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ABSTRACT

Mathematics educators in Japan have traditionally emphasized mathematical perspectives in research and practice. This paper features an account of changes in mathematics education in Japan that focus on the possibilities of individual students as well as their mathematical ways of thinking. Students' mathematical thinking, mathematical perspectives, and teaching methods have been integrated as part of a mathematics education reform effort. (Contains 19 references.) (DDR)

Teaching by Open-Approach Method in Japanese Mathematics Classroom

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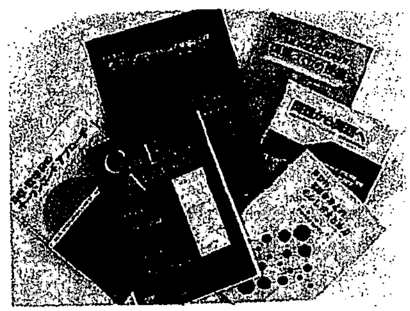
ORIGINS OF OPEN-APPROACH METHOD IN JAPAN

In Japan, mathematics educators have traditionally been emphasizing mathematical perspectives in their research and practice. In these twenty years, more attention has been paid to individual students in the stream of mathematical perspectives emphasized. Some of the representative research results have been published under the titles of "The open-ended approach," "The open approach," "From problem to problem" and "Various ways of thinking" (Shimada, 1977; Nohda, 1983; Takeuchi & Sawada, 1984; Sawada & Sakai, 1995; Koto, 1992). The tradition of posing and solving problems in mathematics class since before World War II served as a base for the emergence of these researches.



Most of these recent researches focus on possibilities of individual students as well as their mathematical ways of thinking. Development of teaching methods that are tuned to a variety of students' ways of thinking is also a major issue. In other words, students' mathematical thinking, mathematical perspectives and development of teaching methods have been integrated, which constitutes a remarkable feature in recent Japanese mathematics education and mathematics education research.

An origin of such trend was the research on evaluation conducted at the beginning of 1970s. One of leading research was the one by Shimada and others concerning the method of evaluating student's achievement in higher objectives of mathematics education. They meant higher objectives as follows:

- To be able to mathematize a situation and to deal with it. (In other words, to be able to bring forth an (important) aspect of the problem into student's favored way of thinking by mobilizing their repertories of learned mathematics, to reinterpret it in order to deal with the situation mathematically, and then to apply their preferred techniques.) (Shimada, 1977)
- To be able to collaborate with others in solving a mathematical problem. (Shimada et al., 1972)

It was here that they developed open-ended problems in order to evaluate students' activity. Open-ended problem refers to the problem that is formulated so as to have multiple correct answers. Shimada and others developed different open-ended problems such as "marble problem" and "water flask problem."

In those days, Japanese Mathematics Course of Study was organized around the idea of Modernization of Mathematics Education. The class activity was so called "issei-jugyou" (frontal teaching). There were 45 students and one teacher in the classroom. The teacher explained new concept to the students and presented examples of concept and solutions of exemplary problems. A series of knowledge, skills, concepts, principles and laws was presented to students in the step-by-step fashion. Under such circumstances, the open-ended problem was expected to serve as a vehicle for changing the lesson organization substantially.

In the beginning, the research was conducted by four researchers, Shigeru Shimada, Toshio Sawada, Yoshihiko Hashimoto, and Kenichi Shibuya. A few years' later, more researchers and teachers in elementary and secondary schools participated in the research. These teachers used the method in their mathematics classrooms. The book "The open-ended approach: A new proposal for teaching mathematics" (Shimada, 1977) was published as the result of this collaborative work. Recently, the book was translated into English and published by NCTM (Becker & Shimada, 1997). The research has been continued and developed in the ways as mentioned above.

At present, there are still many schools in Japan that have 40 students and a teacher in one classroom. However, ways of teaching became more variable compared with 30 years' ago, and came to emphasize ideas of each student together with the traditional mathematical perspectives. As indicated by the production of many books above, the idea of "openness" in teaching and evaluation has been developed and extended in various ways through collaboration between researchers and schoolteachers and has been realized in actual mathematics classrooms in Japan (see Note).

In this short presentation, I will describe the idea of open-approach method and show an example of teaching situations that realized the idea of open-approach method. Then, I will discuss several perspectives for future mathematics education research from the viewpoint of open-approach method.

IDEAS OF OPEN-APPROACH METHOD

Opening Up the Hearts of Students Toward Mathematics

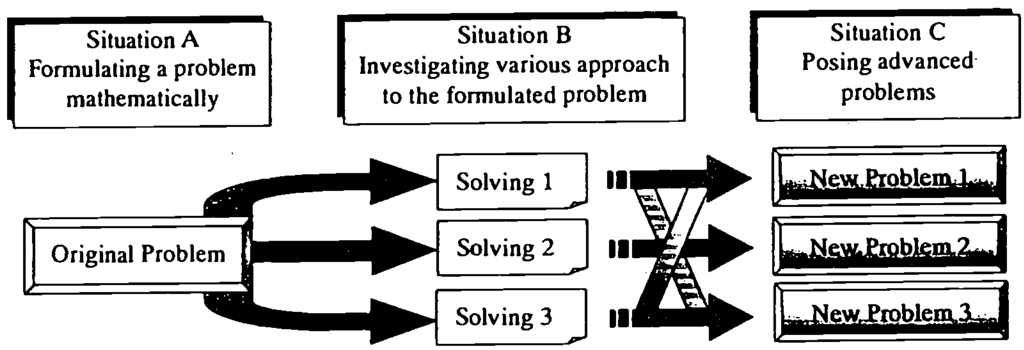
All of the educational activities should open a student's present-day learning to his/her future learning. Thereby, the student can acquire necessary qualifications to make his/her life successful. Even in school mathematics, we should take into account that every student is encouraged to seek his/her own way of life, and has whole mind and body to contribute to his/her community with full force on the basis of mathematical sense, knowledge, skills, the ways of thinking, and so on. Therefore, we should ensure the maximum opportunity and the best environment of learning in any kinds of educational activities as possible. However, it is clear that most students cannot necessarily learn the content of more than the middle grades, because of the "hard" characteristics of school mathematics (its structural, abstract, and conventional phases), even though they can easily learn the content of the lower grades by themselves (Nohda, 1982). Therefore, the appropriate teaching is necessary especially in school mathematics.

In teaching mathematics, teachers are supposed to assist their students in understanding and elaborating their mathematical ideas as far as possible in response to students' achievement, disposition, and so on. However, the teaching only anchored in the logic of teacher never can open up the heart of student, even if its process and product are "attractive" for teachers mathematically. On the other hand, the teaching flattered students' ideas is bound to end up the activities of low mathematical quality, and finally never can open toward mathematics.

Teaching by open-approach method aims that all students can learn mathematics in response to their own mathematical power, accompanying with certain degree of self-determination of their learning, and can elaborate the quality of their process and products toward mathematics. In other words, teachers who employ open-approach method in their teaching need to try to understand a lot of students' ideas as possible, to sophisticate the ideas in mathematical activities by means of students' negotiations with others and/or teachers' advice, and to encourage their self-government in elaborating the activity mathematically. Thus, the teaching by open-approach method intends to open up the hearts of students toward mathematics.

The teaching by open-approach method assumes three principles. The first is related to the autonomy of students' activities. It requires that we should appreciate the value of students' activities for fear of being just non-interfering. The second is related to the evolutionary and integral nature of mathematical knowledge. Content of mathematics is theoretical and systematic. Therefore, the more essential certain knowledge is, the more comprehensively it derives analogical, special and general knowledge. Metaphorically, more essential knowledge opens the door ahead more widely. At the same time, the essential original knowledge can be reflected on many times later in the course of evolution of mathematical knowledge. This repeated reflection on the original knowledge is a driving force to continue to step forward across the door. The third is related to teachers' expedient decision-making in class. In mathematics class, teachers often encounter students' unexpected ideas. In this bout, teachers have an important role to give the ideas full play, and to take into account that other students can also understand real amount of the unexpected ideas.

Teaching by open-approach method consists of three situations in general; Situation A: Formulating a problem mathematically, Situation B: Investigating various approach to the formulated problem, and Situation C: Posing advanced problems.



In Situation A "Formulating a problem mathematically," teachers show students the original situations or problems, and students try to formulate them as mathematical problems in response to their own learning experience. In Situation B "Investigating various approach to the formulated problem," students are expected to find their own solutions on the basis of their experience. Teachers direct students to discuss the relations of wide variety of solutions proposed, and lead them to integrate seemingly unrelated solutions into a more sophisticated one. In Situation C "Posing advanced problems," students try to pose more general problems on the basis of their activities in Situation B.

Through solving these problems, they are expected to find more general solutions.

Openness and Types of Problems

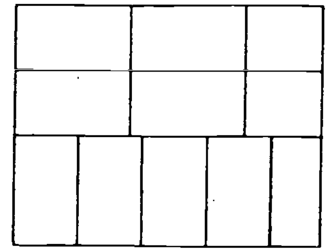
In the open-ended approach proposed by Shimada, emphasis was placed on the problem whose end was not closed to one answer. He and his colleagues intended to organize class by making use of multiple correct answers positively. In the open-approach method, the meaning of openness is considered broadly than the open-ended approach. Here, in addition to the problem whose end was open, the problem that produces multiple correct solutions and the problem that produces multiple problems are included. By this extension, the difficulty of constructing the open problem is overcome. Moreover, it becomes possible to provide more opportunities for students with different abilities and needs to participate in the class. After getting multiple solutions by his/her own, it also becomes possible to lead students to sum up their solutions from the viewpoint of mathematical ideas (Nohda, 1983).

Problems used in the open-approach method are non-routine problems. Furthermore, based on the openness described above, it is reasonable to classify the problems into three types: "Process is open," "End products are open" and "Ways to develop are open." Several researchers use these names. The types are described below with typical examples.

Process is open. This type of problem have multiple correct ways of solving the original problem. Needless to say, all mathematical problems are inherently open in this sense. However, the issue is that many school problems require only the answers or do not emphasize the process aspect of the problems. It is therefore important to verbalize that the process is open and ask for teachers to look at the problems at hand from such a viewpoint. The "card problem" below is one example of this type.

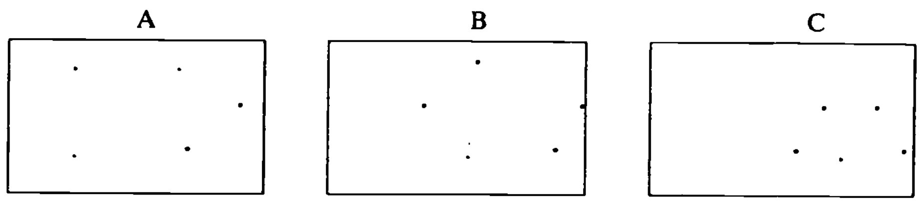
"As 37 pupils will make birthday cards for the teacher 'Matsui Sensei' in the classroom meeting, it has been decided that everyone will make cards. Then, they have to make some small cards (in the shape of a rectangle 15 cm long and 10 cm wide) from some bigger rectangular sheet (45 cm long and 35 cm wide). The problem will be, "How many small cards can you make from the bigger one?"

Here, students may dissect the rectangular sheet into the size of card and get the arrangement as shown in the figure at right. Students also may calculate (35 x 45) $\div$ (15 x 10) and get the answer 10.5 numerically. Another student may calculate (7x9) ÷ (3x2) by noting the ratios.



Multiple solutions enable students to carry out the activity according to their abilities and interests, and then through group discussion, to seek a better process of problem solving.

End products are open. This type of problem has multiple correct answers. As stated above, Shimada and his colleagues have been developed this type of problems (e.g., Shimada, 1977). In Europe, Christansen & Walter (1986) talked about the importance of investigation problems, which is similar to the problems that the end products are open. An example of this type, "marble problem," is shown below, which is well known as a representative problem in the open-ended approach.

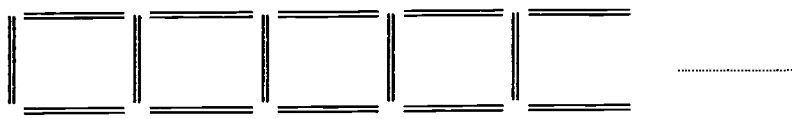


"The figure shows scattering patterns of marbles thrown by three students A, B and C. In this game, the student who has the smallest scatter is the winner. In these examples, the degree of scattering ranges from A to C. In such a case, it is convenient if we have some numerical measure to indicate the degree of scattering. Then, think about it from various points of view, and show ways of indicating the degree of scattering by itemized statements. After that, think of the best answer for this problem."

Students may discover "measure the area of a polygonal figure" as a measure of degree of scattering. Another students may think of "measure of the length of all segments connecting two points," and still another may do "measure the radius of the smallest circle including all points." These methods of measure have advantages and disadvantages. The teacher can help students see both the advantages and disadvantages in generalizing the proposed methods.

Ways to develop are open. After students solved the problem, they can develop new problems by changing the conditions or attributions of the original problem. When we emphasize this aspect of "from problem to problem" (Takeuchi & Sawada, 1984), the problem can be said that ways to develop are open. An example below, "matchstick problem," is taken from problems used in the US-Japan comparative study on mathematical problem solving (Miwa, 1992).

"Squares are made using matchsticks as shown in the picture below. When the number of squares is eight, how many matchsticks are used?



(1) Write a way of solution and the answer to the problem above.

(2) Make up your own problems like the one above and write them down. Make as many problems as you can. You don't need to find the answers to your problems.

(3) Choose the one problem you think is best from those you wrote down above, and write the number of the problem in the space. Write down the reason(s) you think it is best."

Here, students may develop problems by changing the number of squares. They may further change the condition "square" to "triangle" or "diamond," for example. They may also develop problems to ask for the number of squares when the number of matchsticks used is given (inverse problem). Students can enjoy developing their own problems. Furthermore, by comparing with their peers they can discuss mathematical structures of the problem and generalizability of their solutions in the lesson.

Evaluation of Students' Responses

It would be worthwhile to mention here how to evaluate student's activity in the open-approach method. This is because the aim of the method is not to produce correct answers but to promote student's mathematical ways of thinking and creativity. Indeed, it is not easy for the teacher to evaluate a variety of student's responses being produced.

Student's response is evaluated according to the following criteria (see Shimada, 1977)

- Fluency – how many solutions can each student produce?
- Flexibility – how many different mathematical ideas can each student discover?
- Originality – to what degree is student's idea original?
- Elegance – to what degree is student's expression of his or her idea simple and clear?

These criteria need to be evaluated by both quantitatively and qualitatively. Here, especially the first two criteria can be evaluated by counting the number of responses.

In Nohda (1998), a model in the form of a matrix has been constructed to evaluate responses by the criteria of "diversity" and "generality." In the matrix, an item (Aij) shows the number of responses by the student. "Diversity" is expressed by (Aij) where j is a constant, in which different mathematical ideas correspond to different (Aij). "Generality" is expressed by (Aij) where i is a constant, in which different level of generality correspond to different (Aij). For instance, responses of "marble problem" described above can be evaluated in the following way.

Diversity — A1j: Ideas of length, A2j: Ideas of area, A3j: Ideas of variance

Generality — Ai1: Concrete example, Ai2: Semi-concrete example, Ai3: Abstract example

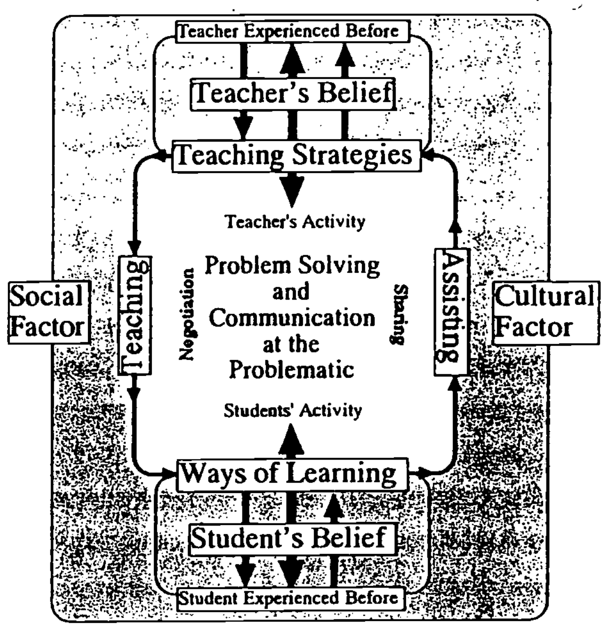
A11: Max. or min. length of two points.
A12: Circumference of 5 points.
A13: Sum of all lengths of 5 points distances.
A21: Min. square covering 5 points.
A22: Min. circle covering 5 points.
A23: Sum of the areas of triangles formed from 5 points and so on.
.....

$$P:\begin{bmatrix}110\\000\\000\end{bmatrix} Q:\begin{bmatrix}121\\011\\000\end{bmatrix}$$

According to this evaluation, we can say that the student Q has a more diversified and a more general approach than student P. On the other hand, supposing that P and Q indicate the states of the same student prior to and after the lesson, respectively, then it is possible to know how the student has changed through teaching using the open problems by comparing the two matrices.

TEACHING SITUATION BY OPEN-APPROACH METHOD

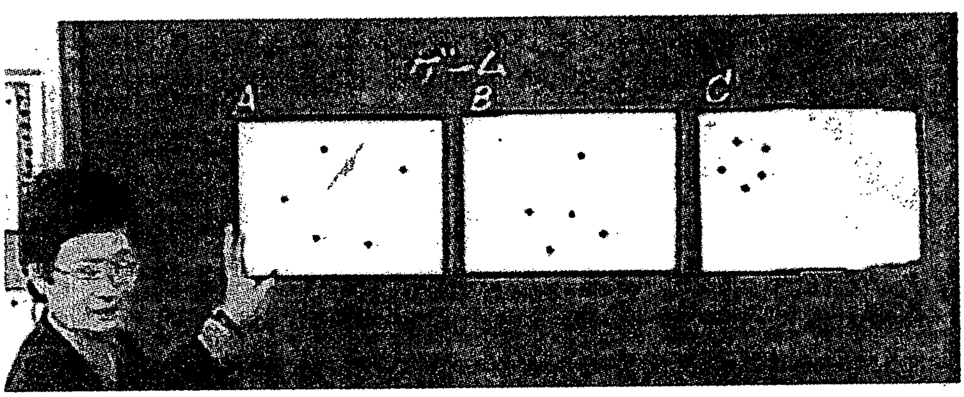
In this section, I will describe how mathematics teaching proceeds by using open-approach method in class. Figure below shows a characterization of Japanese teaching of mathematical problem solving (Nohda & Shimizu, 1989). In Nohda (1982), I investigated the process in terms of pedagogical tactics by Herbert. The figure also shows several features of Japanese class in that problem situation that contains important mathematical ideas is presented to students, and students challenge the situation collaboratively and finally reach their solutions (see also TIMSS results by US Department of Education, 1996). However, it becomes more difficult to make such process happen, as students become older and their abilities and beliefs vary far more. Therefore, in the open-approach method, it is intended to provide students with rich situations by using open problems that have possibility to serve for individual differences among students both in their abilities and interests and in the development of mathematical ways of thinking, and to support the investigative process of solving and generating problems. Through such activities, students are expected to learn not only mathematical knowledge but also important basis of mathematical problem solving such as mathematical ways of thinking, beliefs, and meta-knowledge of "how to learn."



Here, an example of teaching situation is shown. Mr. Tsubota who is a mathematics teacher in the Elementary School Attached to University of Tsukuba, Tokyo, conducted a class by using the "marble problem" (Tsubota, 1988). The students were in grade 6 (11 to 12 years old). I will illustrate the flow of class according to the three situations described earlier.

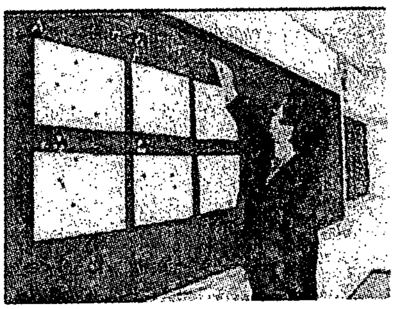
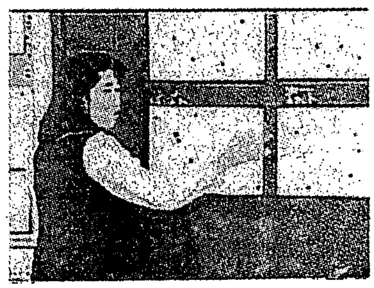
Situation A: "Formulating a problem mathematically." The "marble problem" was presented to students not by sentences but by a game situation as follows:

Teacher: We will play a game of throwing marbles on a piece of paper and comparing how much the marbles scattered. Winner is the one whose marbles scattered most. Let's see, each of the three people, A, B and C, threw marbles, and the marbles scattered in this way (teacher shows the students figures). Who do you think is the winner?



Then, Mr. Tsubota let the students experience the marble-throwing-game by themselves on their desks. After a while, he asked several students to present their results on the blackboard. By looking at presentations by their peers, the students began to realize that there are a variety of scattered ways. Mr. Tsubota then asked the students how to decide which marbles are scattered more and how to convince others that is a reasonable decision. Several students raised their hands and made proposals. Many of them capitalized on their knowledge as sixth-graders and put their focuses on lengths and areas. Discussion was gradually shifted to the differences among proposed ways of making decision.

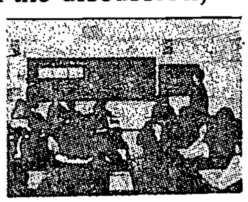


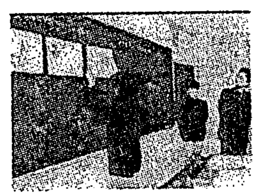



This way of presenting problem is often seen in mathematics teaching in Japan. In the case of Mr. Tsubota, he cultivated students' mathematical words such as "length" and "area" in the earlier part of the discussion. Based on these naturally verbalized words by the students, he then posed the essence of the problem, "Is it possible to use number for making a good decision?"

Situation B: "Investigating various approach to the formulated problem."

In the later part of the discussion, the students came to present different ways of



using number. Other students seemed to share strong points of each way. Still, some students proposed counter-examples and indicated that some of the ways cannot be applied to extreme cases. Through intense discussion, the students came to integrate variety of solutions into more sophisticated ones.

Situation C: "Posing advanced problems." After all presentations were made, Mr. Tsubota let the students go back to the original game situation and decide the winner by using someone's proposed way. The students said that they liked simpler ones, and finally decided to use "area" to make comparison. They measured the "areas" of their own scattered marbles and decided the winner. At the same time, it became apparent that in some cases the "area" did not give reasonable measures for the purpose of comparing the degree of scattering. At the end of the class, the teacher and the students recognized that the "area" method needed to be revised further and reflected on today's class. The teacher concluded the class by saying, "Today, we learned how to measure and compare ambiguous objects."

In sum, Mr. Tsubota's class illustrates that it is possible that students (i) pose problems in the problematic situation, (ii) formulate their own approaches by themselves, (iii) accept that there are a variety of solutions, and (iv) closely examine, justify and refute different solutions. It shows that the open-approach method enables the construction of vital mathematical activities in the classroom.

PERSPECTIVES FOR FUTURE MATHEMATICS EDUCATION RESEARCH

Although the theory of open-approach was constructed around 1980, the above discussion shows that it has many contact points with the ideas discussed in the mathematics education community today.

In the open-approach method, teachings are required to be open to student's mind. Such requirement can be found in constructivist approach to teaching, which was raised in mid 1980s. Teachers following the open-approach try to orchestrate their lessons by taking advantage of students' thoughts, even when those thoughts are unexpected for the teachers. This seems closely related to the idea of "learning trajectory" that M. Simon has proposed (Simon, 1995). The openness of approaches to one problem is also an important aspect of the open-approach. The class discusses student's various ideas and thoughts, and develops them mathematically through sophistication by the peer group and appropriate advises by the teacher. Thus, the open-approach class may share the common interest with the class that emphasizes mathematical discussion and communication. Furthermore, the evaluation in the open-approach method, where the emphasis is laid on students' ways of mathematical thinking and their creativity rather than correct answers, reflects the common expectation with the research that facilitate students' attitudes and beliefs in problem-solving oriented classes (Nohda, 1993).

These similarities imply that the recent research findings in these areas can enrich the open-approach method, while the ideas underlain the open-approach method can be used as a global framework for integrating the fruits in the research areas. Considering that the open-approach style is also open to mathematics, the idea of open-approach can present a viewpoint from which we can reexamine how mathematics is located in the research or proposed teaching methods. For example, the open-approach presented some viewpoints in its evaluation, like flexibility, originality, and elegance, which reflect the nature of mathematics or "doing mathematics." This means that it tries to evaluate not only students' positive or active attitudes to mathematics, but also mathematical nature in students' thinking.

The number of people who are interested in Japanese mathematics classes has been increased since the mathematics education reform movements around the world in 1990s. The open-approach method is based on the tradition of Japanese mathematics education community in a sense that it made good characteristics of the tradition explicit and extended them. Therefore, its basic spirit, "be open both to students and to mathematics," can be a perspective for investigating Japanese lessons. In fact, this is consistent with the analysis of TIMSS Videotape Studies. When Stigler and Hiebert characterized the lessons in the US, Germany, and Japan in terms of relationships among students, teachers, and mathematics, they stated that students and mathematics were dominant in Japanese lessons (Stigler & Hiebert, 1999, pp. 25-26). Because of being open to both of students and mathematics, problem solving oriented lessons would result neither in teachers' demonstration of the best solutions nor in mere presentation of students' various opinions.

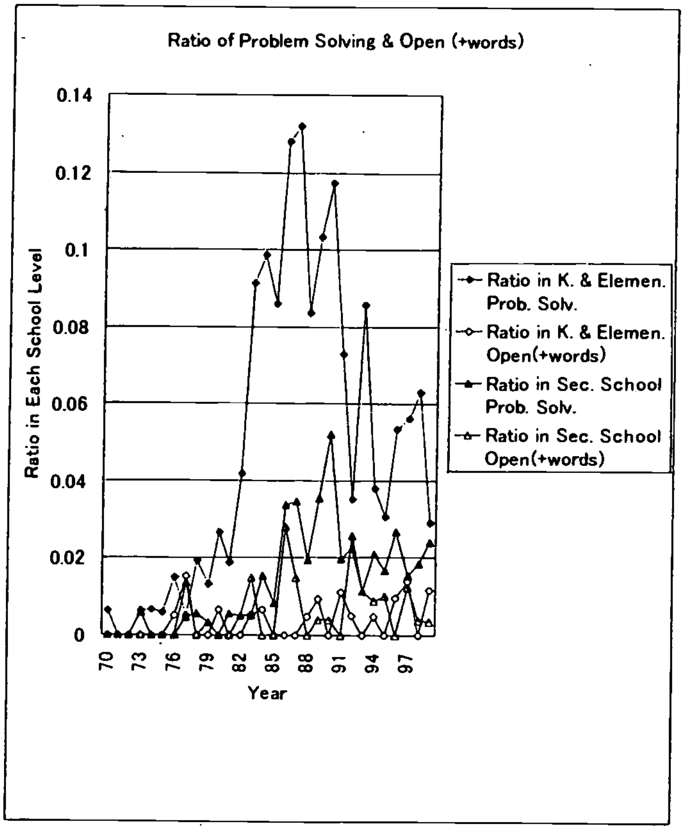
I would like to conclude this paper with two research problems to be studied further. First, we need to develop more good problems, especially open-ended type problems. As stated above, it is most difficult to construct open-ended type problems among the three types of open problems. There is not sufficient stock of such problems even in Japan, so we need to develop them because they are very valuable to mathematics education today. We examined the "marble problem" as the example of open-ended problem. In that problem, students are expected to mathematically make sense of the situation where marbles are scattered. This suggests that some open-ended problems can be related to mathematical modeling. It may be possible to get hints for developing good open-ended problem by referring to the research on mathematical modeling.

Second, we need to study changes in students' mathematical ways of thinking more closely. Many teaching practices following the open-approach were implemented, and the changes in students' understanding and positive attitudes have been explored. But we do not fully understand how each student's creativity and mathematical thinking can develop and what is the cue for such development. Investigation of such issues is also needed for improving mathematics classes through the open-approach method.

NOTE

The table below shows the ratio of presentations at the Annual Meetings of Japan Society of Mathematical Education from 1970 to 1999 that included the words "Problem Solving" or "Open" in their titles by the levels of education. The graph below shows the ratio in each year.

	K. &Ele.	Lower Sec.	Upper Sec.	Tertiary
Total Num. Of Presentation	5246	3586	3213	435
Ratio of "Prob. Solv."	5.4%	2.6%	0.4%	1.1%
Ratio of "Open (+words)"	0.4%	0.9%	0.2%	0



The table shows that the ratio of presentations whose titles include "Open" is high in secondary education levels. It is aligned with the aim of open-approach, i.e., to contribute to a variety of students' differences at these levels. The figure shows that problem solving oriented class was pervaded during 1980s especially in elementary schools. This is consistent with the trend by NCTM at that time.

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