DOCUMENT RESUME

ED 469 111 SE 066 902

AUTHOR	Barton, Mary Lee; Heidema, Clare
TITLE	Teaching Reading in Mathematics: A Supplement to "Teaching Reading in the Content Areas Teacher's Manual (2nd Ed.)."
INSTITUTION	Mid-continent Research for Education and Learning, Aurora, CO.
SPONS AGENCY	Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE	2000-00-00
NOTE	64p.; For "Teaching Reading in the Content Areas," see ED 402 546.
CONTRACT	RJ96006101
AVAILABLE FROM	Mid-continent Research for Education and Learning, 2550 S. Parker Road, Suite 500, Aurora, CO 80014-1678. Tel: 303-337-3005; Fax: 303-337- 3005; e-mail: info@mcrel.org; Web site: http://www.mcrel.org.
PUB TYPE	Guides - Non-Classroom (055) -- Reports - Descriptive (141)
EDRS PRICE	EDRS Price MF01/PC03 Plus Postage.
DESCRIPTORS	*Communication (Thought Transfer); Critical Thinking; Elementary Secondary Education; *Mathematical Vocabulary; *Mathematics Instruction; *Reading; *Teaching Methods; *Textbooks

ABSTRACT

This document explores what literacy in mathematics involves and presents suggestions and strategies teachers can share with their students to help them become more proficient in reading and communicating in mathematics. The book examines what the research says about the role of the reader, the role of climate, and the role that text features in mathematics as well as their implications for instruction. It also presents math-specific examples of the strategies included in "Teaching Reading in the Content Areas (TRCA) Teacher's Manual" so that mathematics teachers can see how to use and apply these strategies in their classrooms. (KHR)

Teaching Reading in Mathematics

A Supplement to *Teaching Reading in the Content Areas*

by Mary Lee Barton

Clare Heidema

McREL Mid-continent Research for Education and Learning

Teaching Reading in Mathematics

A Supplement to *Teaching Reading in the Content Areas Teacher's Manual (2nd Ed.)*

Mary Lee Barton
Clare Heidema

McREL
Mid-continent Research for Education and Learning
2550 S. Parker Road, Suite 500 Aurora, CO 80014-1678
303.337.0990 • 303.337.3005

McREL

Acknowledgments

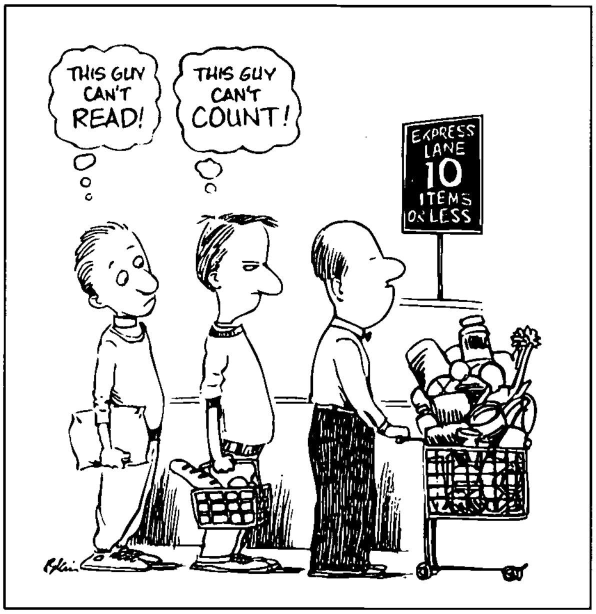
A number of people contributed to the production of this document. In particular, the authors would like to acknowledge Mid-continent Research for Education and Learning staff, specifically Brian McNulty, John Sutton, John Ristvey, and Arlene Mitchell, for their thoughtful review and feedback. The authors would also like to thank Barbara Gaddy and Andrea Jachman for their editing expertise and Judy Schlecte for her desktop publishing skills.

To order a copy of *Teaching Reading in Mathematics*, contact McREL:

Mid-continent Research for Education and Learning
2550 S. Parker Road, Suite 500
Aurora, CO 80014-1678
phone: (303) 337-0990
fax: (303) 337-3005
e-mail: info@mcrel.org
web site: http://www.mcrel.org

This publication is based on work sponsored wholly, or in part, by the Office of Educational Research and Improvement, Department of Education, under Contract Number #RJ96006101. The content of this publication does not necessarily reflect the views of OERI or any other agency of the U.S. Government.

Table of Contents



Reprinted with permission from *Not Strictly by the Numbers*

Rationale

As this cartoon suggests, part of the challenge of teaching reading in mathematics stems from confusion over what "reading mathematics" actually means. Is it being able to comprehend numerical data? Or is it being able to comprehend worded passages in, say, a mathematics textbook? The National Council of Teachers of Mathematics (NCTM) has articulated the importance of students knowing how to communicate about mathematics. In *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), they state:

> The development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem-solving situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking. (p. 6)

As outlined in the second edition of *Teaching Reading in the Content Areas: If Not Me, Then Who?* (*TRCA Teacher's Manual*), reading is a constructive process in which the reader interacts with text, using her prior knowledge and experience to make connections, generate hypotheses, and make sense of what she reads. In this supplement to the *TRCA Teacher's Manual*, "reading mathematics" means the ability to make sense of everything that is on a page — whether the page is a worksheet, a spreadsheet, an overhead transparency, a computer screen, or a page in a mathematics textbook or journal — in other words, any resource that students might use to learn and apply mathematics.

As such, reading mathematics requires the same skills as reading in other content areas. These include

- decoding and comprehending what is read;
- planning for and monitoring the effectiveness of one's reading;
- analyzing and evaluating the content in light of one's prior knowledge, experiences, and schemata; and
- making inferences and generating conclusions based on the reader's unique interpretation of what is read.

If reading mathematics requires the same skills one uses to read other content-area material, why is reading a mathematics textbook so challenging for students?

The fact is that mathematics text demands that readers use additional, content-specific reading skills. For instance, mathematics students must be able to read not only from left to right, as they do in other subject areas, but also from right to left (consider an integer number line), from top to bottom or vice versa (with tables), and even diagonally (with some graphs)!

Second, mathematics texts contain more concepts per word, per sentence, and per paragraph than any other kind of text (Brennan & Dunlap, 1985; Culyer, 1988; Thomas, 1988). In addition, these concepts are often abstract, so it is difficult for readers to visualize their meaning.

A third difference is that authors of mathematics texts generally write in a very terse or compact style. Each sentence contains a lot of information, and there is little redundancy. Sentences and words often have precise meanings and connect logically to surrounding sentences. Students who want to read mathematics texts quickly — as they might a short story in their English language arts class — may miss significant details, explanations, and the underlying logic.

Mathematics also requires students to be proficient at decoding not only words but also numeric and nonnumeric symbols. Consequently, the reader must shift from "sounding out" words like *plus* or *minus* to instantly recognizing their symbolic counterparts, + and –.

Even the layout of a mathematics text can inhibit comprehension. Students often scan a page of text looking for examples, graphics, or problems to be solved, skipping worded passages filled with crucial information.

Finally, many mathematics textbooks are written above the grade level for which they are intended. Therefore, the vocabulary and sentence structure in a mathematics textbook are often especially difficult for the students at the grade level for whom the book is intended.

No wonder reading mathematics presents such unique challenges! In this manual, we explore what literacy in mathematics involves. We also present suggestions and strategies teachers can share with their students to help them become more proficient in reading and communicating in mathematics.

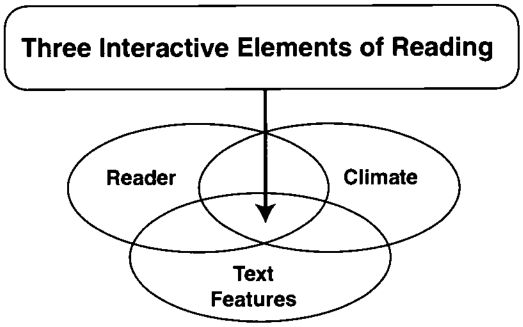
Specifically, we

- examine what the research says about the role of the reader, the role of climate, and the role of text features in mathematics as well as their implications for instruction;
- present math-specific examples of the strategies included in the *TRCA Teacher's Manual* so that mathematics teachers can see how to use and apply these strategies in their classes; and
- present additional strategies to help students become more proficient in reading mathematics.

Section I

Notes

Three Interactive Elements of Reading



Teaching reading in the content areas is not so much about teaching students basic reading skills as it is about teaching students how to use reading as a tool for thinking and learning. The instructional model used in this manual assumes that three elements work interactively to determine the meaning a reader constructs from text: the **reader**, the **climate**, and the **text features**. See the *TRCA Teacher's Manual*, pages 1–5, for a discussion of these three interactive elements of reading.

The Role of the Reader

"Don't tell me why. Just tell me how," the student urged, as the teacher tried to explain the reasoning behind how to solve the mathematics problem he was working on in study hall. He didn't want to be bothered by what he considered "extra" input. He was eager to get the answer. Like many readers of mathematics text, he wasn't eager to do the work of constructing meaning. He was more interested in finding a procedure he could use than in learning how to explain his process or communicate his discoveries.

Yet, comprehending mathematics, like any other subject, is a constructive process. Research (e.g., Siegel & Borasia, 1992) has shown that "in order to acquire mathematical expertise in a durable and useful form, students need to construct mathematical knowledge and create their own meaning of the mathematics they encounter" (p. 19). Learners make sense of new information by considering how this information fits into their prior knowledge and experience. Students need to understand and be able to articulate the "why" as well as the "how."

Sometimes, well-intentioned parents try to help children be more efficient by teaching them algorithms — tried and true ways to get answers using procedures like carrying and borrowing. These algorithms, devised as paper-and-pencil procedures, were designed to be quick and efficient. However, they often do not help people understand why they work. In fact, algorithms can make understanding harder to achieve (see Mathematics for Parents Newsletter on Place Value, Wisconsin Center for Education Research found online at http://www.wcer.wisc.edu/MIMS/Parent_Newsletters/Place_Value/newsletter14.html).

The role of the reader in mathematics — as in other content areas — is to activate and use prior knowledge. Mathematics relies heavily on conceptual understanding; therefore, the effective reader has a clear understanding of mathematics concepts, how they build on one another, and how they are related.

Unfortunately, one feature of many mathematics texts is that concepts are introduced but not discussed again for several chapters. As Smith and Kepner (1981) point out, mathematics concepts may be "developed in a spiral curriculum in which concepts, words, and symbols are developed and practiced, then followed by a period of disuse. When returning to previously learned words and symbols, teachers should verify that students know their meanings" (p. 10). Smith and Kepner also recommend that teachers ask students to define a term in their own words rather than expecting students to parrot those presented in class. In that way, teachers can ensure that students truly grasp the meaning of a concept.

Because many mathematics concepts are abstract, it is essential that students practice working with abstract ideas. Even students in the early grades can begin to learn new skills and concepts by starting with concrete items and then moving to the more abstract. For example, the abstract concept of sets that students work with in algebra can be introduced to first graders using manipulatives, such as crayons, jelly beans, or nuts, and asking students to group these items into sets according to a descriptor, such as color or size or shape. As their cognitive ability develops, students can move to classifying

Notes

elements in representational settings, such as pictures or photographs, into sets and subsets. For example, students might look through their science texts to find pictures of rocks or clouds, each of which could be divided into types of rocks or types of clouds pictured.

The importance of teachers activating and assessing students' prior knowledge in mathematics cannot be overstated. Strategies teachers can use to activate prior knowledge include K-W-L and Semantic Mapping (see pages 116 and 134 in the *TRCA Teacher's Manual*). An additional strategy included in this supplement (p. 49) is the Knowledge Rating Chart (Davis & Gerber, 1994). Here, students analyze their level of familiarity with particular mathematics concepts. The information gleaned from this strategy can assist teachers in planning how much to review prior to introducing new information.

The Role of Climate

As discussed in the *TRCA Teacher's Manual*, children learn best in a classroom climate in which they feel safe taking risks. Unfortunately, many students do not feel like taking risks in the mathematics classroom. This may be a consequence of past mathematics instruction that focused on the *product* rather than the *process* of doing mathematics. Students who have a history of errors and wrong answers may have grown to hate mathematics. The very thought of working out a problem in front of the class can raise intense anxiety in these students. When confronted with a word problem, these students shut down.

Fortunately, reports published by the National Research Council (1989, 1990) and NCTM (1989, 1991, 2000) propose a vision for mathematics education that emphasizes mathematical power through active, flexible, and resourceful problem solving. No longer should instruction focus on imitating and memorizing what is presented by the teacher, but rather on "students' problem-solving strategies, including their ability to generate and define problems, as well as their mathematical reasoning and communication" (Siegel & Borasia, p. 19). These reports also discuss the importance of students' attitudes toward mathematics, recommending that

Notes

teachers place greater emphasis on the affective elements involved in learning mathematics.

One way to shift instruction to focusing on process is to point out to students that there is more than one way to solve a problem. Model for students how you reach a solution, but also have students discuss in groups the steps they went through to reach their solution. Another technique is to ask questions that allow for more than one response. For example, you might set up a word problem in which students are told that they have a certain amount of money to spend for camping supplies or for party decorations. Provide a list of items with prices marked, and allow students to "shop" so that they purchase what they think they will need for this scenario, spending an amount that uses their funds yet is within their budget. Then ask students to write an explanation of their purchases and their calculations.

Another method of easing learners' fears of making mistakes is to give students credit for the effort they put into solving a problem. "Show your work" is becoming a common directive in many mathematics classrooms, and many state assessments require that students "show or explain" so that students can earn credit for problem-solving techniques. At first, students may resist what they perceive as additional work involved in writing out their thought processes. Once they recognize the value of focusing on methods of solution and become used to articulating *their* process, resistance should ease.

The Role of Text Features

The sentence structure, vocabulary, organization, and organizational patterns of mathematics textbooks can present unique challenges to inexperienced readers. In this section, we examine each of these features and offer some suggestions on how to help students become more successful readers.

Sentence Structure

In a recent Internet discussion among calculus educators, one instructor suggested that the reason students may have such difficulty comprehending mathematics is that written passages often use complex sentence structure.

Notes

He contended that students today lack a background in English grammar and mathematics syntax:

> If students don't really understand what verbs are, and how verbs link subject and object; if they don't understand what descriptive phrases modify, is it any wonder that students don't understand how equals signs and inequalities link mathematical phrases, and that students don't understand how mathematical quantifiers modify mathematical phrases?†

Teaching grammar and syntax is beyond the scope of the typical mathematics curriculum. How, then, can mathematics teachers help students understand their text? One response to this instructor's concerns has merit:

> Perhaps we could take some actions with regard to reading (and writing) in our own courses... . How extreme would it be to ask students to read and paraphrase... specific definitions and/or theorems and supply... a labeled diagram with description. One or two (or even daily!) assignments of this type might shed more light on individual difficulties students are encountering than asking them to do a template problem.†

Other authors have discussed at length how syntax — the way an author puts words together to form phrases, clauses, and sentences — can make reading mathematics difficult (Linville, 1976; Shuard & Rothery, 1984). Let's examine the way mathematics is written and how this can affect comprehension.

The simplest structure an author might choose is subject-verb-object. Examples of this structure are the following:

Juanita buys a ticket.

Draw a rectangle.

The answer is 37.

† These comments originated on the CALC-REFORM listserv, May 25, 2000.

Notes

This structure becomes somewhat more complicated for young readers when the author adds adjectives, adverbs, or phrases, or uses conjunctions such as *if, and, but, or,* or *nor* to connect two ideas:

> *If the cost of one ticket is $35, and Juanita buys four tickets, how much will she spend?*

Or

> *To calculate the area of the figure below, find the areas of the square and the triangle separately, and then add these areas together.*

Other features that writers use that can cause readers difficulty are the use of passive voice:

> *The football <u>was caught</u> on the 35-yard line.*

subordinate clauses:

> *The field <u>that lies behind our new house</u> is 20 acres in length and 30 acres wide.*

and the use of the comparative:

> *Is the radius of circle A <u>greater than</u> the diameter of circle B?*

Although this is not meant to be a lesson in syntax, we want to emphasize that students do find *how* something is written just as challenging as *what* is written. Strategies such as asking students to write out their answers in full sentences and to construct their own word problems have been used successfully to help students become more familiar with communicating and reading mathematics.

An algebra teacher we know recommends the following strategy to familiarize students with the sentence structure of mathematics. She writes a different equation on each of 30 notecards — enough to distribute one notecard equation to each student in her class. Once students have read the equation on their cards, they must write three different word problems for which the equation can be used. They write the equation and the three word problems on a separate sheet of paper. After students have completed the word problems for one card, students swap their card with another student, and repeat the process. Students continue to swap notecards until they have written word problems for equations on four or five notecards. The teacher

Notes

Notes

then collects students' papers. From the word problems each student wrote, she selects at least one "winner" (best word problem for an equation).

During the next class meeting, the teacher returns students' papers and asks students to copy their winning word problem onto a notecard. Students again exchange cards, this time writing the equation and solution to the word problems they get. In this way, they obtain practice in using mathematics vocabulary and become more familiar with the sentence structure in a word problem.

Another strategy is to reproduce the word problems from a textbook so that students can highlight or underline the key words in these problems that tell what information is needed, what the context is, what is being asked, and so on. First, model for students how you determine which words in the problem are essential to understanding. Then ask student volunteers which words they believe are critical to a problem.

Finally, we recommend that mathematics teachers assess the sentence structure used in their text materials. It may be necessary to adapt these materials or to take time to instruct students in how to tackle complex writing styles (Linville, 1976).

Vocabulary

Mathematics vocabulary is one feature of mathematics text that can challenge even able readers. Numerous studies (see those cited in Earp, 1970, 1980; and Helwig et al., 1999) reveal that a knowledge of mathematics vocabulary affects achievement in arithmetic, particularly problem solving. "Reading comprehension and arithmetic achievement tend to be positively related. Almost without exception instruction in vocabulary and/or reading skills in arithmetic paid off in terms of greater achievement, especially in the area of problem solving" (Earp, 1970, p. 531).

Vocabulary instruction in mathematics holds some unique challenges. First, certain concepts in mathematics are embedded within other concepts to be defined and understood.

Notes

Figure 1 illustrates this point. As this figure indicates, integers are understood in relation to whole numbers and to rational numbers. That is, to understand the concept of integers, students must comprehend the relationships involved. Similarly, rational numbers are understood in relation to integers. So, a natural prerequisite to operating with rational numbers is to work with integers. Real numbers are understood in relation to rational numbers, and so on. In such instances, vocabulary development in mathematics can require a kind of hierarchy, or an understanding of how one concept is embedded within another.

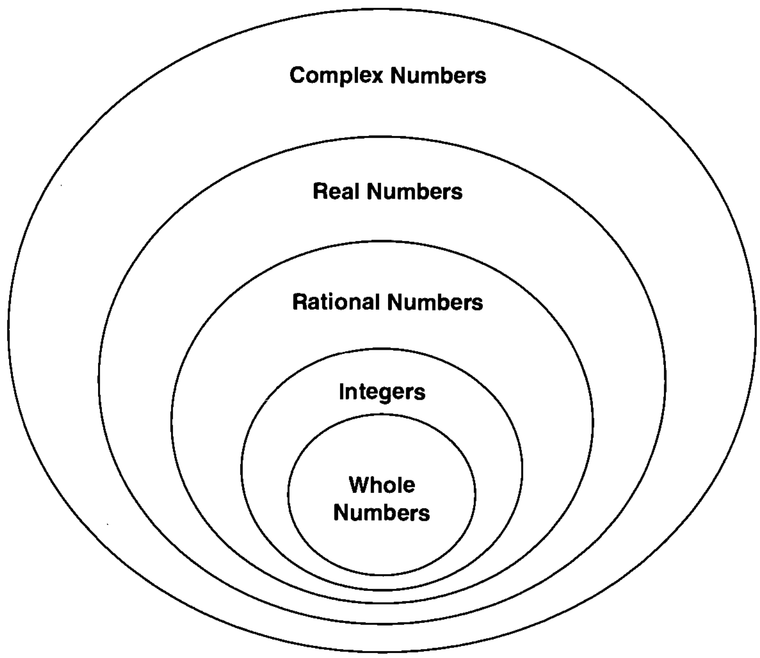


Figure 1. Number systems

Another aspect of mathematics vocabulary that holds potential difficulty for students is the complex overlap of this vocabulary with the vocabulary used in "ordinary" English. Shuard and Rothery (1984) maintain that teachers need to explain to students that there are three categories of words used in mathematics:

- Words that have the same meaning in Mathematical English (ME) and Ordinary English (OE);
- Words that have meaning only in ME; and
- Words that have different meanings in ME and in OE.

Words that have the same meaning in ME and OE

Students are not likely to have difficulty with this category. Examples of such terms include *dollars*, *cents*, *because*, *balloons*, and *driving*. Terms like these appear regularly in "real-life" word problems. Nevertheless, teachers need to be sensitive when choosing text. In one inner-city classroom, students balked at solving a "real-life" word problem that described a situation in which two boys went kayaking. Although students had the computational skills to complete this simple distance problem, they could not see beyond this unfamiliar OE word.

Words that have meaning only in ME

Terms that are unique to mathematics cause students difficulty when they are not part of their prior knowledge or experience. These are words (e.g., *hypotenuse*, *cosine*, *coefficient*) that are not typically used at home or in everyday speech. It is critical that mathematics teachers use effective techniques to introduce and reinforce meanings of new terms! Strategies such as those on pages 70–89 of the *TRCA Teacher's Manual* and techniques such as repeated definitions (even flash cards), vocabulary journals, and the use of simple sketches and pictures can help students relate the words they are learning to their definitions.

A word of caution: Often, teachers and students use informal terminology in place of more formal terms used in the text. Consequently, students do not gain familiarity with the technical vernacular that they will be expected to know. For instance, if the teacher continually substitutes the phrase "number on the top" for the term *numerator* in an attempt to make the meaning clearer for students in the short run, he may be doing them a disservice in the long run. Students need to learn the technical terms they will encounter again in future course work.

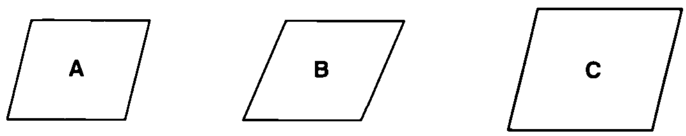
Words that have different meanings in ME and in OE

Some words have a meaning in ME that is completely different from or unrelated to their meaning in OE. For example, the word *difference* could easily confuse the young student faced with the question, "What is the

Notes

difference between 4 and 7?" A student might answer, "Four is even, but seven is odd," when the intended correct response is "three."

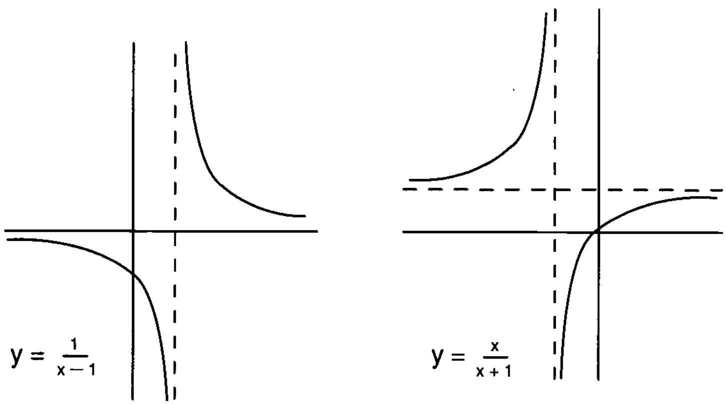
In other instances, the mathematical use of the term might have a more specialized meaning than its OE counterpart. The terms *average* and *similar* are two examples. In Figure 2, the first two shapes look similar as that term is used in ordinary English. However, the mathematical meaning of *similar* is more specialized: In order for polygons to be similar, they must have corresponding angles that are equal.



A and C are similar, whereas A and B are not.

<u>Figure 2.</u> Polygons

At other times, meanings of mathematical terms are introduced in a manner that is less clear. Consider the example in Figure 3.



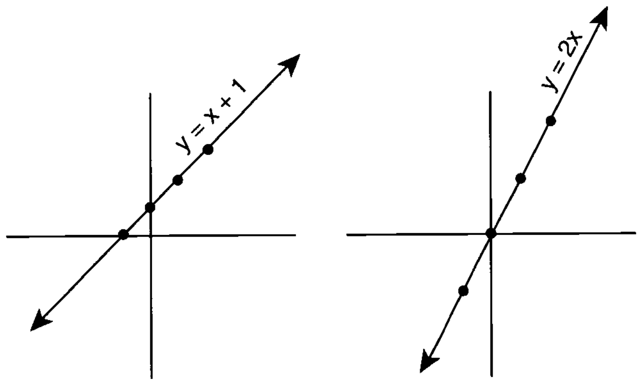
These graphs have *asymptotes.* As x approaches 1 in the first example, or as x approaches -1 in the second example, $|y|$ gets very large. The lines $x = 1$ and $x = -1$ are vertical asymptotes, respectively, of $y = \frac{1}{x-1}$ and $y = \frac{x}{x+1}$. In the second example, the line $y = 1$ is a horizontal asymptote of $y = \frac{x}{x+1}$.

<u>Figure 3.</u> Asymptotes

In this case, the reader has to extract the definition of *asymptote* from the examples given.

Notes

A third way authors may introduce vocabulary terms is even more indefinite, and therefore difficult, for students to comprehend; the definition may be implied within the text. For example, see the "definition" of *linear equation* in Figure 4.



The points in each graphed set are on a straight line. The first set of ordered pairs are solutions to the linear equation y = x + 1. The second set are ordered pairs that solve the linear equation y = 2x.

Figure 4. Linear equations

Reading mathematics means decoding and comprehending not only words but also mathematical signs and symbols. This means that students must switch between the skills they use to decode words and the skills needed to decode signs and symbols. Decoding words is generally based on sounds; thus, we say "sound it out" when students have trouble reading an unfamiliar word. In contrast, mathematical signs and symbols may be pictorial, such as an arrow, or they may refer to an operation (e.g., $\div$) or an expression (e.g., $\geq$). Consequently, students need to learn the meaning of each symbol and to connect each symbol, the idea the symbol represents, and the written or spoken word(s) that corresponds to that idea.

To complicate matters, these symbols can have more than one meaning. For example, the symbols $12 \div 4$ could represent the concept of sharing ("twelve divided into four equal parts") or could represent grouping ("How many groups of fours in twelve?").

Moreover, the same idea and the same *translation* or wording of the idea can be represented by different symbols:

$$12 \div 4,\ {}^{12}/_{4},\ 4\overline{)12}$$

Notes

Division, therefore, can be conveyed in a variety of ways, and the student must understand that the meaning is the same.

Notes

Shuard and Rothery assert that

> a child needs eventually to understand that there are two ideas (grouping and sharing) and several sets of symbols attached to the word *division*, including the notation $^{12}/_{4}$ which makes the link between division and fractions explicit. A child has a fully operational concept only when all these ideas are seen as aspects of the concept of division, rather than as isolated ideas and notations. (p. 38)

Similarly, the same multiplication fact can be expressed in words:

multiply three by five; or *three times five;* or *the product of three and five.*

and in symbols: 3 x 5 or 3(5) or 3 • 5.

Shuard and Rothery note that symbols rarely appear alone and can be combined in a variety of ways to communicate different meanings. Thus, the position and order of symbols convey specific meaning. However, irregularities in the rules for combining symbols can cause confusion for some students. Just when a student has learned how to decode 3x in algebra as "3 multiplied by x," she moves into calculus, where she must grasp the idea that δx does not mean "δ multiplied by x," but rather "an increment in x."

Smith and Kepner (1981) also discuss how this irregularity affects mathematics students as they progress through their studies:

> Initially, "–" represents the binary operation in subtraction; i.e., 5 – 3 means 5 subtract 3. With the introduction of integers, –7 represents a particular seven, namely "negative 7." In this context, the horizontal bar describes which 7 is being considered. Finally, –x refers to the "opposite of x" or the "additive inverse of x." Here –x identifies the monary operation, i.e., an operation of a single number. If students are confused about the exact meaning of a word, symbol, or number, the solution to a sentence like "What is the value of –x when x = –7?" becomes an exercise in frustration. (p. 9)

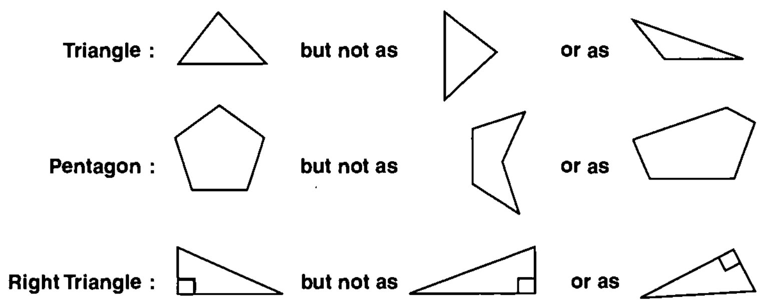
Children sometimes struggle with where to start to read a combination of symbols. When reading their social studies or language arts textbook, they read from left to right, and from the top of the page to the bottom. But when they open their mathematics books, this convention doesn't always apply. For instance, both of these combinations

$$27 \div 3 \text{ and } 3\overline{)27}$$

convey the same idea, but the order of the symbols is reversed. Consequently, it is not unusual for young mathematics students to mistakenly conclude that in all division, "the larger number is the one you divide by the smaller number."

Although most students learn how to recognize the relationships between ideas and symbols, teachers need to be aware of the confusion that can occur when students are constructing meaning. They also need to adapt instruction for students who become confused when directions change, for example from "solve" to "multiply" or to "do these multiplication problems."

Another caution specific to mathematics is that students may confuse illustrations with definitions especially if a common picture is used almost exclusively. So, for example, a student may view:



Teachers should help students avoid confusing the illustration with the complete definition.

Other forms of graphics that students must learn to read in mathematics are diagrams, graphs, and tables. The ideas expressed in these forms often cannot be expressed as easily in words. Again, students reading mathematics must shift their reading skills from decoding and comprehending the prose text on the page to decoding and comprehending the ideas represented in tabular

Notes

or graphic form. Constructing meaning becomes even more complex when the textbook page also includes photographs and other kinds of illustrative material. Here the student must determine which information is essential for understanding. Again, we urge mathematics teachers to offer students specific instruction in how to navigate the kinds of print and graphics typically used by the authors, editors, and publishers of their textbook series.

Notes

Two vocabulary strategies that can help students identify and examine relationships among concepts are List-Group-Label (see page 34, "Vocabulary Development") and the structured overview. The former is a student-centered strategy; the latter is teacher directed. An example of a structured overview for plane figures appears in Figure 5.

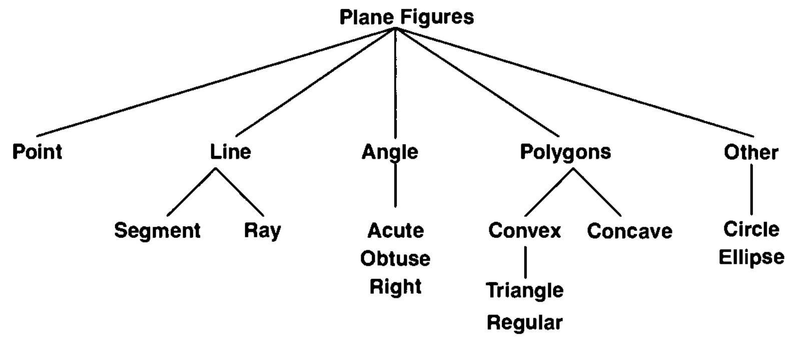


Figure 5. Structured overview for plane figures

As noted in the *TRCA Teacher's Manual*, content-area terms often are semantically related. Structured overviews offer learners a visual picture of how semantically related terms are logically sequenced and connected. Seeing these relationships helps students make the connections necessary between these concepts and their prior knowledge. Creating structured overviews also can aid teachers in sequencing and planning instruction. Finally, graphic organizers such as this can be used as assessment tools to evaluate students' ability to analyze and internalize conceptual relationships.

Organization

Typically, five types of writing are found in mathematics text: exposition, instructions, exercises and examples, peripheral writing, and signals (Shuard & Rothery, 1984). Instruction in how to recognize and comprehend each type of writing will enhance students' mathematics reading skills.

Notes

Exposition is the explaining of concepts and methods, vocabulary, notation, and rules. The reader is meant to comprehend this material, but not necessarily use this knowledge right away. At times, vocabulary terms are introduced and defined through exposition:

> In reading mathematical exposition, the reader often needs to work out steps in the argument for himself, so it is essential for him to use pencil and paper as he reads, rather than passively reading the text as he would a novel. In text for children, some exposition is not distinguished from exercises — indeed, exposition is often presented as exercises in order to force the reader into an active mode of reading. (Shuard & Rothery, p. 11)

Instructions tell the reader to perform a task, such as to draw, complete, or solve. Although text instructions may seem straightforward to the teacher, students can make mistakes when unfamiliar words are used to identify a process or operation. It's a good idea to familiarize students with different ways that instructions are given in a new text. For instance, some authors use the phrase "solve the following" while others use "complete…," or "find the …," or "evaluate … ." To the experienced teacher or student, such varied instructions are clearly synonymous; however, to students whose prior knowledge is limited, a change in instructions can be confusing.

Exercises and **examples** can involve computations, simple problems, or more complex problems. In any case, an exercise may require students to practice the idea just as it was introduced or to apply their knowledge and extend their understanding of the idea presented. It is helpful to point out to students which kind of skills they will need to use in completing the exercises or reading the examples.

Peripheral writing includes introductory remarks, clues, and observations or remarks that help the reader move through the text but are not crucial to understanding or learning. For example, at the beginning of a chapter, the author may remind students, "In the last chapter we learned how to add and subtract fractions. In this chapter, we will learn how to multiply and divide fractions." This introduction helps activate students' prior knowledge and also links that knowledge to the new information students can expect to learn in this chapter.

Sometimes, authors include rhetorical questions within the text. For example, after an expository section on polygons, the author may write, "Can you think of other shapes that would fit into this category?" Inexperienced students may find this confusing. Are they supposed to reply to this question in an active way by taking out paper and pencil and making a list of other shapes that could be in the category under discussion? Or is the author simply asking the reader to think about this subject? Since students often are expected to answer questions in their mathematics text, they may find the purpose of these rhetorical questions unclear.

Signals are reader aids that help guide the reader through the page, such as headings and subheadings, bullets, arrows, and the like. They are an extremely important feature in mathematics text — one that teachers should introduce students to early in the school year. Each publisher selects certain fonts, graphic symbols, and text boxes to signal specific kinds of information. These graphic aids also indicate relationships among ideas and their superordination and subordination. All of these devices help readers navigate through text that at first glance may appear to be disorganized and jumbled. Students need to learn how their text uses typographical signals so that they can become more expert readers.

Organizational Patterns

As we discuss in the *TRCA Teacher's Manual*, authors of informational text attempt to organize their ideas in logical fashion, so that readers can recognize main ideas and differentiate between these and information offered as evidence, support, and illustration.

When students first read mathematics text, they are confronted with an unfamiliar organizational structure. Younger students are acquainted with chronological story patterns. They may even have encountered informational text written in the traditional format of generalization and support, in which paragraphs begin with a topic sentence that can guide or give purpose to their reading.

Notes

However, many mathematics texts do not conform to these organizational patterns. Instead, they may follow a pattern used with guided discovery. Here, authors do not begin their discussion with a general topic sentence or thesis. Moreover, not all of the steps in their arguments are necessarily stated explicitly in the passage. In guided discovery, authors provide activities and questions through which students are supposed to discover the ideas or infer them on their own. Active participation in constructing meaning is essential. Although guided discovery can be an effective instructional technique, novice readers can struggle with text written in this format. What if students do not make accurate inferences or reach the same conclusions as the author intends? And what if students cannot identify the main and supporting ideas? Research indicates that when the main idea of a passage is not clearly stated, even college-level readers have difficulty articulating it (Dickson, Simmons, & Kameeni, 1995). Therefore, mathematics teachers should preview their text material to identify whether the main ideas are clearly written and appear in a consistent location throughout the text.

Like text written for guided discovery, arithmetic story problems are not organized in a manner familiar to young students. Instead, facts and details often appear at the beginning of the problem and the thesis or topic sentence appears at the end (Reutzel, 1983). Consequently, students are not sure what their purpose for reading is until the end of the problem. They may forget or just not attend to important details of the problem before they find the thesis or topic sentence. As students reach the intermediate and upper grades, more distractors or irrelevant details are included in word problems, which is further cause for confusion. Students need experience in reading story problems and in analyzing the main idea or question, as well as in identifying supporting and irrelevant details.

An added challenge in comprehending and solving story problems is determining the precise meaning of the topic sentence — that is, the question asked or the problem posed. Even students who are able to read the problem and understand its situational context still can have trouble identifying what the problem is asking them to do (Denmark, 1983).

Notes

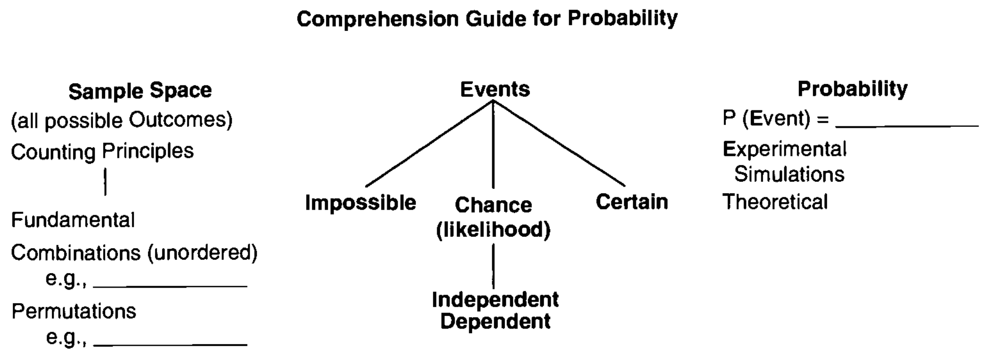
Notes

Although research indicates that teachers define and implement problem solving in a variety of ways, mathematics educators, textbooks, and classroom resource materials typically utilize a view of problem solving based on Polya's four-step process for problem solving (Gray, 1999).

1. Understand the problem. Here, students determine the data given, the unknown, the information needed or not needed, and the condition or context of the problem. It may be helpful to draw a sketch including the known information.
2. Devise a plan. This may be as simple as selecting the correct operation demanded by the problem. In more challenging instances, students may need to examine ways this problem is similar to others they have solved successfully in the past. If an immediate connection can not be found, they may need to consider the problem from a different perspective. That is, they may need to find a related problem—one more general but having similar features—and examine how solving that sort of problem can give them hints about how they might attack this problem. They also may restate the problem to make sure that they understand terminology, as well as to ensure that they have understood the whole situation, and have taken into account all essential components involved in the problem. Eventually, they should settle on a plan to come up with a reasonable solution to the problem.
3. Carry out the plan, checking (or proving) that each step is correct.
4. Examine the solution obtained. Check the result to make sure that it is reasonable or solves the problem.

We offer several additional strategies for working on word problems in this supplement on pages 44–54: Five-Step Problem Solving, K-N-W-S, Three-Level Guides, and Word Problem Roulette.

Davis and Gerber (1994) recommend helping students understand how information in a chapter fits together by going over a graphic organizer for the chapter before they read. See the example in Figure 6. The organizer can have portions that students need to fill in as they read. This task can aid them in processing the information in a logical fashion.



Directions: Save this sheet. Fill in the definitions, diagrams, and examples as we work through Chapter 11. This will be a useful study guide.

Figure 6. Graphic organizer for chapter on probability

Finally, Shuard and Rothery (1984) state that

> In considering text as a whole, attention needs to be paid to the clarity and flow of meaning of the whole passage. Some mathematics text seems to stress obvious points while omitting important steps which the writer takes for granted, though their absence may prevent the reader from comprehending what is written. It is comparatively easy for the teacher to discover these gaps in the material he writes for his own classes, and to remedy omissions orally. The task of the published writer is more difficult, and the teacher should not take it for granted that, simply because material is published, it contains all the necessary explanation.... Clarity and flow of meaning are perhaps the most important features which enable pupils to 'get the meaning from the page,' and so to read mathematics with understanding. (p. 135)

Notes

Section 2

Strategic Processing

Clearly, reading mathematics requires the strategic processing of information. What specific skills and thinking processes do effective readers of mathematics use? As in all content areas, these skills include the ability to plan for, monitor, and evaluate one's comprehension.

One way that students can practice analyzing their thinking processes is by learning from their wrong answers (Moss, 1997). Walter Szetela, an education consultant, suggests using a problem like this one: "A penny weighs about three grams. A nickel weighs about five grams. Which is heavier—30 cents worth of pennies or 30 cents worth of nickels? Abby's answer is that 30 cents worth of nickels is heavier because a nickel is heavier than a penny. Is Abby correct?" After students have studied a problem like this one, Szetela suggests asking questions. For example:

- Does the answer in the problem make sense considering the facts that are given? Why or why not?
- Was the strategy that was used to solve the problem a good one? Why or why not?
- Is there another way of solving the problem? Explain.
- Were mistakes made in solving the problem? If so, explain them.

K. Denise Muth (1988) contends that we can help students become more successful at mathematics problem solving by helping students see that the same comprehension monitoring skills used by effective readers will help them become more successful in solving word problems: planning, monitoring, and remediation. She notes that "teaching these processes to students has been shown to make them better problem solvers (p. 61)."

Muth says the planning phase includes identifying one's goals and selecting the strategies that will help the reader achieve those goals. For example, when reading a textbook, a reader might determine that her goal is to locate a specific piece of information. The strategy she chooses might be to skim the text to find that information. Just as effective readers plan their reading

Notes

Notes

by setting specific goals and selecting appropriate strategies to achieve these goals, mathematics students can approach a mathematics word problem by first reading the problem to determine their goal. A goal a student might have when solving a word problem is to produce an answer or to demonstrate a method of solution. In this case, strategy selection involves selecting the right operation to use. Teachers should ask students to select the appropriate strategy for solving the problem and to explain why they chose it. The explanation should encourage students to check the strategy they have chosen against the problem goal.

Monitoring is a continuous process of checking whether one's goals are being met. In social studies class, this might mean verifying predictions made prior to reading about events that will be discussed or questioning oneself about whether the text makes sense. Similarly, the mathematics student monitors comprehension by estimating and verifying whether answers appear sensible in light of information given in the problem, by rechecking that the correct numbers have been entered into a formula, and by double checking one's work, perhaps with alternate methods.

Remediation includes identifying the causes of comprehension breakdowns and selecting appropriate remedies. Often, this occurs at the same time as monitoring. In English class, a student might check on the meaning of unknown words, modify his reading rate to ensure comprehension, or reread confusing portions of the text. When solving word problems, the mathematics student verifies that formulas used or operations performed were correct or considers whether there is a better way to solve the problem.

In any case, Muth recommends that teachers demonstrate that these strategies are applicable across content areas and texts. Since these skills are used in particular when students encounter difficult text, Muth recommends that teachers use examples that students are likely to find difficult. Students need to know why these skills are valuable in addition to how and when to use them. Finally, teachers should model these strategies for students, so they can see how they are used in real-life settings. Think-alouds (see page 139 of the *TRCA Teacher's Manual*) are useful both in reading text and in working through the problem-solving process. As always, students will use strategies when they acknowledge their value and when their teachers do not simply talk about them, but model their use.

Kresse (1984) asserts that modeling effective metacognitive behaviors during problem solving, and then offering students practice in these behaviors, is essential for students to become independent learners. She cites research (Knifong & Holtron, 1977) on students solving word problems incorrectly that revealed that while students could read all of the words in the problems correctly, knew the situation presented in the problems, and even understood what the problems were asking, still they could not identify how to work the problem.

Kresse recommends explaining to students that the information needed to solve a problem lies in the wording of the problem. Therefore, students need to learn how to look for keys to retrieve that information. She offers a two-part strategy. The first part involves teacher analyzing and preparing for the problem-solving task. She points out that mathematics teachers often explain how a problem is solved, but rarely explain "how they know how." Through their own extensive practice with solving word problems, teachers have formed subconscious generalizations about problem-solving techniques. Therefore, Kresse contends that teachers should spend time reflecting on and analyzing the keys to process comprehension:

> First, ask yourself how you know the correct process. Try to select just the words necessary to work the problem, to be sure of the correct process. After you select the process words, use a mental cloze procedure†. Read the problem with only the words you've chosen. Can you still perform the correct operation? What else can you delete and still work the problem correctly? (p. 599)

The next step in the preparation/reflection process is checking to see if any of the words or symbols in the explanation are specific concepts or vocabulary that should be reviewed with the class. Then, visualize the problem. Is there a way to work the problem visually that will help solve it?

---

†The cloze procedure deletes some words from a text and replaces them with blanks. The reader must supply the missing words.

Notes

The last step in analyzing the process is to practice explaining how you know how to work the problem.

Notes

After the teacher has modeled his metacognitive process, Kresse recommends giving students instruction in these behaviors through a technique she calls the S-Q-R-R strategy:

- Survey the problem. Read the question sentence first.
- **Q**uestion yourself. "What is this asking me to find?" (This provides a purpose for reading the problem word-by-word.)
- **R**ead the problem aloud in its entirety, and explain how you determine which information is key and which information is extraneous. If appropriate, draw a sketch and label it using the key information.
- Ask, "What is the correct process to solve this problem?"
- Work the problem.
- Check your **R**easoning. Ask, "What process did I use? Why did I choose that process? Was my reasoning correct?"

Teaching students to use this strategy requires some scaffolding of learning. That is, after students see this technique modeled, they should practice by working a problem and then having the teacher help them check their reasoning. After students demonstrate that they understand how to use this technique, they can practice these behaviors independently.

Section 3

Strategic Teaching

Notes

When developing an instructional framework for presenting new information to students, teachers typically analyze both the content they want to cover and the best process to use to convey that information.

Content analysis involves the selection and organization of content. It includes examining the ideas, concepts, and learnings for the unit of study — in the standards-based classroom, this includes relevant standards and benchmarks. Once identified, this content is then organized into a logical sequence, one that will help students develop a solid understanding of the material.

Process analysis entails selecting the most effective learning activities and the appropriate sequence for these (Smith & Kepner, 1981). In general, process analysis yields a lesson introduction, an assimilation component, and follow-up activities. The introduction should include activities to assess and activate prior knowledge. It should also outline the purposes for the lesson and teach essential new vocabulary. If necessary, it should identify any directions for reading and organizing what is read with a graphic organizer. The assimilation phase includes the actual instruction of information selected in the content analysis. The follow-up portion of the lesson offers students activities that help reinforce, internalize, or extend knowledge.

Smith and Kepner suggest a specific outline (shown in Figure 7) as a general instructional framework that synthesizes the content/process analyses.

Instructional Framework

I. Introductory Activities—student motivation, preparation, and direction
   A. Selecting appropriate motivational activities
      1. Inclusion of teacher-dominated or student-dominated activities
      2. Use of a media presentation
   B. Reviewing pertinent background information
      1. Teacher-led discussion of prior learnings
      2. Student awareness of learning as related to real-life situations
      3. Discussion of important concepts related to the lesson
   C. Setting purposes for the lesson
      1. Discuss with students the types of problems they will be solving.
      2. Give students a list of questions to answer as they do the lesson.
      3. Identify the types of follow-up activities students will perform.
   D. Giving directions for reading
      1. Discuss strategies useful in reading word problems.
      2. Identify key sentences or paragraphs in the text.
      3. Eliminate unnecessary reading passages.
      4. Set reasonable time limits for reading, problem solving, and follow-up activities.
      5. Discuss important tables, diagrams, and charts needed during the lesson.
   E. Teaching essential vocabulary
      1. List important terminology (words and symbols) on the chalkboard.
      2. Select representative words and symbols which will be taught for meaning or pronunciation or both.
      3. Pronounce remaining words and symbols for students.

II. Assimilation Activities—instruction in the major understandings
   A. Lecture style discussion
   B. Silent reading
   C. Supplementary instruction (e.g., media presentation)
   D. Supervised study
   E. Guided discovery lesson

III. Follow-up Activities—activities intended to extend, internalize, or reinforce student learning.
   A. Small group discussions
   B. Teacher-made content tests
   C. Vocabulary games or puzzles
   D. Mathematical project (individual or group)
   E. Additional teaching in the meaning or pronunciation of words and symbols
   F. Demonstrations of the mathematical principle being studied in different contexts
   G. Written assignments
   H. Model construction
   I. Additional practice in reading word problems
   J. Additional instruction in strategies useful in solving word problems
   K. Discussion of related student experiences
   L. Additional reading or study in supplementary textbooks

Figure 7. Instructional Framework

From *Reading in the Mathematics Classroom* (pp. 13–14), by C. F. Smith and H. S. Kepner, Jr. (with R. B. Kane), 1981, Washington, DC: National Education Association.

Note that the author's purpose is to present a comprehensive list of activities from which teachers may select those they think will best support learning. They also recommend that "teachers who have been frustrated when students fail to read would do well to reconsider the directions they give when making an assignment. [We] urge teachers not to say, 'Go read,' but to give purpose and direction to the assignment" (Smith & Kepner, p. 14).

Notes

Section 4

Six Assumptions About Learning

Six Assumptions About Learning

Learning is:
1. Goal-oriented
2. The linking of new information to prior knowledge
3. The organization of information
4. The acquisition of cognitive and metacognitive structures
5. Nonlinear, yet occurring in phases
6. Influenced by cognitive development

Jones, Palinscar, Ogle, and Carr (1987) have proposed six assumptions about how students learn. Because these assumptions stem from significant research and have important implications for reading instruction including reading in mathematics, they warrant study by educators wanting to examine how learning theory aligns with and undergirds reading theory. Refer to Section 4 in the *TRCA Teacher's Manual* (pages 61–67) for a discussion of these assumptions.

Notes

Section 5

Reading Strategies

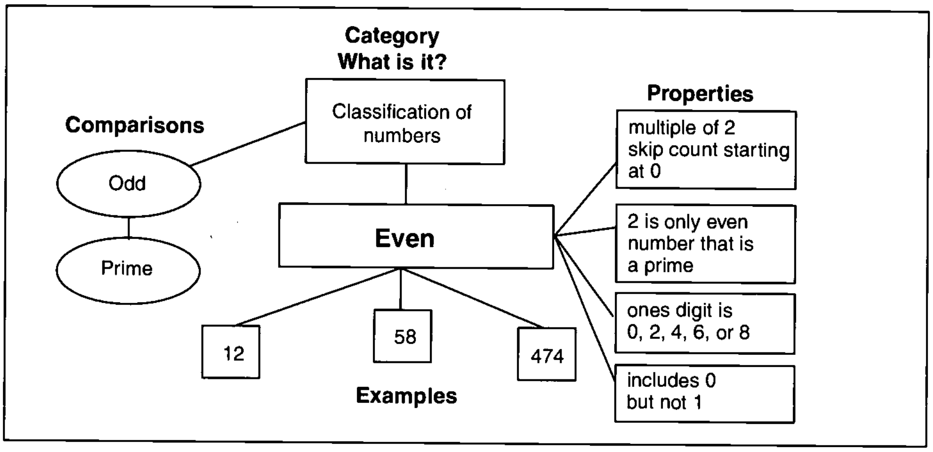
The strategies in this section are particularly applicable to the mathematics classroom. You will note that most of these strategies are described in the *TRCA Teacher's Manual*, however, here we offer examples of these strategies as they apply to mathematics. We have added some strategies in this supplement that do not appear in the *TRCA Teacher's Manual*. Therefore, the page numbers that appear below refer to the contents of this supplement. The format used in discussing each strategy is the same as that used in the *TRCA Teacher's Manual*.

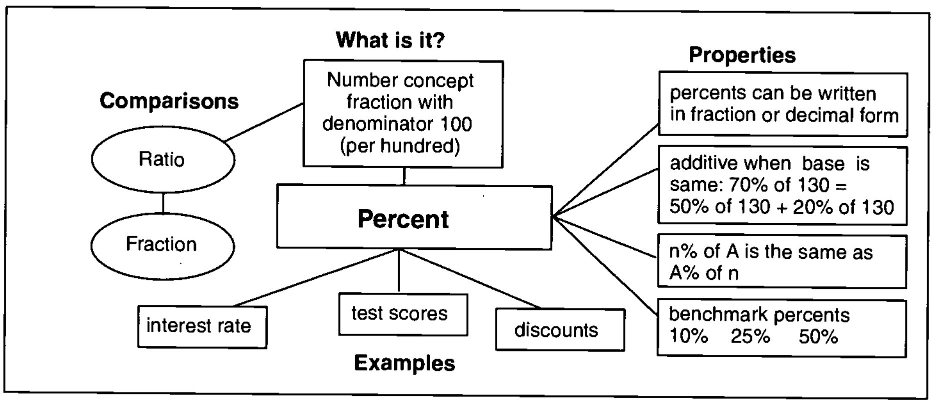
Notes

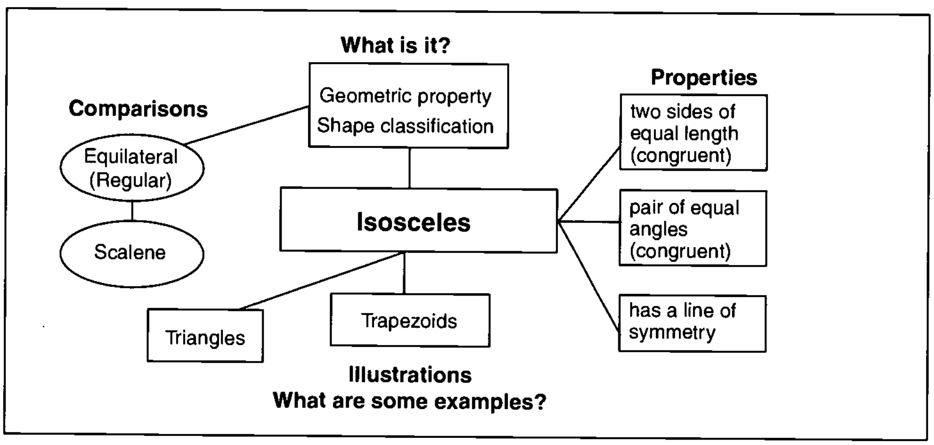
1. Concept Definition Mapping

See pages 70–73 in the *TRCA Teacher's Manual.*

Examples of Visual Representations: **Concept Map**







2. Frayer Model

See pages 74–77 in the *TRCA Teacher's Manual.*

Examples of Visual Representation: **Frayer Model**

Definition A whole number with exactly two divisors (factors).	**Facts** • 2 is the only even prime number. • 0 and 1 are not prime. • Every whole number can be written as a product of primes.
Prime	
Examples 2, 3, 5, 7, 11, 13, ···	**Non-examples** 1, 4, 6, 8, 9, 10, ···

See also the example on page 77 in the *TRCA Teacher's Manual*, which uses geometry vocabulary.

Notes

Notes

3. List-Group-Label

What is it?

Similar to Semantic Mapping and Word Sorts (pages 82 and 89 in the *TRCA Teacher's Manual*), List-Group-Label helps students examine the relationships among subject-matter concepts. Taba (1971) and Fraenkel (1973) note that this strategy also involves students in their learning because they are responsible for contributing the vocabulary they associate with a particular concept rather than manipulating vocabulary provided by the teacher. As such, this strategy can activate prior knowledge and help learners make essential connections between their experience base and new understandings.

How to use it:

1. Write a content-area term on the board or an overhead transparency. Explain to students that this term has something to do with the next unit (or chapter).
2. Ask students to generate words and phrases that they associate with this term. As students volunteer responses, they will stimulate others in the class to contribute their ideas.
3. After you have developed a list of 15–30 words or phrases, ask students to consider what the words have in common and to organize them into categories. Remind them that these categories should identify significant relationships among the terms, something that extends their learning. Grouping words by their initial letter, for example, is not a mathematically significant relationship.
4. Once students have completed classifying these terms, ask them to explain the rationale behind their groupings.
5. Use this discussion as an opportunity to broaden students' understanding of these concepts and how to apply this understanding when solving problems.

Notes

4. Prereading Predictions

See page 78 in the *TRCA Teacher's Manual.*

With some adaptation, this strategy might be used effectively in mathematics. For example, before a lesson on Pascal's Triangle, the teacher could

- list terms (e.g., *rows*, *diagonals*, *sum*, *triangular numbers*, *symmetric patterns*, *combinations*, *binomial coefficients*);
- review meanings of terms in other contexts as appropriate;
- ask for predictions of how these terms might apply in this context; and
- revisit with students (after the lesson) the terms and how they relate to Pascal's Triangle.

5. Semantic Feature Analysis

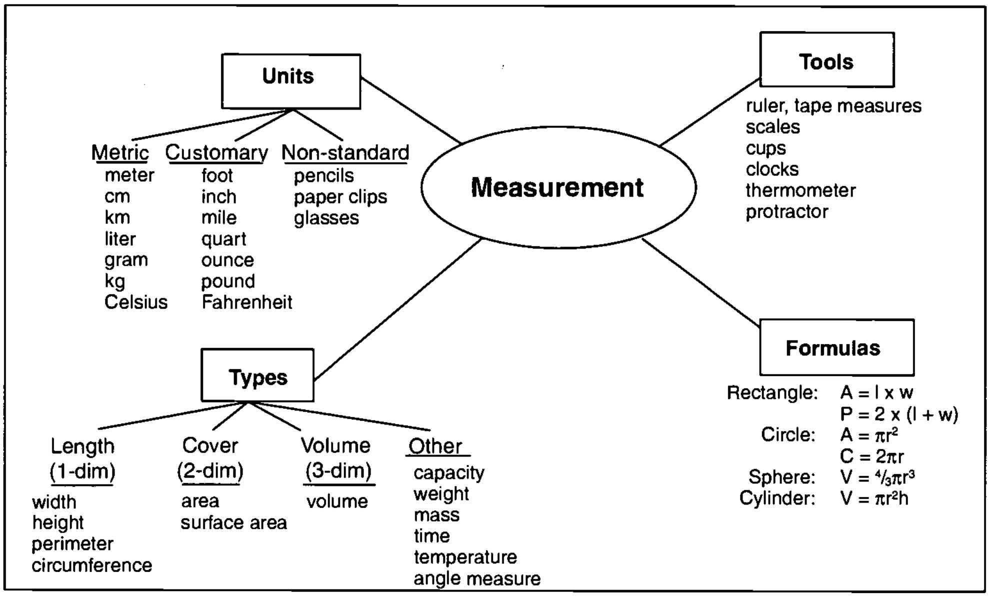
See pages 79–81 in the *TRCA Teacher's Manual.* The example on page 80 uses Semantic Feature Analysis in a geometry context.

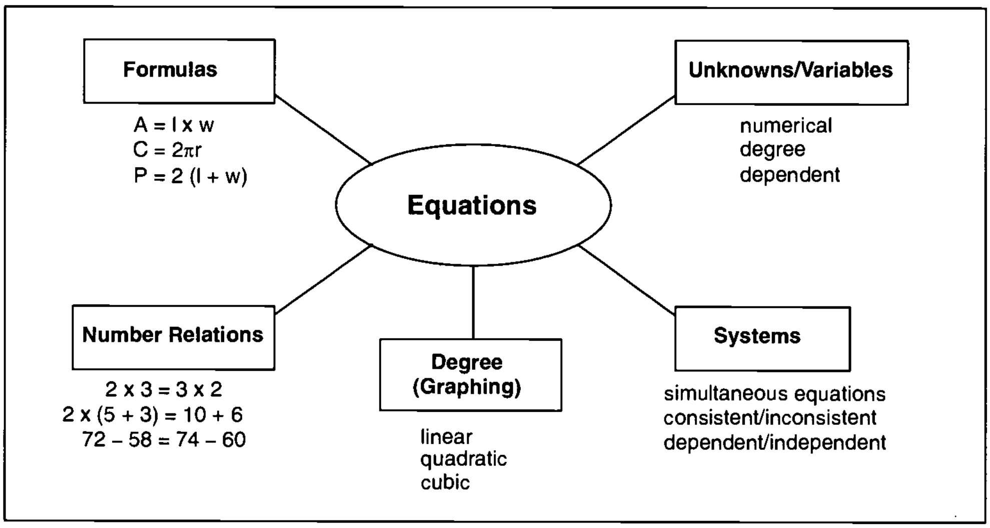
Notes

6. Semantic Mapping

See pages 82–84 in the *TRCA Teacher's Manual.*

Examples of Visual Representations: **Semantic Maps**





7. Stephens Vocabulary Elaboration Strategy (SVES)

See pages 85 and 86 in the *TRCA Teacher's Manual.* This strategy illustrates how vocabulary meanings can vary in different contexts. In mathematics, there are words that take on somewhat different meanings depending on the topic area. For example, consider *dimension* in measurement and coordinate geometry, *dependent* in probability and algebra, *symmetric* and *equivalent* in algebra and geometry, and *degree* in number theory and in angle measurement.

8. Student VOC Strategy

See pages 87 and 88 in the *TRCA Teacher's Manual.* This strategy helps students analyze word meanings from context.

Notes

Notes

9. Word Sorts

See page 89 in the *TRCA Teacher's Manual.* As an example, students can sort vocabulary words used in geometry into categories such as the following:

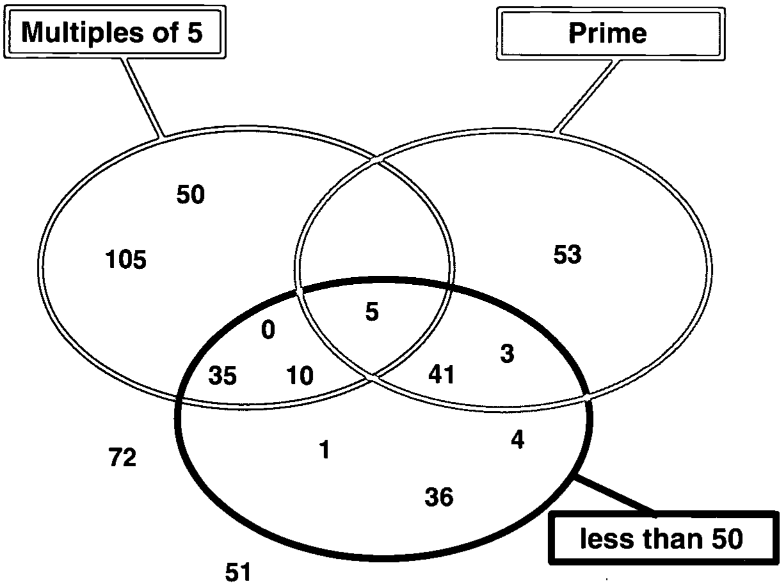
Parts of Shapes	Shapes		Measures	Relations
	Plane figures	Solid figures		
diagonal	triangle	spheres	length	parallel
vertices	square	cubes	perimeter	perpendicular
edges	circle	prisms	volume	adjacent
lines	hexagon	cones	circumference	opposite
points	parallelogram	cylinders	radius	symmetry
rays	rhombus	pyramids	area	intersecting
angles				congruent
				bisector
				similar

A variation of Word Sorts is to provide sets of four or five terms. In each set, three of the four terms (or four of the five terms) are related in some way. The student explains the relationship and identifies one term that is **unrelated** to the others. For example, each list of terms below has one unrelated term once the relationship among the other terms is identified.

length	cubic	acute	similar
perimeter	linear	prime	reflection
volume	quadratic	scalene	rotation
radius	variable	equilateral	translation
width		right	

Notes

Number Sorts (a variation of Word Sorts)



This graphic example could be an exercise if students are asked to place the numbers correctly in the picture.

Vocabulary Development

Notes

Symbols are part of the vocabulary of mathematics. The next two strategies are really games, but these games have a vocabulary development feature with respect to symbols. They can be adapted for use with other topics.

10. Concentration

What is it?

Many teachers adapt the format from the TV game show "Concentration" to help students review concepts they have learned. Adapted from a strategy developed by the National Education Association, "Metric Concentration" is an example of how mathematics teachers can help students store what they have learned about metric equivalents. The components of the game include not only pairs that match but also distracting pairs. Consequently, the game goes beyond asking players to rely on memory alone. To win, students must think about metric principles.

How to use it:

1. Create a deck of cards of paired metric equivalents (e.g., 120 m and 0.12 km) and distractors (e.g., 120 cm and 12 m).
2. Shuffle the deck and place the cards face down. (See the sample deck illustration on page 41.)
3. Ask one student, or player, to turn over two cards. If the student identifies cards that are equivalent, allow the player to keep those cards and take another turn. If the cards are not equivalent, the player should turn them face down again and relinquish his turn.
4. Players alternate turns until all of the cards except the distractors are gone. The winner is the student with the most pairs of cards.

Vocabulary Development

Sample deck for **Metric Concentration.**

8.5 m	29 cm	150 mm	0.4 km	0.7 m
70 mm	1.5 km	370 dm	96 cm	96 mm
15 cm	4 m	7 cm	29 m	0.96 dm
400 cm	85 cm	3.7 m	37 m	400 m
0.029 km	8.7 dm	0.85 km	3.7 km	370 cm
3700 m	8500 mm	0.29 m	0.96 m	1500 m

Notes

Vocabulary Development

Notes

11. Number Cubes

What is it?

The purpose of Number Cubes is for players to order pairs of fractions or decimals. To complete this task, the player must decode and comprehend mathematical symbols, compare their values, and then justify the relationship between them.

How to use it:

1. Construct a pair of cubes with fractions or decimals on each face. For example:

Cube A	Cube B	Cube A	Cube B
$\frac{1}{2}$	$\frac{3}{8}$	0.3	0.49
$\frac{2}{3}$	$\frac{1}{3}$	0.04	0.5
$\frac{4}{7}$	$\frac{5}{6}$	0.055	0.07
$\frac{1}{5}$	$\frac{3}{5}$	0.91	0.061
$\frac{3}{4}$	$\frac{1}{4}$	0.709	0.706
$\frac{7}{8}$	$\frac{2}{7}$	0.09	0.88

2. Explain to players that the goal of the game is to earn points by rolling cubes and correctly identifying which cube shows the greater value. A correct response earns a player two points. The first player to reach the score of 30 (or some predetermined score) wins the game.
3. Begin play by having players roll a cube to determine the order of play.
4. The first player then rolls the cubes, identifies which cube shows the larger value, and explains her answer. The discussion after each play of correct and incorrect answers ensures that players do not win by guessing and also helps reinforce understanding. Play rotates until there is a winner.

Notes

12. Anticipation Guide/Prediction Guide

See pages 104–106 in the *TRCA Teacher's Manual.*

Examples of Visual Representations: **Anticipation Guides**

Anticipation Guide
Statistics

Directions: In the column labeled "Me," place a check next to any statement with which you agree. After reading the text, compare your opinions on those statements with information contained in the text.

Me	Text	
___	___	1. There are several kinds of averages for a set of data.
___	___	2. The mode is the middle number in a set of data.
___	___	3. Range tells how far apart numbers in a data set can be.
___	___	4. Outliers are always ignored.
___	___	5. Averages are always given as percents.

Anticipation Guide
Multiples and Divisors

Directions: In the column labeled "Me," place a check next to any statement with which you agree. After reading the text, compare your opinions on those statements with information contained in the text.

Me	Text	
___	___	1. Multiples relate to multiplying and divisors relate to dividing.
___	___	2. 0 is a multiple of any number.
___	___	3. 0 is a divisor of any number.
___	___	4. Multiples of 2 are called even numbers.
___	___	5. Multiples of 1 are odd numbers.
___	___	6. Every number is a multiple of itself.
___	___	7. Every number is a divisor of itself.

Notes

13. Five-Step Problem Solving

What is it?

Braselton and Decker (1994) assert that students' comprehension of word problems can be enhanced by teaching them to read word problems as meaningful passages — as short stories from which students can construct meaning based on their prior knowledge and experience. Students are presented with a graphic organizer that leads them through a five-step problem-solving process (similar to Polya's four-step process): restate the problem question; determine what information is needed to solve the problem; plan the steps (calculations) to be performed; carry out the plan (perform the calculations); and evaluate the reasonableness of the solution.

How to use it:

1. Using a transparency and overhead projector, introduce students to the layout and design of the graphic organizer (see Figure 8 on page 46). Point out that the diamond shape of the graphic reinforces the fact that each student begins with the same information about the problem and should arrive at the same conclusion, if he has been successful at solving the problem. Between the top and bottom point of the diamond, students will use a variety of problem-solving strategies, depending upon their reasoning and experience.
2. Explain each of the steps outlined in the graphic.
3. Present students with a word problem, reading it aloud and asking students about their prior knowledge of the situation and elements included in the "story."
4. Model for students how to complete the first step of the organizer, restating the question in a number of ways. Ask students to identify which version is the clearest and to explain the reasoning behind their choice. Once students know how to approach the problem, they will feel more confident about solving it.
5. Model how to complete the remaining steps in the graphic organizer.

Notes

6. When students understand the steps in the graphic organizer, offer them opportunities for guided practice: Select another word problem and lead them through each step of the process. Ask students to discuss their thinking as they read the problem and to articulate the reasons for the responses they give. Encourage divergent thinking. Point out to students that there may be several different approaches to the same problem.
7. Let students work in small groups to discuss and complete several more problems using the five-step graphic organizer.
8. Allot a few minutes over the next several days for students to practice solving word problems using the organizer. Point out that the graphic organizer is a guide, not a set formula for solving problems. Students should note that several approaches to the same problem can still result in the same solution.

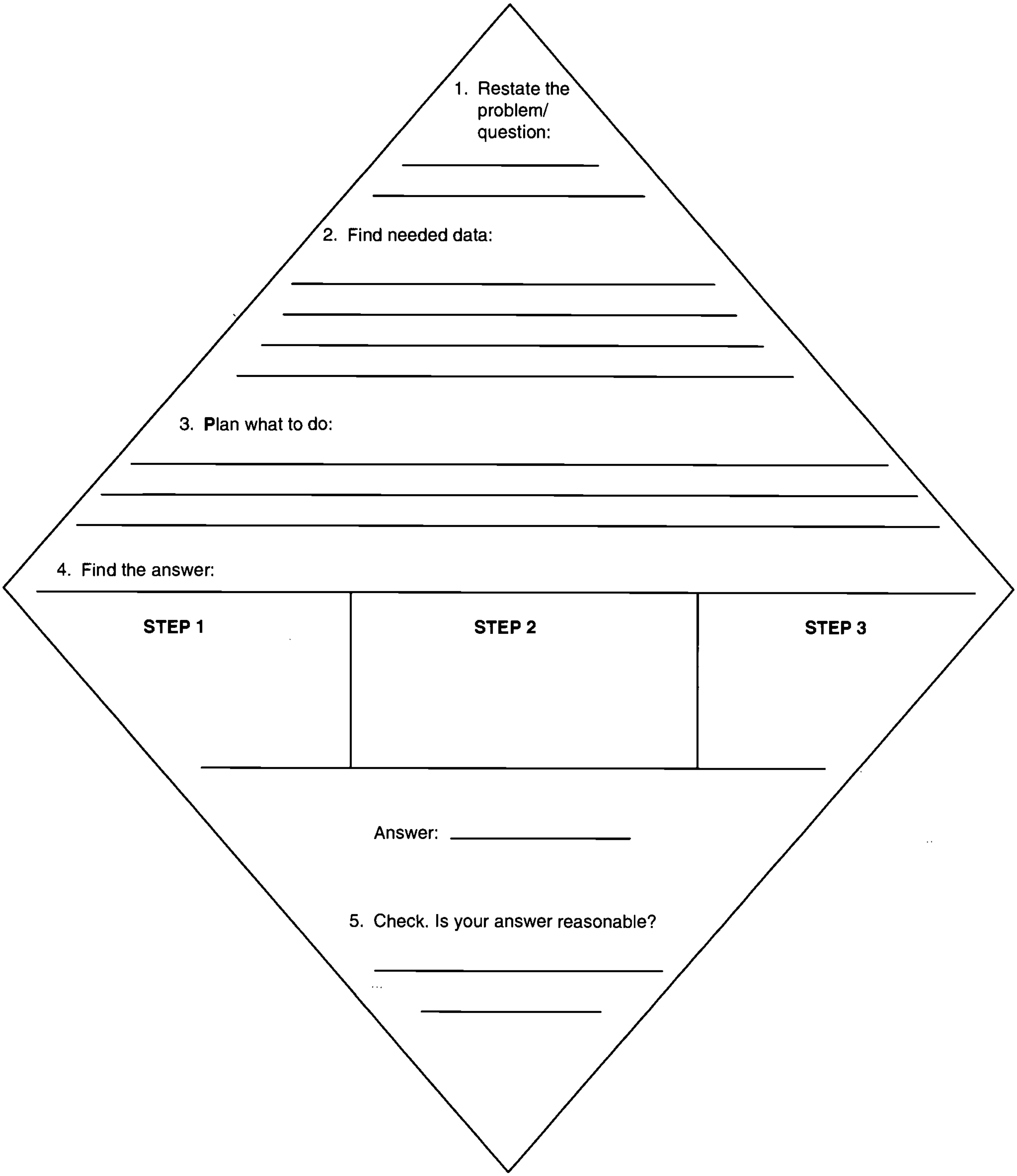


Figure 8. Graphic organizer for Five-Step Problem Solving

Reprinted with permission. Figure adapted from "Using Graphic Organizers to Improve the Reading of Mathematics, " by S. Braselton and B. C. Decker, 1994, *The Reading Teacher, 48*(3), p. 277.

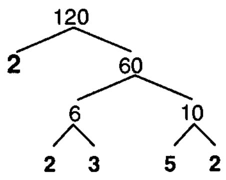
Notes

14. Graphic Organizers

See pages 109–111 in the *TRCA Teacher's Manual.*

Graphic organizers are commonly used in mathematics, as in the following examples:

Factor trees



$120 = 2^3 \times 3 \times 5$

Multiplication Table

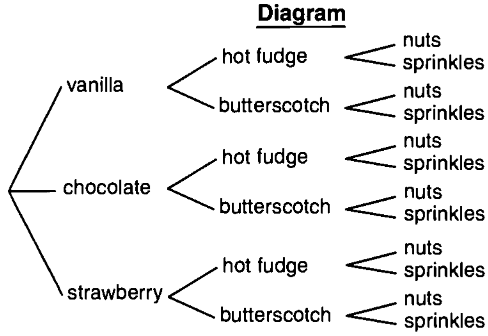
×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

The following list and diagram are examples of graphic organizers for solving the problem of counting the number of possible sundaes with three choices of ice creams, two choices of syrups, and two choices of toppings.

Organized list

Ice Cream	Syrup	Topping
1. vanilla	hot fudge	nuts
2. vanilla	hot fudge	sprinkles
3. vanilla	butterscotch	nuts
4. vanilla	butterscotch	sprinkles
5. chocolate	hot fudge	nuts
6. chocolate	hot fudge	sprinkles
⋮	⋮	⋮

Diagram



See also the examples of Semantic Maps on page 36 of this supplement.

The structured overview described on page 14 is another kind of graphic organizer.

15. Group Summarizing

Notes

See pages 112 and 113 in the *TRCA Teacher's Manual.*

Example of Visual Representation: **Group Summary.**

Sample Organizational Format for Group Summarizing Activity on Pascal's Triangle	
Description 0 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1 1 6 15 20 15 6 1	**Patterns** —symmetry —sum of a row is a power of 2 —diagonals: 1's, counting numbers, triangular numbers
Uses —combinatorial patterns —binomial coefficients	**Interesting Facts** This arithmetic triangle was known to the Chinese as early as 1100 A.D. Italian mathematicians investigated properties of the triangle in 1550. Pascal identified it in 1650.

16. Knowledge Rating Chart

What is it?

Davis and Gerber (1994) suggest that mathematics teachers can assess students' prior knowledge by using a knowledge rating. This strategy differs from the anticipation/prediction guide in that students rate their working knowledge of the concepts to be covered in the lesson. Teachers can use the information students provide to inform instruction.

How to use it:

1. Present students with a list of the major concepts to be covered in the unit, chapter, or lesson.
2. Ask students to rate how familiar they are with these terms.

Examples of Visual Representations: **Knowledge Rating Chart.**

Directions: Rate the following statistics terms as follows:

1. I've never heard of the word before.
2. I've heard the term, but I don't know how it applies to mathematics.
3. I understand the meaning of this term and can apply it to a mathematics problem.

mean ____________________
median ____________________
mode ____________________
weighted average __________

line of best fit __________
correlation ______________
range __________________

normal distribution ______________
bimodal distribution ____________
skewed distribution _____________
flat distribution ________________

Notes

Notes

17. K-N-W-S (K-W-L for word problems)

What is it?

In this strategy students use a worksheet similar to K-W-L to analyze and plan how to approach solving a word problem.

How to use it:

1. Using a transparency and overhead projector (or on the board or chart paper), introduce students to the four-column worksheet.
2. Present students with a word problem and model how to fill in information in each of the columns. Explain **how you knew** what information should be included in each column; teachers often show "how" but don't explain "how you know."
3. Ask students to work in groups to complete the graphic. Ask students to discuss with their groups how they knew what to put in the graphic.
4. Give students ongoing independent practice in using the graphic organizer to solve word problems. Periodically ask students to write an explanation of their reasoning process.

Examples of Visual Representations: **K-N-W-S Worksheet.**

K What facts do I KNOW from the information in the problem?	N Which information do I NOT need?	W WHAT does the problem ask me to find?	S What STRATEGY/ operation/tools will I use to solve the problem?

See pages 116 and 117 in the *TRCA Teacher's Manual* for a discussion of the K-W-L strategy.

Notes

18. Search Strategy

See pages 132–133 in the *TRCA Teacher's Manual.*

Examples of Visual Representations: **Search Strategy.**

SEARCH STRATEGY FOR RESEARCH ON FIGURATIVE NUMBERS

S	**FIGURATIVE NUMBERS**
E	Know Think Want to Know squares & triangles patterns & uses
A	Are other kinds of figurative numbers useful? How would hexagonal or pentagonal numbers be defined?
R	Research
C	Share in groups
H	Discuss learnings Ask new questions

19. Semantic Mapping

See pages 134 and 135 in the *TRCA Teacher's Manual*. The purpose of the "Semantic Mapping" strategy is to encourage students to identify relationships among key concepts and related terms in the reading.

20. "Think-Alouds"

See pages 139–141 in the *TRCA Teacher's Manual.* The purpose of the "Think-Alouds" strategy is to let students see how the teacher attempts to construct meaning for unfamiliar vocabulary, engages in dialogue with the text author, or recognizes when she is not understanding so must select a fix-up strategy.

Notes

21. Three-Level Guides

What is it?

Davis and Gerber (1994) recommend their "three-level guide" to help students analyze and solve word problems. Using a teacher-constructed graphic organizer, students must evaluate facts, concepts, rules, mathematics ideas, and approaches to solving particular word problems.

How to use it:

1. Construct a guide for a given problem similar to the one shown on the next page. The first level (Part I) should include a set of facts suggested by the data given in the problem. The students' goal will be to analyze each fact to determine if it is true and if it will help them to solve the problem.
2. The second level (Part II) of your guide should contain mathematics ideas, rules, or concepts that students can examine to discover which might apply to the problem-solving task.
3. The third level (Part III) should include a list of possible ways to get the answer. Students will analyze these to see which ones might help them solve the problem.
4. Introduce students to the strategy by showing them the problem and the completed three-level guide. Explain what kind of information is included at each level.
5. Model for students how you would use the guide in solving the problem.
6. Present students with another problem and guide. Have them analyze the information you have included to determine its validity and usefulness in solving the problem.
7. With advanced students, ask them select a word problem from the text and complete a three-level guide to be shared with the class.

Notes

Examples of Visual Representations: **Three-Level Guides**.

A three-level guide to a math problem

Read the problem and then answer each set of questions, following the directions given for the set questions.

Problem: Sam's Sporting Goods has a markup rate of 40% on Pro tennis rackets. Sam, the store owner, bought 12 Pro tennis rackets for $75 each. Calculate the selling price of a Pro tennis racket at Sam's Sporting Goods.

Part I

Directions: Read the statements. Check Column A if the statement is true according to the problem. Check Column B if the information will help you solve the problem.

A (true?)	B (help?)	
________	________	Sam's markup rate is 40%.
________	________	Sam bought 12 Pro tennis rackets.
________	________	Pro tennis rackets are a good buy.
________	________	Sam paid $75 for a Pro tennis racket.
________	________	The selling price of a Pro tennis racket is more than $75.

Part II

Directions: Read the statements. Check the ones that contain math ideas useful for this problem. Look at Part I, Column B to check your answer.

________________ Markup equals cost times rate.

________________ Selling price is greater than cost.

________________ Selling price equals cost plus markup.

________________ Markup divided by cost equals markup rate.

________________ A percent of a number is less than the number when the percent is less than 100%.

Part III

Directions: Check the calculations that will help or work in this problem. Look at Parts I and II to check your answers.

____________	0.4 x $75	____________	12 x $75
____________	$75 x 40	____________	40% of $75
____________	1.4 x $75	____________	$75 + ($\frac{2}{5}$ x $75)

Notes

22. Word Problem Roulette

What is it?

Davis and Gerber (1994) posit that students should discuss and write about the content of word problems. The game "Word Problem Roulette" gives students the chance to collaborate on solving a problem and then communicate their thought process and solution in writing.

How to use it:

1. Divide the class into collaborative groups, and provide each group with a word problem.
2. Explain to students that they are to solve this problem verbally. No writing or drawing may be done at any time during this step.
3. After the groups have discussed the problem and agreed how to solve it, members should take turns writing the steps to the solution in words rather than in mathematical symbols. Each group member must write one sentence and then pass the solution sheet to the next group member so he or she can add the next sentence.
4. After the groups have finished writing down all of the steps, ask each group to have one member read the solution to the class, while another writes the symbolic representation of this solution on the board.
5. Solicit volunteers from the other groups to write their version of this mathematical sentence on the board for the class to review.

Examples of Visual Representations: **Word Problem Roulette**.

Directions: Read each problem below and discuss a solution with your group. Do not write anything during the discussion. This should be an oral discussion only. When the group is satisfied with the solution, write a group report of the solution sentence by sentence. Each person in the group writes one sentence and then passes the solution report to another person to write the next sentence. Use only words (no symbols) to write the solution report. 1. A family of three adults and three children go to an amusement park. Adult admission fare is twice as much as a child's. The family spends \$81. How much is the adult admission fare? 2. A sale on swimsuits reduced the price on a two-piece suit from \$45 to \$27. What percent decrease does this represent?

23. Learning Logs

See pages 148–150 in the *TRCA Teacher's Manual.*

The following are possible learning log topics, adapted from Brudnak (1998).

Before learning — to activate and assess prior knowledge:

- Why do we use rulers (or scales or other measuring devices)?
- What do these symbols mean?
- Describe instances when you use addition at home.
- How is multiplication similar to addition?
- Make a web to describe some uses of fractions.

During learning — to help students identify how well they understand what is being covered in class:

- Explain how you know that $7 + 3 = 11 - 1$
- How do you know what a story problem is asking you to do?
- Write a story problem where you need to calculate 5×7.
- Find examples in our classroom of the geometric shapes we are studying.
- Draw three pictures that demonstrate the concept of multiplication.

After the lesson — to help students reflect on their learning:

- I have trouble understanding....
- Write a note to a student who was absent from class and explain what we learned in class today about right triangles.
- Write a note to your parents explaining how you know when a shape has a line of symmetry.
- My favorite kind of story problem is....
- Explain how you could do the calculation $65 - 19$ in your head.

Notes

Notes

24. Question-Answer Relationships (QAR)

See pages 145–147 in the *TRCA Teacher's Manual.* The purpose of the QAR strategy is to provide teachers and students with a common vocabulary to discuss questions and sources of information for answering questions.

25. Writing-to-Learn

See pages 154–157 in the *TRCA Teacher's Manual.* Writing-to-Learn activities can be used to help students reflect on and explore ideas and concepts they are learning about or reading about in class.

Bibliography

Braselton, S., & Decker, B. C. (1994, November). Using graphic organizers to improve the reading of mathematics. *The Reading Teacher, 48*(3), 276–281.

Brennan, A. D., & Dunlap, W.P. (1985). What are the prime factors of reading mathematics? *Reading Improvement, 22,* 152–159.

Brudnak, K. A. (1998, January/February). What works in math: Math communication. *Learning, 26*(4), 38–49.

Culyer, R. C. (1988). Reading and mathematics go hand in hand. *Reading Improvement, 25*, 189–195.

Davis, S. J., & Gerber, R. (1994, September). Open to suggestion: Content area strategies in secondary mathematics classrooms. *Journal of Reading, 38*(1), 55–57.

Dickson, S. V., Simmons, D. C., & Kameeni, E. J. (1995, February). *Text organization and its relation to reading comprehension: A synthesis of the research.* Technical Report 17. National Center to Improve the Tools of Educators, University of Oregon.

Denmark, T. (1983, January). Improving students' comprehension of word problems. *Mathematics Teacher, 76*(1), 31–36.

Earp, N. W. (1970, April). Observations on teaching reading in mathematics. *Journal of Reading, 13*, 529–33.

Earp, N. W., & Tanner, F.W. (1980, December). Mathematics and language. *Arithmetic Teacher, 28*(4), 32–34.

Fraenkel, J. R. (1973). *Helping students think and value: Strategies for teaching the social studies.* Englewood Cliffs, NJ: Prentice-Hall.

Gray, A. Susan. (1999, September). Is problem solving in middle school mathematics "normal"? *Middle School Journal, 31*(1), 41–47.

Helwig, R., Rozek-Tedesco, M., Tindal, G., Heath, B., & Almond, P. (1999, December). Reading as an access to mathematics problem solving on multiple-choice tests for sixth-grade students. *Journal of Educational Research, 93*(2), 113–125.

Jones, B.F., Palinscar, A.S., Ogle, D.S. & Carr, E.G. (1987). *Strategic teaching and learning: Cognitive instruction in the content areas.* Alexandrea, VA, and Elmhurst, IL: Association for Supervision and Curriculum Development and North Central Regional Educational Laboratory.

Knifong, J. D., & Hotran, B. D. (1977, May). A search for reading difficulties among erred word problems. *Journal for Research in Mathematics Education, 8*(3), 227–30.

Kresse, E. C. (1984, April). Using reading as a thinking process to solve mathematics story problems. *Journal of Reading, 27*, 598–601.

Bibliography

Linville, W. J. (1976). Syntax, vocabulary and the verbal arithmetic problem. *School Science and Mathematics 76*, 152–158.

Moss, J. (1997, January/February). Math that makes sense. *Learning*, *25*(4), 52–56.

Muth, K. D. (1988). Comprehension monitoring: A reading — mathematics connection. *Reading Research and Instruction, 27*(3), 60–67.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Professional standards for teaching mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Research Council, Mathematical Sciences Education Board. (1989) *Everybody counts: A report to the nation on the future of mathematics education*. Washington DC: National Academy Press.

National Research Council, Mathematical Sciences Education Board. (1990). *Reshaping school mathematics: A philosophy and framework for curriculum.* Washington DC: National Academy Press.

Reutzel, D. R. (1983, October). C^6: A reading model for teaching arithmetic story problem solving. *The Reading Teacher, 37*, 28–35.

Shuard, H., & Rothery, A. (Eds.). (1984). *Children reading mathematics*. Athenaeum Press: Newcastle upon Tyne, England.

Siegel, M., & Borasia, R. (1992). Toward a new integration of reading in mathematics instruction. *Focus on Learning Problems in Mathematics, 14*(2), 18–36.

Smith, C. F., & Kepner, H. S., Jr. (with R. B. Kane). (1981). *Reading in the mathematics classroom.* Washington, DC: National Education Association.

Taba, H., Durkin, M. C., Fraenkel, J. R., & McNaughton, A. (1971). *A teacher's handbook to elementary social studies: An inductive approach.* Reading, MA: Addison-Wesley.

Thomas, D. A. (1988). Reading and reasoning skills for mathematics problem solvers, *Journal of Reading, 32,* 244–249.

BEST COPY AVAILABLE

McREL

2550 S. Parker Road, Suite 500
Aurora, CO 80014-1678
303.337.0990 • www.mcrel.org