This paper explores the practical and theoretical implications of work related to modeling teaching. The main concerns are questions of what theories and models of cognitive and behavioral phenomena such as teaching-in-context might look like and what standards should be used to judge work of this sort. (Contains 26 references.) (DDR)
ON THEORY AND MODELS: THE CASE OF TEACHING-IN-CONTEXT

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The Teacher Model Group at Berkeley has, for some years, been working toward the development of a theoretically driven model of the teaching process. The idea is to characterize, with the kind of precision typically used in cognitive models, how and why teachers do what they do "on line"—that is, during the act of teaching. The main questions involved in constructing this kind of model of teaching-in-context are as follows: What knowledge does the teacher have? What beliefs? What goals? How does the teacher perceive unfolding events in the classroom? What decisions does he or she make, for what reasons? How does all this fit together, in fine detail, at a level of mechanism?

We see this kind of work on modeling teaching as having both practical and theoretical implications. I will not pursue the practical side of things in this paper, although I hope some of the pragmatic implications are obvious. Simply put, the better you understand any process, the more potential you have to make it work better. Doing so may be far from easy—consider how much work it has taken to translate research on problem solving into productive problem-solving instruction—but as the case of problem solving shows, improved understanding can indeed yield improved performance. Details regarding the pragmatic implications of our work in modeling teaching, and on what it may take to translate this kind of theoretical work into practice, may be found respectively in Schoenfeld (in press-a) and van Zee and Minstrell (in press).

My focus in this paper is primarily metatheoretical. Main concerns are questions of what theories and models of cognitive/behavioral phenomena such as "teaching-in-context" might look like, and establishing standards by which to judge work of this type. Within the space allotted for this paper, I can only suggest the dimensions of the model itself and of the cases we have worked through in detail. This will (just barely) convey some of the flavor of the work. Then I shall suggest how well the model measures up to the standards that have been elaborated. Though there is scant room for detail here, extensive detail can be found in a forthcoming volume of Issues in Education, which includes an extended discussion of the model (Schoenfeld, in press-a), a series of commentaries on it, and a response to the commentaries (Schoenfeld, in press-b), and in two papers that offer case studies (Schoenfeld, Minstrell, & van Zee, 1996; Zimmerlin and Nelson, 1996). I begin by providing some brief examples of situations that we have modeled.
Cases in point: Typical situations that we try to model.

Example 1: Jim Minstrell. James A. Minstrell teaches physics at Mercer Island High School in Washington state. Minstrell is an award-winning teacher who has written extensively about his goals and practices (see, e.g., Minstrell, 1989, 1992; van Zee & Minstrell, 1997a, 1997b). It is the fourth day of the school year. Minstrell is teaching a lesson of his own design, one of a sequence of introductory lessons carefully constructed to introduce students to some of the important themes underlying the course. He wants students to experience physics as a sense-making activity, and to understand that even in rather simple and ostensibly "objective" judgments, there are multiple issues of discretion — e.g., in deciding which data to collect, which data to "count" after they have been collected, and how to combine and interpret those data.

The topic under discussion appears simple: what is the width of a particular table in the classroom? Eight students in the class have taken measurements, in centimeters, and have produced the following numbers: 106.8; 107.0; 107.0; 107.5; 107.0; 106.5; 106.0. The class has discussed various issues, such as: Should all or only some of the data be included? How might the data be combined, and which method of combining them would yield the "best number" to represent the width of the table? In the classroom discussion, students have mentioned and discussed the possibility of using the arithmetic average (defined by a student as "Add up all the numbers and then divide by whatever amount of numbers you added up") and the mode ("the number that shows up most frequently"). At that point a student says: "This is a little complicated but I mean it might work. If you see that 107 shows up 4 times, you give it a coefficient of 4, and then 107.5 only shows up one time, you give it a coefficient of one, you add all those up and then you divide by the number of coefficients you have."

Here are the key questions in terms of the model. Assume we have studied Minstrell carefully — read his papers, interviewed him, perhaps even seen him teach previous versions of this course. We have a good sense of what he thinks is important, what his agenda for the class that day is, and what he knows. He is in the middle of teaching, and something unusual has just happened. Can we say how Minstrell is likely to respond? More importantly, can we say what leads him to respond that way — what beliefs, goals and knowledge shape his decision, and how their interplay results in his choosing to act the way he does?

Examples 2, 3, and 4: Mark Nelson, Deborah Ball, and Alan Schoenfeld. Here are some parallel cases, covering a wide variety of teaching "territory." Mark Nelson is a student teacher teaching an algebra lesson on reducing exponents in expressions like \((x^3y^5/xy^2)\). This is the first time he is teaching the lesson, so he has little by way of pedagogical content knowledge (Shulman, 1986) related to the topic, though his knowledge of the mathematics is secure. He has had students work some problems at their
desks, and is about to convene the class for a whole-class discussion of the problems. We know his intentions and expectations, as well as his classroom routines. If you “feed” us the class’s responses to his questions, one by one, can we predict what he will say, and how the discussion will go? Failing prediction – the toughest standard for any model – can we at least explain, post hoc but in a principled way grounded in the mechanisms of the model, why things evolved the way that they did?

Expanding the problem space, consider a lesson taught by Deborah Ball (the “Shea Numbers” tape of her third grade classroom on January 19, 1990). Ball enters the classroom with a specific item high on her agenda – to have the students reflect about the ways they learn and what they take as evidence for mathematical “truth” – as a follow-up activity to a meeting they had had the previous day with a fourth grade class. The classroom discussion keeps tending away from this kind of “meta-level” conversation to mathematical specifics: is the number zero even, odd, or special; can a number be even and odd; and so on. How will she act, and why? Or, consider the opening days of my problem solving course (see Arcavi, Kessel, Meira, & Smith, 1998). The course is largely interactive, with many of the ideas we work with generated by the students. Is it possible to model my teaching – to say in advance, on a principled basis, how and why I will react to the comments and suggestions made by students? Can this be done in such a way that it “explains” my actions, from the moment I enter the class on any given day to the moment the class session ends? [N.B. The presentation at the conference will allow for elaboration in detail, including a line-by-line discussion of transcripts, that is precluded here by space constraints.]

How the model works

What follows is a brief suggestion of the mechanism by which the model works – for detail on the specifics of the case presented see Schoenfeld (in press-a, in press-b) and Schoenfeld, Minstrell, & van Zee (1996). As noted, the core idea is that the decisions made by the model of the teacher are a function of the teacher’s attributed beliefs, goals, and knowledge. Here is how they play out in the case of example 1 described above.

Figure 1, which represents a small part of the complete parsing of Minstrell’s lesson, provides a rough characterization of what Minstrell did and why in response to the student’s suggestion of a “complicated” way to arrive at a best value for the width of the table. The whole of our lesson representation starts with a box representing the lesson, marked [1] in its upper left-hand corner. In this case the analysis indicates that the lesson can be decomposed into four major “chunks” (segments of the lesson that cohere phenomenologically in some way), which are denoted [1.1], [1.2], [1.3], and [1.4] respectively. The labeling continues in that way. Here, the segment of the lesson catalyzed by the student’s comment is labeled [1.2.2.3]
already fairly deep in the nested structure of the lesson. In the upper right-hand corner of each box in Figure 1 we identify the numbers of the lines of transcript corresponding to each transcript chunk. Chunk [1.2.2.3] extends from lines 164 through line 271 of the transcript, which is 517 lines long. It is further decomposed into chunks [1.2.2.3.1], [1.2.2.3.2], etc. Inside each box, which represents a chunk of the lesson, we briefly describe the following: triggering and terminating events (what caused the teacher to embark on this path, what caused it to be terminated); high priority beliefs related to this episode; goals that the teacher’s decision was intended to achieve; relevant knowledge on which the teacher’s actions are based and decisions are made; the nature of the chunk (e.g., standard pedagogical routine or script).

Here is a summary description of Minstrell’s initial decisions and actions in response to the student comment. In terms of content, Minstrell believes that the class should serve as a sense-making community, in which students explore physical phenomena in reasoned ways. In terms of pedagogy, he believes that he should be responsive to student initiatives that are “in the ballpark.” Here the student comment, a proposed way to compute the “best value,” is reasonable and germane. Thus the model says that Minstrell will decide to pursue it – even if the short-term cost is to defer other topics he’d planned on doing next in the lesson. But, how will he pursue it? First, it is important to note that Minstrell recognizes that one possible interpretation of what the student says is the standard formula for “weighted average” of a collection of numbers. Hence there is the potential to relate the student’s suggestion directly to an earlier discussion of “average.” It is also important to know how Minstrell tends to introduce issues into discussion. Minstrell employs a rhetorical device he calls “reflective tosses” in which he “catches” the intellectual content of an idea and “tosses” it back to the students for clarification, elaboration, or comment. Thus the model predicts that, having decided to attend to the issue and having the relevant knowledge, Minstrell will first ask the student to clarify her statement (thus making it public, and open for classroom discussion) and will then work with the class to explore it. This is what he does – in fact, by asking the student who had first proposed a definition of “average” to comment on this new proposal, setting the stage for a comparison of the usual definition of average (“Add up all the numbers and then divide by whatever amount of numbers you added up”) and the formula for weighted average that he has written on the board. When this is resolved (bringing chunk [1.2.2.3.1] to a close), a comment by the student leads (as the model predicts) to a second round of clarifications, where the class compares weighted and unweighted averages. At that point, having dealt fully with the student’s comment, Minstrell returns to his agenda for the lesson. [For more detail on this and the other cases, see the papers cited above.]
Theoretical underpinnings

The Teacher Model Group’s work is situated in the “cognitive science” approach to cognition – specifically in what Greeno (in press) calls “the standard framing assumptions of cognitive theory.” Our intention (for now) is to construct the architecture of a model that, in some meaningful way, captures the thinking and decision-making that teachers make “on line.” The specific goal of any particular model (of a particular teacher-in-context) is to delineate the beliefs, goals, and knowledge of the teacher, and, using these constructs, to characterize the decision-making of the teacher as events unfold in the classroom. We are, then, studying what goes on “in the head” of particular teachers. Our constructs are mental entities – in the model, representations of beliefs, goals, knowledge (in the form of action plans or other schemata), etc. The decision-making mechanism is akin to that of AI-like models: one can think of a goal-driven architecture using a spreading activation network. (Rough translation into everyday English: When one or more goals that a teacher has are of highest priority at the moment, and some action or sequence of actions within the teacher’s repertoire is likely to do the best job of meeting those goals, then that is the action or sequence of actions the model says the teacher will take.)

Our modeling work draws upon the vast literature on teaching (see, e.g., Borko & Putnam, 1996; Calderhead, 1996; Clandinin, 1986; Clark & Yinger, 1987; Fenstermacher, 1994; Grossman & Stodolsky, 1994; Shulman, 1986, 1987; Thompson, 1992) and a more specific, cognitively-oriented corpus of research that attempts to describe the mental constructs that support teaching and how they interact (see., e.g., Berliner, 1994; Clark & Peterson, 1986; Leinhardt, 1993; Leinhardt & Greeno, 1986; Shavelson, 1986). I see the Teacher Model Group’s work as a logical extension of the past few decades’ work on thinking, learning, and problem solving – as one point on a continuum where the ultimate goal is to explain (individuals’) thoughts and actions in complex social settings. This work is in many ways a direct extension of my work on problem solving, and a reflection of the field’s increasing capacity to model complex behavior. In the early years we brought people into the laboratory to watch them working on problems, in isolation—the reason being that the tools researchers had for understanding cognition were so limited that we needed to control the environment as much as possible. As the field’s understandings of things such as the knowledge base, strategy use, metacognition, and beliefs grew, it moved toward the study of cognition in more “natural” settings, e.g., in classrooms. As the capacity to model interactive decision-making grew, studies of tutoring and teaching-in-context became feasible. We are now, as the research under discussion shows, capable of modeling such complex behavior. Yet, this work is still quite constrained, and its limitations should be noted.

From an “internal” perspective (that is, living within the framing assumptions of cognitive theory), there are at least two major issues to con-
Figure 1. A Representation of Part of Minstrell's Decision-Making
Reprinted with permission from Schoenfeld, in press-a

\[ 1.2.2.3.1 \] (164-225)
Impromptu Excursion: Clarifying the nature of a complicated formula proposed by a student

Initiating event, beliefs, goals, method, and chunk type are all as identified in Chunk 1.2.2.3.
Specific content goal:
\* Have students come to conclusion that the complicated formula yields the arithmetic average.
Terminating event:
\* The specific content goal immediately above is achieved.

\[ 1.2.2.3.1.1 \] (164-225)
Student Comment

\[ 1.2.2.3.1.2 \] (166-169)
Clarifying What the Student Suggested
Specific (emergent) content goal:
\* Make sure the class understands the nature of the proposed formula.
Method:
\* Interactive elicitation using reflective tosses.
Terminating event:
\* The content goal is achieved.

\[ 1.2.2.3.1.3 \] (200-225)
Showing the "Complicated" Formula is the Arithmetic Average
Specific content goal:
\* Have class conclude the formulas are the same.
Method:
\* Interactive elicitation. calling on a specific student to provide content.
Terminating event:
\* The content goal is achieved.

\[ 1.2.2.3.2.1 \] (226)
Student Comment

\[ 1.2.2.3.2.2 \] (227-241)
Framing and Clarifying the Comparison
Specific (emergent) content goal:
\* Clarify the difference between the two formulas.
Terminating event:
\* The specific content goal immediately above is achieved.

\[ 1.2.2.3.2.3 \] (242-271)
Framing and Clarifying the Comparison
Specific (emergent) content goal:
\* Work through compelling example to make sure the difference is understood.
Method:
\* Interactive elicitation using reflective tosses.
Terminating event:
\* Content goal clearly achieved; teacher summarizes with mini-lecture.

Note:
The next level of detail, which would consist of elaborating on Minstrell's use of interactive elicitation to achieve the goals specified in the following chunks: [1.2.2.3.1.2], [1.2.2.3.1.3], [1.2.2.3.2.2], and [1.2.2.3.2.3], is not represented here. A detailed description of the ways in which Minstrell interacts with the students using that method is given in Schoenfeld, Minstrell, and van Zee (1996).
sider. The first is whether such modeling makes unwarranted assumptions about the phenomena being studied, and thus distorts them. As Leinhardt (in press) observes, physics makes progress by virtue of idealizations: “consider a spherical cow” is not a bad assumption with which to begin solving some physics problems. But the same hypothetical spherical cow, in a biology lab, might be problematic. Is there the danger of introducing such beasts into the classroom, via models such as ours? The second has to do with very stringent constraints on the model, which should be stressed – the model is a model of teaching-in-context, in the “here-and-now.” We do not yet model history – except as in the mind of the teacher, whose knowledge includes his or her memories of previous experiences with the content, with these students, etc. We do not model context – except for the teacher’s perceptions of the context, and of the supports and constraints within it. We do not model mechanisms of change – for example, how and why the teacher thinks differently after a lesson, or a unit, or the year is over. All of these are limitations of the current model – but the kinds of things that might be overcome, within the framing assumptions of cognitive theory, over the next few decades.

From an “external” perspective, the challenge can be raised (see., e.g., Greeno, in press) that the lens through which this kind of model views the classroom – the teacher’s – is all too distorted. The classroom is a highly interactive environment in which there are multiple actors; the teacher is only one (albeit an important one) whose view may or may not “explain” much of what takes place. Moreover, the totality of the classroom may supersede the perspectives of the individual actors, rendering individual perspectives inadequate as versions of what takes place. In short, I agree. The issue here is to see how far we can push this kind of model, and how much it can explain under various circumstances – not to claim that what the teacher sees, and how much of it we can model, represent “reality.” In a fashion similar to Greeno, others may argue that the “interior lens,” which only accounts for the teacher’s perspective of context (constraints, supports, etc.) and not for the “real thing,” must perforce be inadequate. Perhaps so – but again, the teacher’s view of context (including the teacher’s sense of what materials might or might not be accessible, what flexibility there is with regard to curriculum, and what the “abilities” of the students might be) is surely a significant factor in shaping what happens in the classroom. The goal is to see what can be explained with this kind of model, and then to transcend it.

Metatheoretical Notes

If only for a moment, it is worth stepping outside the space of current assumptions to point out that the terms “theory” and “model” have very different meanings in different fields. Consider Table 1, for example. People with backgrounds in mathematics and physics expect theories and models to have very specific entailments. In those domains, a theory
(e.g., the mathematical theory of equations or an inverse-square law of gravitational attraction) is a precise statement of "what counts," and a model embodies that theory in very specific computational terms. Moreover, in both domains, theories and models support a precise form of prediction. By that standard, educational/psychological theories and models are often found seriously wanting—a though the rejoinder, that spherical cows don't necessarily represent real objects very well, should not be lightly dismissed. In my opinion, theories and models from the biological sciences (which may also be disdained by some mathematicians and physicists) may provide quite appropriate parallels to the kinds of theories and models that are appropriate in psychology and education. Consider theory, for example. The theory of evolution is not "provable" in the mathematical sense, but evidence can be brought to bear on its validity. And, the theory can be held to strict scientific standards, for example a kind of a posteriori "prediction": while evolution moves too slowly for predictions of the future to be tested, the theory does imply that as yet undiscovered fossil records will have certain properties, and will not have others. Equally important is the stance toward models. One can take biological models (whether of predator-prey relations, or of specific organs such as the heart or even of the human body) as approximations, in the sense that actuarial tables are approximations—what they predict may best be thought of as a range of outcomes, with probability values attached. (Such a distribution is, of course, the precise form of genetic predictions using Punnett squares.) In many contexts, it may be that the appropriate form for the predictions of educational and psychological models can most productively be thought of as probability distributions of outcomes.

Standards for judging models and theories

In keeping with the above comments, I propose that four major criteria are appropriate for judging theories and the models that embody them: descriptive power, explanatory power, predictive power, and scope. De-
Descriptive power refers to how well the theory and model seem to capture the situation being characterized. Are important aspects of the situation represented, and do they interact in the theory and model ways that seem to correspond to the ways they interact in “reality?” Explanatory power takes things a step further. Do the theory and the model provide a sense of mechanism that explains how and why things fit together, above and beyond providing descriptions of their interactions? The notion of predictive power is almost self-explanatory. What is obvious is that the more accurately the theory and models derived from it predict outcomes, the more confidence one will have in the robustness of the theory. Somewhat less obvious is the nature of appropriate predictions – see the comment in the preceding paragraph about psychological predictions being conceptualized as probability distributions of outcomes. Finally, on scope: the issue is, what range of phenomena do the theory and model cover? A theory of equations that covers only linear and quadratic equations is not of much interest; nor is a theory of teaching that applies only to didactic lectures.

**A preliminary assessment of the theory and the model**

Lacking the space to examine Examples 1 through 4 in detail, I can only argue here by assertion. The detail does exist. Minstrell’s lesson segment is analyzed in depth in Schoenfeld (in press-a) and in Schoenfeld, Minstrell, & van Zee (1996); Nelson’s in Schoenfeld (in press-a) and in Zimmerlin & Nelson (1996); Schoenfeld’s in Arcavi, Kessel, Meira, & Smith (1998) and in Schoenfeld (in press-a); and Ball’s in Schoenfeld (in press-b).

Broadly speaking, the model does well on the criteria of descriptive and explanatory power. In all of the examples above, the teacher being modeled has been an informant on the research and has provided substantial information regarding the work. In some cases, such as Minstrell’s, we did a preliminary analysis and then ran it by the person being modeled – providing that person the opportunity to say that the assertions we made were wrong, or that we had missed something important or emphasized the wrong things. Thus far the analyses have held up rather well. They seem to take into account what is important, both from the perspective of cognitive theory (after all, the constructs in our models are derived from the main constructs of cognitive theory) and from the perspective of our informants/colleagues. In the case of Nelson, for example, the model predicts that he will run into difficulty when an explanation he offers the students does not clear up their (expected) confusion as he thinks it will. Moreover, the model explains why he gets into that difficulty by providing a detailed description of the specific cognitive and pedagogical resources Nelson has at his disposal, and showing how those resources are insufficient to deal with the situation he finds himself in.

At a “face value” level, the model does fairly well by way of prediction – at least in those cases (Minstrell, Nelson, Schoenfeld) where we have felt
confident that the model captures the teacher's decision-making. Here the issue of scope becomes central. On the one hand, the three cases just mentioned cover a fair amount of territory: Minstrell is an experienced high school physics teacher who was teaching an innovative lesson of his own design, Nelson a beginning high school mathematics teacher working through a traditional lesson for the first time, and Schoenfeld (like Minstrell) an experienced teacher working through a college mathematics class of his own design. I feel comfortable asserting that the model covers mathematics and science, secondary and collegiate, traditional and innovative – as long as the lesson is agenda-driven. In all of these cases, the teachers had fairly clear ideas of where they wanted the lessons to go. Although there was wide variation in how and with what success these teachers deviated from the original agendas in response to classroom contingencies, there is no question that, by and large, the teachers' agendas were the primary driving forces in shaping what took place in the lessons modeled. I have little doubt that agenda-driven instruction, in general, can be modeled – and that when it is, the models will fare rather well with regard to prediction.

Things get more complex, however, when one considers some of the things that happen in Deborah Ball's January 19, 1990 third grade class. There the teacher is highly sensitive to developmental as well as content concerns, making for a more complex initial agenda than is apparent in the lessons that we have analyzed at more advanced levels. Perhaps more importantly, the directions of that lesson evolve substantially in response to unpredictable issues that arise during the class session. This kind of emergent agenda has been much more difficult for us to model; it may, ultimately, be where the model will break down. It is not yet clear that it will: recently (Schoenfeld, in-press-b) we have had some success in analyzing why (we believe) Ball makes some of the choices she does in that class, and we may ultimately become successful at modeling that lesson. If we do, we will have shown that the model has very large scope – the teaching in these lesson segments spans a pretty large teaching space. If we do not succeed, so be it. Where it is known to work – which already covers a fair amount of territory – the model does well along the dimensions of descriptive, explanatory, and predictive power. And when we discover where it doesn't, we will have an important set of phenomena to explore further.

References


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