DOCUMENT RESUME

ED 474 046		SE 066 777

AUTHOR	Angiama, R.O.
TITLE	Teaching Adult Students Mathematical Investigations-6.
PUB DATE	2000-00-00
NOTE	7p.; In: Proceedings of the International Conference on Adults Learning Mathematics (ALM-7) (7th, July 6-8, 2000, Medford, MA). For full proceedings, see SE 066 771.
AVAILABLE FROM	Peppercorn Books, P.O. Box 693, Snow Camp, NC 27349.
PUB TYPE	Reports - Research (143)
EDRS PRICE	EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS	Adult Education; *Adult Programs; Cognitive Style; Concept Formation; *Educational Strategies; Experiential Learning; Higher Education; *Mathematics Instruction; *Numeracy; Relevance (Education); *Teaching Methods

ABSTRACT

Teaching Adult Students Mathematical Investigation is based on the continuing research work carried out for the last ten years of teaching on the Foundation Course in Mathematics at Goldsmiths, University of London. Teaching Mathematical Investigation to adult students is a very challenging and often rewarding experience for adult educators as well as for the adult students. The theme of the investigation is that adult students are asked to make Ice-Cream Cones containing two scoops of two different flavors. First, they are asked to decide the flavors, then make as many different combinations of two scoops of Ice-Cream Cones, using each flavor only once. The paper concludes that many adults already have a wealth of experience. Teaching methods such as "mathematical investigation" and discussion with adults can provide a bridge to new learning initiative for adult students. Clearly, the evidence in this investigation suggests that adult students gain more when they know what work they are personally accountable for and what to do when they have finished, with the "Ah, Ha!" experience. (Author)

Teaching Adult Students Mathematical Investigations – 6

R. O. Angiama
Goldsmiths College, University of London, UK

Abstract

Teaching Adult Students Mathematical Investigation is based on the continuing research work carried out for the last ten years of teaching on the Foundation Course in Mathematics at Goldsmiths, University of London. Teaching Mathematical Investigation to adult students is a very challenging and often rewarding experience for adult educators as well as for the adult students.

The theme of the investigation is that adult students are asked to make Ice-cream Cones containing two scoops of two different flavours. First, they are asked to decide the flavours, then make as many different combinations of two scoops of Ice-cream Cones, using each flavour only once.

The paper concludes that many adults already have a wealth of experience. Teaching methods such as "mathematical investigation" and discussion with adults can provide a bridge to new learning initiative for adult students. Clearly, the evidence in this investigation suggests that adult students gain more when they know what work they are personally accountable for and what to do when they have finished, with the "Ah, Ha!" experience.

Introduction

> The idea of "mathematical investigation" is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to solve problems in very many fields. (The Cockcroft Report, 1982)

Mathematical Investigation (MI) is a source designed to provide effective practice for teachers of mathematics in some basic concepts in mathematical education and the development of adults' learning skills in a form that generates interest and enthusiasm. Many adults already have a wealth of experience, and teaching methods such as mathematical investigation and discussion with adults can provide a bridge to new learning initiatives for adult students.

In this investigation, adult students are asked to make Ice-Cream Cones containing two scoops of two different flavours. First, they are asked to decide on the flavours, then make as many different combinations of two scoops of Ice-Cream Cones, using each flavour only once. In their different groups, students are encouraged to discuss their investigation work with each other and use constructive criticism to develop ideas.

Two Flavours

Strawberry

Chocolate

Only one combination is possible
1

Three Flavours

Toffee

Strawberry

Chocolate

Three possible combinations
3

On observation, one group introduced the problem by assuming that an ice-cream seller has different flavours for making cones and that each cone must just contain two different flavours. The group raised the following question: How many different cones can the seller make with the following 9 flavours: lemon, vanilla, strawberry, toffee, chocolate, orange, apple, and peach?

The group then went on to identify the purpose of the investigation, which is to find the maximum number of choices they can have, given a number of flavours and the number of combinations. A combination consists of two scoops of two different flavours

They let the flavours be represented respectively by their first letter. In order to avoid confusion between Chocolate and Coffee, which have the same initial letter, Coffee was represented with a K.

L, V, S, T, C, K, O, A, P.

Method of Investigation and Analysis

We suggested starting the investigation with the two first flavours and then increasing one by one until exhaustion of the list. That is to say, we will work out from L and V, and then for L, V, and S, ..., and so on. At the end of the list, we will collect the results in a table and will try to find the rule which allows us to work out the number of combinations for a given numbering of things if we know how to combine them.

Let us now start.

For 2 flavours L and V, how many ice-cream cones:

As we can use each flavour just once, we have

L,V → L+V → 1 cone

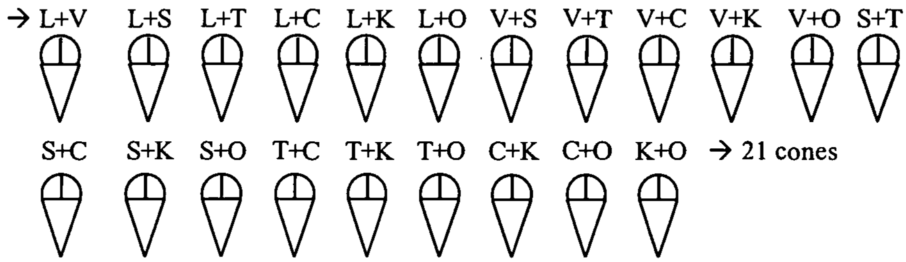
For 3 flavours
L,V,S → L+V L+S V+S → 3 cones

For 4 flavours
L,V,S,T → L+V L+S L+T V+S V+T S+T → 6 cones

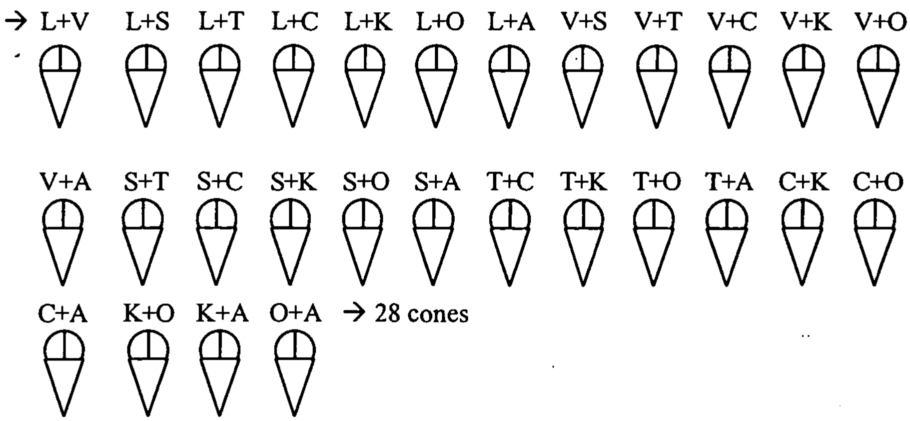
For 5 flavours
L,V,S,T,C → L+V L+S L+T L+C V+S V+T V+C S+T S+C T+C → 10 cones

For 6 flavours
L,V,S,T,C,K → L+V L+S L+T L+C L+K V+S V+T V+C V+K S+T S+C S+K
T+C T+K C+K → 15 cones

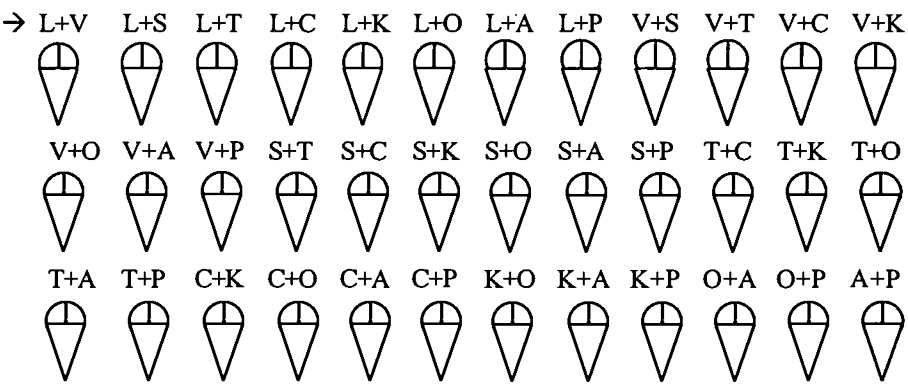
For 7 flavours
L,V,S,T,C,K,O →



For 8 flavours
L,V,S,T,C,K,O,A →



For 9 flavours
L,V,S,T,C,K,O,A,P →



→ 36 cones

Now let us collect the results altogether in a table; f stands for flavour and c for cone.

TABLE ROA/1

f	c
2 →	1
3 →	3
4 →	6
5 →	10
6 →	15
7 →	21
8 →	28
9 →	36

What do we learn by observing this table?

First Observations
The results in the table show that:

When f = 2, c = 1, and when f = 3, c = 3. Thus the difference between c-1 and c equals f-1, i.e., 2.

When f = 4, the difference between c-1 and c equals 3, which is equal to f-1, c = 6, c-1 = 3.

When f = 5, c = 10. Here also there is a difference of f-1 between c and c-1. f-1 = 4 → c-1 = 6.
So, c = (f-1) + (c-1)

When f = 6, c = 15. Again c is different from c-1 by f-1, i.e., 5.

When f = 9, c = 36. Once again c-(c-1) = f-1 = 8.

Second Observation
If c is always different from c-1 by f-1, what about the first row of the table, i.e., the case where f = 2 and c = 1? Is the first observation also applicable to this case? Let's try!

c must be different from c-1 by f-1.
c = 1, then c-1 = 0 and the difference is 1.
Now f = 2, then f-1 = 1, therefore c is different from c-1 by f-1.

Hence the first observation is applicable to the case where f = 2 and c = 1, the first row of the table.

Third Observation
c increased by arithmetical progression, which means by addition. For example, when

f = 2, c = 1
f = 3, c = 1 + 2 = 3
f = 4, c = 1+ 2 + 3 = 6
f = 5, c = 1 + 2 + 3 + 4 = 10
f = 6, c = 1 + 2 + 3 + 4 + 5 = 15
f = 7, c = 1 + 2 + 3 + 4 + 5 + 6 = 21
f = 8, c = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28
f = 9, c = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36

Fourth Observation
f = 2, c = f-1
f = 3, c = (f-1) + (f-2)
f = 4, c = (f-1) + (f-2) + (f-3)
f = 5, c = (f-1) + (f-2) + (f-3) + (f-4)
f = 6, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5)
f = 7, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5) + (f-6)
f = 8, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5) + (f-6) + (f-7)
f = 9, c = (f-1) + (f-2) + (f-3) + (f-4) + (f-5) + (f-6) + (f-7) + (f-8)

So if f = 1, c = f-1, which is nought; since each cone must necessarily have two different flavours, there is no combination and c = 0.

What does the fourth observation tell us?

It tells us that for any number of f, provided $f \geq 2$, c will be equal to the sum of f_i, f_{i+1}, f_{i+2} ... until $f_{i+(n-1)}$ [where i = 1]. For example, if f = 3, then n = 2, f_i = f-1, and $f_{i+(n-1)}$ = f-2.

So $\sum_{i=1}^{n-1} f_i$ is one rule to work out the number of combinations

The Σ is called sigma and means sum

$\sum_{i=1}^{n-1} f_i$ is read sigma f_i from i = 1 to n - 1

Another way to work out the possible combinations is to apply the following combination formula.

$$^{m}Cp = \frac{m!}{p!\,(m-p)!}$$

Where m stands for number of elements to combine, C for combination, and p for number of elements in a combination.

This formula is read as combination of m elements taken p by p equals factorial m divided by factorial p, factor of factorial m minus p.

Let us open a parenthesis to show how the factorial works.

2 ! = 1 x 2
3 ! = 1 x 2 x 3
4 ! = 1 x 2 x 3 x 4
n ! = 1 x 2 x 3 x .... x n
1 ! = 1

And by convention, 0 ! = 1

To use this formula for our ice-cream investigation, we just have to substitute f for m to get

$$^{f}Cp = \frac{f!}{p!\,(f-p)!} = c$$

Combination of f flavour taken p by p equals factorial f divided by factorial p factor of factorial f minus p, equals c.

So, for f = 2 and p = 2, we have c = $\frac{2!}{2!\,(2-2)!} = \frac{2}{(2 \times 1)\,0!} = \frac{2}{2 \times 1} = \frac{2}{2} = 1$

for f = 3, p = 2 c = $\frac{3!}{2!\,(3-2)!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} = \frac{3 \times 2}{2 \times 1} = \frac{6}{2} = 3$

for f = 5, p = 2, C $\frac{5!}{2!\,(5-2)!} = \frac{5x4x3x2x1}{2x1x3x2x1} = \frac{5 \times 4}{2 \times 1} = \frac{20}{2} = 10$

for f = 9, p = 2, C $\frac{9!}{2!\,(9-2)!} = \frac{9x8x7x6x5x4x3x2x1}{2x1x7x6x5x4x3x2x1} = \frac{9 \times 8}{2 \times 1} = \frac{72}{2} = 36$

Conclusion

We have shown through this investigation two ways to work out the possible number of combinations of a set of elements without repetition, if we are given the number of elements in each combination. The first one is by arithmetical progression using the sigma

$$\sum_{i=1}^{n-1} f_i$$

and the second one is the combination formula using the factorial notation: the product of all the positive integers from n down to 1 is denoted by n!

$$^{f}Cp = \frac{f!}{p!\,(f-p)!}$$

In this investigation, both ways are equivalent. However, I think that the combination formula is easier to use than the first one for a large number of elements. Nevertheless, we can use the sigma formula for a smaller number.

From the pictorial representations and observations made, it can be observed that the combination columns follow that of the Blaise Pascal Triangle of numbers. The numbers in Pascal's triangle also occur when we consider the number of ways we can combine different flavours. There is need for more research into adults returning to study mathematics and numeracy, focusing on both cognitive and affective domains. These need to be complemented by a third, the ontological dimension.

I would argue that whilst teaching techniques in mathematics education of the past have shown that adult students can be trained to use their minds and not to think, teaching techniques in mathematics education today (e.g., mathematical investigation) should require adult students to think, as well as young people in the education system. This has wider educational implications for improving professional teaching standards and skills for adult educators, improving learning strategies, improving learning objectives, resources, learning activities, and outcomes. Implications for teaching adult students should be drawing attention to the professional development, both initial preparation and continuing education, of more practitioners in the field.

References

Angiama, R. O. (1994). Adults re-learning investigations. In D. Coben (Ed.), *Proceedings of the first international conference of Adults Learning Mathematics—A Research Forum*. London: Goldsmiths College, University of London.

Angiama, R. O. (1995). Teaching adult students mathematical investigation. In D. Coben (Ed.), *Mathematics with a human face* (Proceedings of the second international conference of Adults Learning Mathematics—A Research Conference). London: Goldsmiths College, University of London.

Benn, R. (1997). *Adults count too. Mathematics for Empowerment.* National Institute of Adult Continuing Education (England and Wales).

Cockcroft, M. W. H. (1982). *Mathematics counts* (Report on the Committee of Inquiry into Teaching of Mathematics). London: HMSO.

Frankenstein, M. (1989). *Re-learning mathematics*. London: Free Association Books.

Isatsarom, A., et al. (1996). Mathematics: Problematical notion of closure. In P. Ernest (Ed.), *Mathematics education and philosophy: An international perspective* (p. 1994b). London: Falmer Press.

Skemp, R. (1971). *The psychology of learning mathematics*. Harmondsworth, Middlesex, England: Penguin Books.

SE066771

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

ERIC
Educational Resources Information Center

Reproduction Release
(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: ADULTS LEARNING MATHEMATICS: A CONVERSATION BETWEEN RESEARCHERS AND PRACTITIONERS	
Author(s): MARY JANE SCHMITT AND KATHERINE SAFFORD-RAMUS (EDS.)	
Corporate Source: NCSALL, HARVARD GRAD SCHOOL OF ED.	Publication Date: 2001

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, Resources in Education (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign in the indicated space following.

The sample sticker shown below will be affixed to all Level 1 documents	The sample sticker shown below will be affixed to all Level 2A documents	The sample sticker shown below will be affixed to all Level 2B documents
PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY SAMPLE TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE, AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY, HAS BEEN GRANTED BY SAMPLE TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY SAMPLE TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
Level 1	Level 2A	Level 2B
☒	☐	☐
Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g. electronic) *and* paper copy.	Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only	Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits.
If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche, or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.	
Signature: Katherine Safford-Ramus, Ed.D.	Printed Name/Position/Title: KATHERINE SAFFORD-RAMUS ASSOCIATE PROFESSOR
Organization/Address: ST. PETER'S COLLEGE 2641 KENNEDY BLVD JERSEY CITY, NJ 07306	Telephone: 201-915-4930 Fax:
	E-mail Address: SAFFORD_K@spc.edu Date: 9/17/02

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
4483-A Forbes Boulevard
Lanham, Maryland 20706
Telephone: 301-552-4200
Toll Free: 800-799-3742
e-mail: ericfac@inet.ed.gov
WWW: http://ericfacility.org

EFF-088 (Rev. 2/2001)