Little research has assessed the role of student-initiated metacognition in the learning of mathematics. In this study, secondary school students (junior high school and high school) were asked to consider their own thinking in terms of how they learn and when they know that they know (the "aha" experience). Students were able to define and articulate some of their metacognitive processes and to illustrate three paths to knowledge. Knowledge was student-defined as the ability to use newly learned information in novel settings and to describe problem-solving strategies even if the student is unable to solve fully the problem at hand. Students suggested that memorization and conceptualization work symbiotically and the "aha" experience occurs when the learned material becomes a useable and internally explainable concept. (Contains 1 figure, 2 tables, and 13 references.) (SLD)
That "Aha" Experience: 
Meta-cognition and Student Understanding of Learning and Knowledge

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ABSTRACT

Little research has assessed the role of student-initiated meta-cognition in the learning of mathematics. This study asked secondary school students to consider their own thinking in terms of how they learn and when they know that they know (the "aha" experience). Students were able to define and articulate some of their meta-cognitive processes and illustrated three "paths to knowledge." Knowledge was student-defined as the ability to use newly learned information in novel settings and to describe problem solving strategies even if the student is unable to fully solve the problem at hand. Students suggested that memorization and conceptualization work symbiotically and the "aha" experience occurs when the learned material becomes a useable and internally explainable concept.
INTRODUCTION

During the last several years, there has been much discussion regarding students' abilities to effectively use higher order thinking skills. Improvement of student ability to think and problem solve (and thus "to know") has been the thrust of a series of educational reforms and has further influenced the development of new curricular standards, curricular guidelines, and teaching methods. Higher level thinking skills go hand-in-hand with mastery of basic skills and a symbiotic relationship develops between them (Oster, Graudgenett, McGlamery, and Topp 1999, Caine 2000).

Much current research on student understanding and cognition focuses on how to help students explain their problem-specific thinking: what they did, how they did it, and why they did it that way. Mathematics education has moved to a more constructivist approach (NCTM 2000, Working Groups of the Commission on Standards for School Mathematics 1989) which offers students greater opportunity to delve more deeply into important mathematical concepts and ideas while they learn to communicate their understanding in more descriptive, natural language rather than in traditional mathematical formats (Washington Commission on Student Learning n.d., NCTM 2000). Thus, mathematical success, the ability to describe, explain, and use these mathematical concepts, is largely dependent on student recognition and use of their own higher order thinking processes (Pogrow 1999, Sheppard and Kanevsky 1999).
Recent research on student understanding and cognition focuses on how best to support students in their academic learning activities (Pogrow 1999, Sheppard and Kanevsky 1999). Given a problem-specific situation, students are taught to explain what they did, how they did it, and why. However this ability to explain is by no means universal and both successful students and experts in a variety of fields are often unable to describe their reasoning process (Sheppard and Kanevsky 1999, McCaslin 1996, Iran-Nejad 1995).

One aspect of "knowing" is what occurs when one observes the "common student reaction of 'Aha'" (Ellsworth and Sindt 1994, ¶29). In this case, metacognition, the ability to describe what led to that "Aha" experience, seems to be the link between knowing and explaining. The ability to solve a problem, which may be a mechanistic process even when addressing high-level solution strategies, is not the same as knowing how one thinks about solving problems, the alternatives explored, and the insights relied on (Sheppard and Kanevsky 1999, Ellsworth and Sindt 1994). Thus, knowledge relies both on the ability to complete a task and the ability to explain one's thinking about how the task was considered and completed (Ellsworth and Sindt 1994, Schoenfield 1991).

There is evidence that "knowing" is a variety of separate entities (Mason and Spence 1999): "Knowing-why" suggests the background to an idea or question, "knowing-how" suggests a mechanical process or a way of completing task, "knowing-that" suggests a reasoning process or an association between ideas. These types of knowing suggest the ability to solve problems and thus
are testable and concrete. "Knowing-to" is the ability to shift one's perspective to allow for new insight that leads to the exploitation, development, and exploration of new ideas using the more prosaic types of knowing. We may say that someone "knows-to" when one can recognize what needs to be done to solve a problem or what direction should be explored to reach a conclusion, although one may not have the skills to complete the work (Mason and Spence 1999).

If a student can ascertain what drives the shift in perspective in his or her own mind in any specific context, then "knowing-to" and its associated depth of insight and understanding may be more easily carried into a different context. Ultimately, it is this "knowing-to" that is most valuable in terms of thinking tools and it is this "knowing-to" that we are trying to encourage students to develop. Yet "knowing-to" and the meta-cognitive processes leading to it are the least understood of the types of knowing and thus are the least teachable and testable.

"Knowing-to" can only be taught and/or supported if we understand how students envision their own thinking and how the cognitive processes of students are similar to and different from the processes of adults. Most researchers have relied on adult interpretations of student explanations of their reasoning process (Pogrow 1999, Maher and Martino 1996) because it is assumed that students are not developmentally prepared to effectively articulate their own understandings.
However, the goal of meta-cognitive thought is to be aware of how one thinks in a way that can be understood by oneself and explained to others (Sheppard and Kanevsky 1999, Mason and Spence 1999) without being interpreted by a third party.

This paper explores reflections about thinking (meta-cognition) among students studying mathematics in middle school and high school to determine if they have the communication skills to begin to define "how they know when they know." If students are self-reflective and communicative, then we can ask what brings them to that state of knowing. In terms of mathematics, we can begin to explore the roles of conceptualization and basic skills in reaching the "Aha" experience and begin to get a sense of what students find most valuable as they move from learning to knowledge.

METHODS

The primary questions asked in this research were "how do you know when you know?" and "what factors lead you to being confident in your knowledge?" Of greatest interest was what students themselves could reveal about their own insights in knowledge and understanding. With that in mind, several focus groups were held in the early part of 2000 to elicit student understanding of how they learn. All twenty-nine students in grades 7 through 12 attending an American-curriculum international school in West Africa were invited to participate in a series of focus groups organized to discuss how students understand math.
Students were not obligated to join the groups and the purpose of the group discussions was fully explained to them at the time of invitation. Students questioned the researcher closely about a variety of concerns prior to making their decisions whether or not to join the groups; ultimately twenty-seven students participated in the discussions. Two students, a male in Grade 8 and a female in Grade 11, did not participate due to absences on the day the groups met.

Four groups were convened based on grade level (Grade 7, Grade 8, and two groups for students in Grades 9 through 12) and sessions lasted about an hour. Students of similar ages and maturity levels met together to ensure cohesion in recent mathematics studied and in general understanding of the issues involved. Another concern was that if mixed age/grade groups met together, younger students, less sure of their own thinking and understanding, would be less able to contribute to the discussions (Sheppard and Kanevsky 1999). There is evidence that younger students, while experiencing similar strengths and weaknesses in terms of their mathematical understanding as older students, may not have the ready-access to the vocabulary to quickly explain their thinking. Thus, separating the groups based on age and grade level allowed for all students to contribute to the best of their verbal ability without the additional pressure of older, more verbal, students "stealing their thunder." Two students who questioned their own command of spoken English chose to answer questions as an aside to the group moderator or to other students; their
answers were repeated to the group as a whole, who then commented on their responses and asked them to elaborate on a variety of points.

Based on student participation in these focus groups, response pattern analysis, discussion trends, and group dynamics are reported. Also, student interpretations of knowledge and learning are described. Table 1 outlines the major questions asked during the focus group sessions.

RESULTS

GROUP CHARACTERISTICS: As Table 2 suggests, the focus group population favored males (n=19, 70%). However, due to the small size of the total student body, gender was not an issue of concern, as all the students socialized and felt comfortable together. The majority of participating students (n=17, 63%) had attended this school for at least 2.5 years and most (n=16, 55%) had attended similar English-language, American- or Canadian-curriculum schools in other countries, including the United States and Canada. Native English speakers (n=13, 48%) hailed from the United States (n=3), Canada (n=6), Nigeria (n=3), and Cameroun (n=1) (where French and English are both official languages) and half the remaining students were fluent in social and academic English (n=7, 24%), having completed all their education in English language schools. Six other students were fluent in social English and were acclimating to academic language expectations. Only one student, the oldest child in Grade 7, had never been introduced to academic English prior to this academic year; he was repeating the year having completed this same academic program in a French-
language school the year before. His knowledge of mathematical concepts and ideas was strong and his ability to understand spoken English had improved markedly over the five months he had been studying in English. He tried to answer questions in English but did, at times, resort to answering in French or asking a friend to give his English answer a better grammatical representation. One other student, a Grade 10 student in Group 3B, was shy about her English ability (her social and academic English was excellent; she had attended an English language school for three years, but had spent the last two years at a German gymnasium). She chose to whisper her responses to the group moderator who then relayed them to the full group for further discussion.

**LEARNING AND KNOWLEDGE:** Students in all four groups agreed that learning was both a process (acquiring information in an organized manner) and a tool (the ability to use information to "find out things I didn't know"). They differentiated between learning, which is what is done in school, and knowledge, which they defined as unteachable, occurring almost spontaneously. They recognized that new learning could not progress until and unless previous learning became knowledge because learning was not useful until it was internalized and could be used almost without thinking. The internalized information was what was described as knowledge. Memorization, whether recommended by the teacher or not, became the primary method for these students to bring learning into the realm of knowledge. They recognized that memorization in and of itself was *not* knowledge but memorization seemed to be
an important step in the process of bringing learning into the realm of knowledge. Information could be memorized yet not become knowledge. However, if information was not memorized, it could never become knowledge.

Memorization was identified as a tool that was both resented and disliked. All groups made it clear that the act of memorizing was key to knowledge and the ability to use newly-learned information in novel situations was unobtainable without memorizing basic rules, facts, and information.

"I don't memorize things because they will help me later on. I realize later that what I memorized helped me."

(Female, Grade 7)

"Learning is to crack a code. Memorization doesn't help you understand but once you understand, memorization makes it go faster."

(Female, Grade 10)

Knowledge has "slower input" than memory (Male, Grade 8) but if what was memorized was "important enough," it could turn into knowledge. Thus, once students were able to use new information and integrate it with what had been previously memorized, they began to use the learned material independent of teacher direction or guidance. They "knew" what had been learned. While Grade 7 students in general had more trouble articulating their ideas about
knowledge, a female student characterized the integration by suggesting that "knowledge is having memorized something but being able to use it so well that you don't need to use what you memorized anymore." Another young woman who had been struggling with math for several years, was particularly insightful when she suggested that she "[did] not like learning rules by heart but it help[ed] [her] understand the hard stuff" (Female, Grade 7).

Although much class time was spent developing context for new mathematical ideas, the theory and conceptual background of mathematical ideas were deemed irrelevant at the time taught. Students reported memorizing what was necessary to go where the teacher was guiding them next, suggesting that proficiency in advanced work was based on what had been memorized previously. However, as they became more skilled at following the rules and using the routines, the students also described an "aha" experience. The theory and concepts that had been ignored became clear and once the routines were integrated into their new understanding of the concepts, students unanimously suggested that the memorized information then took a back-seat role.

"First we memorize, which allows us to process information that becomes "what we learn" and then we apply this learning and it becomes knowledge"

(Three Males, Grade 8, working together)
Students in all grades recognized their hierarchy in the struggle to develop a deeper understanding of mathematical ideas. They believed that "knowing," especially in the mathematical sense of being able to solve unique problems independent of teacher guidance, required that they learn facts and procedures that seemed initially irrelevant. They also believed that in order to ensure that the goal of "knowledge" would be met, certain information had to be memorized. They memorized information blindly only to understand later how the memorized information fit into the overall picture and how the memorized information fit into the theory and concepts being taught.

There came a point of "epiphany" which students described as "really knowing" because the new information fit into "what I already know I know" (Female, Grade 11). This epiphany was characterized as being a more rewarding experience than the process of learning itself. To make the leap from learning to knowledge, students perceived that in order to use any new process in the solution of a problem (e.g., solving a problem involving the height of a bouncing ball at a given time, which involves solving a quadratic equation), information (e.g., the various ways to factor an equation) has to be quickly accessible to be effectively utilized. While, in this example, one could argue that students who understood the concept of factoring could rely on notes or a text to guide them in the actual factoring, students suggested that this was not efficient and reliance on written text impeded their ability to independently problem-solve.
However, once students "saw the reasons, the logic, the memorization is not important" (Male, Grade 11).

One group, 3A, developed a diagram to explain what they understood as their path to knowledge (Figure 1). Students began by characterizing the process of learning as "unpleasant" and something they tended to "dislike." They suggested that memorization was a necessary evil, repetition was disliked, working problems and hearing explanations was not an enjoyable experience, and for the most part, they did not participate in the learning process willingly. However, they accepted that some students did begin by "seeing the point;" they understood that there was some underlying reason that they were being asked to learn something and were willing to do the work ("struggle" with the task of learning). Some students saw no point in the subject at hand but were willing to accept that there might indeed be a point and were willing to "play the game" in order to satisfy teachers and/or parents. Both groups identified an epiphany: "you know you know it when you compare it to the world around you and it corresponds to life" (Female, Grade 11). The consensus was that regardless of whether the teacher gave specific information and the students developed the general rules (inductive learning) or the teacher suggested the general rules and the students defined the specifics of a mathematical situation (deductive learning), the epiphany came when they put the new learning inside of what they already know and found a way to apply it to their lives broadly, both in their actual day-to-day lives and their in-school academic careers.
Several struggling students described a negative pattern that was integrated into this model: they were aware that they did not have the requisite knowledge on which to stack new learning and they volunteered that they had chosen not to learn the information that would help them develop the skills that could lead to knowledge. They were quick to point out that this lack of knowledge was not due to teacher inadequacy but in their own disinterest and their disbelief that "any of this" was really important. They recognized their own negative cycle: they disliked the subject matter, they were unwilling to accept that there could be any point, they made minimal (if any) effort to improve their skills and/or understanding and thus were frustrated with their inability to follow the steps and routines, so their dislike of the subject matter increased. Ultimately, they found themselves unable to proceed and faced continuous, unrelieved frustration in mathematics. They characterized themselves as unwilling at any time to try the one step that they perceived would help them, memorization of procedures and basic facts.

As mathematically successful students begin to integrate the information they earned in previous years with new ideas and concepts, knowledge begins to grow and when questioned, students could almost visualize the connections. At times, the question "what happened?" or "how did you know to do that?" helped the student realize that something major had occurred and that this new knowledge was now a part of them and would be key to future learning and understanding.
CONCLUSION

Learning is a necessary precursor of knowledge but students dispute that learning always leads to knowledge. According to the students, the relationship between learning and knowledge is tenuous, with learning being a public, teachable (and therefore testable) construct. Knowledge is the end result of an internal series of events that integrates disparate ideas, facts, and information, merges them with new ideas, and creates novel connections that were not previously apparent. Students perceived that the goal of education was to teach the process of learning, while little attention was placed on the development of knowledge. Although memorization was not a particular focus of these interviews, students indicated that reliance on memorized routines was important in their own evolution to knowledge.

Student recognition of memorization as a necessary part of learning, if learning is to lead to knowledge, suggests that a teaching method that focuses on conceptual understanding may be less conducive to encouraging knowledge development simply because the ability to use the concepts effectively and to learn from the concepts requires easy access to prior and current information, facts, and formulas. Similarly, a teaching method that limits concept development and stresses routine problem solving and memorization of routines may not give students the opportunities needed to integrate conceptual ideas into their routine skills.
Students in Grade 7 had difficulty articulating their thoughts but they explained their ideas in simple, concrete language that did describe differences in how they learned and how they understood. Students in Grade 8 and in High School were progressively more actively involved in the discussions, often talking over one another and excited about sharing ideas and thoughts. While the concepts of learning and knowledge were things they had clearly thought about before, the students had not juxtaposed them in this way and the possibilities, connections, and ideas seemed to jump at them at a faster and faster rate as each hour-long discussion progressed.

While the older students had a stronger vocabulary with which to describe their experiences and understandings, the descriptions offered by the younger students mirrored that of the older students but in simpler language. In other words, the process leading from learning to knowledge appears to follow a similar path among Middle and High School students, although the verbal connections and ability to explain the cognitive processes may be limited among younger students. Pogrow (1999) has suggested that helping students understand their own meta-cognitive processes will help their academic performance in all subject areas. These focus groups suggest that modeling the meta-cognitive processes of secondary school students using the language of age-appropriate peers and/or high school students would offer a number of possibilities to assist younger students strengthen their own understanding of their own processes. A limitation to current meta-cognitive based teaching
practice may be its reliance on problem-specific explanation: students may be better able to identify generalized thinking patterns and integrate conceptual realities if the focus is shifted to meta-cognition as a more general tool that leads to different representations of "knowing-to" in disparate situations.

Students suggest that routine is an important part of concept understanding and the ability to use and integrate ideas, and to see innovative solutions to problems, requires broad knowledge and routinization of skills. While we, as teachers, are able to encourage students to master routines and develop processes for conceptualizing and understanding, this work suggests that helping students develop "knowledge" is more elusive.
Table 1

Questions Asked During the Focus Group Sessions

What do you like about mathematics?
Is this different from what you find "easy" in mathematics?

What do you not like about mathematics?
Is this different from what you find "difficult" in mathematics?

Define "learning."

* How do you decide what to memorize?
* Does memorization help you understand?

Define knowledge.
How do you "know when you know?"

What type of contexts (frameworks) do you put your math knowledge into?
Does that help you understand?
Does a context help you better learn the mathematics?

* Questions not originally part of the Focus Group protocol but included because students identified memorization as a key to learning, understanding, and knowledge.
Table 2
Focus Group Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Group 1 Grade 7 (n=6)</th>
<th>Group 2 Grade 8 (n=8)</th>
<th>Group 3A Grades 9-12 (n=7)</th>
<th>Group 3B Grade 9-12 (n=6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Age Range</td>
<td>12y, 3m (10y 0m - 13y 2m)</td>
<td>13y, 4m (12y 5m - 13y 3m)</td>
<td>15y 8m (14y 5m - 17y 10m)</td>
<td>15y 10m (14y 9m - 17y 3m)</td>
</tr>
<tr>
<td>Gender</td>
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<td>7 Male 1 Female</td>
<td>6 Male 1 Female</td>
<td>3 Male 3 Female</td>
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<td>0</td>
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<tr>
<td>&lt; 1 Yr Eng</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mean Yrs at this School Range</td>
<td>2.7 yrs (.5 yrs - 6.5 yrs)</td>
<td>2.9 yrs (.5 yrs - 7.5 yrs)</td>
<td>4.3 yrs (1.5 yrs - 9.5 yrs)</td>
<td>2.7 yrs (.5 yrs - 9.5 yrs)</td>
</tr>
</tbody>
</table>
Figure 1

"The Path to Knowledge"
(as described by students in Group 3A)
REFERENCES


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