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ABSTRACT

Sociocultural perspectives of education and educative processes occupy center stage in educational research and mathematics education, and this is a natural reaction to the recent dominance of reductionist psychological theories. This paper wants to make public an ongoing discussion related to the compatibility of the psychological and sociocultural perspectives. It also discusses how either theory can be reconceptualized to be more compatible with the other. (Contains 28 references.) (DDR)

ON RELATIONSHIPS BETWEEN PSYCHOLOGICAL AND SOCIOCULTURAL PERSPECTIVES

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Discussions of sociocultural perspectives of education and educative processes continue to occupy center stage in educational research and mathematics education. In part, this is a natural reaction to the not-so-distant predominance of strongly reductionist psychological theories drawing upon a correspondence between mental representation and an external mathematical reality. Also, one of us has written extensively about the difference between claiming that a perspective "has it right" and acknowledging the possibility of adopting different stances in regard to a given observation (Cobb, 1990, 1991, in press; Cobb, Yackel, & Wood, 1992). But tensions do exist in trying to reconcile the two perspectives so that one need not suffer "split-brain" syndrome in order to use both without contradiction (Confrey, 1991, 1995; Steffe, 1995).

In this paper we make public an ongoing discussion related to the compatibility of psychological and sociocultural perspectives, and our discussions of how either might be rethought to be more compatible with the other.

Pat

The movie *Contact* opens showing us the earth as seen from an orbiting satellite. The camera backs away slowly at first, then increasingly rapidly, showing the moon in orbit around the earth, then the earth-moon pair orbiting the sun. Mars appears to our left, then Jupiter, then Saturn. The planets and sun diminish in size as we leave the solar system, which itself becomes a speck against a sparkling background as we back away further, passing Alpha Centauri. We pass through interstellar dust as we approach the Milky Way's edge, then we exit the Milky Way and continue backing away until we see thousands of galaxies, then nebulae, and so on.

To me, a significant aspect of this opening was that at no moment did I feel like I'd made a jump in perspectives. I always had the feeling of moving through a continuous transition. Not once did I wonder about what I was seeing or how it fit within the overall transition. It was only when I thought of fixed states within a larger overall transition, such as from an image of a single cell to an image of thousands of galaxies, that I was startled by a sense of apparent disconnection. But the sense of apparent disconnection dissipates when we can imagine "zooming" continuously from one state to the next, keeping a coherent image of the transition.

We might describe any one perspective in isolation of the others in structural terms related to human experience (e.g., swirls, columns, dust, clouds, etc.). At the same time, it would be a challenge to describe the mechanics of an exploding star using cell-level vocabulary. But we can aspire to develop theories which articulate well enough across observations differing in orders of magnitude that we can translate among them while keeping a sense of underlying or overarching phenomena.

I find this image, of "zooming out continuously", to work metaphorically for making a distinction between a unified perspective and the coordination of multiple perspectives. A unified perspective is one which enables us to transition among seemingly disparate phenomena — phenomena which seem to need their own theories. Thermodynamics is one example of a unified perspective. For many years, heat energy, energy of falling objects, and the work of physical labor were treated as unrelated quantities. The genius of thermodynamics was that it reconceived the idea of energy so that measures of one form could be transformed into an equivalent number of units appropriate for another form (Klein, 1974).

The distinction between unified and multiple perspectives isn't a huge distinction, and it isn't new. The unification of quantum mechanics and the general theory of relativity was one of Einstein's major, unfulfilled efforts (Fritzsch, 1994; Hawkings, 1988), and that one of the obstacles to the unification is an absence of appropriate imagery in which phenomena in both might be grounded (Miller, 1987). Newell (1973) pointed out that what one takes as an object versus what one takes as process varies with one's grain of analysis. Paul's enormously powerful work on coordinating psychological and social perspectives (Cobb, 1990, 1995) makes a parallel point about coordinating different background theories as a way of looking at classrooms from different points of view.

What might be a little new in the above is an orientation toward establishing ways of thinking about phenomena that enable shifts between perspectives to be more like continuous zooming. Making the attempt to think of unifying metaphors may be useful for framing current questions about psychological versus social perspectives in mathematics education. To achieve a unification of psychological and social perspectives would mean that we become able to "zoom" out or in with respect to a set of problems without loosing sight of where we started. In particular, we could "zoom in" from what we see as patterns of sociocultural activity to seeing that same activity as an expression of a hugely complex set of interactions among reflectively acting, cognizing, remembering, interpreting, feeling individuals. Also, we could zoom out from what we see as a collection of individuals who vary (or not) with respect to some set of characteristics we've deemed of interest, and who we imagine interacting by various means and with various resources, to seeing that same collection as having stable and persistent characteristics that appear to be independent of individual participants.

Paul

Pat, I find your metaphor of zooming for coordinating perspectives (or perhaps levels of analysis) helpful. I assume that you have something like this in mind:

1. A student,
2. located in ongoing small group interactions,
3. located in an emerging classroom microculture,
4. located in the activity system of the school,
5. located in the practices of the local community,
6. located in the broader policy environment.

To start the conversation, I want to make two observation sparked for me by the metaphor. The first is to differentiate zooming and the nesting of settings from an alternative slant on the coordination of perspectives. As an illustration, imagine that we are analyzing video-recordings of a one-on-one teaching session between a researcher and a student. We might focus on the ways in which the student reorganizes his or her mathematical reasoning while interacting with the researcher. A psychological constructivist analysis of this type is, in effect, made from inside the interaction and is concerned with the student's interpretations of the researcher's actions. Alternatively, we might analyze the same video-recording by focusing on patterns and regularities in the ongoing interaction, and on the taken-as-shared meanings that the researcher and students jointly establish rather than on the student's (or teacher's) personal interpretations. A symbolic interactionist interpretation of this type is established from the outside and makes the interaction between the researcher and student and explicit object of analysis. As a further possibility, we might view the researcher and student representatives of different cultural traditions who are attempting to communicate. For example, we might contrast the suppositions and assumptions that the student makes as a consequence of her history of participation in particular cultural practices with those that the researcher makes about the teaching session as a consequence of her induction into a particular research tradition in graduate school. In an analysis of this type, which might be characterized as sociohistorical in nature, our position is not merely outside the local interaction, but is outside entire communities of practice.

My point in giving this example is to illustrate a case of coordinating perspectives in which the scale of the phenomenon to be explained does not change (at least on the surface). To be sure, zooming is still involved — from inside the ongoing interaction, to outside the local interaction, to outside broad cultural traditions. However, it is accomplished (usually implicitly) by the analyst as she switches from one perspective to another. If we think about the coordination of theoretical perspectives in such cases, the challenge of developing a "unified theory" involves integrating a number of well-established theoretical perspectives such as psychological constructivism, symbolic interactionism, and sociohistorical theory. The result would be something akin to a cosmology that purports to provide a way of explaining almost everything independently of situation and purpose. Aside from the issue of feasibility in light of conflicting epistemological assumptions, the quest for an over-arching theoretic scheme of this type is of little interest to me as a mathematics educator. To explain why, I turn to the second observation sparked by your zooming metaphor.

As I read the analogies you draw with the development of theory in physics, I found myself reflecting on a characteristic of official, public scientific (and mathematical) discourse that is incidental to your argument. As we are both aware, this discourse assumes an agent-less voice that masks the interests and purposes for which a theory was developed, and instead portrays the theory as the result of reading of the Book of Nature. At times, when I read attempts to synthesize, say, Piagetian and Vygotskian theory, I have a sense that this same orientation is involved. In my view, this orientation, which Shotter (1995) referred to as the lure of cosmology, should be avoided by mathematics educators. The type of work we do as we seek to contribute to the continual improvement of the learning and teaching of mathematics is not a spectator sport. Instead, co-participation is at the core of work in our field. We might, for example, co-participate in mathematical reasoning with a student during a one-on-one teaching session, or we might co-participate with a teacher and his students is the learning and teaching of mathematics during a classroom teaching experiment, or we might co-participate with a group of teachers in the development of a professional teaching community, or we might co-participate in the restructuring of a school or school system as we attempt to forge a common agenda with teachers and administrators. For me, it is essential that the theoretical constructs we use to make sense of what is happening in any of these cases capture our co-participation in the process of educational improvement. To put the matter even more directly, we have to avoid what might be termed split-brain syndrome wherein we co-participate in the educational process with students, teachers, and administrators, but then describe the experience of doing so in the agent-less voice of the ultimate observer.

Against the background of these observations (some would say diatribes), let me return to your zooming metaphor and formulate the issue as I see it. I hope it is clear that the various forms of co-participation listed above can easily be brought into correspondence with nested settings that I listed at the outset. On my interpretation, the issue you raise is that of developing a coherent set of interrelated theoretical constructs that enable us to make sense of the various levels of activity in which we might co-participate as we seek to contribute to the ongoing improvement of mathematics teaching and learning. In this context, coherent means that analyses of one level of activity can, at least in principle, be recast in terms of analyses of activity at other levels. This is, I believe, consistent with the spirit of your proposal. I would also add that an important criterion for me is that analyses of any level of activity feed back to inform our own (and hopefully others') decisions and judgments as we strive for improvement. The theoretical constructs used to develop such analyses therefore have to do work. They might best be viewed as conceptual tools that are specifically designed to support the ongoing process of change and innovation. And, in this process, the theoretical constructs would be modified and adapted in response to the pragmatic concerns and interests that are encountered. In addition to grounding theory to the multiple settings of mathematics learning and teaching, this openness to pragmatic concerns serves as an antidote to the lure of cosmology.

Now it's your turn at the plate. Does the issue as I have formulated it provide an adequate basis for our continuing conversation, or do you want to tweak it a little?

Pat

Thank you, Paul, for helping me elaborate my original idea. In doing so I think you take the conversation in interesting and productive directions I hadn't considered. I'd like to begin with your last point, on co-participation, then your early example of analyzing a videotaped interview from multiple perspectives, then my original zooming metaphor. This will be with the aim that we tease out some of the details needing attention if we're to actualize the connections we have in mind.

I remember learning in college that, to write scientifically, I should write with authority, and to write with authority often translated into writing in the passive voice. It is by employing this simple grammatical trick, writing in the passive voice, that we, as researchers and observers, turn our (certainly powerful!) personal insights into seemingly general truths read from "The Book of Nature." You said:

> For me, it is essential that the theoretical constructs we use to make sense of what is happening in any of these cases capture our co-participation in the process of educational improvement. To put the matter even more directly, we have to avoid what might be termed split-brain syndrome wherein we co-participate in the educational process with students, teachers, and administrators, but then describe the experience of doing so in the agentless voice of the ultimate observer.

This reminded me of Steir's (1991, 1995) proposal that people doing research in social settings attempt to capture their own contributions to the phenomena they investigate, being open to the possibility that there might not be anything to investigate had they not participated in creating the phenomena being studied. But even more, I read your statement as a call that we always attempt to speak in ways that allow readers to know where we, as observers, have positioned ourselves relative to what we are describing. Put another way, I interpret your suggestion as one that calls on us, as researchers, to always make clear in our text for whom we imagine ourselves speaking. In regard to viewing a videotape of a researcher and child and reporting our observations and analyses, we could imagine ourselves speaking for:

- a participant in a dialog, conveying that person's meanings and motivations;
- an observer of a dialog who has access to the participants' personal meanings and to each participant's ongoing interpretations of the other;
- an observer of a dialog who has access to the participants' personal histories and to the histories of groups with whom they identify themselves.

Your suggestion is especially powerful in its implications for making research easier or harder for outsiders to read. By striving to make clear for whom our text speaks, we help readers position themselves as they build images of the phenomena we describe. This, with one small exception, is consistent with your notion that sometimes we choose from among different theoretical perspectives without "zooming" between scales of analysis. The exception is one you noted yourself — that in moving from one perspective to another "...zooming is still involved — from inside the ongoing interaction, to outside the local interaction, to outside broad cultural traditions. However, it is accomplished (usually implicitly) by the analyst as she switches from one perspective to another."

So, in proceeding it might be useful to state what I see is our common position and restate the issue I'd hoped to raise with the metaphor of "zooming" among perspectives. I see our common position being that we need a way to imagine the amalgam of settings in which we situate our activities, observations, and analyses so that we can move across levels of analysis — across what you've described as psychological constructivism, symbolic interactionism, and sociohistorical theory — without lapsing into the passive voice, and thereby avoiding "agentless descriptions given by a universal observer". The issue I'd hoped to raise is that these "ways of imagining ..." will not come free. We must discuss and debate possible "ways of imagining" explicitly (Miller, 1987). I should also agree explicitly with another point you made — that, as mathematics educators, we should not lose sight that *our* actions are tightly bound up with a set of core problems having to do with the improvement of individuals' mathematics education. This, I believe, will keep us from trying to develop, as you say, a theory of everything.

I do not have in mind a particular "way of imagining", but I can suggest a starting point for the discussion. In fact, you suggested it by way of example:

> We might contrast the suppositions and assumptions that the student makes as a consequence of her history of participation in particular cultural practices with those that the researcher makes about the teaching session as a consequence of her induction into a particular research tradition in graduate school. In an analysis of this type, which might be characterized as sociohistorical in nature, our position is not merely outside the local interaction, by is outside entire communities of practice.

I suspect that analyses of participation and of practice will be particularly rich in possibilities for elaborating "ways of thinking" which enable one to move between levels of analysis in ways that insights drawn at one level inform our analyses at another. The reason I think this is that, in its common usage, "to participate," in its intransitive form, suggests an interface between an actor and a setting. At the same time, "practice", as a noun, suggests a stable form of activity within a group which need not be a common form of activity among members, but rather is a state of dynamic equilibrium among its inter-acting members. So, to me, by focusing on how we might understand or come to understand the ideas of participation and practice we address explicitly the question of how we can imagine individuals' activities and groups' characteristics in mutually supportive, compatible ways.

In closing this piece, I'd like to make explicit to persons reading our exchange something said by Salomon (Salomon, 1993). It is that sociocultural and scientific theorists tend to think of explanations differently. Sociocultural explanations tend to be oriented toward descriptions of intact systems having certain observed characteristics, where descriptions tend not to appeal to internal mechanisms which produce the observed characteristics. Scientific explanations tend to be more mechanistic — in the sense of aiming to produce models having components that interact according to certain principles and which produce, through interaction, the observed phenomenon. This is not to be confused with strong information processing models. Maturana captured the essence of modeling when he described it as rethinking the observed phenomenon so that you imagine from whence it arose.

> As scientists, we want to provide explanations for the phenomena we observe. That is, we want to propose conceptual or concrete systems that can be deemed intentionally isomorphic to the systems that generate the observed phenomena. (Maturana, 1978, p. 29)

I, personally, find the modeling perspective to be useful in that explanations we give tend to be less ad hoc than the former. The scientist's production of models also reflects the high value scientists place on explanations which support prediction. A byproduct of adopting a modeling point of view is that it forces us to examine our basic constructs, to ask "what do we mean" by such basic terms as "participation" and "practice."

I suppose I leave my part with the question to you, Paul, of whether you see this as a productive direction.

Paul

Pat, judging from your comments, I think that we are very much on the same page. In this response, I am going to address the last you point you raise first by discussing what we might means by such basic terms as participation and practice. My motivation for doing so is to attempt to clarify as much for myself as for others what we might be talking about when we throw these terms around. Given that this is an ongoing project, I would certainly welcome further probing and pushing on your part. Having taken a stab at addressing this issue, I will then focus on the points you make about modeling and explanation.

As you know. I and several colleagues[1] have been trying to develop the notion of a classroom mathematical practice for the last few years. Our motivation for doing so stems directly from the problems and issues we have encountered while conducting classroom teaching experiments. For example, in preparing for a teaching experiment, we outline a possible sequence of instructional activities by envisioning how students' mathematical learning might proceed as the potential sequence is enacted in the classroom. In doing so, we develop testable conjectures about both 1) possible learning trajectories, and 2) the specific means that might be used to support and organize that learning (Gravemeijer, 1994). The important point for our discussion is that these conjectures cannot be about the anticipated learning of each and every student in a class for the straight-forward reason that there are significant qualitative differences in their mathematical thinking at any point in time. In my view, descriptions of planned instructional approaches written so as to imply that all students will reorganize their reasoning in particular ways at particular points in an instructional sequence involve, at best, questionable idealizations. A problem that has arisen for us is therefore that of figuring out how to characterize the envisioned learning trajectories that are central to our work as instructional designers. In particular, if it does not make

[1] These colleagues include Janet Bowers, Koeno Gravemeijer, Kay McClain, Michelle Stephan, Joy Whitenack, and Erna Yackel.

sense to view them as trajectories for the learning of individual students, then what might they be trajectories of? Our current (and potentially-revisable) solution is to view a hypothetical learning trajectory as consisting of conjectures about the collective mathematical development of the classroom community. This proposal in turn indicates the need for a theoretical construct that enables us to talk explicitly about the mathematical learning of a classroom community.

- If we think of theoretical constructs as tools that are developed for particular purposes and interests, then additional design specifications for the theoretical tool that we need include. It should enable us to think about communal mathematical development over the extended periods of time that are covered by instructional sequences.
- It should enable us to make sense of what might be happening in classrooms over these time periods in such a way that the resulting analyses feed back to inform the ongoing instructional design effort.
- It should enable us to relate the collective mathematical activity of the classroom community to both the developing mathematical reasoning of the participating students and to the broader activity system of the school (see the nesting of settings discussed earlier).

My reason for suggesting this last criterion is again pragmatic. For example, when we make pedagogical decisions and judgments during a teaching experiment, we find it essential to attend to students' qualitatively different ways of interpreting and solving tasks, and in fact view that diversity as a primary resource upon which to capitalize when attempting to advance our pedagogical agenda. Further, we are all too aware from personal experience that events in the classroom do not occur in a social vacuum. Influences that we have found it necessary to take into account over the years include the students' prior instructional histories, the institutionalized procedures for assessing both students and teachers, the established norms of participation for teachers within the school community (i.e., their obligations to other teachers, administrators and parents), and the students' developing identities as members of groups within the student body. In light of these issues, I hope it is clear that while I am interested in coordinating levels of analysis, the purpose for me is not to develop an encompassing theoretical scheme as an end in itself. Instead, it is to come to grips with the types of issues that we find ourselves addressing in the course of our work.

It is against the background of these and other considerations that we have attempted to "hammer out" the notion of a classroom mathematical practice. Described in terms of this construct, an envisioned learning trajectory consists of an anticipated sequence of classroom mathematical practices together with conjectures about the means of supporting their evolution from prior practices. To clarify what we might mean by a mathematical practice, I will focus of three interrelated aspects: 1) The taken-as-shared purpose, 2) the norms for mathematical argumentation, and 3) the taken-as-shared ways of reasoning with tools and symbols. In doing so, I am going to give a brief example from a seventh-grade teaching experiment with which you are familiar that are focused on statistical data analysis. During this experiment, the students routinely used computer minitools prototyped in Java to compare univariate data sets. In an analysis of this experiment, we argued that the first mathematical practice that emerged as the students used one of these minitools involved exploring qualitative characteristics of collections of data points. In giving this characterization, we are claiming that the taken-as-shared purpose for analyzing data sets in this classroom was to identify qualitative trends or patterns. For example, in one instructional activity, the students attempted to determine which of two brands of batteries was superior by analyzing data on the life spans of ten batteries of each brand. A pattern identified by one student that was treated as significant by the teacher and other students was that all the batteries of one brand lasted more than 80 hours whereas two batteries of the other brand lasted considerably less than 80 hours. As this illustration indicates, part of the challenge when describing a mathematical practice is to clarify what mathematical activity might be about in a particular classroom at a particular point in time.

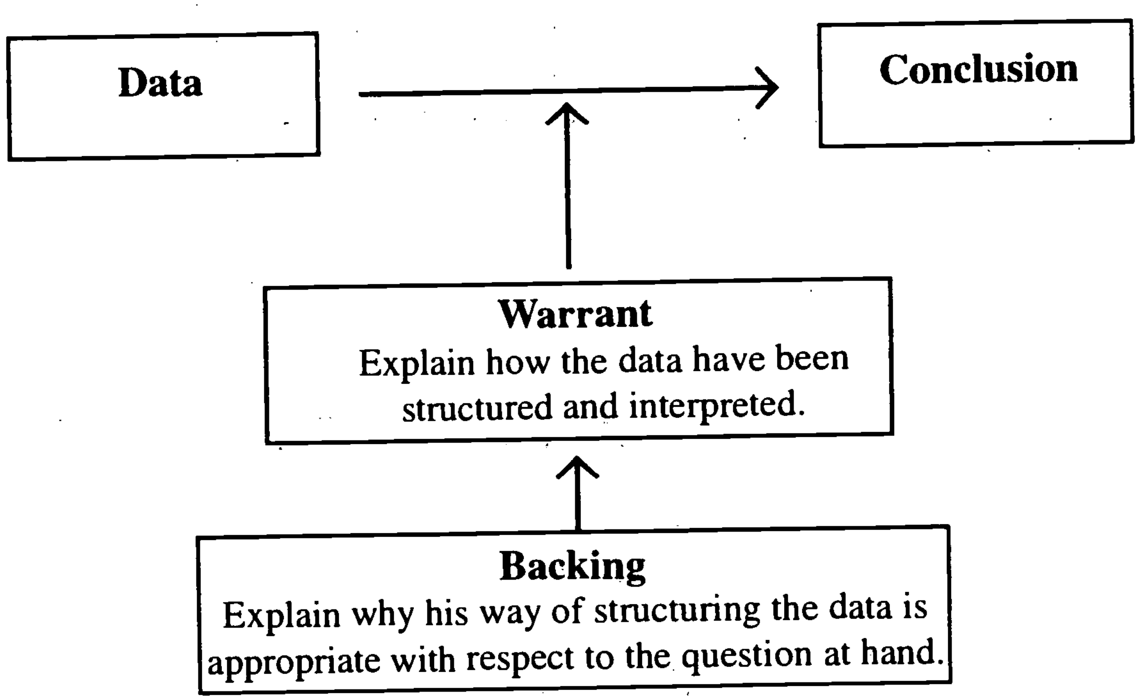

Figure 1. Toulmin's Justification Scheme

In this same analysis, we also argued that the teacher and students negotiated particular norms for mathematical argumentation. In terms of Toulmin's (1969) scheme, this can be represented as shown in Figure 1.

In the case of the sample solution, the student concluded from the data that one of the brands of batteries was superior. He also gave a warrant that explained why the data supported the conclusion when he said that he had used the computer minitool to partition the data sets at 80 hours and had noticed that two batteries of one brand lasted less than 80 hours. In addition, he gave a backing to indicate why his warrant and thus his method for comparing the two data sets should be accepted as having authority when he explained that he wanted a consistent battery that would last at least 80 hours. The norms of argumentation exemplified by this explanation both reflect the taken-as-shared purpose for analyzing data outlined above and serve to further clarify that purpose. In particular, searching for patterns was not an end in itself. Instead, the patterns identified by structuring data in a certain ways had to be justified with respect to the question at hand.

The norms for argumentation also relate to the third aspect of the mathematical practice we analyzed, the taken-as-shared ways of reasoning with the computer minitool, in that the students were developing arguments for a decision or judgment when they used the minitool to analyze data. In general terms, this last aspect of a mathematical practice is concerned both with the ways of using tools and symbols that are treated as legitimate in the classroom, and with what is reasoned about while doing so. In the case of the statistics teaching experiment, taken-as-shared ways of using the minitool to organize data included partitioning data sets, bounding the data points in particular intervals, and bounding clusters of data points. Further, the taken-as-shared ways of reasoning about data that was organized in these ways appeared to be additive rather than multiplicative (see Cobb, in press, for a more detailed discussion). It was for this reason that I in fact spoke of the students exploring qualitative characteristics of collections of data points rather than, say, of distributions. In our estimation, as we looked at public classroom discourse at the beginning of the teaching experiment, there was no indication that the teacher and students were concerned with how data sets were distributed in a statistical sense (cf: Konold, Pollatsek, Well & Gagnon, 1996). Instead, classroom discussions focused on the number of data points in particular intervals, or above or below a particular values.

Well, Pat, that is the best that I can currently do to say what I mean by a classroom mathematical practice. I should clarify that we have refined this notion as we have conducted a number of specific analyses. Part of the difficulty is therefore that of trying to explicate what we actually do in action while making sense of what might be going on in the classrooms in which we work. I would therefore anticipate that there are a number of implicit suppositions and assumptions that we are yet to dig out. In your language, how adequate is the above account in helping you build imagery for the phenomena we are attempting to describe (and how adequate is the construct itself given the purposes for which it is being developed)?

In considering the other term you mention, "participation", we have to address the issue of coordinating individual and communal perspectives on classroom events head on. In the way that I currently look at what is going on in classrooms, a student's mathematical reasoning *is* his or her way of participating in communal classroom practices. Obviously, this statement needs to be unpacked. When I speak of a student's mathematical reasoning, I am taking a psychological constructivist perspective that brings qualitative differences in students' thinking to the fore. In contrast, when I speak of participation in communal practices, I am taking a social perspective that situates the student's reasoning within an evolving classroom microculture. The above statement therefore involves a claim about how these two perspectives on a student's mathematical activity might be coordinated (and thus how the collective mathematical activity of the classroom community might be related to the developing mathematical reasoning of the participating students). We in fact take the relation between the two perspectives to be reflexive. This is an extremely strong relationship and does not merely mean that individual students' reasoning and the practices in which they participate are interdependent. Instead, it implies that one literally does not exist without the other (Mehan & Wood, 1975). What, from one perspective, is viewed as an individual act of reasoning is, from the other perspective, viewed as an act of participating in the communal practices of the classroom community.

I hope that it is clear from this brief account that the coordination at issue is not between individual students and the classroom community viewed as separate, sharply defined entities. Instead, the coordination is between two alternative ways of looking at and making sense of what is going on in classrooms. In other words, we are coordinating different ways in which we can interpret classroom events. What, from one perspective, are seen as the norms and practices of a single classroom community is, from the other perspective, seen as the reasoning of a collection of individuals who mutually adapt to each others actions. Whitson (1997) articulates this point as clearly as anyone when he proposes that we think of ourselves as viewing human processes in the classroom, with the realization that these processes can be described in either social or psychological terms. This formulation is, I believe, consistent with your discussion of the need to allow readers (or conversation partners) know where we have positioned ourselves relative to what we are describing.

In turning to consider your comments about modeling and explanation, I should clarify that my commitments as I try to understand what might be going on in classrooms are more than just to understand the particular case at hand. Instead, specific classrooms serves a paradigm cases as I try to develop ideas that might have more general relevance and yet remain rooted in the settings in which we co-participate with teachers and students. In the case of the statistics teaching experiment, for example, our more general concerns were to further develop the notion of a classroom mathematical practice and to deepen our understanding of the role that tool and symbol use can play in students' mathematical development. I would note that in an approach of this type, theorizing is not an abstract, esoteric game. Instead, it is a means of attempting to be more effective in supporting students' mathematical learning. As a consequence, the perennial problem of bridging the gap between theory and practice fails to materialize in that the resulting theoretical ideas do not stand apart from practice but are instead developed in the context in which they will be used.

As a further point, I want to question whether models of the type you describe that involve "components that interact according to certain principles and which ... produce the observed phenomenon" are necessarily the most useful for our purposes as mathematics educators. I assume that the primitives in such a model of a classroom community might be the teacher's and students' ways of interpreting each others' actions. When the model is "turned loose", broad pattern such as those that we point to when we speak of classroom mathematical practices might then emerge as epiphenomena in much that same way that patterns emerge in statistical data at the macro-level. A difficulty for me concerns what might be taken as a primitive in such a model. Earlier, I clarified that I take the relation between psychological and social perspectives and thus between individual students' reasoning and the practices in which they participate to be reflexive. Given this theoretical commitment, teachers' and students' reasoning are not seen to exist apart from their participation in communal practices, just as the practices are not seen to exist apart from their continual regeneration as teachers and students mutually adapt to each others' activities. Thus, in adopting this view, I would not treat individual students' reasoning as being more primitive than communal mathematical practices, or vice versa. Just as I would have difficulty with a theoretical position that portrayed students' reasoning as being determined by their participation in communal practices, so I would question an approach that treats communal practices as mere epiphenomena. In classroom teaching experiments, for example, our understanding of students' history of participation in classroom mathematical practices helps us explain their reasoning in exit interviews. This attention to history does not appear to be as relevant to the concerns of physicists and biologists when they think through how the primitives in the systems that they study will behave.

I think that I have said more than enough at this juncture. As you can see, I certainly found you comments both stimulating and provocative. Hopefully, this response will serve to move the conversation along.

Pat

Well, Paul, you certainly did move the conversation forward. Let me see if I can recap what I last said and your response to it. I suggested that it might be productive for us to discuss "ways to imagine" social and psychological phenomena that would support our ability to "zoom" between individual and social perspectives so that each truly becomes background for the other. I suggested that the ideas of participation and practice might be productive sites for this discussion, because the idea of participation seemed to entail a relationship between individuals and a group in which they are members, and activity of the group in which the individuals participate. I also drew a distinction between explanations common to sociocultural theories, which tend to describe social systems as unanalyzed wholes having various properties, and scientific explanations which are more analytic, breaking a system into component parts and postulating how those components might interact to produce the observed phenomena.

You agreed with the general thrust or my proposal, elaborating your ideas of practice and participation, and then you went further to explain by way of example that the idea of practice is important in your own work because it supports your goal of understanding and affecting what happens in classrooms. You closed by wondering what utility the activity of scientific modeling might have for mathematics education, stating your strong dislike for any approach that proposes communal practices as "mere epiphenomena".

To respond to all this is a challenge! I'll jump around a bit by first touching on the example of practice as employed in your current project and its affiliation with the notion of "taken as shared". Then I'll respond to your question about the utility of analytic models, and in doing so try to explicate a confusion I have which stems from your remarks about epiphenomena.

If I understand you correctly, you use "practice" in two senses — one which supports your attempt to express what you hope an instructional sequence produces. I had a horrible time formulating the previous sentence so that I wouldn't say something to which you would take immediate exception. My original inclination was to say, "... what you hope students learn," not meaning that you expect every student to learn what you state, but rather that you would be delighted if they did. In my understanding, you use "practice" in the context of instructional design almost heuristically — as a way to finesse the sticky problem of saying what you hope students learn without committing yourself to the impossible objective that every student learn it. It provides a way to imagine "the class, collectively" as if it were one person who could participate in every setting you might imagine being pertinent to the ideas and dispositions you want to address. If my interpretation is consistent with your intention, then this sense of "practice" is consistent with what I would, in other settings, call "cognitive goals of instruction" — the imagery, orientations, dispositions, mental operations, schemes, etc. that would enable a person to contribute to and partake of classroom conversations, activities, and tasks productively. I must stress again, though, that I am talking about *intention*, not expectation. I find this sense of "practice" quite powerful, for it allows me to think about not just what I want students to know, but also to think about ways they might think of the settings in which they find themselves that will be supportive of their desire to participate in ways which will contribute to other students' intellectual growth.

But I need help understanding what you mean by communal mathematical development and collective mathematical activity. On one hand, I can understand these phrases as referring to phenomena I might observe within the confines of a classroom which, when I leave them unanalyzed, strike me as having certain features. I see this as being parallel with observing a particular house not as a structure that evolved over time, emerging from the joint efforts of its builders and designers, but as an object having a certain color, shape, and size, and having certain accoutrements. A consumer could function quite adequately with the latter perspective; a designer could not. I also suspect that an experienced designer *could not* look at a house without a background image of the activity producing it. That is, I suspect that the notion of collective mathematical activity has at its center the characteristic of being an epiphenomenon. I'll return to this later. But first I'd like to point to another example.

Bransford et al. (in press) described two boys, one of whom could not read and one of whom suffered attention deficit disorder. The two cooperated in a cooking club by each relying on the other to compensate for his own deficit. In this setting, we could view the two as, communally, constituting a pretty effective cook. How does this example differ from the example of a house emerging from the communal efforts of its builders. In two ways. First, the house is the product of a group's activity, but we never thought of the house as somehow constituted by the crew. The house is analogous to a meal the boys produce. But another difference is that we view the house as having a permanence that the two boys acting together do not. The two boys acting together is more like the crew which produced the house. We imagine the crew as having, too, a permanence in the sense that we expect some members to leave and others to join without affecting the crew's overall competence. That is, we attribute a permanence to the crew — at least in terms of continuing competence and skill. But we don't attribute the same permanence to the communal competence of Bransford's boys, largely because we don't expect them to stay together in settings other than the cooking club, nor do we expect either boy's deficiency to be redressed by his activities in the cooking club.

My confusion, I think, is that I don't know how to think about communal activities as *other* than epiphenomena, at least in regard to the goal that instruction have some lasting effect. Being epiphenomenal, then, I won't know whether the communal activity is valuable unless I know something about the changes taking place within individual children so they may contribute to it. It seems possible, in principle, that a desired communal practice emerges, but few students are affected in ways that will allow them to contribute in other settings to making it emerge again.

In the same way that I don't know how to think of communal practice as other than epiphenomenal, at least to think of it in ways that matter to instructional design and students' learning, I'm afraid I don't know how to think of "taken as shared". On one hand, we could, like Voigt, (1994, 1996), mean that it is a statement about what an individual person thinks. In Voigt's usage, an idea is "taken as shared" when an individual person presumes other people think the same way as she does about some meaning or idea. It is the observed actor who we claim is doing the taking.

On the other hand, we could mean something in line with Lave (Forman, 1996; Lave, 1991), that when we imagine some meaning or practice as being "taken as shared", that we are making no claim at all about what members of a group think, believe, or mean. Rather, the claim that something is "taken as shared" is a claim that the group, as a single entity, seems to act *as if* it were one entity which thinks in some particular way. In other words, it is the *observer* who does the taking. "I take this group's behavior as if ..."

The examples from your statistics teaching experiment are helpful in one way, in that they clarify for me the kinds of things you see happening communally which inform your assessment of potential learning trajectories (i.e., they inform your evaluation of instructional design). But they are unhelpful in a very important way. On one hand, you contend that we cannot think of classroom mathematical practices as constituting something that each and every student will learn. On the on the other hand, you offer one student's activity as being illustrative of a practice you claim developed. It is in this sense that I see a misfit between theory and implementation. I would think that, to implement your idea of classroom mathematical practices in the conduct of mathematics education research, we would attempt to identify in classroom activities aspects of the class' taken-as-shared (in Lave's sense) activity that emerges because of a collage of behavior emanating from an interaction among students' taken-as-shared (in Voigt's sense) meanings. But this seems to point again to the need to think of mathematical practices as epiphenomena. Now, it may be that we must clarify our personal meanings of "epiphenomena". To me, it points to thinking of an observation as being the *result* of something else. I must be careful lest you think that I'm attributing reality to individuals in interaction and not to communal activity. Far from it. In that regard, I think Bishop Berkeley's (1963) famous dictum, "To exist is to be perceived", is very helpful. When we *see* communal activity, it exists.

In closing, I must say I couldn't agree more with your characterization of the reflexive relationship between social and psychological perspectives. However, for our purposes I think that, with respect to modeling and designing, the psychological perspective is more fundamental. This is for the simple reason that the groups within which children act do not persist. Students act within many groups, and they will join many others over their life. Therefore, we would be remiss not to address the question of how we hope to affect individual children so they are able to act productively in a variety of settings. That is, it is individual children who will persist over time, not the classes in which we view them or in which they act for relatively short periods of time. That is why while I agree completely with your characterization that, as perspectives, psychological and social perspectives are mutually constitutive — one perspective cannot exist without the other —I choose to view the psychological perspective as more fundamental. It aligns more explicitly with what I take as our fundamental goal of making a positive, lasting difference in students' lives after they leave our classrooms.

Paul

Wow, Pat, my immediate reaction is to disappear for a month and to develop a position paper as a means of clarifying my own thinking on the issues you raise. However, as we are under that gun, I will try to give quick responses to the various points you raise.

In talking about our use of the term practice in the context of instructional design, you say that:

> It provides a way to imagine "the class, collectively" as if it were one person who could participate in every setting you might imagine being pertinent to the ideas and dispositions you want to address. If my interpretation is consistent with your intention, then this sense of "practice" is consistent with what I would, in other settings, call "cognitive goals of instruction" — the imagery, orientations, dispositions, mental operations, schemes, etc. that would enable a person to contribute to and partake of classroom conversations, activities, and tasks productively. I must stress again, though, that I am talking about *intention*, not expectation.

Here, I believe that there is a mismatch in our interpretations in that, from my point of view, you have recast the notion of a classroom mathematical practice in individualistic terms. To tease out these difference, I will given an example from our ongoing work. Earlier, I mentioned a seventh-grade teaching experiment that focused on statistical data analysis. We are in fact currently in the process of planning for a follow-up eighth-grade teaching experiment that we will conduct with the same group of students in fall 1998. One of the mathematical ideas that we will focus on is that of co-variation, which includes but is not limited to correlation. An image that I have in mind as I think about possible instructional goals concerns how scatter plots might be talked about and used in public classroom discourse. In particular, we (currently) want scatter plots to be talked about and referred to as texts about the situations from which the data were generated. If this occurs, then it will be taken-as-shared that the aspects of a situation that were judged to be significant and were measured when generating the data co-vary, and that the specific nature of that co-variation is shown by the graph. This formulation of the instructional goal provides an initial orientation for myself and my colleagues as instructional designers and teachers. For example, it suggests that the cloud of dots on a scatter plot should explicitly be spoken about in classroom discussions as measures of aspects of a situation that are distributed in a (two-dimensional) space of values. We therefore have an initial, provisional sense of the types of conversations that we might want to support in the latter part of the teaching experiment.

I hope it is clear that in stating our instructional intent in this way, I am not thinking about the classroom community as if it were one person. Instead, I am thinking about what the teacher and students might be doing collectively. And in doing so, I am attempting to articulate my (potentially-revisable) image of the immediate social situation of individual students' mathematical development at the end of the experiment. In addition to formulating goals, part of the challenge when planning an experiment is to think through possible means of achieving these goals. In this regard, I noted earlier that this involves outlining both 1) a learning trajectory that might culminate with the mathematical practices that constitutes the envisioned goal, and 2) the specific means that will be used to support and organize that learning. In the case of the eighth-grade experiment, for example my colleague Koeno Gravemeijer has sketched such a trajectory and, at the time of writing, we are programming two computer-minitools that we hope will be effective means of supporting the mathematical learning of the classroom community and of the students who participate in it. I mention this to stress that, in contrast to your example of the house, we take a developmental point of view when we think of classroom mathematical practices. Consequently, in the planning process, we attempt to envision how practices might emerge as reorganizations of prior practices. This developmental emphasis is, I hope, also evident in our analyses of what actually transpires in the classroom when we conduct a teaching experiment. For me, an analysis that merely lists a number of practices without describing the process of their emergence from prior practices would be woefully inadequate given that a primary objective when conducting a teaching experiment is to investigate the means of supporting the development significant mathematical ideas.

Pat, a second observation you made allows me to be a little more specific about the process of analyzing classroom events in terms of mathematical practices. You say that the examples I gave from the seventh-grade teaching experiment in my last response:

> are unhelpful in a very important way. On one hand, you contend that we cannot think of classroom mathematical practices as constituting something that each and every student will learn. On the on the other hand, you offer one student's activity as being illustrative of a practice you claim developed. It is in this sense that I see a misfit between theory and implementation.

You are right, I did focus on one student's explanation. However, in doing so, I indicated that this explanation was treated as legitimate by the teacher and other students. Thus, for me, it was an example of what counted as an acceptable explanation in this particular classroom. My focus was not on the reasoning of the student who gave the explanation (psychological perspective), but on the status of the explanation in this classroom community (social perspective). And, I contend, its constitution as a legitimate explanation was a collective accomplishment. As a caveat, I should add that we would not in practice (that word again) claim that certain norms of argumentation had been established on the basis of one isolated case. For example, from what I said, you do not know whether the other students were bored and had no interest in the discussion, or whether they did not view it as their role to question each others' contributions. In general, when we make the inference that something is normative in a classroom (e.g., a particular form of argumentation or a way of reasoning with tools and symbols), we are claiming that members of the classroom community will object when they perceive that those norms have been breached. Thus, methodologically, when we conjecture that something is normative in a classroom, we look for instances where a student's contribution violates those norms and examine whether or not that contribution is constituted as legitimate by the classroom community. In the case of the seventh-grade teaching experiment, there were in fact occasions when students objected when they perceived that the scheme of argumentation I illustrated had been violated (Cobb, in press). This constitutes reasonably strong evidence that the standards of argumentation I described were normative.

A third point that you make brings us to the core issue, the types of theoretical tools at might facilitate our attempts to contribute to the continual improvement of the learning and teaching of mathematics. Framed in this way, the issue at hand is not to decide whether communal practices are epiphenomena in an ontological sense. Instead, it is to clarify whether it more useful for our purposes to think about them as epiphenomena or as phenomena in their own right. In this vein, you comment that you will not know whether communal activity is valuable unless you:

> know something about the changes taking place within individual children so they may contribute to it. It seems possible, in principle, that a desired communal practice emerges, but few students are affected in ways that will allow them to contribute in other settings to making it emerge again.

Later, you reiterate this point when you say that "we would be remiss not to address the question of how we hope to affect individual children so they are able to act productively in a variety settings". From this, you conclude that "the psychological perspective as more fundamental, because it aligns more explicitly with what I take as our fundamental goal of making a positive difference in students' lives". I could not agree more strongly with your statement of our overall goal as mathematics educators. It is for this very reason that we have struggled so hard to develop a way of talking about the mathematical learning of classroom communities. I contend that what we need if we are to continually improve our instructional designs are accounts of students' learning that are tied to analyses of what happened in the classrooms where that learning occurred. An analysis of the classroom mathematical practices established by a classroom community provides a way of describing what transpired in the classroom over an extended period of time. In addition, it enables us to specify the evolving social situations in which the students' mathematical development occurred. To be sure, this analysis of communal learning should be coordinated with a psychological analysis of the qualitatively different ways in which students participated in communal practices and what they learned when doing so. What we then end up with is a situated account of students' learning, one that directly relates the process of their learning to the means by which it was supported. As a consequence, we can immediately develop testable conjectures about how we might be able to improve those means of support. This in turn enables us to engage in educational reform as an ongoing process of improvement in which we continually learn from our experiences of experimenting in classrooms in collaboration with teachers.

In your argument for the primacy of the psychological perspective, I was also struck by your suggestion that although a desired communal practice could emerge, only a few of the participating students might learn in significant ways. First, I should clarify that the establishment of a classroom practice is, for me, a collective accomplishment to which students actively contribute by reorganizing their reasoning. Consequently, a practice cannot, by definition, become established if only a few students learn. Particular purposes, ways of arguing, and ways of reasoning with tools and symbols simply would not become taken-as-shared. It could, however, be the case that the ways in which the students reorganize their reasoning are not as significant as we had intended. This is a question that has to be addressed empirically. In the case of the seventh-grade teaching experiment, for example, we claim that a particular practice that involved reasoning multiplicatively about data emerged during the last part of the experiment. Our classroom observations of the students' reasoning as they participated in this practice indicate that most came to think about data in relatively sophisticated ways. To check the validity of this inference, Cliff Konold, in his role as an independent evaluator, is currently analyzing the individual exit interviews that we conducted with the students. His findings could well lead us to revise our interpretations of our classroom observations. In addition, his analysis, when combined with the analysis of the classroom mathematical practices, will enable us to consider how our instructional design might be improved.

In closing, I want to make a final comment that draws on my involvement in the teaching during the seventh-grade teaching experiment. This proved to be an significant experience for me in that I had not taught in a school classroom for 15 months. I found that I was making sense of what was happening in the classroom differently than I had done previously. In particular, I truly saw a classroom community in which I was a participant. For example, I found that I was looking at ongoing discussions as collective social events. In doing so, I was able to think about and influence the taken-as-shared ways of talking about and reasoning with graphs and other inscriptions, and thus the issues that emerged as topics of conversation. This gave me a greater sense of efficacy as I sought to achieve a pedagogical agenda in that I could attempt to influence the social situation of all the students' learning.[2] I should stress that in doing so, I tried to build on the students' thinking (although a viewing of the video-recordings indicates that there is considerable room for improvement in this regard). However, rather than looking at students contributions in purely individualistic terms, I found

[2] The conjectured learning trajectory for the instructional sequence was also important in that it, in effect, provided a big picture that served to frame the local pedagogical decisions and judgments that I and my colleagues made.

that I was viewing them as acts of participation in ongoing social events. This seemed to make the challenges of teaching more tractable. Perhaps you can keep track of and attempt to influence the reasoning of 30 different individual students simultaneously. Such a feat is beyond my limited capabilities. However, I found that I could (to some extent) monitor both what we were doing as a community and three or four qualitatively distinct ways in which the students were participating in these collective activities. As a consequence, classroom situations that seem almost overwhelming complex when viewed in purely individualistic terms became more manageable.

Conclusion

This conclusion is written by just one of us (PT). Paul and I were supposed to write the conclusion while attending a meeting in Amsterdam. Unfortunately, I sat, stranded, in Washington Dulles airport while Paul was in Amsterdam, and the deadline for final manuscript passed. As such, Paul can't be held accountable for whatever conceptual errors I reveal in these last few paragraphs.

We began this paper with a metaphor — zooming between perspectives without losing the overarching or underlying phenomena revealed at various scales of observation. We then explored how we might think of psychological and sociocultural perspectives regarding mathematical understanding/learning/activity to realize promising directions for resolving conceptual conflicts between them, and in the process revealed what appear to be different commitments to forms of explanation and justification.

We did agree that the notions of practice and participation seemed promising sites for making a connection between psychological and social perspectives, but we ran into problems on how to view social activity. Pat prefers to think of interaction and mutual influence among individuals as being fundamentally constitutive of social activity. This is in the same way that we think of chemical and molecular interactions as being fundamentally constitutive of organic matter. We may not understand the interactions in all their details, nor may we keep track of them in real time. But we never pretend that perspectives of molecular interactions and of organic matter are mutually, reflexively constitutive. Organic matter "emerges" through special types of molecular interaction.

At the same time, Paul prefers to think of perspectives of individual interaction and sociomathematical activity as being mutually, reflexively constitutive. This is not to say that individuals and groups are mutually constitutive. Rather, he prefers to adopt a perspective of social activity that is fundamentally individualistic and adopt a perspective of individual activity that is fundamentally social. One cannot conceive either without having adopted the other.

We still agree that continued efforts to explicate the ideas of participation and practice will be productive. These are ideas that seem to embody, at heart, significant aspects of both psychological and sociocultural perspectives simultaneously.

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