DOCUMENT RESUME

ED 472 720 SE 067 151

AUTHOR	Kennedy, Mary
TITLE	Defining Optimal Knowledge for Teaching Science and Mathematics. Research Monograph.
INSTITUTION	National Inst. for Science Education, Madison, WI.
SPONS AGENCY	National Science Foundation, Arlington, VA.
REPORT NO	No-10
PUB DATE	1997-04-00
NOTE	24p.
CONTRACT	RED-9452971
AVAILABLE FROM	National Institute for Science Education, University of Wisconsin-Madison, 1025 W. Johnson Street, Madison, WI 53706. Tel: 608-263-9250; Fax: 608-262-7428; e-mail: niseinfo@education.wisc.edu; Web site: http:// www.wcer.wisc.edu/ nise/publications.
PUB TYPE	Reports - Descriptive (141)
EDRS PRICE	EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS	Educational Change; Elementary Secondary Education; *Mathematics Instruction; *Pedagogical Content Knowledge; *Science Teachers; Standards; Teacher Attitudes

ABSTRACT

This paper raises the question of what K-12 teachers need to know in order to teach mathematics and science well. It begins by examining reform proposals for K-12 science and mathematics teaching with an eye toward defining what "good" teaching practice consists of. It then examines a wide range of literature to delineate the varieties of knowledge that have been associated with this kind of teaching. While the focus is on subject matter knowledge, the paper addressed the character of that knowledge rather than the content of that knowledge. Types of knowledge identified in the literature include conceptual understanding of the subject, pedagogical content knowledge, beliefs about the nature of work in science and mathematics, attitudes toward these subjects, and actual teaching practices with students. The literature is incomplete with respect to which of these is relatively more or relatively less important. Although the United States does not have a national curriculum, many organizations are working together to achieve an agreed-upon set of goals for science and mathematics teaching and learning. Contemporary education leaders in general, and science and mathematics leaders in particular, have distinct ideas about the best directions for K-12 science and mathematics education and about teachers and teacher education that follow from these goals. This paper examines these proposals and outlines the kinds of subject matter knowledge that teachers need to learn during higher education in science and mathematics. For the analysis that follows, I am less interested in science and mathematics curriculum proposals than in science and mathematics teaching proposals, for embedded in these proposals are indications of what future science and mathematics teachers should be learning from their college-level science and mathematics courses. I shall review the national standards to determine what they define as good science and mathematics teaching and then shall review the associated literature to derive some ideas about what good science and mathematics teachers would need to know or think to teach in the ways reformers demand. (Contains 47 references.) (Author)

Research Monograph No. 10

Defining Optimal Knowledge for Teaching Science and Mathematics

Mary Kennedy

National Institute for Science Education (NISE) Publications

The NISE issues papers to facilitate the exchange of ideas among the research and development community in science, mathematics, engineering, and technology (SMET) education and leading reformers of SMET education as found in schools, universities, and professional organizations across the country. The NISE Occasional Papers provide comment and analysis on current issues in SMET education including SMET innovations and practices. The papers in the NISE Research Monograph series report findings of original research. The NISE Conference and Workshop Reports result from conferences, forums, and workshops sponsored by the NISE. In addition to these three publication series, the NISE publishes Briefs on a variety of SMET issues.

The research reported in this paper was supported by a cooperative agreement between the National Science Foundation and the University of Wisconsin–Madison (Cooperative Agreement No. RED-9452971). At UW–Madison, the National Institute for Science Education is housed in the Wisconsin Center for Education Research and is a collaborative effort of the College of Agricultural and Life Sciences, the School of Education, the College of Engineering, and the College of Letters and Science. The collaborative effort is also joined by the National Center for Improving Science Education, Washington, DC. Any opinions, findings, or conclusions are those of the author and do not necessarily reflect the view of the supporting agencies.

Research Monograph No. 10

Defining Optimal Knowledge for Teaching Science and Mathematics

Mary Kennedy

National Institute for Science Education
University of Wisconsin-Madison

April 1997

About the Author

Mary M. Kennedy is a professor at Michigan State University. Her scholarship tends to focus on the relationship between knowledge and teaching practice, on the nature of knowledge used in teaching practice, on how research contributes to practice. She has won four awards for her work on the nature of knowledge needed for teaching. She has published two books addressing the relationship between knowledge and teaching. Prior to joining Michigan State University in 1986, her work focused mainly on policy issues and on the role of research in improving policy. She has authored numerous journal articles and book chapters in these areas and has authored reports specifically for policy audiences, including the United States Congress.

Contents

Abstract

This paper raises the question of what K-12 teachers need to know in order to teach mathematics and science well. It begins by examining reform proposals for K-12 science and mathematics teaching with an eye toward defining what "good" teaching practice consists of. It then examines a wide range of literature to delineate the varieties of knowledge that have been associated with this kind of teaching. While the focus is on subject matter knowledge, the paper addressed the character of that knowledge rather than the content of that knowledge. Types of knowledge identified in the literature include conceptual understanding of the subject, pedagogical content knowledge, beliefs about the nature of work in science and mathematics, attitudes toward these subjects, and actual teaching practices with students. The literature is incomplete with respect to which of these is relatively more or relatively less important.

Although the United States does not have a national curriculum, many organizations are working together to achieve an agreed-upon set of goals for science and mathematics teaching and learning. Contemporary education leaders in general, and science and mathematics leaders in particular, have distinct ideas about the best directions for K-12 science and mathematics education and about teachers and teacher education that follow from these goals. This paper examines these proposals and outlines the kinds of subject matter knowledge that teachers need to learn during their higher education in science and mathematics. For the analysis that follows, I am less interested in science and mathematics *curriculum* proposals than in science and mathematics *teaching* proposals, for embedded in these proposals are indications of what future science and mathematics teachers should be learning from their college-level science and mathematics courses.

I shall review the national standards to determine what they define as good science and mathematics teaching and then shall review the associated literature to derive some ideas about what good science and mathematics teachers would need to know or think to teach in the ways reformers demand.

Good Science and Mathematics Teaching as Defined by National Standards

The education field is subject to many fads, and what counts as a good idea varies over time and across locations. At present, most people are persuaded that the key to educational improvement lies in developing a coherent and integrated system for governing education, such that tests, texts, licensing decisions, and other educational rules all are based on the same set of ideas. These ideas have come to be called standards. Within mathematics, the National Council of Teachers of Mathematics (NCTM) took the lead by defining both curricular standards and professional teaching standards (1989, 1991). In science, there are at least two major statements of standards, one from the American Association for the Advancement of Science (AAAS, 1993) and one from the National Research Council of the National Academy of Sciences (1996).

Let's examine the standards for pedagogy that these organizations put forward to understand how pedagogy relates to subject matter. Here I concentrate especially on standards for teaching itself—not for planning or evaluation, or for curriculum, but for the act of teaching science and mathematics subjects. Here are statements of teaching standards from the three main science and mathematics standard-setters.

> Teaching Standard B: Teachers of science guide and facilitate learning. In doing this, teachers
>
> - Focus and support inquiries while interacting with students.
> - Orchestrate discourse among students about scientific ideas.
> - Challenge students to accept and share responsibility for their own learning.
> - Recognize and respond to student diversity and encourage all students to participate fully in science learning.
> - Encourage and model the skills of scientific inquiry, as well as the curiosity, openness to new ideas and data, and skepticism that characterize science. (p. 32)
>
> Teaching Standard E: Teachers of science develop communities of science learners that reflect the intellectual rigor of scientific inquiry and the attitudes and social values conducive to science learning. In doing this, teachers
>
> - Display and demand respect for diverse ideas, skills, and experiences of all students.
> - Enable students to have a significant voice in decisions about the content and context of their work and require students to take responsibility for the learning of all members of the community.
> - Nurture collaboration among students.
> - Structure and facilitate ongoing formal and informal discussion based on a shared understanding of rules of scientific discourse.
> - Model and emphasize the skills, attitudes, and values of scientific inquiry. (pp. 45-46)
>
> National Research Council,
> *National Science Education Standards* (1996)

AAAS included descriptions of a number of characteristics of good teaching about science.

- engage students actively [in doing experiments, measuring, etc.]

    - concentrate on the collection and use of evidence
    - provide historical perspectives
    - insist on clear expression
    - use a team approach
    - do not separate knowledge from finding out
- deemphasize the memorization of technical vocabulary
- welcome curiosity
- reward creativity
- encourage a spirit of healthy questioning
- avoid dogmatism
- promote aesthetic responses (pp. 201-204)

American Association for the Advancement of Science,
Science for All Americans (1989)

Though there are some differences between these two sets of teaching standards, there are also some similarities. Both encourage active learning, but AAAS's definition seems to imply that the activity is physical—collecting data, carrying out experiments, etc., whereas the NRC emphasizes conversations in the classroom, suggesting that the activity is more intellectual than physical—more "minds-on" than "hands-on." Still, neither set of standards excludes the other; they merely differ in their relative emphasis. Both want students working in teams, both want them raising questions and exploring ideas for themselves, both want students to learn to evaluate ideas using evidence. The pedagogy for science teaching, then, is one that actively engages students in reasoning about scientific phenomena.

Now let's consider the NCTM standards.

Standard 2: The teacher's role in discourse:
The teacher of mathematics should orchestrate discourse by—

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas; . . .
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate. (p. 35)

Standard 3: Students' role in discourse:
The teacher of mathematics should promote classroom discourse in which students—

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counter examples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity. (p. 45)

National Council of Teachers of Mathematics,
Professional Standards for Teaching Mathematics (1991)

Like the science standards, the mathematics teaching standards emphasize a classroom in which student are not being told, but instead are being asked. In fact, in both science and mathematics, the tilt goes even further than simply asking students questions, toward encouraging students to ask their own questions.

These various statements of standards differ in many of their details, but are remarkably similar in their general tenor. For instance, two of the three sources put forward premises that are worded identically: "What students learn is greatly influenced by *how they are taught* [emphasis added]" (NCTM, 1991; NRC, 1996). This statement in itself is remarkable and represents an important shift in thinking about science and mathematics teaching. In the past, the principal tension in science and mathematics teacher education was how much time should be spent learning the subject and how much should be spent learning pedagogy. Advocates for more attention to subject matter assumed that good teaching depended largely on the teachers' ability to correctly present the content. Advocates for more attention to pedagogy assumed that good teaching depended on the ability to keep students orderly and attentive.

The statement that what students learn depends on how they are taught introduces a remarkable new idea to educational thought: that the method by which one teaches a subject itself conveys important information to students about the subject matter. How a subject is taught tells students whether the subject is interesting or boring, debatable or authoritative, clear or fuzzy, applied or theoretical, relevant or irrelevant, challenging or routine. Thus pedagogy is no longer defined as a set of techniques that enable teachers to maintain discipline or to entice students to pay attention, but instead is defined as integral to the substantive goals of teaching.

If teachers were to implement these standards, they would substantially decrease the predictability of events within their classrooms. When students begin to pose their own questions, raise their own hypotheses in response to their own or others' questions, and argue the merits of their own or others' hypotheses using their own understanding of the evidence or of the rules of inference, the range of ideas that may come up in class is unlimited. Moreover, some ideas will be wrong, or at least inappropriate to pursue. Students may make inappropriate analogies, generate questions or hypotheses that are beyond their capabilities to pursue, or generate ideas that, if pursued, will lead them astray, down dead-end alleys or into trivial pursuits. Roth (1989) noted that the questions her students had about biology were not questions she was prepared to answer when she first completed her bachelor's degree. Students wanted to know such things as whether blood is really blue, what caused hiccups, and how long it takes for oxygen to get from the lungs to the toes. Teachers need to be able to respond to questions and hypotheses that they might not have anticipated, provide students with guidance when they get in over their heads, clarify confusions, and assure that misconceptions aren't perpetuated. Certainly teachers are not expected to move in *any* direction students want to go, but to manage classroom discussions of the sort reformers envision, teachers would need enough knowledge of the subject to recognize which questions are likely to be fruitful and which are likely to be dead ends. That in turn suggests that they must understand how the various ideas in a subject are interrelated and which ideas are relatively more important than others. The standards are silent on how teachers' judgments about fruitful or not-so-fruitful pursuits are to be made; presumably, these would be based on teachers' understanding of the ideas on the table and their relationship to the ideas she wants students to grasp.

To pull off this kind of teaching, then, teachers need a different stance toward mathematics and science than many contemporary teachers apparently have. Evidence suggests that many teachers present these subjects to students as vast collections of facts, terms, and procedures with little connection among them. Moreover, they present these facts and procedures as if they were self-evident givens that students should accept and remember without much thought. If teachers are to engage students in reasoning about important ideas in these subjects, they must themselves have a grasp of these ideas, and they must have a healthy respect for the difficulties of developing and justifying knowledge in these fields. The nature of teachers' knowledge, understanding, and/or attitudes toward these subjects has received considerable attention in the literature of the past decade, as researchers and analysts have struggled to define the special character of knowledge needed for teaching.

Knowledge, Skills, and Attitudes Needed for Reform-oriented Science and Mathematics Teaching

Interest in this new approach to teaching has led to extensive discussions about the kind of knowledge teachers would need to teach in this way. Many authors have tried to define the knowledge and skills needed for good teaching, and most attention has been on the character of that knowledge rather than on the content per se. In my review of this literature on teacher knowledge, I ignore literature having to do with such issues as classroom management, the identification of handicapping conditions, the management of cultural diversity, and a number of other aspects of teaching that may be important but are not necessarily linked to the teaching of academic subject matter. I consider such articles only when the authors explicitly link these issues to teaching academic subject matter.

Although knowledge of the subject matter is probably the most self-evident kind of knowledge needed to teach, the *amount* of subject matter knowledge really needed to help children learn is a contested issue. For those who believe the best source of knowledge is the curriculum materials, not the teacher, the most important knowledge for teachers to have is the ability to read and follow directions (see, e.g., Lawson, 1991). Some state assessments require teachers to know only the subject matter actually covered by the curriculum, reasoning that this knowledge is exactly what teachers will be teaching. However, if students can ask questions that extend far beyond the formal curriculum, and if teachers must respond to those questions, teachers need knowledge that goes far beyond the content officially being taught (Hilton, 1990). On the other hand, one could also argue that teachers don't even need to know the content in the official curriculum if they are able themselves to reason from evidence and make reasonable inferences. Many parents manage to help their children with their homework without having much content knowledge of their own—in fact, many parents choose to educate their children entirely at home. They do this by studying the textbook themselves, trying to discern what it says, and then trying to translate this for their children. Of course, most parents don't make decisions about what to teach, and many are probably wrong in their inferences about what the most important points of a lesson are. Still, if parents can succeed in their endeavors—and the home-schooling movement suggests that many parents believe they can—then we have still a lot to learn about the relationship between subject matter knowledge and teaching.

Separate from questions about the volume of knowledge needed to teach a subject is a growing interest in the character of subject matter knowledge. As a starting point to this discussion, let me introduce the term *recitational* subject matter knowledge, the kind of knowledge that has traditionally been tested in achievement tests in the past. By recitational knowledge, I mean the ability to recite specific facts on demand, to recognize correct answers on multiple-choice tests, to define terminology correctly, and so forth. It is not clear that traditional science and mathematics courses were really limited to recitational knowledge, but it is clear that this is what most reformers think and it is clear that their aim is to extend classroom science and mathematics instruction well beyond recitational knowledge. The term *recitational knowledge*, then, will be used here to refer to the narrow type of straw-man outcomes that reformers believe dominate traditional instruction.

With respect to teachers' knowledge, there is a pervasive belief that recitational knowledge is not sufficient to enable teachers to manage the type of inquiry-oriented classrooms described in the standards. Instead, knowledge of a different character is needed. Listed below are several distinctions that have been made regarding the unique character of subject matter knowledge needed by teachers.

Conceptual Understanding of Subject Matter

Because the main goal of reformers is to instill a deeper understanding in students of the central ideas and issues in various subjects and to enable students to see how these ideas connect to, and can be applied in, real world situations, it therefore makes sense to require that teachers themselves also understand the central concepts of their subjects, see these relationships, and so forth. But what exactly is conceptual understanding? I have found at least five distinct ideas that fall within the general idea of conceptual understanding. One notion of conceptual understanding, for instance, is that we have the sense of size or proportion of things. Paulos (1988), for instance, wants people to be able to grasp large numbers when they are used to describe the size of a population or the size of the federal deficit, to be able to understand the differences in risk associated with traveling via car or plane, and to understand the weather report when it says there is a 50% chance of snow tomorrow. But Paulos is actually writing about the kind of conceptual understanding he would like all lay citizens to have. Most writers who address teachers' subject matter knowledge want much more than this.

The second definition of conceptual understanding, and one that is relatively widely recognized, has to do with attending to central ideas in each subject rather than to its minutiae. The idea of focusing on big ideas has been advocated by Prawat (1991, 1993), but is tacitly implied in many of the standards above. One, for instance, specifically says teachers should "de-emphasize the memorization of technical vocabulary." Researchers in science and mathematics higher education have taken an interest in teachers' understanding of specific ideas, but the specific ideas of interest are quite diverse. They include fractions (Khoury & Zazkis, 1994), diffusion and osmosis (Odom & Barrow, 1993), mathematical functions (Evan, 1993), group theory (Dubinsky et al., 1994), force and energy (Summers & Kruger, 1994), optical image formation (Galili & Goldberg, 1993), and multiplicative relationships (Simon & Blume, 1994), among others.

The third meaning attached to the phrase *conceptual understanding* has to do with the relationships among ideas in a discipline. Teachers (and others) should see that some ideas are more fundamental than others, and that some are needed to justify others, that some encompass others. The argument for understanding these relationships is twofold. First, if teachers are to focus students' attention on the big ideas in a subject, rather than on its minutiae, they themselves need to understand which ideas are "biggest," and they must have a deep understanding of these ideas. Second, if teachers encourage discussions, and encourage students to generate their own hypotheses and speculations, they need to be able to judge whether or not a student's idea should be pursued. If teachers have an idea of what they are hoping students will figure out, and if they know how various ideas connect to one another, they can also have a sense for whether an idea that takes the class toward point A can eventually be used to bring students back to point B, where the teacher wants eventually to be. Without knowing how the various ideas in a discipline relate to one another, support one another, parallel one another, or subsume one another, teachers would have difficulty knowing whether students' questions and hypotheses will lead to greater understanding or instead to confusion and dead ends.

One problem with this definition of conceptual understanding is that relationships among ideas are often extremely subtle, and sometimes the nature of these relationships is not agreed upon even by experts in the field. For instance, is natural selection a *cause* of evolution, a *mechanism* for it, or *necessary condition* for it? Biologists argue about such questions themselves, and similar disputes appear in other fields of science and mathematics as well. Given these disputes, it would be difficult to define the specific relationships teachers should understand.

Researchers who are interested in teachers' understandings of the relationships have, however, devised some interesting strategies for getting at these. One particularly popular idea is concept mapping. Concept mapping consists of asking subjects to graphically show the main ideas in a field and show the relationships among these various ideas. A map of "mammals" might include land, air, and water domiciles, eating habits, reproductive systems, and so forth. Maps can be scored for the number of concepts employed, the number of correctly defined relationships among them, the number of branches, number of levels of hierarchies, and so forth. Some researchers have also used concept maps as a way of getting teachers to outline the domains they believed they were teaching (e.g., Lederman et al., 1993; Shymansky et al., 1993). One problem with concept maps is that they focus researchers' attention on things that teachers volunteer, rather than on things teachers failed to mention. That an idea was not generated does not mean the teacher is unaware of it, nor that the teacher would not know where to place it. At the same time, the ideas that are missing from a concept map may be the ideas that are most muddled in teachers' minds.

The fourth meaning sometimes attached to the phrase *conceptual understanding* is that knowledge must be highly elaborated—that is, an individual who has a strong understanding of some domain is an individual who has knowledge of lots of details and lots of examples within that domain. This idea has been most forcefully advocated by cognitive psychologists, who have argued that understanding, reasoning, and problem solving are all dependent on detailed specific knowledge. This point seems worth mentioning here, in part because it is often forgotten in reform rhetoric: Most reformers have emphasized the fact that it is possible for someone to have detailed recitational knowledge without any understanding of the central ideas. Less often considered is the question of whether one can understand the central ideas without having a large store of detailed knowledge. How could someone understand concepts of kingdom, phylum, genus and species, for instance, without having specific knowledge of many species within these categories and specific knowledge of why they were assigned their particular taxonomic classifications? Can one understand arguments about the proper taxonomic classification of the platypus, for instance, without knowing the most salient features of the platypus, and without knowing the specific variables that are used to distinguish one species from another or one genus from another? Reformers tend to avoid the problem of ensuring that teachers have extensive detailed knowledge because they do not want to confuse this kind of knowledge with recitational knowledge. The problem is that presence of extensive detailed knowledge does not necessarily mean that the knowledge is organized into a framework that enables deep understanding. If it is not, it is merely recitational.

Perhaps because reformers have avoided discussion of elaborated knowledge, there is relatively less literature on how elaborated teachers' knowledge is or should be in any domain. One could argue, though, that concept maps include attention to detailed elaboration as well as to conceptual relationships, in that researchers can score the total number of discrete ideas volunteered in a concept map and can score the numbers of branches that are generated in a map. To the extent that a teacher's concept maps include numerous nodes or numerous examples within a node, one could say that the teacher's knowledge is both elaborated and conceptually organized.

Finally, the term *conceptual understanding* is often used to refer to an ability to reason about phenomena, develop arguments, solve real problems, and justify one's solutions. The evidence shown in the video *A Private Universe* (Schneps, 1989), for instance, suggests that many college graduates cannot determine how the movement of the earth contributes to seasonal climate changes. Most graduates tried to attribute seasonal changes to the earth's orbit around the sun, rather than to its tilt in relationship to the sun, and several drew very odd orbital paths as they tried to generate an orbit that could account for seasonal climate changes.

Interestingly, although much of the literature on what K-12 students should be learning focuses on reasoning and problem solving, very little of the empirical literature on teachers' knowledge focuses on this issue. Greene's (1990) study of students' understanding of natural selection is a good example of research on teachers' reasoning, and Bennett and Carre's (1993) study of the effectiveness of teacher education uses teachers' reasoning about practical problems as outcome measures for teacher education. One reason that studies of teachers' problem-solving and reasoning abilities might be rare is that such problems are time consuming to use and difficult to score, just as they are when used in K-12 classrooms. And the results are equally difficult to interpret.

Pedagogical Content Knowledge

The phrase *pedagogical content knowledge* was introduced by Shulman (1986, 1987) to refer to the ability to represent important ideas in a way that makes them understandable to students. It is pertinent to reformers because, as Shulman intended the term, pedagogical content knowledge was what enabled teachers to translate complex or difficult ideas into concepts that students, as novices, could grasp. Shulman was interested in the use of metaphors and other devices to explain, illustrate, or illuminate important substantive ideas. Pedagogical content knowledge depends heavily on conceptual understanding, of course, for a good metaphor is one that captures the essence of the original idea. For instance, in one of Richard Feynman's lectures on physics, he gives a sense for the size of an atom by saying, "If an apple is magnified to the size of the earth, then the atoms in the apple are approximately the size of the original apple" (Feynman, 1963/1995, p. 5). This metaphor gives novice students an immediate sense for the size of atoms and the number of them that must, therefore, be present in an object such as an apple. The ability to generate such metaphors is, for Shulman, at the heart of pedagogical content knowledge. And it is, presumably, important for any teacher who aims to teach important ideas rather than lists of facts and procedures.

As Shulman uses the term, *pedagogical content knowledge* is clearly different from the kind of *recitational* knowledge that is often assumed to dominate contemporary American education. College students might be able to recite knowledge of atoms, for instance, by noting that atoms are typically 1 or 2 angstroms in radius, and that an angstrom is equal to 10^{-8} cm. Being able to recite such facts can yield a high test score, a high grade point average, and a strong diploma. But being able to recite such facts does not assure that the student (a soon-to-be-teacher) could explain to younger students how big an atom is—to explain it in a way that could be understood by, say, high school students. Because high school students are novices to virtually all of the terms in the recitation, they need help grasping the meaning of the sentence. They are not familiar with atoms, with angstroms, or even with centimeters. They may not be comfortable with the notation of 10^{-8}. Assuring that college students can recite such sentences, therefore, does not assure that they can explain the meaning of the sentence to younger students who are novices in science. To help novices understand complex ideas, teachers need to be able to provide metaphors such as Feynman's apple metaphor.

Shulman has also argued that pedagogical content knowledge also differs from ordinary conceptual understanding in that one's choice of metaphors depends not only on the correctness of the metaphor but also its comprehensibility to the particular audience. That is, pedagogically good metaphors are those that *both* capture the essence of the idea *and* are within the realm of understanding of the students at hand. Using the solar system as a metaphor for the structure of an atom might not work with kindergartners, for instance, because their knowledge of the solar system may not be accurate. Knowing which metaphors and analogies will help students learn, then, requires both strong conceptual understanding of the ideas in the discipline and knowledge of students—what they currently think about the subject, what misconceptions they have, and what knowledge they lack.

Another way in which pedagogical content knowledge may differ from conceptual understanding is that pedagogical content knowledge must be explicit rather than intuitive. Just as I may know the way to the grocery store, but be unable to give you directions, it is possible for someone who works in a given field to have deep and detailed knowledge of that field and yet have difficulty outlining the major domains in the field for others. Similarly, even if a teacher has a good grasp of the nuances of a subject and can solve problems and reason abstractly about issues within that field, that teacher might not be able to help students understand these issues unless his or her own knowledge is explicit (Wilson, Shulman, & Richert, 1987). Having explicit knowledge is important in part because it enables better explanations, but also because it better enables teachers to decide what is most important to teach, what should be taught now rather than later, or what kind of problems could be posed to students that would most likely facilitate their understanding of some particular ideas. To help you find your way to the grocery store, I not only need to know the way, but I need to know how to outline it for you, to identify important landmarks along the way, to predict places where you are likely to become confused or disoriented, and so forth. To make these numerous teaching decisions, I need to be explicitly aware of how my knowledge is organized and be aware of the details that you are likely not to know.

Researchers at the National Center for Research on Teacher Learning have examined teachers' and teacher candidates' representations of certain mathematical ideas and have found that the ability to generate representations is, indeed, quite different from recitational knowledge. In one problem (Kennedy et al., 1993), for instance, teacher candidates were asked the following:

> Imagine that you are teaching in a fifth-grade classroom and you are teaching your students division with fractions. You want to create a story problem to illustrate the following mathematical problem:
>
> $$1\frac{3}{4} \div \frac{1}{2}$$

Virtually all teacher candidates had recitational knowledge of this type of problem. They knew the rule of "invert and multiply," and could readily apply that rule and find a correct answer to the problem. However, few could generate a story problem that correctly *represented* the problem. A typical story problem might look like this: "My roommate and I have 1¾ pizza to share. How much can each of us have?" Their story problems usually illustrated a situation in which 1¾ was being divided by 2, rather than by ½ (Ball, 1990a).

The notion that there might exist a special type of knowledge called pedagogical content knowledge is relatively new, and only a few articles have directly addressed it. Ball (1990a, 1990b, 1991), building on the research of the National Center for Research on Teacher Learning, has written extensively about the pedagogical content knowledge needed to teach elementary school mathematics, as has Leinhardt (e.g., Leinhardt & Smith, 1985). Approaches to documenting pedagogical content knowledge include asking teachers to pose story problems (Silver & Burkett, 1994), asking them to write out lesson plans (Sherman, 1990), and asking them to generate analogies to explain ideas (Wong, 1993).

Beliefs About the Nature of Disciplinary Work

Because reformers want students to reason about mathematical and scientific ideas and to learn to evaluate arguments and evidence, many authors have suggested that teachers must understand that these activities are part of the work of the disciplines. This view has led to a considerable interest in teachers' beliefs about the nature of disciplines they might teach. Many commentators on teaching have discussed the need for teachers to understand the nature of the subject itself—how knowledge is generated, tested, argued about, and justified; what is taken for granted, what makes something anomalous, what makes something important, how deviations from expectations are treated, and so forth. Collins and Pinch (1993), for instance, want all adults—not just teachers—to understand that science, on one hand, is rigorous, but that, on the other hand, it often proceeds in an awkward, stumbling manner. They describe several examples of scientific controversies and argue that these controversies occurred precisely because no one yet knew the right answer. Whether the findings from a scientific study are due to something anomalous in the research procedures or due to the hypothesized phenomenon is not known at the time of the dispute. These authors suggest that K-12 students could quickly experience this process if they were to each estimate the boiling point for water by boiling water. Students would likely get many different boiling points and would then have to argue among themselves to figure out why their solutions differed and which solution was right.

Most references to "classroom discourse" in the standards literature reflect the view that, if students learn only through lectures, even if the lectures were as clear and compelling as Feynman's, they might erroneously perceive science as a subject that is finished and undisputed rather than in process and contentious. If so, then the lecture itself misrepresents the subject matter, for it can't convey the struggles scientists have gone through trying to reveal all these things. Similarly, Copes (1996) has pointed out that mathematicians do not spend their days solving repetitive computational problems, though many students might think so, given the mathematics they do in school. Finally, Ball (1991) has suggested that an important reason for classroom discussion and argumentation is that, if students reason through mathematical quandaries themselves, they become validators of their own knowledge. That is, they do not have to accept scientific and mathematical ideas as received truths, but can reason about them for themselves. This is an important outcome for students, for it helps them recognize that there are standards for knowledge claims and that they themselves are capable of evaluating knowledge.

So Shulman's original concern about how particular substantive ideas are represented to students has extended into a broader concern for how the character of the subject as a whole is represented to students. If we want students to understand that mathematical and scientific ideas did not spring forth in perfect form, but instead had to be sorted out, developed, and justified, students need to understand how such knowledge is created. Through their pedagogy, then, teachers are representing the character of the subject, just as they represent its ideas through their sentences. And that, in turn, means that they themselves need to understand the nature of mathematical and scientific knowledge. In fact, several studies have indicated that teaching practices are indeed influenced by teachers' beliefs about the nature of the subject (Brickhouse, 1990; Smith & Neale, 1991; Stodolsky, 1988; Stodolsky & Grossman, 1995; Thompson, 1984).

Attitude Toward Science and Mathematics

Closely associated with beliefs about the nature of the subject are teachers' attitudes toward it. When Herbert Clemens (1991) was asked to write about what teachers needed to *know* to teach mathematics, he responded by writing about what mathematics teachers needed to *be*. Even if teachers had an acceptable understanding of the nature of knowledge in science or mathematics, we might still not be satisfied unless they demonstrated a certain respectful attitude toward the work. For instance, we would probably not be satisfied with a high school physics/astronomy teacher who seemed to understand how knowledge is generated and justified in his field, but who also attended to his horoscope every day. Nor would we be satisfied with a biology teacher who understood the arguments and evidence involved in, say, the National Academy of Sciences' (1984) discussion of evolution, but who, outside of the classroom, subscribed to a creationist view of the origin of the species. Even if the astrologist and the creationist claimed to understand the way knowledge was generated and tested in their respective subjects, their failure to value these norms and to extend them to their own lives might make us worry about their ability to convey to students an appropriate attitude toward these subjects.

Interestingly, the literature on attitudes toward mathematics and science tends to focus less on teachers' respect for the quality of knowledge in these subjects and more on their positive or negative regard for it. More and more, scholars have found strong fears among mathematics students, a phenomenon now labeled *mathematics anxiety.* Similarly, students may perceive science as impersonal, alienating, and irrelevant to "real" life. These negative—and occasionally even hostile—attitudes would not be desirable in teachers who taught these subjects.

Summary

Reform commentaries include numerous ideas about the qualities of knowledge, beliefs, and attitudes that teachers need in order to teach mathematics and science in the way reformers want these subjects taught. These qualities include a sense of size and proportion, an understanding of the central ideas in the discipline, an understanding of how these ideas are related to one another, knowledge of a variety of details that accompany these big ideas, an ability to reason, analyze, and solve problems within the discipline, an ability to generate metaphors and other representations of these ideas, an understanding of the nature of work in the disciplines, and an attitude of respect for the processes by which knowledge is generated through these disciplines.

The ideas that underlie the reform movement are important. They are carefully reasoned; they take into account close examinations of the disciplines; and they thoughtfully consider students' needs and society's needs for an educated citizenry. Yet if each of these qualities of knowledge, belief and attitude is considered to be a desired outcome for college-level mathematics and science teaching, the sheer variety of potential outcomes would make evaluation of college-level programs both difficult and expensive. Moreover, the devices used to measure these different outcomes are various, and there seems to be no agreement in the field about the best ways to capture these different kinds of understandings and beliefs.

But these qualities of knowledge are rarely derived from empirical examinations of teaching and learning. Much of the literature reviewed here was based on *stipulations* about the qualities of knowledge that seem important to the kind of teaching that is desired. Now that such a body of thought is available, we are responsible for vigorously pursuing empirical studies of these ideas. In my review of the arguments for what teachers need to know or be able to do, I have tried also to indicate the availability of empirical work.

References

American Association for the Advancement of Science. (1989). *Science for all Americans: A Project 2061 report on literacy goals in science, mathematics, and technology.* Washington, DC: Author.

American Association for the Advancement of Science. (1993). *Benchmarks for science literacy.* New York: Oxford University Press.

Ball, D. L. (1990a). The mathematical understanding that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 450-466.

Ball, D. L. (1990b). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education, 21*(2), 132-144.

Ball, D. L. (1991). Teaching mathematics for understanding: What do teachers need to know about the subject matter? In M. M. Kennedy (Ed.), *Teaching academic subjects to diverse learners* (pp. 63-83). New York: Teachers College Press.

Bennet, N., & Carre, C. (Eds.). (1993). *Learning to teach.* London: Routledge.

Brickhouse, N. W. (1990). Teachers' beliefs about the nature of science and their relationship to classroom practice. *Journal of Teacher Education, 41*(3), 53-62.

Clemens, H. (1991). What do mathematics teachers need to be? In M. M. Kennedy (Ed.), *Teaching academic subjects to diverse learners.* New York: Teachers College Press.

Collins, H., & Pinch, T. (1993). *The Golem: What everyone should know about science.* Cambridge, England: Cambridge University Press.

Copes, L. (1996). Teaching what mathematicians do. In F. B. Murray (Ed.), *The teacher educator's handbook: Building a knowledge base for the preparation of teachers* (pp. 261-276). San Francisco: Jossey-Bass.

Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (1994). On learning fundamental concepts of group theory. *Educational Studies in Mathematics, 27*, 267-305.

Evan, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal of Research in Mathematics Education, 24*(2), 94-116.

Feynman, R. P. (1963/1995). *Six easy pieces: Essentials of physics explained by its most brilliant teacher.* New York: Addison-Wesley.

Galili, I., Bendall, S., & Goldberg, F. (1993). The effects of prior knowledge and instruction on understanding image formation. *Journal of Research in Science Teaching, 30*(3), 271-301.

Greene, E. D., Jr. (1990). The logic of university students' misunderstanding of natural selection. *Journal of Research in Science Teaching, 27*(9), 875-885.

Hilton, P. J. (1990). What teachers need to know about mathematics. In D. Dill (Ed.), *What teachers need to know: The knowledge, skills, and values essential to good teaching.* San Francisco: Jossey-Bass.

Kennedy, M. M., Ball, D. L., & McDiarmid, G. W. (1993). *A study package for examining and tracking changes in teachers' knowledge.* East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Khoury, H. A., & Zazkis, R. (1994). On fractions and non-standard representations: Preservice teachers' concepts. *Educational Studies in Mathematics, 27*, 191-204.

Kuhn, T. S. (1970). *The structure of scientific revolutions.* Chicago: University of Chicago Press.

Lawson, A. E. (1991). What teachers need to know to teach science effectively. In M. M. Kennedy (Ed.), *Teaching academic subjects to diverse learners* (pp. 31-59). New York: Teachers College Press.

Lederman, N., Gess-Newsome, J., & Latz, M. (1993). *Becoming a teacher: Balancing conceptions of subject matter and pedagogy.* Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.

Leinhardt, G., & Smith D. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247-271.

National Academy of Sciences. (1984). *Science and creationism: A view from the National Academy of Sciences.* Washington, DC: National Academy Press.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Research Council. (1996). *National science education standards*. Washington, DC: National Academy Press.

Odom, A. L., & Barrow, H. (1993). *Freshman biology majors' misconceptions about diffusion and osmosis.* Paper presented at the annual meeting of the National Association for Research in Science Teaching, Atlanta, GA.

Onslow, B., et al. (1992). Developing a teaching style: A dilemma for student teachers. *Alberta Journal of Educational Research, 38*, 301-315.

Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences.* New York: Vintage Books.

Prawat, R. S. (1991). The value of ideas: The immersion approach to the development of thinking. *Educational Researcher*, *20*(2), 3-10.

Prawat, R. S. (1993). The value of ideas: Problems versus possibilities in learning. *Educational Researcher*, *22*(6), 5-16.

Roth, K. J. (1989, March). *Subject matter knowledge for teaching science, or: How long does it take oxygen to get to the cells?* Paper presented at the annual meeting of the American Educational Research Association, San Francisco.

Schneps, M. (1989). *A private universe* [Video]. Santa Monica, CA: Pyramid Film and Video.

Sherman, H. W. (1990). A comparison of three methods of teaching rational number concepts to preservice teachers. *Educational Research Quarterly*, *14*(4), 48-55.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*, 1-22.

Shymansky, J. A., Woodworth, G., Norman, O., Kunkhase, J., Matthews, C., & Liu, C. (1993). A study of changes in middle school teachers' understanding of selected ideas in science as a function of an in-service program focusing on student preconceptions. *Journal of Research in Science Teaching*, *30*(7), 737-755.

Silver, E. A., & Burkett, M. L. (1994). *The posing of division problems by preservice elementary school teachers' conceptual knowledge and contextual connections.* Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Simon, M. A., & Blume, G. W. (1994). Building and understanding multiplicative relationships: A study of prospective elementary teachers. *Journal of Research in Mathematics Education*, *25*(5), 472-494.

Smith, D. C., & Neale, D. C. (1991). The construction of subject-matter knowledge in primary science teaching. In J. Brophy (Ed.), *Advances in research on teaching. Vol. 2: Teachers' knowledge of subject matter as it relates to their teaching practice* (pp. 187-243). Greenwich, CT: JAI Press.

Stodolsky, S. (1988). *The subject matters: Class activity in mathematics and social studies*. Chicago: University of Chicago Press.

Stodolsky, S., & Grossman, P. (1995). The impact of subject matter on curricular activity: An analysis of five academic subjects. *American Educational Research Journal*, *32*(2), 227-249.

Sullivan, P. (1989). Factors inhibiting change—A case study of a beginning primary teacher. *Mathematics Education Research Journal, 1,* 1-16.

Summers, M., & Kruger, C. (1994). A longitudinal study of a constructivist approach to improving primary school teachers' subject matter knowledge in science. *Teaching and Teacher Education*, *10*(5), 499-519.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*, 105-107.

Wilson, S., Shulman, L. S., & Richert, A. E. (1987). "150 different ways" of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring teacher thinking* (pp. 104-124). London: Cassell.

Wong, E. D. (1993). Self-generated analogies as a tool for constructing and evaluating explanations of scientific phenomena. *Journal of Research in Science Teaching*, *30*(4), 367-380.