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ABSTRACT

In this workshop, we will continue to reflect on a models and modeling perspective to understand how students and teachers learn and reason about real life situations encountered in a mathematics and science classroom. We will discuss the idea of a model as a conceptual system that is expressed by using external representational media, and that is used to construct, describe, or explain the behaviors of other systems. We will consider the types of models that students and teachers develop (explicitly) to construct, describe, or explain mathematically significant systems that they encounter in their everyday experiences, as these models are elicited through the use of model-eliciting activities (Lesh, Hoover, Hole, Kelly, & Post, 2000). During the workshop we will continue to explore these aspects of learning, teaching, and research by continuing our work in smaller groups focusing in: Student Development, Teacher Development, Assessment, Curriculum Development, Problem Solving, and an emphasis on Research Design. (Author)

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MODELS AND MODELING

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In this workshop, we will continue to reflect on *a models and modeling perspective* to understand how students and teachers learn and reason about real life situations encountered in a mathematics and science classroom. We will discuss the idea of a model as a conceptual system that is expressed by using external representational media, and that is used to construct, describe, or explain the behaviors of other systems. We will consider the types of models that students and teachers develop (explicitly) to construct, describe, or explain mathematically significant systems that they encounter in their everyday experiences, as these models are elicited through the use of model-eliciting activities (Lesh, Hoover, Hole, Kelly, & Post, 2000). During the workshop we will continue to explore these aspects of learning, teaching, and research by continuing our work in smaller groups focusing in: Student Development, Teacher Development, Assessment, Curriculum Development, Problem Solving, and an emphasis on Research Design.

Highlights of a *Models and Modeling Perspective*

According to this perspective, models are conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other systems. A mathematical model focuses on structural characteristics of the relevant systems (Lesh & Doerr, in press). These models reside inside the minds of the learners *and* are embodied in the equations, diagrams, computer programs, or other representational media. Because models are conceptual systems, they are partly internal and are similar to the conceptual systems that cognitive scientists refer to as cognitive structures. Nevertheless, the conceptual systems that are most powerful and useful seldom function in sophisticated ways unless they are expressed using spoken language, written symbols, concrete materials, diagrams or pictures, computer programs, experience-based metaphors, or other representational media.

Students have preconceived notions about many important mathematical constructs. However, it is impossible for anyone to know exactly what's inside a students' mind. But, when students are asked to develop a model (which is expressed in some kind of representational media), many inferences can be made about the nature of their mathematical knowledge and its development. These representational systems encourage students to externally present their ideas on paper or other media. Therefore, the external representations that these students create are a means for researchers and

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teachers to view how students are thinking. In an analogous situation, when teachers are asked to develop a model about their students' ways of thinking, this model becomes a powerful window to view how teachers are thinking about this situation.

In order for students and teachers to externalize their understanding of complex situations, we have created different tools that elicit this type of thinking. Model-eliciting activities require students or teachers: (1) to develop a model that describes a real-life situation, (2) that encourages the solver to describe, revise, and refine their ideas; and (3) that encourages the use of representational media to explain (and document) their conceptual systems. These activities are similar to many real life situations in which mathematics is useful. Model-eliciting activities can be designed to lead to significant forms of learning because they involve mathematizing –by quantifying, dimensioning, coordinating, categorizing, algebraizing, and systematizing relevant objects, relationships, actions, patterns, and regularities.

For example, model-eliciting activities designed for students are intended for the solvers to reveal the way they are thinking about a given real-life situation that can be modeled through mathematics. These activities present real-life problem, to be solved by small groups of 3-5 students. The solution calls for a mathematical model to be used by an identified client, or a given person who needs to solve the problem. In order for the client to implement the model adequately, the students must clearly describe their thinking processes and justify their solution. Thus, they need to describe, explain, manipulate, or predict the behavior of the real world system to support their solution as the best option for the client. Like in real life, there is not a single solution, but there are optimal ways to solve the problem.

Model-eliciting activities are designed to engage students in developing math concepts. They set the students into a familiar context in which they are able to understand a need to develop powerful math ideas in order to solve a problem that is meaningful to them. That is, they are given a purpose to develop a mathematical model that best explains, predicts, or manipulates the type of real-life situation that is presented to them. Thus, students are forced into a cognitive situation where they can refine their mathematical ideas iteratively until they develop a construct that is useful and meaningful for them and for their client. Students' descriptions, explanations and constructions given to the client reveal how they are interpreting the mathematical situations they encounter by disclosing how these situations are quantified, organized, coordinated, and interpreted. In this way, model-eliciting activities allow students to document their own thinking and learning development.

A more complete description of *a models and modeling perspective*, written by many of the leading participants in this workshop, can be found in the book *Beyond Constructivist: A Models & Modeling Perspective on Mathematics Teaching, Learning, and Problems Solving* (Doerr & Lesh, in press). This book including chapters that focus on student development (Aliprantis & Carmona, in press; Dark, in press; Johnson & Lesh, in press; Kardos, in press; Lehrer & Schauble, in press; Lesh & Carmona, in press; Lesh, Zawojewski, & Carmona, in press; Oakes & Rud, in press;), teacher development (Doerr & Lesh, in press; Koellner Clark & Lesh, in press; McClain, in press; Schorr & Lesh, in press; van Reeuwijk & Wijers, in press), curriculum development (Carlson, Larsen, & Lesh, in press; Cramer, in press; Lamon, in press; Lesh, Cramer, Doerr, Post, & Zawojewski, in press; Shternberg & Yerushalmy, in press), and problem solving (English & Lesh, in press; Harel & Lesh, in press; Kehle & Lester, in press; Lesh, Lester, & Hjalmarson, in press; Lester & Kehle, in press; Middleton, Lesh, & Heger, in press; Zawojewski & Lesh, in press; Zawojewski, Lesh, & English, in press).

Research Design from *a Models and Modeling Perspective*

A models and modeling perspective has proved to be a rich context for research and development. One of the main points of convergence from the conclusions achieved by each of the groups in the past workshops resided on the need for innovative designs for research and assessment that can help answer questions that involve the understanding of complex situations that are dynamic and iterative. Through the work that main participants of this workshop have been doing in each of the areas: Student Development, Teacher Development, Assessment, Curriculum Development, and Problem Solving, collaborative work has been done in innovative research design.

The *Handbook of Research Design in Mathematics and Science Education* (Kelly & Lesh, 2000) describes a variety of innovative research designs that have been developed by mathematics and science educators to investigate interactions among the developing knowledge and abilities of students, teachers, and others who influence activities in mathematics and science classrooms. This handbook has helped in setting the foundations to identify several characteristics that distinguish the type of research design needed to answer they types of questions we are most interested in, which at the same time, lead to the need of new research designs in mathematics and science education:

First, it is important to radically increase the relevance of research to practice –often by involving many levels and types of practitioners in the identification and formulation of problems to be addressed- or in the interpretation of results, or in other key roles in the research process. So, instead of having only one-way transmission of research into practice, research methodologies that are proving to be most useful in mathematics and science education often involve bi-directional interactions and iteratively evolving feedback loops among many levels and types of participants (students, teachers, researchers, curriculum designers, policy makers).

Second, in mathematics and science education, most of the things that need to be understood and explained are complex systems –not necessarily in the strict mathematical sense, but at least in the general sense that they are dynamic, interacting, self-regulating, and continually adapting. That is, they do not simply lie dormant until they are stimulated; rather, they initiate action, and, when they are acted on, they act back. In particular when they are observed and when information is generated about them, changes often are induced that make researchers (and assessments) integral parts of the systems being investigated. Furthermore, among the most important systems that mathematics and science educators need to investigate: (a) many do not occur naturally (as givens in nature) but instead are products of human construction, (b) many cannot be isolated because their entire nature tends to change if they are separated from complex holistic systems in which they are embedded, (c) many are not observable directly but are knowable only by their effects on other agents or events, and (d) most include a variety of interacting communities of agents whose interactions lead to:

- Feedback loops that produce second-order which may outweigh or change the influence of first-order effects,
- Emergent characteristics of the system-as-a-whole that cannot be derived from characteristics of the agents within these systems.
- Behaviors that are often inherently unpredictable.

Third, the mathematical models that are needed to describe and explain the preceding systems are not restricted to linear equations or other kinds of simple input-output rules that presuppose the existence of independent variables that can be isolated, factored out, or controlled (Lesh & Lamon, 1992). For example, because of recent advances in fields such as those focusing on *geographic information systems*, there has been an explosion of new software and technologies that are capable of using graphic, dynamic, and interactive multimedia displays to generate simple (but not simple minded) descriptions of complex systems ranging from weather systems, to traffic patterns, to biological systems, to dynamic and rapidly evolving economic systems. Consequently, it is no longer necessary for educational decision-makers to rely on reports that involve nothing more than simple-minded unidimensional reductions of the complex systems that characterize the thinking of students or teachers –or relevant communities.

Fourth, research is about knowledge development; and, not all knowledge is reducible to a list of tested hypotheses and answered questions. In particular, in mathematics and science education, the products that emerging new research designs are intended to emphasize often focus on the development of models (or other types of conceptual tools) for construction, description, or explanation of complex systems. When producing these latter types of products, distinctions are being made between: (a) model development studies and model testing studies, (b) hypothesis generating studies and hypothesis testing studies, and (c) studies aimed at identifying productive questions versus those aimed at answering questions that practitioners already consider to be priorities.

Based on the term and the characteristics described by Brown (1992) and Collins (1992), we will call such research design a "design experiment". This type of research design can be characterized through four general principles that apply to Design Experiments focusing on the development of constructs and conceptual systems used by students, teachers, or researchers.

We will use the term "participant", to refer, generically, to students, teachers, curriculum developers, program developers, software developers, and other types of researchers, developers, or practitioners. This terminology is appropriate because, to investigate the nature of the developing constructs and conceptual systems used by any of these participants, the following principles should be expected to apply.

1. **The Externalization Principle**. Situations should be identified in which the relevant ways of thinking that are desired to investigate (and/or develop) are expressed in forms that are visible to both researchers and to relevant participants. Design activities naturally tend to lead to *thought-revealing artifacts*, like the model-eliciting activities (Lesh, Hoover, Hole, Kelly, & Post, 2000) described in the previous section. That is, the underlying design often is apparent in things that are designed; the underlying constructs often are apparent in complex artifacts that are constructed; and, the underlying models often are apparent in conceptual tools that embody them. In other words, in the process of designing complex artifacts and conceptual tools, participants often externalize their current ways of thinking in forms that reveal the constructs and conceptual systems that are employed. In this sense, then, the products may be referred to as *embodiments* of the relevant conceptual systems. Therefore, as the tools or artifacts are tested, revised, or refined, the underlying ways of thinking are also tested, revised, and refined. This tends to be true especially if the products are *conceptual technologies* in the sense that they include not only procedures for *doing* something, but also conceptual systems for *describing* and *explaining* the situations in which the artifacts or conceptual tools are intended to be useful. That is, the reason for developing these tools has to do with interpretation, description, explanation, or sense-making –as much as transformation, construction, or computation.
2. **The Self-Assessment Principle.** Design "specs" should be specified as criteria that can be used to test and revise trial artifacts and conceptual tools (as well as underlying ways of thinking) –while discerning products that are unacceptable, or that are less acceptable than others. The design "specs" should function as Dewey-style "ends-in-view". That is, they should provide criteria so that formative feedback and consensus building can be used to refine thinking in ways that are progressively "better" based on judgments that can be made by participants themselves. In particular, ends-in-view should enable participants to make their own judgments about: (a) the need to go beyond their first primitive ways of thinking, and (b) the relative strengths and weaknesses of alternative ways of thinking that emerge during the design process. Productive ends-in-view also should require participants to develop constructs and conceptual systems

that are: (a) powerful (to meet the needs of the client in the specific situation at hand), (b) shareable (with other people), (c) re-usable (for other purposes), and (d) transportable (to other situations). In other words, both the tools and the underlying ways of thinking should be shareable and generalizable.

3. **The Multiple Design Cycle Principle (or the Knowledge Accumulation Principle).** Design processes should be used in such a way that participants clearly understand that a series of iterative design cycles are likely to be needed in order to produce results that are sufficiently powerful and useful. If design processes involve a series of iterative development>testing>revision cycles, and if intermediate results are expressed in forms that can be examined by outside observers as well as by the participants themselves, then auditable trails of documentation are generated automatically; and, this documentation should reveal important characteristics of developments that occur. In other words, the design processes should contribute to learning as well as to the documentation and assessment of learning.
4. **The Diversity and Triangulation Principle.** Design processes should promote interactions among participants who have diverse perspectives; and, they also should involve iterative consensus building –to ensure that the knowledge, tools, and artifacts will be shareable and reusable- and so that knowledge accumulates in ways that build iteratively on what was learned during past experiences and previous design cycles. In general, to develop complex artifacts and tools, it is productive for participants to work in small groups consisting of 3-5 individuals who have diverse understandings, abilities, experiences, and agendas. By working in such groups, communities of relevant constructs tend to emerge in which participants need to communicate their current ways of thinking in forms that are accessible to others. Once diverse ways of thinking emerge, selection processes should include not only feedback based on how the tools and artifacts work according to the ends-in-view that were specified –but also according to feedback based on peer review. In this way, consensus-building processes involve triangulation that is based on multiple perspectives and interpretations. So, the collective constructs that develop are designed to be shareable among members of the group; and, they are designed in ways so that knowledge accumulates.

These characteristics and principles are not at all unfamiliar to scientists who investigate complex systems in fields such as astronomy, biology, chemistry, or physics –or in design sciences such as architecture or artificial intelligence. Thus, a new project has been supported by the National Science Foundation, in which input from leading scientist from outside the fields of mathematics and science education who are experienced in the use of Design Experiment methodologies in their fields will be considered for appropriateness to answer some of the questions math and science educators are demanding from research. This new project will produce a sequel to the earlier

Handbook of Research Design in Mathematics and Science Education (Kelly & Lesh, 2000). Not only will the new book focus on design research methodologies, but it will also describe on new types of dynamic and iterative assessments that are especially useful in design research –where rapid multi-dimensional feedback is needed about the behaviors of complex, dynamic, interacting, and continually adapting systems.

The Working Group at PME-NA XXIV

It is clear that mathematics and science educators have become increasingly aware of the fact that few of the most important problems they need to address are going to be resolved by only a single isolated study. Instead, understanding the types of complex systems we are interested in research nearly always require communities of researchers and practitioners, representing a variety of theoretical and practical perspectives, working together collaboratively over extended periods of time and across a variety of sites. The discussion during this workshop of the use of Design Experiments in Mathematics Education in each of the groups: Student Development, Teacher Development, Assessment, Curriculum Development, and Problem Solving, will provide an opportunity for all of the participants to begin or continue the development of these greatly needed communities of researchers and practitioners, to expand our focus of research and answer the types of questions we are being challenged by our field for this new century.

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