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ABSTRACT

This paper explores questions about linear relationships through the use of pattern. Participants in the session generated and extended patterns and used calculators to investigate the relationships. Examples from current Texas Instruments (TI) books and U.S. textbook series, plus examples from proposed TI materials are shared. These activities support the middle grades algebra strand suggested in the National Council of Teachers of Mathematics (NCTM) Principles and Standards document. (Author/KHR)

Exploring Patterns, Functions and Algebraic Reasoning with a Calculator

Melfried Olson

Exploring Patterns, Functions and Algebraic Reasoning with a Calculator

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Description: This session will explore questions about linear relationships through the use of patterns. Participants will generate and extend patterns and use calculators to explore the relationships involved in the specific situations. Examples from current TI books and US textbook series plus examples from proposed TI materials will be shared. These activities support the middle grades algebra strand suggested in the NCTM Principles and Standards document.

Patterns, functions, algebraic reasoning and the use of calculators in school mathematics

Overview of patterns, functions and algebraic reasoning

In the document *Principles and Standards for School Mathematics* (NCTM, 2000) the topics of patterns, functions, and algebraic reasoning are primarily described under the strand of Algebra. As with many topics in mathematics, it is difficult to separate the content strand from the process stands. Hence, many examples of patterns and functions can also be identified in the strands of Number and Operations, Geometry, Data Analysis and Probability, as well as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.

Specifically, the Algebra Standard states:

"Instructional programs from prekindergarten through grade 12 should enable all students to—

• understand patterns, relations, and functions;

• represent and analyze mathematical situations and structures using algebraic symbols;

• use mathematical models to represent and understand quantitative relationships;

• analyze changes in various contexts" (page 37)

Early experiences that provide children with the opportunity to employ the processes of classifying and ordering can naturally lead to expressing patterns either with pictures or orally. It is common for children to express the relationship in each of the following sequences 2, 4, 6, 8, ... and 1, 3, 5, 7, ...as 'add two'. In doing so, they are focusing on how each term is obtained from the previous term, a very important component of recursive thinking. Later, however, we want students to be able to express these two sequences as '2n', and '2n – 1, respectively. As students explore patterns and relations in the middle grades, they should engage in generating and validating equivalent expressions.

Along with considering the mathematics elementary school children are to learn, we must also consider what mathematical content teachers of elementary school mathematics learn. While examining the appropriate content for teachers, the Mathematical Association of America notes, "Once one looks below the surface, mathematics at the elementary school is conceptually complex" (MAA, Chapter 3, page

1)) It becomes important for instructors of prospective teachers to engage their students in studying these complexities and having them reflect on how they learn mathematics. In discussing the role of Algebra and Functions for prospective teachers or mathematics at the elementary level, MAA states:

> "Algebraic notation is an efficient means for representing properties of operations and relationships among them. In the elementary grades, well before they encounter that notation, children who are encouraged to recognize and articulate generalizations will become familiar with the source of ideas they are later to express algebraically. In order to support children's learning in this realm, teachers first must do this work for themselves. Thus, they must come to recognize the importance of generalization as a mathematical activity. In the context of number theory explorations (e.g., odd and even numbers, square numbers, factors) they can look for patterns, offer conjectures, and develop arguments for the generalizations they identify. Moreover, the arguments they propose become the grist for investigating different forms of justification. If, in this work, teachers learn to use a variety of modes of representation, including conventional algebraic symbols, the algebra they had earlier experienced as opaque symbol manipulation can be invested with meaning." (Chapter 3, page 3)

Teachers of mathematics at the middle level must have a deeper understanding of concepts and procedures related to algebra and functions. The scope of algebra at the middle level expands so teachers must be able to move comfortably between and among differing general ideas related to algebra. MAA suggests "Prospective middle grades teachers must understand and experience algebra as a symbolic language, as a problem solving tool, as generalized arithmetic and generalized quantitative reasoning, as a study of functions, relations, and variation, and as a way of modeling physical situations" (MAA, Chapter 4, page 3)

The use of calculators in teaching and learning patterns, functions, and algebraic reasoning

A calculator can be successfully used to address several of the components of the content mentioned above. With appropriate use of a constant key, a calculator can be used to generate recursive patterns. Patterns as simple as adding one constantly can be used with children to learn counting. When a pattern has been determined, a calculator with graphing capability can be used to give a visual picture of the pattern. Furthermore, a calculator with statistical capability can be used to generate an equation for the pattern. Of course, a calculator can be used to perform computations related to predictions or to extend patterns that would not be made otherwise.

Exploring Patterns, Functions, and Algebraic Reasoning with the Calculator

1. Connecting Cube Activity

A. Angelica was making a train with connecting cubes. She began making her train using the following pattern of colors (R stands for Red, B for Blue, Y for Yellow, and G for Green): R R B R R B R R B R R B R. If Angelica continues this pattern, what will be the color of the 20th cube?, the 88th cube?, the 147th cube?

B. How would your solution change if the colors went B R Y B R Y B R Y?, If they went R R Y B R R Y B R R Y B? How will you organize your work for this? Describe at least three patterns you see in each.

C. For the pattern B R Y B R Y B R ....

1. We see that the numbers of the blocks that are blue are: 1, 4, 7, 10, ... (That is, these numbers when divided by 3 leave a remainder of 1.)
2. The numbers of the blocks that are red are: 2, 5, 8, 11, ... (That is, these numbers when divided by 3 leave a remainder of 2.)
3. The numbers of the blocks that are yellow are: 3, 6, 9, 12, ...(That is, these when divided by 3 leave a remainder of 0. Another way to say this is these numbers are multiples of 3.)

D. Consider the pattern R R Y B R R Y B R R Y B......What patterns do you observe?

2. Skip Counting Activity

Roshawn was practicing skip counting by using the constant operator key. Unfortunately while she was pushing buttons she did not always record the appropriate amount of information to recall what she had done. In fact, before she realized it she had written down the result of six different skip counting practice trials. She listed these in A - F below. However, as can be seen, for each trial she only wrote down the two numbers showing last on the calculator display. That is, she did not write down other possible useful information such as what numbers were being used to skip count and from what numbers was she skip counting.

A. For each of the pairs of values below can you help determine what number Roshawn was using to skip count and also the number where she started? (Recall that in each of these pairs, the first number indicates the number of times the skip counting was done. The second number indicates the result after the skip counting.)

1. = 21   177
2. = 15   99
3. = 22   158
4. = 19   481
5. = 32   99.2
6. = 36   5

B. In a table form, let us examine the list all the integer-valued entries that can be found for the start and skip for **Number 1 in Part A**.

[Start at	Skip Count by
9	8
30	7
51	6
72	5
93	4
114	3
135	2
156	1
177	0]

What relationships do you notice in the above table that refers to the problem in Part A (21   177)? If you could skip count by 6.5, at what value would you start? If you start at 17.3, what value would you use to skip count? What equation expresses the relationship between the numbers in the two columns?

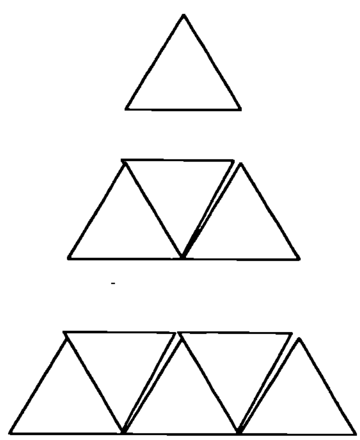
3. Adapted from *Stadium Walls* (Texas Instruments, 1998)

An architect designs roof trusses for a building with the following design. Trusses are made from steel beams arranged in the form of equilateral triangles, as in the diagram below. Each line segment represents a steel beam.

Truss of length 1

Truss of length 2

Truss of length 3



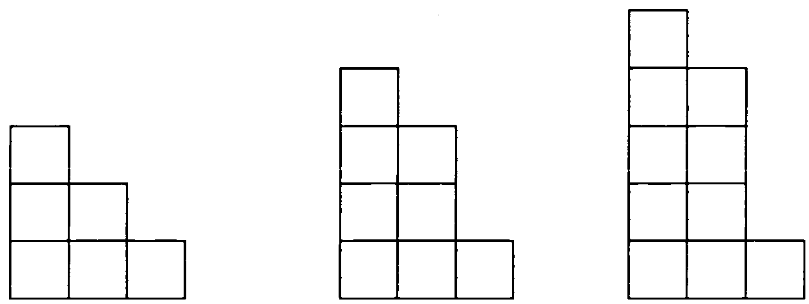
Using toothpicks or other materials, make a table to investigate this situation.

A. How many beams are needed for a truss of length 7?

B. How many beams are needed for truss of length 10?

C. How many beams are needed for a truss of length 23? Of length n?

4. **Adapted from *The Twin's Towers*, Texas Instruments, 1998**
Consider the towers that are made below.



1 story 2 story 3 story

A. What is a recursive pattern that would generate the pattern for the number of blocks needed for the towers?
B. How many blocks are needed for the 8 story tower?
C. How many blocks are needed for the 99 story tower? The 150 story tower?
D. If 4000 blocks were used, which story would be built?
E. How many blocks are needed for an n-story building?

5. **Stacking Cups – Adapted from Navigations through Algebra in Grades 6 – 8, NCTM and Bob Mann, Western Illinois University**

You are working at the Math Munchie Market. The market sells various hot and cold drinks that come in several different-sized cups. One day, while gathering inventory data, you decide there must be a better way to know how many cups are in a stack besides counting all of them.

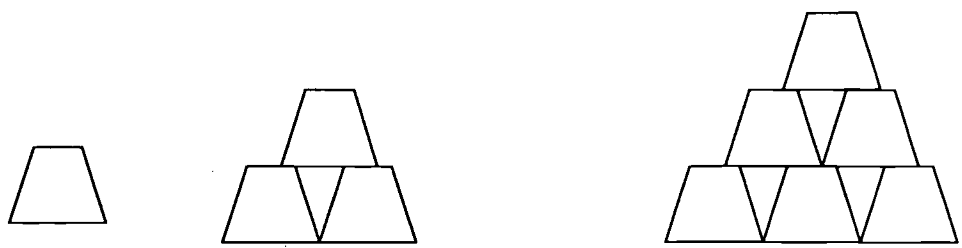
A. Gather data from the cups available to see if you can make a prediction about the number of cups there would be for any given number of cups in a stack.
   1. Describe the cup(s) with which you are working.
   2. Start stacking cups of each type and measure the height of the stack after each cup has been stacked. After stacking 8, plot your data on a graph.
   3. What relationships do you see?
   4. Can you predict the height of 50 cups? Of 100 cups? Will a stack of 100 cups be twice as high as a stack of 50 cups?
   5. If you knew the stack was about 100 cm tall, could you predict how many cups were in the stack? 200 cm? Will a stack 200 cm tall have twice the number of cups as a stack 100 cm tall?
B. Use the stat feature of the TI – 73 to determine the equation for each cup.[Enter number of cups in L1 and height of cups in L2. Create a Stat-Plot of L2 versus L1. Determine the regression line, Stat→Calc→LinReg(ax+b)]
C. Stacking Chairs. If the chairs in the room stack, explore the height versus the number of chairs in the stack.
D. What other explorations could be generated?

6. Stacking Cups Plus

Number relationships and explorations can show up in ways that sometimes surprise us. You are invited to investigate a stacking cups problem and determine all you can about the exploration. Here are some things that might get you started (and also might sidetracked you from more important ideas).

Contemplate the situation given to you, answer the questions given below, and add questions and observations to the list. **Do all your estimations first**—*ON YOUR OWN*, then work with others to determine reasonable solutions.

Some children were stacking cups in a row. The first few stacks looked like the following:



Stack 1 Stack 2 Stack 3

Question	Estimated answer	Computed answer	Happy with estimation? Yes/No
1. How many rows will there be before the cups reach the ceiling when measured from the tabletop? From the floor?			
2. How many more cups will be in the bottom row when building from the floor than when building from the table?			
3. How many cups would you need to stack the cups as high as this building? 1 kilometer?			
4. If your cups are stacked 10 rows tall, how tall will this be? 25 rows tall? n rows tall?			
5. If your cups are stacked 10 rows tall, how wide will the bottom layer be? 25 rows? n rows?			
6. If your cups are stacked 10 rows tall, how many cups are in your stack? 25 rows? n rows?			

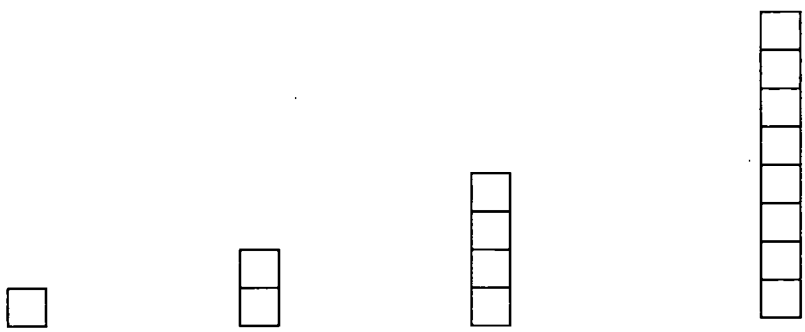
7. If you knew how wide the bottom row of the stack was, how would you predict the height of the stack?			
8. If you knew how tall the stack was, how would you predict how wide the bottom row of the stack was?			
9. List other question you may like to ask.			

List observations you have to share:

7. A caper with cubes

Number relationships and explorations can show up in ways that sometimes surprise us. You are invited to investigate the "Caper with Cubes" problem and determine all you can about the exploration. Here are some things that might get you started (and also might sidetracked you from more important ideas).

Students were busy stacking cubes on a 'chalk' tray (you can use the tabletop). The first three patterns looked like the following.



Stack 1 Stack 2 Stack 3 Stack 4

Contemplate the situation given to you, answer the questions given below, and add questions and observations to the list. **Do all your estimations first—*<u>ON YOUR OWN</u>***, then work with others to determine reasonable solutions.

Question	Estimated answer	Computed answer	Happy with estimation? Yes/No
1. How many columns will there be when the cubes reach the ceiling when measured from the chalk tray? From the floor?			
2. How many more columns will be in the arrangement when building from the floor to the ceiling than when building from the table?			
3. How many columns would you need to stack the cubes as high as this building? 1 kilometer?			

4. If your cubes are in 10 columns, how tall will this be? 25 columns? n columns?			
5. If your cubes were stacked using with purple and gold cubes in every other column (with purple in column 1, 3, 5, etc., and gold in columns 2, 4, 6, etc.) after 10 columns, what is the ratio of purple cubes to gold cubes? After 80 columns? What if you alternated columns of red, white, and blue, in this order?			
6. If you knew how many columns you were using, how would you predict the height of the stack?			
7. If you knew the height of the tallest column, how would you know how many columns there were?			
8. If you knew how many cubes were in the tallest column, how would you determine the number of cubes in the total arrangement?			
9. List other question you may like to ask.			

List observations you have to share:

Other Activities for Using the Calculator to Explore Patterns, Functions, and Algebraic Reasoning

Stadium Walls (From *Using the TI-73: A Guide for Teachers,* Texas Instruments, 1998) Students can use the constant key to help generate the pattern necessary to find the number of beams for walls of lengths 1 to 10. With the TI - 73 the values generated could be entered into lists and a linear regression used to determine the y = ax + b equation.

The Twin's Towers (From *Using the TI-73: A Guide for Teachers,* Texas Instruments, 1998).

Students can either use the suggested keystrokes for the TI - 73 or determine how to use the operator keys on the TI - 15 to generate the patterns desired. This problem provides an interesting look at how to use the TI - 73 to generate ordered pairs to represent the steps in a table. You may wish to only use the constant or operator key to focus on the recursive nature of the growth of the buildings.

Patterns From Concrete to Abstract (from *Patterns, Patterns, Patterns - Patterns Everywhere!, Math Teacher Educator Short Course for College Professors Teaching Pre-service Teachers with an Elementary School Math Focus,* Edited by Margo Lynn Mankus, and Mark Klespis, Technology College Short Course Program. 1999.)

Students investigate concrete patterns and use the information to predict what would happen in a general case. The CONS or OP feature can be used to extend patterns and the TI - 73 can be used to find formulas for some expressions.

A Very Famous Numerical Pattern by Leonardo de Pisa (from *Patterns, Patterns, Patterns - Patterns Everywhere!, Math Teacher Educator Short Course for College Professors Teaching Pre-service Teachers with an Elementary School Math Focus,* Edited by Margo Lynn Mankus, and Mark Klespis, Technology College Short Course Program. 1999.)

Students can investigate and extend numerical patterns similar to the Fibonacci sequence by using the CONS or OP features. The TI - 73 can be used to present a visual representation of the patterns formed.

Let's Play Video Games (from *Patterns, Patterns, Patterns - Patterns Everywhere!, Math Teacher Educator Short Course for College Professors Teaching Pre-service Teachers with an Elementary School Math Focus,* Edited by Margo Lynn Mankus, and Mark Klespis, Technology College Short Course Program. 1999.)

Students can use a variety of calculators to examine the patterns involved by the problem posed that related to how many different ways you can buy tokens to play video games at an arcade

Building Swimming Pools (from *Algebraic Thinking Module, Math Teacher Educator Short Course for College Professors Teaching Pre-service Teachers with a Middle School Math Focus,* Edited by Debbie Crocker, Margo Mankus, and Sharon Taylor, Technology College Short Course Program. 1998.)

Students can use the graphing feature of the TI - 73 to examine the number of tiles needed to enclose a square swimming pool. In addition to other ways that students can obtain the same information, from the data entered in a list, a regression equation for this relationship can be obtained.

How Totally Square, Part I and Part II (from *Discovering Mathematics with the TI – 73: Activities for Grades 5 & 6*, Edited and Revised by Melissa Nast, Texas Instruments, 1998.)

Students will make a variety of sizes of squares, discover patterns, and make predictions based on the patterns observed. The graphing, list, and the ability to obtain equations from the regression function on TI - 73 can be used in this investigation.

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