This study investigated individual achievement change over time in mathematics for African American students, whether this differs from student to student, and whether individual growth parameters for this domain were related over time. The study also sought to gain a deeper understanding of individual change in student academic achievement through covariance structure analysis in the Statistical Analysis System PROC MIXED Procedure. The study used panel data from the Louisiana State Department of Education files for students tested with the state's Norm Referenced Tests in grades 4, 6, and 7. Complete scores were available for 11,627 students in the 3 waves. The variety of data analyses and model fitting procedures used with the panel achievement data in mathematics showed that initial differences in achievement were evident among these students at grade 4, with those not on the lunch program, and consequently of higher income groups, showing somewhat higher mean scores in mathematics. Mathematics intercepts for the lunch-receiving and nonlunch program students were statistically significant. Results show the presence of true individual differences among students' learning growth rates. Results also indicate that where a student starts in domain achievement is related to future growth in mathematics. Two appendixes contain figures illustrating change in mathematics achievement. (Contains 53 references.) (SLD)
A Study of Students' Academic Change in Mathematics Achievement: A Case for African American Students

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A Study of Students' Academic Change in Mathematics Achievement: A Case for African American Students

In order to understand growth in mathematics, a basic knowledge of learning theory, language acquisition process and cognitive processes is essential. Several theories have emerged in the realm of psychology and education suggesting that individual learners employ various strategies when processing information during classroom experiences (Morgan, 1997, Rugutt, 2000). Among the basic components involved in learning are: perception, sensory organs, short-term memory, long-term memory and motoric systems. These components work in a complex interactive way through the human central nervous system. Piaget (1929) presented five stages of cognitive development that postulate that as children grow older, their abilities to conceptualize develop. These stages are a) sensori-motor, where the infant learns to differentiate self and objects in the external world (0 and 2 years of age), b) pre-operational thought, which is between 2 and 4 years of age, is characterized by egocentricism and classification of objects in the external world by the child, c) the intuitive stage which occurs between the ages of 4 and 7. In this stage, the child thinks in classificatory ways but may be unaware of classifications, d) the fourth stage, characterized by concrete operations, takes place between 7 and 11 years. During this stage, the child is able to use logical operations such as reversibility, classification and serialization and, c) the developmental stage is punctuated by growth of formal operations. This takes place during ages 11 through 15. This stage is characterized by trial steps towards abstract conceptualization.

In a similar developmental model Cramer (1978b), describes five stages in the language acquisition process. Stage one is marked by babbling and random
experimentation with sounds. The child produces all sounds relevant to his native language as well as sounds significant in languages other than his own. Stage two sets a beginning of recognizable behavior. The child responds to verbal language signals and begins to produce sounds to express needs. Later, utterances such as “bye-bye, da-da, ma-ma” become common as the child’s vocal mechanism and mental development grow. Stage three is described as “telegraphic” because of the preponderance of nouns and verbs over other words (articles, prepositions, conjunctions, and auxiliary verbs). In stage four, acquisition of syntactic structures of language, rules for the generation of the same, and rapid expansion of new vocabulary items are experienced. In stage five, the child has internalized native language grammar. Generation of grammatical sentences becomes evident.

Bandura (1977) contends that, as a process, learning involves functionalism, interactionism, and significant symbolism. He stresses the depth of how individuals are capable of self-regulation and self-direction. Bandura’s theory is based on concepts such as response, conditioning, stimulus, reward, imitation, conformity, deviance among others, in relation to personal development (Jarvis, Holford, & Griffin, 1998).

The notion that a problem or a particular subject matter is difficult to solve is a key organizing concept in the design of any math activity by teachers and that difficulty is a quantitative concept (Ohlsson, Ernst, & Rees, 1992, Rugutt, 2000). Case (1985, 1992) investigated how working memory develops in relationship to Piagetian stages of cognitive development and found that working memory is domain specific for mathematics in 12- to 14-year-olds in both traditional and gifted students.
Dark and Benbow (1990, 1991) report similar results on working memory and growth in mathematics skills. Most past research has concentrated on the early acquisition of mathematical skills with a focus placed on pace and sequence of skill acquisition. There are few studies that included individual differences and rates of change other than those labeled disabilities (Robinson, Abbott, Berninger & Busse, 1996).

Mathematics and language go hand in hand in setting a stage for an understanding of learning aspects in student academic life. This could be because of the intricate nature they both offer in affecting each other and on affecting other domains. Language and mathematics are the cornerstones of student academic growth. A student with a strong foundation in both of these domains is more likely to do well in many other disciplines. Knowledge of how mathematics and language relate and how students grow in them is crucial not only for pedagogical reasons but also for the health of education of any educational system and for the prosperity of student welfare.

It is therefore important that key factors that impact on math achievement are understood and researched on. McLeod (1988) stated that students often report frustration or satisfaction when they work on non-routine problems and that affective responses are an important factor in problem solving.

Within the extant literature on the early acquisition of mathematical skills, many studies have focused on the pace and sequence of the skills acquisition, with very few extending to individual differences and the rate of development. Williamson, Appelbaum, & Epanchin (1991) used individual growth curves to study academic growth in reading and mathematics and found out that the correlation between rate of
change and ability test scores range from 0.534 to 0.700 for grade 3, in mathematics achievement and mathematics ability.

Mathematics is a language that uses symbols and signs. For a mathematical problem, say in an examination, a student will face a situation expressed in a combination of words, symbols, data and diagrams. The student's first task is to translate the problem into what could be called the language of mathematics. In the language, figures are translated into mathematical grammar. Since mathematics is typically a problematic subject for beginners, it is necessary to introduce mathematics during the early years of schooling. Lesh and Zawojewski (1992) discussed problem solving strategies such as drawing a picture, thinking of a related problem and working "backward. These strategies help the learner break the problem into smaller and easier steps that are easily built into a cognitive process. In knowledge organization and problem solving strategies, Krutetskii (1976) argued that different systems of thought used by gifted students are inaccessible to those who do not have highly organized knowledge. Talented students can skip intermediate steps and generalize broadly and faster than the average students who may need to develop new ways of thinking which involves reorganization of their knowledge and evident in Piaget's concrete operational level of reasoning from normal operation levels.

Reys (1990) while discussing the key areas in mathematics estimation pointed that the variety of possible approaches to an estimation problem creates an open-ended, problem-solving-oriented atmosphere in a learning environment and in effect presents unique instructional problems. Reys (1990) summarized that computational
estimation, much like the problem solving, calls on a variety of skills, which is built over a long period of time.

**Conceptual and Empirical Issues in Studying Change**

Why should change be studied in education? A focus on the study of change enables an in-depth investigation of how key elements of learning in and other variables exert an influence on student achievement outcomes. A study of change in education lends itself to an in-depth evaluation of the extent differences in schooling experiences; in particular, differences in classroom environment and instructional quality, contribute to the development of interindividual differences in achievement. While many theorists have presented models to describe growth and change, these models are infrequently tested with data (Magnusson, 1985). It is apparent that lack of familiarity with many quantitative methods for estimating learning growth curves appears to be a major obstacle to the empirical testing of growth models (Burchinal & Appelbaum, 1991). Bryk and Raudenbush (1992) amplified the same problem by noting that research on change has been plagued by inadequacies in conceptualization, measurement, and design and has long perplexed behavioral scientists. In many situations, instruments used to assess the subjects are developed for fixed points in time, yet individual academic growth is dynamic. These instruments have not adequately captured individual differences in the rate of change. The study of change requires more than two waves of data but frequent studies have utilized only two data points and are thus not able to adequately address the issue of growth (Bryk & Raudenbush, 1987; Bryk & Weisburg, 1977; Rogosa, Brandt, & Zimowski, 1982). When there are only two waves of data on each subject, there is no way to know the
exact shape of individual growth over time (Willett, 1988). It has also been stressed that data from two time points and the difference score are less than optimal for the study of change but three or more waves of data are preferable (Olweus & Alsaker, 1991).

The difference score that was initially employed and continues to be used as a measure of change because of the concentration of two-waves measurement has restrictive assumptions and its continued use as a measure of change has been condemned by many researchers (Cronbach, Furby 1970; Lord, 1963; O’Connor, 1972; Thorndike, 1966). These researchers have instead recommended other statistical techniques of evaluating change.

The study of this change in education is important because it is through change that the effectiveness of a curriculum can be assessed and improved. In recent research on individual change, investigators have used individual growth modeling in order to make use of the enormous volume of longitudinal data available in academic and related institutions, while providing better methods for investigating interindividual differences in change (Bryk & Raudenbush, 1987; Rogosa et al., 1982; Rogosa & Willett, 1985; Sayer & Willett, 1998; Willett, 1988; Willett & Sayer, 1994, 1996). Further, recently, pioneering researchers have shown how the analysis of change can be conducted conveniently by the methods of covariance structure analysis (Tisak & Meridith, 1990; Sayer & Willett, 1998; Willett & Sayer, 1994, 1996).
Purpose/Objectives

The purpose of this study was to investigate individual achievement change over time in mathematics for African American students and whether this differs from student to student and if the individual growth parameters for this domain were related over time. Further, the study sought to gain a deeper understanding of individual change in student academic achievement through the covariance structure analysis in the SAS PROC MIXED procedure. The study was designed to answer the following questions: (a) are the growth parameters (intercepts and slopes) in mathematics related within domain? (b) is the pattern of interrelationships, among the individual achievement growth parameters, the same for African American students with free/reduced cost lunch and without free/reduced cost lunch (SES)? (c) are there marked differences in variability in Mathematics achievement growth parameters within each SES group?

Conceptual/Theoretical Frameworks

In the past many concerns have been raised about the relative low performance of U.S. students in mathematics and science as compared to those of US key economic competitors (Kaplan & Elliott, 1997). Reynolds and Walberg (1992) stressed this fact by citing comparative studies that continue to show the poor performance of U.S. students, especially at the junior and high school levels. However, many of these studies have not been longitudinal in nature.

Literature is replete on the early acquisition of mathematical skills, many studies have focused on the pace and sequence of the skills acquisition, with very few extending to individual differences and the rate of development. Williamson, Appelbaum, and Epanchin (1991) used individual growth curves to study academic growth in reading and
mathematics and found a moderate correlation between rate of change and ability test scores.

Willett and Sayer (1996) studied the growth of change in mathematics and language in healthy, asthmatic and seizure groups of children of ages 7, 11 and 16. Their study established that true growth trajectories for healthy and asthmatic children were similar while those with seizures had low averages in both domains. Positive correlation coefficients between the initial status in reading and initial status in mathematics and between the rate of change in reading and the rate of change in mathematics were established.

Sanders & Horn (1998), utilized longitudinal data to study student academic growth over time, stated that the child serves as his or her own “control” thus allowing the partitioning of school system, school and teacher effects free of exogenous factors that influence academic achievement. Their study found that largest academic gains are in the lowest achievement group. However, limited studies exist that have focused on individual growth trajectories in mathematics and the impact associated with social economic status (SES).

In order to capture details of the component of student academic growth, this study was partitioned into four basic parts. First, it investigated the growth curves to compare two groups dichotomized on the basis of SES (lunch was used as a proxy for SES) levels in mathematics. Second, it investigated the patterns of interrelationship among the individual achievement growth parameters for the two groups--those with and those without free/reduced price lunch. Third, it investigated the variability in learning abilities, observed from academic growth parameters for two groups of learners in
mathematics. Fourth, it utilized a combination of individual academic growth curves and covariance structure analysis, a more flexible and robust technique than the traditional methods in the study of academic growth. The traditional methods have been limited in sensitivity to errors in model parameters.

**Methodology**

**Sampling Procedures**

This study used panel data drawn from the Louisiana State Department of Education (LDE) school data files. The subset of students involved was obtained as follows. Of all the elementary school students in the LDE data files, only those who attended public schools and were of African American ethnic group were sampled. The sampled students were tested on the Norm Referenced Tests (NRTs) in grade 4, 6 and 7. Wave one had 24,030 students; wave two had 22,262 students, while the third and last wave had 23,982 students. The subsets of students who had complete records for grades 4, 6 and 7 were 11,627.

**Instrumentation and Measurement**

The Iowa Tests of Basic Skills (ITBS) and Norm-Referenced Tests (NRTs) as part of the Louisiana Educational Assessment Program (LEAP), was utilized. The NRT measure is a multiple choice scale for mathematics domain and allow the educators to compare individual and group performance results with a national norm. These tests indicate how a given student's knowledge or skill compares with others in the norm group. Reliability data for the ITBS meet stringent psychometric standards where the ITBS Complete Battery average test reliabilities (K-R 20) for grades 3 through 8 are 0.86 and 0.87 for the fall and spring, respectively.
Data Analysis Procedures

This study adopted a multilevel data analysis procedure as provided in the covariance structure analysis technique of Singer (1998), Sayer and Willett (1998) and Willett and Sayer (1994, 1996) for single and double populations. Ordinary least squares (OLS) fitted trajectories summarizing observed growth patterns for both mathematics and language between grade 4 and 7 for a subsample of 13 students selected from sample were completed. The multilevel data analysis techniques carry out analyses at two levels simultaneously; within- and between- individuals (Bryk & Raudenbush, 1992; Kaplan & Elliott, 1997; Yang & Goldstein, 1996).

The complete results of the hierarchical linear modeling approach, utilizing the SAS PROC MIXED routine, as detailed in the works of Littell, Milliken, Stroup, and Wolfinger (1996), and Singer (1998) are presented in Appendices A and B. In utilizing this approach, individual growth models for mathematics were treated as linear functions of time with the individual intercepts and slopes treated as random. Using this technique (hierarchical/random coefficient modeling), "an unconditional linear growth model" with a simple two-level model was considered, in which the level-1 model is a linear individual growth model, and the level-2 model expresses variation in parameters from the growth model as random effects unrelated to any person-level covariates/predictors. The parameters in level-1 (within person) model used \( \pi \) and the parameters in the level-2 (between person) model used \( \beta \). The level-1 and level-2 models were then written as:
\[ Y_{ij} = \pi_{0j} + \pi_{1j} (\text{Time})_{ij} + r_{ij}, \text{ where } r_{ij} \sim N (0, \sigma^2) \] and
\[
\pi_{0j} = \beta_{00} + u_{0j},
\pi_{1j} = \beta_{10} + u_{1j}, \text{ Where }
\begin{pmatrix}
  u_{0j} \\
  u_{1j}
\end{pmatrix}
\sim N
\begin{pmatrix}
  0 \\
  0
\end{pmatrix},
\begin{pmatrix}
  \tau_{00} & \tau_{01} \\
  \tau_{10} & \tau_{11}
\end{pmatrix}
\]
which were written in combined form as:
\[
Y_{ij} = [\beta_{00} + \beta_{10} \text{Time}]_{ij} + [u_{0j} + u_{1j} \text{Time}_{ij} + r_{ij}],
\]
As can be seen above, the multilevel model was expressed as the sum of two parts: a fixed part, which contains two fixed effects (for intercept and for the effect of \text{TIME}) and a random part, which contains three random effects (for the intercept, the \text{TIME} slope, and within person residual \( r_{ij} \)). The time variable for this study was grade level and it appeared in the model line as the predictor for mathematics. The treatment of the intercept and slopes as random effects can be changed, and also the covariates (predictors) of the level-2 components can be introduced depending upon the nature of the particular research question.

**Results**

As a first step in choosing the appropriate mathematical function to represent true individual change, this study conducted a series of exploratory strategies such as inspecting each person's empirical growth record by plotting his or her observed status against time (Sayer & Willett, 1998; Willett, 1989; Willett & Sayer, 1994, 1996). This study also examined wave-by-wave univariate statistics on the dependent variable to check if the normality assumptions were tenable.

An individual-level data exploration is crucial when covariance techniques are being employed and as such a careful inspection of the data in the table suggests that there is variability at the initial point of data evaluation (grade 4 in this case) in
mathematics scores. There is also heterogeneity in the rate at which skills are being
developed (progress) over time—comparing the domain within individual. The inspection
of the individual growth trajectories for mathematics scores for most students increased
as time passed and that there was heterogeneity in observed change across students.
Further, an inspection of the fitted trajectories for mathematics and for each student of a
subsample of 13 randomly selected students shows that a test of strict-stability model
(trjectories parallel to the horizontal line), that is no growth occurred at all—the growth
curves for the entire sample consist of a set of parallel lines is rejected as evident by the
growth curves. Neither a parallel stability model—a model that posits that there is
growth, but everyone grew by the same amount—that is, there was no individual
differences in growth though mean growth levels occurred as was indicated by growth
trajectories. In mathematics and within each SES group the fitted trajectories show cases
of growth heterogeneity.

Table 1 shows mathematics mean scores for students receiving free/reduced cost
lunch and those bearing full costs of lunch. The lunch variable is utilized as a proxy for
social economic status (SES). These results show that students without free/reduced cost
lunch had higher mean values in mathematics. The initial mean differences across the two
groups of learners continued to grow as students advanced in school. Using the mean
values provided in Table 1, it is evident that learners continue to diverge in mathematic
achievement as they advance in school. Within the two groups of lunch categories,
African American students on free/reduced cost lunch program scored lower than their no
free/reduced cost lunch counterparts in mathematics and the differences continue to
widen as students move from grade 4 through grade 7. These results suggest that
students' initial status in mathematics is important. The results suggest that initial mathematics differences among the groups are maintained, and for students without free/reduced cost lunch actually widened, from grade 4 through 7.

Table 1: Estimated Means of Three Waves of Mathematics for African American Students With Lunch and Without Free/Reduced Cost Lunch

<table>
<thead>
<tr>
<th>Grade</th>
<th>With Lunch (WL)</th>
<th>Without Lunch (NL)</th>
<th>Mean Difference (NL-WL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 4</td>
<td>187.6</td>
<td>193.7</td>
<td>6.1</td>
</tr>
<tr>
<td>Grade 6</td>
<td>208.4</td>
<td>213.7</td>
<td>5.3</td>
</tr>
<tr>
<td>Grade 7</td>
<td>217.4</td>
<td>225.6</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Note: The within grade mean difference score was computed by subtracting African American students' with lunch mean score from African American students' without lunch mean score.

In interpreting the output presented in Figure 1 (Appendix A), for the students who were in the lunch program, it can be seen that PROC MIXED converged in four iterations. This relatively rapid convergence is not always the case especially when the data is not balanced or there is a high degree of collinearity in the dataset. Starting with the estimates of the fixed effects, $\beta_{00} = 186.89$. This is the estimate of the average of the intercepts across persons, that is, the average value of the standardized mathematics achievement when $\text{TIme} = 0$ [i.e., at grade = 4] and $\beta_{10} = 10.30$ is the estimate of average slope across persons. To summarize, the average person on lunch program began with a score of 186.89 and gained 10.30 points per testing occasion. The null hypothesis that either of these parameters are 0 in the population was rejected since the t-values, 1120 and 157.45 were large and significant.

The focus of the random effects on the estimated values of the variance-covariance matrix of the growth parameters, which can be written as:
\[
\begin{pmatrix}
\tau_{00} & \tau_{01} \\
\tau_{10} & \tau_{11}
\end{pmatrix} =
\begin{pmatrix}
139.45 & 30.07 \\
30.07 & 12.33
\end{pmatrix}
\]

and the estimated value of \( \sigma^2 \), 124.22. In the SAS output given Figure 1 (Appendix A), it can be seen that all the tests are rejected, including those of the variances of the intercepts and the slopes \((\tau_{00}, \tau_{11})\) respectively. This suggests that there is variation in both the intercepts and slopes, that which could be explained by level-2 (person-level) predictors/covariates.

The standard errors of the estimated growth parameters are also produced by SAS and the results of the test that these population variances (covariance) are 0. The model also provides several goodness of fit statistics that can be used to evaluate this model and to compare the goodness of fit for this model with that of other (nested) models. Some of the models that could be evaluated are those derived by allowing the within person covariance-variance structure to take different shapes such as compound symmetry, autoregressive (1) and totally unstructured.

Figure 2 (Appendix B) presents a summary of the results of random coefficient regression analysis (hierarchical linear modeling) for mathematics and for group of learners without the lunch program. As was the case with the lunch program group, the results show the presence of variability in the intercepts and slopes and the covariance-variances within the mathematics domain.

In interpreting the output presented in Figure 2 (Appendix B), for the students who were not in the lunch program, it can be seen that PROC MIXED converged in four iterations. Like the group in the lunch program, this is also relatively rapid convergence, and is not always the case given the conditions stated previously. Starting with the
estimates of the fixed effects, $\beta_{00} = 191.18$. This is the estimate of the average of the intercepts across persons, that is, the average value of the standardized mathematics achievement when $\text{TIME} = 0$ [i.e., at grade = 4] and $\beta_{10} = 11.41$ is the estimate of average slope across persons. To summarize, the average person not on lunch program began with a score of 191.18 and gained 11.40 points per testing occasion. The null hypothesis that either of these parameters are 0 in the population was rejected since the t-values, 511.08 and 82.53 were large and significant.

Further, the focus of the random effects on the estimated values of the variance-covariance matrix of the growth parameters, which can be written as:

$$
\begin{pmatrix}
\tau_{00} & \tau_{01} \\
\tau_{10} & \tau_{11}
\end{pmatrix} = 
\begin{pmatrix}
173.87 & 36.43 \\
36.43 & 13.06
\end{pmatrix}
$$

and the estimated value of $\sigma^2$, 112.32. In the SAS output given Figure 2 (Appendix B), it can also be seen that all the tests are rejected, including those of the variances of the intercepts and the slopes ($\tau_{00}$, $\tau_{11}$) respectively. This suggests that there is variation in both the intercepts and slopes for the learners not in the lunch program, that which could be explained by level-2 (person-level) predictors/covariates, as was the case with students in the lunch program.

The standard errors of the estimated growth parameters and the results of the test that these population variances (covariance) are 0 are also provided. Further, the model also provides several goodness of fit statistics that can be used to evaluate this model and to compare the goodness of fit for this model with that of other (nested) models. The SAS results presented in Appendices A and B are summarized in Table 2.
Table 2: A Summary of the Random Coefficient Regression Analysis (Hierarchical Linear Model-HLM) Results for both Free/Reduced Cost and no Free/Reduced Cost Lunch groups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Intercept (I)</th>
<th>Slope (S)</th>
<th>Variance (I)</th>
<th>Variance (S)</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA^a</td>
<td>186.89</td>
<td>10.30</td>
<td>139.45</td>
<td>12.33</td>
<td>124.22</td>
</tr>
<tr>
<td>AA^b</td>
<td>191.17</td>
<td>11.41</td>
<td>173.87</td>
<td>13.06</td>
<td>112.32</td>
</tr>
</tbody>
</table>

AA^a African American with free/reduced cost lunch
AA^b African American without free/reduced cost lunch
Note: All entries in table are significant at p < 0.05

The major findings of the study showed that: 1) students vary significantly in knowledge of mathematics at entry into grade 4 and that students not in the lunch program overall initial status in mathematics was higher than that of students with free/reduced cost lunch, 2) the mathematics overall slopes for the two groups of learners were positive and significantly different from zero, 3) the correlation coefficients of the slope and initial status for mathematics and within each group of learners were positive and statistically significant and 4) variance estimates for mathematics slopes were significantly different from zero and showed variance increases at lower grade levels as students advance in school from grade 4 through grade seven.

In this study, the maximum likelihood estimates of the population means of true intercept and true slope in mathematics for the two groups of learners were reported. The entries in the first row of Table 2 estimate the African American (on lunch program) population means of true intercept (186.89, p < 0.05) and true slope (10.30, p <0.05) for mathematics. The estimated population means of true intercept and true slope for the African American learners not on lunch program are 191.17 (p < 0.05) and 11.41 (p < 0.05), respectively. The true intercept and true slope for the respective group describe the
average trajectory of true change in the dependent variable—the average trajectory of true change in dependent variable. On average, African American (on lunch program) students’ true mathematics scores increase by 10.30 per year while true mathematics scores for the group not on lunch program increase by 11.41 per year. Students’ knowledge in mathematics improved over time, and more rapidly for the group not on lunch program.

In both groups of learners (with lunch and with no lunch), mathematics slope parameters were all positive and statistically significant. The domain respective intercepts are initial average achievement scores at grade 4 adjusted for measurement error (Kline, 1998). The intercept is a characteristic of the whole sample while the variance of the same, reflects the range of individual differences in the mathematics domain around the intercept. The mean rate of change, on the other hand, reflects a group-level characteristic—its value indicates the average amount of occasion-to-occasion change in mean levels of mathematics (also adjusted for measurement error). The statistics provided by the slope (rate of change) presents information about the rate of individual differences in linear occasion-to-occasion changes over time.

The variances of mathematics for the two groups of learners were statistically significant. Thus, there is evidence of interindividual heterogeneity in true change in mathematics. Thus, students differed in their growth trajectories in mathematics. Correlation coefficients of intercepts and slopes within the mathematics domain were positive and statistically significant were both positive in direction. These coefficients show that students who had high initial mathematics achievement scores showed greater rates of subsequent change. They tended to progress more rapidly in mathematics over
time. Further, the intercept was related to the slope changes in mathematics. Thus, where a particular student starts in an achievement domain is necessarily related to his or her future growth (mean level) in the domain of interest.

**Brief Summary of Major Results**

The variety of data analyses and model fitting procedures used with the panel achievement data in mathematics for the large sample of African American students in grade 4, 6 and 7 in this study showed the following:

- Initial differences in achievement levels were evident in mathematics at grade 4 with African American students not on lunch program demonstrating somewhat higher mean scores in mathematics.
- Mathematics intercepts for the two groups of learners (the lunch and non-lunch program) were statistically significant, an indication that students vary significantly in their knowledge of mathematics at entry into grade four.
- Mathematics slopes (rate of change) were positive and statistically significant from zero with the non-lunch group students demonstrating somewhat higher slope parameters in mathematics. The heterogeneity in the regression slopes indicates presence of true individual differences among students’ learning growth rates, thus lending support to differences in students’ mathematics learning rates.
- The variance estimates of mathematics slopes were statistically significant with students not on lunch program depicting higher variance parameters in mathematics. African American students not on lunch
program were therefore more variable in their mathematics learning rates than African American students on lunch program and that these differences were observed at lower grade levels.

- The correlations between the intercept and slope within mathematics and for the two groups of learners (lunch programs) were positive and statistically significant from zero, an indication that where a student starts in domain achievement is related to his or her future growth (mean level) in mathematics.

**Discussions/Significance**

Over the past several years, there have been lamentations about the educational performance of U.S. students. This is well reflected in the publication of *A National at Risk in 1983*. This report highlighted important points that left many education researchers asking themselves questions about achievement growth such as: how much does student achievement change during different stages of a students' schooling?

The National Center for Educational Statistics (1997) study on reading and mathematics and reading achievement found that racial disparities in 12th grade achievement reflect differences in achievement prior to entering high school. This study also showed that differences between the two SES groups of African American students become smaller over the years of schooling. The decreasing difference in learning between groups can be explained in part, by the fact that the majority of those who received free and reduce cost lunch come from economically deprived environments that are not as academically nurturing as more economically advantaged environments. The proportion of students who come from economically disadvantaged groups of students
without free and reduced cost lunch is not as large as for those that received free and reduced cost lunch. Once in school, early childhood experiences associated with differing home environments that differentially impact students' academic performances might be somewhat diminished by the effects of schooling over time.

The results of this study differ from the findings of other studies and suggest that initial status differences in achievement levels between the two groups of students are rather stable, and in some comparisons, actually increase over time (from grade 4 through grade 7). These differences as well, might be predictive of later, differential dropout rates between the two groups. The growth curve analyses in these comparisons showed that the growth curve for the African American students who do not received free/reduced cost lunch was higher at all grade levels than the growth curve for African American students who did receive free/reduced cost lunch. These findings may well reflect the differential and interacting influences of the nature of differing home environments among groups, as well as differential impacts of schooling over time. The intercept changes in mathematics and for the two groups of learners were related to their respective slopes. This suggests that where a student starts in domain achievement is related to his or her future growth in the domain of interest. Though a number of individual growth patterns over time were shown in this study within each group, and when comparisons were made within group by SES levels, the total group effects of home and schooling were shown to sustain over time. It is important to note that imbeddedness of poverty within any particular group translates into differential learning environments in terms of per capita learning resources made available at home, which subsequently impacts school learning and achievement. Recent large scale reviews of the literature to identify both proximal and distal factors
impacting student learning and achievement clearly document the importance of proximal factors that include both the school and the educational quality of the home environment (Wang, Haertel, & Walberg, 1993).

Sanders & Horn (1998) and Rugutt (2000, 2001) found differences in classroom teacher effectiveness and prior achievement levels of students to be the two most important factors impacting student gains in learning and achievement over time. Sanders and Horn (1998) further found that students assigned to ineffective teachers continue to show the effects of such teachers even when the students were assigned to very effective teachers in subsequent years. The findings reported in this study are consistent with those of Sanders & Horn.

In the study of individual differences and the learning of mathematics, Fennema, and Behr (1980) suggested that individuals differ on a wide number of cognitive variables such as mathematical aptitudes—numerical ability, mathematical reasoning, and inductive/deductive ability in problem solving process. The results of the present study suggest that these differences are evident in the early school years (grade 4) and are maintained, and may actually increase, over time (through grade 7).

In terms and measurement and theory, the mathematics domain utilized in this investigation was an average composite of the various subscales. These subscales were mathematics concepts/estimation and mathematics problem solving/data interpretation. The factor structure of mathematics could not be examined across the three measurement occasions because a relatively high correlation across domain subscales. One important objective in longitudinal test development is to evaluate the extent to which the same factor structure exists for all measurement occasions, that is, to establish that the same
indicators on different measurement occasions are equally stable over time. These analyses were not completed in this study for the two groups of learners to investigate whether the Louisiana Educational Assessment Program-Norm Referenced Test (LEAP-NRT) instrument works differently for the two groups. Williamson, Appelbaum and Epanchin (1991) stated that the interpretation of growth depends on the assumption that the same attribute(s) are being measured across the investigation period. The validity of the interpretations also depends upon the quality of the metric used. If the scales score metric does not provide "a common metric across all levels of tests used, then measurement of growth is suspect even if substantive content is common across all levels" (Williamson, Appelbaum & Epanchin, 1991; Rugutt, 2000). While not possible in this study, examining the factor structure of measurements at each point in time in longitudinal analyses is recommended. Such analyses allow for a more comprehensive picture of the stability of both measured and latent variables over time.

It also seems important that factors that directly relate to proper and reliable assessment of student achievement in mathematics be observed. Royer (1990), stated that test using multiple-choice items were measuring offline reasoning processes rather than online comprehension processes and extreme care must be observed when using these tests to make grade placement decisions, diagnosing reading difficulty, or assessing educational gain. Royer (1990) argued that standardized reading comprehension tests that utilize multiple-choice questions do not measure the comprehension of a given passage, but rather measures a reader's world knowledge and his or her ability to reason and think about the content of the passage. For mathematics educators need to use multiple data points and multiple forms of assessments of students' knowledge of
mathematics other than relying only on the scores of standardized tests to evaluate students’ learning growth. Both reliability and validity of inferences about student learning and academic progress are enhanced with analyses of longitudinal data.

Due to the rather large data set utilized in this study, a test of whether the patterns of missing data were random or systematic was not completed but an assumption was made that the missing cases in the data set were purely random and that missing data would not adversely affect the sample size. However, students who dropped out of school at each wave are perhaps more likely to come from families with particular characteristics (e.g., low SES, job instability of parents). This obviously can create problems with reliability of the data and the generalizability of the results. Further, the growth parameters computed may not be adequately representative of the true change in achievement for the two groups learners compared over time. It is also important to be cognizant of the fact that when the missing pattern is not random, there is no adequate statistical fix to remedy this problem.

Though this study did not attempt to model the problem of missing data, it employed listwise deletion. The covariance matrix generated by listwise deletion will always be consistent, that is, positive semi-definite (Anderson and Gerbing, 1984). However, if the pattern of missing data is not random, an inconsistent matrix – not positive definite, can result (Rovine, & Delaney, 1990). Despite the fact that listwise deletion can result in a positive semidefinite matrix, it is also known that this technique can present problems for tests of goodness of fit, unless the missing data are missing completely at random (Kaplan, & Elliott, 1997; Muthén, Kaplan, & Hollis, 1987).
Though there have been advancements in statistical computing power, multivariate data are frequently hampered by missing values. The traditional and relatively old methods of dealing with incomplete data, that is, deletion (listwise, pairwise) for cases with incomplete information, substituting plausible values such as means, or regression prediction for missing values continue to be utilized. In this study, listwise deletion was used. With listwise deletion cases with missing observations on any variable in any analysis are excluded from all computations—thus a final sample includes only cases with complete records. Though the recent advances in theory and computational statistics have produced flexible and powerful procedures with sound statistical bases (Likelihood-Based Estimation—Efficient Estimation—EM, Multiple Imputations—MI) (Cohen, & Cohen, 1983; Kline, 1998; Schafer, & Olsen, 1998; Rovine, & Delaney, 1990), the statistical processes involved are above the reach of many researchers who are faced with the problem of missing data on a daily basis. These computational statistical techniques are quite involved and may require equally demanding data preparation procedures which many users of secondary data analysis may see as a nuisance that should be avoided as much as possible.

Furthermore, many techniques for handling missing data rarely account for the patterns of missing observations—whether random or systematic. This is a much bigger problem and compounds that of the proportion of the missing data. There is no clear guideline about how much missing data is too much. Cohen and Cohen (1983) suggested that 5% or even 10% missing data on a particular variable is not large. Irrespective of the method utilized in imputing missing values, the data set would still fail to provide
In terms of practice, it is important that teachers have a better understanding of their students' literacy development. This helps teachers to recognize patterns of behavior, which suggests aspects of students' development behavior out of what is provided in the curriculum. Knowledge of student's literacy development accords teachers an opportunity to develop more flexible curricula to meet the changing needs of specific students or groups of students.

The Louisiana School Effectiveness study (Teddlie, 1994; Teddlie & Stringfield, 1993) discussed areas in which school policies can positively affect teachers' behaviors such as appropriate teacher selection and replacement, frequent personal monitoring of classroom behavior, support for teachers through direct assistance and in-service programs, and overall instructional leadership. These strategies lay a fertile ground for effectiveness in classroom instruction and management. Mendro (1998) discussed equity in student access to a quality education as regards the type of help to provide to students who have had an ineffective teacher in the past. Mendro (1998) stated that students who are placed with an ineffective teacher suffer long-term negative effects and their needs to be a policy issue put in place to allow for more equitable distribution of resources to enhance the quality of teaching and learning. In a recent study that aggregated data at the student level, Sanders and Horn (1998) found that ineffective teachers were ineffective with all students regardless of students' prior levels of achievement while teachers of the highest effectiveness were generally effective with all students. Though Sanders & Horn (1998) found teacher effectiveness to be a dominant factor affecting student gains in academic achievement when compared to other classroom context variables (e.g., class size, classroom heterogeneity), it seems important that schools recognize socioeconomic differences among students in the early years in considering more equitable distribution of educational resources, particularly good teachers.
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APPENDIX A

ML Estimation Iteration History

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Convergence criteria met.

Covariance Parameter Estimates (MLE)

| Cov Parm  | Subject | Estimate  | Std Error  | Z     | Pr > |Z| |
|-----------|---------|-----------|------------|-------|------|| |
| UN(1,1)   | SSN    | 139.45060800 | 4.21846799 | 33.06 | 0.0001 |
| UN(2,1)   | SSN    | 30.07208868  | 1.25561110 | 23.95 | 0.0001 |
| UN(2,2)   | SSN    | 12.33389425  | 0.71833902 | 17.17 | 0.0001 |
| Residual  |        | 124.22274412 | 1.87184479 | 66.36 | 0.0001 |

Model Fitting Information for MATH

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Solution for Fixed Effects

| Effect     | Estimate    | Std Error  | DF  | t    | Pr > |t| |
|------------|-------------|------------|-----|------|------|| |
| INTERCEPT  | 186.89442547 | 0.16673904 | 9421 | 1120.9 | 0.0001 |
| TIME       | 10.29810013  | 0.06540647 | 18E3 | 157.45 | 0.0001 |

Figure 1.: Interindividual Differences in Change in Mathematics for African American Students with Free/Reduced Cost Lunch
**APPENDIX B**

**ML Estimation Iteration History**

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**Model Fitting Information for MATH**

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**Solution for Fixed Effects**

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**Figure 2.**: Interindividual Differences in Change in Mathematics for African American Students without Free/Reduced Cost Lunch
Title: A Study of Students' Academic Change in Mathematics Achievement: A Case for African American Students

Author(s): John Kangengo Rugutt, Chad D. Elliott, Eugene Kennedy

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