DOCUMENT RESUME

ED 464 093 TM 033 773

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TITLE Preservice Teachers' Understanding of Functions: A Performance Assessment Based on Non-Routine Problems Analyzed in Terms of Versatility and Adaptability. Interim Report.

PUB DATE 2002-04-02

NOTE 21p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 1-5, 2002).

PUB TYPE Reports - Descriptive (141) -- Speeches/Meeting Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.

DESCRIPTORS Algebra; *Comprehension; *Functions (Mathematics); Higher Education; *Interviews; *Preservice Teachers; Secondary Education; *Student Characteristics

ABSTRACT

Recent reports have pointed to the need for teachers to have "deep understanding" of concepts and procedures in school mathematics, particularly those related to functions. However, previous research has demonstrated that many students, including preservice teachers, have limited understanding of functions. This paper describes a set of nonroutine tasks designed to assess various aspects of students' understanding of functions, including their use of different representations (algebraic, tabular, and graphic) and their ability to adopt a process perspective or an object perspective on functions. This paper focuses on students' versatility and adaptability, which together constitute flexibility. The nonroutine tasks were used in task-based interviews of two types: contemporaneous interactive interviews and sequential reflective interviews. The paper provides examples of the tasks and discusses the reasoning behind the rationales applied. (Contains 6 figures and 21 references.) (SLD)

PRESERVICE TEACHERS' UNDERSTANDING OF FUNCTIONS: A PERFORMANCE ASSESSMENT BASED ON NON-ROUTINE PROBLEMS ANALYZED IN TERMS OF VERSATILITY AND ADAPTABILITY

INTERIM REPORT

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A paper prepared for Session 20.13: Mathematics Education Paper Discussions 2

At the 2002 Annual Meeting of the American Educational Research Association

In New Orleans, Louisiana, April 2, 2002

Abstract

Recent reports have pointed to the need for teachers to have "deep understanding" of concepts and procedures in school mathematics, particularly related to functions. Yet previous research has indicated that many students, including preservice teachers, have limited understanding of functions. We have developed a set of non-routine tasks designed to assess various aspects of students' understanding of functions, including their use of different representations (algebraic, tabular, graphic) and their ability to adopt a process perspective or an object perspective on functions. In this paper we focus on students' versatility and adaptability, which together constitute flexibility. Our non-routine tasks are used in task-based interviews of two types: contemporaneous, interactive interviews and sequential, reflective interviews. The paper provides examples of tasks and discusses their rationales.

The Conference Board of the Mathematical Sciences recently published a report (2001) containing recommendations for the mathematical education of teachers. This document makes numerous statements about the need for teachers to have "deep understanding" of school mathematics concepts and procedures, and it identifies functions as a topic in which mathematicians and mathematics educators need to collaborate to construct course activities to help future teachers develop deep personal understanding. Although it is complex and can be difficult to achieve, attaining a deep and rich understanding of the concept of function is crucial for success in mathematics courses in high school and college. Romberg, Carpenter, and Fennema (1993) claim that "there is general consensus that functions are among the most powerful and useful notions in all mathematics" (p. 1). Yet despite this importance, numerous studies conducted to describe students' understanding of functions (Breidenbach et al., 1992; Carlson, 1998; Sfard & Linchevski, 1994; Thompson, 1994; Vinner & Dreyfus, 1989) suggest that many secondary and postsecondary students, including preservice teachers (Even, 1993), demonstrate a limited view, or in Skemp's (1978) terms, an instrumental understanding, of the function concept. The participants in this study are all preservice teachers, and our focus is on the depth of their understanding of content they will soon teach in secondary schools.

Purposes of the Research

In previous research (Schroeder, Donovan, Schaeffer & Reisch, 1999; Donovan, 2000; Reisch, 2001; Schaeffer, 2001) we have developed a collection of non-routine tasks for college mathematics students designed to assess various aspects of preservice teachers' understanding of functions. In our analyses of the students' performance we have attended mainly to the representations (tabular, algebraic, graphical) students have used and also to the instances in which students' thinking could be characterized as reflecting a process perspective or an object perspective on functions. In this paper we present an extension and refinement of that work using those constructs, but here we focus more on the versatility and adaptability of the participants' performance (Moschkovich, Schoenfeld, & Arcavi, 1992; Sfard & Linchevski, 1994).

Perspectives and Theoretical Frameworks

This study has evolved from a concern that preservice teachers need to be aware of the differences between instrumental understanding and relational understanding (Skemp, 1978) and from a conviction that to teach mathematics well in the current era of reform, teachers need to have deep understanding of mathematics as well as versatility and adaptability in choosing and using different representations and perspectives of mathematics. This is to say that we value mathematical connections (NCTM, 1989, 2000) and are seeking specific means of assessing (and ultimately promoting) the development of problem solving and mathematical connections on the part of preservice teachers.

Many mathematical concepts such as number, variable, and function can be viewed from two complementary perspectives – as processes and as objects. These perspectives result from an ontological duality inherent in mathematical concepts, namely "the idea that many mathematical concepts can be seen in two different, complementary forms, such as operational and structural, or process and object [respectively]" (Harel & Dubinsky, 1992, p. 19). Sfard (1991) has argued that "the ability of seeing a function or a number [or other mathematical concept] both as a process and as an object is indispensable for deep understanding of mathematics, whatever the definition of 'understanding' is" (p. 5). Harel & Dubinsky (1992) have defined these views (or perspectives or conceptions) as follows:

> *Operational Conception.* One of the two basic ways of approaching an abstract mathematical concept (the other one is called a *Structural Conception*). Operational conception occurs when a person views a given notion as referring to a certain PROCESS rather than to an object. For example, function, although usually defined as a static

permanent construct (a set of ordered pairs), may also be conceived as a computational procedure (p. 20).

Structural Conception. One of the two basic ways of approaching an abstract mathematical concept (the other one is called an *Operational Conception*). Structural conception occurs when a given notion is conceived as referring to an object. Such an abstract object is a metaphor which makes an abstract entity in the image of a physical thing: it appears permanent, it is clearly delineated and highly manipulable. Function, when conceived as a set rather than as a computational procedure, is such an abstract object (p. 21).

Moschkovich, Schoenfeld, and Arcavi (1993) have argued that

developing competency with linear relations [or functions more generally] means learning which perspectives [process and object] and representations [tabular, algebraic, and graphical] can be profitably employed in which contexts, and being able to select and move fluently among them to achieve one's ends (p. 72).

These authors developed the conceptual framework shown in Figure 1 and referred to as the function matrix, combining perspectives and representations. This framework allows researchers not only to characterize students' cognitions (placing them in one or more cells of the matrix), but also to characterize students' versatility and adaptability by noting the variety of cells in which students function and their ability to move from cell to cell connecting their work in the different cells.

	Tabular Representation	Algebraic Representation	Graphical Representation
Process Perspective			
Object Perspective			

Figure 1: The function matrix (Moschkovich, Schoenfeld, & Arcavi, 1993)

We consider that having a flexible understanding means that an individual is not only able to work within each cell of the function matrix, but also to move throughout the cells of the matrix, changing and connecting representations and/or perspectives as needed to complete mathematical tasks. Both Sfard and Linchevsky (1994) and Moschkovich, Schoenfeld, and Arcavi (1993), have discussed flexibility in terms of the use of multiple representations and the adoption of both operational and structural perspectives, and have claimed that flexibility has two distinct and independent components, namely versatility and adaptability. Versatility has been characterized as the "ability to see an expression as a process, [and] in another context ... view it as the product of this process, and in still another situation as a function [i.e. an object]" (Sfard & Linchevsky, 1994, p. 205). A person demonstrates adaptability to the extent that he or she has the capacity to adjust the perspective and representations to whatever task is presented. According to Moschkovich, Schoenfeld, and Arcavi (1993), "flexibility is a hallmark of competence" (p. 97).

Research Methods

The focus of our previous work has been on the development of tasks that would be appropriate (i.e. challenging enough yet also accessible) for undergraduate mathematics students. The non-routine tasks we have been developing are designed to elicit thoughtful response; generally they cannot be solved by the routine application of a taught procedure. They are similar to what Selden et al. (2000) describe as "moderately non-routine tasks," that is, problems that "are not very similar to problems that students have seen before and [that] require known facts or skills to be combined in a novel way, but [they] are 'straightforward' in not requiring, for example, the consideration of multiple sub-problems" (p. 148). Implicitly our tasks involve the use of multiple representations because they are presented using one representation, but work in a different representation facilitates progress toward the goal. Furthermore, they are structured so as to provide evidence about whether the student is adopting a process or an object perspective.

We have developed these tasks for use in task-based interviews (Davis, 1984). As Davis explains, task-based interviews can vary along a number of dimensions, including the nature and amount of intervention by the interviewer, the extent to which participants are asked to verbalize their thoughts as they work at the task, the tools and materials available to them, and the equipment used to make records of the interview. In the present study we have adopted two different types of task-based interviews; we call these types or styles of interviews *contemporaneous, interactive* interviews and *sequential, reflective* interviews, respectively.

In the contemporaneous, interactive interviews the interviewer observes the student as he or she works on the task and may (or may not) ask the participant to think aloud while working. Although the interviewer tries not to intervene very much, follow-up questions are used, including questions that might function as hints. Typically in follow-up questions the interviewer asks the participant to consider the possibility of solving the problem in a different way or to explain how the problem was solved. Sometimes additional questions about the task or extensions of the task are addressed. We tend to use the contemporaneous, interactive interview style when we anticipate that the participants will find it relatively easy to get started and make progress.

The sequential, reflective style of interview is used for the more difficult or complex tasks where we anticipate that more time will be needed to get started or to explore different approaches to the problem. In this type of task-based interview participants are given time to work on the task alone prior to talking with the interviewer about what they did to complete the task. Before setting them to work, the interviewer reminds the participants to document as much of their thinking as possible since they will be asked later for a reconstruction of what they did to solve the problem. Ericsson and Simon (1980) have observed that when think-aloud protocols are used and interviewees "are working under a heavy cognitive load, they tend to stop verbalizing or they provide less complete verbalizations. ... Omissions [are] caused in reports, by requiring subjects to perform intervening tasks concurrently with their verbal reporting" (pp. 242-243). Although retrospective verbalizations might be considered less reliable, less detailed, or less complete than the verbal reports obtained in the contemporaneous, interactive interviews, Ericsson and Simon conclude that

> When clear probes are used for specific retrospective memory and when reports are requested immediately after the last trial(s), informative verbal reports can usually be obtained, although perhaps not always in the case of complex pictorial stimuli. The failure of subjects to report some information does not demonstrate the uselessness of verbal protocols. Incompleteness of reports may make some information unavailable, but it does not invalidate the information that is present. (Ericsson & Simon, 1980, p. 243)

Since very little time passes between the completion of the individual work and the start of work with the interviewer, we believe the information each participant reconstructs generally reflects the participant's original work appropriately. We recognize that additional insights may be gained from the process of reflection, but we do not regard this eventuality as problematic. Prompts or questions that serve as hints allow the interviewer to distinguish between (1) participants who can complete the tasks or answer questions without any sort of suggestion, (2) participants who are able to respond after being prompted, and (3) participants who make little or no progress even after a suggestion is provided. Certainly those participants who are capable of enacting appropriate aspects of their understanding independently have firmer or deeper connections between the concepts involved than those who can only complete the task after receiving a prompt to stimulate their thinking. Similarly, those who make progress as the result of a suggestion or trigger have demonstrated more maturity in their function schema than those who do not move forward with the task even after receiving some help.

The interviews we have conducted (and will continue to conduct) are both audiotaped and videotaped. Transcriptions are made primarily from the audiotapes, because they are easier to work with, but we sometimes find the videos, which focus on the participant's written work, are essential in order to understand what the verbal reports refer to. During and after the interviews the interviewer makes field notes to document information about the interview that might not captured on either of the tapes.

Each participant is provided a sheet containing the statement of the problem along with ample space for written work. Different colored pencils and pens, a graphing calculator, and blank paper are also made available to the participant. All written work is collected for analysis.

Tasks

In this section we present descriptions of and rationales for some of the tasks we developed for use with preservice secondary mathematics teachers. We have developed the tasks and follow-up questions through an iterative process of trials and revisions, and we continue to refine the materials and hone our interviewing skills.

Systems of Linear Equations Tasks. A series of tasks inspired by the work by Sfard and Linchevski (1994) is shown in Figure 2. Problem 1 presents a singular system of equations, one in which the two equations are equivalent to one another, that is, their graphs are coincident. However, it is not immediately obvious that this is the case. While some students may not be sure what to make of the result $0 = 0$, which they may get if they follow the normal routine for solving simultaneous linear equations, or the $7 = 7$, which they may get if use a substitution method, others may know, as a rule, that in such cases the equations are equivalent and there exist an infinite number of solutions. Therefore, follow-up questions concerning why these rules are correct, whether an infinite number of solutions means that the equations hold for all values of x and y, and so on should be explored.

Problem 1:

Solve $\begin{cases} 2(x - 3) = 1 - y \\ 2x + y = 7 \end{cases}$

Problem2:

Is it true that the following system of linear equations

$$\begin{cases} k - y = 2 \\ x + y = k \end{cases}$$

has a solution for every value of k ?

Problem 3:

Is it true that the following system of linear equations

$$\begin{cases} y - 4x = 2 \\ y - kx = 1 \end{cases}$$

has a solution for every value of k ?

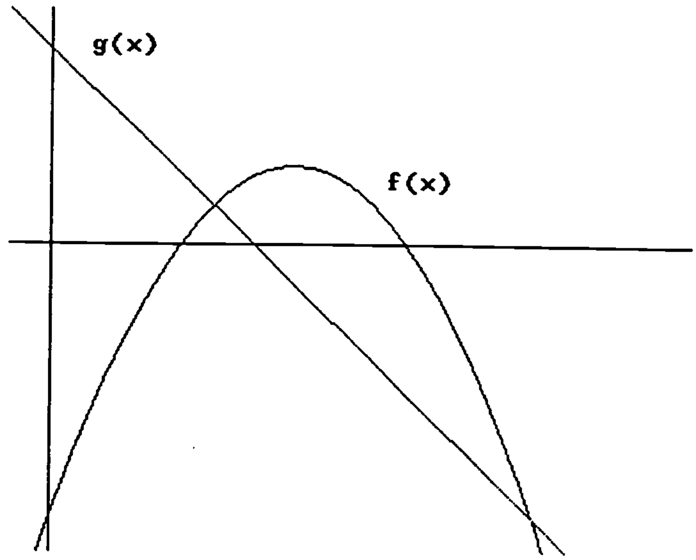
Figure 2: Systems of Linear Equations Tasks inspired by Sfard and Linchevski (1994)

Problem 2 presents an unusual situation in that the solution has $x = 2$ regardless of the value of the parameter k. but the y value of the solution depends on k as $y = k - 2$. If one approaches this problem intending to use the two equations to solve for k, one finds that k "disappears" leaving $x - 2 = 0$. There are a number of possible questions for follow-up, including questions about the role of the k (as a parameter, rather than a variable), questions about the relationship (if any) between the lines representing the given equations and the lines (one vertical, one a family of horizontal lines depending on k) representing the set of solutions, and so on. As usual, the quality of the participant's explanations provides evidence about the depth and flexibility of the understanding.

In Problem 3, the situation is somewhat simpler, and we expect many preservice teachers will readily recognize that when $k = 4$ then the two lines will be parallel but one unit apart because their y-intercepts differ by 1. This should lead to the conclusion that the system has a solution for every value of k except 4, because then the lines will have different slopes and hence intersect. In this problem, however, if one attempts to solve the two given equations for k, one gets the result $k = 4 + 1/x$, and it may not be easy to explain how to interpret this finding.

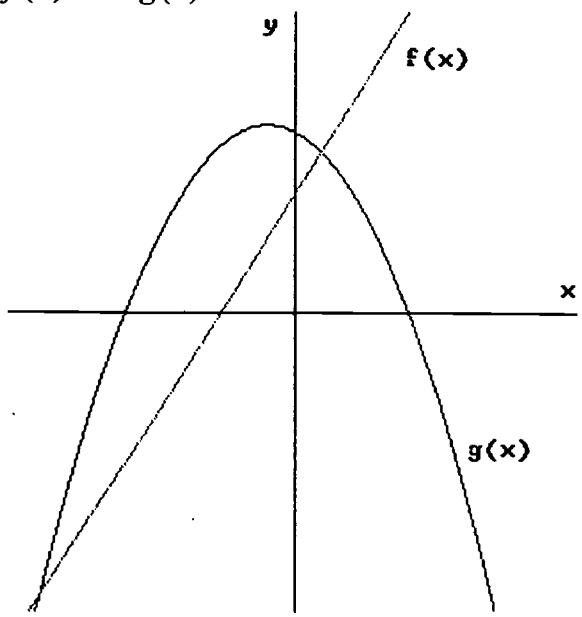
Graphing Tasks. Another series of tasks, developed by Schaeffer, concerns finding the graphs of the sums, differences, and products of functions presented graphically. These tasks, presented in Figure 3, demand visual reasoning in their present form with scale-less axes. When we piloted earlier versions with scales on the axes, we found that many students preferred to use the features of the graphs to determine algebraic representations of the functions, to calculate algebraically the required sum, difference, or product, and then to graph the new function using its algebraic representation and a table of values. While such an approach does demonstrate technical competence converting from graphical to algebraic operations and vice versa, it also reduces the task to a series of routine ones and views the functions primarily as processes rather than objects. Participants who deal with these problems visually tend to treat the functions as visual objects.

The graphs of f and g are provided below. Sketch the graph of $(f-g)(x)$ on the same axes as $f(x)$ and $g(x)$.



The Graphing Task, Part A

The graphs of f and g are provided below. Sketch the graph of $(f \cdot g)(x)$ on the same axes as $f(x)$ and $g(x)$.



The Graphing Task, Part B

Figure 3: Graphing tasks developed by Schaeffer

Composition Tasks. Composition of functions is an advanced topic, but one that can be dealt with algebraically or graphically or perhaps even with tabular representations. Our composition tasks are presented in Figures 4, 5, 6, and 7.

The main purpose of Part A was to determine whether the participants were able to execute the algebraic mechanics of composition. In this regard, the task is very routine in nature. We anticipate participants may have difficulty realizing the domain restriction in the first composition, but if the student has a deep understanding of the process of composition, this domain restriction can be determined and explained. Precalculus and calculus students frequently use the end result of the composition of functions to determine the domain of the newly created function, and this task nicely illustrates a case for which that approach fails to provide a correct domain. In the second half of Part A, participants are faced with a function for which the graph is not likely to be immediately recognized. If a graphing calculator is available, it is likely to be used; however, asking the follow up question about how the graph would be produced without technology provides some notion of the student's approach to curve sketching.

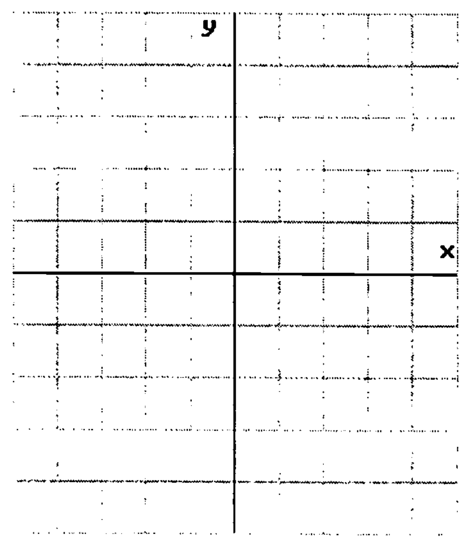
Part B of this task was created to force participants to operate with graphical representations at either the object or process level. The first two parts can be algebraically verified as a horizontal and vertical shift of the y_2 graph (object), but even advanced students may not begin to tackle the task in this way. A pointwise (process) approach to the composition can be conducted at several different values of x, and the pattern that emerges will appear to look much like that of y_2. Students who are dependent on explicit algebraic representations will struggle with this task, as the equation for y_2 is not one they are likely to determine. This would represent a limited adaptability. Flexible students are able to complete and explain this task at the algebraic and graphical object level as well as the graphical process level.

Figure 4: Composition Task Part A

As you work through the task, please attempt to document as many of your thoughts and ideas as possible (whether you used the ideas or not). When you finish, I will ask you to reconstruct what you have done.

Given $f(x)=x^2-2$ **and** $g(x)=-\sqrt{x+1}$ **answer each of the following.**

(a) Determine $(f\circ g)(x)$ in simplified form and sketch a graph of this new function.



(b) Determine $(g\circ f)(x)$ in simplified form and sketch a graph of this new function.

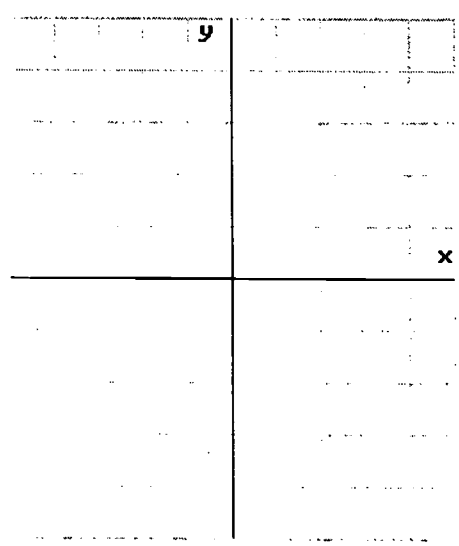
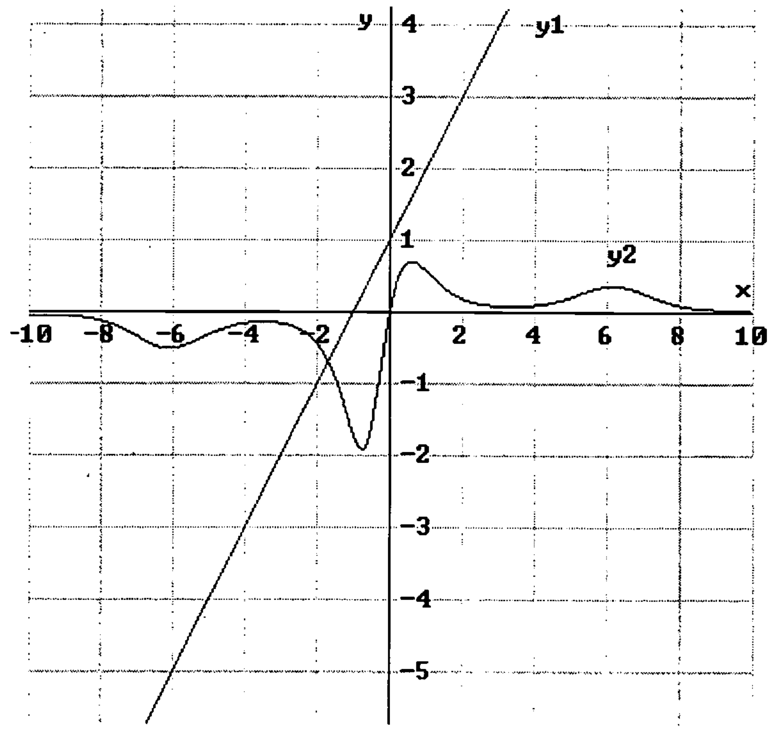


Figure 5:

Composition Task – Graphical version

(a) Use the given graphs to sketch $y_2 \circ y_1$.



(b) Use the given graphs to sketch $y_1 \circ y_2$.

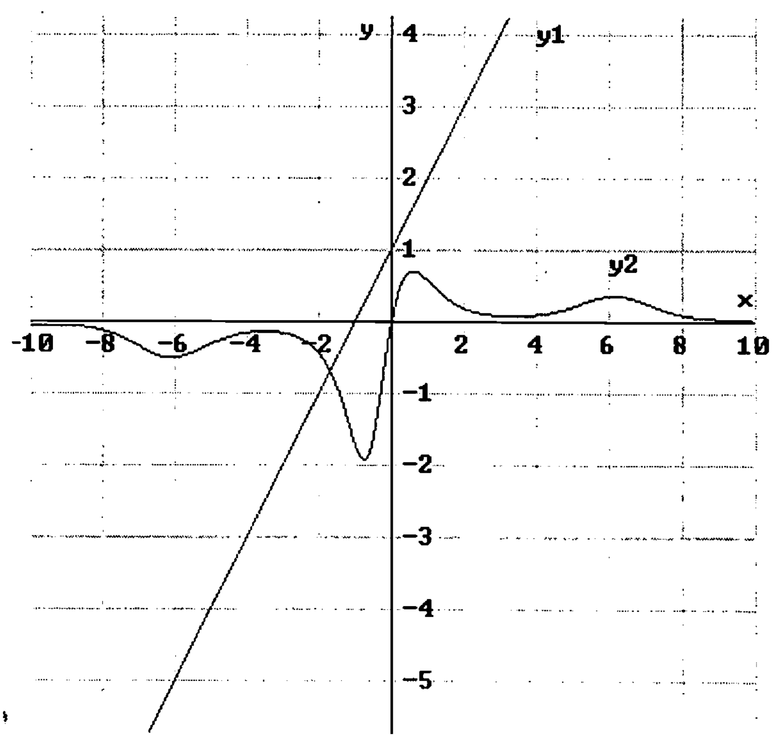
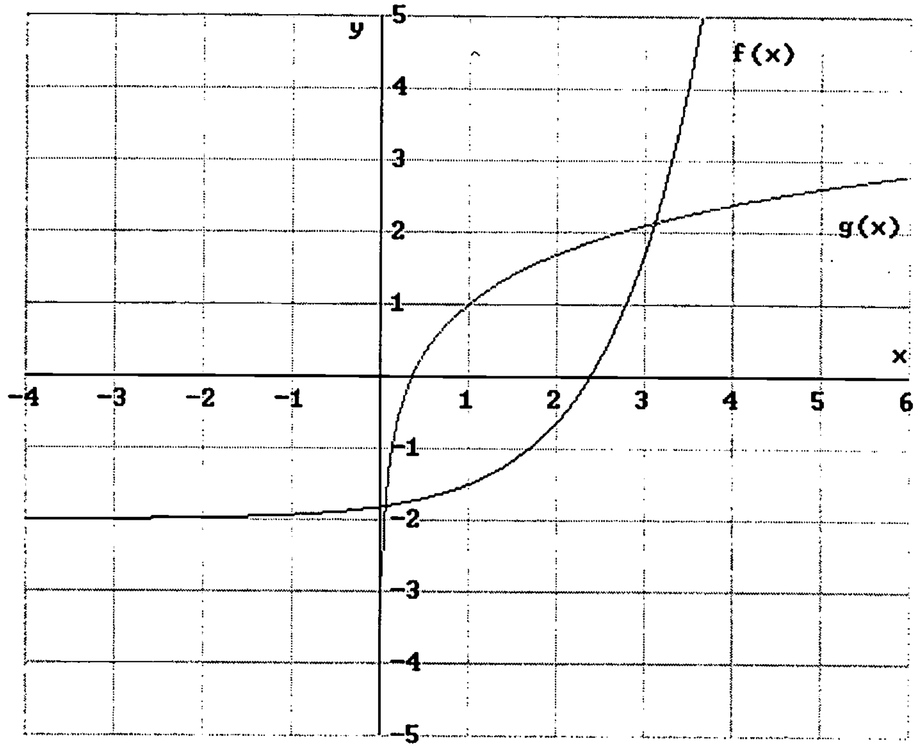


Figure 6: Composition Task Part B, Continued

(c) Use the given graphs to sketch $f \circ g$.



<u>Figure 7: Composition Task Follow-up Questions</u>

<u>Part A</u>

Tell me everything you can about the new function.

What is the domain of $(f \circ g)(x)$? The range? How did you decide?

What is the domain of $(g \circ f)(x)$? The range? How did you decide?

Did you use the calculator to create your graphs? Can you justify what it gave you? If you didn't have that tool, how would you have gone about creating the sketch?

Part B

Tell me everything you can about the new function. (for each case) Domain, range, how you decided?

In the third case, can you estimate the value of $(f \circ g)(1)$? How did you obtain your estimate? What is $\lim_{x \to 0^+} (f \circ g)(x)$? How did you decide?

Data Analysis

The data collected in this study are analyzed in two steps using first the function matrix and then the definition of flexibility. The function matrix is used to record the perspectives and representations being used at particular points in the participant's written and/or oral work. Then, by analyzing the matrix as a whole, flexibility can be documented in the following manner. Instances of versatility are identified when the individual shows the ability to work in many different cells on a single problem, that is when the individual is able to view a function from either perspective (process and object) as well as in any representation (tabular, algebraic, and graphical). Instances of adaptability are identified when the individual shows the ability to access specific cells (particular perspectives combined with particular representations) depending on the task at hand. To summarize, we think of versatility as the collection of tools possessed by an individual, whereas adaptability is the ability to select and use tools required for the job. Together, versatility and adaptability constitute flexibility.

Results

When we submitted the proposal for this paper we had only a few preliminary results, and we planned to present at this conference findings based on at least two to four tasks administered to each of eight or more undergraduate students majoring in mathematics with the intention of completing a program leading to certification as a secondary mathematics teacher. A number of circumstances prevented us from realizing that goal. The results we do have are consistent with the findings of other researchers who have investigated preservice teachers' understanding of functions and have been disappointed with the overall depth of their understanding (e.g. Breidenbach et al., 1992; Carlson, 1998; Sfard & Linchevski, 1994; Thompson, 1994; Vinner & Dreyfus, 1989). Like other researchers we have found instances in which students who appear to have prerequisite skills are nonetheless unable to marshal them to solve non-routine problems, a result discussed by the Seldens and their colleagues in research on undergraduates' understanding of calculus (Selden, Selden, & Mason 1994; Selden, Selden, Hauk, & Mason, 2000). However, we have also documented cases in which students responded to interviewers' questions in ways that indicated that the questions prompted or redirected their thinking, suggesting that with scaffolding preservice teachers are able to make mathematical connections of the type we value.

Conclusions and Implications

We believe that this research project can offers insights for mathematics teacher educators into the nature and extent of preservice teachers' understanding of functions. We also believe that our tasks provide activities that can be adapted to serve as learning activities as well as assessments. We look forward to completing the project and disseminating our results.

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