape and volume relationships. It also created continuous opportunity to develop students' mathematical language. These were positive illustrations of linking "spontaneous" and "scientific" concepts (Bruner, 1984; Vygotsky, 1962).

Yet another unexpected effect of using the familiar domestic and other hands-on materials was the triggering of memories from one session to the other which seemed to be created through the visual impact of the physical artefacts. The lasting memories appeared stronger than any that the words and diagrams traditionally used on worksheets had provoked, since students' speech in later tasks contained many direct references back to prior visual memories. Their hold on the mathematical ideas and relationships seemed to be strengthened by association with previous instances in which they had arisen. For example, the food and drink containers used in the estimation task became synonymous with cylinders and were visualised by the students in later tasks. The string used to model "fencing," or perimeter, was also referred to in later situations.

Students who made connections between the tasks seemed to be showing evidence of greater "ownership" of the relationships and the language. These observations agree with suggestions by Brown, Collins, and Duguid (1989) that exposure to conceptual language in a variety of different situations strengthens and broadens students' understanding of mathematical terms.

This workshop offered a selection of group and pair activities designed to foster discussion, encourage visualisation and estimation, and facilitate a sense of mathematical meaning beyond formulae and symbol manipulation.

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Having Some Fun With Maths – The Aussie Way

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This practical and hands-on workshop presented a number of examples of student activities that illustrate different strategies that can be used to support successful adult numeracy teaching. They are based upon the following approaches to the teaching of adult numeracy.

Maths Language
Encourage and use familiar and relevant language in the classroom. "Talking maths" and sharing words and meanings between students, and with the teacher, is the best way to overcome difficulties in comprehending maths language. Activities that support and encourage this should be used.

Co-operative Work
Get students to work co-operatively together to encourage interaction and discussion, and hence help learn from each other. This can be in pairs, small groups or even as a whole class group. This approach encourages learning through communication from a number of sources, not just the teacher or textbooks. Students will learn by having to explain their ideas to each other.

Enjoyment and Success
Many adult students have had negative experiences of maths teaching and may suffer from maths anxiety. It is important therefore to provide an exciting classroom atmosphere with a range of activities and teaching experiences which stimulate interest, discussion, and active learning. Most of all learning should be fun. Maths activities, including games, can demonstrate concepts whilst providing an opportunity for students to interact in a relaxed and enjoyable way.

There is nothing better than getting students in the group to experience success in solving problems, either in a group or individually. This will build confidence and help overcome any maths anxiety they may have.

Practical and "Hands-on" Materials
Remember that most adults were taught maths by traditional pen and paper methods and were expected to remember rules and formulas by memory without ever really understanding them. Concrete materials are a great way of explaining to adults why things really do work. If you don't understand a concept properly to start with then consequent learning becomes an almost impossible task.

Relevant Contexts
Try to place maths learning in a context relevant to the students, drawing on students' backgrounds, interests, and experiences. This includes placing mathematical ideas into an historical and social context. It could take into account such maths related activities as, for example, shopping and banking, measuring, cooking, the weather, reading timetables and street directories, following directions, and sports activities.

Students can learn about relevant topics while doing mathematics. Many areas of knowledge require an understanding of maths concepts and skills. Areas such as the environment, health and diet, geography, statistics, and public and political processes can all be used as content areas of mathematics.

Teach by Understanding
Teach by understanding, not simply by a reliance on memorisation. Often people have learnt the "wrong" way to solve a problem and until they are shown why their way is wrong and *why* your way works they will have difficulty in coming to terms with the correct methods. Although the ultimate aim may be to know some facts

off by heart (e.g., times tables) the way to achieve that is to ensure that initially the reasons and understanding behind what is happening and why you do something is clearly explained and understood. The memorisation can then follow through practise and reinforcement.

Sample Activities
Activities selected from the following list will be demonstrated during the session.

Data and graphs

- Average wages
- Pumping petrol

Cooperative logic problems

- The flats
- The supermarket
- Cities
- What's the number
- Where is Grandma's House?

Decimals

- It was one of those days
- Double zero three and nine
- Target 100
- Decimal dilemma

Dice games and place value

- Multi-digit
- Double digit
- Dicing with decimals

The above activities are taken from the following Australian adult numeracy resources:

Goddard, R., Marr, B., & Martin, J. (1991). *Strength in numbers.* Melbourne: Adult, Community and Further Education Board.

Johnston, B., & Tout, D. (1995). *Adult numeracy teaching—Making meaning in maths*. Melbourne: National Staff Development Committee for Vocational Education and Training.

Marr, B., Anderson, C., & Tout, D. (1994). *Numeracy on the line*. Melbourne: National Automotive Industry Training Board.

Marr, B., & Helme, S. (1987). *Mathematics: A new beginning*. Melbourne: Northern Metropolitan College of TAFE.

Marr, B., Helme, S., & Tout, D. (1991). *Breaking the maths barrier*. Canberra: Department of Employment Education and Training.

TECHNICAL AND VOCATIONAL EDUCATION

The Cargo-Master: Mathematics in Technical Vocational Education

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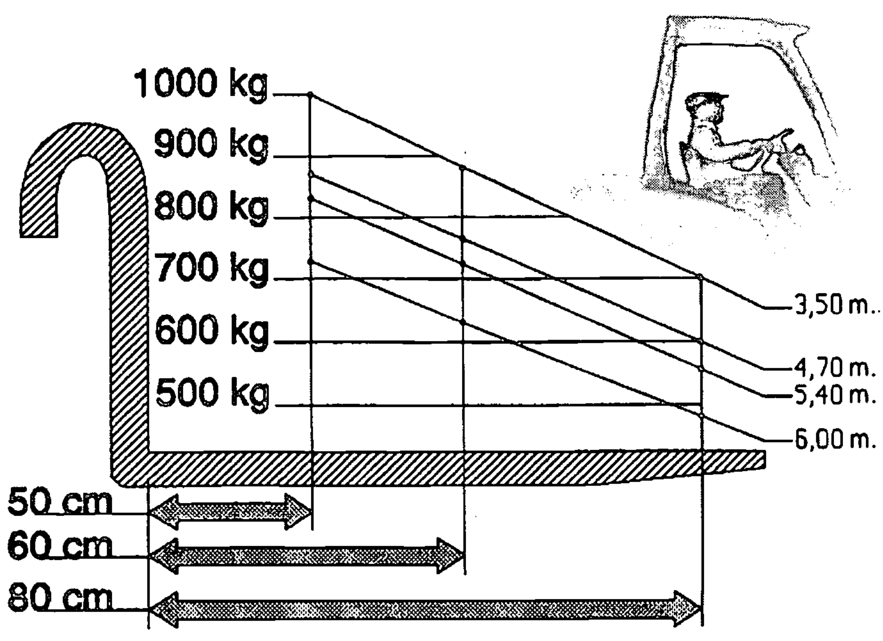
Introduction
Mathematics is an important subject in technical vocational education. In lots of technical professional activities you need mathematics to perform well and so you need to learn a lot of mathematics in technical education. In this paper we agree with the great value that mathematics can have for a technical vocational education. But we don't think that the schools always select the most important mathematical knowledge and skills or that they teach in the best way. We think that a different approach gives better results. By using an example from a forklift driver's course we want to show how a teacher can analyze what kind of math is essential and also how he or she can teach it in the best way.

The Problem
As an example of a vocational education problem we take a problem from a forklift driver's course. For a forklift driver the most important question in his work is how much weight he can lift before his forklift tumbles down. For the answer to this question he needs to know how many kilograms he can lift. For the solution to this problem he can use a graph at the back of his forklift. This is not so easy, because it is a complicated graph with three variables: the height, the weight, and the last distance. If the forklift driver can read this graph he knows how high and how much weight he can lift when he knows the last distance. (See figure.)



In most courses they concentrate on arithmetical problems and don't give a base or the insight into this kind of problem. We did it in another way. Before we started the course we did some research. We tried to find out what kind of mathematics you need to solve this problem as a form of mathematical forklifting.

Mathematical Forklifting

In our analysis of the problems of the analysis we draw a distinction between the more general mathematics that are needed and the more specific mathematical knowledge that is needed in the situation of the fork-lift driver's course.

The most important general skill is that you need some courage to attack the problems with the mathematical figures. You must not be afraid of math in general and graphs in particular. For this aspect it is important the course did not start with only figures. Less educated people are often afraid of this form of mathematics because they could not understand these things at school.

The forklift driver also needs some general problem solving skills. Before he does anything he first has to decide what he will do, then he has to do some calculating, and finally he has to decide what the results of his calculations mean in practice. It is very important that students master this "Plan, do, and review" principle. The students must know that they first have to plan their activities, that they have to decide what is important to know and which figures matter in this situation and what kind of mathematical procedures they have to use. In phase two they have to do the mathematical activities such as the calculations. And in the last phase they have to look back and ask themselves: What is the meaning of the figures that I have to know?

At a more specific level it is in the first place important that they know how to read a table and a graph. They also must be able to make simple graphs by themselves. In this way they get a real understanding of this subject. The students must also be able to put in practice basic mathematical skills like multiplying and dividing, but they are always allowed to use a calculator.

Third, they have to know the metric measures meter, kilogram, and centimeter, but that is not the most important part of measuring. The students also have to know that measurement is a real activity and that the measures have a meaning in real life; 600 kg is not only a number but it's also something that is real and something that you can imagine. That is the reason why a course of measurement has to start in a real life situation.

Didactical Forklifting

In most vocational education there is no time for the student to invent some mathematical ideas. The teacher really starts to explain how it is and then the students have to imitate the behavior of the teacher. The knowledge and skills the students learn in this way are not deep but superficial. We think about another approach with three essential characteristics.

The first one is that the students have to do something with their hands. Mathematics must not be only a pen and paper activity but there have to be also some "hands on" activities. In this way the students get a concrete base for their knowledge. In our forklift course we started with an experiment with the cargo-master, a little model of a real forklift. The students have to find out how many nuts and bolts the little cargo-master can lift. In this way they get a real understanding of the problems at their work.

The second characteristic is that the students have to interact with the other students and with the teacher. In mathematical education it is really important that the students learn how they can put their thoughts into words. For that reason it is important that there is some discussion in small groups and they also can present something to a group. In our experiment with the cargo-master the students first have to do their work in small groups and after that they have to present their results to the whole group.

The third characteristic is that our course does not start with the solution of the problem and the formula but that that is at the end of the course. First we give the students the opportunity to make a base for the understanding of the graphs. They can try out their own solutions and learn in that way what works well and what is not working. So they don't need tricks but they can use their knowledge in practice.

Conclusion

Mathematics is important in technical vocational education. It could improve the professional activities of the students. But then the schools must select for every vocation only the essential math. They also have to choose a didactical direction, which makes the students more active and gives the students more responsibility for their own education.

Women in the Urban Informal Sector: Effective Financial Training in Botswana

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Introduction

Ample evidence indicates that in Botswana, women are not well represented in occupations requiring mathematical competence such as computers, finances, and engineering. The Government of Botswana's Central Statistics Office (1998) reported that of the 345,405 in employment in 1995/6,[1] 189,301 were men and 156,104 were women. The report states:

> The largest employer of females is Education, with 62.3 percent of its workforce being females. Females employed in the Wholesale, Hotels, Restaurants & Trade accounted for 59.3 percent of the total employment in this industry. Private Households is also one of the major employer of females, with 89.7 percent of its employees being females. Health & Social Work was also one of the industries dominated by women. The male dominated industries are construction (68.5 percent), Agriculture (68.9 percent), Mining & Quarrying (84.3 percent) and Businesses Services (75.5).

In the primary schools, girls and boys are nearly equally represented: male 161,497, female 160,771 (Central Statistics Office, 1999), but gender imbalances exist in the secondary and higher educational institutions. The University of Botswana 1999/2000 enrolments (1999) show the following distribution of male and female students:

Undergraduate, full time	**male**	**female**	**total**
Faculty of Business	378	309	687
Faculty of Education	729	900	1,629
Faculty of Engineering & Tech:	877	108	985
Faculty of Humanities	691	932	1,623
Faculty of Science	620	614	1,234
Total	**4,301**	**3,220**	**7,521**

With regard to attitudes about mathematical capacity, literature states that there is an often openly voiced belief that since women are not good in mathematics, they do not and should not pursue careers in which mathematics plays an important role. Walden and Walkerdine (1986) argue that "...there is a particular combination of classroom practices and an understanding of mathematical learning which produces failures in girls, and that in consequence girls are put into the position of being successful but not succeeding" (p. 124).

However, women are increasingly owners of micro businesses in the informal and formal sectors—women own 75% of the estimated 50,000 micro businesses in the country (Jefferis, 1999). They are confronted with the need to maintain accurate financial records, decide on proper pricing of goods and services, and make financial projections. It is assumed that in Botswana the bias against their doing well exists from within—expressed in the form of a lack of self-confidence—and from without—resistance from institutions, the public, other businesses, and training organisations. Adequate training is thus a need for these women, but a recent report of the Government's training policy for the informal sector conducted by PEER Consultants (1997) has noted that quality, access, and relevance of training are major areas of concern.

[1] Employed persons include: Central and Local Government, Parastatal, Traditional Agriculture, Informal Sector, and Other Private.

Training is a significant consideration when taken in conjunction with the Government of Botswana's policies regarding the importance of encouraging sustainable small businesses; it has been actively supporting their formation since the country's independence from the United Kingdom in 1966. There are several reasons for such support: these businesses can help in creating employment for its citizens because of their flexibility, ability to specialise, and their local nature, thus providing more work notably for women and other disadvantaged groups. They also promote the economic diversity needed for a healthy economy.

The most recent Government initiatives aimed at developing a strong business community appear to be comprehensive and cover the issues from several major directions:

- 1997 ***Industrial Development Policy***;
- 1999 ***Policy on Small, Medium and Micro Enterprises in Botswana*** (SMMEs) is interrelated with the *Industrial Development Policy* and is concerned with four areas needed to help form and strengthen the SMMEs: **institutional; financial; education; skills development and training;** and **regulatory.**
- 1998 ***Vocational Training Act.***

Despite the many attempts of the Government, training at this time is still rather eclectic and ad hoc. The VTCs and Brigades provide vocational training for people up to the age of 25. This age restriction can be problematic for many women who may have had to start formal training at a later age because of family responsibilities. Both the VTCs and the Brigades do now offer a short introduction to entrepreneurial skills and financial requirements of a business, and more courses are being added as a result of the above-mentioned new Government Policies and Acts. Many NGOs (non-governmental organisations) provide training of specific vocational and entrepreneurial skills with little co-ordination or certification of the courses. The private sector, which offers a wide range of courses,

Even if women do enter formal vocational training institutions, one finds the more expected distribution along the lines of gender: males predominate in courses like welding and construction, while women are primarily enrolled in clerical and secretarial courses (Government of Botswana, Education Statistics, 1995). The effect is that women become employed in less lucrative jobs, and if they do start a micro business, it is likely to be one which is also less lucrative as the women often form a particular business based on skills gained from formal sector employment. The Women's Affairs Department of the Government (draft document, 1999) notes that men earn more than women in all sectors (p. 126).

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It is reasonable to assume that women will continue to be involved in SMMEs, particularly on the micro level; that training will be an issue; and that they will continue to need mathematical skills and financial knowledge.

Importance of the Research

This research is aimed at developing a guide for financial training programmes of informal sector women business owners, and to help ensure a higher rate of success than is presently being experienced. Present statistics indicate that 80% of small and micro enterprises cease trading within 5 years (Republic of Botswana, Small, Medium and Micro Enterprises Task Force Report, 1998, p. 11). There are an estimated 50,000 micro enterprises in Botswana. For these enterprises, 75% of which are women-owned (p. 9), records are too poorly kept to know definitely if they fail or not, but other factors indicate that at best the business is on a subsistence level.

The Research Questions

The following research questions used in this study are:

1. **What are the underpinning educational and social-psychological theories of how girls and women learn mathematics?**
2. **What financial knowledge is needed for women owners of micro businesses?**

3. **How do women micro business owners in Botswana learn the mathematics and financial management skills necessary to run a business?**
4. **What are the most effective training approaches given the above?** What changes would be most useful both in training the trainers, writing training materials, reconsidering the location, cost, and length of training?

Description of the Research Method and Scope of the Study
This study is qualitative. A sample of 60 businesses in four urban locations meeting specific criteria have been used to address questions 2-4 above. The criteria were: size, location, kind of business, and length of time the business has been in existence. Because the problems of an urban village and an urban town are different, it was necessary to examine both to provide workable solutions that accommodate these separate conditions.

Representative training programmes, training providers, and trainers have been investigated using questionnaires and observation of selected training courses.

Research Design
This is a qualitative study, with data from previous research used to establish certain base points and determine necessary background information, for example, data from previous studies on the informal sector in Botswana, and the analysis of the effectiveness of training methods in teaching financial skills.

The Research Model
Qualitative Research has been identified by the researcher as the most useful research method in discovering the main attitudes, beliefs, and practices of the women and training institutions involved in this study. According to Sherman and Webb (1988), definitions of qualitative research may vary, but there are similarities no matter who is defining the theory. One of the commonalities is *context*. Referring to Shimahara (1988), Sherman and Webb say that "human behavior—experience—is shaped in context and that events cannot be understood adequately if isolated from their contexts" (p. 5). An investigation which is attempting to identify attitudes and practices which are usually not articulated by individuals must be investigated in the context of the person's experiences. Therefore, variables cannot be isolated or stripped from their context, a key factor of science and quantitative investigation.

Another commonality is that of natural inquiry: Sherman and Webb argue that "qualitative research is not verification of a predetermined idea, but *discovery* that leads to new insights. Thus qualitative researchers focus on *natural settings*" (p. 5). This study is attempting to discover how women acquire financial skills and knowledge; this must be discovered—while there are basic assumptions, the process of discovery is key to this research.

Other commonalities of qualitative research—*holistic experience; speaking for oneself;* and *non-intervention*— all relate to this study. It is not possible in the view of the researcher to identify hidden attitudes without both an in-depth interview and observation of what actually takes place in the daily running of the business, nor is it possible to learn how the women owners feel and act without understanding the source of their attitudes.

Research Sample and Procedures
There are 3 main domains of investigation excluding the theoretical information required: first, women business owners and how they acquire financial and mathematical knowledge; second, identification of the financial knowledge needed to operate a successful micro business in Botswana; third, training—curricula, trainers themselves, courses offered. As each domain has its own unique considerations, the methods of research will vary.

Business Owners and Sample
A sample of 60 businesses has been selected: 15 for each of the four localities as described below, chosen from three main sub-sectors which have been defined in the Botswana Adaptation of the International Standard Classification of 1997:

Division 3:	Manufacturing
Division 6:	Wholesale and Retail, Hotels, etc.
Divisions 7 – 9:	Transport, Finance, and Services

Regarding the location of the businesses, an important consideration relates to the possibilities of cultural differences. For the purposes of this study, the places selected are generally grouped as "north" and "south." "South" is broadly meant to represent towns and villages made up of people from similar ethnic backgrounds, with a common language, and, because of closer proximity, who have been more influenced by South Africa. "North" is meant to represent people in the Francistown area who have a different ethnic background, have a different language, and would be more influenced by Zimbabwe. Although all speak Setswana, those in the north are the Kalanga people who have a different language from Setswana.

For the South:

Kanye is a large village located 50 km from Lobatse and 100 km from Gaborone with an estimated population of 40,000. It has a well-developed business sector and has access to training facilities within the village.

Gaborone is the capital of the country with a rapidly growing population currently estimated at over 200,000. As the capital, it provides much of the training facilities in the country, access to government departments, larger markets, better infrastructures such as transportation, but also greater competition and more expensive business premises and accommodation.

For the North:

Francistown was a town founded before independence, unlike Gaborone which was established specifically to become the capital city at the time of independence. The population of Francistown is estimated as 100,000. Like Gaborone, it is on the main tar road and rail line that goes through the country—from Zimbabwe to South Africa. The majority of people in Francistown are Kalanga. It will soon have the first vocational-teacher training college in the country.

Tutume is a village located 60 km Northeast of Francistown towards the Zimbabwe border. The estimated population is 15,000. Tutume has one Brigade teaching basic vocational skills. The student enrolment in 2000 is 194.

Women are operating businesses in both villages and towns, and both have common as well as distinct sets of problems. Problems of premises, licenses, supply, advertising, competition, transport, access to training sources, pool of employees, markets, Government departments, selection of business, and possibilities for growth vary according to the location of the business. Problems vary depending on what kind of business is being run: manufacturers have a different set of problems than do retailers, and these are complicated by factors related to location.

Training organisations: it has been necessary to investigate training organisations—public, parastatal, private, and NGOs—to see what training strategies are already in place, what problems are encountered, and what gender differences may be identified.

Assumptions

One assumption is that most women have poor knowledge of mathematics and a lack of confidence in their ability to learn. As a result, it is hard for them to acquire financial knowledge and skills. Further, it is assumed that they have been adversely influenced by a math bias, originating from both the culture and the schools, and this is the root of the negative attitude towards their ability to learn mathematics.

It is further assumed that financial knowledge and skills are obtained in most instances non-formally, through their own discussions with each other and observations of other businesses. Networking and mutual-help are assumed to be strongly used in women-owned enterprises—they give help to each other routinely. Even when formal training is given, the knowledge would be taken by one/few, translated into understandable concepts, and passed from one to the other through informal discussions.

Another assumption is that if this is correct, these approaches must be incorporated into the training courses and that active steps have to be taken to overcome the self-doubts and lack of confidence assumed. A one-week course is assumed to be inadequate. Mentoring, planned follow-up, and incorporation of approaches that duplicate the effective ways in which they learn should be the basis of the course. Training should be done using problem-solving techniques; with a heavy reliance on case-studies relating to the particular business; field observations of other businesses; of encouraging networking and small business co-operative groups. This needs to be tested by questioning trainers, business owners, and appropriate government sources.

Preliminary Findings

The first stage of the research has been completed, and findings indicate that training, although available in various forms, is often inaccessible because of logistical problems—women cannot leave their businesses for extensive training. Further, the knowledge of effective use of finances is very selective. For example, several owners transport their goods back and forth each day to and from home and business. This is costly and these costs are not included in pricing the services or goods. Another example is that of credit: most people are paid once a month causing a cash shortage towards the end of the month. The owners extend credit to customers without good records and in many instances they are not recovering all payments. These losses are also not included in the costing. The choice of businesses is a random affair, usually based on other success stories. Costing is done on the basis of competition—many of the businesses charge the same price, but it is not known if the price is profitable or not. Learning is accomplished in an extremely informal fashion—through trial and error, example, and networking.

Available training through the SMME programme is only one week long and there is no follow-up to see how the business is doing. Some institutions, notably RIIC in Kanye, do provide more in-depth training and systematic follow-up to the trainees.

The final stage of this research will be to bring the women owners together for group discussions on what they need in relationship to training. This will provide the framework for the recommendations and the draft training curriculum being formulated. On a final note, it was most encouraging to observe that in spite of constraints of training, education, and financial knowledge, a number of women have been able to expand their businesses. It is these successes that can be incorporated into the training programmes and final recommendations.

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The KAM-Project:
Structure of the Swedish Upper Secondary School

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An Example

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Abstract

In Sweden vocational programs have been included in the upper secondary school since 1994. In these programs all students have to study mathematics approximately 100 hours. Nearly 50% of those students fail in the national mathematical tests in this course (Course A). That is after already 900 hours in the compulsory school system. The content for this session is from a development project funded by the Swedish National Agency for Education. Its main focus is on studying the relationship between mathematics and the vocational subjects. Part of this is to improve and develop mathematical models that are relevant and applicable. The outcomes stress changing the approach of learning and teaching and assessing with a focus on using integration while still keeping a door open to further education and to the more formal side of the subject. Since this project started the syllabus has changed in the same direction as this project, which of course is of great importance for our future work.

Structure of the Swedish Upper Secondary School

The Swedish upper-secondary school system is composed of national study programs consisting of different courses. The programs are of three years duration. Students can choose from sixteen national programs. Fourteen programs contain vocational subjects, and in these programs students spend at least fifteen weeks doing work experience at places of work outside school.

Mathematics Courses

Mathematics in upper secondary school consists of five units. The units are progressive, but can also be looked upon as a final course. Most pupils in the natural science program are taking all five courses. Course A has 100 points, Course B has 50 points, Course C has 50 points, Course D has 50 points, and Course E has 50 points. Course A is the lowest. Course A (100 hours), the only compulsory course in mathematics, is in most of the vocational programs.

About 65% of the mathematics lessons are the secondary school in course A, and half of them in the vocational programs. Course B is about 12%, course C about 13%, and course D and course E together about 12% (Grevholm, 1999).

The demands from society and the workplace are harder, and to meet some of those needs course A is compulsory in all programs. This means that it is the same content of Mathematics whichever program you study. Moreover, many students leaving year 9 do not have the mathematical knowledge that they are expected to have. But they might have a pass degree from the compulsory school. Mathematics in the compulsory school consists of 900 hours.

The Grading System

Grades in every course in the upper secondary school are awarded according to the following scale: Pass with special distinction (denoted by the letters MVG), Pass with distincion (VG), Pass (G), and Fail (IG).

New Curriculum, New Problems, New Opportunities

Currently a great number of the pupils who attended the vocational and trade stream at the upper secondary school failed mathematics and got the mark IG. Previously those pupils who had not achieved in mathematics or who were not motivated to learn it could choose not to study mathematics. However, this alternative does not exist according to the new curriculum, Lpf 94, as Course A in Mathematics is compulsory in all programs. For these pupils it is especially important that they are confronted with a kind of mathematics that starts on their own level, which in some cases may be a level corresponding to a year 5 level course. It is also important that the teaching of mathematics be given a new aim and direction so that it will be quite applicable in the vocational subjects and will support them in their vocational skills.

In England, problems with pupils in the preparatory vocational track of the upper secondary school are similar to the problems identified in Sweden (Steedman & Wolf, 1998).

In 1994 a new curriculum was introduced in Sweden, in fact two curricula, one for the first 9(10) years, i.e., Lpo 94, and Lpf 94 for the next 3 years. There is an emphasis on sharing and crossing borders in different areas.

> Development in professional life implies that you need to cross the borders between different areas of different professions, which is a big challenge for planning how to work at school. (Lpf 94)

In the vocational programs the learners are taught mathematics by an academically trained teacher and sometimes in the vocational course by an autodidact. This fact might be a problem. The mathematics teacher does not know what kind of mathematical content the student needs in the vocational course and the vocational teacher has very little and limited knowledge of mathematics and didactics of mathematics. The vocational teacher has never been trained to teach mathematics. (Mathematics is taught according to the content in course A.) The mathematics teacher does not know the content or very little of the vocational subjects and the vocational teacher does not help the learner to link the content that is taught/learned to the course where it could/would be applied.

Upper Secondary School for Everyone

Nearly 100% of the population between 16 and 19 years in Sweden is studying at the upper secondary school. Today all communities in Sweden are by law obliged to offer all students that have completed the compulsory school a place in the upper secondary school. The Swedish upper secondary school will give basic knowledge for professional and social life.

Below are examples of tasks from a national test for Course A. This is to give you an idea of the content and the demands for grade pass in course A. The test has three parts with different characteristics. The examples below are from part 1, a "numbers sense type test" with no help of a calculator and a time limit of 25 minutes. Only an answer is required.

Some Examples From a Swedish National Test Course A.

Write three hundredths in decimal form.

How much is the half of $\frac{1}{4}$?

Mia shall dilute cordial. How much squash will she gets if she follows the instruction on the bottle.

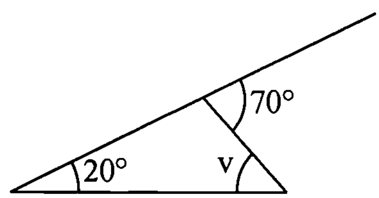
Which of the following expressions can be written as x^3 ?

$3x$ $\quad x \cdot x \cdot x$ $\quad \frac{x^6}{x^2}$ $\quad x + x + x$ $\quad x^2 + x$

Solve the equation $7(x-4) = 49$.

Calculate the value of $5 \cdot 10^4 + 2 \cdot 10^3 =$

How many degrees is angle v?



A hair is growing 0.4 mm per day.
How many days will it take for theto grow 1 cm?

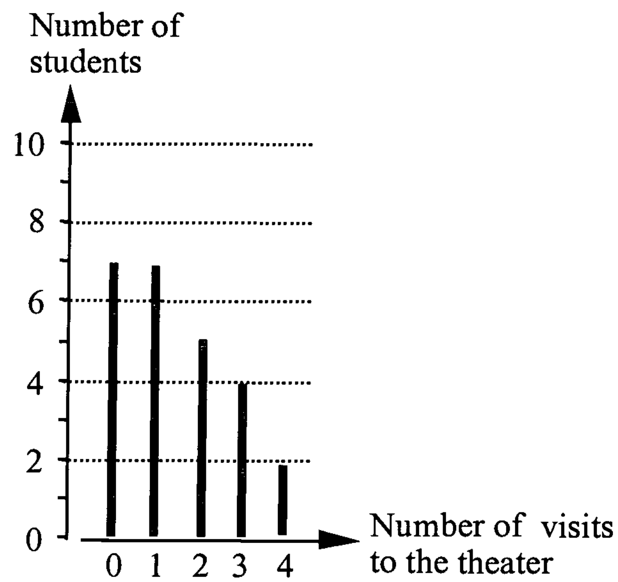
Which of following **calculations** give the highest result ?

$\frac{32}{1,8}$ $\quad \frac{32}{18}$ $\quad \frac{3,2}{1,8}$ $\quad \frac{3,2}{18}$

What's the value of $6x + 8y$
if
$3x + 4y = 17$?

A car drives 9 km in 10 minutes.
Calculate the average speed in km/h.

The diagram shows how many times the students in a class visit the theater during a month.

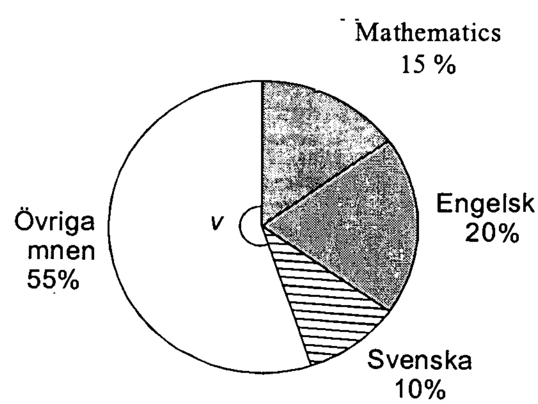


What is the median value for the number of visits to the theater?

Write the following parts in order.

4 ‰ 70 ppm 0,3 %

200 students



How many math students are there?

The KAM Project: An Example

Project Background

Too many of our students in vocational education fail in mathematics. As I started to teach mathematics in vocational education I joined the students in some of their vocational classes. Their vocational teacher complained to me about the students' low performance in basic calculations. The students' attitude can be summarized in the following lines from Hill:

> In the vocational subject even the parts that were theoretical they felt motivated most of the time. In the core subjects they said, a bit surprised, that they recognized most of it. The content, the methods and...the boredom. (translated from Hill, 1999)

Today nearly all Swedish students go to upper secondary school where course A in mathematics is compulsory, but many of the teachers are trained to teach in the old form where the task was to give all the students a more formal mathematics. Old traditions are still alive in a changing upper secondary school. Teachers are lacking support through in-service training.

Official statistics show that nearly 50% of the students in vocational education fail in the national test but half of these students get the "pass" mark anyway. In combination with other reports this can be seen as a trend which for the students is a dangerous development in the long run. Instead of changing teaching methods, some teachers produce their own criteria to give the grade "pass" to a course. As we will show later the students have the intellectual skills to really pass course A but they lack motivation. We know from experience that the teachers want to change their teaching due to their concern for their students, but they need help from in-service training and textbooks.

An Example: The Gearbox

The first foundation course that students in the Transport Program study is "Coach Work," a course studied by the students in the Vehicle Engineering Program. The following model was in one textbook (Brogård & Kristersson, 1989).

Gear ratio; $u = \frac{z_2 \cdot z_4}{z_1 \cdot z_3}$ where z_1 stands for the number of teeth on the gearwheel on shaft A in the gearbox and z_4 for the number of teeth on the gearwheel on shaft B (figure beneath); z_2 and z_3 stand for the number of teeth on the gearwheels on the common shaft.

The gearbox works like this: The wheel with 12 teeth (shaft A) drives the wheel with 48 teeth. And the 25-teeth wheel sits on the same shaft as the wheel with 20 teeth, which in turn drives the wheel with 40 teeth. This means that the number of revolutions of shaft B is less than the number of revolutions of shaft A.

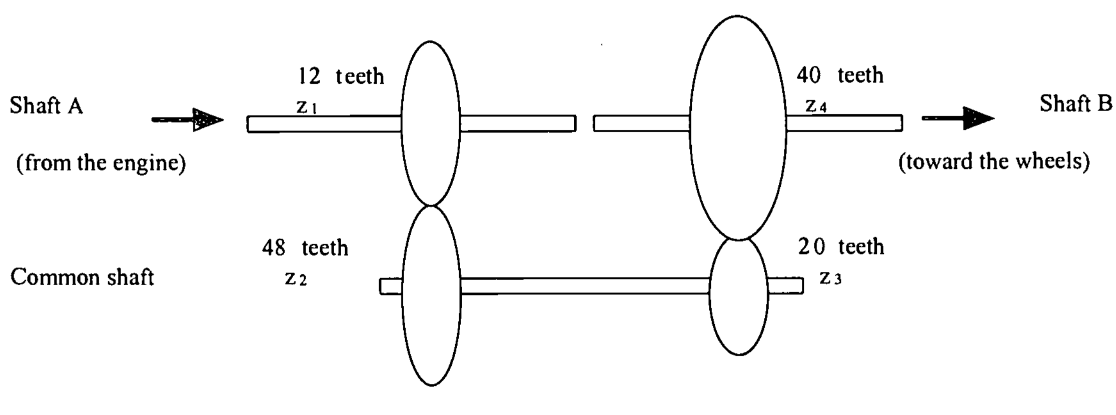


Figure 1

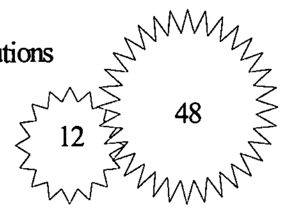
Fractions

From this formula we get a new formula, $u = \frac{n_1}{n_2}$ where n_1 and n_2 stand for the number of revolutions of shaft A and shaft B respectively.

Doing calculations using the formula above causes difficulties for most of our students. The students have difficulty understanding the formulas and calculations. Our problem is to help students meet the mathematical prerequisites of Course A, but on the other hand it is also to have them understand at the same time the function of the gearbox. The teacher has to coordinate instruction in these topics with instruction in topics such as multiplication and division of rational numbers in connection with percentages and scale. An important principle in our work is, in the first phase, to get rid of irrelevant mathematics from the gearbox model above.

We can look at the problem in two ways: either in terms of the number of teeth of the cogwheels that show changes in revolutions or in terms of the gear ratios. These are in fact inverses of each other.

4 revolutions 1 revolutions

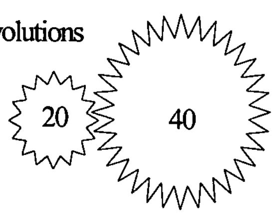


$\frac{12}{48} \Leftrightarrow \frac{1}{4}$

Gear ratio =
4:1

Figure 2

2 revolutions 1 revolutions



Gear ratio =
2:1

$\frac{20}{40} \Leftrightarrow \frac{1}{2}$

The pinion with the 12 cogs is engaged to the pinion with the 48 cogs. As the 12-cog wheel rotates one revolution the big pinion rotates ¼ revolution. The process is repeated once more. The number of revolutions from the engine shaft is reduced in the gearbox and the increase in momentum (gear ratio) is thus inverse to the increase.

Gear ratio = 4:1 & Gear ratio = 2:1

⇓

Gear ratio = 8:1

Four times and two times bigger
=
eight times bigger

The large 48-tooth wheel is on the same shaft as the 20-tooth wheel, hence we can combine the ratios to find the overall effect.

To understand the gearbox we think it is easier to first look at the number of teeth (the circumference) to understand what happens in the gearbox and then via exercises show that the ratio between revolutions (teeth) and the gear ratio (momentum) are inverses of each other.

For the vocational students we need to connect the function of the gearbox with the momentum and we will do this with the help of a balance. In the garage, as an application of this, you take a lever and the function of the moment keys and the bolt to show this.

To explain the role of the transmission of power and the relation of the power to the number of revolutions, in the garage they use a 10-speed bicycle that has been disassembled and set up for demonstration. At this demonstration station, there can be many activities to give the students concrete experience with power, revolutions, and moment. The students then proceed through a station where they work with starter motors, and finally to the gearbox station.

Results

After this the vocational teachers work with the mathematical material and the mathematics teachers work with vocational material. The different lessons are then joined together in an appropriate structure. Both the core teachers and the vocational teacher assess Joint tasks.

The project is running with just one class. This is because of timetable constraints and the teachers' opportunity to actively take part in the project. Thus we chose class A as the experiment group. In order to find the control group we tested all classes in ratio and percentages. We chose class T1A, which had almost the same result as the experiment group.

The two classes chosen were taught in mathematics as well as in the Vehicle/basic course. A teacher who had not actively taken part in the development of the model and the material taught the control group. The difference in the teaching was that the experimental group had more time studying the gearbox and had also specially designed teaching material for fractions and percentages. At this stage we were interested in looking at whether the students' knowledge and thinking about these sections of mathematics had changed, i.e., if we had any success in our teaching strategy.

Written Test

Results from the written test in Mathematics (max 32 points) are shown in the diagram below. The number of students in each class is written in brackets. Test 1 was a pre-test in Mathematics before teaching the concept of ratio. Test 2 was given the students after ratios had been taught. TS1C is the control group and G1A is the project group.

During the test the students were not constrained by time but no calculators were allowed. At the end of the school year both groups took the test again. We were also interested to see whether the quality of the thinking had been affected as a result of the special teaching. For that reason eight students were interviewed after both the initial and the final tests.

Most of the students in these programs have not been successful in understanding mathematics. Part of the reason for this is that the content has not seemed relevant to them. The thinking procedures they have constructed do not enable them to apply their mathematics. In order for the students to change their thinking modes, the prerequisite is that they recognize this problem. They need to be exposed to an intellectual conflict (Piaget). This will not happen if the teaching methods and contents are no different from the previous ones.

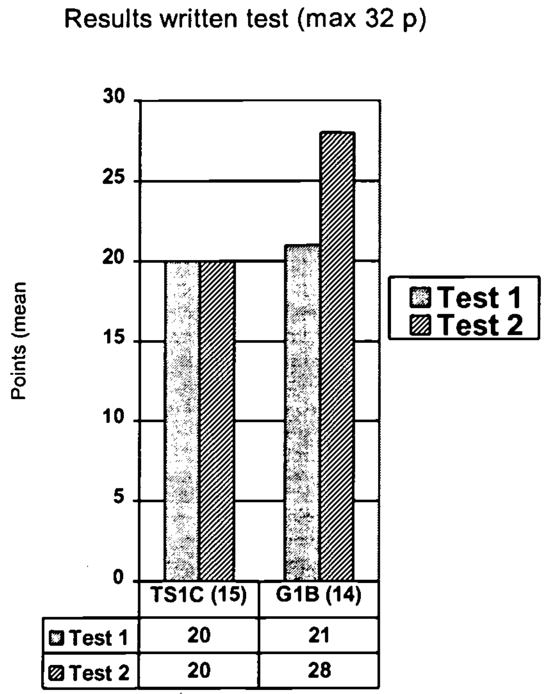


Figure 4

The control group has had traditional upper secondary teaching by another teacher who assumed that the students have the basic prerequisites in mathematics from the compulsory school. The traditional approach includes a lot of repetition and, since the prerequisites have not been questioned, it reinforces unproductive thinking modes. Through practical activity related to their future vocation the experimental group has been forced to question, giving them reason to change their former thinking modes. These new thinking modes are strengthened and deepened through the program in the mathematics class.

In the Swedish version of this report (Skolverket, 1999) all the answers to the questions given in the math test are analyzed, but as this is a concentrated version we know that many of the students in the test group have changed the thinking modes.

Attitudes
An important part of this project is to give the students a positive attitude toward mathematics. Data about experiences of mathematics and attitudes toward mathematics were collected through open questions in semi-structured interviews close to each of the two tests. This allows study of attitude change.

Some of the questions and responses by the students:

- What do you think of the subject of mathematics?
  "As boring, but it has become easier."
- Do you think that you need mathematics in the education here at this school?
  "No"
  This was the answer given by over half of the students prior to the gear ratio course and none afterwards.
- How do you think you will do in the mathematics course A?
  "I will fail the course."
  25% thought this before the course and none afterwards.

The students' self-confidence strengthened and they became more motivated in their study of mathematics because of the relevance of the subject.

Discussion

The teachers from both the mathematics and vocational teachers' education programs have gained from this project. Both groups also feel they have participated on equal terms with no inferiority involved. All have gained a broader perspective on their role as teachers.

For teachers who come from different traditions and, because of long experience, may be firmly set in their ways, negotiation for change and the changing of working methods is difficult.

For the mathematics teachers it is new to take some responsibility for the use of the mathematics they teach in the vocational subjects. The vocational teachers in the past have not had to think about the origin of the mathematics they teach. Often the mathematics used in the vocational subjects has been presented as isolated and rigid facts and formulas. In order for these two groups to work together productively the approach had to be open, allowing critique and questioning from both groups.

The evaluation of the project indicates that we are on the right track. However there are still many questions to be answered:

- Does the knowledge of vocational subjects also improve?
- Is there any linkage to other sections of the course?
- How do the students find this approach?
- Would students' outcomes be improved if the fraction content had been repeated after the gearbox work?
- Does this improve the students' long-term learning?

We hope to get resources to continue this project and to tackle these questions and others during the next years of this project.

Acknowledgments

The authors wish to record their appreciation and thanks to Marjorie Horne, ACU Melbourne, Australia, and Wiggo Kilborn, University of Göteborg, Sweden, who made this paper possible.

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From Classroom to Construction Site

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Editor's Note: The following is a description of a poster session displayed throughout the ALM-7 Conference.

Latinos and minorities have worked for many years in the building trades but often without due compensation or the benefits of union membership. The APTitude program (Apprenticeship Prep for the Trades) was founded at Congreso de Latinos Unidos in 1998 to change this situation. The original union partnership was with the Carpenter's Union but it has expanded to many other trade unions. These include the Painters, Drywall Finishers, Glaziers, Steamfitters, Sheet Metal Workers, Sprinkler Fitters, Electricians, Operating Engineers, and Cement Masons.

Entry to the trade unions' apprenticeship programs requires that an applicant pass an entrance exam consisting almost entirely of math. The fourteen-week APTitude program prepared students for these exams. The class covers basic math, fractions, decimals, percent, basic algebra, and geometry. Each of the unions has its own requirements after passing the entry test such as having a driver's license and passing a drug test. Tests are given at the pleasure of the unions at different times during the year. The apprenticeship schools also start at different times. Because of these variations, APTitude places graduates in other full-time employment pending the start of the union program.

The poster shows a series of photos taken at the Carpenter's Union Apprenticeship school during a class trip. In addition to lessons in a standard classroom the apprentices do hands-on carpentry projects in on-site workshops. The other union training facilities have similar training sites and equipment for actual practice.

Entering the building trades can be even more difficult for Latino and minority women. They face the problems of racism and the traditional prejudices against women in the field. The poster features a female APTitude graduate who is working as an electrician's helper in preparation for entry to the Electrical Workers union. Female graduates report having to endure the usual harassment given to new workers plus a high level of sexual harassment. Those who persevere are highly satisfied with their careers.

Other photos on the poster show graduates employed at construction sites around the city. They are apprentice carpenters, electrical workers, heavy equipment operators, painters, and cement masons. I've included quotes from graduates: "Life is like a math problem, you have to budget yourself to make ends meet. Math is all around from clocks to prices to your pay." "I want a job to support my kids and have benefits. I want to help them in school and keep them off the corner and show them a better way."

The ten-question mini quiz shown on the poster covers fraction equivalents, decimal to fraction conversion, measurement concepts, ratio, percentage, and the geometric formula for the altitude of a right triangle. These skills are mastered by APTitude graduates before they enter the union apprenticeship program of their choice, where they will hopefully begin a long-term career.

TEACHER KNOWLEDGE

Why Understanding 1 ¾ ÷ ½ Matters to Math Reform: ABE Teachers Learn the Math They Teach

Charles Brover, Denise Deagan, and Solange Farina
The New York City Math Exchange Group, USA

It is widely recognized that making significant change in mathematics instruction requires profound change in teaching and teachers. The National Council of Teachers of Mathematics' *Professional Standards for Teaching Mathematics* (1991) declared that its guidelines for professional standards rest on two assumptions:

- Teachers are key figures in changing the ways in which mathematics is taught and learned in schools.
- Such changes require that teachers have long term support and adequate resources (NCTM, 1991, p. 2).

The first of these assumptions is self-evident. The second, however, remains a distant goal for most schools. The lack of long term support and adequate resources for teachers of mathematics is particularly evident in Adult Basic Education (ABE). Still, even in this environment of scarcity we should consider what models of professional development actually work best, deserve support, and will lead to the changes we seek. The seven-year experience of The New York City Math Exchange Group (MEG) and some recently published cross-cultural research suggest that teacher-researcher collaboratives may provide the best model for changing both teachers and teaching in ABE programs.

Despite all that has been written about the need for reform in mathematics education, the culture of teaching in the U.S. remains stubbornly unchanged (Hiebert, 1999; National Research Council, 1989). If teachers are key figures in changing the way mathematics is taught, the prospect for reform in ABE is particularly fraught with difficulties. Adult Basic Education teachers share the wider teaching culture of mathematics in the U.S. that emphasizes isolated procedural skills and repeated practice. The marginal status of the study of mathematics in many literacy programs and the non-math background of many ABE teachers called upon to provide math instruction compound the problem of professional development. The limited mathematical content and knowledge of ABE teachers constitutes a decisive barrier to meaningful reform because without it they lack the confidence and agility to make significant changes. Instead, this deficiency leads to restricted and reductionist views of mathematics and math teaching.

In this problematic context, MEG developed in the 1990s as a volunteer, ongoing teacher collaborative committed to improving mathematics instruction in the ABE classroom. MEG's mission is to help teachers learn more math, as "It seemed self-evident that if teachers didn't know and do math, they could not teach it effectively." (See Brover, Deagan, & Farina, 2000.) The acquisition of math content is generally viewed as a sequential and cumulative accomplishment most often measured by college coursework. As Thomas J. Cooney observes, the level of difficulty is conflated with the level of understanding (Cooney, 1994, p. 11). The question most often asked of teachers' math knowledge is: How far? But we should also ask: How deep? Does the successful completion of advanced coursework necessarily correlate to a profound understanding of fundamental mathematics and problem solving?

For MEG, developing a deeper understanding of the math we teach is organically connected to our mission of reforming classroom mathematics instruction. MEG seeks to effect change in the culturally embedded activities of teaching mathematics, so its model for professional development goes beyond "telling" and "showing" teachers what to do. "Changing the beliefs about mathematics teaching and learning that teachers possess requires giving them powerful experiences in mathematical thinking and conceptual understanding." "Professional development programs should actively engage teachers in doing mathematics, with the leaders facilitating experiences that model what teachers should do with their own pupils" (Hyde, Ormiston, & Hyde, 1994, p. 50). As an ongoing mathematical community, MEG bases its work on the idea that teachers can learn the way we expect our students to learn—by constructing mathematical knowledge and understanding socially. Although this model of learning is sometimes suggested for students, it is rarely recognized as a professional development model for teachers.

Two books published in 1999 engaged this question of the relationship of mathematics content and knowledge to pedagogy and change by stressing the need for teachers to have a deeper understanding of fundamental mathematics: *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*, by Liping Ma, and *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*, by James W. Stigler and James Hiebert. These books created controversy and interest in the wider mathematics education community not only because they closely examined the content-pedagogy connection, but also because they relied on cross-cultural data.

Liping Ma based her study on a small sample of interviews with U.S. elementary school teachers conducted as part of the 1980s study by the National Center for Research on Teacher Education (NCRTE); for the rest of her data she returned to interview teachers in China where she had been an elementary school teacher before becoming a professor at Berkeley. Stigler and Hiebert based their book on carefully observed and coded videotapes of classroom teaching that were part of the TIMSS research. At the heart of both of these books is the intuitive idea that knowing math deeply is critical to teaching math well. Content knowledge and confidence support flexibility and pedagogical effectiveness.

These books strongly suggest that U.S. teachers don't prepare and present lessons that are as rich, textured, coherent, and challenging as Asian teachers' lessons. Stigler and Hiebert emphasize the culture of teaching; they see in their video study "a distinctly American way of teaching, which differs markedly from the German way and from the Japanese way":

> What we can see clearly is that American mathematics teaching is extremely limited, focused for the most part on a very narrow band of procedural skills. Whether students are in rows working individually or sitting in groups, whether they have access to the latest technology or are working only with paper and pencil, they spend most of their time acquiring isolated skills through repeated practice. (pp. 10-11)

What's the problem in the U.S.? Ma says that "in the United States, it is widely accepted that elementary mathematics is 'basic,' superficial, and commonly understood. The data in this book explode this myth. Elementary mathematics is not superficial at all..." (p. 146). Ma says that Chinese teachers are more able to see the connectedness of math concepts and to draw from complex "knowledge packages" to inform their pedagogy. Unlike their U.S. counterparts, experienced Chinese elementary school teachers, she says, are more likely to possess "PUFM."

> Profound understanding of fundamental mathematics (PUFM) is more than a sound conceptual understanding of elementary mathematics—it is the awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual understanding and instill those basic attitudes in students. (p. 124)

Ma provides extensive reports from experienced Chinese teachers who see many different ways to understand a problem and present it effectively to students. When U.S. elementary teachers were asked to solve a subtraction problem by regrouping, for instance, they simply applied a rule about "borrowing" whereas Chinese teachers entertained different approaches and aimed for the understanding that the standard algorithm is rooted in number system and place value.

When U.S. teachers were asked to compute 1 ¾ ÷ ½ and to create a word problem they might use in class for that numerical representation, only 43 percent could compute correctly, and *none* could invent appropriate word problems. Their problems represented division by 2 or multiplication by ½; some teachers just gave up. It was clear that even those U.S. elementary school teachers who could do the computation did not understand the process of division of fractions. All of the Chinese teachers interviewed by Ma, on the other hand, were able to compute the problem and provide appropriate word problems representing the computation. Moreover, most of the Chinese teachers were able to demonstrate alternative computational approaches and invent problems that showed a deep understanding of different models of division.

MEG decided to research this question among our colleagues in ABE in New York this summer, and we brought a model of this mini-research project to our workshop at the Adults Learning Maths Conference 2000 in Boston in July. We posed to our colleagues Ma's problem about division of fractions.

In our mini-research we gave the problem models to two groups of MEG members. The first group was comprised of "new" MEG members (<1 year in MEG) who were relatively new to teaching math and/or participating in professional development in mathematics, as represented by MEG participation. The second group was comprised of more "veteran" MEG members (>5 years), who had much experience participating in and presenting staff development in mathematics.

Of the first group, 87.5% of the teachers were able to compute the problem, but only 25% could create an appropriate story problem, and only 12.5% could create more than one story problem illustrating more than one model of solving the problem. Of the 87.5% who could compute the problem, all but one teacher used the traditional "invert and multiply" algorithm, with little or no understanding of why this method worked. Of the second, more experienced MEG group, 100% could do the problem and create an appropriate story problem.

At the ALM Conference we presented the same two tasks ("Divide 1 ¾ by ½" and "Write an appropriate story problem") to 4 groups of teachers, researchers and practitioner-researchers in our workshop.

Each group worked collaboratively (as opposed to participants in Ma's research and in MEG's mini-research). Each group was able to compute the problem; some examples of story problems from each group are as follows:

Group 1
We have 1 ¾ pizzas left over from last night. If each person in our family likes to eat ½ a pizza, how many people can we serve?
Our challenge: Can we think of a situation where the remainder (of ½) matters?

Group 2
1. I want to put 1 ¾ liters into ½ liter bottles and I want to know how many bottles we need. Answer: 4
2. Mary has a piece of wood that is 1 ¾ ft. long. She wants to make signs that are ½ ft. long. How many signs can she make? Answer: 3
3. Sheila has 1 ¾ cans of cat food. If she gives her (fat) cat ½ can per meal, does she have enough food to give her cat 4 meals? Explain your answer. Answer: No

Group 3
1. I have 1 ¾ meters of ribbon. How many ½ meter pieces can be cut off? How many "full" pieces?
2. I have 1 ¾ acres of land to sell in plots for cash but zoning requires ½ acre plots.
   How many plots can I make? How many can I sell?

Group 4

1. A movie is 1 ¾ hours long, with ½ hour parking time [per quarter]. How many quarters do I need?
2. Ribbon: 1 ¾ yard ribbon. How many ½ yard pieces can I cut?

The measurement or quotitive model, represented by finding how many ½ lengths there are in something 1 ¾ lengths long, was used in 89% of the problems. The partitive model, represented by finding how long the whole is if half a length is 1 ¾, was used in 11% of the problems. The product and factors method, represented by finding the length of a side of a 1 ¾ square foot rectangle if one side is ½ foot long, was not used at all.

We found that among both the experienced MEG members and the self-selected ALM attendees, the abilities to compute the division problem, create story problems, and reason mathematically and abstractly were higher than in Ma's sample of U.S. teachers and our sample of new MEG members.

This suggests that it is only through comprehensive and ongoing staff development that *all* teachers can better understand, apply, and teach mathematics to their students. The Chinese teachers understand what many American teachers do not—that fractions and other "elementary" mathematics are *not* "basic" and "elementary" but represent rigorous and connecting strands of an integrated mathematics curriculum. By applying comparable intellectual rigor to these strands as to later "advanced" mathematics, students are able to build strong foundations and make connections in mathematics throughout their lives. This works not only in support of professional development for teachers, but in support of adult and elementary school learners, who too often get the message that they are incapable of understanding even "easy" mathematics, and who are not provided with the appropriately trained teachers necessary to explore and construct knowledge of complex subjects such as fractions and other "elementary" mathematics.

The books by Ma and by Stigler and Hiebert underscore the proposition that mathematics reform in ABE will require teachers to have a deeper understanding of fundamental mathematics. They also offer strong support for teacher-researcher collaboratives as an effective model to develop that understanding. Both books insist that for substantial change to reach the classroom, teachers must drive the engine of change. And they recognize substantial obstacles in the U.S. system: Stigler and Hiebert stipulate that "A requirement for beginning the change process is finding the time during the workweek for teachers to collaborate" (p. 144). "We must empower teachers to be leaders in this process" (p. 127). Ma notes that Asian teachers have much more opportunity than U.S. teachers do to collaborate with peers. Further, Chinese and Japanese teachers have much more non-teaching time. Chinese teachers spend most of the day working collaboratively to develop a deeper understanding of their specific subject matter and engage in "materials study." Japanese teachers work together on what they call "Lesson Study" (Stigler and Hiebert, 1999, pp. 110-127), in which they develop, edit, and polish particular lessons. According to Stigler and Hiebert, a Japanese teacher-researcher collaborative may spend an entire year developing a single lesson.

If the development of teachers' "PUFM" is critical for math reform in ABE, the prognosis for meaningful reform is grim but not altogether hopeless. Although many ABE teachers do not have sophisticated academic math experience, they may become better math teachers while teaching. For Asian teachers, significantly, profound understanding of fundamental mathematics develops mainly *after* they start their professional careers. "...[T]he key period during which Chinese teachers develop a *teacher's* subject matter knowledge of school mathematics is when they teach it—given that they have the motivation to improve their teaching and the opportunity to do so" (Ma, 1999, p.147). The same may well be true for ABE teachers *if teacher-researcher collaborative models of professional development gain long-term institutional support.*

Deepening teachers' understanding of fundamental mathematics is key to bringing reform into the classroom. This should be a central goal of mathematics professional development in ABE. Though teachers' efforts to pursue coursework in higher math education should be supported, it may not be the most productive path to meaningful improvement in ABE math teaching. The best opportunity to realize professional growth in ABE may be to put teacher-researcher collaboratives at the heart of professional development.

This implies bridging the gap between researcher and practitioner and raises a number of interesting research questions: What mathematics do ABE teachers know, and how does this knowledge affect their instructional flexibility and effectiveness? What mathematics do ABE teachers need to know and *understand* to be effective math instructors? There are many barriers to professional development in ABE, and the overarching question in the U.S. is political: Will support and resources be provided for the professional development of teachers whose students have been socially and economically marginalized?

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Professional Numeracy for Teachers

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Context

In the UK this year for the first time, student teachers were required to take a numeracy test in order to gain qualified teacher status (QTS). This follows increasing emphasis on raising the standards of performance of teachers and the establishment of a national framework of standards for newly qualified teachers (NQTs), subject leaders, headteachers, and others. At the same time, there has been a significant increase in the collection and analysis of pupil performance data. Teachers and schools have been encouraged (fuelled by external inspection of the quality of school provision) to use such data to plan and implement strategies for raising pupil achievement.

The rationale for the introduction of the numeracy tests is unclear. All teachers are required to have a mathematics qualification and all primary (3-11) teachers are required during their training to demonstrate competence on a prescribed mathematics curriculum. The aim of the test (TTA, 2000) is described as "intended to ensure that everyone qualifying to teach has a good grounding in the use of numeracy in the wider context of their professional role as a teacher."

Opposition to the tests has come from both student teachers and their trainers (Neumark, 2000). Whilst supporting the view that teachers should have the professional numeracy skills they need to fulfil their roles competently, there has been concern amongst trainers over the means of assessment of those skills. Indeed, there is consistent research (for example, Nunes, Schliemann, & Carraher, 1993; Steen 1990) indicating that the capacity of individuals to perform mathematical tasks in a written test context differs significantly from their capacity to perform such tasks in their everyday life context. Furthermore there is evidence that individuals use different approaches in their everyday contexts from those they learned (usually at school) and use in written assessment contexts.

Data Collection

In common with many Universities, we offered support sessions for students and the opportunity to participate in a "mock" numeracy test. Here they could practice being in a timed test environment and they could identify knowledge and skills for further development and practice. The test was identical for both intending primary and secondary teachers. Contexts in which questions were set were usually sector specific so it was important for some students to ensure that they were clear about the context of the sector in which they were *not* training. The first section was a mental arithmetic test (Appendix 1) given orally. Students were allowed to jot notes down around the edge of a pre-printed answer sheet. They were not allowed to use a calculator. 10 or 15 seconds were allowed for each solution. The second section was a paper and pencil test of about 15 questions. Calculators were allowed. Students had about 45 minutes. Students were asked to volunteer their responses to the mock test for analysis. About 30 students out of the 40 participating did so.

In addition, a sample of 15 practising teachers was interviewed in their workplace both about their actual use of numeracy in their daily work and about their views on the need for and relevance of a selection of the mock test items.

Two questions dominated the study. Could student teachers do the questions set in the test? What were the actual numeracy demands on teachers in their professional roles?

Analysis of Mock Test Responses

The results show poor scores by some students. A separate study (Boylan, Elliott, Povey, & Stephenson, 2000) suggests that some students were extremely anxious about the numeracy test and suggests that high levels of anxiety were bound to affect their performance.

An analysis of the student responses to the mental test indicates that some of the difficulties were:

- Running out of time, often after starting one way, realising you got stuck and trying a different approach.
- Inability to quickly recognise simple fractions in either their lowest terms or as percentages necessitating time consuming repetitive cancelling or unsuccessful long winded attempts at formal written algorithms.
- Many students reported lack of instant recall of multiplication table facts.
- Excessive use of formal written algorithms instead of shorter methods even with, for example, x by 2 or 3 and addition/subtraction.
- Some confusion between raw scores/data and percentage data.
- Rigidity of methods for calculating percentage—either 10% as a base or 1% as a base or written algorithm, rather than variability depending on numbers.
  For example, Find 45% of 20
  Tried 1% of 20 – awful, tried 5% got stuck and then ran out of time
  10% = 2, 40% = 8, 5% = 1 so altogether 9
  Well I said 50% is half = 10 then I got stuck.
- Simple arithmetical errors in addition or subtraction (due to time pressure?).

Written questions were similar in nature to those shown in Appendix 2. More than one third of the students achieved less than half marks. Particular difficulties occurred with:

- Confusion over presentation of information, reading tables the wrong way round.
- Forgetting to calculate differences as percentages, or not being able to do so.
- Being unable to select relevant data.
- Working with currency conversions—this was very poorly done by almost all students.
- Time pressure—many incompleted responses.

Discussion of Mock Tests

It was clear from the mock tests that over half the students sitting the test would have failed to achieve the required pass mark at that stage. In one sense this is of grave concern, although one can suppose that those electing to sit the mock test were those students who were most worried about their performance. Evidence from other studies (e.g., Nunes et al., 1993) also points to the fact that failure to pass the test does not of itself imply inability to handle numeracy demands on the job. Indeed, one could question whether teachers ever in practice need to perform mental calculations of this nature at all.

Perhaps of more concern was the fact that some students did not appear to have confidence and skill in personal numeracy demands such as knowledge of multiplication tables, ability to cancel fractions, etc., or the flexibility to alter their approach to problem solving when this is currently, for primary teachers, a teaching demand—as opposed to a professional work demand.

However, does any of this matter in relation to the ability to do one's teaching job effectively? When, if ever, do teachers need to perform mental calculations quickly and under pressure (apart from when they are teaching mathematics/numeracy)? Does it matter whether they have several strategies for performing calculations or are stuck with one, even a paper-based one, provided that they can get the right answer when needed?

In order to gain some knowledge of the actual demands we set out to interview teachers at different stages of their careers in both secondary and primary schools.

Interviews

Following pilot discussions and interviews with a headteacher and a newly qualified teacher in her first year of teaching, a semi-structured interview schedule was developed. Fifteen teachers agreed to be interviewed and were sent, in advance, a copy of the interview schedule together with a sample of mock test questions (Appendix 2). Interviews were conducted at the end of the school year in July. The sample comprised:

	Primary	Secondary
Headteacher	2	1
Experienced teacher	3	2
Newly qualified teacher (NQT)	4	3

Each teacher was asked about the relevance of 6 mock test questions. The responses to this were

Question	Not relevant	Yes, highly relevant	Qualified Yes
1	11	2	2
2	10	3	2
3	9	4	2
4	8	4	3
5	7	5	3
6	1	5	9

Analysis and Discussion

Pilot discussions with the NQT had indicated that almost no numeracy skills were either needed or used on a day to day basis beyond simple counting. Analysis of the 7 NQT interviews confirms this.

None of the secondary NQTs had ever had to make any financial calculations at all—all of this sort of work being done by administrative staff. The Primary NQTs reported having to collect dinner money or photograph money or money for trips, etc., but again, totalling was completed by administrative staff. Two NQTs referred to the fact that they received a copy of the school annual report in which financial summaries were presented but that there was no explicit expectation on them to be able to interpret this.

All NQTs reported a requirement to work with attendance figures at some level. The demands were largely to total attendance and/or absence over a period of time (a week, half a term, a year). In some cases these data were presented to them having been calculated by administrative staff or having been computer generated.

No NQTs reported having to perform time calculations exactly, as assessed by the numeracy test. Three stated that they had to engage in approximate calculations with regard to time planning and time management

Percentage calculations were tested is several places on the numeracy test itself. Interestingly, the use of percentages by NQTs was mixed, with some reporting no use of percentages either because marks were always converted to grades, and not to percentages or because percentage calculations were completed by a more senior member of staff. One NQT reported that *"I'll give boys percentage results in tests because the competitive element encourages them but its not necessary....we don't use percentages...its in grades...the kids are getting used to not having percentages. Percentages have totally gone out of the window, they don't exist anymore in schools."* Similarly, a Primary NQT reported that *".. its in levels, not percentages.."* All NQTs were all engaged in basic data collection. Analysis (more frequently done by primary teachers) was largely either item analysis to identify which task items were done poorly in order to shape future teaching or score scanning to pick out individual pupils with low scores. In most cases the data collected were expressed in terms of level or grade rather than numerical score. Where there was any more detailed analysis this was invariably completed by the headteacher, subject coordinator, or member of staff with responsibility for assessment.

NQTs had pupil performance data, but the analysis was performed by senior management in the school—*"So we get the raw data but we also get the senior management's analysis of this data or where it refers particularly to the department usually the KS3, 4 or 5 coordinator will go through the data and explain its relevance."* There was even a disincentive to engage in analysis in one case *"So you get the data, you spend two weeks trying to work it out before its time for the meeting about it so I've just learned 'OK have a look through it, leave it, someone will explain it to me shortly.'"*

Discussion in both cases was minimal and limited to whole staff meetings. The presented interpretation of data was rarely justified and NQTs emotional involvement was hard to disguise—*"it's the interpretation that we usually have a problem with, nobody seems to pick out the good points"* or *"so for example whereas English is doing very, very well in GCSE and A level, the KS3 results are below the national average so that's used to say –well obviously you're not teaching them properly at KS3...nobody seems to realise Hmmm, maybe if they come in a little weak and that by the time they get to GCSE we've boosted them up again. It's the interpretation that we usually have an argument with."*

The validity of data was judged by reference to the teachers' knowledge of individual pupils and the wider significance issues were largely ignored. For example, there was no discussion by any NQT of the sample size for their school in relation to national data, rather an acceptance that if senior management said these were the implications then they must be so. The analysis by senior management was largely unquestioned.

In relation to the sample mock test questions, the first 5 were overwhelmingly rejected as relevant by almost all NQTs. The last question was the only one viewed by NQTs as relevant, and then with some qualification—one would see such data and analysis but would not do it oneself.

Experienced Teachers

More experienced teachers reported more examples of where they had needed to engage in simple money calculation, some mensuration, and conversions of numerical values to percentages than was the case for NQTs. In all cases they reported seeing more data in a variety of formats. However, only those experienced teachers with specific responsibility for undertaking analysis reported having to do anything with the data. In particular it was noticeable that, by comparison with headteachers, they largely felt that, whilst being impressed with the contextualisation of the sample test questions, in practice they were not calculations that they as teachers would perform, certainly not in that way. All thought the last question was relevant but, like the NQTs, commented that the data would be presented with a summary/conclusion for them to look at, rather than them having to draw conclusions themselves. Those teachers who did engage in data analysis had not had any training and relied on others to help if they got stuck. The levels of awareness of the significance of data appeared low, with comments like *"well, I decide myself, whether I think it's significant"* (not using technical definitions). Primary teachers were much more likely to keep their *own* data on pupil performance and to use this together with detailed monitoring records on individual pupils to develop strategies for raising achievement. In that context, the overall school, LEA, and national data were seen as less relevant.

Headteachers

Two of the three headteachers interviewed were highly positive about the sample tests, thought that more of such assessment was desirable, and were generally very supportive of their introduction. The third headteacher was more qualified in her support as was the headteacher interviewed in the pilot discussions. All headteachers were involved personally in extensive data analysis as well as financial data management. The latter they had invariably "learned on the job" with one having funded herself through accountancy courses during her holidays to ensure she had the skills and the confidence to manage this aspect of her work effectively.

All headteachers emphasised the importance of being able to understand data, being able to make judgements about how it is used and misused, its significance, and its relevance. All pointed to the need to examine national comparative data in detail in terms of the school in question given the limited criteria for matching groups of schools.

> *You've got to look at yourself against LEA and national averages and understand the issues around that, how representative your school population is by comparison ... that's where you can either understand or not understand ... so you need to be able to make judgements about how the data is being used and I think that's where a lot of people fall into deep waters ... they don't understand that sort of thing, so its not just about manipulating figures its about understanding the use and misuse of statistics, the significance of statistics the relevance of them.*

They talked about the need for a high level of understanding going well beyond what was tested in the sample mock questions—*"You've got to understand inter-quartile ranges and you've got to understand median and mode, normal distribution, significance, levels of significance...,"* and of the need to relate the data to the specific context in which you were operating in order to make appropriate decisions about future actions.

These priorities were reflected in all the schools by the existence of at least one member of staff with overall responsibility for data analysis and the expectation (at varying levels) that subject coordinators, heads of department, year coordinators would take greater responsibility than classroom teachers for raising data analysis issues with staff. There was no provision in any of the schools for training in data analysis—it was left to individuals to get to grips with, with support from the headteacher if needed.

In contrast to the NQTs and most of the experienced teachers, two of the three headteachers felt that all the mock test questions were relevant, even highly relevant. This is interesting in the light of their conviction that data must be interpreted in the context of the school, class, etc. in which you are working and also in relation to their emphasis upon "sense making" with data, not just reading data.

Concluding Remarks

This has been a pilot investigation on a relatively small scale and, as such, care must be taken in drawing conclusions. The numeracy test focused on mental arithmetic and data interpretation, largely of pupil performance, and on students' capacity to perform calculations related to money, time, and measurement. Evidence from those interviewed suggests that whilst newly qualified teachers do get presented with pupil performance data, they are not required or expected to analyse it. Moreover, they rarely need to perform the sorts of calculations presented in the test. More extensive work is needed to establish whether or not this is a general picture in England. If it is then the relevance of the test can be challenged.

Individual pupil tracking, targeting, and monitoring *was* expected of most teachers. The real context, employed as a full teacher in a school gave meaning to this activity, with teachers utilising considerably more than their numeracy knowledge to make judgements about progress, appropriate targets, gains, losses, etc. This mediation of professional and numeracy knowledge is similar to that reported by Noss, Pozzi, and Hoyles (1999). So, for example, like their nurses, averages and global data (as presented in performance tables) can mask the individuality of the performance of each pupil taught which professionally is of greater significance.

> *... its also very useful in the beginning when you don't know the children to identify students who are possibly more able or students who have done extremely better in one type of exam, for example multiple choice, so they might have better vocabulary but might not do so well when its reading comprehension ... (however!) its usually completely different to the reality, you have one child who got on exceptionally well in a multiple choice vocabulary test ... so*

> *the results of this test would suggest that this child could be able to achieve a very high level ... whereas the reality is, ok yes they can tick a box so their reading skills might be quite advanced, obviously someone's been working with them, but their writing skills are extremely poor ... so you take all this statistical evidence with a pinch of salt. (NQT)*

Further in depth study is needed to examine the ways in which teachers use numeracy knowledge professionally and the relationship between numeracy and professional knowledge.

The tests do not assess teachers' understanding of important concepts such as "significance." Teachers interviewed appeared to accept the analysis and conclusions presented to them—in some cases even being dissuaded from questioning such analyses. However, as headteachers pointed out, a critical understanding of data interpretation is vital:

> *I wonder whether in teacher training there shouldn't be a module of stats, related to education, but to give a real understanding of the statistics because it isn't just the manipulation of the figures, it is the understanding, the significance of them and the relevance of them in context. (Headteacher)*

If we are serious about data interpretation, use, and management should we be insisting that student teachers acquire higher level statistical skills? Given that there appears to be little training offered to existing teachers, and given that newly qualified teachers appear to need the least numeracy skills of all those interviewed, perhaps numeracy is something that should be considered as a mandatory training and test later in one's career. Of course it would be politically unacceptable to suddenly drop teachers from the profession just as it has been politically unacceptable to make the numeracy test a more significant test of understanding. With its current format, over 95% of student teachers passed the test the first time. What that means is less clear.

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Appendix 1

Mental Arithmetic Test

Each question was read aloud to students twice from a tape recording. They had pre-printed sheets on which to respond. No calculators were allowed. 15 seconds were allowed for each question.

1. In a class of thirty pupils, eighty five per cent achieved grade C and above and five per cent achieved grades D to G. How many pupils did not achieve a grade?
2. A teacher had a budget of one hundred and twenty pounds. Half of the budget was spent in term one and thirty six pounds spent in term two. What proportion of the budget remained at the end of term two? Give you answer as a fraction in its lowest terms.
3. A class of twenty four pupils had access to eight computers. Each pupil needed fifteen minutes' computer time. How much time needed to be booked per computer?
4. A school day finished at fifteen thirty. The two afternoon lessons were fifty five minutes each, with a fifteen minute break between lessons. What time did the first afternoon lesson begin?
5. A school play cost one hundred and sixty pounds to produce. Eighty parents paid two pounds fifty each to attend the one performance. How much profit did the school make?
6. In class A twenty one out of twenty eight pupils passed a test. In class B, eighty seven per cent of pupils passed the same test. What was the difference between the two classes in the percentage of pupils passing the test?
7. A careers teacher gave a talk to four classes each of twenty six pupils and three classes each of twenty eight pupils. How many pupils attended the talk?
8. What is one point two minus zero point eight five? Give your answer as a decimal.
9. A class of twenty seven pupils sat an end of Key Stage two test. Eighteen pupils achieved Level three and above. What proportion of the class did not achieve Level 3 and above? Give your answer as a fraction in its lowest terms.
10. A department ordered seventy books costing five pounds each. The supplier gave a discount of ten per cent. How much was the total order?
11. In a class of thirty pupils, three fifths were boys. How many were girls?
12. A pupil scored thirty five out of seventy in one test and forty two out of seventy in a second test. What was the percentage increase in the pupil's score in the second test compared with the score in the first?

Appendix 2

Sample written test questions used in interviews.

1. A school report required a single mark to be recorded for GCSE coursework for each pupil. A pupil achieved the following marks for different coursework components

Component A	74
Component B	62
Component C	57

The total mark is calculated by the formula:
Total mark = Component A x 0.5 + Component B x 0.3 + Component C x 0.2
What is the total mark for this pupil?

2. For a GCSE subject, 20% of the marks were allocated to coursework and 80% to the final examination. The coursework was marked out of 95 and the exam out of 75. A pupil scored 68 for coursework and 65 for the exam. What was the pupil's final percentage mark to the nearest whole number?
A 74%
B 75%
C 83%
D 84%

3. A teacher travelled a 47mile round trip to attend a training course. The teacher attended 15 sessions to complete the course. Mileage costs can be claimed for the total distance travelled for the whole course. The rate of payment is:
29p per mile for the first 100 miles
22p per mile for the remaining distance travelled
What is the total mileage claim for the course?

4. A teacher arranged for 5 groups of pupils to give short presentations in a session lasting 1 ½ hrs. The pupils were given 15 minutes to prepare their presentations at the start of the session. Each presentation lasted 10 minutes. There was a changeover time of 2 minutes between presentations. How much of the session remained after the last presentation had finished?

5. The annual school report included a grade for each pupil's attainment. Grade A was awarded for a test mark of 72% and above. The table below shows the results of this test for the boys in a class.

Pupil	Mark	Pupil	Mark
A	31	I	7
B	12	J	15
C	17	K	23
D	29	L	27
E	19	M	18
F	24	N	23
G	30	O	17
H	35	P	26

If the test was marked out of 36, how many boys in the class have achieved a grade A?

6. COMPARATIVE PUPIL ATTAINMENT DATA AT AGE 7 KS1 READING; TESTS/TASKS

			Percentage of Pupils	
Level achieved	**Sex**	**Year**	**County**	**National**
2 and above	All	1995	80.8	78.5
2 and above	All	1996	81.3	78.0
2 and above	All	1997	82.4	80.1
2 and above	All	1998	82.6	80.1
2 and above	Boys	1995	77.4	73.9
2 and above	Boys	1996	76.6	73.2
2 and above	Boys	1997	78.0	75.7
2 and above	Boys	1998	78.0	75.4
2 and above	Girls	1995	84.3	83.3
2 and above	Girls	1996	86.1	83.0
2 and above	Girls	1997	87.2	84.7
2 and above	Girls	1998	87.4	84.8

A teacher is looking for trends in county and national KS1 reading tests/tasks. Indicate which of the following statements are true.

1 County performance is consistently above national average
2 Girls outperform boys in KS1 in the County
3 The percentage of boys achieving Level 2 and above increased annually in the County

A 1 and 2
B 2 and 3
C 1 and 3
D 1, 2 and 3

Math In-Service Training for Adult Educators

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I.- Introduction

The paper reports an experience developed in the south of Argentina with adult educators. We will concentrate on a series of workshops on math education. The workshops were part of a larger State project aiming at constructing a unified curriculum for adult basic education and vocational education. The workshops were seen as a tool to make explicit the actual teaching strategies in math education and, at the same time, as a way of creating conditions for teachers from adult basic education and vocational education to work together.

The aim was to create conditions to make explicit and bring into reflection the teaching strategies used by teachers of adult basic education and vocational education. Teachers (with formal training) and teachers of vocational courses (without formal training) participated in a series of workshops where they were asked to design a teaching situation on a particular content in the area of mathematics.

We will present some examples of the situations designed during the workshops. Then, we will discuss the situations considering the limits and possibilities of the teaching strategies regarding the particular content involved. For the analysis we will follow the theory of Brousseau.

II.- Didactic Theoretical Background

Education is a social event in which individuals "share meaning" in such a way that they understand the reality that surrounds them, consequently fostering their own development in everyday life. If we admit that school must prepare citizens for a satisfactory integration into the life of the society to which they belong, this conception claims for an adequate understanding of those realities of the culture in which they live and the capacity to make a profitable use of the necessary techniques of a certain work, "Every social project for teaching and learning is dialectically constituted with the identification and designation of *savoir*'s contents as the contents to be taught" (Chevallard, 1997).

The relationships students establish throughout the teaching-learning process depend, among other factors, on the interactions they have with such notions. At school, that relationship is generally segmented and algorithmic. It is essential that students are given concepts they consider of interest. At the same time, these contents must have *sens*. As Brousseau states (1993), the student builds the *sens* of notions, since he knows when to use them, whenever he utilizes them as problem solving tools, that is to say, taking the relationships he establishes with the mathematical notion and the teaching situation as a starting point.

Brousseau declares that all knowledge involved can be distinguished by one or more situations in which the *sens* of that notion is preserved. Adequate situations must be situations in which the mathematical notion to be taught is the best solution. If the *sens* of notions is not contemplated, the teaching-learning process is reduced to a mere circulation of official *savoir*.

Depending on the way the problem is posed to the students, the situations given in the classroom may have different "status": to take decisions, to solve or communicate something new, to justify knowledge. Brousseau (1993), accordingly, identifies situations of action, formulation, validation, and institutionalization.

The study of a situation, of the conditions that will allow some notion to work in the classroom, demands the organization of notions in such a way that they can explain their origin, questions, and problems that have been posed. It does not have to do with the reproduction of its historical development, but it is a matter related to the organization of an environment where this *savoir* can "live."

Within the framework of the Theory of Didactic Situations the work of teachers is, to an extent, inverse to that of the mathematician's: he must look for situations that provide *sens* to the notions he has to impart and he must create the conditions (Llorente, 1998) that make those notions form part of the knowledge of a student at a given time. In order to transform the students' answers and knowledge into a cultural and communicable *savoir*, students, with the help of teachers, will have to go back on their steps and take their productions out of context and make them impersonal in such a way that they can identify them with the *savoir* that is being developed within the cultural and scientific community of their time.

Taking into account all these general aspects the teacher will have to fulfil certain conditions:

- the context that will provide *sens* to teaching. In adult education, loom, cooking, or hairdressing classes can offer a network of referential situations that teachers can use to organize teaching situations.
- the knowledge to be taught constitutes a solution adapted to the problem.
- didactic variables that lead to the development of knowledge in a fruitful way.
- the student's previous experiences so that he/she can try a resolution method, although he/she cannot solve it completely. This condition is especially relevant in adult education since they reach formal education with an amount of knowledge that has to be taken into account. If this previous knowledge is not taken into account, there is a serious risk of changing the *sens* of knowledge through the posing of childish situations.

III.– The Analysis of Two Teaching Situations Outlined During the Workshops

We will base our analysis on two teaching situations outlined as from the reference situations of a hairdressing class and of a cooking class, respectively. These situations, which were outlined for teaching purposes, capture important aspects of the notion of proportionality, even though they can be approached by intuitive actions. We will try to make explicit, as best we can, the scope of these situations, considering the different aspects of knowledge that are involved in the solution process being practiced. We will also point out some of the conditions that are related to the study of the modeling process of the notion by means of functions, based on the *sens* of the latter.

It is evident that proportionality occupies an important position in culture outside the school. This is the reason why we favor productions based on this subject of *savoir* in the workshops, in order to try and demonstrate how this instrument may be a valid one to recover teaching strategies.

We can say that the reference situations proceeding from cooking, knitting, sewing, and hairdressing classes may provide a range of teaching situations that could be modeled through linear functions, proportional functions.

In order to select the situations that will be put forth in the classroom, one should take into account issues related to:

- how proportional situations are recognized;
- and, once they have been deemed to be proportional, how they will be modeled effectively.

The teaching situations designed by the teachers would fit into the distinction made by Brousseau on action situations. These situations may contribute to a first approach to a teaching content.

Modeling Through Functions

A proportionality relation is a bivariate function f: $A \rightarrow B$, where A and B are subgroups of the real[1] numbers so

$f(x) = k \, . \, x$ with k as a real constant other than zero.

This function verifies the following properties:

Property 1: $f(\alpha \, . \, x) = \alpha \, . \, f(x) \quad \alpha \in R, x \in A, (\alpha \, . \, x) \in A$

Property 2: $f(x_1 + x_2) = f(x_1) + f(x_2) \quad \forall \, x_1, x_2 \in A$ and $(x_1 + x_2) \in A$

III. 1.- Analysis of the First Situation

The situation taken as reference on topics related to hairdressing practices would offer a possible context for dealing with dependency and variability, two aspects that characterize functions.

In the situation presented by the teachers, the question is to find out the necessary volume in cm^3 of hydrogen peroxide needed for a tube of dye, in order to dye an average sized head. The figures given in the situation do not provide clarity regarding the variation of hydrogen peroxide content in relation to the tubes of dye used; that is, the concept of "one magnitude being directly proportional to another" is vague.

In solving this situation, operations with rational numbers are involved. It is probable that the students' experiences with rational number calculus allows them to apply scalar procedures that account for property 1.

If this experience does not show what is expected, it is possible to make use of the well-known "reduction to the unit" procedure or the "rule of three" technique. In the reduction to the unit procedure, the search for the image of 1 is distinctly expressed, but the relation between the two spaces (cm^3 of hydrogen peroxide-dye tubes) is not included, but it remains as a scalar procedure. The reduction to the unit procedure is often mistaken for the functional one (that expresses a relation between two magnitude spaces) because the means of calculating are the same. The difference between these is that, in the reduction to the unit procedure, what is sought is the image of 1, while in the functional procedure it is a k (constant). This k allows you to go from one magnitude space to another one. When applying the "rule of three" some "rites" are followed that in most cases are based on learned techniques rather than on the resolution of the question. How can a correspondence rule be recognized as proportional?

Number selection can be considered as a didactic variable as it enables the development of different procedures. In this case, changing the numbers to favor the use of procedures of the functional type would turn out to be "unnatural." To avoid this, a ratio of price and volume (in cm^3) could be used.

For example, it would be convenient to count on data that ease the choice between buying a 40 cm^3 dye tube at \$4 or a 47 cm^3 tube at \$4.50, an ordinary situation in which it is easier to find the relation between 40 and 4 than between others.

Dye volume (cm^3)	Price \$
40	47
47	

$\boxed{\div 10}$

Dye volume (cm^3)	Price \$
47	4.70
40	

[1] In compulsory schooling, the proportionality situations are generally restricted to N or Q^+.

The selection of these numbers contributes to the construction of *sens* in the law of correspondence. In this case it is the correspondence between two magnitude spaces: volume of the dye and price.

III. 2.- Analysis of the Second Situation

The situation designed in the cooking class can be dealt with by means of intuitive actions, that is, doubling, tripling, and quadrupling the quantity of each ingredient. We can infer that these actions correspond to property 1.

The numbers proposed in the second instruction can also be solved intuitively. In this case properties 1 and 2 are combined and there are different proportional relations between each of the ingredients and the number of cookies. Although it is possible that properties 1 and 2 may be involved in each relation, it is senseless to carry out a mathematical analysis to explain them since an intuitive solution would suffice.

The situation would have to be conditioned so that the work in the classroom is not only intuitive but so that there is also room for the study of the properties as well as the laws of correspondence, that is, how and when some things change in relation to others.

The different relations between ingredient content and number of cookies could be presented in order to recognize proportional situations. The questions posed in each situation should not only consider the relation between the amounts used of each ingredient and the number of cookies that can be obtained, but they should also drive at establishing a relation among all the ingredients used. This conforms to the need for a deeper analysis of how and when proportional relations are present or not.

For instance, taking the basic recipe we can pose the following question:

A: For 1 kg of flour, how much butter is needed?

In the solution, at least two procedures could arise that express different relations:

The amount of butter used may be calculated on the basis of the variation of the amount of flour used.

Amount of flour	amount of butter
400	200
1000	

Or the amount of butter can be calculated using the ratio between the amount of flour and the number of cookies, and then using the result obtained to find out the amount of butter needed based on the number of cookies previously obtained. It is highly probable for this procedure to turn up if questions were previously formulated regarding the amounts of different ingredients and the number of cookies that can be obtained, or vice versa.

Amount of flour	number of cookies
400	50
1000	125

number of cookies	amount of butter
50	200
125	

B: How many eggs are needed for 150 g of butter?

The question poses the relation between number of eggs and amount of butter.

For 200 grs. butter, 2 eggs;
For 100 grs. butter, 1 egg;
For 125 grs. butter, 1 egg;

We can see that sometimes the relation established is proportional (as in A). Instead, the number of eggs is not proportional to the change in the amount of butter for making cookies, what will give elements to discern a proportionality situation from others.

If in the resolution we wanted to establish a relationship between the amounts of the two measures in question, giving rise to the law that governs the dependency between these two measures, we would need to pose a different situation.

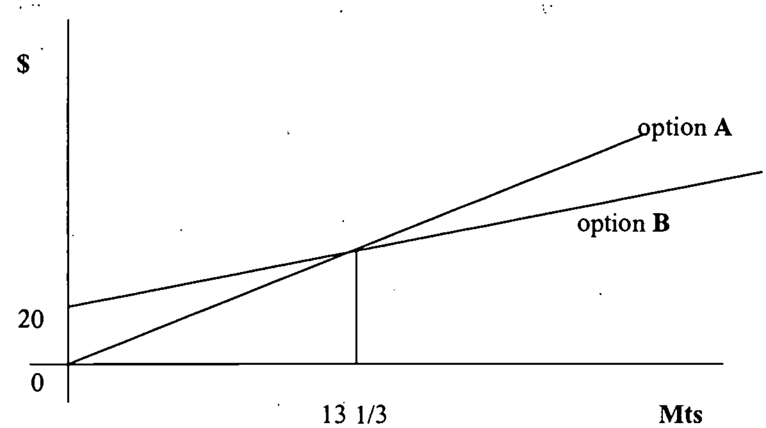
A situation should be put forth, whereby its solution promotes the inclusion of the correspondence law in order to establish predictions and to take decisions.

For example:
The facade of a house is to be repaired. There are two possibilities:

A: The cost per m^2 of repair work is $5.
B: The cost per m^2 of repair work is $3.50 plus $20 invested in a tool needed for the job.

In which cases should we choose A and in which B?

In example B there is a cost which is proportional to the extension in m^2 that need to be repaired, and $20 of initial cost should be added to each example. In example A total expenditure is proportional to the m^2 that have been repaired.



Procedures involving the rule of three are not appropriate due to operating cost. The optimal procedure consists of finding the answer in the chart where the relation of the two options is shown. In order to choose the most convenient option, it is necessary to carry out an analysis, the scope of which should not be limited to obtaining results by calculating the particular figure variations. The use of a chart is interesting because it allows us to make predictions and take decisions comparing figures.

This situation tries to show the weakness of the application of the rule of three. In order to make a decision we need a series of organized data. To achieve this we may draw a chart or a graph expressing these relations. It is necessary to develop a comprehensive study of the functions that model the relations expressing the options.

Remarks

The selection of situations for dealing with proportionality should foresee that through retrieving intuitive procedures we allow for the development of knowledge. An analysis of the comprehensive characteristics of the function is preferred, rather than a segmented study based on techniques of resolution such as the "rule of three."

It remains the teacher's responsibility to continue with the teaching of each content, and therefore making the students understand the social significance of the knowledge skills involved (definitions, properties, graphic and colloquial representations, etc.).

As Brousseau (1995, p. 27) states:

> The resolution of the problem may give the student the idea that there was nothing new to learn. Yet, being aware of having replaced an old and culturally identified strategy for another "invented" by himself, he will find it difficult to state that this innovation is a new learning. Why must it be identified as a method if it seems to come so easily when needed? How could an individual on his own identify from all the decisions he has taken those that derive from the situation and that could be useful in other situations from those which are purely local and occasional?
>
> The social conditions of learning by adaptation, rejecting the principle of knowledge intervention of a third person to obtain an answer, tends to suppress the identification of this answer as something new, and therefore as something that derives form the acquisition of knowledge.

IV.- Conclusions

Finally we would like to point out the potentialities of the workshops as a tool for narrowing the distance between prescription and practice, that is, between normative curriculum and actual teaching or real curriculum.

We understand that any attempt to build a unified curriculum demands that two communities of practitioners—adult basic educators and vocational educators—be brought together. Therefore the workshops constitute common spaces of interaction for jointly designing teaching situations and create common discourses between the two communities of practitioners.

Another point we would like to make is that the intervention of a specialist in mathematics during the workshops proved to be relevant in controlling the logical structure of the content but avoiding didactic suggestions that could darken the educators' teaching strategies.

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Appendix

First Situation

1)

The dying of an average head demands the use of 75 cm^3 of hydrogen peroxide per 1.50 tubes of dye. How many cm of hydrogen peroxide are needed for a base dye of one tube?

Possible resolution:

Hidrogen Peroxide (cm^3)	Dye Tubes
50	1
75	1.5
100	2
150	3
200	4

1.5 tubes ———— 75 cm^3
1 tube ———— $75 \div 1.5 = 50$ cm^3

2)

With one liter of hydrogen peroxide (1000 cm^3), how many base dyes can we prepare?

Second Situation

1)
200 grs. butter
200 grs. sugar
2 eggs
400 grs. flour

To make 50 cookies of 6 cm diameter

If I wanted to make 100 cookies, how much would I need of each ingredient?

- For 150 cookies?
- For 200 cookies?

2) According to the ingredients given in this example:

500 grs butter
500 grs sugar
5 eggs
1 kg flour

How many 6 cm diameter cookies will be obtained?

Communicating the Message: The Math Practitioner

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Editor's Note: The following is a description of a poster session displayed throughout the ALM-7 Conference.

The Adult Numeracy Network (ANN) relies on three vehicles to carry information to its members—a newsletter, *The Math Practitioner*, published 3 times a year; an electronic discussion forum, the Numeracy List, where members and non-members can share ideas and good practices; and the ANN Web Page. Participants at the ALM7 Conference had an opportunity to pick up samples of some of the best back issues of *The Math Practitioner*, some of the best classroom activities and articles from the newsletter archives, and to read about listserv discussions around instructional concerns. Our purpose in the Poster Session was to encourage ALM7 participants to contribute to *The Math Practitioner* so that a wider audience could benefit from their work.

People who stopped by the display picked up copies of all the handouts available, but very few expressed an interest in submitting material to be published in the newsletter. The work that the researchers in attendance have been doing has relevance to instruction in the United States. In fact, one of the strongest impressions as presenters talked about their work was the global nature of the issues. If workers in the United States are under prepared to handle the Numeracy of the 21st century workplace, there is work being done in Sweden and Australia and other countries that can provide ideas and solutions. If materials in the U.S. adult classrooms are not reflective of the actual tasks they purport to train adults to do, then the work being done in the Netherlands and Sweden can offer some insights. The presentations of math around topics such as family literacy, citizenship education, distance education, the development of frameworks for math education, and others illustrate the global relevance of these topics. So by the end of the conference, the question that remained was, "How do we get the researchers to share their work in print so the conversation with practitioners can continue?"

Participating in the conference through a Poster Session afforded us the opportunity to meet with many presenters and to share the mission of the Adult Numeracy Network. As a result of conversations during the Poster Session we have already obtained permission to print materials from some ALM7 presenters, and the next issue of *The Math Practitioner* will contain those items. The next step is to review the Proceedings of the conference and to select presentations that have particular meaning to practitioners. One idea for sharing this information is to feature a Research Corner in the newsletter. Another possibility, more in keeping with the format of the newsletter, is to continue the conversation by printing both a synopsis of the research and a practitioner response. This response might take the form of questions for the researchers, or a practical idea for implementing the findings in classroom instruction. No matter what form the conversation takes, it is important that it continue in *The Math Practitioner* and on the Numeracy List so that more practitioners can benefit from the breadth of work being done in Numeracy education worldwide.

Numeracy: You and Me Together in Numeracy

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Editor's Note: The following is a description of a poster session displayed throughout the ALM-7 Conference.

A select group of Massachusetts Adult Educators recently met for a pilot certification course focusing on collaborative efforts with learners to increase mathematics literacy. Through sharing experiences, best practices, and examining recent theories and studies of learning, the teachers developed curriculum units that focus on the needs of their learners.

The pictures shown on the poster highlight the Massachusetts ABE (Adult Basic Education) standards in action: Mathematics as Communication, Problem Solving, Reasoning, and Connections. The learners are engaging in hands-on activities ranging from manipulatives to computer technology to increase their mathematical power. Real-life activities, such as planning a cookout, investing in the stock market, or using coupons, are the backdrop for concepts normally taught and reinforced as meaningless "drill and kill."

Mathematics as Communication is an integral part of adults learning mathematics. They need to be able to articulate their understanding of concepts and in turn inform others of their newfound knowledge. Many adults also find it extremely rewarding to be able to help their children with math homework.

Problem Solving within mathematics is an everyday activity. In order to achieve mathematical power, these learners need to develop their problem solving skills in a variety of different settings. Lessons that focus on daily tasks, such as planning a cookout or finding the best deal in the grocery circulars, are learner-centered and generate much enthusiasm.

The learners that were engaged in the stock market simulation used reasoning skills to justify purchases or sales of their investments. They tracked their portfolios over many weeks and generated spreadsheets and graphs of the data in support of their conclusions. One learner proudly brought in his 401K retirement plan statement to show his peers that he was a stockholder. He stated that he now could understand this information.

Mathematics as Connections is exhibited in many aspects of these lessons. The idea of a learner-centered approach presupposes that there will be connections made to the learners' home, work, and community, as well as across the curriculum. Lessons must have meaning or relevance; otherwise the educator runs the risk of turning off the learner. Adults have been alienated by math most of their lives. It is the job of the educator to make mathematics fun and less threatening, while still facilitating the learning of traditionally difficult concepts. The developers of the Massachusetts ABE Certification Numeracy Pilot Course instilled the importance of incorporating the Standards in lesson planning. By using a learner-centered approach, the facilitators provided an open forum for the participants to share classroom issues and ideas.

The success of this pilot course will continue within the classrooms of these learners and instructors as they explore ways to work together in learning and using numeracy in their daily lives. Special thanks to the developers of the pilot course (Esther Leonelli, Marilyn Moses, Barbara Goodridge, and Mary Jane Schmitt) and for the efforts of SABES and the Massachusetts Department of Education.

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