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ABSTRACT

The National Council of Teachers of Mathematics emphasizes that young children need play-based opportunities to develop and deepen their conceptual understanding of mathematics. From a social-constructivist perspective, learning is more likely to occur if adults or more-competent peers mediate children's learning experiences. Emphasizing both the developmental and the curricular perspectives, this article focuses on the role of the teacher in guiding preschool children's mathematical learning while they play with everyday materials. Professional growth in three areas was identified as critical in teachers' learning to guide young children's learning of mathematical concepts. The three areas are: the ability to recognize children's demonstrated understanding of mathematical concepts; the ability to use mathematical language to guide their progress from behavioral to representational understanding of mathematical concepts; and the ability to assess systematically children's understanding of mathematical concepts. Checklists tracing the development of three fundamental mathematical concepts--one-to-one correspondence, classification, and seriation--are suggested as tools for teachers to monitor preschool children's learning of mathematical concepts and plan appropriate learning experiences within children's zones of proximal development. The article asserts that creating an environment that is mathematically empowering and mediating children's experiences in this environment establish the foundation for constructing, modifying, and integrating mathematical concepts in young children. The three checklists are appended. (Contains 26 references.) (Author/HTH)

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Learning to Guide Preschool Children's Mathematical Understanding: A Teacher's Professional Growth

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Abstract

The National Council of Teachers of Mathematics emphasizes that young children need play-based opportunities to develop and deepen their conceptual understanding of mathematics. From a social-constructivist perspective, learning is more likely to occur if adults or more-competent peers mediate children's learning experiences. Emphasizing both the developmental and the curricular perspectives, this article focuses on the role of the teacher in guiding preschool children's mathematical learning while they play with everyday materials. Professional growth in three areas was identified as critical in teachers' learning to guide young children's learning of mathematical concepts. First is the ability to recognize children's demonstrated understanding of mathematical concepts, second is the ability to use mathematical language to guide their progress from behavioral to representational understanding of mathematical concepts, and third is the ability to assess systematically children's understanding of mathematical concepts. Checklists tracing the development of three fundamental mathematical concepts—one-to-one correspondence, classification, and seriation—are suggested as tools for teachers to monitor preschool children's learning of mathematical concepts and plan appropriate learning experiences within children's zones of proximal development. Creating an environment that is mathematically empowering and mediating children's experiences in this environment establish the foundation for constructing, modifying, and integrating mathematical concepts in young children.

Introduction

Laura has just finished reading the story "Goldilocks and the Three Bears" to her preschool class. She announces that it is now time for free play. Four-year-old Rachel looks around the room for a while and walks over to the dramatic play/housekeeping center. Today this center is equipped with dolls, other soft toys, cups, plates, plastic silverware, plastic food items, a table,

> *chairs, and some dress-up clothes. Rachel picks up an oversized shirt and slips her feet into "mummy shoes." She then brings out three stuffed bears of different sizes from the collection and places them around the table. As she seats the bears on three chairs, she mutters under her breath, "You are Papa Bear" (picking the largest bear), "you are the Mummy Bear" (picking the medium-sized bear), "and you are Baby Bear" (picking the smallest bear). Rachel then walks to the shelf and pulls out one plate and places it before Papa Bear; she walks back to the shelf to get a second plate and places it before Mama Bear; and then she makes one last trip to pick up a plate to place before Baby Bear. Next Rachel walks to the shelf and picks up a collection of spoons of different sizes. She is now joined by 5-year-old Tiffany, who tells her that the biggest bear needs the biggest spoon, the medium bear the medium spoon, and baby bear the smallest spoon. "Remember, like the bears story Ms. Laura read us." Rachel looks at Tiffany and then at the spoons, then randomly places a spoon before each bear. Tiffany immediately takes over and rearranges the spoons according to the size of the bears. Rachel watches for a few seconds and then walks away.*

Although observing a play episode like this one would not be unusual in many preschool classrooms, it had a particularly strong impact on how Laura understood her students' mathematical knowledge. As a new member of the local chapter of the National Council of Teachers of Mathematics (NCTM), Laura became particularly interested in the development of mathematical concepts in her students. She realized that the most remarkable growth of mathematical knowledge occurs between the pre-kindergarten and grade 2 levels and that it was especially important at this stage to focus on guiding children's development of fundamental mathematical concepts. Yet the lack of an agreed-on math curriculum for preschool made it difficult for Laura to decide which concepts were the most appropriate for her preschool children. Like many other teachers, Laura struggled to make sense of the development of her students' mathematical learning and relate it to her instructional decisions (Franke & Kazemi, 2001). She wrote in her journal:

> Teaching math has always been outside of my "comfort zone." Many commercial and teacher-made math games, including sets of animals, fruit, vehicles, shapes; board counting game; board classification games; and various spinners and large dice, are useful in reinforcing one-to-one correspondence, classification, and seriation. However, while used randomly and in isolation, these games may not help children fully grasp the math concepts they are built on. I have to go beyond providing some form of mathematical learning; I really need to have a well-thought-out math curriculum. I have tried math activities that I hoped would promote learning. I graphed with the children on a large mat. I had them each take off a shoe and decide by color where it should be placed. It was an activity that seemed to me that would be fun and hands-on, but the children were restless and bored. I set out small manipulatives with similar attributes and let the children explore and sort in bowls. I encouraged them to bring in leaf collections for the science table and discussed color and shape. Although the children were exploring the materials, I was challenged to find a way to assess what the children were learning and how to further develop their knowledge.

As is evident from this journal entry, Laura felt the need for a strong conceptual framework that would take into consideration the developmental characteristics of preschool children and would indicate environments that would foster children's natural mathematical abilities. Such a framework could help Laura to decide which mathematical concepts were appropriate for her students and the order in which they should be taught. Laura realized that these decisions needed to be based on her knowledge of the development of mathematical concepts and on an appropriate assessment of children's mathematical knowledge. She also realized that preschool programs needed to expand and deepen the conceptual knowledge that young children have already developed by 3 years of age (Payne, 1990). The NCTM's (2000) new standards emphasize that all preschool children need opportunities to explore their world and experience mathematics through play. Knowing that, however, left Laura with more questions than answers. She wrote in her journal:

> How do I use play and play materials to enhance children's learning of primary math concepts? As a facilitator of learning, how can I engage the children in activities that would enable them to further construct mathematical concepts? What is the order in which math concepts develop? What are the primary math concepts and skills that preschool children need to develop in order to build a solid foundation for their later success in math in school? How do I ensure that I provide opportunities for each child as an individual to learn at his or her own rate? What kind of an ongoing assessment will be most helpful in planning developmentally appropriate math curriculum? How can I further expand the children's math knowledge and skills by improving my own practice and develop my knowledge in teaching mathematics?

Laura's observation of the play episode that involved Rachel and Tiffany helped her focus her work on the following specific questions:

- What mathematical concepts did Rachel and Tiffany exhibit during their play?
- How can I guide their learning so that their understanding of these concepts progresses to a higher level?
- Are other children in my class at the same stage as Rachel with regard to some of these concepts?

With these questions in mind, Laura began her master's project. Because we had a research interest in early learning of mathematical concepts, we became Laura's supervisors. At that time, our own research was at the stage of developing a series of teacher-friendly assessment tools that would facilitate curriculum planning in the content area of mathematics. This project was an exciting opportunity for Laura to deepen her understanding of young children's learning of mathematics. For us, Laura's project was as an opportunity to implement and document the use of these tools in a preschool classroom and to receive her feedback on their appropriateness and usefulness for an ongoing assessment of young children's development of primary math concepts. As Laura's supervisors, we were able to document, through observations and analysis of her journal entries, how her thinking about young children's learning of mathematical concepts developed, and how her understanding about the need to align curriculum, instruction, and assessment grew. In this article, we will focus on the main areas of growth in Laura's professional development that we believe could be helpful to other preschool teachers' growth.

Learning to Recognize Children's Demonstrated Understanding of Mathematical Concepts

The first and most important stage in Laura's professional growth was her enhanced ability to identify children's demonstrated understanding of mathematical concepts. Her observation of Rachel and Tiffany's reenactment of the story of "Goldilocks and the Three Bears" directed Laura's attention to the "impressive informal mathematical strengths " (Baroody, 2000, p. 61) that young children bring to the classroom. She saw that in this episode Rachel demonstrated her behavioral knowledge, that is, knowing how to enact procedures and roles and to implement several mathematical concepts (Katz & Chard, 2000). For example, choosing only the bears from a larger collection of dolls and plush toys demonstrated her behavioral knowledge of the mathematical concept of classification. Providing a plate for each bear and a bear for each chair demonstrated her knowledge of one-to-one correspondence; ordering the bears in size from biggest to smallest showed her behavioral knowledge of seriation. Tiffany also demonstrated her behavioral knowledge of double seriation by rearranging the spoons to correspond with the size of the bears after Rachel had placed the spoons randomly. More important, however, Tiffany demonstrated her ability to verbalize what needed to be done so that each bear received the appropriate size of spoon. Laura's raised awareness of the mathematical context of the interaction between the two children helped her to recognize the different stages they had reached in the development of their knowledge of seriation. She also became aware that young children express their mathematical knowledge in a variety of contexts that are not necessarily related to "math activities." As a result, she could plan individually appropriate learning experiences for them as well as joint experiences where they could learn from each other. She could also encourage informal mathematical learning by creating a math-rich environment and engaging children in mathematical conversations as they interact with the environment.

Learning to Use Language to Guide Children's Construction of Mathematical Concepts

The next stage of Laura's professional growth was marked by a change in her understanding of the role of teachers in preschool children's learning of mathematical concepts. Traditionally, the emphasis in preschool settings has been on how concepts are acquired, not on what should be taught. Kagan (quoted in Jacobson, 1998, p. 12) pointed out, "We've approached [early education] more from developmental perspectives and not from curricular perspectives. We need both."

The constructivist paradigm based on Piaget's theory of cognitive development has long provided the theoretical framework for educational practice in which children acquired concepts through active involvement with the environment and constructed their own knowledge as they explored their surroundings. Applying this theory to mathematics has led to the use of manipulative materials that enable young children to count, engage in active learning, and develop concepts (Kaplan, Yamamoto, & Ginsberg, 1989). The teacher has been seen to take the role of providing a variety of materials and arranging an environment that is rich in materials and choices. However, in the revised version of the principles of developmentally appropriate practice (Bredekamp & Copple, 1997), the National Association for the Education of Young Children (NAEYC) leaders acknowledged that the emphasis on providing a variety of choices in the classroom and avoiding teaching children specific skills has been misinterpreted. As a result, in preschool settings, manipulative materials were typically used in a nonsystematic way that permitted double randomization: one to do with the appearance of the manipulative material per se and the other determined by variations in the readiness of the children to register them (Feuerstein & Feuerstein, 1991). This randomization may have prevented real conceptual learning from occurring for a number of children who could have otherwise been included in planned activities for learning. Although high-quality learning in the preschool years is often informal, this informality does not imply an unplanned or unsystematic program. Mathematics learning in preschools should be thought provoking, should include opportunities for active learning, and should be rich in mathematical language. More recently, the NCTM's (2000) standards addressed the issue of mathematical content, mathematical process, and the importance of introducing young children to the language and conventions of mathematics.

Thus the role of the teacher in active learning has been seen more recently as being crucial. The teacher is the facilitator who creates a learning environment that is mathematically empowering (NCTM, 1991). The theoretical framework that informed this change was Vygotsky's (1978, 1986) social-constructivist theory of cognitive development. In this theory, learning is more likely to occur if adults or older children mediate young children's learning experiences (Baroody, 2000). Vygotsky believed in a learning continuum characterized by the distance between a child's ability to solve a problem independently and his or her "maximally assisted" problem-solving ability under adult or more-experienced peer guidance. He called this area where real learning occurs the "zone of proximal development" (ZPD). The role of the teacher, therefore, is to provide "scaffold assistance" (Berk & Winsler, 1995), which entails a continual modification of the tasks so as to provide the appropriate level of challenge that enables the child to learn. The adult changes the quality of the support over a teaching session, adjusting the assistance to fit the child's level of performance (Berk & Winsler, 1995). Children learn through meaningful, naturalistic, active learning experiences. The adult must build on this knowledge and take the children to higher levels of understanding.

Having embraced the Vygotskian view of learning, Laura began to realize she must decide what further opportunities—not only materials, but more important, interactions—she needed to provide for Rachel, Tiffany, and the rest of the children in her classroom. Only then could she develop and expand their understanding of mathematics meaningfully. She wrote in her journal:

> I need to make the physical environment in my classroom more math rich. The furniture is child sized and easily adaptable to accommodate cooperative work. There is adequate and comfortable space on the partially carpeted floor to explore, construct, and work with concrete materials. Math materials and manipulatives are stored in clear bins on picture-labeled, open shelves and are in easy reach of the children. It is my intent now to increase the children's mathematical comprehension by assisting their construction of knowledge in one-to-one correspondence, classification, and seriation.

To guide children's learning of the concepts demonstrated during the free play episode, Laura began to see the need to become involved in a variety of situations that create a common language related to mathematics (Franke & Kazemi, 2001). For example, we were able to observe her daily discussions with children that involved comparisons of opposites during choice time. The children and the teacher talked about which blocks were bigger or smaller, which blocks fit into the shelves the best: small, medium, or large. They also made it a daily habit to discuss order: who was the first person in line, who was the second person in line, who was the last person in line or the caboose, the snack person.

Language allows the acquisition of new information as well as the appropriation of complex ideas and processes (Bodrova & Leong, 1996). Open-ended questioning can encourage expanded thinking. "What else?" and "I wonder what would happen if" can draw children's attention to new ways of thinking and interacting. Kamii (1982) explains that it is important to allow children who are constructing their own mathematical knowledge to do so without the teacher reinforcing the "right-ness" or correcting the "wrong-ness" of the child's answer. Disagreement with peers can help the child reexamine the correctness of his or her own thinking. Social interactions through group games are an excellent source of constructing new mathematical ideas and can lead children to make new connections and expand their own reasoning. This interaction helps them to become more independent and less reliant on the teacher as the sole source of answers.

If learning situations were organized and based on the developmental sequence of mathematical concepts, then the curriculum would reflect the children's present stage of understanding and would provide possibilities for further development at each child's pace. According to Katz and Chard (2000), understanding "how knowledge develops, what they [children] can understand, and how they understand their experiences as development proceeds is another basis for curriculum planning" (p. 26). Thus to take both Rachel and Tiffany from behavioral to representational knowledge (i.e., mental or symbolic representations of the concepts abstracted from direct and/or indirect experiences), Laura needed to carefully plan not only the physical layout of her classroom but most importantly her interactions with them so as to help their progress through the stages of the representation of mathematical concepts.

Learning to Assess Children's Understanding of Mathematical Concepts

Like most educators, Laura was looking for ways to improve the alignment of curriculum, instruction, and assessment. While working on her master's project, she began to think at a higher level about the connection between curriculum and assessment. She realized that if the purpose of assessment was to enable teachers to make appropriate decisions to improve students' understanding and learning of mathematical concepts, then her own deep knowledge of these key concepts, facts, principles, and processes was essential for planning appropriate curriculum and classroom experiences. Thus to be able to guide children's learning of mathematical concepts, she needed to be thoroughly grounded in the developmental sequence of the concepts the children learn. Only then could she assess the current level of children's understanding of mathematical concepts and plan experiences in their zone of proximal development.

Laura realized, however, that theoretical knowledge alone was insufficient for effective teaching; she would need appropriate tools to assess such learning. Assessment and documentation of children's work could help her plan developmentally appropriate and, more important, individually appropriate experiences that would promote children's learning. It is well accepted among early childhood professionals that observation is the most appropriate method of assessing preschool children and that play offers a perfect context for observing children and determining their knowledge and understanding (Garvey, 1990; Howes, 1992).

The following sections outline how Laura used the theoretical knowledge about the developmental sequence of mathematical concepts that were demonstrated by Rachel and Tiffany in the play episode to assess and guide all the students' learning of these concepts. These concepts are (1) matching and one-to-one correspondence, (2) sets and classification, and (3) order and seriation. Children's development of these concepts progresses through several stages. We compiled these stages in a checklist, and Laura used this checklist in her work.

Concept #1: Matching and One-to-One Correspondence

As discussed above, Rachel's placing of one plate for each bear demonstrated her understanding of the concept of matching and one-to-one correspondence. Typically, children between 2 and 4 years of age develop this understanding through the relationships of "more-less-the same" (Brush, 1972; Gelman & Gallistel, 1978). Matching is a prerequisite for conservation; it is one of the earliest mathematical concepts to develop and forms the foundation for the development of logical thinking. One-to-one correspondence is the fundamental component of the concept of number. It is the understanding that one group has the same number of things as another. It is preliminary to counting and basic to the understanding of equivalence and the concept of conservation of number (Charlesworth & Lind, 1999; Montague-Smith, 1997). Once children understand basic one-to-one correspondence, they can apply this concept to higher-level activities that involve equivalence and the idea of "more or less" (see Appendix I).

Using the checklist, Laura was able to identify the stage of Rachel's understanding of the concept of one-to-one correspondence as being at "matching even sets of items that are related or go together but are not alike." To support and guide Rachel's learning to the next level of the same concept, Laura provided opportunities for her to match uneven sets of five or more items. She used every opportunity to join Rachel in the housekeeping play area. Using everyday objects (both in even and odd quantities) with which Rachel was familiar, like cups and saucers, spoons and forks, shovels and pails, or sets of plastic animals, Laura was able to identify Rachel's ability to match items that are alike or not alike. When Rachel's use of these materials did not indicate a clear pattern, Laura asked specific questions. For example, Laura brought in some plastic animals to add to the teddy bears and used small containers. At an opportune time in play, she asked Rachel to find one animal for each container. After repeated interactions of this nature, Laura observed Rachel playing at the water table, placing one frog on each plastic leaf in the water. Laura also noted that at the snack table Rachel carefully placed a cup next to a paper napkin for each child.

To take Rachel from behavioral to representational knowledge, Laura was careful to use language related to the concept of matching and one-to-one correspondence. Rich social interactions with teachers and more-competent peers can contribute to children's opportunities for learning and developing behavioral knowledge into representational knowledge. Children's ability to use words such as not enough and too many would show the highest level of their understanding of matching and one-to-one correspondence. The use of children's literature also facilitated the development of language related to mathematical concepts.

Because one-to-one correspondence means that one group has the same number of things as another, Laura's goal was to help not only Rachel but all the children in her classroom to see the relationship in any set of materials. As a result, Laura converted clean-up time to an important "math time" by introducing a matching game. She asked the children to place one object in a container or on a shelf. In doing this activity, they were to match object to object, object to picture, and picture to picture (see Appendix I). She also introduced various commercial games and teacher-made matching activities available to the children at choice time. The teacher-made activities included baskets of small objects, divided trays, tongs (optional, depending on the individual child's fine motor skills), and a one-through-three or one-through-six die. These activities introduced the concept of matching: one object goes into one section of the tray. One of the activities that Laura's children really enjoyed was taking marbles from a basket with a melon scoop and putting one marble in each compartment of an ice cube tray. Laura wrote in her journal, "This activity is so popular that I have to take names for a waiting list for those children who want to do the marble game over and over."

As the children became more proficient in their one-to-one-correspondence skills, Laura introduced grid and short path games. Grid games are bingo-type cards (without letters or numbers) used in combination with dice or spinners and counters (Moomaw & Hieronymus, 1995). Teddy Bear Bingo and Candy Land Bingo are examples of commercial grid games. These games allowed Laura to observe the different levels at which the children were with regard to the level of development of matching and one-to-one correspondence. For some children, counting the pips on the dice was a challenge; they either double-counted or skipped pips. Rachel, for example, counted six as " one, two, three; one two three." For others, counting did not present a problem. They were even able to use mathematical language to not only explain what they were doing but also to predict what they needed to win the game. Megan said, "I got six, now I only have three more to go," and Tiffany said, "One and two is three, now I need four more." Having observed Tiffany, Laura asked her if she would like to play the grid game with Rachel. Tiffany, who enjoyed the game tremendously and was looking for all available opportunities to play it, readily agreed. During the interaction between the two children, Tiffany said to Rachel, "This is not how you count these! Look. You go like this (pointing to each pip with a pencil and saying one, two, three, four, five)." After several repetitions, Rachel was able to count on her own to six.

In path games, children roll a die or dice to advance a mover on a path of distinctly separate spaces. Moomaw and Hieronymus (1995) assert that "path games incorporate the thinking strategies needed for grid games at a more difficult level and place additional emphasis on social interactions with teachers and peers" (p. 117). The first short path game covered the path with bingo chips to help the squirrel find some nuts. The one-through-six dice were used (Figure 1). All of the children were able to understand the concept of the short path game with a start and finish.

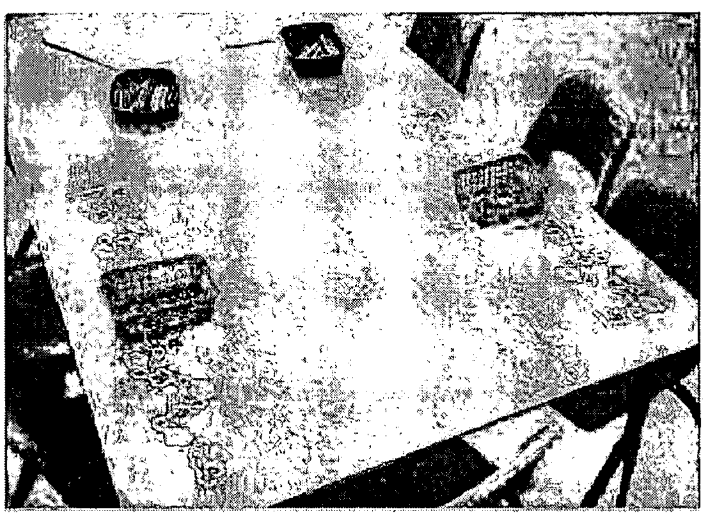

Figure 1. The math table is set up for the short path squirrel game.

The next short path activity was more complex. The snake game used unifix cubes as counters and used the one-through-six spinner. The snake game was more difficult for the children who had not yet mastered the ability to match uneven sets with five or more items. Rachel, for example, had difficulty matching the unifix cube with its corresponding square. The squares followed an "s" shape, and the shape confused her. She skipped squares and lost count when she was adding cubes. She could not finish the game. Tiffany, on the other hand, was already able to predict, "I have three, and now I just need one more!" She also counted squares to see how many she had left before she was finished. She played the game several times with great enthusiasm. Knowing that Tiffany was successful in helping Rachel learn how to count the pips on the dice to six, Laura once again ask her to play with Rachel. This time Tiffany used a different strategy to show Rachel what she needed to do. She said, "Rachel, you just put your finger on the next square and then move the cube." Although Rachel learned quickly how to follow the curved path, recognizing the numbers on the spinner remained a problem. Tiffany decided that she would have to tell her how many squares she needs to move her cube. Rachel was happy to have Tiffany help her.

Concept #2: Early Classification: Creating Sets

In her reenactment of the story of Goldilocks, Rachel demonstrated her understanding of classification when she saw the sameness of the bears regardless of their size. According to Sugarman (1983), "Classification exists when two or more discrete events are treated as equivalent" (p. 4). This classification leads to the recognition that one group of objects is part of a larger group. However, some people may treat some objects or groups of objects as equivalent for different reasons.

Using the checklist, Laura determined that Rachel had behavioral knowledge of classification by association and that she demonstrated some knowledge of class inclusion. Thus to guide Rachel's learning of this concept, Laura needed to engage Rachel in an activity that would help her understand the concept of class: inclusion. Snack time presented such an opportunity. While making a fruit salad, Laura asked Rachel, "We have apples and bananas in this fruit salad; could we add any other fruit?" Clean-up time also provided Laura with an opportunity to ask Rachel to put all the animals in one box. A few days later, the children were pretending to go on a picnic, and Laura overheard Rachel tell the children, "We need to put all the food in the picnic basket." As one of the other children put the food in the basket, Rachel picked up a variety of toys and placed them in another box to take to the picnic. During the "picnic," Laura "accidentally" placed a ball in the picnic basket, and she was reprimanded by Rachel, who said, "That does not go in the picnic basket."

Laura realized that at each of the levels of the development of the concept, it was important that she talk to Rachel and ask her to describe and then explain what she had done. Vygotsky believed that children become capable of thinking as they talk (Bodrova & Leong, 1996). When a child demonstrated behavioral understanding of a concept and described what she or he had represented, Laura made sure that she talked to the child to determine that she was also able to explain her actions. This discussion ensured that the child had truly understood the concept and was not merely repeating words with no real understanding. The use of language in shared activity allows the child to construct meaning and also to demonstrate a higher level of understanding of the concept.

Most very young children have the ability to classify objects. However, young children do not necessarily know the names of colors, shapes, materials, and so forth. This lack of vocabulary may be mistaken for lack of knowledge or ability to classify by one attribute. So the teacher should ask young children to classify not using a specific color or shape but rather using general questions such as "Can you find something that is the same color (or shape or size or material, etc.) as this one?" By the time children demonstrate that they can classify by two or more attributes, they have already acquired the vocabulary to describe the specific characteristics of the object. So it is then appropriate for the teacher to ask the children, "Can you find something that is red and long?"

To help Rachel develop the ability to classify by function or association, during clean-up time Laura asked her, "Can you put the things that you draw with together in this box, please?" or "Can you find in the play center all the things a doctor uses and put them in one place, please?" During dramatic play, Laura asked the children to gather everything necessary to set up a grocery store so that Goldilocks could buy more groceries to make porridge for the bears. Although it is not typical for preschool children to have a clear understanding of class inclusion and exclusion, when asked specific questions, some may demonstrate partial understanding of the concept. They are particularly likely to understand when class inclusion is related to personal experiences such as visiting a doctor's office, going to the grocery store, or gardening with a parent (see Appendix II).

Graphing is a more complex way of classification. Simple group bar graphs are developmentally appropriate for preschool and enable the children to work together and learn from each other. Bar graphs that distinctly display information give the children practice in creating and comparing sets:

> A good graph arises out of the children's natural desire to share information with their peers, quantify the results, and compare the outcomes. Graphs can be especially motivating to cognitively advanced children since they provoke a high level of thinking. (Moomaw & Hieronymus, 1995, p. 170)

As Halloween approached, Laura engaged the children in graphing based on predictions. She introduced pumpkins with a graph titled "How Do Pumpkins Grow?" (Figure 2). Pumpkins growing various ways illustrated the choices: on a pumpkin tree, on a pumpkin bush, on a vine, or under the ground. The children's names were on cardboard rectangles and available for them to choose. Laura called the children over individually and presented each choice again and asked them to put their name by how they thought pumpkins grew.

This activity showed again that young children think differently or do not have knowledge assumed by adults. The majority of the children chose correctly that pumpkins grew on vines. Sid, however, stated, "Pumpkins grow underground like potatoes." Jamie also chose underground but could not explain her choice. When questioned, she said, "Because they [pumpkins] do." After the children and the teacher completed their discussion, Laura showed the class some pictures of a pumpkin patch and pumpkins on a vine. She asked if anyone could see how the pumpkins were growing. All the children agreed that pumpkins did indeed grow on vines.

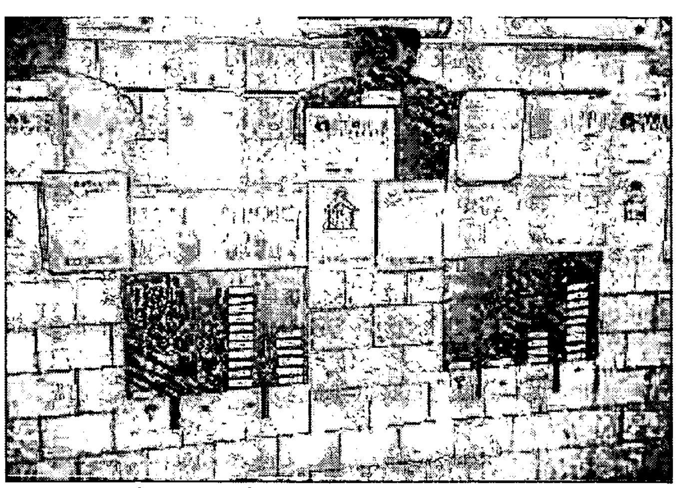

Figure 2. Graphing display of "How Do Pumpkins Grow?"

Concept #3: Order and Seriation

In the play episode described above, Rachel also demonstrated her behavioral understanding of seriation by systematically placing the bears from largest to smallest. Ordering is a higher level of comparing (seeing differences) and involves comparing more than two objects or more than two sets. Ordering or seriation involves putting more than two objects or sets with more than two members into a sequence. Ordering also involves placing objects in a sequence from first to last, and it is a prerequisite to patterning. Ordering is the foundation of our number system (e.g., 2 is bigger than 1, 3 is bigger than 2, etc.).

Laura saw from the checklist that the next stage in the developmental sequence of that concept is double seriation. During the play episode, Rachel did not understand this concept, as she demonstrated when she placed the spoons randomly and not according to the size of the bears. In fact when the older child, Tiffany, reminded her that the biggest bear needed the biggest spoon, Rachel ignored her, and when Tiffany continued, Rachel walked away. Stories like "Goldilocks and the Three Bears" are frequently used to illustrate the concept of double seriation. Yet because Rachel did not grasp the concept from the first reading, Laura decided to provide cups and spoons, animals, and bowls of various sizes that could be used for double seriation. Later in the school year, Laura noticed Rachel explaining the concept of double seriation to Emily in the same way that Tiffany had attempted to explain the concept to Rachel. Laura heard Emily finally exclaim, "I get it—the big bowl goes with the big dog!" Competent peers can model concepts and guide learning for the less-competent child during shared activities. Shared activity forces the participants to clarify and elaborate their thinking (Bodrova & Leong, 1996).

Laura also engaged all the children in learning experiences that could help them gain both behavioral and representational knowledge of the concept of order and seriation. These included asking children to line up by height before going out to play, putting characters in their paintings according to their size, seriating sounds from loudest to softest, and coloring objects according to their hue from lightest to darkest or vice versa. Sequencing of events while on a field trip was another learning experience Laura provided for her students that was related to understanding seriation. In addition, Laura was conscientious about using mathematical language when the children were playing with blocks, nesting cups, and so forth. Some specific questions she asked were, "Can you find a block that is smaller than this?" or "Can you find something that is bigger than this cup?" While playing with toy vehicles, she asked the child to put the cars in order from biggest to smallest or smallest to biggest. Laura also brought to the classroom her own collection of 17 pinecones—from giant Sequoia cones from California to very tiny pinecones from sapling evergreen trees. The children were excited to learn where she collected them and how the different types of pine trees have different-sized pinecones. They enjoyed putting them in order from the smallest to the biggest and vice versa. Although most children used trial and error to put them in order, almost all of them were able to seriate at least 9 of the cones from biggest to smallest. One child was even able to seriate all 17 of them. Seriating in reverse order was more challenging and needed a lot of verbal cueing on the part of the teacher. The inclusion of vocabulary like first, second, third, and so forth helped the children to develop representational knowledge of seriation (see Appendix III).

The Use of the Checklists

When teachers continually monitor and evaluate children's understanding, they can build on the children's knowledge in contexts that are meaningful to the children. The checklists provided a means for charting children's understanding of some mathematical concepts in Laura's preschool classroom. Laura used these checklists not to evaluate or determine mastery but to gather information that could be used for curriculum development. She used these checklists to identify the specific stages of development of the concepts in each child and then to plan appropriate materials and learning experiences to scaffold children's learning in the zone of proximal development of that concept. Laura was careful to note that in addition to demonstrating behavioral understanding the children were also able to describe and explain their actions. Children's explanations of their actions helped Laura determine that there was true understanding of the concept and that they were not merely repeating words without real understanding. The ongoing assessment allowed her to monitor individual children's progress and thus focus on guiding children's learning of these concepts. The checklists helped Laura make decisions about providing developmentally appropriate activities for the children she worked with. She wrote in her journal:

> The checklist helped me arrange my lessons in a logical manner from simple to more complex. I learned to be a careful observer and listener to the children not only at the math table but also during free choice and playtime. I was able to adjust to children's individual needs in various pre-math activities. I aligned curriculum and assessment to give me a more solid grasp of the stages of development of the mathematical concepts of matching and one-to-one correspondence, classification, and seriation.

Systematic yet flexible use of checklists in any preschool classroom can facilitate teachers' decision making about how to set up the classroom, what questions to ask, and what resources to provide for the development of each child (Helm, Beneke, & Steinheimer, 1997). Like Laura, other teachers can use these checklists while observing small groups of children working together or individual children participating in an activity. The checklists can also be used for individual interviews to assess children who do not demonstrate understanding while working independently or in groups. In addition, the checklists can be used for performance assessments to determine how children carry out specific tasks that replicate real-life experiences (Billman & Sherman, 1996).

Teachers can use the checklists as frequently as they consider necessary to chart children's development and understanding of concepts. To determine the level of understanding at the beginning of the year, the checklist can be used in the first few weeks of the program. It would be helpful to carry out this assessment for all children during free-choice activities. The role of the teacher could then be to provide a variety of materials that enable the children to demonstrate spontaneously and naturally their behavioral knowledge of mathematical concepts. This initial information could then be used in deciding what experiences could be helpful for individual children and for small groups of children who need similar experiences. After providing opportunities for children to demonstrate their behavioral knowledge through active involvement with materials, teachers need to interact with the children. When teachers use the language of mathematics in such interactions, children are helped to progress from one level of behavioral knowledge to the next, or from behavioral to representational understanding of the concept. Laura noticed that the children's overall increased awareness of math led to many more spontaneous uses of math skills in the classroom. She recorded in her journal:

> Plastic animals were classified and seriated. Colored blocks were used to make intricate geometric patterns. The building blocks were used in increasingly more complex ways. Building with blocks at the beginning of the school year was single-leveled and linear. As the project progressed and the children became more proficient, building with blocks became multileveled and more abstract. The calendar numbers were counted many times during the day with more-able children helping their less-able friends identify the names of number symbols. This increase of mathematical awareness carried over to some children's homes. Several parents told me that their children had become very interested in math outside of school. Megan's mother, for example, told me that she was patterning "everything": the family's shoes, cans in the cupboard, cereal, candy, and even her little brother's toys.

Periodic and systematic use of the checklists is necessary for monitoring the development of the concepts in each child. Dating observations while using the checklists provides a record of each child's growth and development and helps identify children with similar levels of understanding at any given time. This process informs the teacher's decisions about the need to guide the learning process for each child. "Quality assessments inform instructional decisions and allow teachers to monitor individual children's progress while focusing on how children are thinking about mathematics" (NTCM, 2000, p. 6). When teachers know what mathematical concepts they wish children to understand and the stages through which they develop, they can plan meaningful learning experiences and assess children's progress (Richardson & Salkeld, 1995). As teachers plan for children's development, they must also take into consideration children's interests and stages of development. It is critical to allow children time for free play that enables them to explore mathematical concepts. While children are engaged in an activity, the teacher can observe and then become active in guiding their learning. This interaction will help the children's progress from behavioral to representational understanding of mathematical concepts. Thus the flexible yet systematic use of the checklists provided here can facilitate preschool teachers developing children's mathematical knowledge. They also provide a means for the teachers to systematically examine their own practice and make informed decisions about meeting individual children's mathematical learning needs. The following journal entry clearly communicates Laura's own sense of professional growth:

> During this project, I developed skills as a researcher. I systematically studied my own practice and made many adjustments to accommodate my newfound mathematical abilities. I became adept at planning lessons and producing developmentally appropriate math activities for children. As I became more knowledgeable and I gained some confidence, I began to develop my professional voice. Most of the children, their parents, and the administration of my school very enthusiastically received the entire project. The children's excitement about math was continuous.

Acknowledgment

All quotes from the teacher's journal are included with her permission.

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Appendix I

Checklist for Preschool Pre-mathematical Concepts
Matching and One-to-One Correspondence

Name of the Child ____________________

Concepts/Stages of Development	Sept.-Oct.	Dec.-Jan.	April-May
Matching related items that are not alike			
1. Matches different but related items that are not alike			
2. Matches even sets—with 5 or fewer items			
3. Matches uneven sets with 5 or more items			
4. Uses appropriate vocabulary while matching sets (e.g., too many, not enough)			
Matching similar items			
5. Matches 2 similar items			
6. Matches even sets—with 5 or fewer items			
7. Matches uneven sets—with 5 or more items			
8. Uses appropriate vocabulary while matching sets that are alike (e.g., too many, not enough)			

KEY TO CHECKLISTS	
✔	Demonstrates behavioral knowledge of the concept
✔✔	Demonstrates behavioral and representational knowledge of the concept
0	Demonstrates partial behavioral knowledge of the concept
00	Demonstrates partial representational knowedge of the concept
X	Does not demonstrate any kind of knowledge of the concept

Appendix II

Checklist for Preschool Pre-mathematical Concepts Sets of Classification

Name of the Child ___________________

Concepts/Stages of Development	Sept.-Oct.	Dec.-Jan.	April-May
1. Able to group identical objects			
2. Sorts objects by 1 attribute—color, shape, size, material, pattern, texture			
3. Classifies by 2 attributes			
4. Classifies by 3 attributes			
5. Describes what has been done while classifying by 1, 2, or 3 attributes			
6. Explains what has been done while classifying by 1, 2, or 3 attributes			
7. Classifies according to function			
8. Describes and/or explains what has been done			
9. Classifies according to association			
10. Describes and/or explains what has been done			
11. Understands class exclusion			
12. Understands class inclusion			
13. Describes and/or explains what has been done			
14. Classifies by number			

KEY TO CHECKLISTS	
✔	Demonstrates behavioral knowledge of the concept
✔✔	Demonstrates behavioral and representational knowledge of the concept
0	Demonstrates partial behavioral knowledge of the concept
00	Demonstrates partial representational knowedge of the concept
X	Does not demonstrate any kind of knowledge of the concept

Appendix III

Checklist for Preschool Pre-mathematical Concepts

Order and Seriation

Name of the Child ___________________

Concepts/Stages of Development	Sept.-Oct.	Dec.-Jan.	April-May
1. Comparison of opposites (e.g., long/short, big/small, etc.)			
2. Orders 3 objects in random order			
3. Orders 3 objects by trial and error			
4. Orders 3 objects in a systematic manner			
5. Seriates in reverse order			
6. Performs double seriation			
7. Describes what has been done			
8. Explains what has been done			

KEY TO CHECKLISTS	
✔	Demonstrates behavioral knowledge of the concept
✔✔	Demonstrates behavioral and representational knowledge of the concept
0	Demonstrates partial behavioral knowledge of the concept
00	Demonstrates partial representational knowedge of the concept
X	Does not demonstrate any kind of knowledge of the concept

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