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ABSTRACT

This document contains the following papers on mathematics from the SITE (Society for Information Technology & Teacher Education) 2002 conference: (1) "Teachers' Learning of Mathematics in the Presence of Technology: Participatory Cognitive Apprenticeship" (Mara Alagic); (2) "A Fractal Is a Pattern in Your Neighborhood" (Craig N. Bach); (3) "Redesigning College Algebra Delivery from Direct Instruction to a Computer Environment" (Brian Beaudrie); (4) "Mathematical Modeling of Motion Captured with a Calculator-Based Ranger" (John E. Bernard); (5) "General Logic Actions and Database Oriented Methods of Their Development in Teaching Advanced Calculus" (Mikhail M. Bouniaev); (6) "Mathematical Treats Form the Stars: Integrating Curricular Elements through Partnerships between NASA and Math Methods Faculty" (Caroline M. Crawford and others); (7) "Virtual Manipulatives in Mathematics: Addressing the Conceptual Dilemmas" (Lawrence O. Cannon and others); (8) "Enhancing Statistical Content in Pre-Service Elementary School Teachers' Web Constructions" (Jack A. Carter and Beverly J. Ferrucci); (9) "Graphing Calculators and Algebra I, Algebra II, IPC, and Chemistry Teachers' Perceptions of Change" (Scott W. Slough and Gregory E. Chamblee); (10) "Actions on Objects: Useful Internet Locations" (Michael L. Connell); (11) "Using Technology Tools To Revitalize Mathematics Teaching: Perspectives from the United States and Namibia" (Thea K. Dunn); (12) "Using Animations in the Teaching of Calculus Concepts" (Jose H. Giraldo); (13) "'Warrick's Secrets': Teaching Mathematics through an Internet-Based 3D Massively Multi-Player Role Playing Game" (Tracy Goodson-Espy and others); (14) "Are Teachers 'TechReady?' Evaluating the Technology Competencies of Preservice Teachers" (Delwyn L. Harnisch and others); (15) "Using Visualization To Make Connections between Math and Science in High School Classrooms" (Delwyn L. Harnisch and others); (16) "Visualization and Collaborative Learning in Math/Science Classrooms" (Delwyn L. Harnisch and Ronald J. Shope); (17) "Using Technology To Reduce Mathematics Anxiety in Preservice Elementary Teachers" (Nicholas A. Holodick and Denise M. Reboli); (18) "PDA's: The Swiss Army Knife of Handheld Technology for Mathematics Classrooms" (Elliott Ostler and Neal Grandgenett); (19) "Using Online

Discussion Forums To Develop Teachers' Understanding of Students' Mathematical Thinking" (Jeffrey C. Shih); and (20) "Technological Dilemmas: A Guide to Selecting and Implementing Resources for Secondary Mathematics Instruction" (Tobin White). A brief summary of a conference presentation on Project Interactivate is also included. Most papers contain references. (MES)

Mathematics (SITE 2002 Section)

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MATHEMATICS

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Coming in the year of 9-11 and Tropical Storm Allison it is perhaps not surprising that the submission process being piloted this year seemed to be ridden with gremlins. But, as the old adage states, "Real boats rock". Growth and improvement bring their own growing pains which are quickly forgotten once the new has become old. We would like to urge SITE members to be patient as the wrinkles are worked out of the new submission processes. Due to some of these constraints readers of the annual will notice that the section introductions this year are abbreviated, but the sections they serve to introduce remain as crucial and important as ever. We would also like to reemphasize that each paper accepted within the mathematics section was carefully peer reviewed – although the introduction might be abbreviated, the bar for acceptance was held high!

We would like to thank all of those who submitted papers to the mathematics section this year. Although space and time does not allow us to provide the annotated introduction of previous years we feel that you will be pleased with the rich and diverse representation of mathematical themes and teaching approaches and the enhancements which technology has enabled. As was the case in last year, the large number of PT3 related presentations have strengthened not only the SITE conference, but the entire field of technology and teacher education. It is wonderful to finally be able to seriously examine technologies impact upon teaching and learning within mathematics across a broad context of methods, situations, and populations. It's quite exciting to think about what the years ahead might bring when we are able to focus upon substantive issues without having to spend huge efforts in maintaining a stable workplace within which to perform our research.

In looking ahead, we would like to restate last year's themes as organizing frameworks within which to consider and evaluate research efforts. The first area concerns itself with the nature of the content to be taught. Mathematics in a technology-enhanced world looks substantially different than it does when the sole tools to think lived are calculator, paper and pencil. Clearly, content issues need to be addressed in a markedly different fashion in teacher preparation. Secondly, the role of teacher and the interaction between teacher, technology and student will need to be carefully addressed. One might expect to see a growing number of papers dealing with these interactions. The third area of potential change lies in that of the student as they use this new technology enhanced environment to explore mathematics. We feel more strongly then ever that these organizing frameworks will go far in helping readers integrate the various research findings in technology enhanced mathematics education, not only from this conference but from other sources as well.

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Teachers' Learning of Mathematics in the Presence of Technology: Participatory Cognitive Apprenticeship

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Abstract. The purpose of this paper is twofold: (a) to present the theoretical background and development of an evolving design of the learning environment for the technology-based mathematics course for K-12 teachers, and (b) to explore the qualitative features of the instructor-teacher and teacher-teacher interactions, while integrating technological and mathematical teaching/learning through the participatory cognitive apprenticeship instructional methods.

Introduction

Shifts in the philosophy and theory of learning as well as emerging technologies support the view that a paradigm shift in teaching and learning mathematics with the use of information and computing technologies (ICT) is taking place. The existence of increasingly efficient ICT tools lends support to the view that the learning environment in school mathematics is changing into a more technological one. Teachers are aware of current changes and are involved in the processes of these changes in their schools.

Many teachers are disillusioned by their experience with technology integration so far. Marcinkiewicz (1991) points out that teachers are often not sure that the skills and experiences they acquire in available technology training will be easily transferable to classroom instruction. High-quality training, sufficient resources and awareness of necessary change are some of the critical factors necessary to regain the trust (Cafolla & Knee, 1995). To build confidence, teachers need successful experiences and ongoing pedagogical and technological support when integrating technology into their curriculum (Byrom, 1997).

Finding ways to apply modern teaching theories integrating technology within the confines of the traditional classroom poses many challenges. Within the field of instructional design there is a growing support for addressing these challenges (Lebow, 1995). One of these is the need to help students develop mastery of their own learning, and provide teachers with the tools they need to facilitate that development. One of the implications, the shared responsibility for student learning co-created by both student and instructor, termed as an apprentice-mastership (Winn, 1995) indicates a possible direction in instructional design. Several studies have used the cognitive apprenticeship model as a means to support and enhance technology training (e.g. Cash et al.1997).

"Technology in the Mathematics Classroom K-12" is a course that pre-service and practicing teachers take to advance their knowledge of technology integration. This paper describes the evolving design of a learning environment model during facilitation of that course. Learners' interactions (instructor-teachers, teacher-teacher) played a significant role in developing this course. The qualitative features of these interactions will be explored. Other results have been reported elsewhere (Alagic & Langrall, 2002). Data collected is comprised of online interactions, questionnaires, assignments, and interviews of the teachers (students in the course).

Participatory Cognitive Apprenticeship in the Technology-Based Learning Environment

Cognitive apprenticeship model is developed within the situated learning paradigm. Learners participate in a community of practice that is developed through sequenced guided activity and interaction, in ways similar to that in craft apprenticeship, with more emphasis on the development of cognitive skills (Lankard, 1995). A cognitive apprenticeship instructional model merges the components of Schoenfeld's model for teaching mathematical problem solving and Treisman's collaborative workshop model (Johnson & Fischbach, 1992). These models appear to successfully develop not only the cognitive, but also the metacognitive, skills required for true expertise. The cognitive apprenticeship can be viewed as a representation of Vygotskian "zone of proximal development." Cognitive apprenticeships suggest project or problem-based group-work for students with close scaffolding of the teacher. An expert does modeling of a task. Learner performance and reflections are accomplished with coaching. Students' tasks are more difficult than students can manage independently, but only so that they require the aid of peers and instructors' scaffolding guidance. Knowledge and skill are made meaningful by the context in which they are acquired. Learning from modeling, coaching, fading, articulation, reflection, and exploration of ideas are the most significant phases of this process (Cash et al., 1996).

Since instructional strategies in developing technology-enhanced material are often complex in natural school settings, a cognitive apprenticeship model is a more applicable means of facilitating technology integration than traditional apprenticeship (Cash et al., 1996). Collins (1991) perceives technology to be the resource-intensive mode of education and a supportive environment for cognitive apprenticeship.

The impact of technology on students' achievement is linked with the way the technology is used: Grade appropriate use of computers, for example, has been found to be more important in producing increased learning than the amount of computer use. Yet as research findings regarding the use of technology in classrooms are domain-specific and often reflect a narrow set of conditions, they require careful interpretation. Jonassen (1999) gives five ways in which instructional technologies have been used to support learners' internal negotiations and meaning making, as:

1. Tools to support representing learners' ideas, understandings, and beliefs,
2. Information vehicles for exploring knowledge,
3. Support for simulating meaningful real-world problems, situations and contexts,
4. Social media to support learning through conversation, and
5. Intellectual partners to support learning-by-reflecting.

Empowering teachers through the use of technology in open-ended problem solving process, interpreting mathematics and developing conceptual understandings is at the heart of mathematics education. Mathematics teachers need high quality and on-going opportunities to experience and do mathematics supported by diverse technologies (Dreyfus & Eisenberg, 1996; Schoenfield, 1991).

Emerging research and teaching practice increasingly focus on the role and effects of classroom interaction to the learning process. The effects of social interaction on learning have support in Vygotsky's view of cognitive socialization and Piagetian ideas of cognitive conflict. The developmental research inspired by Vygotsky's view has focused on collaborative cognitive activity emphasizing interaction between learners as a source of development. The theoretical concept of interactions has novel characteristics in a technology-based mathematics learning environment. This view characterizes learning as *participatory activities*, and includes both peer and student-teacher interactions. The thinking processes behind learning are extending beyond individual cognition to include features of both the groups as well as the technological tools employed. Group interactions and communications converge to the joint task to be solved. A simulation of complex mathematical phenomena or reflections on the learner's thinking processes assists in reaching a joint goal. This common goal helps students in explaining their difficulties, while providing new opportunities for teacher to scaffold. The teacher-students' interaction episodes in this environment may involve qualitatively new formats.

The participatory cognitive apprenticeship in the technology-based learning environment (PCATLE) is a variation of the cognitive apprenticeship learning model developed as a result of two years of teaching "Technology in the mathematics classroom K-12", author's work with prospective and practicing teachers as well as teaching experiences in mathematics and ICT. It has been evolving in association with the teachers/networking partners involved. This paper describes the main characteristics of the development of this model (up to this point) and some of the qualitative features of class interactions during that process.

The view of *learning for understanding* is a backbone for the PCATLE model. We recognize understanding through a "flexible performance criterion" (Perkins 1993). Knowledge and skills are important, but if they are not understood the student cannot make a good use of them. Learning for understanding requires thinking in number of ways with what we "know", practicing and negotiating our thinking until we can make

the right connections flexibly. That also means that the pillar of learning for understanding must be actual engagement in those performances.

"Technology in the Mathematics Classroom K-12"

"Technology in the Mathematics Classroom K-12" is a three credit hours summer course which teachers take either as a part of their graduate coursework or to advance their knowledge of technology integration. Assuming that common overarching goal of teaching and learning mathematics with and for understanding, guiding questions for the course are the general ones: How do I reach a necessary understanding of my audience and how do we together reflect about potential audiences that participants in this class will be teaching? Where do we want to be at the end of this course? How are we going to get there? How are we going to know if we are there?

We focus on what teachers (students in this course) bring in and on the negotiated learning goal in the existing context (where they want to be and where we would like to take them) by the end of the course. We take into account the learner's original ideas, we stage discrepant or confirming experiences to stimulate questions and encourage the generation of a range of responses with the opportunity to apply these in various situations, as suggested by Berryman (1993).

The second cohort, one this paper is focusing on, had 19 teachers (two primary, five elementary, six middle, five high school teachers, and one pre-service teacher). The underlying themes are:

1. Experiencing and doing mathematics as problem solving, reasoning, connecting and communicating through a variety of representations in the technology-based learning environment.
2. Recognizing how conceptual understanding and procedural knowledge are developed together and that their mutual development is enhanced (reinforced) through technology.
3. Reinforcing awareness of changes in the School mathematics brought both by current school reform for standards-based teaching that supports integration of technology and by the development of ICT;
4. Evaluating a variety of computer programs and web resources for learning and doing mathematics; and
5. Organizing class so those teachers take the first steps towards becoming self-reliant, self-regulating, and self-evaluating learners in their technology based design of mathematics activities. The course environment is structured so that it helps students reach these goals. Both the didactical approach of the course as well as the time available for personal interaction between teacher and instructor ask for the embedding of scaffolding into the course site.

Daily assignments included metacognitive reflections via e-mail with the facilitator and either a mathematical task or reporting on teaching strategies in classrooms that integrate technology. Part of class time was spent on critical evaluations of available software and Web resources based on evaluative resources and teachers' experiences. The computer lab used during the course had state-of-the-art equipment, including wireless technology and most of the mathematics software available on the market. During class time, the facilitator also had technical support. Teachers chose to complete mathematics projects using the following resources: LOGO (n=2); CAS (n=4); Maple (n=2); dynamic geometry (n=5); spread sheets (n=6). Initially, everyone explored all of the above software, as well as concept mappings, graphic organizer software, and Internet resources.

The entire group was motivated to try possibilities and share their experiences with the contributions of emerging technologies to mathematics education. Nurturing, self-reliant, self-regulating and self-evaluating environment requires ongoing class negotiations where instruction-authority is based both on pedagogical content knowledge related to mathematics and technology, and expert-utilization of what the class participants bring in terms of both knowledge/understanding and classroom experiences. Learning and instruction in both cohorts was based on reciprocal understanding between the participants. That was accomplished by establishing shared goals based on common interests in learning about technology-based mathematics phenomena. The following learning environment rules have also being helpful in keeping everybody engaged in their tasks: Try to find answers on your own before asking questions; Ask your partners first, specific area-experts next, ask your instructor the last.

Activities are brought about in three levels: *Learner's level.* The first task for learners is to get oriented to the context and determine a reasonable but challenging goal for their studies based on available opportunities that include not only existing mathematical software but also level of available expertise. The learners can study different things depending on their individual objectives related to the field. The final "product" is realized through the portfolio that includes lessons/activities of technological representations of mathematical

phenomena, a (group) project/presentation on the mathematics topic using appropriate technology for developing the concepts, and reflective online journaling. *Network level.* The lecture materials are available for students on the Web, which offers a possibility for the discussion on these materials. The students' group projects are also available on the network level. Reflective discussions are usually initiated by instructor's reflective question, but often, they take life of their own. On the *local level* the studies consist of lectures and activities on integrating technology in mathematics teaching usually done by instructor or experts in the field. Teachers are encouraged to share their classroom experiences in integrating technology. Discussions about student's study products are based on the shared idea of collaborative learning for understanding.

During discussions, although very enthusiastic when preparing lessons for the coming fall, teachers talked about day-to-day obstacles such as class management, absence of appropriate ICT-based curriculum materials and "covering material" -- *capturing the struggle* (Alagic & Langrall 2002) between wanting to promote technological innovations and everyday realities. They indicate that integrating technology require both curricular and scheduling adjustments. Some teachers expressed need for more training. On the positive side, including technological representations of mathematical ideas in developing conceptual understanding through scaffolding was well supported by most of the teachers; well planned participatory activities were most often mentioned. An agreement to keep in touch via online discussion was supported by everybody.

These deliberations clarify for us the need to identify research questions and appropriate methods for both development and implementation of PCATLE model and further investigations of learning opportunities provided by participatory activities and reciprocal interactions in ICT-based differentiated mathematics instruction.

Implications

The PCATLE model has been experienced by the participating teachers as a well-designed and a fruitful way of organizing the course, as it makes possible, for example, to share expertise and also to develop the model and the curriculum together. The combination of different kind of activities including lectures, collaborative activities in the lab and on the Web, and individual studying are for many teachers a new way of studying. This kind of learning model is challenge for all involved, but also for the organizing network, because it does not necessarily fit very well to the traditional teaching/learning structures.

For me, the researcher, this report reflects (1) a "term introduction" stage of the concept *participatory cognitive apprenticeship in technology-based environment,* and (2) beginning stages of this concept expansion. The PCATLE as an orienting framework for the teaching practice is an effort to support teachers as they were learning to interpret and interact in the new mathematical culture they were immersed in.

Although a number of change theorists insist that reform efforts start the process with a clear, specific vision of what they want to accomplish, approaches based on postmodern epistemologies as well as chaos theory suggest that the reverse is preferable (Fullan, 2001). Research outcomes suggest that the PCATLE is an inviting direction/framework for supporting teachers as learners. The following stage of the learning cycle, expansion, awaits new challenges. In believing that a computer-supported learning environment creates an optimal environment for participatory cognitive apprenticeship, we are going to continue revising our technology-based learning environment focusing on analyzing learning (mathematics) for understanding and social interaction in that context.

Attention has to be brought to the fact that this kind of learning environment requires far more individualized attention and interaction than a full-time teaching practice allows. The researcher had a particular affinity for teaching this course, a great support from colleagues, and an excellent technical support in the lab. My belief at this point is that a technology-supported learning environment creates an optimal situation for the PCATLE-based learning. Such an environment provides multiple opportunities for students to negotiate meanings concerning different abstract mathematical phenomena. Appropriate and modest scaffold assistance either from me or my lab-assistant seemed to enhance the learning activity and move the projects forward.

How to create optimal conditions for PCATLE and its interaction challenges future studies. Focus from traditional theories on mutual understanding needs to be broadened to study the other possible aspects of the social interaction. Relevant viewpoints in future studies will be students' motivational interpretations in a computer environment and embedded opportunities for differentiating instruction. Reaching every student at an appropriate, yet challenging cognitive level will increase learning for understanding, recognizing understanding through a flexible performance criterion (Perkins, 1993) and providing better conditions for differentiated instruction which ultimately provides better individualized learning for ALL learners (Postlethwaite, 1993).

References:

Alagic, M., & Langral, R (2002, in press). Differentiating Mathematics Instruction through Technology. In M. Borovcnik & H. Kautschitsch (Eds), *Proceedings of the ICTMT 5 in Klagenfurt 2001:* Vol. 25. *Technology in Mathematics Teaching.* Vienna, Austria: Schriftenreihe Didaktik der Mathematik.

Berryman, S., E., et al. (1993). *Building in the middle* (Vol. 5). New York: Education and the Economy.

Byrom, E. (1997). *Review of the Professional Literature on the Integration of Technology into Educational Programs.* Available online: http://www.serve.org/technology/litreview.html

Cafolla, R., & Knee, R. (1995). *Factors limiting technology integration in education: The leadership gap*. Paper presented at the SITE95 - Sixth Annual Conference of the Society for Information Technology and Teacher Education (SITE), San Antonio.

Cash, J. R., Behrmann, M. B., Stadt, R. W., & McDaniels, H. (1996). Effectiveness of cognitive apprenticeship instructional methods in technology classrooms. *Journal of Industrial Teacher Education*, 34(2), 29-49.

Collins, A. (1991). Cognitive apprenticeship and instructional technology. In Idol L.& Jones B. F. (Eds.) *Educational values and cognitive instruction: Implications for reform* (pp121-138). Hillsdale, N.J. Lawrence Erlbaum Associates.

Dreyfus, T., & T. Eisenberg (1996). On different facets of mathematical thinking. In R. Sternberg & T. Ben-Zeev (Eds.), *The Nature of Mathematical Thinking*. Hillsdale, NJ, USA: Lawrence Erlbaum Associates.

Jonassen, D., Peck, K. & Wilson, B. (1999). *Learning with technology: A constructivist perspective.* Upper Saddle River, NJ: Prentice Hall.

Johnson, S. D., & Fischbach, R., M. (1992). *Teaching problem solving and technical mathematics through cognitive apprenticeship at the community college level.* Berkeley, CA: National Center for Research in Vocational Education.

Lankard, B., A. (1995). *New ways of learning in the workplace.* Columbus, OH: ERIC Clearinghouse on Adult Career and Vocational Education.

Lebow, D. (1995). Constructivist values and emerging technologies: transforming classrooms into learning environments. *Proceedings of the 1995 Annual National Convention of the Association for Educational Communications and Technology.* Anaheim, CA

Marcinkiewicz, H. R. (1991). Levels of use of the innovation: A framework for analyzing innovation adoption. *The Journal of Teacher Education,* 26(1), 52-56.

Perkins, D. (1993). Teaching for understanding. *American Educator: The Professional Journal of the American Federation of Teachers*. 17(3). 28-35.

Postlethwaite, K. (1993). *Differentiated science teaching: Responding to individual differences and to special educational needs.* Open University Press: Philadelphia.

Schoenfeld, A.H. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J.F. Voss, D.N. Perkins, & J.W. Segal, (Eds) *Informal Reasoning and Education*, (pp. 311-343). Hillsdale, NJ: Erlbaum.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Winn, W. (1995). Virtual Reality Roving Van Project. *T.H.E. Journal*, 5(12).

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A Fractal is a Pattern in Your Neighborhood

http://www.pages.drexel.edu/~bachcn/Fractals

Fractal geometry is not just another chapter of mathematics, but one that helps every person see the same world differently."

– Benoit Mandelbrot

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Abstract

In recent years, the development of fractal geometry and the study of chaotic structures have gained widespread attention. The study of these "new" geometries has influenced recent work in almost all areas of intellectual endeavor (e.g., computer science, mathematics, physics, art, systems analysis, management, meteorology, and biology). The patterns of scaling geometries also have been found in the hair-weaving practices, architecture, and artwork of several African and Southern Indian cultures (Eglash, 1999). In addition, given that students often find the study of mathematics to be an isolated collection of arbitrary rules and abstract objects disconnected from their experiences, the beauty of fractal imagery provides a unique aesthetic element to the study of mathematics that can have a positive effect on student interest and motivation (Bach, 2002). In short, fractal geometry is a field with wide application, relevance and pedagogical promise.

However, with all of its potential for improving mathematics education, there has been insufficient attention given to teaching the basic concepts of fractal geometry to secondary students (see references for notable exceptions), and providing them with tools that assist and encourage hands-on exploration of these geometries within their local experience, within different cultures and societies, and within other areas of study. The web-based mathematics environment, A Fractal is a Pattern in Your Neighborhood, was developed to address this concern.

The tool was designed with several goals in mind: 1) to present fractal geometry in a carefully scaffolded manner, starting with the most basic properties of fractal structures and building to more sophisticated properties of chaotic structures and complex numbers, 2) to implement a method of teaching mathematics that first introduces mathematical concepts as they can be found in familiar environments, 3) to build into each lesson a writing component that encourages students to seek connections between fractal properties and other areas of human inquiry, 4) to provide students with a communication tool that allows them the opportunity to share their ideas with other students, teachers and experts, as well encouraging self-reflection (e.g., e-mail, online discussions, Web searches, online portfolios), and 5) to facilitate student use of various Web technologies (e.g., QuickTime VR, Java Applets, Javascripted functionality, animated GIFs, and PDF files).

A Fractal is a Pattern in Your Neighborhood was implemented during a NASA-funded project directed at preparing high school instructors to teach fractal geometry, use the fractal site in their classes, gain proficiency Web-based technologies, and utilize online NASA resources. In this session, I will demonstrate the online fractal lessons, describe their pedagogical foundations, and discuss some of the challenges that arose during the implementation of the lessons conducted with in-service teachers over the past year.

References

Bach, C. (2002). *The pedagogical promises of fractal geometry*. Manuscript in preparation.

Bach, C. (n.d.) *A fractal is a pattern in your neighborhood*. Retrieved December 20, 2001, from Drexel University, School of Education Web site: http://www.pages.drexel.edu/~bachcn/Fractals

Center for Polymer Studies. (n.d.) *Patterns in nature*. Retrieved December 20, 2001, from Boston University, Department of Physics and the Science and Mathematics Education Center Web site: http://polymer.bu.edu/ogaf/

Eglash, R. (1999). African fractals : Modern computing and indigenous design. New Jersey: Rutgers University Press.

Lanius, C. (n.d.) *A fractals unit for elementary and middle school students*. Retrieved December 20, 2001, from Rice University, Department of Mathematics Web site: http://math.rice.edu/~lanius/frac/

The Shodor Education Foundation, Inc. (n.d.) *Introduction to fractals: infinity, self-similarity and recursion*. Retrieved December 20, 2001, from http://www.shodor.org/interactivate/lessons/frac1.html

Redesigning College Algebra Delivery from Direct Instruction to a Computer Environment

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Introduction: In the summer of 2001, the Department of Mathematics and Statistics at Northern Arizona University received a grant from the PEW foundation to radically redesign its delivery of College Algebra. The course would change from a traditional lecture course to one that would be web-based in nature. Part of the reason for making the change is to provide a learning atmosphere that is more active in nature, and one that would allow students of differing abilities to progress at their own rate. A web-based format would also allow for flexibility of use on the part of the students; they could work on the course materials whenever it fit into their schedule. The location would be flexible too; they could work on the course in the computer lab in the mathematics department, in other computer laboratories close to their dorm rooms, or even at home. Another reason for making the change is to attempt to stem the tide of students who fail the course. Approximately thirty to forty percent of College Algebra students withdraw from the course or receive failing grades; of these, almost half (47%) will leave the university entirely.

During the fall of 2001, the committee in charge of implementing the transition began to meet and discuss all of the necessary steps that would have to be taken before the project could be implemented for the Spring Semester of 2002. The committee consisted of an Assistant Dean of the College of Arts and Sciences and three members from the Department of Mathematics and Statistics: a tenured professor, a tenure-track assistant professor, and a full-time lecturer. The following report details the some of the work that has been performed by the committee up to this date concerning the transition.

The Redesigned Course

Software: Once the proposal was formally accepted, it was decided that the department should begin testing various software programs that were written specifically for College Algebra. During the summer sessions, different software packages were tried out in the College Algebra courses that were run during that time. Students were asked to spend a minimum amount of time (about one hour per week) trying out the software and evaluating its functionality from their perspective. The use of the software was not made a requirement in the courses; rather it was made available for the students if they wished to use it.

Based on this, during the fall semester two different software packages were selected to be evaluated by the committee: ALEKS, which is published by the ALEKS corporation through McGraw-Hill, and My Math Lab, an Addison-Wesley product. It seemed to the committee that there existed two possible courses of action. One route was to plan out the course, while at the same time testing each software product to see how well it "fit" the planning, and making a decision on software at the end of the process. The second option reversed the process: choose the software package first, and design the course to best fit the software. It was decided to follow the second course of action, and after roughly one month of testing by individual committee members, ALEKS was selected.

Scheduling: As part of the grant, a new computer lab was created dedicated to the College Algebra classes. Three graduate teaching assistants and several undergraduate students serving as computer laboratory assistants would staff the computer lab. These staffers would be responsible for helping support and enhance the students' learning of College Algebra. However due to constraints in both budget and the staff's own class schedules, the lab could only be open approximately 32 hours per week. And it was decided that part of this time would include the regularly scheduled class meeting times.

It was quickly realized that some initial instruction concerning the proper use of ALEKS would have to be provided to maximize efficient student use of the software. Therefore, it was decided that students would meet at their regularly scheduled meeting times (as listed in the class schedule) for the first week of the course. During this time the students would work through a tutorial, take an initial assessment exam

provided by the software package (used to determine the level each individual student should begin work in College Algebra), and receive additional instruction from the computer laboratory staff.

Since the students would first come to the computer lab during their regularly scheduled class time, it seemed natural that the computer lab should remain open during these times throughout the semester. It guaranteed that the lab would be open for all students at a time when they had no other classes, thus ensuring that all students had access to it. It was also hoped that if the students knew the lab would always be open at the times when their classes were supposed to meet, some would establish the routine of always working on the College Algebra materials at these times.

By keeping the computer lab open during these times (mostly between 9:00 a.m. and 3:00 p.m.), a majority of the allotted hours were used up. The remaining time was designated for Sunday evenings.

Assessment: The committee considered several different ideas on how to best assess the students. After much debate, it was decided that traditional paper-and-pencil testing would be used. While this has the disadvantage of not being aligned with instructional method, the advantage lies in accountability; in other words, it would be known that the person taking the test would indeed be the same person who should be taking the test. The students could take these tests at any time they believed they were properly prepared for them. However, it was decided that a deadline should be set for each test, to ensure that students did not wait until the last week of the semester to begin taking them. There would be five tests during the semester; each would be worth 50 points. The final test would be worth 100 points, for a total of 350 points.

It was also decided that the use of ALEKS should in some way be calculated into their assessment. While discussing many options, it was determined by the committee that students would be given credit for working 6 hours each week on ALEKS, and making adequate progress. They could earn up to 4 points per week (for 12 weeks), and if they met established goals for at least 10 of those weeks, they would receive two bonus points, for a total of 50 points.

Because students could take the tests at any time (up to the deadline), it would be necessary to have several versions of tests available. For this purpose, it was decided to create a large database of College Algebra questions. These questions would be created and stored electronically on a computer; if a student wished to take a test, the lab assistant could use the program to randomly select the appropriate questions for each test from the various databases.

Finding an appropriate software product to handle this proved to be a difficult task. Many software packages investigated could not handle the complicated mathematical symbols and graphs necessary. Others only allowed for a multiple choice/true false/short essay type of question. In the end, a data base program (File Maker Pro) was chosen since it had the ability to handle most of the demands that would be placed on it. The committee then hired three other instructors to write questions for these test databanks. Each question will have 15 different versions of it, making it virtually impossible for anyone to be dishonest when studying for the tests.

Assessing Impact: During the Spring Semester of 2002, half of the College Algebra classes will be taught traditionally, and half using the web-based curriculum. The numbers of students in both designs should be roughly the same. For comparison purposes, all students in both designs will be given a pre-test; and throughout the semester students in both classes will have identical (i.e., similar in content) questions in their exams. These matched questions will allow for the comparisons of student performance on specific outcomes, as well as compare student performance on those outcomes in both the traditional and web-based course. The pre-test data will be used to help control for variations across the different sections. Questionnaires will also be used to assess the affective variables concerning how the students feel about using a web-based curriculum in general, of ALEKS in particular, and of the entire redesigned course structure. All of the assessment measures will look at the overall effectiveness of the redesigned course when compared to the more traditional method of teaching College Algebra.

Mathematical Modeling of Motion
Captured with a Calculator-Based Ranger

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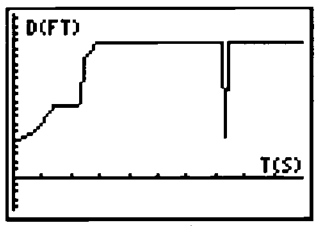
Abstract: The National Council of Teachers of Mathematics (1996) has stipulated that a most exciting potential of technology is its effect on increasing the amount of time that can be devoted in developing conceptual understanding and reasoning processes that lie at the heart of mathematical problem solving. In fact, whether it be with the use of spreadsheets, graphing utilities, or instructional software, technology permits students to explore situationsenabling them to visualize relationships readily and to test ideas quickly. In general, the same is true in using a Calculator-Based Ranger (CBR), a motion detecting device. A CBR enables real data from real experiments to be captured in real time. In turn, using a CBR with graphing calculators (e.g., TI-83 Plus graphing calculator) permits rapid production of visual displays, with statistical summaries possible leading to prediction models. For this reason, a CBR should be a standard instructional tool in mathematics classrooms 8th-12th grades. This equipment shows promise in making algebraic and statistical ideas accessible to all students. With CBR-based lessons, simulations and modeling, in particular, make it possible to study mathematical situations in a less abstract way.

One role of mathematics teachers is to help students interpret and analyze situations from mathematical perspectives, as well as to assist them in seeing interrelationships of mathematics (e.g., unifying ideas such as set, relation, function) that tie various concepts together. However, even students who display a high level of mathematical skill often fail to see any relationship in what they may be learning or doing to real-life situations. Furthermore, in recent years greater emphasis has been placed on teaching problem solving in the mathematics curriculum. Nonetheless, research shows that students= problem-solving skills are still considerably weaker than their computational skills. Although most educators agree that much more needs to be done to improve students= abilities to solve problems and to think critically, as well as to ascertain that mathematics not be learned in isolation from its applications, seemingly the level of maturity required to work with data, formulas, graphs, and other such mathematical concepts, generalizations, and procedures can be a hurdle for many students especially when data collection and analyses is cumbersome and time-consuming. Yet, if one feels that problem-solving skills and an understanding of the usefulness of mathematics is important, shouldn=t one teach students to transfer the mathematics they have learned or are learning to applied areas despite the toil or challenges? Is it possible that with a little technological help, opportunities that make applications and connections accessible and that help students make discoveries can and will make a difference? ***We believe such is the case.*** With a Calculator-Based Ranger (CBR) and a TI-83 Plus graphing calculator, students can collect, view, and analyze motion data without tedious measurements and manual plotting. Motion data is essential in many applied fields of mathematics (e.g., physics, engineering), so rather than having this skill be a stumbling block to students= continued success, studying motion data, even in mathematics classes, can be a real benefit then and later as students encounter an awareness of mathematical concepts and unifying ideas, and their applications in this context.

From a curricular perspective, a Calculator-Based Ranger lets students explore mathematical and scientific concepts such as: (1) motion: distance, velocity, acceleration; (2) graphing: coordinate axes, slope, intercepts; (3) functions: linear, quadratic, exponential, sinusoidal; (3) calculus: derivatives, integrals; and (4) statistics and data analysis: data collection methods, statistical analysis. Indeed, the Calculator-Based Ranger sonic motion detector can also be used with TI-82, TI-83, TI-85/CBL, TI-86, and TI-92 and easily brings real-world data collection and analysis into the classroom making it possible for students to also explore relationships between studied concepts. Fortunately, for most people, a Calculator-Based Ranger is easy-to-use, especially with the TI-83 Plus Graphing Calculator which includes the RANGER, MATCH, and BOUNCING BALL programs as standard built-ins. Consequently, getting started with CBR activities does not require extensive calculator or programming experience. To exemplify how to operate a CBR, the activity entitled A**Experiments about Motion Captured with a Calculator-Based Ranger**@ shall be demonstrated. This activity acquaints students with basic functions and properties of motion under various settings (e.g., people, cars, and pendulum in motion). The purpose for elaborating on this activity is to demonstrate that a CBR enables real data from real experiments to be captured in real time, helping students to develop the skills to apply mathematics to more realistic settings, as well as to help them see interrelationships between concepts studied. This is demonstrated by using a CBR with a graphing calculator (e.g., TI-83 Plus graphing calculator) which permits rapid production of visual displays, with

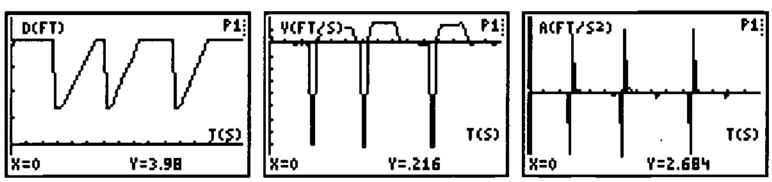
statistical summaries possible leading to prediction models. Critical to this demonstration is the belief that when students actively participate in helping to shape their own understanding of an idea, as facilitated with technology, they are learning how to learn. Consequently, their view of mathematics as a tool for answering questions that are meaningful to them can only be strengthened.

As an example, the diagram from a TI-83 Plus graphing calculator display shown below shows the motion detected by the CBR of a person walking away from the motion detector from a distance of about 5 feet. The person is walking rapidly away from the motion detector for about 3 seconds, and data is collected for about 10 seconds altogether. The diagram depicts these continuous actions. The second diagram depicts the motion of three distinct battery-operated "toy" cars. Here, the velocity of the cars are detected by thc CBR and diagrammed in a TI-83 Plus graphing calculator environment. By using technological tools, students are able to obtain skills via experiments which show them how to look for applications, and how to see unifying ideas in mathematical context. Indeed, this equipment shows promise in making algcbraic and statistical ideas accessible to all students.

Start at about 5 feet from the motion detector and walk **rapidly** away from the motion detector for 3 seconds. Collect data for 10 seconds.		CBR Ranger Settings Realtime: No Time(s): 10 Display: Dist Begin on: [Enter] Smoothing: None Units: Feet

Who=s the FASTEST? Data Collection on Three "Toy" Cars

Collect data for 15 seconds. Run each car in turn, with a few seconds in between. Interpret the graphs generated by the motion detector on the TI-83 Plus Graphing Calculator. What do the graphs suggest?



References

National Council of Teachers of Mathematics (1996). *Algebra in a Technological World.* Addenda Series: 9-12. Reston, VA: NCTM.

Texas Instruments TI-83 Graphing Calculator Guidebook (1996), Texas Instruments.

Texas Instrumcnts Getting Started with CBR™, Texas Instruments Incorporated, 1997.

TEXTEAMS (Texas Teachers Empowered for Achievement in Mathematics and Science) ARethinking Middle School Mathematics: Algebraic Reasoning Across the TEKS for Grades 6-8", TEXTEAMS Institute, May, 2000.

TEXTEAMS (Texas Teachers Empowered for Achievement in Mathematics and Science) AAlgebra II/PreCalculus Institute@, TEXTEAMS Institute, May, 2000.

General Logic Actions and Database Oriented Methods of Their Development in Teaching Advanced Calculus

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Abstract: This paper discusses how databases can be used in developing general logic actions in teaching Advance Calculus. The methodology of teaching advanced calculus that we are currently developing has a special importance for training mathematics teachers. Actually teachers of mathematics and those who are going to pursue careers in other related to mathematics fields are going to use knowledge and skills from this class differently. For mathematics teachers the most important thing is to learn how to structure calculus theorems, how to use previously proved theorems in proofs of other theorems, how to use examples and counterexamples to substantiate different issues, etc. The methodology that we have been developing for the last four years is designed specifically for teaching this kind of things.

Introduction. Functional and operational composition of action.

As theoretical and experimental research shows (Bouniaev, 1993) that databases and bases of knowledge (complete with a designed interface) can serve as construct frameworks in organizing interactive computer oriented instruction.

The methodology of teaching advanced calculus that we are currently developing is of particular importance for training mathematics teachers. Actually teachers of mathematics and thosc who are going to pursue careers in some other related to mathematics fields are going to use knowledge and skills from this class differently. For mathematics teachers the most important thing is to learn how to structure calculus theorems, how to use previously proved theorems in proof of other theorems, how to use examples and counterexamples to substantiate different issues, etc The methodology that we have been developing for the last four years is designed specifically for teaching this kind of things. .

Our analysis of using information technologies is based on the theory of stage-by-stage development of mental actions (SSDMA theory) developed by the Russian school of psychology (Galperin &Talizyna, 1979), (Talizyna, 1975) as applied by the author in teaching advanced mathematics (Bouniaev, 1991&1996).

According to Talizyna (Talizyna, 1975) all actions can be referred to two categories: general logic actions and specific actions. General logic actions are inherent in every subject field and are different only in objects at which they are directed. Examples of such type of actions are qualification, break up into classes, comparison, contributing to a concept, action of proof. For example, qualification as a type of action exists in mathematics (qualification of the conics and differential equations) as well as in other disciplines. Specific actions are basically inherent to a given subject field. For example, in mathematics they are arithmetic operations, differentiation, etc.

The SSDMA theory specifies four independent characteristics of any action used to judge the level of development of an action. An action can be in a materialized (material), speech or mental form. Mental form of action is the highest form of action development. The mental form of action also means that its objects are representations, notions, concepts and all operations are performed in the mental form. The ability to perform a whole action in the mental form indicates that it has gone through all the stages of development and interiorization.

As a rule, the performed actions themselves consist of other, more primitive actions and in their turn can be part of other actions. Actions that are part of a given whole, are called operations. That is, operations are also actions, hence the term emphasizes only the hierarchial subordination among actions.

Databases in studying advanced calculus

The structure of any database significantly depends on what kind of information this database is used for. Therefore as points of reference for constructing databases we offer students typical questions that will be part ofthe tcst (Bouniaev, 2001).

1. Formulate the theorem. 2. What is the theorem's premise? 3.What is its conclusion? 4. What notions are used in the theorem's formulation and what is their meaning? 5.What theorems were used in the proof of the given theorem? 6. Give examples of problems, which are based on solving this theorem. 7. What theorems need to be used to do this problem? 8. Is the conclusion true in absence of any of the premises? 9.Are all the premises necessary for the conclusion to be true? 10. If you answered, "yes" in 9, demonstrate it. 11. Give an example of an object that can be attributed to this concept. 12. Give an example of an object that cannot be attributed to this concept. 13.Prove this theorem. 14. Solve a proof problem.

At our first experiment the students themselves determined the structure of databases, fields, records, etc Theoretical analysis conducted after the experiment led us to the conclusion that teaching was based on the first type of the orientation part of an action (Talizyna, 1975), the learning process was not fast enough and required considerable efforts on the part of a student. Thus taking into account this fact at our next stage we made a decision to base instruction on the fourth type of the orientation part of an action

We believe that fields in the databases can serve as this generalized system of points of reference in developing the action of theorem proof. Since the structure of proof of any mathematical statement is practically identical, the system of reference points developed by us can be successfully used both in Advanced Calculus and Abstract Algebra as well as in any other course with one of the main goals being to teach proofs. Thus as a generalized system of reference points the students were offered tables "Theorem Statements", "Concepts and Their Definitions", "Theorem Proofs".

In table, "Theorem Statements" the following fields were established: "Name of the theorem" (for example "Intermediate value theorem"); "Object of the Theorem" (for example "A function f"); "Premise 1" (for example "Domain is an interval <a, b>"); " Premise 2" (for example " Continuous on the interval"); " Premise 3" (for example "Domain is a closed set"); "Premise 4" (For example "f (a)<C<f (b)");"Premise N"; Conclusion 1" (for example "There is a point "c" bctween "a" and "b" such that f (c)=C); Conclusion N"; " Concept 1 Used in the Theorem Statement" (for example "Continuity on a set");"Concept N Used in the Theorem Statement"; "Model Problem 1";...;"Model Problem N".; "Theorem Used for the Proof of..."(For example, Theorem used for the proof of Theorem 5-11, Theorem 7-3, etc.)

It should be noted that filling in fields while developing a particular action takes place at different time periods of study.

The structure of table "Theorem Proofs" is determined by the following fields. "Name of the Theorem"; "Object of the Theorem"; "Definition of Concept 1 Used in the Proof";" Definition of Concept N Used in the Proof"; "Theorem 1 Used in the Proof"; " Theorem N Used in the Proof"; "Transformation 1 of the Proof" "Transformation N of the Proof"; " Proof Structure".

Table "Concepts and Their Definitions" contains the following fields (as an example we use the definition of the least upper bound). "Name of the Definition" (in our example "least upper bound"; "Object of the Definition" (number L); "Auxiliary objects (in our example it will be set A, since we analyze the definition of the least upper bound of a set); "Concepts, Used in the Definition" (in our example "upper bound"); "Requirement 1" (L is an upper bound); "Requirement 2" (for any upper bound L', L≤L'); Requirement N"; "Example of the Object that Can be Attributed to the Concept" (1 for interval (0,1)); "Examples of the Object that Can Not be Attributed to the Concept" (½ for interval (0,1) & 2 for interval (0, 1)); "Definition Used in the Proof of Theorem..." (Theorem3-4, Theorem8-2, etc.); "Definition Used in the Statement of...."(Theorem 2-1, Definition3-4, etc). «Hierarchical Relationship with Other Concepts (in our example "least upper bound" implies "upper bound"; "maximum" implies "least upper bound"

As we already noted, any action developed in the process of instruction can be presented as a sequence of general logic and specific actions or operations. In the study of the majority of disciplines development of general logic actions is not the goal of instruction. In this respect advanced math courses

significantly differ from any other disciplines. One of the main goals of instruction of Advanced Calculus is development of the action of theorem proof and the action of using the theorem in solving practical problems. In order to achieve this goal we need to develop certain "elementary" general logic actions. These "elementary" actions include among others such actions as comparison, generalization, concretizing, attribution to the concept and drawing conclusions.

Developing of elementary logic actions

The action of comparison. As a rule, development of the action of comparison is directly connected with the development of concepts. Comparison implies singling out criterions of comparison. Knowledge of the system of possible criterions of comparison and the chronology of their appearance in the process of instruction demonstrates the level of development of basic mathematical concepts.

At the initial stages of instruction criterions of comparison can be presented to students. As observations show, in studying every new concept a student makes a comparison using only the studied criterions. Psychologically it is understandable and logical. However, in the process of instruction it is necessary to make sure that every new criterion does not stay isolated but gets incorporated into the already existing system of criterions. That means that every new criterion first should be added to the set of already existing criterions and, second, that a strong hierarchical connection of the newly added criterion with the previously existing criterions be established in the mind of a student.

Using databases helps to solve this problem in the process of independent construction of databases, during their utilization and updating. Probably, the most important role in incorporating a new criterion into the system of the already existing ones is played by the fields "Object" and «Hierarchical Relationship with other Concepts" in table «Concepts and Their Definitions».

At the initial stages of instruction, students often experience considerable difficulties in understanding what is the object of the definition. Thus the object of the notion "least upper bound" is often described as a set which means that in analyzing the definition the students don't focus on the fact that 'least upper bound" is a number. The existence of the field "Object" makes it possible not only to make an explicit note of this very important element of concept but also to make a query about what concepts studied before had the same object, i.e. incorporate a new criterion of comparison into the already existing system. The necessity to fill in the field "Hierarchical Relationship with other Concepts" not only makes it necessary to make such a query but also to analyze the relationship of the new added criterion of comparison with the already existing ones. The logic of presenting a new material as a rule implies that the newly introduced concept involves theorems connecting this concept with the already existing ones. Therefore in most cases the hierarchical relationship of the new and existing criterions will be established when the students compile the table "Theorem Statements".

Action of generalization. Development of the action of generalization while working with databases is connected first of all with generalization of the earlier applied ways and methods regarding a new class of problems. A significant role in this is played by the field "Transformation" with the table "Theorem Proof" and students actions while working with this field. In the process of filling in this field it is required not only to identify what transformation (inequality, identity, idea, etc.) was used in the theorem proof but also to indicate in the proof of what theorems this or similar transformation was previously used. As an example let us consider Bernoulli inequality and generalizations related to it. Probably it is used in advanced calculus for the first time while proving that the limit of the sequence $a^{1/n}$ is equal to one. The proof that the sequence $(1+1/n)^n$ is monotone in most cases is based on binomial theorem. The proof of Bernoulli inequality can also be based on binomial theorem. That's how the idea of using binomial theorem in " convergence proofs" can be generalized for a wider class of problems.

The next example is the use of triangle inequality in proofs related to convergent sequences (series), uniformly continuous functions, etc. The triangle inequality is used for the first time in advanced calculus while proving that the limit of the sequence is unique: $|a-b| = |a - a_n + a_n - b| \le |a - a_n| + |b - a_n| \le d/2 + d/2 = d$. Later t is used in various proofs. In the chain of inequalities that is written above two beautiful ideas are used. The first one is triangle inequality, the second is the idea of adding zero in the form $a_n - a_n$ to the algebraic expression. Both these ideas are then generalized for a wide class of problems. At the same time despite the fact that the use of the second method as a rule involves the use of the first one these two are different. If a student cannot distinguish between the two and views them only as one entity (what happens in a traditionally organized instruction) then the development of each of them goes very

slowly. In this respect it is difficult to overestimate the use of the fields "Transformation # N" in the table "Theorem Proof". First, while filling the fields in "Transformation" a student has to separate these two methods. Second, later on when these methods come up in the study of some new material a student has to analyze the history of using them. This means that a student develops in his/her mind a set of problems that require the use of these approaches together and another set of problems where these approaches are used independently of each other.

Action of attributing to the concept. Performing the action of attributing to the concept implies answering the question whether this object belongs to this concept or not. For example," Is this sequence monotone?" " Is this function differentiable?", etc. While performing this action it is important to understand what objects in principle can be attributed to this particular concept. As our observations of the traditionally organized process of instruction indicate a student very often simply does not understand the meaning of this question. In this regard the presence of the field "Object of the Definition" and the table "Concepts and their Definitions "play a very important role. The necessity to frequently answer the question about the object first of all develops a general understanding of the importance of the object and secondly, connects the object with the system of its possible criterions/attributes. The action of attributing to the concept can involve working both with the Table "Concepts and their Definitions " and "Theorem Statements". In the first case attributing to the concept is based on the definition of this concept while in the second it deals with the use of the theorem. In both cases the student is required to define characteristic criterions of the given concept. A set of characteristic criterions is incorporated into the corresponding record. In the process of work at this record a student is required to single out each of these criterions, so we see the development of the action of attributing to the concept in the step-by-step form. If in proving by definition some of the characteristic criterions present any difficulties then the action of attributing to the concept can be based on using the theorems. In this case a model of proof is developing in the mind of a student with the initial stage being not the analysis of the given but the analysis of the conclusion. For example, it is necessary to decide whether a given sequence has a finite limit or not. To solve this problem it is expedient to analyze the premises of all the theorems with the conclusion "sequence converges". Maybe one the possible sets of premises that leads to the required conclusion matches the characteristics of the given sequence? The existence of a database allows us to easily do this. Naturally, we do not assume that every time when there is a need to use a theorem in order to attribute an object to a concept the student goes to the databases. Our goal is quite the opposite, i.e. to develop this action in the mental form. However, as practical experience shows at the initial stage of abstract mathematics study a simple explanation that there is a possibility to start with the conclusions and after that go to the required premises is far from being sufficient. It is necessary for a student to associate this mental action with some external activity. Working with databases can become such external activity

Action of drawing conclusions. As in the case of attributing to the concept action, the structure of the action of drawing conclusions involves singling out a set of characteristic criterions. Another similarity is that systems of characteristic criterions are actually presented to students in the explicit form. In attributing to the concept this is presented as a system of criterions of the single given concept while in the case of drawing conclusions it is given in the form of a set that is a combination of systems of attributes (criterions) of one or several concepts.

The action of drawing conclusions can be broken up into three classes. The action of drawing empirical conclusions belongs to the first class; the second class contains drawing conclusions with the given premises based on the logically substantiated rules of drawing conclusions and previous knowledge. The third class encompasses drawing free (not given) conclusion with the given premises. Development of the third class is closely connected with organizing the instruction process based on the learning search activities. We believe that this kind of instructional process while studying abstract math disciplines at the baccalaureate level does not fit the level of students learning skills developed by that time. Development of the action of drawing conclusions as empirical conclusions is one of the goals of studying Calculus at the intuitive level. Therefore we will focus on the second class of drawing conclusions.

The very activity of building tables "Theorem Statements" and "Theorem Proofs" is a material model of the action of drawing conclusions. While making a record in the table "Theorem Statements" a student learns to make clear distinctions between all the given premises. Learning to analyze the premises of a theorem is essential for learning how to prove a theorem.

In a traditionally organized instruction the process of reading theorem proofs as a rule is not yet performing an action. The situation radically changes with databases oriented instruction. In the process of creating records in the table "Theorem Proofs" structuring of the whole proof in the material form takes

place. All the used theorems, definitions, transformations and the order of their use are presented in the explicit form.

The action of concretizing. One of the most important problems in organizing instruction of advanced calculus is that of developing the skill to use the proven theorems to solve concrete problems. We can not consider a theorem to be learned until a student knows how to apply it. Even if a student knows how to prove it.

In the development of the action of concretizing in database oriented instruction practically all the fields in the table "Theorem Statements" play a very significant role.

The first step in the development of the concretizing action is a search for model problems of the studied theorem. By model problems we mean problems that significantly rely on this theorem for solution. Let us consider the theorem that any monotone bounded sequence converges. When students begin to deal with this theorem their technical skills are not quite developed yet and the number of concrete sequences that are worth discussing without distracting students from the major topic of study is extremely limited.

Formally, the problem to prove that $(1+n)/n$ converges can serve as an illustration (a model problem) for the theorem that monotone bounded sequence converges. At the same time this problem can be easily solved just by using the definition of a limit. Moreover, this problem looks as an ideal model problem for the proof by definition. A «good» model problem for this theorem is the problem that $(1+1/n)^n$ converges.

Taking into account the fact that this kind of situation occurs quite often in the process of instruction we advise students not to use the same (or similar) problem as a model for different theorem. Development of the action of concretizing can be viewed as a sequence of actions of attributing to the concept. In the action of concretizing an object and theorem premises are given. The given set of premises determines a certain set of objects. The problem of concretizing boils down to the attribution of a given object to the concepts defined by each particular premise. The possibilities of using databases in developing the action of attribution to the concept were discussed earlier.

References

Bouniaev, M. (1993). *Data Bases for training purposes.* (Monograph) Prometey Publishing House, Moscow;

Bouniaev, M. (1995). Some psychological aspects of developing computer based instruction in undergraduate advanced mathematics. *In International Conference on Computers in Education, Proceedings of UCCE 95*, Singapore; December 5-8,1995, (pp.583-591). Charlottesville, VA: Association for the Advancement of Computing in Education.

Bouniaev, M. (1996) Stage-by-Stage Development of Mental Actions and Computer Based Instruction. *In B. Robin, J.D.Price, J.Willis, & D.A. Willis (Eds.), Technology and Teacher Education Annual* (pp.947-951). Charlottesville, VA: Association for the Advancement of Computing in Education.

Bouniaev, M (1999). Computer Based Instruction and General Logic Action Development In Teaching Mathematics. In David A. Thomas (Ed.), International Conference on Mathematics/ Science Education &Technology 99, Proceedings of M/SET 99 - March 1-4, 1999, San Antonio, Texas, 211-215, Association for the Advancement of Computing in Education

Bouniaev, M (2001). Using Databases in Teaching Advanced Mathematics Courses, *World Conference on Educational Multimedia, Hypermidia & Telecommunication, June 25-30*, 2001, Tampere, Finaland

Galperin, P. & Talizyna, T. (1979) Contemporary condition of SSDMA theory. In *Vestnik MGU, V.14, #4 Psychology (p p.54-63).*

Talizyna, T. (1975). *Control of a knowledge acquisition process.* Moscow: Mosc. St. Univ. Publishing House.

Mathematical Treats form the Stars: Integrating Curricular Elements Through Partnerships Between NASA and Math Methods Faculty

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Abstract: Mathematics methods coursework can be an innovative environment through which to emphasize the integration of real-world data structures and opportunities. These opportunities can create instructionally informative opportunities for learners, as well as inform teacher candidates of innovative teaching tools at their fingertips. NASA offers numerous curricular opportunities to the mathematical methods coursework, with work focusing on both PreK-12 learners as well as university learners. Such a wide array of interest levels integrate numerous learning objectives, depending upon the needs and desires of the instructors and learners whom they serve.

Introduction

Teacher candidates with a specialization area focus in mathematics maintain a working knowledge of the learning environment due to their superior university methods faculty; however, the integration of opportunities that will be available within the field are of utmost importance. NASA offers the opportunity to maintain educational excellence through partnerships with PreK-12 instructors. Such partnerships are available through out the United States of America as well as around the world. The focus of this presentation, as well as proceedings paper, will focus upon the innovative opportunities that the NASA educational entity offers to the PreK-12 as well as university mathematical methods courses. The integration of NASA's superior real world curricular abilities into math methods coursework, which offers teacher candidates opportunities to work with real-world data structures and environmental elements that would otherwise be unavailable to the majority of teacher candidates in methods courses, present professional development and learning that will then be integrated into curricular scope and sequence for future PreK-12 learners.

National Standards

The "Technology Principle" is one of six principles that the National Council of Teachers of Mathematics (NCTM) designate as imperative for all teacher candidates to master (NCTM, 2000). The "Technology Principle" states that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, http://www.nctm.org/standards/principles.htm, paragraph 28). However, the key to the success of mathematics teaching is the mathematics teachers, not the technological tools that may support the educational endeavors (Garofalo, Drier, Harper, Timmerman & Shockey, 1000; Kaput, 1992; NCTM 1991, 2000). Through the collaboration that can arise between PreK-12 education environments, higher education environments that support teacher candidates and the National Aeronautics and Space Administration (NASA), the possibilities towards supporting and realizing NCTM's "Technology Principle" are available.

NASA Educational World Wide Web Sites

There are numerous Web sites that are supported by distinct arms of NASA. Each of these Web sites delineate the orientation for each NASA campus, such as the following Web sites:

- Practical Uses of Math and Science: The On-line Journal of Math and Science Examples for Pre-College Education
  http://pumas.jpl.nasa.gov/
- InfoUse's PlaneMath
  http://www.planemath.com/
- NASA Spacelink
  http://spacelink.nasa.gov/.index.html
- NASA-JSC Distance Learning Outpost
  http://learningoutpost.jsc.nasa.gov/
- The Space Place
  http://spaceplace.jpl.nasa.gov/teachers_page.htm
- NASA Human Space Flight Metric Converter
  http://www.spaceflight.nasa.gov/station/reference/calc/index.html
- NASA-AMATYC-NSF Mathematics Explorations I and II
  http://cctc.commnet.edu/lta/
- NASA KIDS
  http://kids.msfc.nasa.gov/
- LTP Glenn Learning Technologies Project
  http://www.grc.nasa.gov/WWW/K-12/airplane/index.html
- Space Science Data Operations Office of NASA/Goddard Space Flight Center: Space Science Education
  http://ssdoo.gsfc.nasa.gov/education/education_home.html

The examples presented in the above Web sites are developed by and in collaboration with scientists, engineers, educators, instructional designers, and other professionals. The sole intent of these endeavors being the support of education pertaining to the emphasis of mathematics and cross-curricular, real-world support in PreK-12 educational environments.

Conclusions

As delineated in NCTM's "Connections" section (NCTM, 2000), "Mathematics is an integrated field of study, even though it is often partitioned into separate topics. Students from prekindergarten through grade 12 should see and experience the rich interplay among mathematical topics, between mathematics and other subjects, and between mathematics and their own interests. Viewing mathematics as a whole also helps students learn that mathematics is not a set of isolated skills and arbitrary rules" (NCTM, http://www.nctm.org/standards/standards.htm, paragraph 30). NASA has made available interactive workshops, real-world data sets and lesson plans focused upon specific levels of mathematical principles to support the educational endeavors of our education profession, and are to be commended.

References

Garofalo, J., Drier, H., Harper, S., Timmerman, M.A., & Shockey, T. (2000). Promoting appropriate uses of technology in mathematics teacher preparation. *Contemporary Issues in Technology and Teacher Education* [Online serial], *1*(1). Available: http://www.citejournal.org/vol1/iss1/currentissues/mathematics/article1.htm

Kaput, J.J. (1992). Technology and mathematics education. In A.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: Macmillan.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author. Retrieved from the World Wide Web on December 18, 2001: http://www.nctm.org/standards/

Virtual Manipulatives in Mathematics: Addressing the Conceptual Dilemmas

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Abstract: The authors are co-principal investigators for an NSF project to create a web-based National Library of Virtual Manipulatives for learning mathematics in the elementary grades (K-8 emphasis). Our virtual manipulatives are tightly focused, highly interactive Java applets, some based on physical manipulatives commonly in use in the schools (e.g. geoboards, tangrams, pattern blocks, fraction bars, etc.); others are concept manipulatives especially designed to teach or reinforce basic mathematical concepts.

These virtual manipulatives, designed for children's learning, are at least as important for the instruction of teachers. They provide unprecedented opportunities and a relatively non-threatening learning environment for teachers. Teachers, as students, "learn by doing." At the same time they give teachers tools with which they can communicate, we believe, a more correct and mathematically accurate understanding of key concepts.

The fact that these teaching tools are web-based also opens entirely new ways to assess mastery in distance learning environments.

Background and Setting

This paper is in some sense a follow-up of, or at least related to, a paper presented at a previous conference, SITE 2000. At that time we were just beginning our three-year National Science Foundation funded National Library project. We are now nearing the end of the funding period, but as with any project of substance, we see as much more to be done as we may have already accomplished.

Our goal in this project is the creation of a substantial library of interactive Java applets that can be used by teachers and parents anywhere to increase conceptual understanding of mathematics. The entire library will continue to be freely accessible from the web at www.matti.usu.edu. The library must be viewed at present as a work in progress; all the virtual manipulatives (applets) presently available at the MATTI site are functional prototypes, subject to continued refinement. (Software has been said to share an aspect of an oil painting in that it is "never done, just at a point where it can be left.") Many of the manipulatives, even if not in finished form, are also being made commercially available from several sources, and we anticipate additional sponsored versions in the future. Early in our project, we worked with the Electronic Format Group and the writers of the NCTM Principles and Standards of School Mathematics ("Standards 2000"). Many of the E-examples that appear in the web version of the Standards are early versions of our virtual manipulatives.

As we continue to create new applets and work with those on our site, we are actively working with teachers and students. Their explorations and experience make a significant contribution to our evaluation and refinement process.

We have been particularly pleased with the reception of our materials by teachers. We intended from the beginning that our applets should serve both elementary students and their teachers, both as instructional tools and to strengthen the mathematical preparation of pre-service and in-service teachers. It

has been a pleasure to work with several groups of in-service elementary teachers. In both web-based distance instruction (in Utah and in North Carolina) and in on-site classes (in Utah and North Carolina and Ohio), we have used our virtual manipulatives as part of our technology-supported mathematics courses.

A more broadly based thrust comes from the incorporation of a CD-ROM containing a number of our virtual manipulatives in the updated version of the Fifth Edition of *Mathematics for Elementary Teachers* by Musser, Burger, and Peterson (John Wiley & Sons, 2001). A revision for the forthcoming Sixth Edition (2002) will contain substantially more applets.

It is this work with teachers that supports our claim that virtual manipulatives provide a relatively non-threatening learning environment. By having teachers working individually, or in very small groups at a shared computer, we can encourage them to explore without exposing any perceived areas of ignorance. One of our primary goals in applet design is to maintain a tight focus. We don't want an applet that "does too much." Keeping a single goal in mind and giving the user control, we permit each individual to explore the consequences of various choices at a comfortable pace.

Another critical design objective is to help the user, student or teacher, to develop a clear understanding of a single mathematical idea. An animation or demonstration can perhaps encourage memorization of a rule or algorithm, but a rule seldom helps understanding. It is often not easy for beginners to see why a particular mathematical relationship holds or for them to make a connection to more basic principles. Anthony Ralston has articulated this distinction well (Ralston 2001):

> ...many Americans believe that elementary school mathematics consists of nothing more than a lot of rules to be learned ("the algorithms of arithmetic") ...there is much more to arithmetic that should be learned by children in elementary school: *not just the how but the why, not just individual procedures but the connections between procedures and the concepts underlying them,* and the *value of viewing problems from multiple perspectives*. (Italics added.)

Why Virtual Manipulatives?

In addition to the above-listed advantages, research into the value of physical manipulatives for student learning is increasing. They have numerous advantages that apply across the spectrum of student ability and interest, from those disadvantaged by economics or language or minority status, to especially gifted, curious students who can direct their interest into unanticipated explorations. Recent studies are supporting our contention that an electronic format, with student interaction with software on a computer screen, shares many of the same advantages, with some additional capabilities not available with physical manipulatives.

Our concern here, however, is teacher preparation. Although not yet quantified, the advantages of virtual manipulatives are real. It has been a delight to have experienced teachers exclaim over the possibilities they see for the use of our Library materials in their classrooms: "Oh, *that's* why that works!" or "This will help them understand *why* common denominators."

Math methods courses, and content courses for prospective elementary teachers, often involve physical manipulatives for demonstration or exploratory purposes, to familiarize teachers with important teaching tools. For virtual manipulatives, working with the computer has a respectable cachet. They are freely available at home, on the web and in the classroom. Furthermore, virtual manipulatives often have features that are impossible to duplicate with their physical counterparts. The computer allows real-time dynamic control. The user can change colors, save, and even print, the state of an applet after creating an interesting or illuminating example. Even when the computer activities are child-intended and designed, controlling their use and exploring consequences is somehow more acceptable than just handling physical objects. There is also the acknowledged value of "learning technology" even if it is really mathematical concepts that are being learned.

With the level of engagement necessitated by interaction with our virtual manipulatives, we get away from what Cuoco (2001) has called "watch-and-do pedagogy" that is so pervasive in college and university classes (and which young teachers often try to carry into their elementary classrooms): the teacher works out a problem, the students mimic the procedure on very similar problems, and then practice on a larger set of worksheet or homework problems. This teaching (learning?) style infects too many current mathematics textbook writers and has been called, in even less complimentary terms, "monkey-see, monkey do." Virtual manipulatives can be used by imaginative instructors to encourage fairly extensive individual explorations, in the best situations using student discoveries to stimulate class and small group discussions.

And because we recognize the importance of drill for learning, the number of variations on simple themes that we can build into our applets encourages repetition without excessive "sameness."

We are learning that virtual manipulatives also have versatility and applicability in the areas of assessment and distance learning. The potential for using our manipulatives in a distance education format is essentially the same for students five feet or five hundred miles away, as long each person is in front of a computer and has real-time access to our manipulatives on the web. We are developing an on-line testing and computer-grading procedure, where the person taking the test also has access to all of the tools that have been used for learning. Thus the computer activities can furnish versatile concept-based test questions. This is important future work, and we will explore these at some length in another paper. Here we more closely examine some specific examples.

Examples from Fractions

Manipulation of fractions is perhaps as frustrating as any single topic in elementary mathematics. The perplexity and apparent difficulty continues into college, interfering with the learning of calculus as the same mistakes are repeated by those who have ostensibly mastered the techniques years earlier.

Liping Ma (Ma, 1999) has raised questions of serious import for mathematics educators. One of her best-known examples deals with the question of dividing by fractions. She worked with a set of 23 American and 72 Chinese teachers, interviewing them, individually and in groups, on several topics in the teaching of elementary mathematics. The question posed was to compute the quotient of two fractions, $1\frac{3}{4} \div \frac{1}{2}$. The lack of general understanding of the concept of division was reflected by a question that came from the father of one of our elementary students, "How come when Brent divides by ½, he gets a bigger answer? Aren't things supposed to get smaller when you divide?"

Not only were there differences of opinion among Liping Ma's interviewees about how to teach the concept, only 9 of the 23 Americans teachers were able to compute the quotient correctly; all 72 of the Chinese teachers succeeded. And several of those who had an algorithm to accomplish the task (usually some variation of "invert the denominator and multiply"), to get the correct answer, didn't have an adequate explanation to give to students as to why their rule should work. In subsequent chapters, Ma talks of the need for a "Profound Understanding of Fundamental Mathematics," that embodies "an understanding of the terrain of fundamental mathematics that is broad, deep and thorough."

We are under no illusions that we have a magical "fix" for the deficiencies observed by Ma or that the proper use of virtual manipulatives will necessarily create a Profound Understanding of Fundamental Mathematics. What we do assert is that we have a collection of tools that can contribute to a better understanding of some fundamental principles. We focus on a couple of ideas to illustrate.

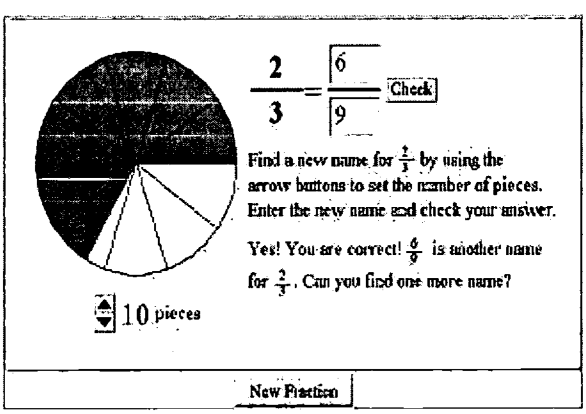

Figure 1 Equivalent Names for Fractions

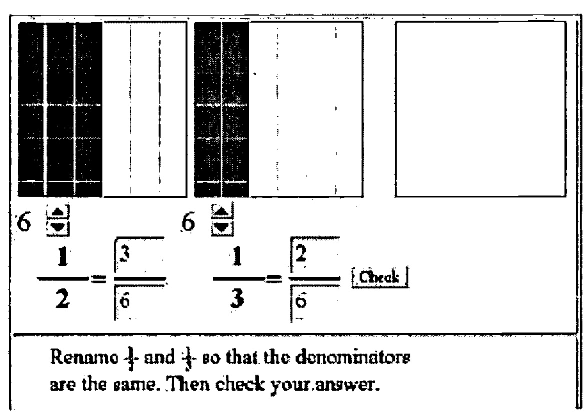

Figure 2 Common Denominators

Figure 1 is a snapshot of an applet in which the user is presented with a shaded fractional portion of a whole and is asked for an equivalent name. The up and down arrows change the number of divisions ("pieces"), and as is obvious in the figure, 10 divisions don't line up to divide 2/3 evenly. Any multiple of

3 clearly works, and the computer will accept any numerical answer of the form $2m/3m$, allowing the user to *observe* the validity of cancellation (or multiplying by 1 in the form of m/m). The user can provide as many equivalent names for each fraction as desired, with or without changing the number of pieces. It also becomes apparent that the only possible denominators are multiples of 3. Pressing the New Fraction button shows up a new fractional portion of one of several different shapes.

This is the first of several applets in which the user changes the grid or the number of pieces to show a simple idea. The next one we show examines the ideas necessary for the addition of fractions and why a common denominator makes sense of the process. The goal is to add the fractional parts shown. The first step is to rename the fractions with the same denominator, again using the up and down arrows to change the grid so that both copies of the whole are divided and the shaded part is also divided evenly, as shown in Figure 2.

When the Check button is pressed and the computer verifies that the answers are legitimate (without requiring a *least* common denominator), a new page appears. In Figure 3 we show the result of following the instructions. Initially, the shaded pieces are still located as in Figure 2, but they are now movable and can be dragged to the sum square, where they snap together as in Figure 3. Again, the answer is checked for numerical accuracy, not simply as strings. The pieces cannot be moved until the user has provided common grids that make sense of the idea of putting them together.

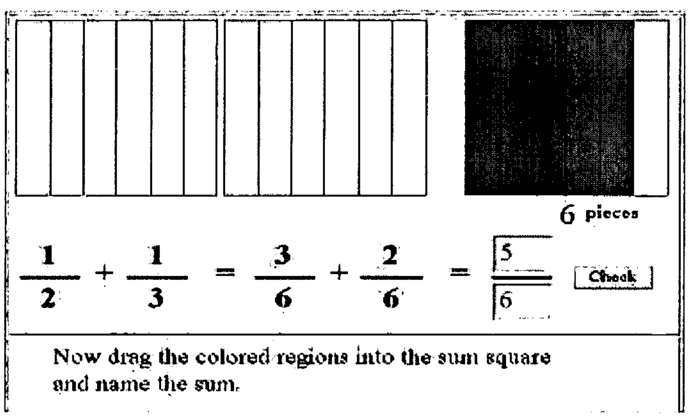

Figure 3 Fractions successfully added

What we hope to accomplish with this applet is the idea that while fractional pieces of a common whole can conceptually be put together to make a larger part of the whole, we cannot reasonably name the sum without proper names (common denominators) for both pieces. There are several observations that may be relevant. First, we only attempt to add fractions of the same whole. This is not explicitly pointed out to the user, but the initial seedings always show the same whole for each part and for the sum. We expect the teacher or parent to observe that kind of distinction, that we are adding numbers. While we use fractions to model parts of physical items, it makes no sense to apply a fraction name to what we get if we add half of an apple to a third of an orange. Secondly, the only sums we illustrate with this applet are less than a whole. We could have allowed other options, but each applet is intended to focus on a single idea. We consider it more important to work repeatedly with the simpler situation, which does, after all, clearly illustrate the utility of a common denominator.

We regret that space limitations do not allow us to more than sample an applet for the division of fractions. It is our hope that a description will encourage interested readers to visit our web site and explore this and many other virtual manipulatives in the NSF Library. We have applets to illustrate multiplication of whole numbers, and of fractions, by showing shaded rectangular regions where the user controls the size of the two factors. Division of whole numbers is similarly illustrated, where dividing 23 by 5 shows 23 squares on a grid, with 4 copies of the set of 5, and 3 left over. Again, the user can change the divisor at will, each time seeing an answer to the question, "How many copies of 5 (or whatever) are there in 23?"

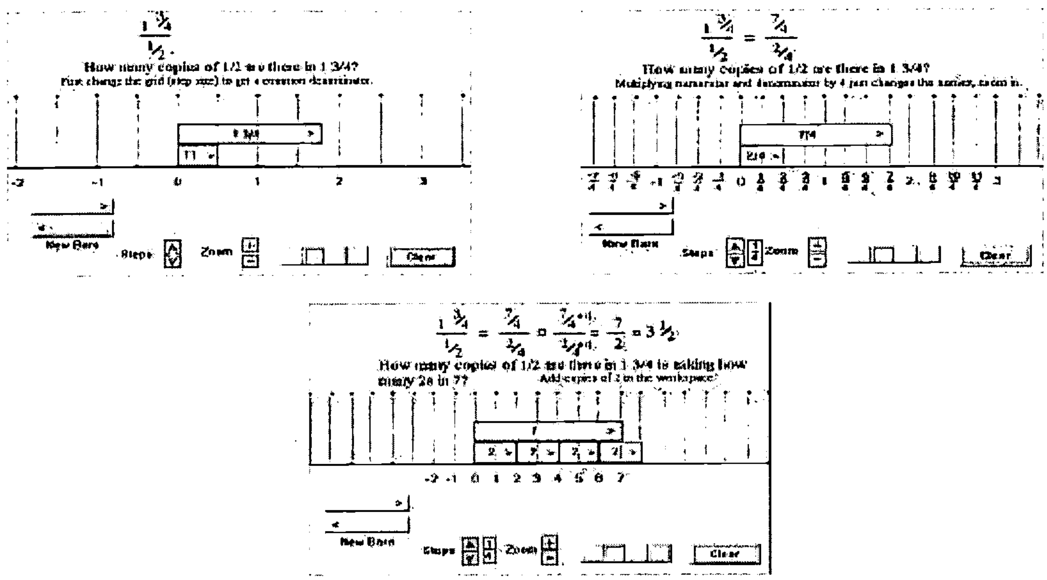

Figure 4 Division of Fractions

With this kind of experience at hand, Liping Ma's question of dividing 1¾ by ½ becomes, "How many copies of ½ are there in 1¾?" We have a new fraction division applet that allows the user to produce an arrow (rectangular region) of length 1¾ and another, immediately below the first, of length ½. When the user changes the grid size (much as is done in the figures above) to represent a common denominator, the labels on the rectangular regions become, respectively, 7/4 and 2/4, and the user sees that the question is equivalent to "How many copies of 2 are there in 7?" To complete the division, the user lines up three more ½-length (yellow) arrows below the longer one. The labels change, but the number of copies of ½ in 1¾ is shown visually as 3, plus ½. While these activities are underway, they are accompanied by the following sequence of fractions, providing a robust symbolic algorithm that can be used for any division of fractions.

$$\frac{1\frac{3}{4}}{\frac{1}{2}} = \frac{\frac{7}{4}}{\frac{2}{4}} = \frac{\frac{7}{4}\square 4}{\frac{2}{4}\square 4} = \frac{7}{2} = 3\frac{1}{2}$$

The power of this manipulative, as with the others in the library, is that the user is engaged, in control, and can repeat the experience with as many fractions as desired, reinforcing both the concept and the symbolic algorithm.

Literature Cited

Cuoco, Al (2001). *Mathematics for Teaching*, Notices of AMS, 18, #2. 168-174.

Liping Ma (1999). *Knowing and Teaching Elementary Mathematics*, 1999, Lawrence Erlbaum Associates, Mahwah NJ.

Ralston, Anthony (2001). *Review*, American Mathematical Monthly, 108, Dec. 2001. 998-992.

Enhancing Statistical Content in Pre-Service Elementary School Teachers' Web Constructions

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Abstract: Research on future teachers' mathematical Web site constructions shows there is a tendency for these constructions to emphasize design aspects rather than the mathematics content of the sites. In this study, students received more detailed, knowledge-based guidelines for completing Web construction projects. A cognitive analysis of the statistics content in 38 student Web sites constructed before, and 43 Web sites constructed after the more-detailed guidelines were used, showed that the later Web sites contained more items at a procedural applications level of cognition. There was also evidence of small increases in the later Web site items at a conceptual applications cognitive level, but little change was evident in the number of items at the highest of the cognitive levels, problem-solving applications. The findings suggest that further improvements in the cognitive levels of the students' examples and exercises on their Web pages are feasible.

Introduction

Ivers and Barron (1999) reported that there were nearly 10,000 US schools connected to the Internet in 1998, and half of these were elementary or primary schools. The authors further predicted that virtually all US public schools would be connected to the Web by 2001. With this proliferation of Web access there is a growing trend for schools to use their presence on the Web for interactive communication and for more teacher Web pages. Moreover, access to Web site creation programs makes it increasingly possible for non-programmers to create effective Web pages. However, these rapid advances in Web technology may serve to indicate that the instructional purposes of elementary school Web pages have yet to be well defined. As in the introduction of computers into the classroom, it may take some time for elementary school teachers to utilize the full potential of the Web.

The Study

A prior study of future elementary school teachers' mathematical Web site constructions (Carter & Ferrucci 2000) showed that there were tendencies for these students to focus more on the design aspects of Web sites rather than on the statistical content. The present study was consequently undertaken to encourage prospective teachers to emphasize the statistical content of their constructions. This encouragement involved the use of more detailed knowledge-based guidelines in the instructions to the students for completing a Web site construction project. After students completed their projects, the statistical content of the completed Web sites was analyzed to determine the cognitive levels of the examples and exercises that the future teachers incorporated into their sites. Particularly, the examples and exercises that the future teachers used in reviewing the statistical concepts of the mean, median, and interquartile range were analyzed with respect to their cognitive levels. To facilitate the analysis, a

cognitive taxonomy was used to classify the examples and exercises from the Web sites. The categories in the taxonomy were essentially the comprehension and applications levels of the well-known Taxonomy of Educational Objectives (Bloom, Englehart, Hill, Furst, and Krathwohl, 1956). The comprehension level of the taxonomy was adapted directly from the Bloom taxonomy, but the applications level in the Bloom taxonomy was refined into three levels to elaborate the complexity of examples and exercises that students used in their Web pages. Hereafter, the designations Mean1, Median1, and IQR1 indicate the comprehension level (Level 1) of cognition about the mean, median, and interquartile range, respectively. Similar notations designate procedural, conceptual, and problem-solving applications cognitive levels (Levels 2, 3, and 4) for each of the three statistics for which students prepared Web pages.

The revised instructions for completing the construction project stipulated that students should include examples and exercises that involved problem solving and higher-level cognitive processes, and samples of these types of examples and exercises were included in the guidelines. Students received a handout of the guidelines in class, and a copy of the guidelines was also included both on the students' and on the instructor's Web sites. To complete the Web site project, students replaced given, blank Web pages for the mean, median, and interquartile range with Web pages intended to help their own future students in reviewing these statistics. Web pages about the mode and the range were included in the students' Web sites as further examples. Students' Web sites also contained a homepage that was customized, as part of in-class instruction, to show the teacher's name, school and grade level. Also in the sites was a Web page that showed assignments and other information about the mathematics program in the future classroom, as well as another page that listed Web resources that the future teachers might use in completing their Web site constructions.

To prepare the prospective teachers to complete the Web site projects, separate class sessions dealt with modifying and editing text on Web pages, inserting backgrounds and images into Web pages, and inserting links onto Web pages. In each of the class sessions examples were used that illustrated images, objects, or text that students could use in preparing their own Web sites. For instance, students practiced inserting images of box-and-whisker plots and links to electronic examples (e-examples) within their own Web pages. In all, the instructional component for the Web site project consisted of four 70-minute class periods in a lab where each student worked individually at a computer. Students were also informed that they should expect to spend 10-15 hours to complete the project.

In all, 38 students from two classes completed the Web site project using an original, unmodified set of guidelines (Carter & Ferrucci, 2000), and the students in these classes are referred to as the "earlier classes" in this report. The cognitive levels of the examples and exercises in the earlier classes' Web sites were compared with the examples and exercises in the Web sites created by two later classes (with a total of 43 students) who completed the Web site project after receiving the revised instructions. The students in these later classes are referred to as the "later classes", hereafter. Aside from the different guidelines, efforts were made to ensure that other instructional materials and activities about the Web site projects were the same for the four classes.

To analyze the examples and exercises in the Web sites from the earlier and later classes, a panel of two instructors and two graduate students rated the examples and exercises using the taxonomy developed for this study. The panel reviewed the Web sites and categorized the exercises and examples based on the taxonomy, and there were no unresolved differences with respect to the cognitive levels of the examples and exercises.

Figure 1 shows the number of items (exercises and examples) on the mean, median, and interquartile range in the Web pages prepared by the earlier and later classes. Although the number of items on each of the statistics is greater for the later classes, the number of items on the interquartile range is approximately half the number of items on the mean and the median.

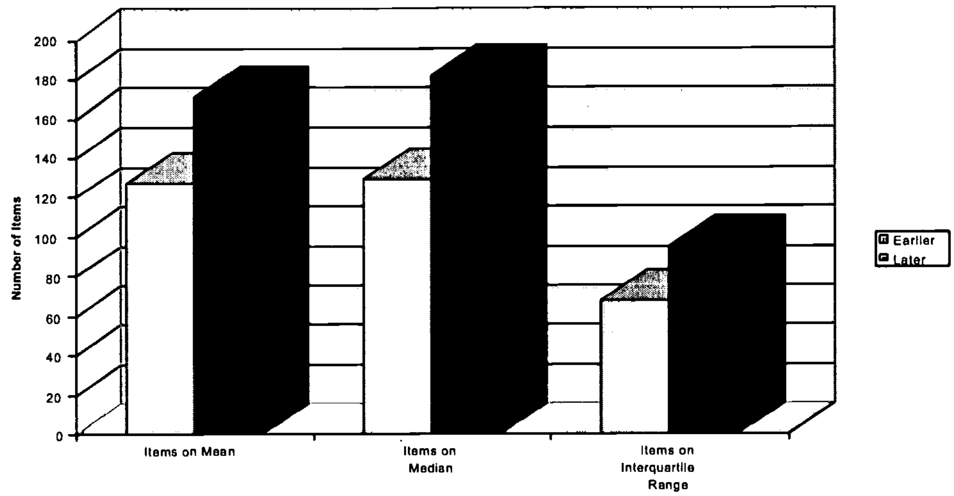

Figure 1: Number of Items on the Mean, Median, and Interquartile Range in the Web Pages Prepared by Earlier and Later Classes

Figure 2 shows the earlier and later classes' distributions of Web page items on the mean by cognitive levels. Mean2, Mean3, and Mean4 in figure 2 refer to the levels of cognition concerned with procedural, conceptual, and problem-solving applications. At each of these levels of cognition, the average number of items increased from the earlier to the later classes. Notably, the increases at the levels of the conceptual and problem-solving applications were small (about .25 of an item) compared to the increase at the procedural applications level (about 1 item). Figure 2 also shows that the average number of items at the comprehension level of cognition (Level 1) was essentially zero for both the earlier and later classes. Moreover, there were comparable distributions for the earlier and later classes' Web page items on the median and the interquartile range.

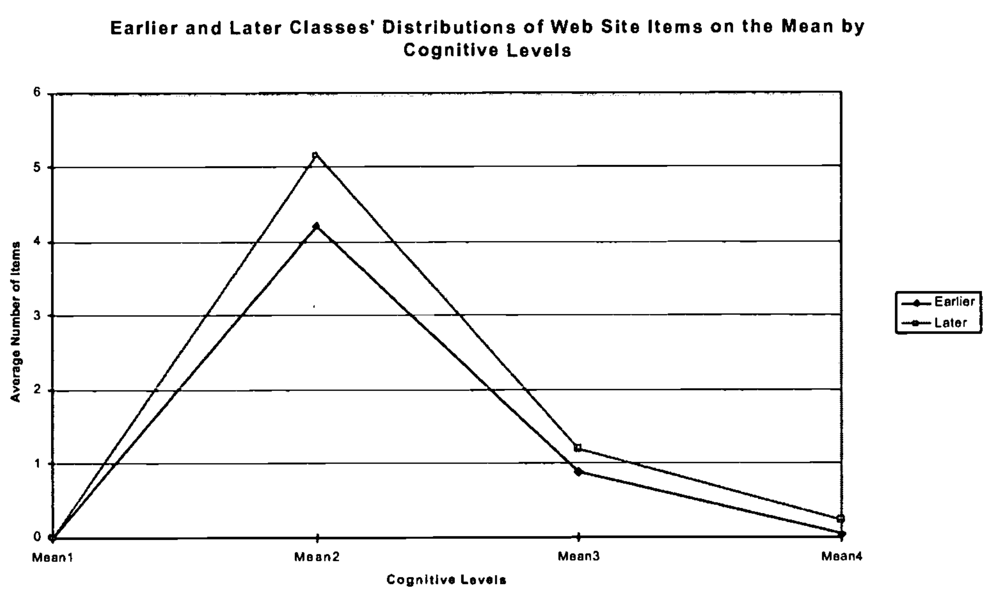

Figure 2: Earlier and Later Classes' Distributions of Web Page Items on the Mean by Cognitive Levels

Figure 3 shows earlier and later classes' average number of Web page items by the entire range of cognitive levels for these items. Particularly, at the procedural applications level, the average number of items increased from about 4 to 5 items for each of the mean and median, while the average number of items increased from about 2.5 to 3.25 items for the interquartile range. At the conceptual applications cognitive level, the average number of mean and median items increased from less than 1 to more than 1 item, while the average number of interquartile range items increased from about 0 to less than .25. For each of the three statistics, the number of items at the problem-solving applications level (Level 4) was virtually negligible for both the earlier and later classes. A possible exception to the later finding may have occurred for items on the mean written by the later classes. The average number of these items at the problem-solving applications level was almost 1/4 of an item.

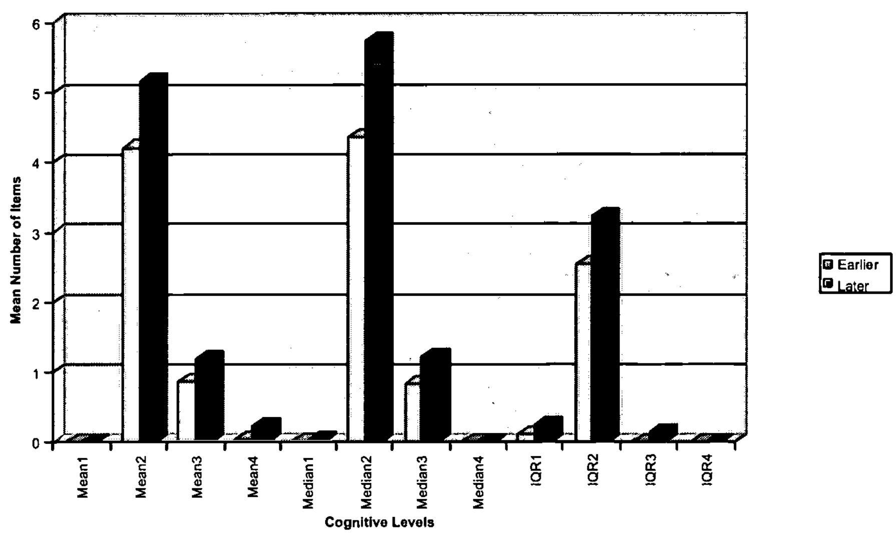

Figure 3: Earlier and Later Classes' Mean Number of Web Page Items by the Cognitive Levels

Conclusions

The results of the study give evidence that the number of items on the mean, median, and interquartile range increased from the earlier to the later classes. This indicates that the revised guidelines for the students for completing the Web site project may have increased the number of examples and exercises prepared by students in completing their projects. The results also showed that the number of procedural, conceptual, and problem-solving applications involving the mean increased between the earlier and later classes. However, the increase was less pronounced in the case of the conceptual and problem-solving applications. In both the earlier and later classes' Web site constructions, there was no evidence of items on the mean written at the comprehension level of cognition. This level of cognition may be applicable to the mean in only the most trivial instances when merely an observation of data or a graph can identify the mean. With respect to Web page items on the median, there was some evidence that the number of items at the procedural and conceptual applications levels increased from the earlier to the later classes. At the comprehension and problem-solving applications levels of cognition, there was no evident increase between the earlier and later classes. For the earlier and later classes' distributions of Web page items on the interquartile range, there was also some evidence that the average number of these items increased from the earlier to the later classes. Particularly, this increase was more evident at the procedural applications level.

An examination of figure 3 shows that there was little or no difference between earlier and later classes on the average number of Web page items at the conceptual and problem-solving applications level of cognition for all three of the statistics. This indicates that the revised guidelines for the Web site project probably resulted in, at most, an increase in the number of items at the procedural applications level of cognition. Correspondingly, the revised guidelines did not appear to affect much change in higher-level thinking. That is, items at the higher cognitive levels (conceptual and problem-solving applications) increased little compared to items at the procedural applications level. Future work in this area may find it feasible to increase the number of higher-level items that students include in their Web site constructions.

Figure 3 indicated that the average number of items at the procedural applications level increased by about 1 item for each of the mean, median, and interquartile range. Particularly, this increase was from an average of 4 to about 5 items in the case of the mean and median, and from 2.5 to 3.25 items in the case of the interquartile range. Consequently, future work on pre-service teachers' Web constructions about statistics may endeavor to increase the number of items at the procedural applications level for the interquartile range. Figure 3 also indicated that the average number of items at the conceptual applications level increased from about .8 to about 1.2 items for the mean and median. The comparable figures for the interquartile range showed that the average number of items increased from 0 to less than .2 of an item. As a result, Web site construction projects in the future might also be aimed toward increasing the average number of interquartile range items to at least match those for the mean and median. Finally, Figure 3 further showed that the major increases between the earlier and later classes occurred mainly at the lower as opposed to the higher levels of cognition for all of the statistics. Notably, the average number of items at the problem-solving applications level of cognition for the mean, median, and interquartile range (Mean4, Median4, IQR4) were almost zero. This result further indicates that work remains to be done to increase the number of higher cognitive level items included by prospective teachers in their Web pages.

A major motivation for this study related to prospective teachers' tendencies to focus more on appearances than on content in Web site constructions. By revising the guidelines for completing the project, there was evidently some increase in the number of items at the procedural applications level of cognition for each of the statistics in the students' Web pages. Consequently, the revised guidelines apparently provided some impetus for students to move from learning about a technological tool to statistical learning with that tool. Put another way, the findings of this study produced some evidence that the powerful technology of Web sites did less to distract the later classes from the underlying statistics than was the case with the earlier classes. Clearly, ensuring that the content is not compromised is an important consideration whenever technology is used in mathematics education.

The Web certainly offers unlimited possibilities for improving statistical understanding, and there are increasing indications that future teachers will need to use sophisticated Web-based systems to post information and to close communication gaps between teachers, parents, and students. Aiding teachers to become more proficient in their Web skills should lead to the creation of more effective and dynamic displays of mathematical information, and projects such as the Web site construction project outlined in this study are apt to facilitate these proficiencies.

References

Bloom, B. S., Englehart, M.D., Hill, W. K., Furst, E. J., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives: The classification of educational goals*. New York: McKay.

Carter, J. & Ferrucci, B. (2000). Mathematical Web Site Constructions for Prospective Elementary School Teachers. In D.M. Watson and T. Downes (Eds.), *Communications and Networking in Education, Learning in a Networked Society* (pp. 255-264). Boston: Kluwer Academic Publishers.

Charischak, I. (November 2000). A Look at Technology's Role in Professional Development of Mathematics Teachers at the Middle School Level. *School Science & Mathematics*, 100(7), 349-354.

Ivers, K.S. & Barron, A.E. (1999). The Presence and Purpose of Elementary School Web Pages. *Information Technology in Childhood Education Annual, 1999.* Association for the Advancement of Computing in Education: Charlottesville, VA. 181-191.

Graphing Calculators and Algebra I, Algebra II, IPC, and Chemistry Teachers' Perceptions of Change

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Abstract: The purpose of this study was to determine Algebra I, Algebra II, IPC, and Chemistry teachers' initial perceptions of change prior to their participation in a year-long professional development program that emphasized integration of the math and science utilizing graphing calculators. The results indicate that as a group, the teachers (1) exhibited high information stage concerns, high personal stage concerns, and collaboration stage concerns; and (2) are more aware of the graphing calculator and its potential than previous groups. Algebra I and Algebra II teachers reported higher technological proficiency, but their stage concerns were not statistically different from the IPC and Chemistry teachers.

Mathematics and science educators include the use of technology as a common goal in their most recently developed standards. The National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (NCTM, 2000) suggests a framework for the types of technology-based activities and content that should be taught. Similarly, the National Research Council's *National Science Education Standards* include suggestions for science education reform in technology-based content and professional development (NRC, 1996). Further, both documents point toward significant increase in the integration of math and science. Reading between-the-lines, technology is encouraged as a tool that can facilitate such integration.

Graphing calculators show promise for integrating mathematics and science. There is a growing body of recent research into the use of graphing calculators in the teaching of algebra (Beckman, Senk, & Thompson, 1999; Dunham & Dick, 1994; Embse & Yoder, 1998; Milou, 1999;); in both chemistry and physics (Adie, 1998; Roser & McCluskey, 1999; Taylor, 1995); and even the integration mathematics and science (Tharp et. al.).

The State of Texas has developed the Texas Knowledge and Skills (TEKS) (http://www.tea.state.tx.us/teks/). The TEKS clearly extend the national reform documents by specifically indicating the use of graphing calculators in algebra:

> Students use a variety of representations (concrete, numerical, algorithmic, graphical), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities and model mathematical situations to solve meaningful problems (§111.32. Algebra I (One Credit) #5).

Beyond the explicit statement that graphing calculators be used in the teaching and learning of mathematics, the new Texas Assessment of Knowledge and Skills (TAKS) (http://www.tea.state.tx.us/student.assessment/taks/index.html) **requires** the use of graphing calculators on the state-mandated tests at the ninth, tenth, and eleventh grades for math and at the eleventh grade for science.

Theoretical Framework

The implementation of technology will require change in the classroom. One model that has been utilized to inform the decision-making process when innovations are introduced is the Concerns-Based Adoption Model (CBAM). CBAM states that successful implementation of an innovation is a process not an event (Hall & Hord, 1987; Fullan, 1991; Friel & Gann, 1993), developmental in nature (Hall & Hord, 1987), and a highly personal experience for each teacher (Hall & Hord, 1987). Hall, George & Rutherford (1986) define concerns as the feelings, thoughts, and reactions that individuals have about an innovation or a new program that touches their lives. To measure these concerns, Hall, Wallace & Dossett (1973) developed the Stages of Concern Questionnaire (SoCQ). Initial research on the instrument construction verified the existence of seven stages in the concerns process: awareness, informational, personal, management, collaboration, and refocusing, with internal reliability for individual scales ranging from $r=0.64$ to $r=0.83$ (Hall, George & Rutherford, 1986).

Participants

The participants in this study are high school math and science teachers from a single large urban school district in Texas participating in a year-long professional development program that is ultimately aimed at improving math and science achievement. Within the professional development program, the teachers are divided into two sub-groups. The two subgroups are Algebra I (ALG I) with Integrated Physics and Chemistry (IPC) and Algebra II (ALG II) with Chemistry I (CHEM). The teachers are paired because the vast majority of students who are enrolled in ALG I will also be enrolled in IPC and because there is significant overlap in the knowledge and skills that are taught in each course. There is less overlap with students for the ALG II and CHEM group, but curricular overlap is strong enough to warrant the pairing. The teachers are recruited through their building principal and science department chairs and must participate as pairs, one from math and one from science.

The focus of the professional development is on increased communication and collaboration between math and science teachers within the district and specifically in individual schools, with a heavy emphasis on technology. Graphing calculators (TI-83s), Calculator-Based Laboratories (CBLs), and Calculator-Based Rangers (CBRs) with multiple probes were provided to all teachers at the beginning of the program to be used in all subsequent workshops. Each teacher will ultimately receive a minimum of 125 hours of professional development that culminates in a two-week summer institute. Upon the completion of the 125 hours, each teacher will receive a set of ten calculators, CBLs, and CBRs, an overhead panel for display purposes, and up to four additional probes for use in their classrooms.

Methods

Research questions

1. Are there significant differences between the holistic stage concerns profiles for ALG I, ALG II, IPC and CHEM teachers?
2. Are there significant differences between the mean stage score profiles for ALG I, ALG II, IPC and CHEM teachers?
3. Do the demographic profiles differ for ALG I, ALG II, IPC and CHEM teachers?

Data was collected from a total of forty-three secondary mathematics and science teachers during their respective introductory sessions in September 2001. The largest subgroup was the ALG I with IPC and consisted of twenty-five participants. Twelve participants taught ALG I and eleven participants taught IPC, with two participants teaching both courses. Eighteen participants were enrolled in the second subgroup, ALG II and CHEM. Ten participants taught Algebra I. Eight participants taught chemistry.

All participants were administered the Stages of Concern Questionnaire (SoCQ) on the first day of the in-service. The SoCQ is a thirty-five item Likert-scale instrument that contains seven levels of responses. The responses range from 0 = irrelevant to me, 1 = not true to me now, to 7 = very true to me now. A demographic survey also was administered at this time. The survey collected two types of information: background and technology-using history. Background information collected included gender, years teaching, highest degree earned and age. Technology-using information collected included self-rating of the ability to integrate graphing calculators and computers in the classroom, in-service training received, and number of years integrating a graphing calculator and computer in the classroom.

Mean stage scores and total concerns score were calculated for ALG I, ALG II, IPC and CHEM teachers. To determine overall concerns levels, two analyses were performed. First, mean stage scores were converted to percentile ranks based on the norms presented by Hall, George & Rutherford (1986). Second, a peak stage score analysis was calculated for each group. Peak stage scores are defined as the stage at which an individual has his or her highest percentile rank score on the seven concern stages (Hall, George & Rutherford, 1986). Finally, analyses of variance (ANOVAs) were performed on mean stage scores and total concerns score to determine subgroup differences. Since there are seven stages of concern, the significant p-level for mean stage score ANOVAs was $p=0.007$ ($p=0.05/7$). Total score ANOVAs used a significant p-level of $p=0.05$.

Results

ALG I teachers had the highest percentile concerns at the information stage and their lowest percentile concerns at the refocusing stage (Awareness=77, Information=84, Personal=70, Management=43, Consequence=43, Collaboration=68, and Refocusing=38). IPC teachers had the highest percentile concerns at the information stage and their lowest percentile concerns at the management stage (Awareness=81, Information=90, Personal=76, Management=47, Consequence=54, Collaboration=84, and Refocusing=52). ALG II teachers had the highest percentile concerns at the awareness stage and their lowest percentile concerns at the consequence stage (Awareness=84, Information=75, Personal=83, Management=56, Consequence=48, Collaboration=68, and Refocusing=52). CHEM teachers had the highest percentile concerns at the information stage and their lowest percentile concerns at the consequence stage (Awareness=86, Information=93, Personal=83, Management=65, Consequence=59, Collaboration=76, and Refocusing=69). Overall, the percentile scores demonstrate that all groups were very aware of graphing calculators and their uses and wanted to learn more about how this technology impacts their classroom. The profile also demonstrates that the participants were users of the technology in the classroom but still needing information on how to best integrate the technology to impact student achievement.

No significant differences ($p<0.007$) were found between ALG I, ALG II, IPC and CHEM teachers group means. Thus, while percentile scores varied slightly as to highest and lowest percentile concerns, all participants entered the in-service with similar expectations.

Demographic analysis found that (1) teachers in each subgroup were predominantly female (>60% for each subgroup), (2) almost all CHEM teachers had a masters degree or higher while the other groups had more teachers with only a bachelor's degree, (3) years of teaching experience was related to course taught (CHEM vs. IPC; ALG I vs. ALG II), (4) few ALG II teachers had used computers in instruction while almost half of the ALG I teachers had integrated computers, however, most science teachers regardless of level had integrated computers in their instruction, (5) all ALG II teachers and almost all ALG I teachers had integrated graphing calculators in the classroom while few CHEM and IPC teachers had used graphing calculators in instruction, and (6) CHEM and IPC teachers self-rated themselves as novice users of graphing calculator and intermediate (almost experts) at using computers in the classroom while ALG II and ALG I teachers considered themselves novices at integrating computers in the classroom and intermediate (almost experts) at using graphing calculators in the classroom.

Summary

All groups entered this in-service wanting to learn more about the how to successfully integrate technology in the classroom (high information stage concerns). These needs focused primarily in the areas of how will this curricular change impact me and my teaching (high personal stage concerns) and how can I work with others to help bring about this curricular transition (collaboration stage). This early awareness of graphing calculators and interest in the integration process is atypical for those entering previous math, science, and technology professional development programs (Chamblee, 1996; Chamblee & Slough (in press A); Slough 1998;). This warrants further investigation.

Demographic data present distinctly different groups. Group differences can be classified according to discipline more than subject taught. This fact agrees with previous research in this area (Chamblee & Slough (in press A)). Of particular note, both groups of math teachers, ALG I and ALG II, rated their technical proficiency high with regard to the graphing calculator and the science teachers, IPC and CHEM, rated themselves as novices. Yet, there was no statistically significant difference between the groups mean stage concerns. At first, this may appear to be problematic with regard to either the self-reporting or the CBAM model. When, in fact, mere technological proficiency does not make one immune to the same concerns. This warrants further investigation.

Similar research questions were utilized to analyze the differences between all math and all science teachers who participated in the professional development program. Due to the limited space, the results are discussed in a separate paper (Chamblee and Slough (in press B)).

References

Beckman. C. E., Senk, S. L., & Thompson, D. R. (1999). Assessing Students' Understanding of Functions in a Graphing Calculator Environment. School Science and Mathematics, 99(8), 451-456.

Chamblee, G. E. (1996). Using graphing calculators to teach first-year algebra: North Carolina mathematics teachers' concerns. (The University of North Carolina at Chapel Hill, 1995). Dissertation Abstracts International, 56, 3869A.

Chamblee, G. E. & Slough, S. W. (in press A). Implementing Technology in Secondary Science and Mathematics Classrooms: A Perspective on Change. *Journal of Computers in Mathematics and Science Teaching.*

Chamblee, G. E. & Slough, S. W. (in press B). Science and Mathematics Teachers Perceptions' of Graphing Calculators and Change. *Technology and Teacher Education Annual, 2002.* Association for the Advancement of Computing in Education, Charlottesville, VA.

Dunham, P. H., & Dick, T. P. (1994). Research on graphing calculators. Mathematics Teacher, 87(6), 440-445.

Embse, C. E. & Yoder, V. W. (1998). Multiple Representations and Connections Using Technology. Mathematics Teacher, 91(1) 62-67.

Erb, T.O. & Doda, N.M. (1989). Team organization: Promise – practices and possibilities. Washington, D.C.: NEA.

Friel, S. N. & Gann, J. H. (1993). Making change in schools. Arithmetic Teacher, 40, 286-289.

Fullan, M.G. (1982). The meaning of educational change. New York: Teachers College Press.

Hall, G., & Hord, S. (1987). Change in schools: Facilitating the process. Albany NY: SUNY Press.

Hall, G. E., George, A. A., & Rutherford, W. A. (1986). Measuring stages of concern about the innovation: A manual for the use of the SoCQ questionnaire. Austin: Research and Development Center for Teacher Education, The University of Texas at Austin.

Hall, G. E., Wallace, R. D., Jr. & Dossett, W. A. (1973). A developmental conceptualization of the adoption process within educational institutions (Report No. 3006). Austin: Research and Development Center for Teacher Education (ERIC Document Reproduction Service No. ED 095 126).

International Technology Education Association (ITEA) (2000). Standards for Technological Literacy: Content for the Study of Technology. Reston, Virginia: Author. [also available on-line at http://www.iteawww.org//TAA/TAA.html]

National Council of Teachers of Mathematics (2000). Principles and Standards for School Mathematics. Reston, VA: Author.

National Research Council (NRC). (1996). National Science Education Standards. Washington DC: National Academy Press.

Mauer, R. E. (1994). Designing interdisciplinary curriculum in middle, junior high, and high schools. Needham Heights, MA: Allyn & Bacon.

Milou, E. (1999). Thc Graphing Calculator: A survey of classroom usage. School Science and Mathematics, 99(3), 133-139.

Slough, S. W. (1998). High school science teachers' perceptions of telecommunications utilizing a Concerns-Based Adoption Model (CBAM). Unpublished doctoral dissertation, University of Houston, Houston, Texas, USA.

Tharp, M. A., Fitzsimmons, J. A., & Ayers, R. L. B. (1997). Negotiating a Technological Shift: Teacher Perception of the Implementation of Graphing Calculators.

Actions on Objects: Useful Internet locations.

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Abstract: The modern classroom computer has an unparalleled ability to implement both graphical and procedural components of mathematics understanding in a single unified object. This dual encapsulation allows students to see both the form of representation and their actions upon the representation simultaneously. An extension of activity theory, Action on Objects (Connell, 2001), has been developed incorporating this ability which has proven of great worth in instructional planning and effective utilization of technology to enhance mathematics learning.

During the summer of 2001 the author lead a graduate course where we carried out an extensive review of existing websites focusing upon those best fitting this action upon objects model. This paper will provide a brief overview of the model and the rubrics used in the course of this website evaluation together with annotated links to the resources themselves.

Background

A major conceptual framework used in my research and writings has been that of *action upon objects* (Connell, 2001). This has in turn lead to some foundational questions surrounding the nature of the technology-enhanced objects and the types of actions that one might be expected to perform upon them. I have created a conceptual framework of actions upon objects that has proven to be very powerful in both experimental settings as well as in school classrooms. I feel that it also captures recent thinking on object reification in mathematics (Sfard, 1994).

It is clear, however, that if this approach is to be effective that the objects to think with must be developmentally appropriate for the student. The following model captures my current "best attempt" to put this into an easily presentable format.

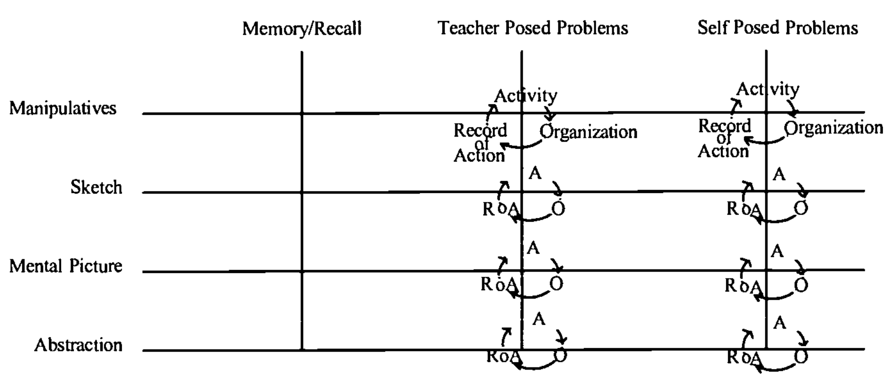

Figure 1. Model illustrating the type of *actions* as performed upon the *objects of thought*.

Evaluation Rubrics

During the summer of 2001 the author lead a graduate course within which we carried out an extensive review of existing websites focusing upon those best fitting this action upon objects model. The following illustrations serve to outline the evaluation template used. The final paper will develop the rationale for these in more detail and provide a copy of the template for inclusion on the SITE CD. It will also include some selected sites and their annotations so that a preview of the database might be achieved.

Summary of Detailed Evaluation		
	Maximum Score	Evaluated Score
Accuracy:	20.0	0.0
Appropriateness:	10.0	0.0
Scope:	7.0	0.0
Content Subtotal	37.0	0.0
Navigation:	21.0	0.0
Presentation:	25.0	0.0
Quality:	7.0	0.0
Technical Subtotal	53.0	0.0
Documentation	10.0	0.0
Total Score	100.0	0.0

Figure 2. Summary of Evaluation

Section I - Content					
I.a	Accuracy:				
I.a1	Error-free information	2	0 or 1		0.0
I.a2	Current information	1	0 or 1		0.0
I.a3	Objective, balanced presentation of information	1	0 or 1		0.0
I.a4	Bias-free viewpoints and images	1	0 or 1		0.0
I.a5	Cultural, ethnic, gender and racial groups presented without bias	1	0 or 1		0.0
I.a6	Correct use of grammar, spelling, and sentence structure	1	0 or 1		0.0
I.a7	Updated within last 12 months	0.5	0 or 1		0.0
I.a8	Recent "Last" Update	1	0 or 1		0.0
I.a9	Authority of author/designer	1	0 or 1		0.0
I.a10	Contact information for author/designer	1	0 or 1		0.0
I.a11	Expertise/reputation of host site	0.5	0 or 1		0.0
I.a12	Information relevant to Local Standards and/or NCTM Standards	3	0 or 1		0.0
I.a13	Contains useful information that cannot be found more readily available elsewhere	2	0 or 1		0.0
I.a14	Time table to progress from one level of complexity to another	1	0 or 1		0.0
I.a15	Clearly identified objectives are stated at the beginning	1	0 or 1		0.0
I.a16	Identified Objectives Met	1	0 or 1		0.0
				Subtotal	0.0
I.b	Appropriateness:				
I.b1	Concepts and vocabulary relevant to students' abilities	1	0 or 1		0.0
I.b2	Interaction compatible with the physical and intellectual maturity of intended audience	2	0 thru 2		0.0
I.b3	Cost Effective	1	0 or 1		0.0
I.b4	Use as student activity or teacher illustration material	1	0 or 1		0.0
I.b5	Is reading level appropriate for the grade of target audience	1	0 or 1		0.0
I.b6	Subject matter maintains student interest	2	0 thru 2		0.0
I.b7	Provides student a sense of accomplishment when a certain level of expertise is [illegible]	2	0 or 1		0.0
				Subtotal	0.0
I.c	Scope:				
I.c1	Information of sufficient scope to adequately cover the topic for the intended audience	1	0 or 1		0.0
I.c2	Logical progression of topics	2	0 or 1		0.0
I.c3	Variety of activities, with options for increasing complexity	2	0 or 1		0.0
I.c4	Adequately addresses the background of the student	1	0 or 1		0.0
I.c5	Allows fast progression more rapidly if expertise is exhibited	1	0 or 1		0.0
				Subtotal	0.0

Figure 3. Detail of data collection 1.

Section II - Technical Aspects					
II.a	Navigation:				
II.a1	Rapid retrieval of information and screen transitions	2	0 or 1		0.0
II.a2	Intuitive icons, menus, and directional symbols that foster independent use	2	0 or 1		0.0
II.a3	Controllable pace, including options for stop/pause/exit	1	0 or 1		0.0
II.a4	Controllable sound and narration that can be turned off separately	1	0 or 1		0.0
II.a5	Easy access to cross references, additional information	0.5	0 or 1		0.0
II.a6	Two levels of searching	1	0 or 1		0.0
II.a7	Bookmark and highlighting features	1	0 or 1		0.0
II.a8	Ready access to site, site not overloaded	0.5	0 or 1		0.0
II.a9	Inlinks that allow easy navigation throughout the site	1	0 or 1		0.0
II.a10	Standard multimedia formats	3	0 or 1		0.0
II.a11	Logical options for printing/downloading all or selected text and graphics	2	0 or 1		0.0
II.a12	No "bugs"	2	0 or 1		0.0
II.a13	Objects can be manipulated easily/user friendly	3	0 thru 2		0.0
II.a14	Has links to other useful sites that may fit the same criteria	1	0 or 1		0.0
				Subtotal:	0.0
II.b	Presentation:				
II.b1	Multimedia functionality throughout site	3	0 thru 2		0.0
II.b2	Information presented in a manner to stimulate imagination and curiosity	3	0 thru 2		0.0
II.b3	Activities that provide opportunities for creative problem solving	3	0 thru 2		0.0
II.b4	Use of appropriate and supportive feedback	2	0 thru 2		0.0
II.b5	Options for help, tutorial segments	1	0 thru 2		0.0
II.b6	Uncluttered screen displays	2	0 thru 2		0.0
II.b7	Concise screen displays	1	0 thru 2		0.0
II.b8	Captions, labels, or legends for visuals	1	0 thru 2		0.0
II.b9	Legible text and print size that is appropriate for the intended audience	1	0 or 1		0.0
II.b10	Site follows good graphic design principles	1	0 thru 2		0.0
II.b11	Graphics and art functional, not merely decorative	1	0 thru 2		0.0
II.b12	Product advertising not intrusive	1	0 thru 2		0.0
II.b13	Interactivity promotes student motivation	3	0 thru 2		0.0
II.b14	Esthetically pleasing	1	0 thru 2		0.0
II.b15	Focus is on the concept, not bells and whistles	1	0 thru 2		0.0
				Subtotal	0.0
II.c	Quality:				
II.c1	High quality visuals relevant to the content	2	0 thru 2		0.0
II.c2	High quality audio and video consistent with state-of-the-art reproduction capabilities	2	0 thru 2		0.0
II.c3	Sound that is clearly understandable and consistent in quality and volume	2	0 thru 2		0.0
II.c4	Sound and music that is relevant to screen displays	1	0 thru 2		0.0
				Subtotal	0.0
Section III - Documentation					
III.a	Instructions for search and help features	1	0 or 1		0.0
III.b	Toll free technical support available	1	0 or 1		0.0
III.c	Description of target audience	1	0 or 1		0.0
III.d	Summary of the contents of the application	2	0 or 1		0.0
III.e	Instructional and/or behavioral objectives	2	0 or 1		0.0
III.f	Suggestions for classroom use, lesson plans, related activities	3	0 thru 2		0.0
				Subtotal	0.0
				Total Weighted Value	0.0

Figure 4. Detail of data collection 2.

Conclusion

I am planning on including both the template file and the developed database of all reviewed websites (94+) with this submission. The inclusion of the database will allow others to have access to a peer review of excellent materials that support not only the action on objects model, but also the NCTM emphasis upon student-centered learning and problem solving. It is my hope that SITE members will avail themselves of the opportunity to contribute to this developing database using the provided template.

References

Connell, M. L (2001). Actions upon objects: A metaphor for technology enhanced mathematics instruction. In D. Tooke & N. Henderson (Eds). Using information technology in mathematics (pp. 143-171). Binghamton, NY:Hawarth Press.

Sfard, A. (1994). Reification as the Birth of Metaphor. For the Learning of Mathematics. 14(1), 44-55.

Using Technology Tools to Revitalize Mathematics Teaching: Perspectives from the United States and Namibia

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Abstract: This study reports on a research project whose goal was to provide secondary mathematics teachers in the United States and Namibia with opportunities to utilize technology tools to teach mathematics for understanding. Qualitative and quantitative methods were utilized to examine the responses of the teachers to the introduction of technology tools in the mathematics classroom and the ways in which the teachers utilized the technology tools in their teaching. The teachers were provided with extended opportunities to teach mathematics in an environment supported by diverse technology tools, including dynamic geometry software, graphing calculators, virtual manipulatives, and general-purpose tools such as word processing, paint programs, and spreadsheets. Throughout the study, the researcher provided a mentoring component by offering technology support, instructional ideas, classroom activity samples, and suggestions to enhance instructional ideas developed and implemented by the teachers. Examples of instructional practices illustrate that technology tools encouraged the teachers to pursue more conceptually focused teaching strategies. The analysis suggests that the technology tools facilitated the teachers constructing an image of technology -enhanced mathematics teaching and learning.

Introduction

Technology tools have a powerful role to play in education. As Bransford, Brown, and Cocking (1999) note, "What has not yet been fully understood is that computer-based technologies can be powerful pedagogical tools...extensions of human capabilities and contexts for social interactions supporting learning" (p. 218). The research reported in this paper is part of a project to revitalize mathematics teaching and learning by providing teachers with extended opportunities to do and experience mathematics in an environment supported by technology tools. An overarching goal for the project examined in this study was to promote innovative practices in the use of technology tools to enhance secondary mathematics teaching and learning. A secondary goal was to promote the use of technology tools to facilitate the teachers' own understandings of mathematics while enhancing their ability to utilize the technologies to teach mathematics in the spirit of educational reforms of both nations. The part of the project relevant to this paper examines the responses secondary mathematics teachers in the United States and Namibia to the introduction of technology tools in the mathematics classroom. It explores the ways in which the teachers utilized the technology tools in their teaching. These tools included dynamic geometry software, graphing calculators, virtual manipulatives, and general-purpose software such as word processing, paint programs, and spreadsheets. The teachers were provided with extended opportunities to experience mathematics as problem solving, communication, reasoning, and building connections (President's Committee of Advisors on Science and Technology, 1997) and to teach mathematics in an environment supported by diverse technology tools.

Theoretical Framework

While the existent literature does not directly address the issues raised by this study, several works provided a lens through which the data in this study were viewed. Research on the appropriate use of technology reveals that students can enhance their mathematical knowledge and conceptual understandings with technology tools (Groves, 1994; Dunham and Dick, 1994). Because they enable students to visualize and experience mathematics, engage in real-world problem solving, and generate representations of their own learning, technology tools are important resources for teaching and learning mathematics (International Society for Technology in Education, 2000). According to the National Council of Teachers of Mathematics' *Technology Principle*: "Mathematics Instructional programs should use technology to help all students understand mathematics and should

prepare them to use mathematics in an increasingly technological world." (NCTM, 2000, p. 40). This suggests that teachers should be provided with adequate preparation and support to implement technology tools in their classrooms. Teachers should be provided with ongoing mentoring and should have time and support to familiarize themselves with software and content to incorporate technology into their lesson plans (President's Committee of Advisors on Science and Technology, 1997).

When Namibia became an independent nation in 1990 it mandated reforms based upon the goals of access, equity, quality, and democracy (MEC, 1993). The reformed educational system rejects authoritarian, teacher-centered instruction, emphasizes student-centered learning, and takes into account students' prior experiences. The redesigned school curriculum is structured upon a constructivist view of knowledge, learning competencies in content areas, and developing a reflective attitude and creative, analytical and critical thinking (NIED, 1998). In the United States, implementation of reforms regarding mathematics curriculum and instructional practice efforts requires that teachers and students engage in problem solving, reasoning, communicating, and making mathematical connections (NCTM, 2000). The technology-rich environment for teaching and learning mathematics in this project supports the implementation of these standards which assert, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24). The research study described in this paper sought to combine the findings of these earlier works by providing mathematics teachers with extended opportunities to experience and do mathematics in environments supported by diverse technologies (Dreyfus and Eisenberg, 1996). The essence of the pedagogical theme underlying this study is to empower teachers through the use of technology tools in mathematics explorations, open-ended problem solving, developing understandings, and communicating about mathematics (Bransford, et al, 1996; Schoenfeld, 1982, 1989, 1992; Silver, 1987).

Methods

Participants and Context

The participants for this study were four secondary mathematics teachers in the United States and six secondary mathematics teachers in Namibia who voluntarily enrolled in one of two professional development workshops taught by the researcher. A fundamental goal of both workshops was to assist teachers in becoming familiar with technology tools available for teaching and learning secondary school mathematics. The teachers met in the workshops for one week, then returned to their respective schools. During the workshops, the researcher modeled the ways in which technology tools can improve teaching and learning (Knapp and Glen, 1996). The teachers engaged in activities that included the integration of technology tools with, or application to content areas in the secondary mathematics curriculum: geometry, including constructions; algebra, including graphs of relations; patterns and functions; estimation and computation; and, data display. The participants spent time engaging in mathematical investigations utilizing Geometer's Sketchpad, TI-82 calculators, and Excel Spreadsheets and in discussions of those explorations. The activities examined in this paper focused on exploring the capabilities as well as the limitations of the technology tools. The teachers were required to explain and justify their results. Included in the spreadsheet explorations was creating a spreadsheet to find a solution to a system of two linear equations. Another activity involved designing a spreadsheet to show the calculations that occur in computing a square root using the divide-and-average method. One investigation with the dynamic geometry software involved constructing square ABCD, with point E as the midpoint of AD. After constructing segment BE, the teachers were to construct point F on segment BE such the CF would be perpendicular to segment BE. The teachers were to determine how the area of quadrilateral CDEF compared to the area of square ABCD and explain their reasoning. Another exp loration required constructing a trapezoid with three sides of 6 cm and then determining what length the fourth side would have to be in order to maximize the area of the trapezoid.

Data Collection

In order to enhance the validity of the findings, mult iple data sources were utilized during the research process (Lincoln and Guba, 1985). Data were triangulated via multiple sources of evidence including informants, events, and documents. The methodological underpinning of this study is derived largely from orientations to research that draw attention to the importance of detailed qualitative fieldwork and the observation and analysis of teachers in context. Several interrelated research strategies facilitated the analysis of the teachers' experiences. These included: (1) participant observations, audio and videotaping, and field notes of the professional development

workshops and the mathematics classrooms; (2) structured and semi-structured, open-ended interviews with each of the teachers; and, (3) collection of teaching artifacts, including the teachers' technology-based problems, activities, and visual representations. Each participant completed questionnaires designed to probe their mathematics content knowledge and their knowledge of mathematics-specific pedagogy. The questionnaires were completed at the beginning of the workshops, at the conclusion of the workshops, and after returning to their classrooms to teach with technology. In addition to the questions pertaining to pedagogy and content knowledge, the teachers were asked to respond to the following questions, in writing and during their interviews, at the onset and the conclusion of the study: (1) how do technology tools influence decisions about what mathematics should or should not be taught? (2) how do technology tools impact teaching? (3) how do technology tools affect students learning? and, (4) how do classrooms that employ technology tools differ from the classrooms in which you may have learned mathematics?

Data Analysis

In order to gain insights into the ways in which the teachers interpreted and implemented technology tools, the developmental research cycle was utilized to organize, analyze, classify, and consolidate the data (Spradley, 1980). Major themes were developed using taxonomic and thematic analytic strategies (Spradley, 1979). Findings were shared, discussed, and compared by the researcher and the teachers. In order to provide a measure of external validity (Goetz and LeCompte, 1994), the researcher reviewed transcripts and analyses with the participants and allowed them to react to analyses and clarify and elaborate on their responses. This process also enabled the teachers to reflect upon what transpired in their classes when they used technology tools for teaching and learning mathematics.

Findings

This study sought to examine the responses of the teachers to the introduction of technology tools in the mathematics classroom and the ways in which the teachers utilized the technology tools in their teaching. Several themes emerged from the analysis of the data. Four will be summarized here: (1) modeling; (2) problem solving; (3) student-centered learning; (4) collaboration; and (5) education reform.

The researcher utilized technology tools to model for the teachers the ways in which the tools could be utilized to enhance mathematics teaching and learning (Barron and Goldman, 1994) in their own classrooms. Conceptual knowledge and procedural knowledge were emphasized together and technology tools were used to reinforce their mutual development. The data from this study support the notion that effective modeling of the ways in which technology tools can enhance teaching and learning should be an integral component of teacher education programs (Knapp and Glenn, 1996). During the workshops, all of the teachers mastered the use of the technology tools and explored the potential of the tools for doing mathematics. As the teachers engaged in teaching with the tools, they began to design innovative and effective ways in which to integrate the tools into their mathematics teaching. At the conclusion of the study, Z, a teacher from the United States wrote on her final questionnaire, "I understand what you were doing. When you taught us the technology, you showed us the way to teach."

One of the most widely noted insights by the teachers was the recognition of problem solving as a method of teaching and learning mathematics. Across both sites, the majority of teachers utilized technology tools to create visual representations and began to introduce mathematical concepts within the context of problems. All of the teachers used the technology tools to design pedagogical presentations of mathematical concepts. However, several noted that this process was more difficult than they had anticipated. At the conclusion of the study, Ms. H, a Namibian teacher noted, "One important thing I have learned is that the technology was fairly easy to learn. Teaching with it will take more time. It requires many changes for me."

The teachers developed their knowledge of how to use the tools to enhance their own as well as students' mathematical learning. When they returned to their respective classrooms, the teachers engaged their students in technology-enhanced mathematical investigations in which the students were active participants. The technology did indeed influence what and how mathematics was taught and learned (NCTM, 2000). Observations revealed that the teachers moved toward a student-centered approach as they engaged their students in technology-based mathematical explorations. For example, after interacting with and observing their students, several teachers restructured the planned activities to accommodate students' interests and experiences. This suggests that working with the technology tools challenged the teachers' pedagogical thinking. During a final interview, a teacher from the United States, Mr. K, stated, "I took an idea from the workshop but when I saw that it was going nowhere with the kids, I had to make some adjustments to make it work for them and keep them involved."

Mentoring and collaboration facilitated the ability of the teachers to create visual representations to model and explore mathematical concepts. The high ratio of teachers to technology made it necessary for the participants to work cooperatively in groups of three or four as they engaged in open-ended problem solving tasks. It appears that this collaboration, because it encouraged discussions about both the technology and the mathematical concepts, may have facilitated the teachers' construction of mathematical understandings as they developed proficiency with the technological tools. The data from this study suggest that the introduction of technology tools transformed teaching and learning in fundamental ways, including facilitating communication about mathematical ideas. At the conclusion of the workshop, a Namibian teacher, Mr. K., said, "I never imagined learning maths in this way – talking about problems, working together as one."

Significant differences between the teachers in the United States and the teachers in Namibia did not emerge. The findings from this study suggest that utilizing the technology tools enhanced the teachers' ability to teach mathematics in the spirit of educational reforms of their respective nations. With regard to the teachers in Namibia, an unexpected observation was that staff room discussions about the technology tools often cited *Toward Education for All* (MEC 1993) and its implications for teaching mathematics. One Namibian teacher, Mr. T, stated, "As I have done this kind of teaching, the reforms are now understandable." It appears that participating in the process of teaching and learning mathematics with technology tools enabled the teachers to recognize connections between this seminal policy statement of the independent Namibian government and the way in which the technology tools changed how they taught mathematics. In particular, the discussions centered on a critical element contained within this document, that "teaching methods allow for the active involvement and participation of learners in the learning process" (MEC, 1993, p. 60).

Conclusions

At the core of the project examined in this study is the development of mathematical power--understanding, using, and appreciating mathematics. The results of this study point to the conclusion that technology tools enabled the modeling and exploration of collaborative instructional strategies, diverse and creative problem-solving, and investigations of mathematics content areas. This study generated a large body of data pertaining to the teachers' responses to and uses of technology tools in the classroom. These data may be presented to teachers as a basis for reconsidering their own uses of technology tools in the mathematics classroom. The present study revealed that mentoring and collaboration within a mutually supportive environment facilitated the teachers' use of technology tools in mathematics instruction and enhanced their pedagogical practices. This study suggests areas for continued research that will extend our understanding of how teachers construct an image of technology-enhanced mathematics teaching and learning, including examining what conceptualizations of mathematics teaching and learning are useful to teachers as they integrate technology tools into their teaching. The next critical step will be to examine the impact of technology tools on student learning.

References

Barron, L., and Goldman, E. S. (1994). Integrating technology with teacher preparation. In B. Means (Ed.), *Technology and education reform.* San Francisco: Jossey Bass.

Bransford, J. D., Brown, A. L., and Cocking, R. R., eds. (1999). *How people learn: Brain, mind, experience, and school.* Washington, D. C: National Academy Press.

Bransford, J. D., Zech, L., Schwartz, D., Barron, B., Vye, N. and The Cognition and Technology Group at Vanderbilt (1996). Fostering mathematical thinking in middle school students: Lessons from research. In R. J. Sternberg and T. Ben-Zeev (Eds.). *The nature of mathematical thinking* (pp. 203-250). Mahwah, NJ: Lawrence Erlbaum Associates.

Dreyfus, T., and Eisenberg, T. (1996). .On different facets of mathematical thinking. In R. J. Sternberg and T. Ben-Zeev (Eds.). *The nature of mathematical thinking* (pp. 253-284). Mahwah, NJ: Lawrence Erlbaum Associates

Dunham, P. H., and Dick, T. P. (1994). Research on graphing calculators. *Mathematics Teacher,* 87 (September), 440-445.

Goetz, J., and LeCompte, M. (1994). *Ethnography and qualitative design in educational research.* New York: Academic Press.

Groves, S. (1994). "Calculators: A learning environment to promote number sense." Paper presented at the annual meeting of the American Educational Research Association. New Orleans.

International Society for Technology Education (2000). *National educational technology standards for students: Connecting curriculum and technology.* Eugene, OR: International Society for Technology in Education (in collaboration with the U.S. Department of Education).

Knapp, L. R., and Glenn, A. D. (1996). *Restructuring schools with technology.* Boston: Allyn and Bacon.

Ministry of Education and Culture (MEC), (1993). *Toward education for all: A development brief for education, culture, and training.* Windhoek: Gamsberg Macmillan.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: Author.

National Institute for Educational Development (NIED), (1998). *The Basic Education Diploma -Broad Curriculum.* Okahandja, Namibia: Ministry of Higher Education, Vocational Education, Science and Technology and Ministry of Basic Education and Culture.

President's Committee of Advisors on Science and Technology (1997). *Report to the President on the use of technology to strengthen K-12 education in the United States.* (http://www.whitehouse.gov/WH/EOP/OSTP/NSTC/PCAST/k-12.html)

Schoenfeld, A. H. (1982). Some thoughts on problem-solving research and mathematics education. In F. K. Lester and J. Garofalo (Eds.), *Mathematical problem solving: Issues in research* (pp. 27-37). Philadelphia: Franklin Institute Press.

Schoenfeld, A. H. (1989). Teaching mathematical thinking and problem solving. In L. Resnick (Ed.), *Toward the thinking curriculum: Current cognitive research* (1989 Yearbook, pp. 83-103). Alexandria, VA: ASCD.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Silver, E. A. (1987). Foundations of cognitive theory and research for mathematics problem-solving instruction. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education: An overview* (pp. 33-60). Hillsdale, NJ: Lawrence Erlbaum Associates.

Spradley, J. P. (1979). *The ethnographic interview.* New York: Holt, Rinehart, and Winston.

Spradley, J. P. (1980). *Participant observation.* Chicago: Holt, Rinehart, and Winston.

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Using Animations in the Teaching of Calculus Concepts

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SUMMARY

During the last decade a lot of emphasis has been placed on the understanding of calculus concepts. To achieve this purpose the use of technology has been introduced as part of the regular instruction. The visualization of mathematical concepts plays an important role in developing intuition about them and insight in their understanding. With the idea in mind that the visualization helps in understanding of some mathematical concepts, I am implementing animations to teach certain key concepts in the Calculus course with the use of *Mathematica*, as a Computer Algebra System. In this paper I will share my experience in the implementation of these concepts, the reaction from the students, and some qualitative and quantitative results on the learning of those concepts and use of Mathematica as a software package.

BACKGROUND

Through my teaching experience in Mathematics one of the problems I have faced and that has intrigued me throughout the years is the lack of students' insight into mathematical concepts, which leads to confusion and rote memorization of concepts without meaning, with the obvious consequences. Research shows that the proper use of technology, new pedagogical approaches, and a good learning environment are factors, which will favor the learning experience. Even though I have used a variety of techniques to enhance the learning of Calculus such as graphing calculators, projects, group work, computer laboratories, and web support, I noticed that the use of animations brings more attention from the students.

As part of the web material I use to enhance the learning I decided to implement the use of animations for certain concepts to which I will refer later. The main drawback was that in order to accomplish this I had to spend a lot of time learning software itself in order to create the animations. A major disadvantage to this approach was that the students could see the results but without any option to manipulate the information, or explore the concept deeper. However, a year ago as part a project I was involved in, I ran into one the Computer Algebra Systems most broadly used in the teaching of Mathematics, *Mathematica*. With the use of *Mathematica* I have been able to create animations that can be modified as needed to explore the concepts. It is likely that I could have done the same things with another software, but in this case I got the motivation to work with Mathematica.

A Computer Algebra System (CAS) is a software package with which one can perform mathematical manipulations, and computations. Their use is becoming more popular in education as well as in private industry. The type of software used by a particular institution depends greatly on the needs.

THE PROJECT

The Calculus I class is one of the two classes offered by the Mathematics program as part of the Core Curriculum at Texas A&M University- Corpus Christi. As a Core course the class emphasizes on verbal, written, critical thinking, and mathematical skills among others. Students from a variety of majors and with different backgrounds take this class, even though the majority of them are from the College of Science and Technology. The class meets three times a week. Two of the meetings are for the regular lecture, one hour and fifteen minutes each, and the other meeting is for the laboratory component, two-hour block. During the laboratory meeting, the student works on projects aimed to reinforce the concepts learned in class or to explore concepts to be covered soon in class. Each lab has a capacity for thirty students where each student has access to a computer.

In this project I am teaching two classes, one uses *Mathematica* during the lab and the other one uses *Derive* in the lab. With *Mathematica* one can create notebooks (pages), in which one can type regular documents (wordprocessing), as well as mathematical expression. The mathematical capability of this software includes among many of them: solving equations, plotting graphs, create and see animations. The class that uses *Mathematica* in the lab component also has the opportunity to see the software during the class to demonstrate some of the concepts. The other class sees some of the demonstrations but mostly concentrated on the animations, and some times fixed graphs needed for the concepts.

The *Mathematica* notebooks can be made available on the web for demonstration purposes using the Mathematica Reader. As I said they are available only as demos and cannot changed. However, students who have the software installed in their computers can download them to their machines and manipulate them as desired.

These are some of the concepts covered in the notebooks:

1. Transformation of functions. In this case a familiar function is taken and a parameter is changed in the animation. The student can see the effect on the graph when the parameter changes.
2. Intuition about the existence of the derivative of a function at a point. A function is zoomed in at a point. The linear or not linear behavior of the graph locally will indicate the existence or not existence of the derivative.
3. The meaning of the tangent line at a point and its relationship to the derivative of the function at the point. Animations showing the secant lines to a curve from a fixed point are shown. The slopes of those lines are displayed during the animation, to develop the intuition of the concept of the derivative as a limiting process.
4. The graph of a function and its derivative. One of the focal points of calculus I is to interpret the meaning of the first and second derivatives in terms of the original function Through this sequence of simultaneous animations, the original functions, its first, and second derivatives the student can visualize in a more vivid manner those relationships.

Warrick's Secrets: Teaching Mathematics Through an Internet-Based 3D Massively Multi-Player Role Playing Game

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Abstract: The project leverages students' fascination with 3D digital entertainment and medieval fantasy stories to teach mathematics to middle school students by situating the mathematics in an appealing virtual environment--an Internet-based Massively Multi-Player Role-Playing Online Game (MMPORPG) called *Warrick's Secrets*. The development team consists of individuals from the IT industry and researchers from three universities. *Warrick's Secrets* is the result of a Phase I Small Business Innovation Research grant (SBIR) completed for the US Dqpartment of Education in April 2001. Second stage development is on-going following the September 2001 award of a Phase II SBIR by the U.S. Department of Education. An MMPORPG is used as the vehicle to deliver National Council of Teachers of Mathematics *Standards*-based educational content. The system includes advanced 3D computer rendering capabilities and high quality 3D artwork since both have proven crucial to maintaining players' attention in the gaming community. The results of the pilot study are reported along with the progress of the Phase II program.

Background & Theoretical Framework

What are Massively Multi-Player Role Playing Games?

Warrick's Secrets is modeled on currently popular MMPORPG's such as, Ultima Online (UO), Everquest, and Micrsoft's Asheron's Call . Currently, more than 150,000 customers play Ultima Online-including players from 114 countries. (Walton, 2000). Sony's EverQuest and Asheron's Call each have a substantial customer base as well. A MMPORPG is a game with a thousand or more players gaming concurrently in real time. Role-playing indicates that a human player controls an in-game character's speech and actions. The virtual world in the most successful games is like the historical middle ages, including some degree of fantastic creatures (e.g., dragons) and magic. Due to limited resources and time, the Phase I prototype of *Warrick's Secrets* only supports single-player activity. With future funding, we plan to add multiplayer capabilities.

At first glance, some people mistakenly think the educational tool described and the total system planned differs little from existing products which attempt to make learning into game, e.g. Math Blaster, etc. Most existing products are far more successful with younger students. The approach of simply trying to transform learning into a game is less successful with middle school and high school students. Our approach is to tightly integrate the educational content with a MMPORPG. This seemingly subtle difference (integrating educational content with a particular game genre versus attempting to make learning into a game) has an enormous impact on the way students react to the product. There are numerous other differences in our

approach that significantly increases the student acceptance rate, potential learning success rate, and age range for which the teaching tool can successfully be used. Physitron's educational tool is the only one based on a MMPORPG. Additionally, it is one of the few tools to use standards-based mathematical content (NCTM, 2000) and provide teachers a mapping of each mathematical challenge to the corresponding topic/skill within the standard. Other distinctive advantages of Physitron's educational tool are listed below.

- Educational content imbedded into realistic situational context
- Inherent promotion of cooperative learning
- Ability for students to interact with remote students
- Vehicle for covert and overt learning
- Advanced motivation and reward system

Related Research

Areas of mathematics education research for this project are situated perspectives on learning and constructivist views of representation. Mathematics viewed as socially constructed knowledge has strongly influenced other learning theories. Many researchers (Kieren, 2000; Steffe & Thompson, 2000; Sfard, 1998; and Cobb & Yackel, 1996) describe the theoretical underpinnings of their work in terms of "social constructivism." The learning theory that grounds many of these researchers, including those in the present project, may be traced to von Glasersfeld's (1991) radical constructivism, and Piaget's (1970) genetic epistemology. Lave and Wenger (1991) describe situated learning as developing as a function of the activity, context, and culture in which such learning occurs. One of the primary benefits of integrating educational content with a virtual environment and storyline is that educational content is imbedded into a realistic situational context. These ideas are important here because we consider how such theories apply to the development of educational interactive multimedia. An example of such multimedia research is the work of Herrington and Oliver (1997).

Mathematical representation is considered from a constructivist perspective. Representation describes the form in which a problem is encountered- such as graphical, numerical, verbal, or symbolic representations. It also describes the learner's mental construct of the problem. Another advantage of the virtual environment for problem solving is that students have many opportunities to attempt solutions of mathematical tasks. Furthermore, the virtual environment affords a natural mechanism for providing hints and alternative teaching representations based on a student's previous success with a concept or skill. This may increase the number and quality of the mental representations that the students apply to the problems. The project relied on the research of Cifarelli (1988) and Goodson-Espy (1994, 1998) to use the notion of reflective abstraction to explain how, as students work through multiple attempts to solve a problem, they may develop progressively more abstract mathematical conceptions. It has been observed that if students cope with recurrent mathematical themes, they are more likely to develop higher levels of reflective abstraction, and thus attain more powerful mathematical concepts.

Example Problem Included in the Phase I Prototype

The Phase I prototype included on complete problem pathway. This began with a meeting between the mentor magical character, Adel, and the student character, Palus. The first challenges required him/her to solve a series of problems involving percent. If the student was not successful, he or she was provided with a series of hints and provided with the opportunity to try again. The next series of challenges required Palus to interact with a series of talking flasks in the laboratory. The flasks gradually helped Palus learn to read the measurements on a flask and, in a progressive manner, helped him/her learn how to solve a mixture problem such as the following:

Flask #4 asks Palus: "I have 75 milliliters of liquid, which is hotfish soup diluted with the spittle of glowworms. If there is a 30% concentration of glowworm spittle, then how many milliliters of glowworm spittle do we have?"

The prototype including this series of challenges was piloted tested with middle school students, as described below.

Pilot Study Methods

Subjects

The subjects included seven students from a middle school in the southern US. The subjects included two sixth graders, two seventh graders, two eighth graders, and one ninth grader. The subjects included one African-American, six Caucasians, four males, and three females.

Data Sources and Analysis Method

The prototype testing took place at school on April 6, 2001. The individual sessions each lasted between 45-60 minutes. Students attempted all the problem scenarios and completed a questionnaire and participated in a concluding interview. Students were given a pen, paper, and a calculator. The problem set included two problem paths--an initial problem path dealing with percent and a second developing a series of questions about mixture problems. The data sources for the pilot study include: (1) videotapes and transcripts of the prototype testing sessions, (2) student written artifacts, (3) completed questionnaires, (4) observational field notes, and (5) researcher notes from the post-game interviews. Analysis of these data resulted in seven case studies.

Summary of Results

The results of the game prototype testing include three parts: (1) a summary of student reactions, (2) the questionnaire results, and (3) seven case studies. Only excerpts of the results will be discussed below.

- All subjects expressed increased interest in working on mathematics problems that were situated in a computer gaming environment.
- All subjects reported some level of self-confidence in solving mathematics problems but expressed less self-confidence in terms of solving word problems in a classic classroom or textbook setting.
- One of the students specifically stated that he liked the graphics quality of the characters and that the quality was much higher than those of characters in computer game he had at home.
- Several of the students indicated that they enjoyed being an active character in the game and that this improved their ability to interact with the mathematics. Students wanted to control the character's gender, skin color, clothing, ethnicity, character name, and skills.
- Numerous students indirectly noted the fact that the mathematical content was imbedded into a realistic situational context increased their ability to understand and thus solve the problems, particularly the mixture problems. All students indicated that they clearly understood what the problems were asking them to do. Students indicated that it was easier to solve the problems because the in-game flasks were displayed with measurement calibrations. Another student made a similar comment about the collection bins used to represent the percentage problems. The students liked the visual representations of the problems. All of the students attempted all of the problem scenarios. Two of the subjects were able to solve all of the problems on the first try. Three of the subjects were able to solve all but one of the problems on the first attempt.
- All of the subjects were able to attain the levels of reflective abstraction: Recognition and Representation in their problem-solving activities. Three of the subjects were able to attain the level of Structural Abstraction. Three of the students were able to complete the second problem pathway by building on their solution strategies for the previous problems.

Conclusions and Future Directions

Student Enthusiasm for this Genre

The favorite adjective used by the students to describe the game was, "Cool!" The students explained that the game made learning math easier for them because it placed the mathematics into a meaningful context for them. The students indicated that they were motivated to solve problems because they wanted to find out what would happen next in the game or because they wanted to gain access to the next magical tool that was described in the story.

Character Creation and Communication

The students made numerous positive comments about the faces, clothes, and details such as being able to see the characters breathe. The students were intrigued by the ability to assume the role of a character in the

game. They were greatly attracted to the capacity to arbitrarily maneuver their character in the virtual world. Students wanted to be able to choose gender, race, clothing, facial appearance, and other attributes of their character, including the character's name. This is a standard option in most MMPORPG's.

Interactivity In The Game

The students were universally interested in gaining character mobility in the virtual world. They were fascinated by character head motion, lateral motion, climbing stairs, moving underwater, etc. Most of the students wanted to wander about in the game world before beginning the problem sequence. This activity allowed us to gain valuable data about the ease of character controls. It also reinforced our notion that interactivity with objects in the game environment and exploration of new aspects of the virtual environment are crucial factors in gaining and maintaining student interest and represent mechanisms for in-game rewards.

Problem Pathways

The students indicated that they found the collection bin representation for the initial percent problems to be very helpful in solving these problems. The students were very emphatic that the visual representations were extremely useful to them. The students' interactions with the flasks in the laboratory appear to have influenced their problem-solving success with the mixture-related problems. Based on the researchers' experiences in teaching very similar problems to college-aged students in traditional college algebra classes, the success rates of the middle school students on these problems were much higher. In the videotapes and post-session interviews, the students indicated that the visual nature of the representations were also useful in reducing their anxieties about this type of question. The students positively evaluated the hints that were provided for each task. Because of the visual representations of the problems and the scaffolded hints that presented themselves upon incorrect student responses, students were more willing to attempt what they considered to be difficult problems. The students problem solving activities and successes indicate that carefully crafted problem pathways can be a successful teaching tool in assisting students with making necessary conceptual connections via reflective abstraction.

Progress of the Phase II Program

We seek to explore the on-screen 3D conceptual vehicles that can be used to pose interesting mathematical problems in this interactive computer environment. In Phase II, we plan to answer a number of research questions, including the following:

- Does use of this type of an educational tool raise the performance level of students in a regular mathematics classroom?
- Can the notion of reflective abstraction be used to describe mathematical concept building when educational content is integrated with a MMPORPG setting?
- Will our collection of situation-based and "scaffolded" mathematics pathways encourage students to apply problem-solving skills that encourage the development of higher order thinking?
- If allowed to create their own character, what kind of character will they choose in terms of gender, race, ethnicity, facial appearance, and clothing?
- Given that research has found computer gaming to be more popular with males (except for role-playing games, which are essentially as popular with females), will this educational resource be popular and effective with females?
- Do students display an increased motivation to try to solve problems?
- Can standards-based mathematical content be integrated into an MMPORPG in a manner that allows students to discover, learn, and even teach other students without forcing the student to step out of the role of their online character?
- Can the mathematical content be integrated into the MMPORPG innocuously enough so that it does not destroy the enjoyment factor we are trying to leverage?

The number of research questions is obviously large, but warranted, as this area has very little formal investigation.

References

Brown, Collins & Duguid (1989). Situated cognition and the culture of learning. *Educational Researcher, 18* (1), 32-42.

Cifarelli, V. V. (1988). The role of abstraction as a learning process in mathematical problem solving. Doctoral dissertation, Purdue University, Indiana.

Cobb, P. & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*, 175-190.

Goodson-Espy, T. (1994). A constructivist model of the transition from arithmetic to algebra: Problem solving in the context of linear inequality. Doctoral dissertation, George Peabody College of Vanderbilt University. *Dissertation Abstracts International, 55*, 34439.

Goodson-Espy, T. (1998). The roles of reification and reflective abstraction in the development of abstract thought: Transitions from arithmetic to algebra. *Educational Studies in Mathematics, 36*, 219-245.

Herrington, J. & Oliver, R. (1997). Multimedia, magic, and the way students respond to a situated learning environment. *Australian Journal of Educational Technology, 13* (2), 127-143.

Hershkowitz, R., Schwarz, B., & Dreyfus, T. (2001). Abstraction in Context: Epistemic Actions. *Journal for Research in Mathematics Education, 32* (2), 195-222.

Kieren, T. (2000). Dichotomies or binoculars: Reflections on the papers by Steffe and Thompson and by Lerman. *Journal for Research in Mathematics Education, 31* (2), 228-233.

Lave, J. (1988). *Cognition in Practice: Mind, mathematics, and culture in everyday life*. Cambridge, UK: Cambridge University Press.

Lave, J. & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge, UK: Cambridge University Press.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics 2000*. Reston, VA.

Piaget, J. (1970). *Genetic epistemology*. (E. Duckworth, Trans.) New York: Columbia University Press.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27* (2), 4-13.

Sfard, A. (1994). The gains and pitfalls of reification- The case of algebra. *Educational Studies in Mathematics, 26*, 191-228.

Steffe, L.P. & Thompson, P. (2000). Interaction or intersubjectivity? A reply to Lerman. *Journal for Research in Mathematics Education, 31* (2), 191-209.

von Glasersfeld, E. (1991). Abstraction, re-presentation, and reflection. In L. Steffe (Ed.), *Epistemological Foundations of Mathematical Experience* (pp.45-69). New York: Springer Verlag.

Vygotsky, L. (1986). *Thought and language*. (E. Hanfmann & G. Vakar, Eds. and Trans.) Cambridge, MA: MIT Press (Original work published 1934).

Walton, J. (2000). Physitron, Inc. Internal Market Data Report.

Are Teachers "TechReady?"
Evaluating the Technology Competencies of Preservice Teachers

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Abstract: Technology must be essential part of teacher preparation. Technology training also needs to be evaluated against ISTE standards. This paper is a report on the development and initial findings of an online self-reporting instrument that can be used to evaluate the technology competencies of preservice teachers. Data from the field-test of the instrument reveals that while teachers understand how to use the hardware and software, they need more help in knowing how to integrate technology into their instructional strategy.

Introduction

Today's world has grown smaller because of technology. Satellite trucks allow us to view images from anywhere in the world. Cellular telephones allow instantaneous communication. Palm top computers give us powerful tools to plan, store, and present data. Satellites, with power imaging technology provide us with high quality images from hundreds of miles in space.

New technology has created two broad changes in our world today. First, there is a changing economy. New technology is making workers more productive. In addition, financial trade, communications, and assets are being acquired globally. An example is "Sony" who has major holdings in the entertainment business in the U.S.

Second, there is a changing workplace. Globally, we have entered into what some have termed, "The information age." There are many sources of information that are available electronically that can be accessed at the click of a mouse. Therefore, knowledge is quickly becoming a major commodity. I believe that one skill that an educated person must have is the skill necessary to search the vast number of information sources that are available, which address the problems that need to be solved (Report from National Commission on Mathematics and Science Teaching for 21st Century 2000). The skills needed for successful living have altered radically, primarily as a result of the technological revolution and its impact on most jobs and professions. That is to say that technology is changing the way we work and live.

These global changes have two implications for educators. First, we must train our children to use the technology and information tools that are available to them so that they can work effectively in our global society. Schools of education must provide school systems with prospective teachers who can help student's function in our new society, and that means that the teachers themselves must gain new skills and new knowledge. Second, we must use technology to create a more effective teaching learning system to deliver our lessons. If technology is to be used effectively in teaching, it must be used to create a student-centered learning environment. Teachers entering today's classrooms must not only be competent in teaching skills, but they must also be able to integrate technology in their teaching. Technology can be used to help supply five key conditions for learning: (1) real-world contexts for learning; (2) connections to outside experts; (3) visualization and analysis tools; (4) scaffolds for problem solving; and (5) opportunities for feedback, reflection and revision. In short, the focus of the technology evaluation should be on learning and the learners and not the technology. The long-term goal is to build schools as learner-centered communities by expanding the notion of collaboration, which will transform current educational systems. Technology has an obvious role in both of these areas, but it is a role that must evolve from the foundation of research-validated principles and practices.

Because of these global changes, technology has become an integral part of instruction in P-12 schools (NCATE, 1997). New demands are being placed on teachers to use technology in their instruction, which has been brought on by the rapid influx of hardware and software (Matthew & Kimball-Lopez, 2000). Both the National Education Association (NEA) and the National Council for Accreditation of Teacher Education (NCATE) consider

computer technology skills critical for teachers (Gladhart, Carroll, & Ellsworth, 2000). Rosenthal (1999) notes, "Bringing faculty members and America's teaching force up to speed [technologically] is a massive task. . . a problem that will be greatly exacerbated if the teachers entering the profession have not been adequately prepared to use information technologies" (p. 22).

Preservice teachers, therefore, must be able to demonstrate technology competencies that are based on the National Education Technology Standards (NETS) set by International Society for Technology in Education (ISTE). The ISTE standards are an appropriate framework for evaluating technology competencies because they are widely recognized and are comprehensive (Ropp & Brown, 2000). The NETS Standards are divided into six categories which are Technology Operations and Concepts, Planning and Designing Learning Environments and Experiences, Teaching, Learning, and Curriculum Assessment and Evaluation, Productivity and Professional Practice, Social, Ethical, Legal, and Human Issues. In each of these categories are suggested skills that should be mastered before an individual enters the Teacher Education Program, following the completion on an Educational Technology course, following the completion of methods courses and student teaching, and following the completion of the first year of teaching.

The challenge is to properly assess the technology competencies at each of these areas. McCombs (2001) notes the following four essential questions that should be the focus of assessment: (1) How is technology perceived by individuals learners and teachers relative to its teaching-learning support?; (2) What changes in learning and performance outcomes can be observed with different technology uses and with different learners?; (3) What changes in teaching processes can be observed that enhance learning outcomes?; and (4) What changes in the learning context can be observed that create new partnerships and climates for learning?

McCombs (2001) also identifies five data sources that can answer these questions. These are student and teacher self-assessments of technology practices and strategies; students and teacher attitudes toward technology and its specific uses; multiple student motivation measures; multiple students achievement measures; and observational information on learning outcomes, teaching and learning context. I would like to add a sixth, which is student products. While this is related to student achievements, I would like to emphasize that student projects give teachers and other administrators the opportunity to observe the student's ability to apply knowledge to real-world situations.

The Study

To measure the competencies of preservice teachers, a self-reporting web-based form was developed that consisted of Likert type items that were built around each of the six NETS standards for teachers. The six items on the form were as follows: Basic Computer Operations; Basic Communication Tools; Planning Technology-Rich Learning Activities; Plan Multiple Strategies to Facilitate Critical Thinking about Electronic Information; Use Multiple Technologies to Support Learner-Centered Lessons to Meet Diverse Needs of Students; and Use technology to Facilitate Effective Assessment and Evaluation Strategies.

There are two parts to each of the six items on the form. First, respondents were asked to rate their level of professional knowledge proficiency. The "Levels of Knowledge Proficiency" were represented in the following five areas:

1. Converse about the content in general ways
2. Relate the content to broader non-technical issues
3. Give explanations about critical concepts
4. Apply knowledge to challenging practical problems
5. Give expert advice.

Second, on each of the "Levels of Knowledge Proficiency," respondents were asked to rate the "Extent of Professional Knowledge" they had on a Likert type scale from 1-7 where 1 indicates "Not at all" and 7 indicates "A Great Extent."

The survey was administered during the spring 2001 semester to a convenience sample of 36 teacher education students from several colleges and universities in the Midwest. The results show that for each of the six items, the "Level of Knowledge Proficiency" was greatest in their ability to "converse in general ways about the topic" and least in "giving expert advice." This indicates that preservice teachers in the sample have a general knowledge of classroom technology tools, but are not yet comfortable giving expert advice to someone else in any of the six areas. In addition, the "Extent of Knowledge Proficiency" as measured by the Likert scale is highest in "basic communication tools" and least in "planning multiple strategies to facilitate critical thinking about electronic information." In general, the extent of the knowledge proficiency was greatest regarding technology tools, and least

regarding issues of planning and assessment. The data was analyzed for each of our research questions by using SAS.

The Findings

Figure 1 outlines the mean scores in each of the six ISTE Standard areas. The students rated themselves fairly high in all areas in their ability to converse about the content in general ways. In each of the succeeding competency levels, however, the student's ratings of their level of competency went down. This indicates that they had a broad surface knowledge of the content, but the are still lacking in their ability to explain it, apply it, or give expert advice about it.

Another observation is that the students appear to feel fairly competent regarding basic operations and communications tools, but less competent in the areas of planning learning activities, critical thinking strategies, supporting learner-centered activities, and in the area of assessment. What it appears is that while the students are literate in technology, they do not feel competent in using technology in the classroom.

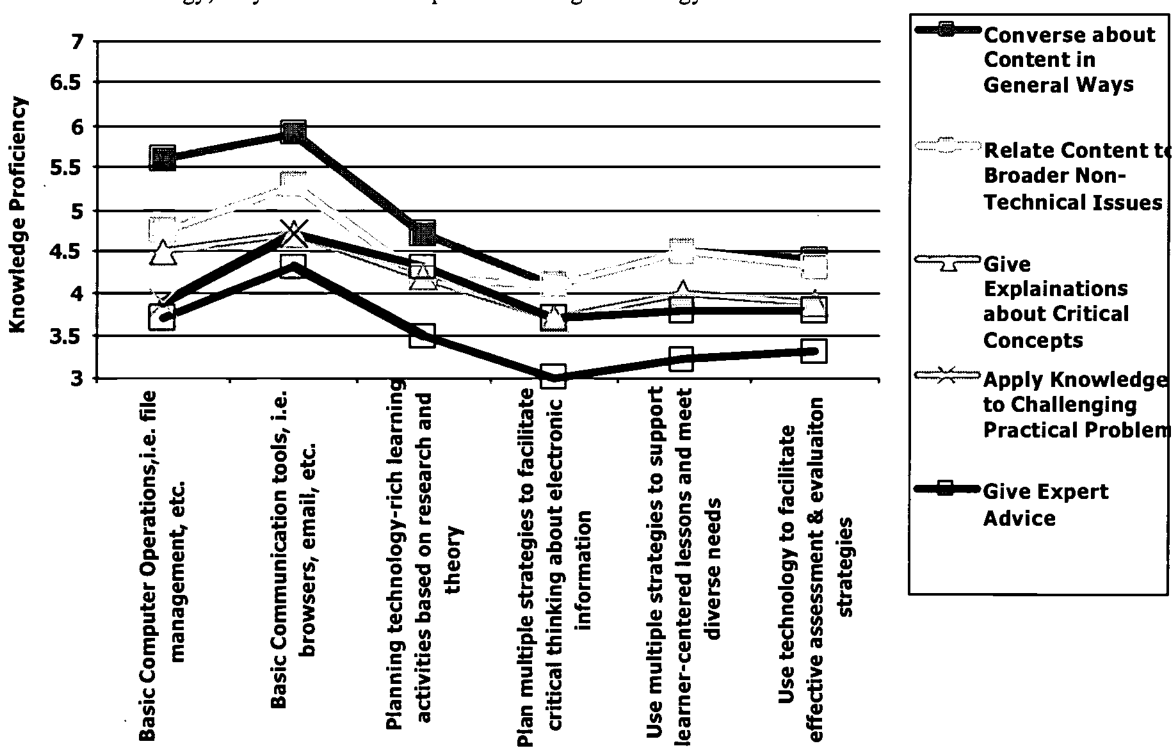

Figure 1: Composite Graph of all ISTE Standard Areas

It might appear, therefore, that the reason for this is that students are not being shown in the methods classes how to use technology as part of an instructional strategy.

The Conclusion

In conclusion, we must begin to rethink the concept of "school" in our technology rich world. We must not think in terms of seat time, and textbooks, and in school libraries. We must add to our thinking learner-centered collaborative activities that give students the tools and the freedom to explore their world and the tools they need to improve their own learning and develop their knowledge base. In addition, we need to better understand the ways in

which technology-supported education represents a melding of the learner and the discipline as framed by inquiry-based learning.

You know, when Sony built the Walkman, they weren't satisfied with being second best. They began to look for ways that they could daily improve their product. Their hard work gave the consumer access to new technology and features. Sony set the standard for portable sound. Just as Sony didn't quit, we as educators should not quit. We should be asking ourselves this question, "What can I do today that will help the students to learn just a little bit better?" It doesn't have to be a big change, but if we do this each day, we will, like Sony, begin to set new standards for education and better prepare our students to live in the 21st century.

Schools in the 21st century must begin to prepare students for a world rooted in information and technology. Such a world calls for students with the kinds of skills and understandings that enable them to function within and contribute to this emerging world. Students must have opportunities to participate in problem-oriented learning activities that are relevant to their interests and worthy of their investment of time and effort. Improving student learning must depend on more than symbol manipulation, generalized learning, pure mentation, and individual cognition. It must lead to competence acquired through authentic activity presented in an environment that encourages collaboration. Students have the right to expect that the knowledge and technology commonplace in their homes and communities are accessible in their schools. They have the right to a learning environment rich with resources both printed and retrievable from the World Wide Web.

References

International Society for Technology in Education (ISTE). (1996). *National Standards for Technology in Teacher Preparation.* [online] Available: http://www.iste.org/Standrads/NCATE/intro.html.

K. Matthew, K. & Kimbell-Lopez K. (2000). *The Proceedings of the Society for Information Technology in Education and Teacher Education 11th International Conference*, USA, 12-16.

Gladhart, M. Carroll, J., & Ellsworth, R. (2000). Getting program and faculty up to speed for NCATE 2000 standards: Project EXCITE. *The Proceedings of the Society for Information Technology in Education and Teacher Education 11th International Conference*, USA, 1487-1494.

McCombs, B.L. (2000). *Assessing the role of educational technology in the teaching and learning process: A learner-centered perspective*. White paper presented at the Secretary's Conference on Technology 2000. [online: http://www.ed.gov/technology/techconf/2000/mccombs_paper.html.

National Commission on Mathematics and Science Teaching for the 21st Century (The). (2000). *Before it's too late: A report to the nation from The National Commission on Mathematics and Science Teaching for the 21st Century.* [Online: http://www.ed.gov/americacounts/glenn/].

National Council for Accreditation of Teacher Education (1997). *Technology and the new professional teacher: Preparing for the 21st century classroom.* [online]. Available: *http://www.ncate.org/specfoc/techrpt.html.*

Ropp M. & Brown J. (2000). Beyond applications to the essential processes of technology integration: Designing an educational technology course to reflect ISTE standards and to model best practices for the future. *The Proceedings of the Society for Information Technology in Education and Teacher Education 11th International Conference,* USA, 335-338.

Rosenthal, I.G. (1999). New teachers and technology. *Technology & Learning*, 19(8), 22-27.

Using Visualization to Make Connections Between Math and Science in High School Classrooms

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Abstract: Today's students are receiving "superficial" knowledge of math and science concepts. According to a report from the National Commission on Mathematics and Science Teaching for the 21st century, in science, students are not mastering the "big concepts," while in mathematics, they are given little information about "how" and "why." To address these concerns, connections need to be made between mathematics and science. Visualization combined with Problem Based Learning (PBL) is one teaching strategy that can be used to address these issues. This paper reviews the findings of a study conducted at two mid-western high schools that used visualization in math/science classrooms. The results revealed that students made meaningful connections between math and science data, connections between math and science language, and connections between math, science and daily life experience.

According to the report from The National Commission on Mathematics and Science Teaching for the 21st Century (2000), entitled *Before It's Too late*, U.S. students are receiving only a superficial knowledge in today's classrooms. The report states, "In an age now driven by the relentless necessity of scientific and technological advance, the preparation our students receive in mathematics and science is, in a word, unacceptable. Despite our good intentions, their learning is too often superficial. Students' grasp of science as a process of discovery, and of mathematics as the language of scientific reasoning is often formulaic, fragile, or absent altogether" (p. 10). The report notes that the problem with current science education is that students are not required to master "big concepts that make science so powerful and fascinating" (The National Commission on Mathematics and Science Teaching for the 21st Century 2000). In mathematics, the content is limited to questions that answer "What" and get little content that addresses "How" and "Why should I care." These two issues could be addressed by making connections in the classroom between science and mathematics. These two subjects and the daily world are linked. Scientists generate data and use mathematics as a tool for data analysis. Yet, in our education system, students see these two subjects as separate and distinct because we have chosen to teach them separately giving students a dichotomous view of science and mathematics

One teaching strategy that could be used to make connections between science and mathematics is visualization. Combined with Problem-Based Learning, visualization uses classroom technology to help the learners create a visual picture of concepts and make connections between mathematics and science through discovery learning experiences.

According to the 1999 report, *How People Learn* (Bransford, Brown, & Cocking 1999), from the National Research Council, technology can be used to help supply five key conditions for learning. These conditions are central to the success of visualization. They include real-world contexts for learning; connections to outside experts; visualization and analysis tools; scaffolds for problem solving; and opportunities for feedback, reflection and revision.

First, technology can support real world context for learning by using simulations, which can form the basis for Project Based Learning. Project Based Learning uses in class projects that are used to cover course content and fulfill certain course objectives. Students have the opportunity to work on the project as teams and report their results to the entire class. One way to do this is through the use of simulation. Simulation exposes students to real-world problems to which they must find solutions. They are looking for answers that are "situation specific" rather than the "right answer" from a textbook (Teaching and Learning 2001).

Second, technology can connect students with outside experts. Through the Internet, students can have access to experts in practically any field all over the globe. They can send e-mail to them and even "chat" with them online. In one of my own classes, the students became very excited when they were able to have their questions answered during a "chat" session with a well-known expert in education. Students can also download documents that are not available in the library, and they can keep up with the latest research.

Third, technology can provide visualization and analysis tools (Harnisch 2000). Rather than talking about concepts, teachers can use technology to visualize them. For example, schools that do not have microscopes can use advanced imaging technology to look at the parts of a flower rather then relying on textbook photos or drawings. Also, 3-D imaging allows chemistry students to construct a three-dimensional model of an atom and animate it so they can see it from all sides. In mathematics, graphing calculators can visually show students relationships between variables. Concept mapping using "Inspiration" software can help students visualize processes and relationships.

Fourth, technology can provide scaffolds for problem solving (Harnisch and Sato, 1990). In today's rapidly changing world students need to learn much more than the knowledge written in a textbook. They need to be able to examine complex situations and define solvable problems within them. They need to work with multiple sources and media, not just the single textbook. They need to become active learners, and to collaborate and understand the perspectives of others. What we are talking about is the ways in which students today need to learn how to learn; that is, they need to learn how to:

- Ask: find problems
- Investigate: multiple sources/media
- Create: engage actively in learning
- Discuss: collaborate; diverse views
- Reflect: learn how to learn

This shift to an inquiry-based mode of teaching and learning is now widely recognized (Bruce & Davidson 1996; Minstrell & Van Zee 2000; Shavelson & Towne 2002; Wells 2001). The National Science Foundation has asked for "research-validated models (i.e., extended inquiry, problems solving)." The Carnegie Foundation's Boyer Commission on Educating Undergraduates in the Research University (1998) has set its number one priority to make research-based learning the standard. The American Association for the Advancement of Science, in its Project 2061, has as its number one goal to have "science literacy for all high school graduates," by which they mean to develop the broad, critical perspective and habits of mind that develop through scientific inquiry.

Fifth, technology can provide collaboration between students, teachers, and outside experts that help students to solve problems. Through email, discussion boards, and web pages, students have access to educators, and to experts who can help them to think through problems. Also, technology can provide students with problem-solving experiences by developing "Inquiry Units." This is available to both teachers and students through a web site at the University of Illinois at Urbana-Champaign at http://inquiry.uiuc.edu/. The "Inquiry Page" is more than a web site. It's a dynamic virtual community where inquiry-based education can be discussed, resources and experiences shared, and innovative approaches explored in a collaborative environment. One example is "Web Quests" which will help them to develop problem solving skills. Based on John Dewey's philosophy that education begins with the curiosity of the learner, we use a spiral path of inquiry: asking questions, investigating solutions, creating new knowledge as we gather information, discussing our discoveries and experiences, and reflecting on our new-found knowledge. We invite you to visit the inquiry page. There are lessons on life that can be downloaded and adapted for use in your classroom. Also, you can place inquiry units on the page and access them from anywhere in the world. Your students can also develop units as part of a lesson and put them on the page to share with others. By doing this, you will become part of a world-video learning community.

The Study

The Visualization teaching/learning strategy was tested in two mathematics and science classrooms in a suburban high school during the 2000-2001 school year. In each school a math/science cohort of 25 learners was formed and a teaching team of math/science teachers. The learners consisted of Honors students taking Algebra II with Trigonometry /Honors Chemistry at one school, and "G" learners having a combination of Algebra II/Physics at the other school. The learners took math and science together as a group rather than changing classes. In addition, the teachers had the opportunity to observe each other's classes. The Project Based Learning (PBL) Coordinator, who was a teacher that specialized in PBL and was selected for the project, trained each teacher in PBL. In addition, the Associate Superintendent of Schools, and the Assistant Principal of each of the schools was kept informed of the progress of the project. This collaboration helped to form a foundation for a culture of learning in math and science at each school.

During the project, students had access to classroom technology including computers, software, web access, graphing calculators, and science laboratory equipment. The Honors class also had the opportunity to upload projects to a secure website for review.

The data for the project consisted of survey instruments, images of projects, field observations, student and teacher reflections, teacher portfolios, interviews with teachers and school administrators, and student focus group interviews. Some of this data was collected electronically using a secure website.

Findings

The use of visualization and the grouping of students into math/science cohorts and teachers into math/science teaching teams helped students make connections between math and science. Through visualization, which also facilitated collaborative learning, students and teachers built a math/science learning community that was formed through their shared classroom experiences. This is illustrated in Figure 1.

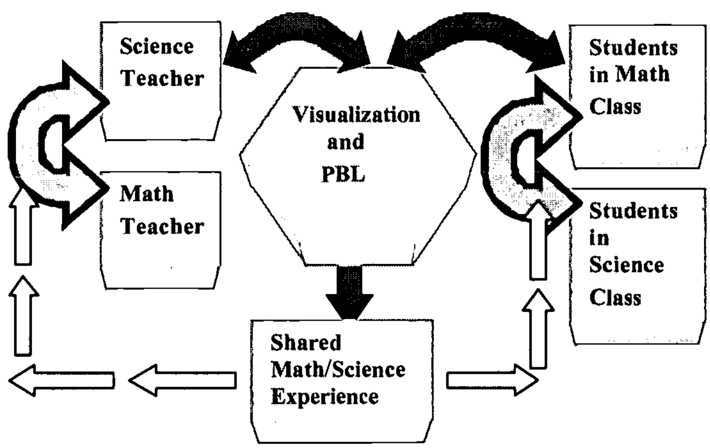

Figure 1: Math/Science Collaborative Learning Community

Visualization helped students and teachers make connections between math and science. There were three types of connections that were made. First, there was a "data" connection, second, there was a "language" connection," and finally, there was a "life" connection.

Data Connection

Students had the opportunity to make connections between math and science by using mathematical formulas and concepts to analyze and draw conclusions from data they had generated in a science class. For

example, the students in one cohort spent the day in research teams throwing softballs at different angles and measured the angles and distance of each throw. The students then used vector concepts and formulas to determine which was the best angle to throw the ball to achieve the greatest distance. In their reflections, students remarked about the connections they saw between mathematics and science. A student wrote, "We created connections from Algebra 2 to chemistry in different ways. For example, when we were learning about log in Algebra 2, at the same time we were learning to determine the pH of different substances. We used 'log' in the equation."

Language Connection

Second, visualization helped students and teachers begin to form a common math/science language. One of the department heads associated with the project noted, "completely integrating a Math and Science course may not be feasible, but speaking a common language in all Math and Science classes would be very powerful." The advantage of a common language to the student is that now students began to see a common set of terms for a concept rather than two sets, one in math and one in science. Teachers also see this as a big advantage. One math teacher said that a common terminology would help the students make connections between the two disciplines because, although math and science have common concepts, they use different terms to describe them. One of the administrators also added that a common language would also serve to make the curriculum more cohesive.

Life Connection

Finally, the students in the project expressed that the teachers connected concepts in Math and science to daily life. One student remarked, "Well, for instance, sometimes when I had math it tended to be a little abstract and you might have wondered what you would use this for?" As result of this experience, however, students expressed they "experienced" a concept rather than just "reading" about it.

Another example was the "Roller Coaster Project." This project allowed students to see the "science" behind roller coasters. It culminated in a trip to a local amusement park. Students did research on roller coasters and then, during their visit to the park, they divided up into teams and collected data during their rides. They used this data to answer questions about the rides and what they experienced. A student in the project summarized it this way: "A teacher that uses technology teaches differently because they don't necessarily teach in the traditional teaching manner. They seem to bring more of the outside world into the school."

Conclusion

In conclusion, visualization is not an easy task. Technology, and access to technology, is an important component for developing visualization. Both students and teachers see visualization as a common link that helps all students have access to the concepts. The group work in the classroom also facilitates learning because it provides a support group and another means to help students learn. Students, who may just "tune out" during a lecture and may be "left behind" as a result, now have the opportunity to be actively engaged in the learning process through visualization and group work.

Technology is also important for teachers as part of their professional development. Teachers need to learn how to use current software packages in their fields and keep up with developments in their field. Technology is one of the factors that is fueling the changes we're seeing in the world by providing an expanded information base. No longer is information confined to physical libraries. The Internet provides access to libraries and resources worldwide. Students and teachers must learn to how to access and use the information that is available if they are to succeed.

While both teachers and students grew academically and professionally, teachers found that teachers who take on this challenge must be willing to view teaching just a little differently. They must be comfortable with restructuring the curriculum, trying different methods, tolerating just a little more "noise" in their classrooms and working through the frustration when the technology just didn't work. Students also faced the challenges associated with discovery learning that sometimes there might not be a "right" answer to a problem, and that the teacher may not have all the answers. Through visualization, students and teachers can learn together to develop shared knowledge that encourages students to develop a sound understanding of the "big concepts" in math and science and how math and science is related to their lives.

References

Boyer Commission on Educating Undergraduates in the Research University. (1998). *Reinventing undergraduate education: A blueprint for America's research universities.* [online: http://notes.cc.sunysb.edu/Pres/boyer.nsf]

Bransford, J.D., Brown, A.L., & Cocking, R.R. (1999). *How people learn: Brain, mind, experience, and school.* Washington, DC: National Academy Press.

Bruce, B.C., & Davidson, J. (1996). An inquiry model for literacy across the curriculum. *Journal of Curriculum Studies, 28*(3), 281-300.

Harnisch, D.L. & Sato, T. (1990). Issues in education in an information age: Trends in cognitive science for instructional design. *Proceedings of InfoJapan '90: Information technology harmonizing with society.* Tokyo, Japan: Information Processing Society of Japan, 365-372.

Harnisch, D.L. (2000, February). *Importance of mathematical visualization: I can see what you mean now.* Presentation as part of Symposium entitled: Moving K-12 Teachers into 21st Century Technology: Building the educational grid for pre-service training. Paper presented at the Society for Information Technology and Teacher Education (SITE), San Diego.

Minstrell, J., & Van Zee, E.H. (Eds.). (2000). *Inquiring into inquiry learning and teaching in science.* Washington, DC: American Association for the Advancement of Science.

National Commission on Mathematics and Science Teaching for the 21st Century (The). (2000). *Before it's too late: A report to the nation from The National Commission on Mathematics and Science Teaching for the 21st Century.* [Online: http://www.ed.gov/americacounts/glenn/]

Shavelson , R.J., & Towne, L. (Eds.). (2002). *Scientific inquiry in education.* Washington, DC: National Academy Press. [Online: http://www.nap.edu/catalog/10236.html?ea_49]

Technology and Learning Magazine (2001, November 15). *Top 10 technology breakthroughs for schools.* [Online: http://www.techlearning.com/db_area/archives/TL/200111/toptech.html]

Wells, G. (2001). *Dialogic inquiry.* New York: Cambridge University Press. [ISBN: 0521637252]

Visualization and Collaborative Learning in Math/Science Classrooms

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Abstract: In many of today's math and science classrooms, learning today is highly "teacher directed." As a result, students often miss the "big concepts" and see little connection between math and science. In a study done at two high schools in the Midwest, math/science student cohorts, and math/science teaching teams, developed a collaborative learning environment that uses visualization and Problem-based learning which formed the foundation for collaboration between math and science teachers and students. Student collaboration took the form of student mentoring. Collaboration among teachers led to faculty mentoring, breaking down some of the walls between science and math, and better classroom help for students.

Introduction

A report from the National Commission on Mathematics and Science Teaching for the 21st Century entitled *Before it's Too Late*, underscores the poor performance of U.S. students compared to other nations in math and science. One reason for the decline in scores is the way that math and science are taught. According to the report, there are two major problems with current math and science teaching methods. First, "Seldom are students asked to master the "big" concepts that make science so powerful and fascinating." Second, the methods used to teach mathematics have "remained virtually unchanged in the last half-century."

In many classrooms, math and science learning is highly teacher directed and focuses on definitions, labels, teacher directed problem solving rather than on discovery learning that is student centered. What is needed is a student-centered learning environment. A student-centered learning environment is centered on eight key instructional and motivational factors. First, instruction must me made meaningful and relevant from the individual learner's perspective. When instruction is learner-centered it implies, according to Margaret Riel (2001) that the learner "is actively engaged in the process of knowledge construction." This means that learning is anything but "boring." In a learner-centered environment, the learners take part in setting the goals, which are then guided by the teacher.

Second, instruction must provide appropriate learning challenges and standards. Classrooms in our technology rich world can no longer reduce learning to memory exercises. While there are areas that require extensive memorization, students must be taught to think and apply that knowledge. In addition, standards of student performance must be established so that students will know what is expected of them. Riel (2001) calls this being "Assessment Centered." Teachers must know "what students are learning and what they need to know. This means that the curriculum needs to be matched to the classroom assessments. The assessments should flow out of the curriculum. A major focus should be on criterion referenced, rather than norm-referenced. One problem with norm-referenced testing is the temptation to "teach to the test." When this is done, the test, rather than the curriculum drives the learning process. In addition to tests, teachers need to find other methods of assessment such as student portfolios to determine the quality of student work.

In addition, the teachers must be what Riel (2001) terms "Knowledge Centered." Teachers must have the knowledge base to be able to evaluate the essential skills and knowledge that students need in a particular discipline. Third, instruction must accommodate needs and be supported in critical thinking and learning skills. This is also related to being "Assessment Centered" Critical thinking involves skills and dispositions. Dispositions are attitudes regarding higher-level knowledge. Tishman et al. (1995) has identified four ways to help students develop higher order knowledge. These are to use real world examples; make comparisons across disciplines; encourage interaction by engaging students in problem solving activities or inquiry; finally, give positive feedback to students when they demonstrate the appropriate use of higher order knowledge that is relevant to the subject being studied.

Fourth, instruction should attend to the climate and context in which learning occurs. Recent brain research suggests, "The richness of early learning experiences affects the physical development of the brain and may be a major cause of intellectual development. If these new theories linking learning experiences with brain development come to be accepted, the optimal match between characteristics of the learner and the learning environment, rather than parental genetic code, might be seen as responsible for school success" (Riel 2001). This means teachers must be concerned not only about children learning, but how they learn it. Teachers must crease an environment that targets a broad range of learning styles.

Fifth, instruction should honor individual needs for choice and control. This is another aspect of being "Learner Centered." McCombs (2000) notes that teachers must value the unique perspectives of the learner. For example, the student could be involved in classroom assessment of their work. Students could be asked to help design the rubric used in evaluation and then asked to apply it to their own work to determine the strengths and weaknesses of their work. They could then be given the opportunity to do the work again. Just as businesses are constantly looking for ways to continually improve their product, involving students in the assessment process not only gives them a measure of control over their work, but it helps them to know how to improve their work.

Sixth, instruction should support individual interests and creativity. The "learner-centered" teacher attempts to learn what interests the student have and allow them to work on projects or use classroom resources that target those interests. For example, students who are interested in drama may be able to use this interest as part of a project. Or, students who are interested in and have access to computer technology may be able to use this interest to make computer presentations.

Seventh, instruction should provide positive social interactions and personal relationships. This means that student-centered learning maximizes collaboration between students and between students and teachers.

Based on these eight factors, a student-centered learning environment must maximize collaboration and discovery learning. One instructional strategy that does this is Problem-Based Learning (PBL). Barbara Dutch of the University of Delaware, describes Problem Based Learning this way: "Problem-based learning (PBL) is an instructional method that challenges students to 'learn to learn,' working cooperatively in groups to seek solutions to real world problems. These problems are used to engage students' curiosity and initiate learning the subject matter. PBL prepares students to think critically and analytically, and to find and use appropriate learning resources" (Dutch http://www.udel.edu/pbl/).

The PBL approach requires teachers to become "facilitators" rather than the primary means of delivery for instruction. As facilitators, teachers structure the situation, make technology and data resources available to the students to solve the problem, and offer advice. The approach is visual, hands on, and student centered, rather than teacher driven. For example, one teacher I know created a PBL situation by accident. While preparing for a biology class she set beakers with various liquids in them and set them aside without labeling them. When the students arrived for class, she realized that she had forgotten what was in the beakers. Instead of throwing the liquids out and starting over, she turned her mistake into a learning opportunity for the students. After she explained to the students what had happened, she said that they were to design tests to see if they could figure out what was in each of the beakers. She said that the students took this as a personal challenge and were very engaged in their work! After about two days, they discovered what was in the beakers! Not only did they learn a lot about science, but, they had an opportunity to solve the problem themselves.

PBL can be enhanced by using visualization. Visualization uses technology to help students "visualize" math and science concepts rather than relying on lecture as the primary classroom delivery system. Visualization enhances PBL because it provides tools such as computer software, the Internet, calculators, and laboratory to help students solve problems. When students learn through PBL and Visualization, both students and teacher generate shared experience because the process is interactive and mediated by technology.

Another form of collaboration that could be built is collaboration between teachers and students in math and science. Collaborative learning between these disciplines would enable students and teachers to make conceptual and practical connections between the two disciplines. This would help students to get what the Glenn Report describes as the "Big Picture."

This could be accomplished by forming math science teaching teams and math/science student cohorts. Teaching teams would enable classroom teachers would be able to share ideas and identify the relationships between their two disciplines and then through visualization allow students to see for themselves these relationships.

Students could also be teamed. Rather than taking math and science separately, math/science cohorts could be created. Students would then be able to learn in an environment that favors the development of group cohesion. Cohesive groups would facilitate interaction, which should result in students feeling more free to share information with one another. This process is illustrated in Figure 1.

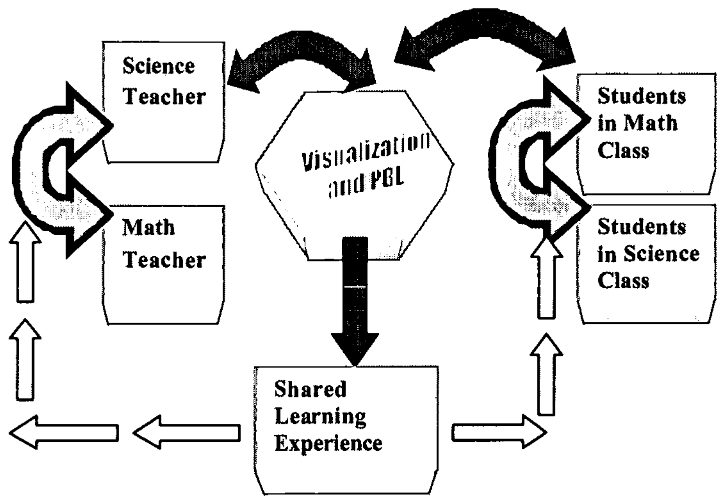

Figure 1: Visualization with Math/Science Teaching Teams

The Study

For the past two years, I've been conducting a research project based on the visualization model using math/science student cohorts and math/science teaching teams. The research was conducted in two high schools to determine the way in which visualization and PBL facilitates collaborative learning in math and science classrooms. The study also explored the problems associated with this approach to learning, the changes in classroom dynamics, and the professional benefits to teachers of this approach.

In each school a science teacher was teamed with a math teacher. One teacher on each team was experienced with at least 10 years of teaching in that field, while the other was a young teacher with less than five years of experience. Each member of the team had teaching credentials in math and science and advanced degrees in their field. In addition to the teams of math science teachers, a teacher was appointed PBL Coordinator for the project. The coordinator helped each of the teachers develop PBL activities for their classrooms.

During the school year, the teachers were given time to visit each other's classrooms and to plan as a team. Periodic visits were made to each school by the primary researcher to observe each class, and hold interviews with the teachers. A website for the project was created to collect survey data, and as a place for students to post projects. Teachers also submitted periodic reflection reports to the web site.

Students at each school were selected to participate in the math/science cohorts. The student cohorts took both math and science classes together for the entire school year Students in the cohorts had a variety of career interests and learning styles. One cohort was composed of Honor Students and the other was "general" learners. At one school the cohort met in different rooms for math and science, while at the other, they met in the same room.

Both quantitative and qualitative data from a variety of sources was gathered for this project. Some of this data was collected and stored at a web-site that was developed for this project. The data includes faculty and school and district administration interviews, student focus groups, surveys, faculty portfolios, faculty journals, faculty and department chair reflections, images of classroom activities and projects, and direct classroom observations.

Findings

There were two major areas of collaboration. There was collaboration between math and science teachers and collaboration between math and science students.

Collaboration Between Math and Science Teachers

Collaboration between math and science teachers was forged through the team approach to teaching. Both math and science teachers spent time in each other's classrooms. There are three advantages of collaboration between math and science teachers. First, math and science teachers learn about each other's discipline. One of the science teachers described his experience with the project as "enlightening" because he had the opportunity to "see how the other side lives." He observed that while math people may know the answer to a problem, science people may not have a single answer to a problem. Also data in science must be generated and is not supplied like the data in Math problems. This is an important difference between the two disciplines. One solution to this problem proposed by one of the science teachers was to find ways to generate data from science projects that could be used in Math Classes. To maximize the collaborative effort teachers must have the time to work, plan, and reflect on how their two subjects come together and how they can conceptually support each other.

A second advantage of collaboration is that teachers were able to provide better help to students in the classroom. While some were a bit nervous with having two teachers present all the time, nearly all students agreed that having two teachers helped them in the learning process. As one student put it they can "help each other." If there was a question about an area that one teacher couldn't answer, the other teacher was there to answer the question.

A third advantage is that collaboration can begin to break down the walls between the two disciplines by providing a platform for teachers to share their experiences with other faculty members in their department and in the district. By sharing these concepts, a team of teachers who are familiar with visualization and PBL can be built.

Finally, collaboration also facilitates faculty mentoring. In each team, the young teacher reported that they had learned a great deal from the experienced teacher. One of the young math teachers for example said that he had begun to think more "visually." In addition, one of the teams said that by working together they had a better idea of what their students were learning.

Collaboration Between Math and Science Students

Collaboration between math and science students took place during the classroom group activities. Heddi Pfister characterized this collaboration between students as a "support group." Generally, students at both High Schools involved in the project liked being together as a group. Many students said that they felt they could get to know others in their class better because they were together for two periods in a row.

This "student" mentoring was an important part of the experience. The students began to see persons in the class they could call on for help. One student wrote, "Since we had the same kids in both of the classes, it let us know what are the kids' strong points and weak points." In addition, some students felt that a peer in the class could explain a concept better than the teacher. A student remarked, "Sometimes you have the teacher explain it and some kids do not understand what they mean by it. So, they ask something else and they explain it in easier terms."

Conclusion

The results of the collaboration between students and teachers mediated by visualization was the development of maturing minds. For teachers, there was professional growth. Teachers experienced growth in learning new methods, restructuring the curriculum and experimenting with new technology and classroom environments.

Students grew in their level of maturity in their conceptual understanding of math and science. In a reflection about one of the classroom projects, a student wrote, "It required me to think in new ways and to work on my own without a lot of help from a teacher. This made it different from any other project I have ever done. You made you decision based on your information, data observations, and personal knowledge on the subject; which means there is no "wrong" answer as long as you have information to support your idea. Because you did it mostly on your own, it was satisfying and rewarding to see how much progress you were making and how much knowledge you were acquiring on your own."

This is what education is all about. It isn't easy. It will take many teachers and students out of their "comfort levels." But, through collaborative learning, students will be developing skills that will be even more valuable than the knowledge in a particular subject. They will experience the joy and frustrations of how to think, reason, and work together in partnership to make decisions.

References

http://www.imsa.edu/team/cpbl/whatis/matrix/matrix1.html

Dutch, B. http://www.udel.edu/pbl/

McCombs, B.L. (2000). *Assessing the role of educational technology in the teaching and learning process: A learner-centered perspective.* White paper presented at the Secretary's Conference on Technology 2000. [online: http://www.ed.gov/technology/techconf/2000/mccombs_paper.html

National Commission on Mathematics and Science Teaching for the 21st Century (The). (2000). *Before it's too late: A report to the nation from The National Commission on Mathematics and Science Teaching for the 21st Century.* [Online: http://www.ed.gov/americacounts/glenn/]

Riel, M. (2000). *New designs for connected teaching and learning*. (2000). White Paper presented at the Secretary's Conference on Technology 2000. [online:http://www.gse.uci.edu/mriel/whitepaper/]

Tishman, S. Perkins, D.N, & Jay, E. *The thinking classroom.* Boston: Allyn and Bacon, 1995.

Using Technology to Reduce Mathematics Anxiety in Preservice Elementary Teachers

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Abstract: The National Council of Teachers of Mathematics in its 1991 publication *Professional Standards for Teaching Mathematics* (NCTM, 1991) and the current *Mathematics Program Standards* for the National Council for Accreditation of Teacher Education (NCATE, 1998) stresses the importance of the disposition of the classroom teacher towards mathematics. Unfortunately, research has reported that many preservice elementary teachers have negative attitudes toward mathematics, are not confident in their own mathematics ability, and claim to have a high level of anxiety towards mathematics. The Education and Mathematics Departments of King's College recognize the phobic attitudes towards mathematics displayed by preservice elementary teachers and have collaborated to establish a plan to hopefully reverse this negative disposition. This paper reviews current strategies implemented and future techniques planned to overcome the negativity and promote a more positive disposition.

Introduction

The National Council of Teachers of Mathematics in its 1991 publication *Professional Standards for Teaching Mathematics* (NCTM, 1991) and the current *Mathematics Program Standards* for the National Council for Accreditation of Teacher Education (NCATE, 1998) stress the importance of the disposition of the classroom teacher towards mathematics. They maintain that if K-12 students are to develop a disposition to do mathematics, it is essential that the teacher communicate a positive attitude towards mathematics. Additionally, teachers need to establish a supportive classroom learning environment that fosters the confidence of students to learn mathematics. Unfortunately, research has reported that many preservice elementary teachers have negative attitudes toward mathematics, are not confident in their own mathematics ability, and claim to have a high level of anxiety towards mathematics (Harper & Daane, 1998; Tooke & Lindstrom, 1998).

The Education and Mathematics Departments of King's College recognize the phobic attitudes towards mathematics displayed by preservice elementary teachers and have collaborated to establish a plan to hopefully reverse this negative disposition. This paper will review current strategies implemented and future techniques planned to overcome the negativity and promote a more positive disposition.

Fall, 2001: Initial Implementations

During the fall semester, the mathematics methods course was taught by a mathematician for the first time. This was done with the hope that someone who actively teaches the subject and is confident in her own ability to do mathematics would model a positive attitude towards mathematics. The methods course is taken after our elementary education majors have taken three math courses. In light of the change of instructor for the methods course, we are reviewing the curricula of the math courses to be sure that they are in line with the techniques that are being stressed in the methods course.

In conjunction with the mathematics methods course, our students participate in a pre-professional field experience. This gives our students an opportunity to observe an elementary classroom, as well as present lessons. The more site-based experiences our students have prior to student teaching, the more confident they become in their ability to teach. Included in this experience is an opportunity to discuss with their classmates the lessons presented and strategies for solving problems encountered in the K-6 schools.

Additionally, we have started to collect information from our students to better understand their attitudes and backgrounds in mathematics. The initial assignment in the methods course was for the student to write his/her mathematical autobiography. This gave us a better idea of how their attitudes were shaped and the concerns they have about teaching. At the end of the semester, we issued the MARS-R to assess the mathematics anxiety that our students still have at the end of the pre-professional semester and to provide a baseline to test how future initiatives influence mathematics anxiety.

Spring and Summer, 2002: Planned Implementations:

At the beginning of the semester, we will again use the mathematical autobiography for an informal assessment of their attitudes towards mathematics. We will also use the MARS-R as both a pre-test and post-test to measure their mathematics anxiety.

We will also be constructing a web site available to students participating in the pre-professional semester and student teaching. This site will include lesson plans that our students developed during previous pre-professional semesters and links to other sites that contain lesson plans. We will be including an S.O.S. button to email the mathematics methods instructor for advice or help. The web site will also contain tips for dealing with mathematics anxiety.

During the summer, we will examine various tutorials that are commercially available to help our students review concepts, as needed, while taking the mathematics courses that are required for the major. These tutorials should also be available to students who wish to use them while in a classroom setting.

Conclusions

The Education and Mathematics Departments recognize the critical need to prepare elementary teachers with a positive disposition towards mathematics. Both departments work collaboratively in designing and evaluating the most effective math curricula for our students. We have begun appropriate interventions to ease the mathematics anxiety of our students and will continue to assess the strategies we employ. Hopefully, our efforts will help prepare teachers who can effectively teach their students mathematics in a positive learning environment.

References

Harper, W.H. & Daane, C.J. (1998). Causes and reduction of math anxiety in preservice elementary teachers. *Action in Teacher Education*. Vol. 19, No. 4, pp. 29-38.

National Council for Accreditation of Teacher Education. (1998) Program for initial preparation of K-4 teachers with an emphasis in mathematics, 5-8 mathematics teachers, 7-12 mathematics teachers. [Online].Available: http://www.ncate.org

National Council of Teachers of Mathematics (1991). Professional standards of teaching mathematics. Reston, VA.

Plake, B. & Parker, C. (1982). The development and validation of a revised version of the mathematics anxiety rating scale. *Educational and Psychological Measurement.* Vol. 42,No.2,p.551-557.

Rodriquez, B., Perez, M. & Mompo, R. (2000). TUTORPEA for adults' teachers lifelong learning. In D. Willis, B. Robin, S. McNeil, & J. Willis, (Eds.) *Technology and Teacher Education Annual, 2000.* (pp. 714-718). Charlottesville, VA:Association for the Advancement of Computing in Education.

Taplin, M. & James, J. (1994). Developing a computer assisted mathematics tutorial for preservice teachers: Some pedagogical issues. *Technology and Teacher Education Annual*, pp. 524-527.

Tooke, D. & Lindstrom, L.C. (1998). Effectiveness of a mathematics course in reducing math anxiety of preservice elementary teachers. *School Science and Mathematics*, Vol. 98, Mar. 1998, pp. 136-139.

Interactivate Your Math Students

Bethany Hudnutt, Shodor Education Foundation, US

Project Interactivate is mathematics courseware developed by the Shodor Education Foundation in collaboration with classroom teachers, content experts, curriculum designers, and education technologists. *Interactivate* has been described by the National Council of Teachers of Mathematics (NCTM) as setting "a new standard for on-line support for math teachers" (see reference, below). The project contains more than 80 classroom-tested interactive activities and tools. Suggested lesson plans and discussions based on various concepts contained in the activities help support standards-based approaches to mathematics education. Supplemental materials for the activities and lesson plans exist in the form of help files, worksheets, open-ended explorations, a dictionary, and links from those pages to related NCTM, NCEE, and DoDEA, standards.
Interactivate runs on any computing platform with any browser that supports Java and can be freely accessed on Shodor's website, http://www.shodor.org/interactivate.

The poster session will center on *Project Interactivate*. After the session, participants will be able to navigate through the site and be inspired to use *Project Interactivate* in their own classrooms. Participants will also learn about the Shodor Education Foundation as a non-profit organization dedicated to integrating computational science and modeling and visualization tools at all levels of education. Bethany Hudnutt, a former high school math teacher and currently the *Project Interactivate* manager, will present.

The presentation will suggest multiple ways of utilizing the courseware in the mathematics curriculum at all grade levels. Also, the usefulness of modeling and visualization to enhancing student understanding of math and science concepts will be discussed.

Shodor's philosophy on the role of technology in the classroom will be an integral part of the presentation. Technology should be used when the computing power of a computer will either make a task less time consuming or make a task which was impossible to complete using other means possible. For example, demonstrating the difference between empirical probability and theoretical probability using dice can be time consuming and uninteresting to students. Asking a student to roll a die one hundred times and keep track of results is a dull and time consuming task where no learning can take place until the experiment is complete. Even then, one hundred times may not be enough rolls for the empirical probability to approximate the theoretical. A computer can be programmed to simulate rolling a die thousands of times and keep track of the results in seconds. *Interactivate* activities are designed with this type of technology use in mind.

Interactivate has received wide-spread recognition and numerous awards including:

- Eisenhower National Clearinghouse Digital Dozen award (twice):
  http://www.enc.org/resources/records/full/0,1240,012956,00.shtm
  http://www.enc.org/resources/records/full/0,1240,018591,00.shtm
- National Council on Teachers of Mathematics' Illuminations:
  http://illuminations.nctm.org/webresources/pinteractive.html
- Forbes Magazine's Best of the Web:
  http://www.forbes.com/bow/b2c/category.jhtml?id=142

Interactivate originated with the Presidential Technology Initiative in partnership with the US Department of Defense Education Activity (DoDEA) with the goal of producing high-quality interactive activities to support a variety of middle-school math texts. Since its inception, *Interactivate* resources have been linked through tables of contents of various standards-based texts, including *Math Thematics, Middle Grade Mathematics, Interactive Mathematics,* and *Mathematics in Action. Interactivate* is in DoDEA schools around the world and in many schools and homes in the US, with more than 25,000 documented users per month.

PDA's: The Swiss Army Knife of Handheld Technology for Mathematics Classrooms

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Abstract: This paper provides an overview of software applications and peripheral devices designed for the Personal Digital Assistant (PDA) that can act as powerful tools in a technology enhanced mathematics curriculum. PDA applications are being introduced at a time when mathematics education reform movements are accentuating the need for functional pedagogical technologies. Attention has been focused on including applications that tend to support traditional curriculum in non-traditional ways. Excerpts from field tests have also been included.

Introduction

The willingness to embrace new technologies as tools for learning can only be justified to the extent that teachers are also willing to embrace, alter, or adapt the curriculum that supports efficient learning through these technologies. Mathematics teachers in general have been fairly willing to experiment with technology applications in an enrichment context, but perhaps less so when it comes down to engaging students in traditional textbook-based topics. Teachers of mathematics commonly report that the most productive technology-based learning environments are those in which students are actively engaged in dynamic mathematical problems involving modeling and problem solving, yet difficulties with management and assessment still have appeared to limit the commitment to creating these learning environments. Unfortunately, special efforts to utilize technology in the traditional mathematics curriculum have often resulted in little more than a series of isolated exercises done on desktop computers, and have ultimately failed to justify why technology was even needed. The Internet has certainly provided new opportunities to learn and express mathematical ideas, and perhaps more importantly created a medium for mathematical discourse. Curricular arguments involving technology however, continue to dissolved into a struggle to justify cost, availability, and time versus the proverbial "payoff" in terms of what students have learned, or at least how well they score on standardized tests. For many mathematics teachers, the solution lies somewhere in the promise that reasonable continuity can be established within and between mathematical topics, and that thoughtful productive learning occurs because of *requisite* technology rather than having technology use appear forced or contrived.

Although the primary factors for effective classroom technology use remain curricular, emerging technologies for education have demonstrated that availability and cost are much less a factor than even three years ago. The Personal Digital Assistant or PDA is a classic illustration of this point. PDA's, which are still considered by many to be day planners, have captured the attention of educators at all levels, largely because of the recent availability of software and peripheral applications. Because PDA's are handheld devices smaller than most graphing calculators, and in many cases costing about the same, they provide students an opportunity to view mathematical ideas in a variety of ways just as graphing calculators do. The many applications now available for PDA's however add a much greater dimension of realism to the mathematics curriculum than is possible with a graphing calculator, and much greater portability than is

possible with a desktop computer. The PDA is in essence a kind of Swiss Army Knife approach to handheld computing where one very small device can act as many different tools. Much of the power of these devices rests in the simplicity with which students can switch back and forth between applications within a single lesson. A lesson might include students collecting data on water samples with probe devices that can feed information directly into the PDA, organizing the data in a database application, calculating and graphing levels of impurities using spreadsheets or graphing calculators, and writing up lab results using a word processing application. The most amazing aspect of this activity would almost certainly have to be that the students could do all of these things while standing ankle deep in the water where they t ook the sample.

The scenario described above would certainly be possible given the right circumstances, as would many other scenarios, given the right software and hardware applications, and most importantly, an imaginative teacher. The next two sections of this paper provide an overview of a few software applications as well as some interesting peripheral devices that with appropriate curricular considerations can be used in some very powerful ways to support traditional mathematics curriculum.

Software Applications

In addition to the standard tools of the PDA (i.e. calendar, calculator, memo pad, "to do" list, etc.), a mountain of software as emerged virtually overnight that will allow students to seamlessly navigate through activities which require them to use data from one application to complete tasks in another. Much of this software is free and can be easily accessed through various web sites. Other software is available at nominal fees.

Graphing Calculators

Perhaps the most widely used technology to date in the mathematics classroom is the graphing calculator. Most graphing calculator models run between $70-$120 and require a significant amount of classroom time to learn the basic functions and to confidently navigate through the menus. Although these tools have a great deal of potential, students can get lost in a sequence of button pushing that may detract from the innate message of the lesson being taught. Conversely, the graphing calculator applications designed for the PDA run about $30-$50 and have a very short learning curve. Most students can have basic functions mastered in a matter of minutes. Many of these applications include the ability to view standard functions and parametric graphs, as well as allowing the user to simultaneously graph multiple equations. Graphs are managed through various types of equation "managers" and can even utilize a number of built-in functions. Functions can be explored through a series of table values as well as the standard graphing mode. One particularly powerful advantage is the zoom feature that allows the user to use the stylus to drag over an area of interest to better examine graphical properties. This includes being able to calculate values for individual points on the highlighted area of the graph which make exercises such as finding points of intersection easy to do. Most standard graphing calculators require these tasks to be managed through a series of menus. Several of these PDA graphing calculators allow users to create their own custom worksheets using the fully integrated equation solvers. These equation solver applications use a pop-up input screen that allows a user to enter a virtually unlimited number of equations and then solve for any desired value or variable. Common built in functions include those for advanced algebra, trigonometry, computer science, physics, chemistry, statistics, Boolean logic and engineering.

Animation

Free applications such as Sketchy (available at hi-ce.eecs.umich.edu) allow users to create a series of pictures that can be animated in such a way as to demonstrate geometric, algebraic, or recursive processes. Tools include basic geometric figures, lines, shading, and freehand stylus drawing. Pages can be easily duplicated and edited as well. In field tests, this program has been particularly powerful in reinforcing concepts in a tutorial setting when students need extra help but are unable to get help from the teacher. The PDA actually acts as the teacher, allowing students to view a specific procedure and stop the

animation at any point they deem necessary. Methods courses at the University of Nebraska at Omaha have included instruction in Sketchy for the purpose of re-teaching processes in mathematics, and also to help illustrate how science and mathematical topics can be better integrated in meaningful ways. Reactions from pre-service teachers of mathematics and science have been very positive. In one case an elementary teacher used Sketchy to help teach the algorithm for long division. However, instead of having Sketchy demonstrate division problems, the teacher had students build an animation showing how long division is done. The students were so motivated and excited that they finished the division unit and tested ten days earlier and scored better on the unit test than the other three classes in the school at the same grade level.

Simulations

Although simulation software is still a relatively new venture in the mathematics classroom, free programs such as Cooties (available at hi-ce.eecs.umich.edu) allow for the study of probability and mathematical modeling in the context of the spread of infectious diseases. The application uses the PDA's beaming feature to have students "meet" each other. Starting with one or more *infected* Individuals, the Cooties spread to other PDAs thus providing students with the opportunity to observe and create mathematical models for predicting how long it will take for everyone to be infected, and to pinpoint where the Cooties originated. Other simulation programs support more traditional textbook topics by allowing the students to adjust variables such as gravity coefficients and then examine the resulting effect on falling objects.

Spreadsheets

One of the more common tools used in mathematics dassrooms is the spreadsheet. Both free and nominally priced spreadsheet packages are available on-line and tend to vary in power. The more expensive versions run approximately $50 and have nearly as much power as common desktop versions. These applications also have the ability to dump data directly into spreadsheets such as Excel or MS Works. Even some of the free versions of PDA spreadsheets have graphing and logic capabilities, which make them an attractive option for linking textbook topics and problem solving. Curriculum designed for spreadsheet use on desktop computers would require very little modification and could be assigned as homework, allowing for more productive use of class time. Secondary level mathematics methods students were encouraged by the ease of which they were able to learn and use a free version spreadsheet, as well as to develop some simple curriculum related to amortization tables. These students were a bit concerned that free versions of spreadsheets tended to be limited in the number functions currently available. Trigonometric, statistical, and logic functions were routinely omitted by the free versions.

Games

In addition to the common drill and practice games, free game applications such as Code Cracker allow students to have fun in a mathematical modeling and problem-solving environment. Secondary level mathematics methods students at the University of Nebraska at Omaha competed in a contest to see who could develop the most efficient algorithm for cracking a four-digit code. Ensuing discussions among class members yielded some interesting revelations about how an effective mathematics curriculum should look. From a methods standpoint, the code cracker activity had some very productive pedagogical byproducts. Other strategy games such as Backgammon, checkers, and various card games are also extremely abundant, but have yet to show any real curricular potential past enrichment.

Peripheral Utilities

One of the most exciting aspects of the PDA is the availability of peripheral devices, which have so much potential for mathematics and science education. It is becoming more and more clear that the manner in which students collect data in the mathematics classroom can be a very motivating force for

learning. PDA's now have peripheral devices including Global Positioning System (GPS), digital cameras, digital voice recorders, a whole line of probes for temperature, turbidity, acidity, sound level, etc., and even robots that can be controlled with various program applications. All of these peripheral devices are attached directly to the PDA using the same pin connector as is used in the HotSync cradle. Managing software is included with each device, and in many cases works in unison with other PDA applications. The probes, used primarily for science activities, are still somewhat costly but allow for dynamic field experiences that would otherwise be impossible using traditional desktop computers. These probes are similar to the probes used for graphing calculators but provide greater flexibility in the collection and interpretation of data. The camera and voice recorder devices are relatively small and are primarily used as qualitative data collection instruments. Free software is available at hi-ce.eecs.umich.edu that allows pictures taken with the PDA camera to be annotated. Finally, with robotics being so pervasive in fields such as manufacturing, law enforcement, and medicine, programming robots in the classroom could provide some interesting avenues for students to investigate mathematical topics. Robot kits are currently available for use with the PDA and cost about $260. Once built, different software applications allow students to steer the robot using the stylus or even to examine how a robot *learns* using sensor technology. The applications for supporting traditional curriculum in the area of robotics are certainly limited, yet innovative teachers may see these as opportunities to motivate students in a more exciting way than ever before in an otherw ise dry subject discipline.

Conclusion

With so many possible features yet unexplored, PDA's can almost certainly be though of as a kind of 21st century Swiss Army knife for classroom technology. The utility and power of these devices goes far beyond any type of technology that has been used in mathematics classrooms to date. No longer will technology based activities need to be contrived, or forced to fit within the parameters of traditional textbook curriculum, but rather they will be supported and made more efficient by a type of technology that allows for seamless transfer of data from one type of application to another and from one subject discipline to another. One would speculate that with appropriate consideration of curriculum, we have begun to tap a source that will revolutionize mathematics education in America.

Using Online Discussion Forums to Develop Teachers' Understanding of Students' Mathematical Thinking

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Abstract: This paper is a report of the findings in a study of the development of online discussion forums to help create learning communities that provide professional development for pre- and inservice elementary school teachers in mathematics. With the support of a PT3 mini-grant, this study was designed to create opportunities for pre- and inservice teachers to build their understanding of number, number sense, computation, problem solving and mathematical communication, as they engaged with their colleagues in both monthly and on-line workgroup meetings. Three findings from this study are: sufficient time is necessary to establish a learning community, technology issues are a potential constraint in the successful development of a learning community, and specific mathematical tasks are better suited for the on-line discussion format.

Introduction

As part of the *Principles and Standards for School Mathematics* (NCTM, 2000), the standard for the responsibilities of colleges and universities includes the need for mathematics education faculty to be a part of school-based mathematics communities. "Teacher educators, mathematicians, and practicing teachers working together can create a rich intellectual environment that will promote veteran teachers' growth and demonstrate to new teachers the value of learning communities." (NCTM, 2000) Until recently, the development of these learning communities was accomplished through face-to-face interaction. This paper will describe the use of online discussion forums to help create learning communities that provide professional development for pre- and inservice elementary school teachers in both content and pedagogy reflected in the Principles and Standards for School Mathematics of the National Council of Teachers of Mathematics. With the support of a PT3 mini-grant, this study was designed to create opportunities for pre- and inservice teachers to build their understanding of number, number sense, computation, problem solving and mathematical communication, as they engaged with their colleagues in both monthly and on-line workgroup meetings.

The Study

Twenty inservice elementary school teachers from an urban, at-risk school participated in the study. The Hispanic population of the school is forty-two percent, while twenty-seven percent of the students participate in English as a Second Language curriculum. Forty-seven percent of the families are in the low-income bracket. Sixty percent of the students fall in the below average or overage group in ability level in mathematics. 35 pre-service teachers enrolled in undergraduate mathematics methods classes also participated in the project.

Each month the participating pre- and in-service teachers were given a problem to pose to their class in preparation for the in-person workgroup meeting. The problem provided mathematical focus, an opportunity to examine trajectories of students' understanding, a challenge to teachers' expectations and a way to talk about details of mathematical thinking. Having a problem to pose to their students also immediately engaged the teachers in thinking about the relationship between their children's mathematical thinking and their classroom

practice. Posing the problem led teachers to see that their classrooms could be a place that they could learn with their students.

Concurrently, the inservice teachers used digital cameras to upload examples of their student work to a private web site. As part of their elementary mathematics methods course, preservice teachers examined the students' solution strategies and related them to the NCTM Standards and research-based frameworks of the development of student understanding (Carpenter, et. al., 1999).

Threaded on-line discussions between preservice teachers, inservice teachers, a mathematics education professor, a mathematics professor and the principal of the school took place on the web site, focusing on both content and pedagogical issues. These experiences, in addition to visits to inservice teachers' classrooms, enhanced the preservice teachers' undergraduate training and focused them on "real" student work and student learning.

All of the participants attended monthly one and a half-hour workgroup meetings. The teachers began the workgroup meeting by filling out a reflection sheet that asked focused questions about their own student work or the examples provided on-line. Discussion of the student work followed for approximately one hour. The discussions were centered on the mathematical strategies evident in the student work. The range of strategies was documented and the discussion focused on how the strategies were mathematically similar or different from one another. The existing research documents the effectiveness of using students' mathematical work to support teacher learning (Franke, Carpenter, Fennema, Ansell & Behrend, 1998; Lehrer & Schauble, 1998; Sherin, 1997). Pedagogical issues were also discussed as they were raised in the workgroups. Each meeting concluded with the teachers summarizing verbally and on paper what they learned during the conversation. As the pre- and inservice teachers developed content knowledge, appropriate pedagogical strategies addressing the process standards of problem solving and mathematical communication within the context of the teachers' classrooms were also modeled.

Findings

There are three main findings that come from the current study of the development of on-line learning communities: sufficient time is necessary to establish a learning community, technology issues are a potential constraint in the successful development of a learning community, and specific mathematical tasks are better suited for the on-line discussion format.

The first finding is common to all professional development projects: sufficient time is necessary to establish a learning community. Even with the best of intentions, instantaneous familiarity between pre-service teachers, in-service teachers, administrators, education professors and mathematics professors is not possible; "sustained, intensive professional development" is very difficult to achieve in one academic year. While things are moving in the right direction at the host school, more time is necessary for change to occur. Fortunately, the current project has been extended for another year.

Though effort was made to facilitate the use of technology in the study, logistical constraints made on-going threaded discussions difficult. The initial workgroup meetings and technology orientation sessions took place in the computer lab of the host elementary school. While the pre-service teachers were regular contributors to the threaded discussion boards, in-service teacher participation was sporadic. At first, the assumption was made that this was because of familiarity with the technology. However, after examining the computers in the in-service teachers' classrooms, the discovery that the discussion board software required 4.5 browsers provided an explanation for the lack of participation: 4.0 browsers were found on all of the classroom computers. The in-service teachers assumed that their computers were simply slow. For them to effectively participate in the discussion boards, they would have to log on either in the computer lab or in the teachers' lounge. The continuity of the threaded discussions was affected by availability of the technology.

Finally, certain mathematical tasks are better suited for the on-line discussion format. There are a limited amount of tasks that can be shared among pre- and inservice teachers that truly serve both populations. Posing a word problem to use as a context for the discussion of the development of children's mathematical thinking is effective for one continuous group of pre- and in-service teachers. In the ongoing study, a new class of pre-service teachers begins in the spring semester. The new pre-service teachers require a shift in the on-line discussions, since they have neither the experience nor the familiarity with the research frameworks on the development of student thinking. Contrast this to the beginning of the project when the pre-service teachers

were more familiar with the research-based frameworks and offered these perspectives to the in-service teachers.

Discussion

On-line discussion forums are promising in the context of professional development and creating on-line learning communities. As with other forms of professional development, there are both logistical and methodological concerns about their success. Time and access to technology are two fundamental concerns in the development of on-line learning communities. Clearly, further study is necessary to better understand the interplay between the use of technology and participation in a learning community.

References

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S.B. (1999). *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

Franke, M.L., Carpenter, T., Fennema, E., Ansell, E., & Behrend, J. (1998). Understanding teachers' self-sustaining, generative change in the context of professional development. *Journal of Teaching and Teacher Education.*

Lehrer, R. & Schauble, L. (1998). *Modeling in mathematics and science.* Unpublished manuscript. Wisconsin Center for Educational Research, Madison: Wisconsin.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Sherin, M. (April. 1997). *When teaching becomes learning.* Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL.

Technological Dilemmas: A Guide to Selecting and Implementing Resources for Secondary Mathematics Instruction

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The NCTM *Principles and Standards Vision for School Mathematics* (2000) posits classrooms where all students have access to computer and calculator technologies, and where those technologies play an essential role in instruction. Examples of the effective integration of such tools into the teaching and learning of mathematics appear throughout the document. But any well-intending teacher or school inspired by this vision and equipped with a budget for such resources, on turning to the array of software titles and calculator resources on the market, faces a daunting pair of questions: How does one know what to buy? Trickier still, once the purchase orders are signed, the calculators removed from their packaging, or the software installed, how will these new tools be best used?

This paper emerged from an effort to provide preservice mathematics teachers with a framework to guide the selection and implementation of new technologies in the classroom. I propose a framework that actively engages teachers in the process of navigating the contested ideological and pedagogical waters of decision-making about technology. I argue that the complexity for practitioners of identifying the most appropriate technological resources for their students' needs, and incorporating those resources into instruction, is organized around several fundamental dilemmas for secondary math instruction. Many authors offer compelling sets of general principles and criteria by which to judge new technologies (e.g., NCTM 2000, Rubin 1999, Linn et al 2000, Nickerson 1995), each representing a clear and coherent vision of the best of contemporary insights into the teaching of mathematics. By virtue of their generality, however, they inevitably fall short of firm answers to many complex questions that can at best only be answered within the highly localized, diverse, and shifting contexts of particular classrooms.

Larry Cuban has distinguished between the "tame" and "wicked" problems faced by educational practitioners (Cuban, 2001). Problems classified as tame are generally resolvable when given proper time, research, discussion, or resources. Wicked problems, or "dilemmas", on the other hand, prove far more intractable. Such dilemmas, rather than ever achieving elegant resolution, demand difficult choices and sacrifices in order to patch together working compromises. More pointedly, while the thorniest dilemmas might be resolved in principle through the application of lofty guidelines, in the reality of school practice they are generally impossible to overcome without giving up at least some principled ground. To address this challenge, I frame the decision-making process for teachers in terms of tensions between competing poles, approaches, and outcomes, each of which potentially represent dilemmas of this sort for selecting and implementing technology. These dilemmas include: developing students' procedural skills versus teaching for understanding; using technology to promote equity versus widening the technological divide; individualizing instruction versus fostering cooperative learning; and informed use of tools versus dependence on tools.

These four sets of tensions reflect the major concerns behind many of the principles and criteria cited above as they apply to technology in math education. They are also intended to evoke some of the research that has framed and the rhetoric that has polarized debate over secondary math instruction in recent years (Battista, 1999; Becker & Jacob, 2000). The scope of these dilemmas is by no means exhaustive, but they do crucially frame many of the decisions teachers must make as they seek to draw technological resources into their practice. Lampert (1985) has argued that the key for teachers selecting between the alternatives forced by dilemmas lies in not choosing at all; in her framework such problems that emerge from practice can only be resolved in practice, through temporary, *ad hoc*, and improvisational moves that depend on the context of a particular classroom and the instincts of a particular teacher. Drawing on the insights offered by exploring these dilemmas, I offer a model for generating technology evaluation frameworks that balance the goals of principle against the necessities of practice. Using a common tool from the contemporary classroom, namely an assessment rubric, I locate the software evaluation process within the bounds of teacher assessment practice in order to highlight the ways technological dilemmas might best be resolved in practice.

Dilemmas

Procedure versus Understanding. A recurring theme in the accounts of computer-supported learning reviewed above is that the ideal contributions of instructional technology should be those which lead to deeper student understanding of mathematical concepts. Many authors make the argument even more strongly, stressing as Andee Rubin (2000) does that new technologies which simply replicate old pedagogies, even by making them more efficient or otherwise more effective, are not worth developing, given the importance of finding ways, technological or otherwise, to improve instruction through radical change and the potential of computer technologies to support such changes. As one author puts it, "given our sorry record of teaching mathematics and science, it is clear that changes in kind are needed. Changes in degree, helpful though they may be, simply are not sufficient" (Perkins et al., 1995).

But what does this mean for a teacher who, for example, believes drill-and-practice software program might shore up crucial skill deficiencies and enhance performance on high-stakes standardized tests for some or all of her students? Should she, say, embrace occasional or even regular use of a computer resource that targets procedural skill development through repetitive multiple-choice problems, and provides tutorial support only through textbook-style lessons? Or should she restrict her use of computer-based technologies to those resources (provided she can find and afford them) which emphasize higher-order reasoning, discovery learning or open-ended problem solving? The kind of procedurally oriented programs this teacher might seek emerge from a lineage grounded in behaviorist learning theory and process-product models of educational research. Such software has evolved over four decades from routine and repetitive exercise delivery that characterized the early use of computers in math instruction to contemporary efforts at intelligent computer-assisted instruction, and still thrives in a variety of more and less sophisticated forms today (Scott et al 1992).

In a comprehensive review of literature on the cultural context of computers in education, Scott, Cole, and Engel (1992) worry that while drill-and-practice programs may be of some value, they are too often used in the absence or at the expense of developing higher-order reasoning and concepts. More recent research and innovation point to a variety of alternative approaches to software-based instruction which promise to engage students in more meaningful learning. Enumerating examples of more effective uses, various authors laud the power of software to facilitate discovery learning for deeper understanding (Nickerson, 1995); to encourage students' moves from conjecture and hypothesis to abstraction and proof (Schwartz, 1995); to embed mathematical concepts in data, simulations and other real-world contexts (NCTM 2000, Rubin 1999); and to mediate students' developing understanding of concepts such as functions through multiple representations (Goldenberg, 1995). These and other alternatives to tutorial and drill-and-practice software certainly show revolutionary promise grounded in learning theory and increasingly supported by examples of successful practice (Bransford et al, 2000). But the idealized learning scenarios envisioned by the proponents of these technological innovations don't necessarily come ready-made for incorporation into the classroom as many teachers know it; they may require corresponding curricular, pedagogical and even institutional revolutions in order to be fully realized in many schools. For many teachers, the massive effort required to bring about such changes may be at cross-purposes with their need to keep up with the daily challenges of a relentless job. And even granted the value of these technological innovations for student learning, they may not erase the challenges for our teacher struggling under district and administrative pressure to raise test scores or solidify basic skills.

Equity: Technological Solutions versus the Technological Divide. While 'Equity' and 'Technology' are listed separately among the NCTM's principles (2000), the elaboration of the equity principle articulates a vision through which "technology can help achieve equity in the classroom" by widening the range of learners to whom high-level mathematics can be made accessible and engaging. Of course, technology cannot increase all students' access to mathematics unless all students have access to technology. Indeed, this claim in the 2000 NCTM standards builds upon a call made by the same organization a decade earlier for calculators and computers to be made available to every student, and for the presence of a computer for demonstration purposes in every classroom (NCTM, 1989). The overlap in NCTM's principles is hardly surprising; technological issues are often difficult to separate from economic ones. Financial resources are a necessary precursor to access to computer technologies, and technological skills arguably enhance students' future access to financial resources by qualifying them for high-paying jobs (Office of technology Assessment, 1988; Stone, 1998). We must endeavor to incorporate technology into instruction in ways that interrupt rather than replicate this closed circuit.

Any truly equitable vision of technological implementation should ideally begin with the assumption that all students have equal access to the technology in question. Such a starting point, sadly, is far removed

from reality. While wealthy or well-funded schools might purchase graphing calculators for every student, and maintain large computer labs, schools in poorer districts may not be so lucky. School resources aside, students from middle and upper class families can more easily afford their own graphing calculators and computers at home, while students from lower income families may only have access to such tools in the classroom, if at all. This disparity means that those students with easier access to calculators and computers will likely feel more at ease and have greater facility working with such tools. Hence, when a socioeconomically diverse group of students encounters instruction with such technologies, those tools may only exacerbate the consequences of unequal access and the resulting disparity in technical skill and comfort among students (Bromley, 1998).

Given this context and the prohibitions imposed by the cost not only of purchasing instructional technologies but also of training teachers in their effective implemention, innovative computer programs risk enhancing the learning only of certain students, namely those students sufficiently privileged to access them. Likewise, even when all students have equal classroom access to software or other tools, socioeconomic or other disparities may mean that some students will benefit more extensively from using those resources than others. Indeed, in an open software market, the more exotic, sophisticated, innovative, and unique the technology, likely the more expensive, and thus quite possibly the more likely it will only land at the fingertips of a select few students.

If computer software and calculators are to help ameliorate rather than accentuate these overlapping technological and mathematical divides, they must be adopted strategically, and often in ways that specifically target the needs of disadvantaged students. The NCTM equity principle emphasizes the power and the potential of technology to remedy equity concerns by making mathematical ideas and representations more accessible, more engaging, and more easily shared by a wide range of learners. Still, such potential never resides in the technology itself, but rather in the form of its implementation, and the most equitable ways to implement technology are no more easily deduced than the most equitable ways to teach mathematics. To return to the problem posed in the previous section, some authors have recently suggested that, by making the codes of the classroom and the rules for successful mathematical performance explicit, instruction which emphasizes repetition of algorithms and procedures may actually be more equitable than pedagogical strategies geared toward more open-ended problem solving and reasoning (Lubienski 2000). Boaler (2001) counters that these arguments, in highlighting the ways students might fail to achieve in classrooms that emphasize open-ended work, overlook the more crucial question of how educators might implement alternatives to procedural instruction more equitably. Seeking support for low SES students in procedurally-oriented software also may lead to a situation in which, as Scott et al (1992) caution on the basis of a 1984 Center for the Social Organization of Schools study, "wealthy and poor schools have equal numbers of computers but poor children spend their time on drill-and-practice exercises while better-off students spend their time in more meaningful activities."

Individual versus Collaborative Instruction. Just as efforts have been made in recent reforms of math curricula to shift focus from procedural or skills-based forms of mathematics instruction to those emphasizing reasoning and open-ended inquiry, those same reforms have likewise noted the value of complementing individual student work with group classroom activities (NCTM 1989). Rather than suggesting that instruction should be organized only around individuals or only around groups, calls for math education reform generally stress the importance of appropriately balancing these different pedagogical strategies (NCTM 1989; NCTM 2000). A recent volume summarizing research in learning points to studies indicating that problem-solving and reasoning strategies may be more effectively developed when students work in groups than individually, but cautions that group work may also inhibit learning in certain contexts as well, especially in early grades. Ultimately, the authors conclude that many questions about the relative benefits of group and individual work in various instructional contexts require further research (Bransford, et al 2000).

The 1989 NCTM standards take a similarly balanced view to these instructional approaches as they apply to the use of computer-based resources, simply stressing that "every student should have access to a computer for individual and group work" (p. 8). But some research suggests a stronger stance on this instructional divide. Scott, Cole, and Engel (1992) cite two early studies which indicate that given the potential benefits of pair and small-group work on computers, the optimal student-to-computer ratio may be higher than one, and that the use of individually oriented software such as drill-and-practice programs is likely to be out of synch with the goals of sophisticated and effective teaching practice. On the other hand, computers are designed primarily for single-user interface, and challenges ranging from the tendency of many software programs to target single users to negotiations over who gets to type at the keyboard or handle the mouse make group work on computers potentially difficult to implement in practice. More practically, implementing computers into both

collaborative and individual work will likely require multiple software packages, and will certainly require multiple schemes for integration into instruction. And of course, while one computer per student may be ideal, financial limitations often stop the achievement of that ideal far short of reality. Once again, teachers are faced with the endlessly complex challenges of resolving different, though potentially complementary, instructional practices against practical constraints.

Dependable Tools versus Tool Dependence. One of the most distinctive new features of the secondary math classroom over the last two decades has been the increasingly widespread use of calculators and computers as problem-solving tools. In addition to those software programs which facilitate either drill-and-practice, adaptive tutoring, or more open-ended exploration of concepts, teachers have made extensive instructional use out of software and graphing calculator features which allow for the exploration of functions, data, and equations through graphical utilities, spreadsheets, probe devices, and symbolic manipulators. These technologies not only facilitate new forms of teacher-led multimedia demonstrations and exploratory activities for students, but also place at students' fingertips the power to easily solve most problems from the pre-calculator and computer high school math curriculum with a few keystrokes. Many advocates of curricular reform see the advent of these technologies as a tremendous opportunity to push classroom activity toward more sophisticated mathematics and deeper student understanding. They argue that working with calculators and computers can foster powerful learning in the context of real data and meaningful models, increase the availability of students' attention for higher-order thinking by reducing the need for rote procedures, and provide students with essential skills for an increasingly technical world (NCTM 1989, 2000). Other educators and policy makers disagree, however, cautioning that scientific and graphing calculators and computer algebra systems may encourage tool dependence rather than skill mastery or sophisticated thinking (California State Board of Education, 2000).

Solutions

Some of these dilemmas may ring in our ears with more authenticity than others depending on the classroom contexts with which we are best acquainted, and that is precisely my point—the dilemmas themselves are as situational and emergent as their solutions. To that end, the approach I propose to educators for navigating these dilemmas as they attempt to make decisions about technology involves categorizing aspects of mathematics instruction that may be addressed by computer-based resources, and that attend to the concerns raised in the preceding section. In keeping with my theme of locating problems in practice, I borrow an educational practitioner's tool, the assessment rubric, to provide evaluative descriptors for a range of potential 'performances' for a given software package. In the sample rubric shown here (Fig. 1), criteria are provided for evaluating two different axes of performance keyed to the features different computer resources might variously emphasize: collaborative learning and individualized instruction. Other useful categories within which to consider software in a complete rubric might include skills and procedures, real-world connections, multiple representations and learning styles, problem solving, interactivity, assessment features, flexibility, and so on.

	Unsatisfactory	Satisfactory	Exemplary
Collaborative Learning	Designed exclusively for use by a single student per computer with no instructionally useful avenues for student-to-student interaction	Includes some lessons or activities which may encourage cooperative investigation or stimulate meaningful discussion among two or more students	Includes many lessons and activities that encourage cooperative investigation by two or more students through networked or multi-user frameworks, stimulating meaningful discussion and shared problem-solving
Individualized instruction	Organizes lessons and work into a highly linear or inflexible structure, offers little freedom for navigating, self-pacing	Offers some curricular flexibility, allows students to work at their own pace and repeat or skip lessons as needed	Features highly flexible, adaptive curriculum which can be tailored to individual student needs, facilitates self-paced work, accommodates a wide array of learning objectives

Figure 1: Sample software assessment rubric

The crucial features are that each category should reflect the various and potentially competing priorities that compose the dilemmas of technology selection, and that educators should have the freedom to select and prioritize these categories in accordance with their instructional goals and school or classroom constraints. In this way, rather than constraining teachers or forcing them into dilemmas, a software evaluation rubric designed

in accordance with global principles and research results should free them to select and implement resources in ways that are most appropriate for their students. To take the opposing poles of one of my dilemmas as an example, collaborative learning and individualized instruction are now posed not as competing alternatives in the quest for the 'best' uses of technology in instruction, but rather and more simply as distinct axes of software performance. Two different teachers might decide to emphasize one or the other of these uses, respectively, in evaluating software, and thus place higher priority on reviewing one category or the other, while a third teacher interested in both collaborative and individualized uses of software might focus on evaluating a package's performance in both categories while perhaps placing less weight on the effectiveness of other features.

One of the keys to the reliability of a rubric is that it be tested against a large sample by multiple judges; the sample here is still only in the earliest stages of such development (Wiggins, 1998). But with continued refinement, a rubric of this sort might serve at least two powerful purposes. By making performance standards explicit along a range of instructional categories, this evaluative mechanism should serve not only as a flexible tool for technological decision making for educators, but also as a guide to software designers that might help them to better account for the realities of the classrooms into which they might insert their products. Just as assessment rubrics in the classroom provide students and teachers with a common frame of reference for exemplary performance standards, the same assessment tool might be used in technology evaluation to raise the bar for design and instructional implementation alike.

References

Battista, Michael T (1999). "The Mathematical Miseducation of America's Youth: Ignoring Research and Scientific Study in Education." *Phi Delta Kappan*, Feb. 1999, p. 424.

Becker, Jerry P. and Bill Jacob (2000). "The Politics of California School Mathematics: The Anti-Reform of 1997-99." *Phi Delta Kappan*, Mar. 2000, p. 529.

Boaler, Jo (2001). Learning from Teaching: Exploring the Relationship between 'Reform' Curriculum and Equity. (Submitted to *Journal for Research in Mathematics Education*).

Bransford, John D., Ann L. Brown, and Rodney R Cocking, eds. (2000). *How People Learn: Brain, Mind, Experience, and School.* Washington, D.C.: National Academy Press.

Bromley, Hank (1998). "Introduction: Data-Driven Democracy? Social Assessment of Educational Computing" in Hank Bromley and Michael Apple, ed., *Education/Technology/Power*. Albany, NY: SUNY Press.

California State Board of Education (2000). Mathematics Framework for California Public Schools. CA: California Department of Education.

Cuban, Larry (2001). *How Can I Fix It? Finding Solutions and Managing Dilemmas.* New York: Teachers College Press.

Goldenberg, Paul (1995). "Multiple Representations: A Vehicle for Understanding Understanding" in David Perkins, Judah Schwartz, Mary Maxwell West, and Martha Stone Wiske, ed., *Software Goes to School.* New York, NY: Oxford University Press.

Lampert, Magdalene. (1985). "How Do Teachers Manage to Teach? Perspectives on Problems in Practice." *Harvard Educational Review, 55*(2), 178- 194.

Linn, Marcia C., James D. Slotta, and Eric Baumgartner (2000). *Teaching High School Science in the Information Age: A Review of Courses and Technology for Inquiry-based Learning.* A report for the Milken Family Foundation.

Lubienski, Sarah (2000). "Problem Solving as a Means Toward Mathematics for All: An Exploratory Look Through a Class Lens." Journal for Research in Mathematics Education, 31(4), 454-482.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: Author

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: Author

Nickerson, Raymond (1995). "Can Technology Help Teach for Understanding?" in David Perkins, Judah Schwartz, Mary Maxwell West, and Martha Stone Wiske, ed., *Software Goes to School.* New York, NY: Oxford University Press.

Perkins, David, Judah Schwartz, Mary Maxwell West, and Martha Stone Wiske, ed. (1995). *Software Goes to School.* New York, NY: Oxford University Press.

Rubin, Andee (1999). Technology Meets Math Education: Envisioning a Practical Future.

Scott, Tony, Michael Cole, and Martin Engel (1992). Computers and Education: A Cultural Constructivist Perspective. *Review of Research in Education, 18*, 191-251.

Schwartz, Judah L (1995). "Shuttling Between the Particular and the General: Reflections on the Role of Conjecture and Hypothesis in the Generation of Knowledge in Science and Mathematics" in David Perkins, Judah Schwartz, Mary Maxwell West, and Martha Stone Wiske, ed., *Software Goes to School.* New York, NY: Oxford University Press.

Wiggins, Grant (1998). Educative Assessment: Designing Assessments to Inform and Improve Student Performance. San Francisco, CA: Jossey-Bass.