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AUTHOR	Dahan, Jean-Jacques
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ABSTRACT

The Cabri programming language is a dynamic geometry software used all around the world by many of teachers, students, and researchers in mathematics. This paper presents examples of using Cabri and graphing calculators as a tool to practice mathematics and provides ways that mathematics could be approached, taught, and received in a way permitting all students to do real mathematics. (Author/KHR)

1

Another way to teach derivative and antiderivative functions with Cabri

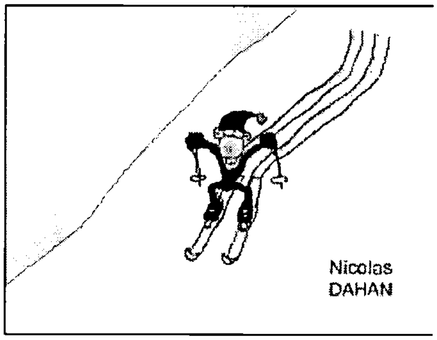
Jean-Jacques DAHAN jjdahan@wanadoo.fr
IREM de Toulouse http://www.irem.ups-tlse.fr
(Institute of Research for the Teaching of Mathematics in Toulouse)

Introduction

Cabri is a dynamic geometry software actually used all around the world by a lot of Teachers (and therefore by a lot of students), a lot of Searchers in Mathematics (for example in hyperbolic or elliptic geometry). More and more persons had been using it in its implemented version in the TI-92 since 1995. In 2002, a more democratic version of this sofware will be available on the TI-83 (Cabri-Junior). It becomes very important to share all our experiences and our creations about Cabri, all the more that, Cabri is really a tool to practice Mathematics in a modern and powerful way. This presentation aims to show you and probably to convince you that Math could be approached, teached and received in a beautiful way permitting to all to do really Math.

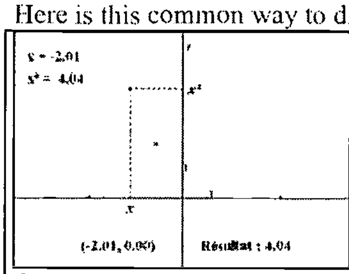
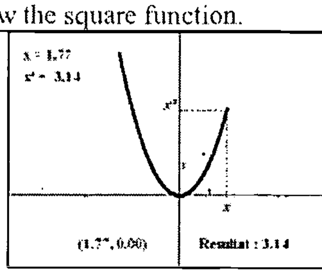
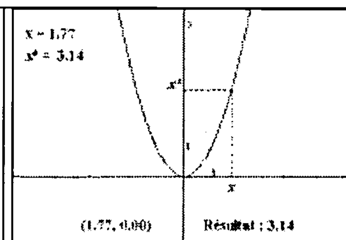
1. How to draw curves of functions?

In Calgary, the first way to draw a curve in Winter is to trace it on the snow of one of the beautiful slopes of the Rocky Mountains. But, it needs special equipments, special outfits and special places in special seasons.
With Cabri, it is possible to do it with only a calculator or a lap top; this way is a way commonly used by our students in french highschools.



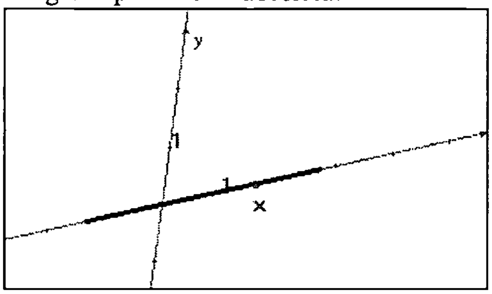
1.1. A library of curves

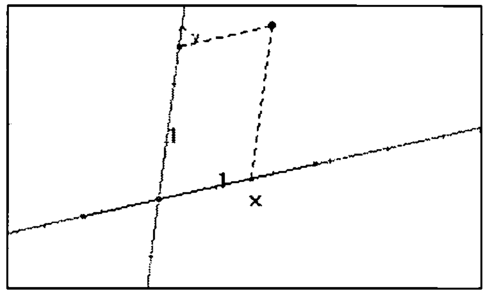
Here is this common way to draw the square function.

		
One calculates the square x^2 of the abscissa x of a point created on the abscissa axis. One creates the point of the square function that can be moved by dragging on the x point.	This curve can appear point after point by dragging on this x point after having actived the trace of the point having $(x ; x^2)$ as coordinates	This curve can also appear in one action: one asks for the locus of $(x ; x^2)$ point when x moves along either the abscissa axis or better on a segment included in the abscissa axis.

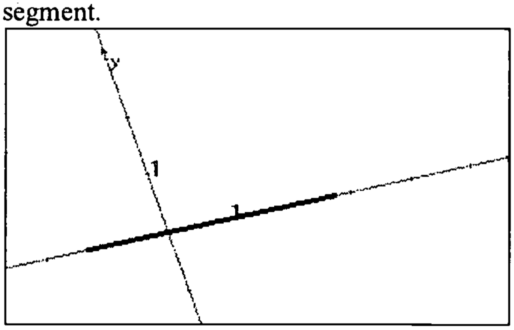
One can also ask Cabri for memorizing our constructions as programs. We have shown 3 macro constructions thare are programs who permit us to get new tools that can be used further in this file and in other files.

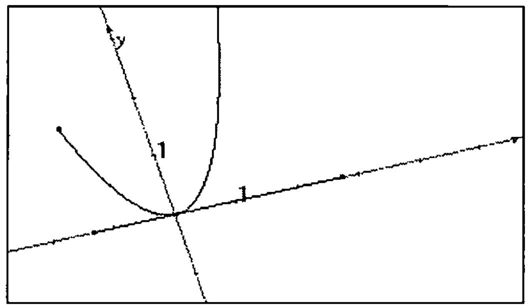
Macro 1: if you click on the initial objects that are, a system of axis, a segment of this axis and a point of this segment, you get as final object, the point of the square function having the first given point as an abscissa.





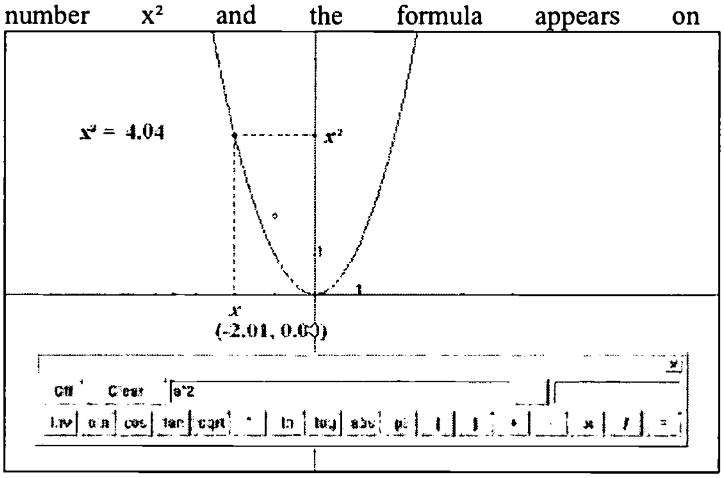
Macro 2: if you click on the initial objects that are, a system of axis and a segment of this axis, you get as final object, the curve of the square fuction having between the bounds of this segment.



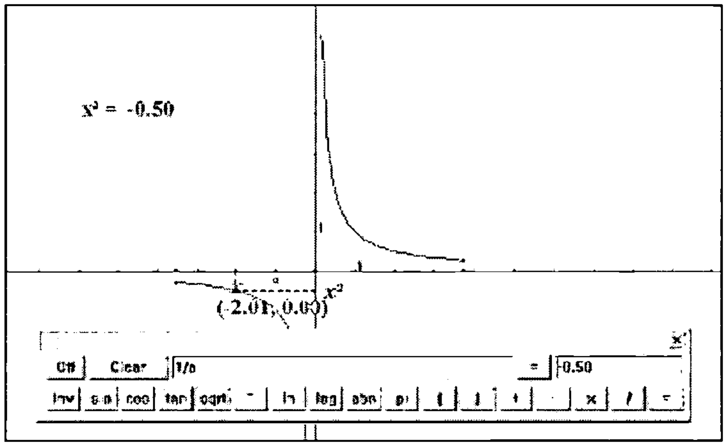


Macro 3: if you click on the initial object that is number x and you get as final object number x^2.

It is possible to get these 3 macros starting from a different function; we need only to get the file giving this new function; let us ask for the calculator of Cabri and double click on the number x^2 and the formula appears on the calculator, $\boxed{a\text{^}2}$.

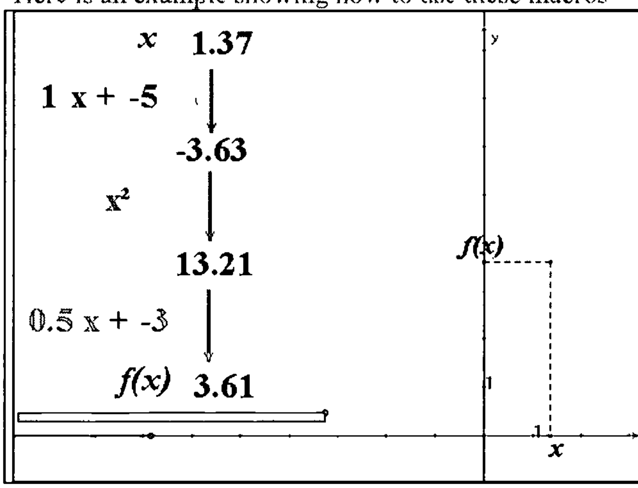


Then, let us turn the formula in $\boxed{1/a}$ and let us click on the $\boxed{=}$ button to get



1.2. Algebraic and geometric composition

Here is an example showing how to use these macros



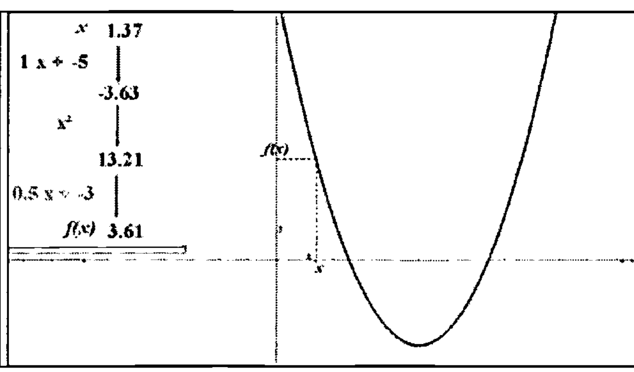
We have here applied to number x = 1.37 representing the abscissa of a point of the abscissa axis macro 3 (corresponding to the $x \to 1x+(-5)$ function) to get –3.63).

We have here applied to this number –3.63 macro 3 (corresponding to the square function) to get 13.21.

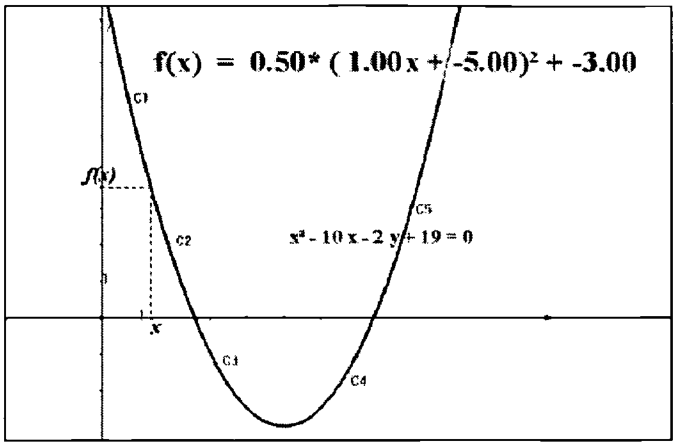
At last, we have here applied to this number 13.21 macro 3 (corresponding to $x \to 0.5x+(-3)$)

The point having (x ; f(x) as coordinates, has been drawn classicaly. Let us remark that the numbers used in the 2 affin functions can be changed

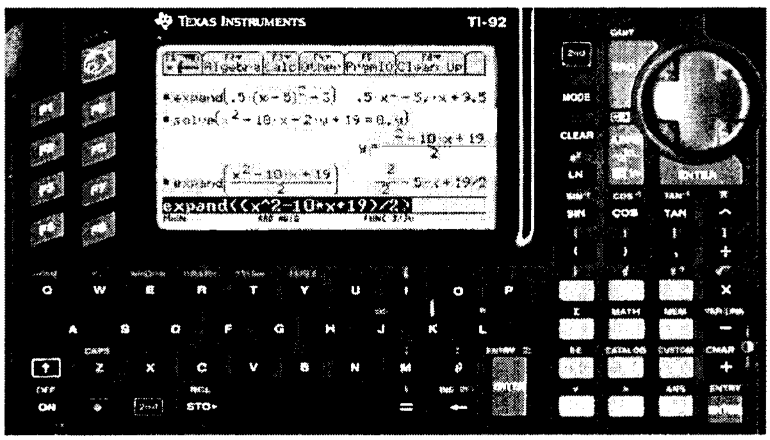
To get the curve of the composed function, we ask Cabri for drawing the locus of point (x ; f(x) when point x moves. It seems to be a parabola.



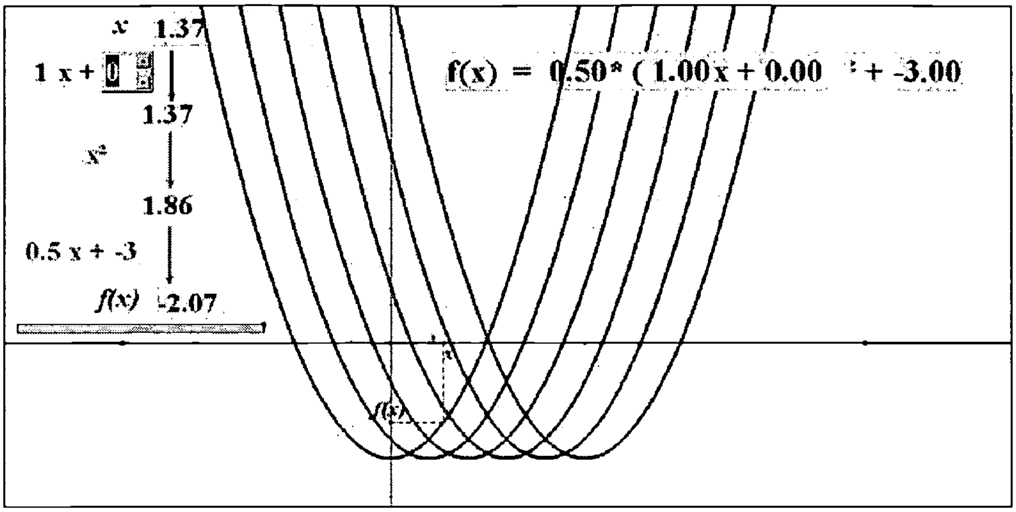
So we can draw a conic passing through 5 points of this locus; this conic seems to be the same curve as ours and this conic is recognized by Cabri as a parabola. Cabri gives us an equation of it (here: $x^2 - 10x - 2y + 19 = 0$).
We have written above the figure one of the formula that we can get in composing the 3 given functions:
$f(x) = 0.5*(1.x + -5)^2 + -3$



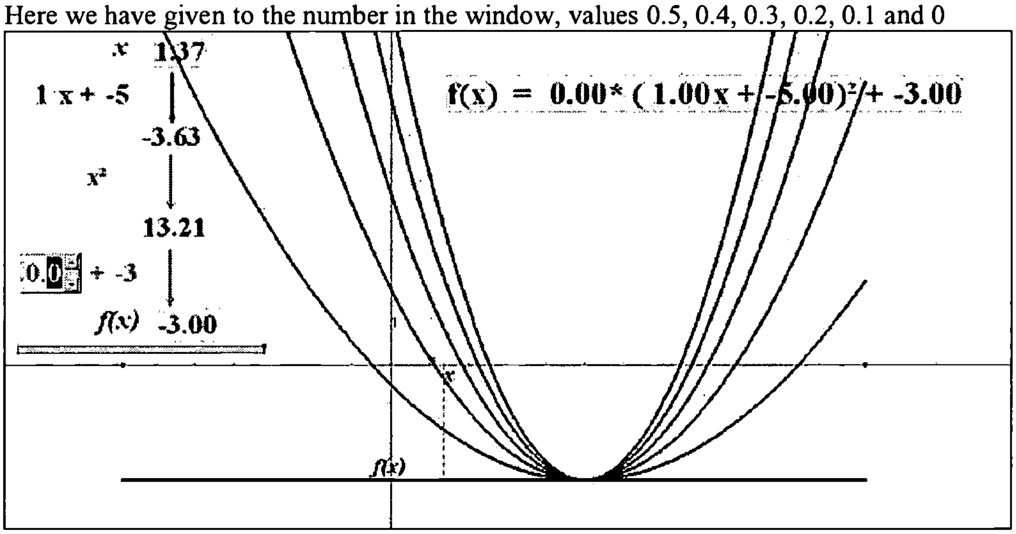
We have shown during the presentation how to use the TI-92 to proove that these two formulas are equivalent.



What about the shape of curves when modifying parameters?
Here we have given to the number in the window, values –5, -4, -3, -2, -1 and 0



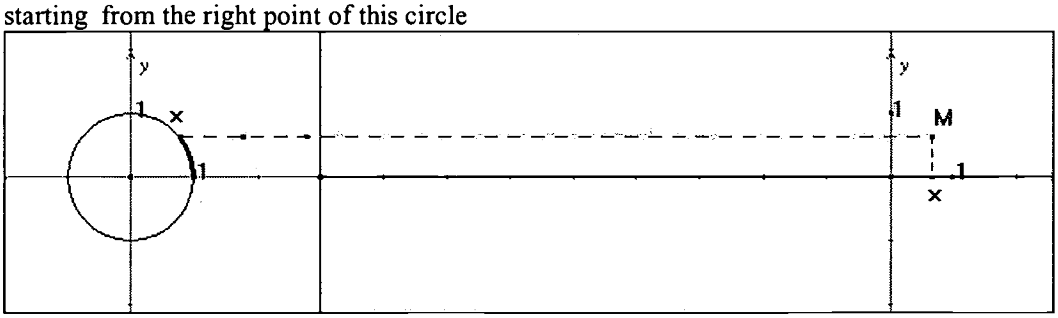
Here we have given to the number in the window, values 0.5, 0.4, 0.3, 0.2, 0.1 and 0



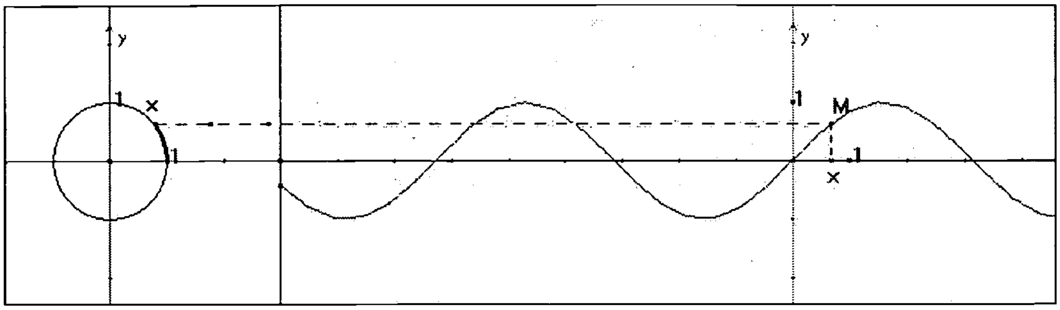
1.3. The special example of trigonometric curves

Curve of the sine function

We have to transfer the x abscissa of a point of the abscissa axis on the tigonometric circle starting from the right point of this circle

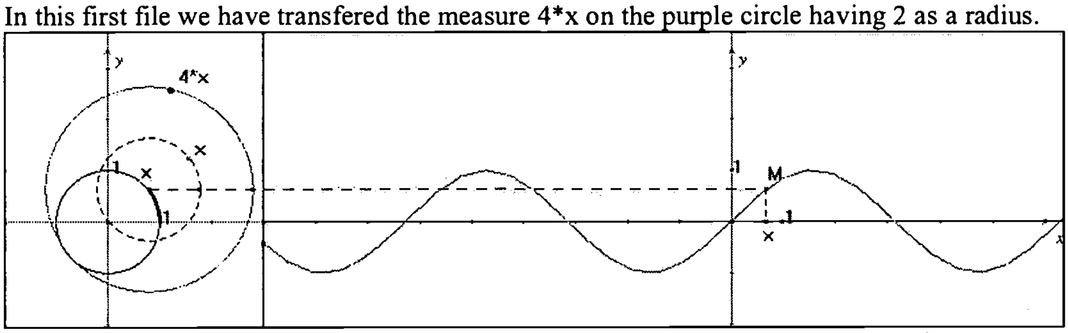


To get the curve we ask for the locus of the point having x as an abscissa and the ordinate of the drawn point on the circle as an ordinate when x moves.

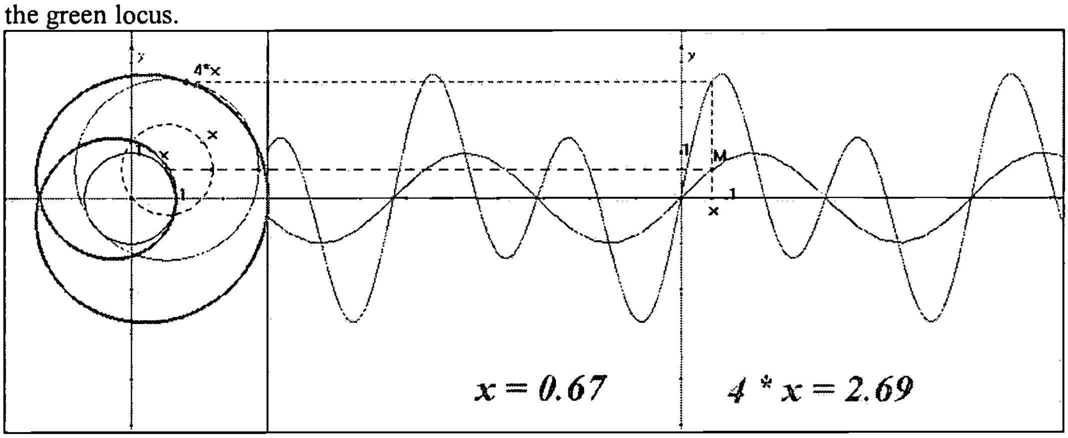


To do more beautiful things, here are constructions given to me by my japanese friend, Ichiro Kobayachi. My students have realised these files without any problems.

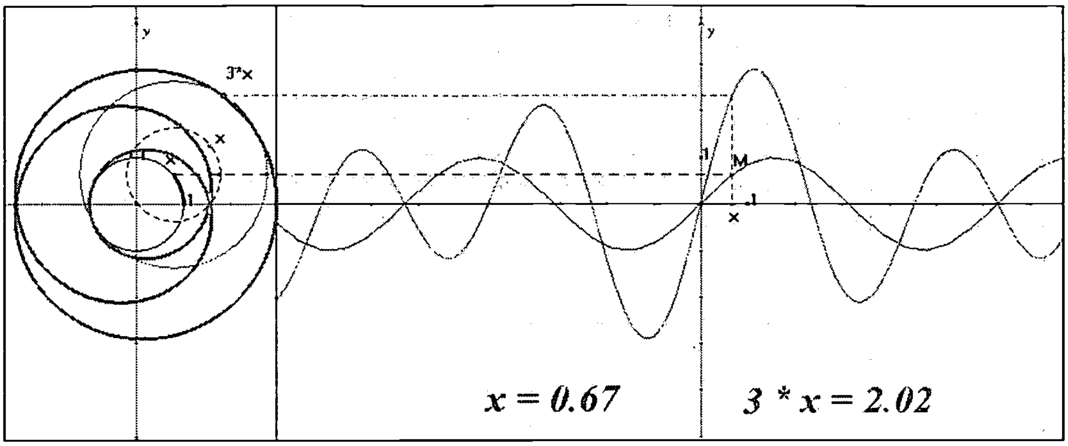
In this first file we have transfered the measure 4*x on the purple circle having 2 as a radius.

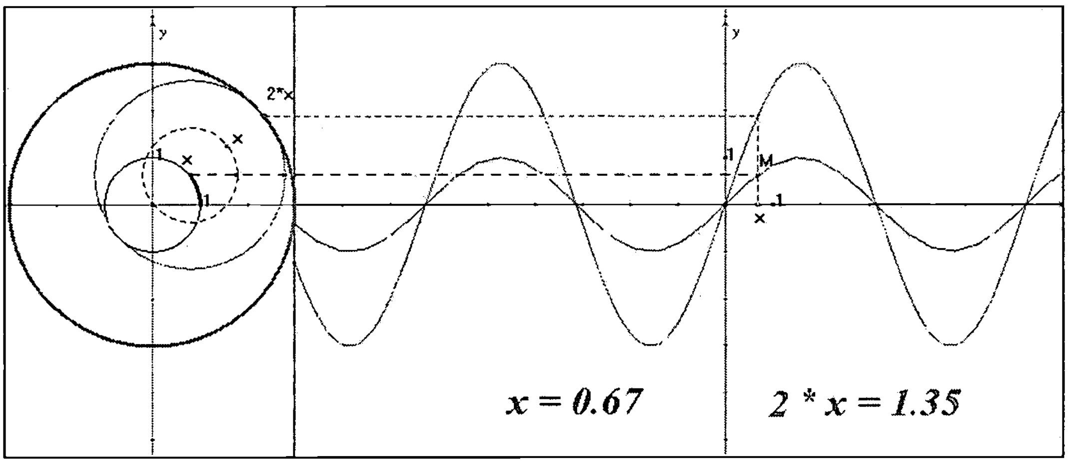


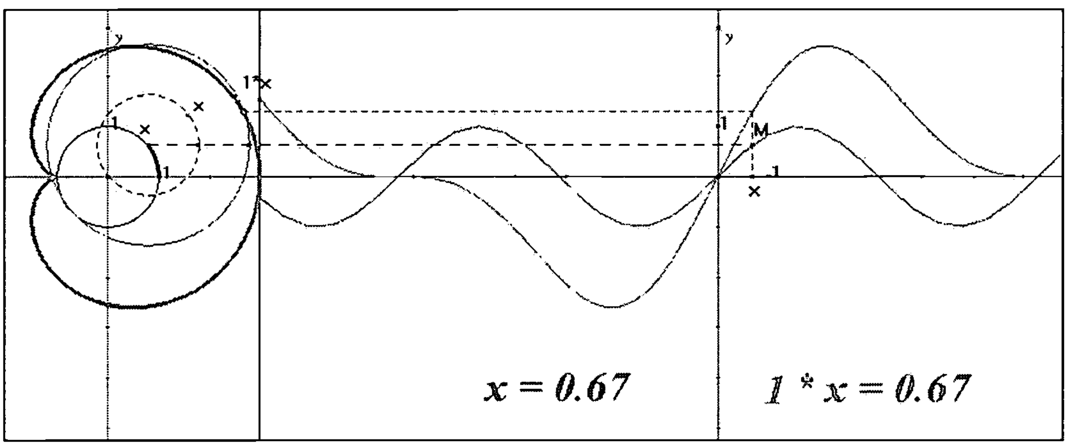
Here we have obtained as a locus a curve similar (the purple one) to the previous but the ordinate is got with the second point: one interesting problem is to find the equation of this curve. If we ask Cabri for giving us the locus of the green point of the purple circle, we get the green locus.



Below, we have modified the value 4 in 3, 2,and 1

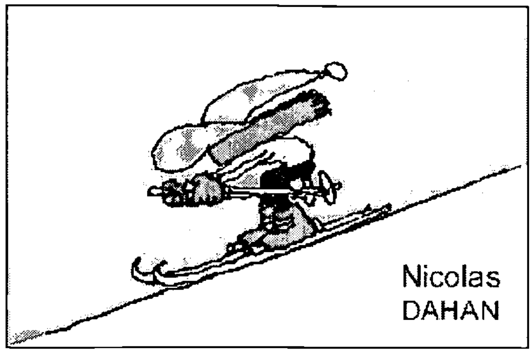






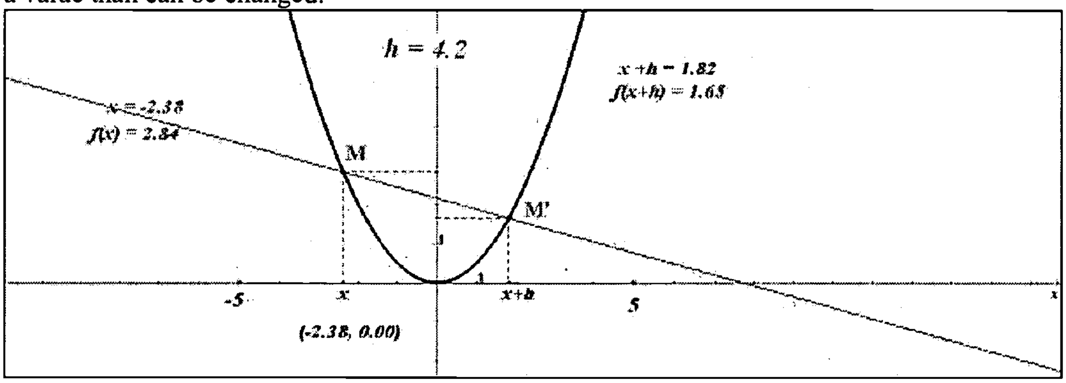
2. How to introduce the tangent line and the derivative function?

We know that the tangent line is in relationship with the slope. When skiing we can have an idea of the slope at each second. Our feelings can give us instaneously an idea of it. It is nice, but with Cabri we can do it so easily without needing gloves and scarves.

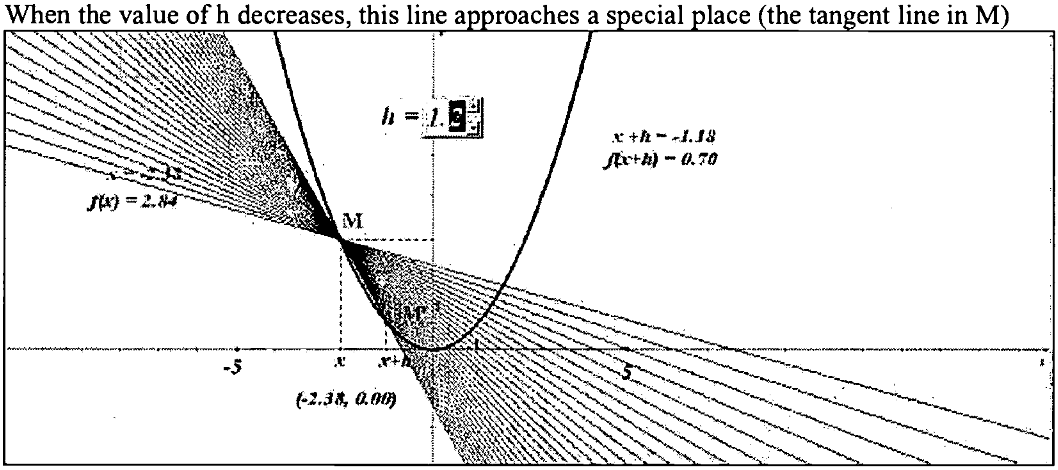


2.1. The Cabri construction

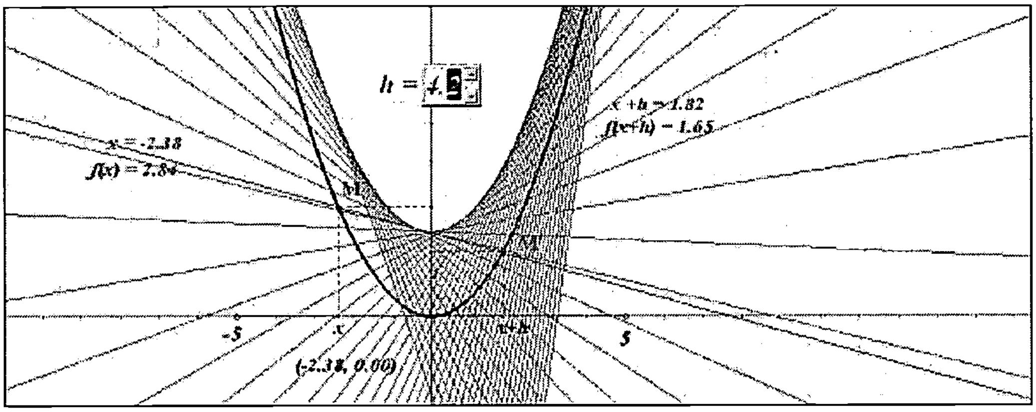
First, let us draw the line (MM') passing through M(x ; f(x)) and M'(x+h ; f(x+h)) where h is a value than can be changed.



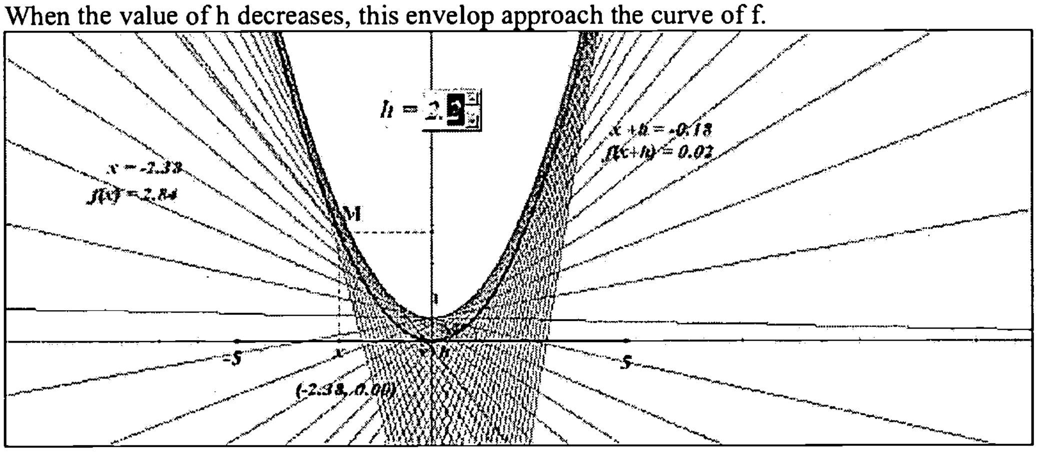
When the value of h decreases, this line approaches a special place (the tangent line in M)



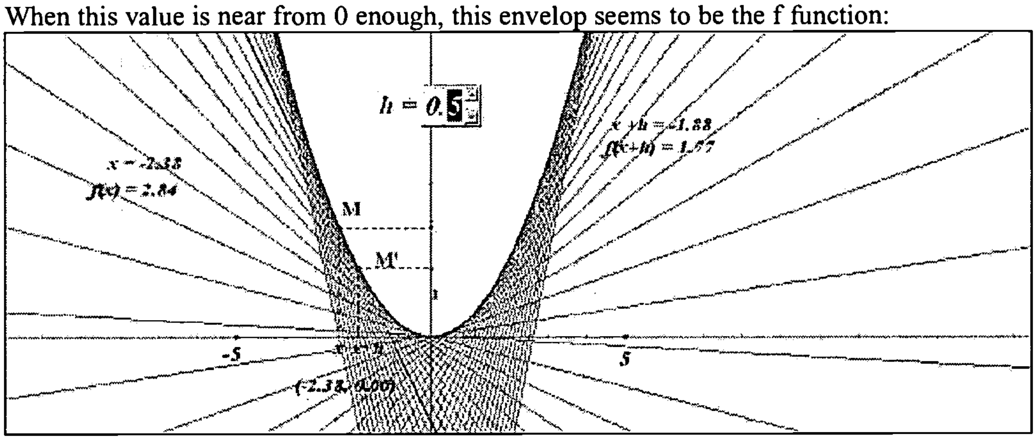
Here is what we obtain when we ask for the locus of (MM'): these lines seem to envelop a curve near the blue one (the blue one is the curve of $f(x) = 0.5.x^2$).



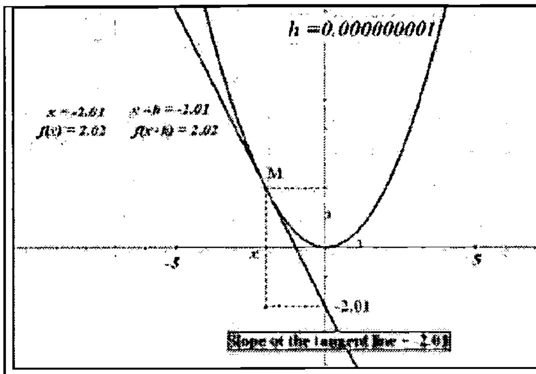
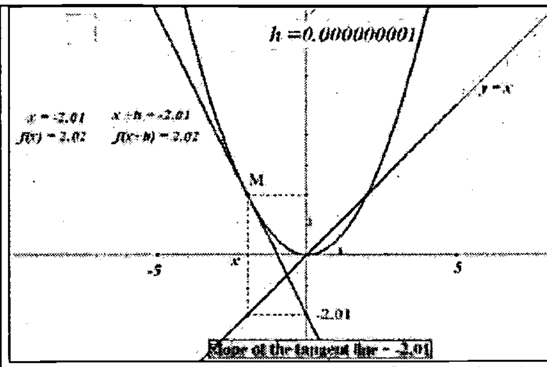
When the value of h decreases, this envelop approach the curve of f.



When this value is near from 0 enough, this envelop seems to be the f function:

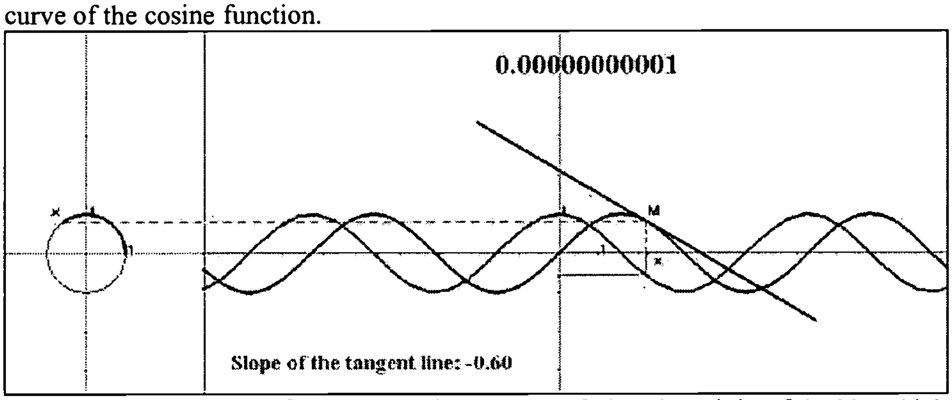


So, we can observe that a (MM') line can be considered as a tangent line of the curve of the f function on point M when h has a value very close to 0.

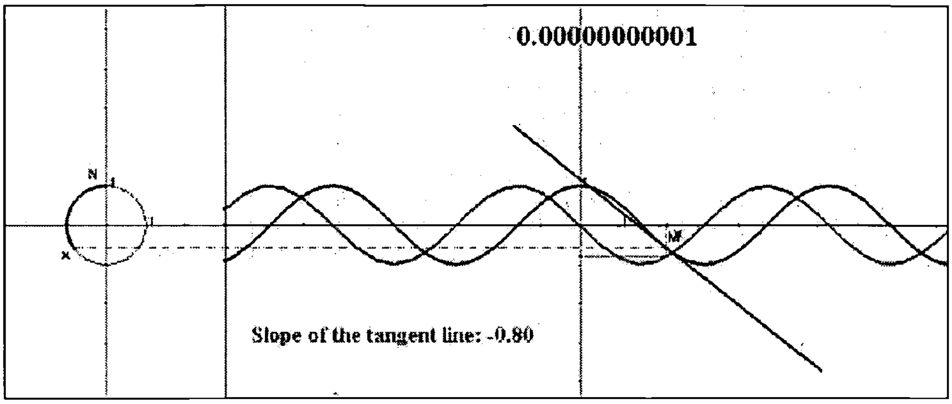
	
Here is the line (MM') that we consider in a Cabri file as the tangent line on M.	Here, we have drawn the locus of the point (x ; f'(x)) where f'(x) is the slope of (MM') with this special value of h. The thick blue curve we get is the curve of the derivative function: it is easy to conjecture that this derivative function can be f'(x) = x (so, we have drawn a line passing 2 points of our locus and asked for the equation of this line).

2.2. Conjecturing easily the algebraic formulas

The blue curve is the curve of the sine function and the red one is the curve of its derivative function built with the previous method: we can easily conjecture that this red curve is the curve of the cosine function.



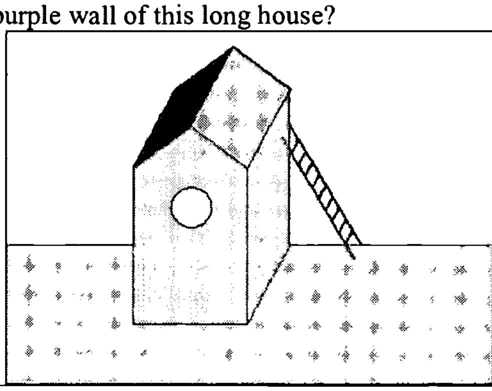
The following file is got from the previous by transfering the origin of the blue thick arc in M. So the sine curve becomes the cosine curve and the derivative function can be conjectured to be the curve of minus sine function



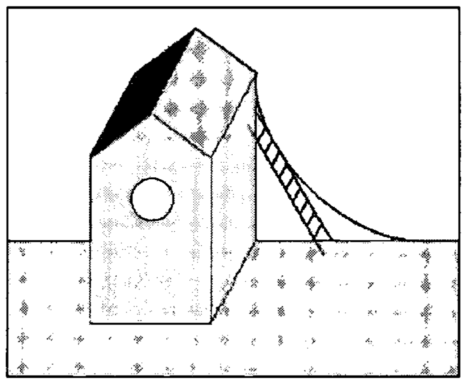
2.3. A curve from its tangents

The following example tries to show that it is easy to imagine with Cabri a curve given with its tangent lines.

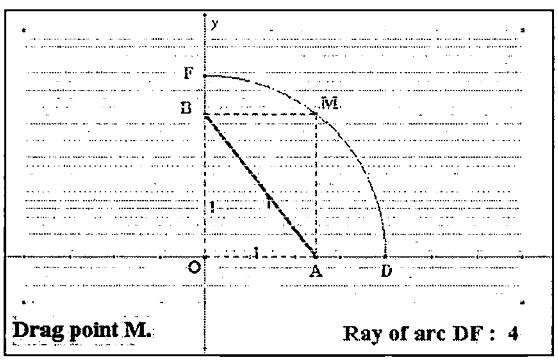
The problem is: what is the curve enveloped by the right part of a scale sliding along the purple wall of this long house?



This curve is obtained as a locus:

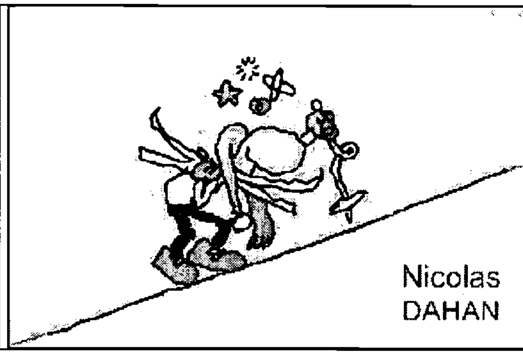


To simulate the movement of the scale AB, we must drag the point M on the red circle. To get the curve we are searching, we must ask the locus of the line (AB) (and not the locus of the segment [AB])



3. How to introduce the antiderivative function?

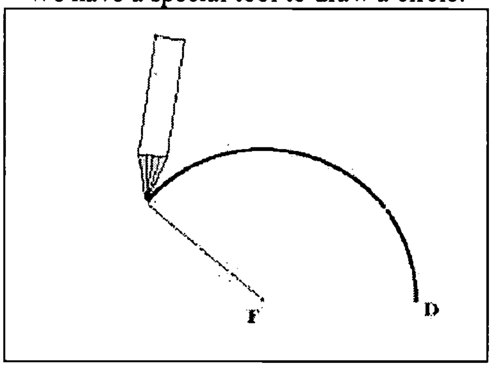
In the mountains, it seems that the inverse way we have to follow, for feeling the tangent line, is the good one, but it is so tiring, as you can see on the right drawing. Cabri will be very powerful to help us feeling this mahematic knowledge.



3.1.Another way to draw a curve from its tangents

The circle is a very particular curve as we know how to draw easily each tangent on each point (perpendicular line to a radius)

We have a special tool to draw a circle:

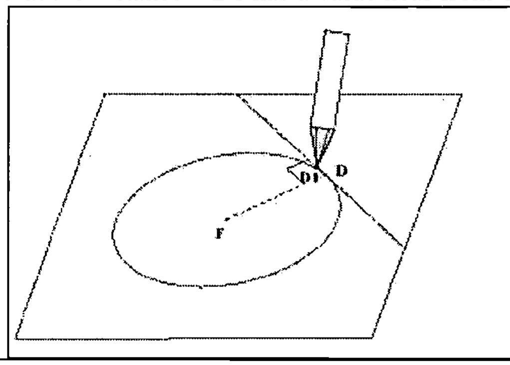


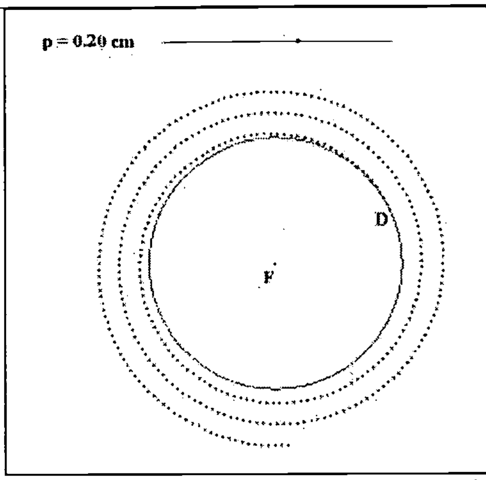
But it is difficult to built such a tool for other curves

Here we can observe that:

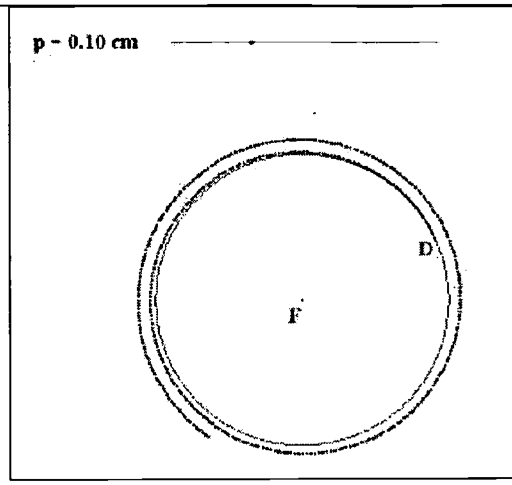
If D is a given point of a circle having a given origin F.

To get an other point D1 of this circle, near from D without using a compass, but only the tangent line to this circle in D (which is the perpendicular line on D to (FD)), we will choose this point D1 on the tangent line so that the distance DD1 has a value near from 0.



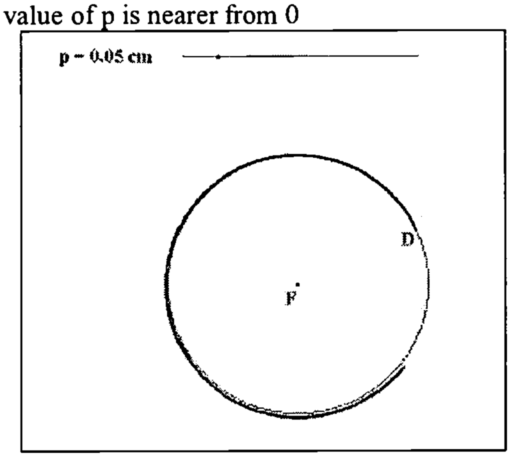


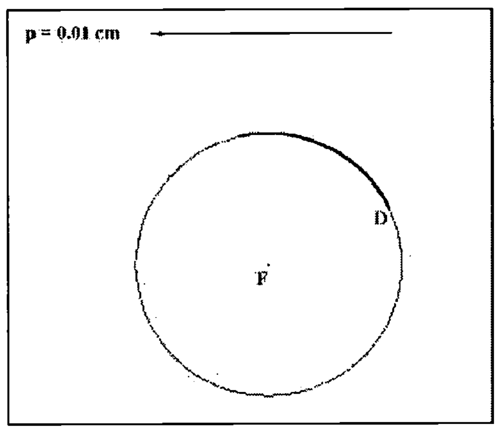
Starting from a point D of a circle and using this method to get other points of the circle, we obtain a chain of red points which does not recover the circle.



This chain is approaching the circle when the choosen distance DD1 = p decreases.

The following files show us that the red chain got with this method is better when the p value is near from 0. The drawing of the circle with this method is all the more accurate that the value of p is nearer from 0





We will use this method to draw curves knowing their tangent lines, that is to say the slopes of their tangent lines. The problem we will solve is to draw the curve of a function f knowing the derivative function f'.

3.2.Euler's method

As an example, we have choosen to draw the curve of the function having 2.x as a derivative function.

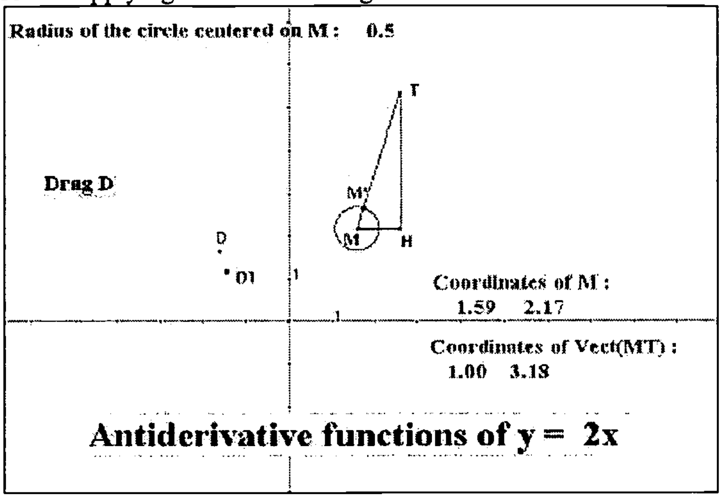
We have done in blue the constructions letting us to draw M' from M such as:

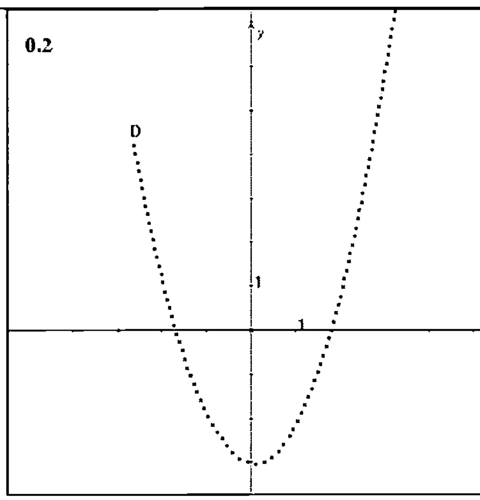
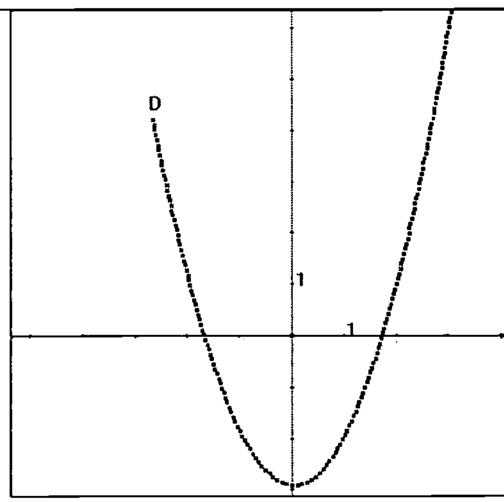
Slope of (MM') = 2x where x is the abscissa of point M and

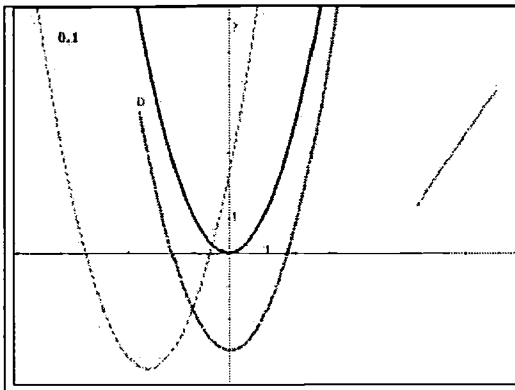
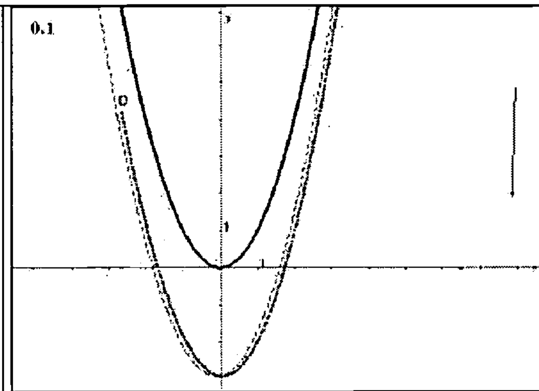
Distance (MM') = a number that can be changed (here this number is 0.5).

After that we have recorded a macro giving us M' as a final object when the initial objects are: the system of axis, the number representing MM' and the point M.

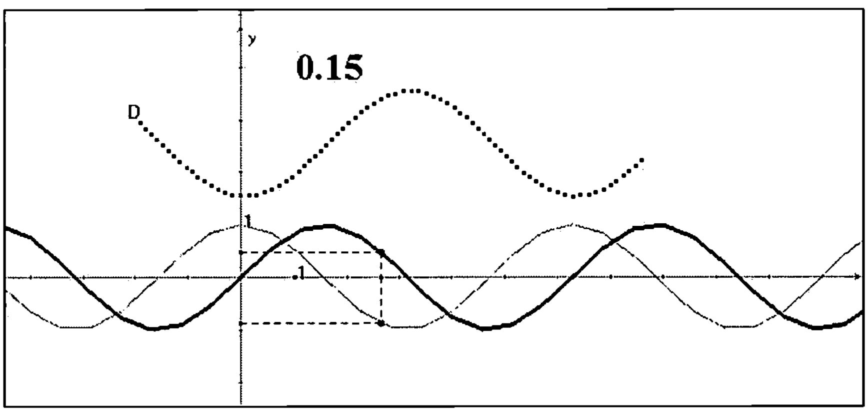
Using this macro and applying it to D we have got D1.



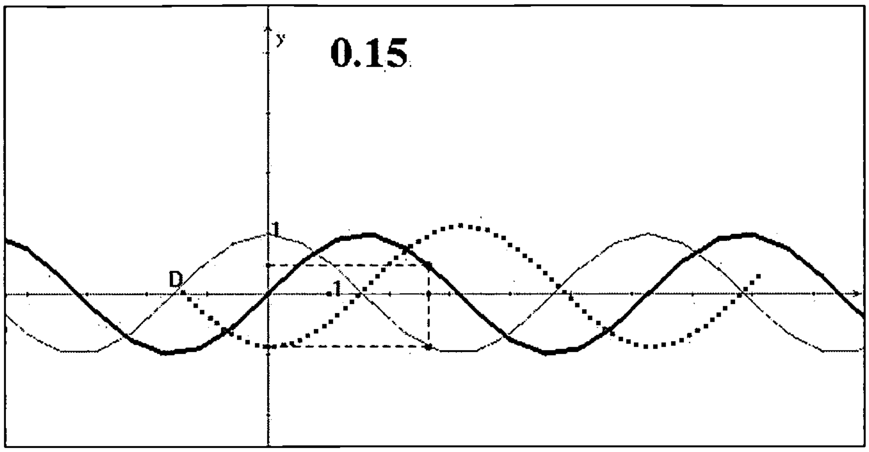
	
The red chain has been drawn applying to one point D the previous macro and again this macro to the obtained point and so on. This chain gives us an idea of the searched antiderivative function	When the number 0.2 is changed in number 0.1, we know that this chain must represent the searched function.

	
Here the dotted curve is the translated curve of the blue thick curve (which is the curve of the square function) using the blue vector.	When we change the blue vector, it is possible to lead the dotted curve on the red chain. So we can conjecture that each antiderivative function is given by the formula $x^2 + k$.

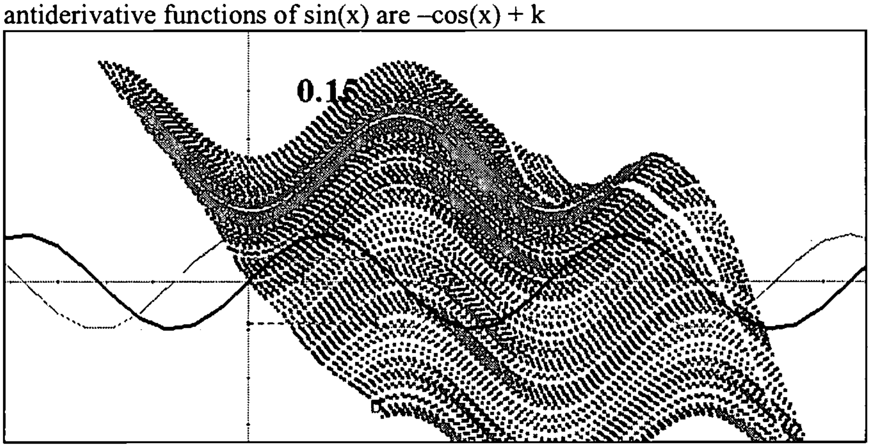
3.3.The power of this method to draw the curve of the antiderivative function
In this part, we have used this method, to determine antiderivative functions of the blue sine function (in purple, we have drawn the cosine curve)



It is possible below to drag point D in order to conjecture that an antiderivative function of the sine function is minus cosine.



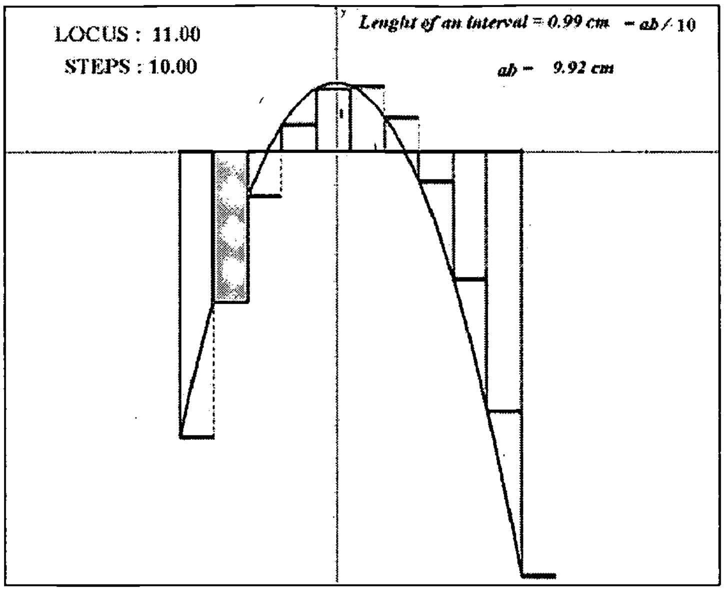
If we drag point D randomly, letting the trace of the red chain, we can conjecture that the antiderivative functions of sin(x) are –cos(x) + k

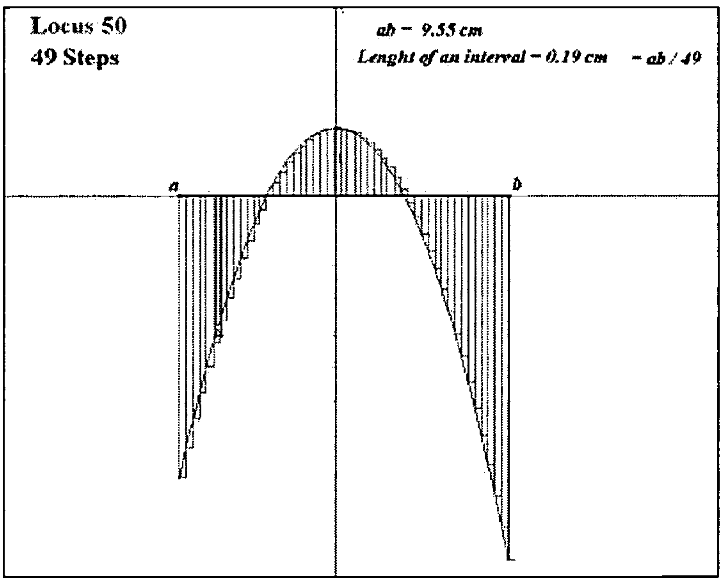


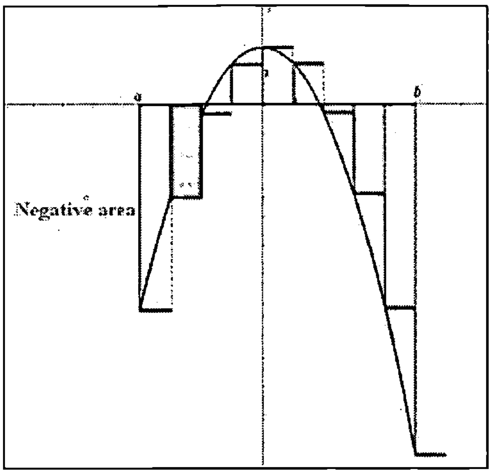
4. Riemann sums and integrals

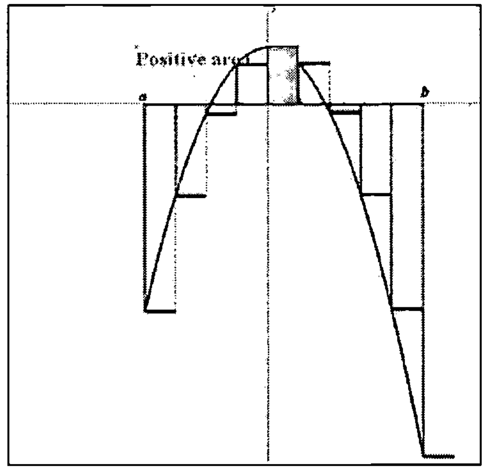
4.1.How to draw Riemann's rectangles?

Playing with the preferences of the loci in Cabri we get the following files

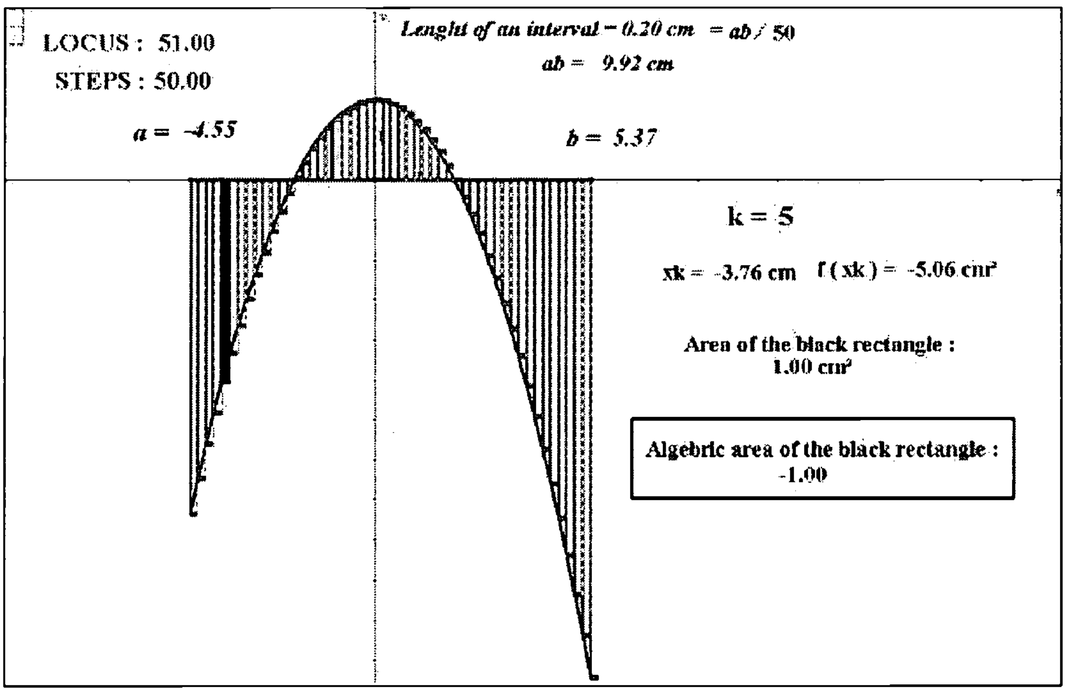

LOCUS : 11.00
STEPS : 10.00
Lenght of an interval = 0.99 cm = ab / 10
ab = 9.92 cm


Locus 50
49 Steps
ab = 9.55 cm
Lenght of an interval = 0.19 cm = ab / 49
a
b

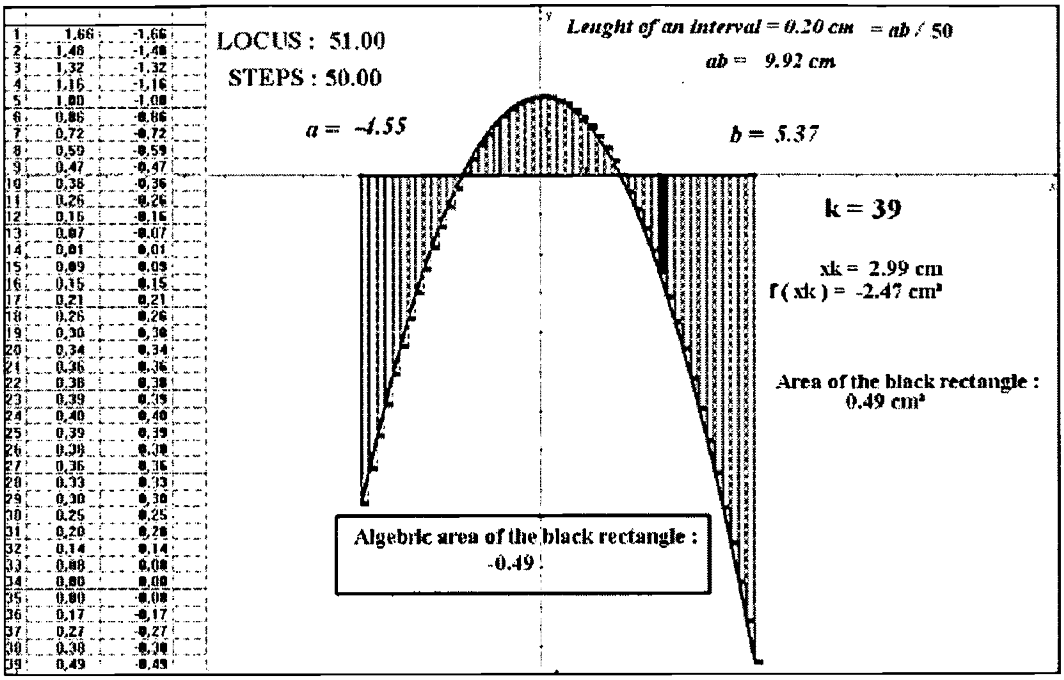


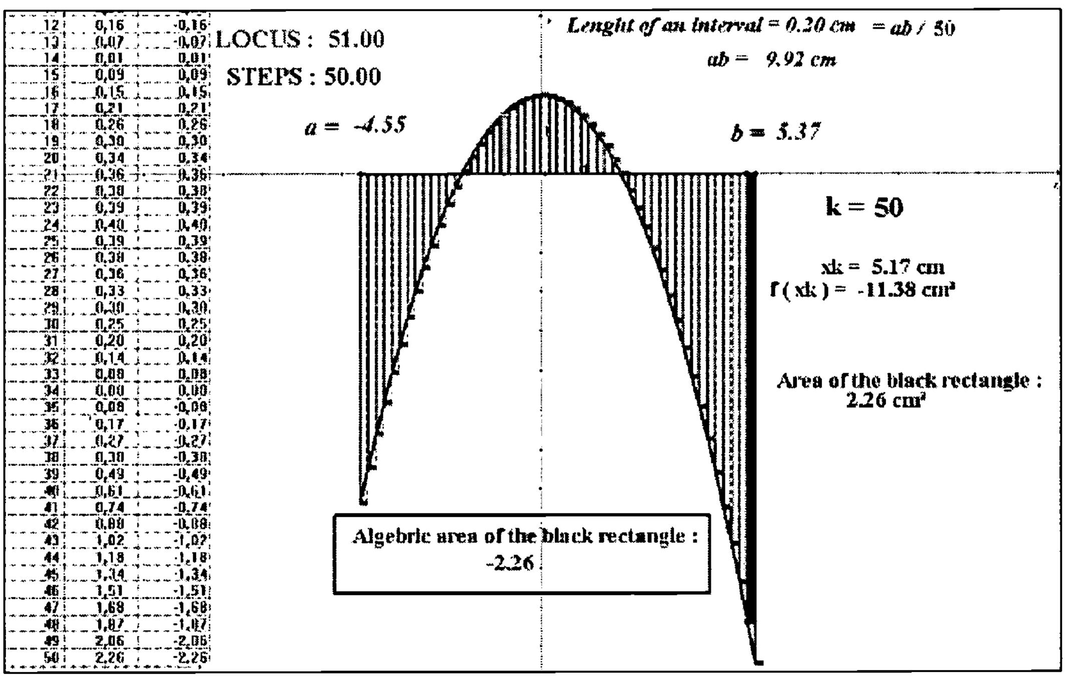


4.2.How to calculate an integral?
Here we modify the number k to change the position of the black rectangle and to modify the value of the algebraic area of this black rectangle



To evaluate the integral of the function $-0.5x^2 + 2$, we will add the algebraic areas of the 50 black rectangles. We have realised an animation of number k from 1 to 50 and we have captured these 50 algebraic areas in the table of Cabri





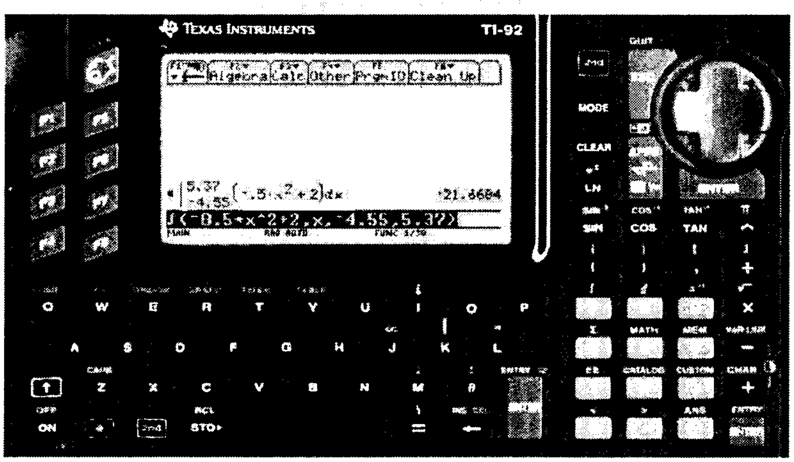
After we have pasted this table in a sheet of Excel in which we have evaluated the sum approaching this integral to get : -21.33

Microsoft Excel - Riemann1ter

Fichier Edition Affichage Insertion Format Outils Données Fenêtre ?

	A	B	C	D	E	F	G	H
1	1	1,66	-1,66					
2	2	1,48	-1,48					
3	3	1,32	-1,32					
4	4	1,16	-1,16					
5	5	1,00	-1		Somme:			
6	6	0,86	-0,86		-21,33			
7	7	0,72	-0,72					
8	8	0,59	-0,59					
9	9	0,47	-0,47					
10	10	0,36	-0,36					
11	11	0,26	-0,26					
12	12	0,16	-0,16					
13	13	0,07	-0,07					
14	14	0,01	0,01					
15	15	0,09	0,09					

We have used the TI-92 to calculate this integral to get this time: -21.67................



Conclusion

Cabri is a tool to practice and to teach Mathematics and not only Geometry. It is possible to approach the classical and basic knowledges of Mathematics with a new creativity and superposing algebraic and geometric fields.

You can find on the website of the IREM of Toulouse this text with Cabrijava applets in order for you to get animated Cabri files illustrating each part of this presentation.
About Cabri: http://www.cabri.net

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