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ABSTRACT

In this study I examined 19 preservice secondary mathematics teachers' solution processes to word problems for which the subtraction or addition of the two given numbers yields 1 more or 1 less than the correct solution. Among the aspects of their solution processes that were examined are: the modeling strategies, the type of errors, and the interpretations of the solutions produced by the procedure. It was found that about 87% of the solution processes to such problems contained formal strategies while about 13% contained counting strategies. It was also found that about 61% of the responses contained errors of which 91% were ± 1 errors. That is, errors due to the interpretation that the answer provided by the addition or subtraction of the two given numbers is the solution to the problem. It is argued that some incorrect interpretations were due, at least in part, to a lack of understanding of the connection between the enumeration process needed to obtain the solution to a problem and the answer provided by the addition or subtraction of the two given numbers. (Author)

# PRESERVICE SECONDARY MATHEMATICS TEACHERS' MODELING STRATEGIES TO SOLVE PROBLEMATIC SUBTRACTION AND ADDITION WORD PROBLEMS INVOLVING ORDINAL NUMBERS AND THEIR INTERPRETATIONS OF SOLUTIONS

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In this study I examined 19 preservice secondary mathematics teachers' solution processes to word problems for which the subtraction or addition of the two given numbers yields 1 more or 1 less than the correct solution. Among the aspects of their solution processes that were examined are: the modeling strategies, the type of errors, and the interpretations of the solutions produced by the procedure. It was found that about 87% of the solution processes to such problems contained formal strategies while about 13% contained counting strategies. It was also found that about 61% of the responses contained errors of which 91% were ± 1 errors. That is, errors due to the interpretation that the answer provided by the addition or subtraction of the two given numbers is the solution to the problem. It is argued that some incorrect interpretations were due, at least in part, to a lack of understanding of the connection between the enumeration process needed to obtain the solution to a problem and the answer provided by the addition or subtraction of the two given numbers.

Current reform documents such as *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) call for students to learn to solve non-routine problems and to establish connections between mathematical ideas and real-world situations. Special types of non-routine problems include problematic story problems involving arithmetic operations. For the purpose of this article, a problematic story problem is a problem for which the result provided by the mathematical operation or procedure with the numbers given in the problem statement does not necessarily represent the solution to the problem. Research studies (e.g., Cai & Silver, 1995; Contreras, 2001; Greer, 1993, 1997; Nesher, 1980; Reusser & Stebler, 1997; Silver, Shapiro, & Deutsch, 1993; Verschaffel & De Corte, 1997; Verschaffel, De Corte, & Lasure, 1994; Verschaffel, De Corte, & Vierstraete, 1999; Verschaffel, Greer, & De Corte, 2000) suggest that students tend to approach problematic story problems mechanically or superficially without paying attention to the realistic considerations of the situational context of the problem or to their modeling assumptions. Some examples of problematic word problems used in some of these studies are the following:

(a) What will be the temperature of water in a container if you pour 1 l of water at 80° and 1 l of water of 40° into it? (Nesher, 1980)

(b) John's best time to run 100 m is 17 sec. How long will it take to run 1 km? (Greer, 1993)

(c) The Clearview Little League is going to a Pirates game. There are 540 people, including players, coaches, and parents. They will travel by bus, and each bus holds 40 people. How many buses will they need to get to the game? (Silver, Shapiro, & Deutsch, 1993)

(d) Lida is making muffins that require 3/8 of a cup of flour each. If she has 10 cups of flour, how many muffins can Lida make? (Contreras & Martínez, 2001)

(e) In September 1995 the city's youth orchestra had its first concert. In what year will the orchestra have its fifth concert if it holds one concert every year? (Verschaffel, De Corte, & Vierstraete, 1999)

Verschaffel, De Corte, and Lasure (1994) used, among others, items a) and b) in their study involving 75 fifth graders in Flanders. Their analysis revealed that only 7 (9%) students provided a realistic and correct response to the first problem and only 2 (3%) provided a response to the second problem that was based on realistic considerations. Silver, Shapiro, and Deutsch (1993) investigated 195 middle grade students' solution processes and their interpretations of solutions to the third problem. They reported that about 22% of the students correctly performed an appropriate procedure but provided an incorrect solution without explicit interpretation. Most of the students interpreted the result of the division (e.g., 13 or 13 with another number) as the number of needed buses. In their study, Contreras and Martínez (2001) examined 68 preservice elementary teachers' solution processes and realistic reactions to the fourth problem. They reported that only 19 (28%) of the participants' responses contained a realistic solution to the problem. They also reported that none of the participants made any comments about the problematic nature of the problem. Finally, Verschaffel, De Corte, and Vierstraete (1999) examined 199 upper elementary school pupil's difficulties in modeling and solving problematic additive word problems involving ordinal numbers. They administered the subjects a paper-and-pencil test consisting of 17 word problems, nine of which were experimental items and eight buffer items. Three of the nine experimental items can be solved by a simple addition or subtraction of the two given numbers. An example of this type of problems is "In January 1995 a youth orchestra was set up in our city. In what year will the orchestra have its fifth anniversary?" The solution to the other six items is either 1 more or 1 less than the answer provided by the addition or subtraction of the two given numbers. An example of this kind of problems is problem e) stated above. Verschaffel, De Corte, and Vierstraete found that the percentage of correct responses for each of the six problematic word problems was less than 25%. They reported that 83% of the errors made on these problems were ±1 errors. That is, most of the pupils' errors were due to their interpretation that the addition or subtraction of the two given numbers provides the solution to the problem.

As argued by Verschaffel, De Corte, and Vierstraete (1999), a problem with all of these investigations and others reported in the literature is that they have involved elementary or secondary students and, thus, a possible generalization to college students and, in particular, to prospective secondary mathematics teachers has not been established empirically. Second, since prospective secondary mathematics teachers are a more mature population from a psychological and mathematical point of view, it would be worthwhile to analyze their solution processes to examine the strategies, errors, and interpretations when solving problematic word problems. Finally, it is important to document, examine, and understand prospective secondary mathematics teachers' strategies, errors, and interpretations when modeling and solving problematic word problems because the teacher is one of the major agents in the classroom. It is the teacher who designs, adapts, or implements the instructional activities. If we want students to model and solve problematic word problems by taking into account the realistic considerations embedded in the problem situation, then it is necessary that the teachers themselves have the experience, disposition, and ability to model and solve such problems realistically. The purpose of this study is to extend Verschaffel, De Corte, and Vierstraete's (1999) investigation. First, I examine prospective secondary mathematics teachers' modeling strategies to solve problematic subtraction and addition word problems involving ordinal numbers. Second, I examine the type of errors, if any, that secondary teachers make when solving such problems. Finally, I examine their interpretations of the solutions provided by the procedure or mathematical model.

## Theoretical Framework

Aspects of reality can be represented by mathematical means. This process of representation is called mathematical modeling. Some physical or real-world problems can also be solved by means of a process of mathematical modeling such as the one depicted in Figure 1 that was proposed by Silver, Shapiro, and Deutsch (1993). There are other models described in the literature (e.g., Verschaffel, Greer, & De Corte, 2000) but Silver, Shapiro and Deutsch's model is appropriate for the present study.

According to Silver, Shapiro, and Deutsch's model, the (simplified) process of mathematical modeling involves four phases. The first phase consists of understanding the structure of the mathematical problem embedded in the story text. During this phase we need to understand the given information, the unknown information, extraneous information, and realistic considerations embedded in the situational context. The second phase consists of constructing a mathematical model or selecting an appropriate procedure, operation, or algorithm whose result will lead us to the solution of the word problem. During the third phase we execute the procedure or algorithm. Finally, we interpret the result provided by the mathematical model or procedure in terms of the realistic context embedded in the story text of the word problem or in terms of the real-world story situation. It is during the fourth phase that we focus on the meaning of the answer produced by the mathematical procedure or computation. Students' responses to problematic word problems could include realistic or correct

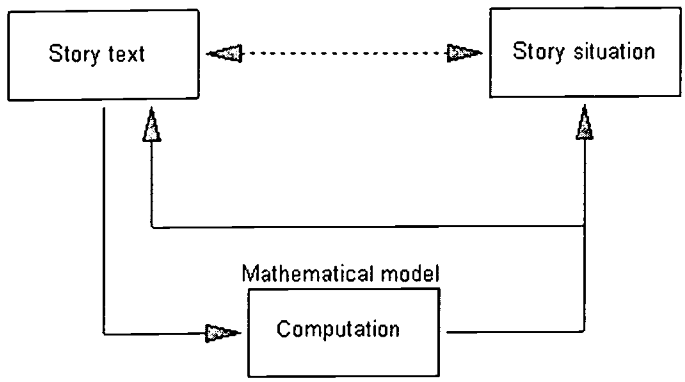


*Figure 1.* Silver et al.'s (1993) referential-and-semantic-processing model for successful solutions.

solutions if they select an appropriate procedure or operation and understand or pay more attention to the meaning of the result produced by the mathematical model.

Silver, Shapiro, and Deutsch's model implies that there are three main potential sources of errors when solving word problems: lack of understanding of the problem, which is suggested when an inappropriate procedure is chosen, incorrect execution of procedures, and incorrect interpretation of the answer produced by the mathematical model or procedure. In their study of the division problem involving remainders stated above, Silver, Shapiro, and Deutsch (1993) found that most of the students' responses, 91% in fact, contained an appropriate procedure (e.g., long division, repeated multiples, repeated addition, etc.) but only 61% of the students who selected an appropriate procedure performed it flawlessly (about 56% of the total number of students). These researchers reported that only 43% of the total number of students provided the correct answer of 14 to the problem but that some of them gave inappropriate interpretations. For example, one student wrote "14 buses because there's leftover people and if you add a zero you will get 130 buses so you sort of had to estimate. Are we allowed to add zeros?" (p. 124-125). About 55% of the students did not get the correct answer because they either failed to interpret the answer produced by the division computation or made computational mistakes that could have been detected if students had interpreted their solutions correctly. The researchers proposed the model exhibited in Figure 2 as a schematic representation of an unsuccessful solution. That is, some students failed to get the correct solution to the problem because they did not map the result produced by the mathematical model (in this case, a division) back to either the story text or the real-world story situation.

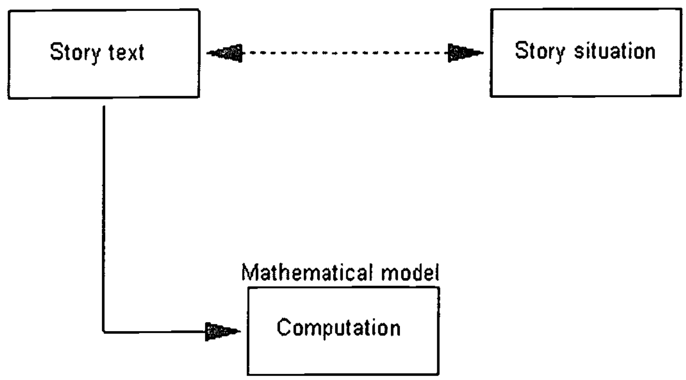


*Figure 2.* Silver et al.'s (1993) referential-and-semantic-processing model for unsuccessful solutions.

### Methods and Sources of Evidence

A paper-and-pencil test was administered to 19 prospective secondary mathematics teachers from three required mathematics classes. Fifteen students were female and four male. All students except one were mathematics majors. At our institution, teachers seeking 7-12 certification in mathematics are required to complete a mathematics major. The non-mathematics major was seeking a supplementary endorsement in mathematics. The written directions included asking students to show work to support their responses. Calculators were not allowed. The test contained nine experimental items and some buffer items. The experimental items were adapted from Verschaffel, De Corte, and Vierstraete's (1999) test. Table 1 displays the nine experimental items. All the experimental items were addition and subtraction word problems involving ordinal numbers. Three of the nine experimental items can be solved by the straightforward addition or subtraction of the two numbers given in the problem statement. The solution of the other six items is 1 more or 1 less than the answer produced by the subtraction or addition of the given numbers.

A difference from some previous research, where the word problems have been designed in an ad hoc way, Verschaffel, De Corte, and Vierstraete's were based on a taxonomy of the possible modeling complexities. The nine experimental items differ in terms of two dimensions: (a) the nature of the underlying mathematical structure and (b) the nature of the unknown information. We can distinguish three categories of problems (Types I, II, and III) based on mathematical structure. The solution of Type I problems can be obtained by adding or subtracting the two numbers given in the problem. The solution of Type II and Type III problems is 1 more or 1 less than the answer produced by the addition or subtraction of the two given numbers. Types

**Table 1.** The Nine Experimental Items

| Type | Item | Required operation(s)* |
|---|---|---|
| I-L | 1. In January 1985 a youth orchestra was set up in our city. In what year will the orchestra have its twenty fifth anniversary? | S + D |
| I-D | 2. Our youth club was set up in September 15, 1970. I became a member in September 15, 1999. How many years had the club already existed when I became a member? | L - S |
| I-S | 3. In March 2000 it had been 34 years since our school had held its first annual school party. In what year was the school party held for the first time? | L - D |
| II-L | 4. In September 1975 the city's youth orchestra had its first concert. In what year will the orchestra have its fiftieth concert if it holds one concert every year? | (S + D) - 1 |
| II-D | 5. Last October (2001) I participated for the first time in the great city running race that is held every year. This race was held for the first time in October 1959. How many times has the race been held? | (L - S) + 1 |
| II-S | 6. In November 1994 the twenty fifth annual school party took place. In what year was the school party held for the first time? | (L - D) + 1 |
| III-L | 7. There was a summer market in our city every summer from 1950 up through 1969. Since then the summer market was cancelled 30 consecutive times. In what year did the summer market restart? | (S + D) + 1 |
| III-D | 8. For a long time the city held a fireworks display every year on the last day of the October festival. In October 1982 we had our last fireworks, and thereafter there was no fireworks display. In October 1999 they restarted the tradition of the annual fireworks display. How many years did we miss the fireworks? | (L - S) - 1 |
| III-S | 9. In December 1999 our sports club held its annual election for its officers. Because of a lack of candidates, there had not been elections for the 23 years preceding 1999. Prior to this election, in what year did the last election occur? | (L - D) -1 |

* L = larger ordinal number
S = smaller ordinal number,
D = difference between the two ordinal numbers.

II and III problems differ in the nature of the enumeration process used to obtain the solution. For Type II problems, the enumeration process begins with the smaller number or the larger number. For Type III problems, the enumeration process needed to obtain the solution does not include neither the smaller nor the larger number. With respect to the nature of the unknown information, three categories of problems can be distinguished: (a) problems for which the larger number is unknown (Type L problems), (b) problems in which the smaller number is unknown (Type S problems), and (c) problems for which the difference between the two ordinal numbers is unknown (Type D problems). Combining the two dimensions in which the problems differ, we can obtain nine possible different types of addition and subtraction word problems involving ordinal numbers.

The main source of data was the written responses provided by the prospective secondary mathematics teachers. I recognize that written responses have some limitations as compared to verbal protocols. However, several researchers (e.g., Hall, Kibler, Wenger, & Truxaw, 1989) have validated the use of written responses to infer cognitive processes. In fact, I did not have any difficulty to determine the strategies that the prospective secondary mathematics teachers used to solve the problems. Nevertheless, I conducted interviews with the students to gain a deeper understanding of the thinking and reasoning that students used to solve the problematic word problems.

## Analysis and Results

Students' written responses were analyzed with respect to four aspects of the process of mathematical modeling represented in Silver, Shapiro, and Deutsch's (1993) model: (a) the strategy, procedure, and operation used by the students to solve each problem, (b) the execution of procedures, (c) the solution to each problem, and (d) the (implicit or explicit) interpretation of the result produced by the mathematical model or procedure. I also conducted an error analysis to determine the type of errors that prevented students from obtaining the correct solution to each experimental item. The students produced a total of 171 responses (57 responses to the non-problematic experimental items and 114 responses to the problematic experimental items). The strategies used by the students were categorized as formal strategies (addition or subtraction of the two numbers given in the problem), or informal (e.g., counting). A total of 55 (96%) responses to the non-problematic items contained a formal strategy, one (2%) contained a counting technique, and another one (2%) contained solving a similar simpler problem. On the other hand, 99 (87%) responses to the problematic items contained a formal strategy and the remaining 15 (13%) contained counting techniques. Overall, 154 (90%) responses contained a formal strategy, 16 (9%) contained counting techniques, and only one (1%) involved solving a similar simpler problem. Students' responses were also analyzed to determine the appropriateness of the procedure, algorithm or operation used to solve the problems. A procedure was judged as appropriate if it could lead to the correct solution or as inappropriate otherwise. Not surprisingly, all students used appropriate procedures. With respect to the execution of procedures, students performed 162 (95%) procedures correctly. Regarding the solutions to the experimental problems, 85 (50%) of the responses contained correct solutions. Specifically, 41 (72%) of the 57 responses of the non-problematic word problems contained correct solutions and only 44 (39%) of the 114 solutions to the six problematic word problems were correct. Considering that all subjects but one were mathematics majors, the percentage of correct solutions was much lower than I expect it. The percentage of correct solutions to each experimental item is exhibited in Table 2.

As we can see from Table 2, the percentage of correct solutions to the problematic word problems varied from 32% (problems 4 and 6) to 47% (problem 8). Even though the participants have probably had extensive experience with routine arithmetic word problems, I was expecting that this sample of prospective secondary mathematics teachers would perform much better on the six problematic word problems because all but one were math majors. Since a high percentage of the procedures was executed correctly, I conducted an error analysis to find out what prevented the prospective teachers from getting the correct answer on the problematic word problems and to further our understanding of students' solution processes. Based on previous research, I predicted that most of the students' errors were ±1 errors. The results are displayed in Table 3.

The error analysis confirmed my prediction. As shown in Table 3, a high percentage of errors for each problematic item was ±1 errors. Overall, the percentage of ± 1 errors made on the problematic items was 91%. A total of 64 (56%) of the solution processes to the problematic items contained ±1 errors and 6 (5%) contained other kinds of errors. The error analysis indicates strongly that the errors on the problematic items resulted from students' interpretation that the addition or subtraction of the two numbers given in such problems provides the solution.

## Discussion and Conclusion

The major purpose of this study was the examination of prospective secondary mathematics teachers' solution processes when solving problematic addition and

**Table 2.** Percentage of correct solutions for each experimental word problem

| Problem | Number of correct solutions | Percentage of correct solutions |
|---|---|---|
| 1 | 14 | 74% |
| 2 | 15 | 79% |
| 3 | 12 | 63% |
| 4 | 6 | 32% |
| 5 | 7 | 37% |
| 6 | 6 | 32% |
| 7 | 8 | 42% |
| 8 | 9 | 47% |
| 9 | 8 | 42% |

**Table 3.** Type of ± 1 Errors for Each of the Six Problematic Word Problems

| Type of problem | Required operation(s) | Type of ±1 error | Percentage of students' ± 1 errors |
|---|---|---|---|
| II-L | (S + D) - 1 | + 1 error | 85% |
| II-D | (L - S) + 1 | - 1 error | 83% |
| II-S | (L - D) + 1 | -1 error | 92% |
| III-L | (S + D) + 1 | -1 error | 100% |
| III-D | (L - S) -1 | + 1 error | 100% |
| III-S | (L - D) - 1 | + 1 error | 91% |

subtraction word problems involving ordinal numbers to determine the nature of their modeling strategies, interpretations, and errors. A paper-and-pencil test was administered to a total of 19 participants. The test included nine experimental items, three of which were non-problematic and the other six were problematic. The solution of the problematic items is 1 more or 1 less than the addition or subtraction of the two given numbers. Overall, it was found that 90% of the responses contained a formal strategy (addition and subtraction with the two given numbers). The other 10% contained informal strategies (counting techniques and solving a simpler similar problem). It is worth to notice that, although prospective secondary mathematics teachers have more familiarity with anniversaries and their mathematical knowledge is more developed than that of the fifth and sixth graders from Verschaffel, De Corte, and Vierstraete's (1999) study, some of their successful solutions to the problematic items were obtained with counting strategies. This suggests that students knew that the addition or subtraction of the two given numbers does not yield the correct answer to the problem and that some adjustment had to be made but they did not know how to make it. It could also indicate that, at least for the subjects using the counting strategies, their knowledge of addition and subtraction involving ordinal numbers was not completely developed. Both conjectures were verified with interviews conducted with the participants.

While the results for the non-problematic items were less than satisfactory (72% of the solutions were correct), the results for the problematic items were alarming (39% of the solutions were correct), especially given that all but one of the participants were majoring in math. An analysis of errors revealed that 56% of the solution processes to the problematic items used by the students contained ±1 errors. That is, it seems that students interpreted that the addition or subtraction of the two given numbers yielded the correct solution to the problematic word problems. Therefore, the model depicted in Figure 2 seems to explain, at least in part, the ±1 errors: students failed to correctly interpret the result of adding or subtracting the two given numbers. However, a deeper, perhaps more important question remains: why did some students interpret the result of the addition or subtraction with the two given numbers as the solution to the problematic items? Several hypotheses could be offered to explain this finding. First, there is the possibility that students approached the problems in a mechanical way because they are used to solve addition and subtraction problems in a straightforward manner. This hypothesis is supported, at least partially, by some students who stated during the interviews that they were not used to think when solving this type of problems because the problems involving addition and subtraction that they have encountered previously had been solved with an addition or subtraction of the two given numbers. A second hypothesis is that some students lack an awareness of informal techniques such as drawing a diagram or solving a simpler similar problem or they may have an underdeveloped repertoire or understanding of such heuristic techniques. I do not offer any direct evidence to support or refute this conjecture. The third hypothesis is that some students do not have a clear understanding of addition and subtraction involving ordinal numbers. This hypothesis is supported by some students who were aware of the problematic nature of the problem but they did not know how to adjust the result produced by the straightforward application of the addition or subtraction of the two given numbers. Some of these students obtained the correct solution by counting techniques. The interviews revealed that these students knew that an addition or subtraction was involved but not how to make the adjustment either on the solution provided by the addition or subtraction of the two given numbers or on the given numbers. It seems then that a plausible explanation for students' lack or interpretation (or misinterpretation) or their use of counting techniques is that they do no have a complete understanding of addition and subtraction involving ordinal numbers. This explanation is in contrast with the one provided by Silver, Shapiro and Deutsch (1993) to understand some middle grade students' solutions to the bus problem when their responses involved 13 or 13 with another number such as a fractional remainder. Silver, Shapiro, and Deutsch reported that about 22% of the students were able to correctly perform an appropriate procedure but did not provide an interpretation for their incorrect numerical answer. These researchers also found that nearly 24% of the students performed the computation procedure incorrectly and provided a numerical solution other than 14 with no interpretation. The researchers argue that both sets of students failed to obtain the correct answer of 14 because they failed to interpret the solution provided by the mathematical procedure. In the case of division problems involving remainders, the lack of interpretation (or misinterpretation) is rooted more deeply on the meaning of the quotient and remainder than on the understanding that the solution of the problem can be represented with a division of the two given numbers. In the present study, in contrast, the lack of interpretation, or misinterpretation, may lie more on an incomplete understanding of the connection between the nature of the enumeration process needed to obtain the solution and the answer provided by the addition and subtraction of the two given numbers. It seems that some of the middle grade students understood that a division with the two given numbers was needed to solve the bus problem but failed to interpret the remainder. In the present study, some prospective secondary mathematics teachers, in contrast, might not have understood that the addition or subtraction of the two given numbers did not provide the solution to some of the subtraction and addition word problems involving ordinal numbers. It seems then that the semantic feature of Silver, Shapiro, and Deutsch's (1993) model does not completely account for the ±1 errors made by the prospective secondary mathematics teachers who chose an appropriate procedure and executed it correctly.

Since a high percentage of errors for the problematic word problems was ±1 errors, 91% in fact, it seems that the prospective secondary mathematics teachers need at least some minimal intervention such as telling them that some of such problems are "tricky" or creating a cognitive conflict by asking them to solve simpler similar problems. The cognitive conflict technique was used during the interviews with some of the students who solved all the problematic word problems by adding or subtracting the two given numbers. Some of them immediately realized that some adjustment had to be made to solve the problems correctly.

While the sample size does not allow to generalize any of the results to larger populations of prospective secondary mathematics teachers from the USA or any other country, this study provides useful practical and theoretical information. From a practical point of view, at the very least, this study suggests that some prospective secondary mathematics teachers might approach some problematic word problems, such as the ones examined here, in a mechanical way. This suggests the introduction of problematic word problems in the school curriculum so that future teachers learn to approach word problems with a realistic perspective. Another contribution of this study is that some prospective secondary mathematics teachers might have an incomplete understanding of subtraction and addition involving ordinal numbers and these teachers will need more than a minimal intervention. Another contribution of this study is related to helping prospective secondary mathematics teachers develop a disposition to provide their students with problematic word problems so they (their students) learn to solve such problems realistically, and, as it is the case with the problems used in this study, develop a deeper understanding of addition and subtraction involving ordinal numbers. From a theoretical point of view, the findings help us to better understand some of the psychological aspects of learning mathematics within the context of problematic word problems. The results also shed some light on some aspects of Silver, Shapiro, and Deutsch's (1993) referential-and-semantic processing model as discussed above.

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