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ABSTRACT

Promoting ACademic Excellence in Mathematics and Science for Workers of the 21st Century (PACE) was a consortium project made up of Indiana University Northwest, the Gary Community Schools, and the Merrillville Community Schools. The focus of this project was to prepare teachers and curricula for Tech Prep mathematics and science courses for the two school districts. The courses and course units prepared by the project are intended to promote the Core 40 Competencies of the Indiana Department of Education for High School courses. This document contains units for Algebra One designed to enable students to select and implement appropriate problem solving methods; collect, analyze, synthesize, and report data; form and test hypotheses; and exhibit the ability to appropriately estimate in all of the above situations. In the units students use graphing calculators and other technologies to develop an understanding of functions and relations. Workplace applications and connections to other courses are integrated throughout the units to further facilitate an understanding of how these functions and relations connect to the world of work. Units include: (1) Measurement; (2) Rational Numbers with Integers; (3) Algebra in a Plane; (4) Solving Equations; (5) Solving Inequalities; (6) Polynomials; (7) Algebraic Fractions; (8) Ratios, Proportions, and Percents; (9) Solving Linear Systems; (10) Radical Expressions; and (11) Statistics, Linear, and Non-Linear Functions. (JRH)

Algebra One

an Applied Approach

Clyde A. Wiles • Kenneth J. Schoon
Editors

PACE **Promoting Academic Excellence In Mathematics, Science & Technology for Workers of the 21st Century.**

Gary Community School Corporation
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Algebra One
An Applied Approach

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PACE

Promoting ACademic Excellence in Mathematics and Science for Workers of the 21st Century

PACE was a consortium project of Indiana University Northwest, the Gary Community Schools, and the Merrillville Community Schools. It was supported by funds from the three institutions and by Eisenhower grants from the Indiana Higher Education Commission.

The focus of the project was to prepare teachers and curricula for Tech Prep mathematics and science courses for the two school districts. The effort took place over 1994 - 1996 and involved more than 70 teachers from seven High Schools. The Director of the project was Dr. Clyde A. Wiles, and the Associate Director, Dr. Kenneth J. Schoon, both of Indiana University Northwest.

Part of the effort was the developing of units and course outlines for use in the first two years of a High School Tech Prep program. Individual schools and faculty will be using these materials in a variety of ways from being a course guide to being a supplement to an already existing program.

We have taken the position that Tech Prep is not a program for the academically deficient. Rather it is an applied approach to curriculum that has the goal of promoting competencies recommended by the State of Indiana for non-remedial high school courses, and which does so in a learning environment that emphasizes applications. We would like students to find within these course materials and instructional approaches immediate and obvious responses to the questions: "What does this look like?" and, "Why would anyone want to know?"

These courses and course units then are intended to promote Core 40 Competencies of the Indiana Department of Education for High School courses. For mathematics, we viewed this as beginning with Algebra One and for science beginning with Biology. The Pre-Algebra course is not a Core 40 course, but does maintain the applied perspective.

Our efforts have had to accommodate to several factors. First there is an Indiana mandate that all high schools have a Tech Prep curriculum that targets the academic and school-to-work needs of the middle 50% of the high school student population. There are on the other hand, persistent beliefs of counselors, teachers, administrators, students, and parents that something called "tech" anything, is just another name for a program intended for "at risk" students who are not expected to acquire competencies at a level that would enable them to pursue post secondary schooling at the college or university level. These beliefs are often supported by admission policies at some universities. We have, therefore, attempted to position Tech Prep courses as courses that meet exactly the same Core 40 competencies (as defined by the Indiana Department of Education) as are to be met by college prep courses of the same name, but to do so in applications-based and problem-centered approaches.

Clyde Wiles, Director
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Division of Education
Indiana University Northwest
Gary, Indiana
May, 1997

ALGEBRA ONE

An Applied Approach

INTRODUCTION

Algebra One, for many students, is the conveyance used to move from arithmetic mastery to the first level of true mathematical sophistication. In this course the student passes beyond the need to simply master skills, and begins to approach, grasp, and manipulate concepts in ways to which they may not be accustomed. Abstractions called variables, while alluded to in previous courses, become the bases for the student's learning, ranging from topics such as solving simple equations to solving systems of linear inequalities. There may not be a more critical time in a student's formative years to root those abstractions firmly on a concrete level. So we look to Tech Prep, an idea that stands with both feet on that concrete level as the base of a series of steps that will lead the student to success.

Using "real world" applications and activities, the Algebra One teacher can lead the students -- sometimes even trick the students -- into using mathematical ideas that they are not yet aware of. Then, the students can be lead to put names and perhaps even definitions to the concepts they have experienced. The teacher is guided by the *Core 40* competencies in the selection of appropriate applications and activities.

To be successful in this course, the freshman students should already be competent in several areas: working with whole numbers, fractions, and decimals, and being able to change one to another; estimating and rounding whole numbers, fractions, and decimals; working with ratios and proportions, leading into percents; measuring length, mass, and volume using both metric and US customary systems of measurement; and using basic problem solving strategies such as drawing a picture, guess and check, modeling the situation, and role-playing.

COURSE GOALS

The successful student will select and implement appropriate problem solving methods; collect, analyze, synthesize, and report data; form and test hypotheses; and exhibit the ability to appropriately estimate in all of the above situations.

Students will use graphing calculators and other technologies to develop an understanding of functions and relations. Workplace applications and connections to other courses will be integrated throughout the units to further facilitate an understanding of how these functions and relations connect to the world of work.

UNIT LIST

The following units have been identified as having the most important ideas to be taught to the Algebra One (applied) student:

1. Measurement
2. Rational Numbers with Integers
3. Algebra in a Plane
4. Solving Equations
5. Solving Inequalities
6. Polynomials (Part One)
7. Polynomials (Part Two)
8. Algebraic Fractions
9. Ratios, Proportions, and Percents
10. Solving Linear Systems
11. Radical Expressions
12. Statistics, Linear, and Non-Linear Functions

SUMMARY OF UNIT COMPETENCIES

The Indiana Core 40, Algebra First Course, competencies are completely represented in the above units. Below is a list of which competencies each unit contains:

Unit One

This is a measurement unit which is important reinforcement for Algebra One but is not a part of the Algebra One Core 40 competencies.

Unit Two

Evaluate algebraic expressions. (1.1)
Use the associative, commutative, and distributive properties. (1.4)
Simplify real number expressions. (2.1)
Determine the additive and multiplicative inverses of a number. (2.2)
Determine the absolute value of expressions. (2.3)
Use exponents with real numbers. (2.4)
Distinguish between rational and irrational numbers. (2.6)
Compare real number expressions. (2.8)

The numbers in parentheses refer to the Indiana Core 40 Competencies.

Unit Three

Graph and locate sets of real numbers on a number line. (4.1)
Graph ordered pairs. (4.2)
Find distance between two points on a number line. (4.3)
Graph a relation on the coordinate plane. (4.4)
Distinguish between a relation and a function. (4.5)
Graph a relation given an equation and a domain. (4.6)
Sketch a reasonable graph for a given relation. (4.7)
Interpret a graph representing a real world situation. (4.8)
Use graphing technology to explore the graph of functions. (4.9)
Determine if data are behaving in a linear fashion. (5.1)
Graph a linear equation in two variables. (5.3)
Graph a line given the slope and y-intercept. (5.4)
Find the slope of a non-vertical line given the graph of a line or an equation of the line or two points on the line. (5.5)
Describe the slope in a real world linear relationship using real world terms. (5.6)
Write the slope of an equation of a line. Find the slope and y-intercept of a line. (5.7)
Write the equation of a line given the slope and one point on the line, or two points on the line. (5.8)
Write an equation of a line which models a set of real data. (5.9)
Use the line which models real data to make predictions. (5.10)
Foreshadow inequalities by describing the location of an ordered pair in a relation to a line. (5.11)
Graph a linear inequality in two variables. (5.12)

Unit Four

Use formulas to solve problems. (1.2)
Solve an equation by using the addition property of equality and the idea of additive inverse. (3.1)
Solve an equation by using the multiplication property of equality and the idea of multiplicative inverse. (3.2)
Solve an equation graphically and by using more than one property of equality. (3.3)
Solve an equation that contains like terms. (3.4)
Solve an equation which has the variable in both members. (3.5)
Solve an equation in which the numerical coefficients and constant terms are fractions or decimals. (3.6)
Solve a formula for one of its variables or find the value of a variable when values of the other variables are given. (3.7)
Use problem solving skills to solve real world and "word" problems which involve a linear equation or formula. (3.8)

Unit Five

Find the solution set for a linear inequality when replacement values are given for the variable. (3.11)
Solve a linear inequality by using properties of order. (3.12)
Use problem solving skills to solve real world or "word" problems which involve inequalities. (3.13)
Find the solution set of combined inequalities. (3.14)
Graph a linear inequality in two variables. (5.12)

Unit Six

Write and simplify expressions involving exponents. (2)
Add and subtract polynomials. (7.1)
Multiply monomials. (7.2)
Find an indicated power of a monomial. (7.3)
Multiply a polynomial by a monomial. (7.4)
Find the product of two binomials. (7.5)
Multiply two polynomials. (7.6)
Solve application problems involving uniform motion. (5)
Solve application problems involving area. (3,4)
Recognize problems that do not have solutions. (4)

Unit Seven

Factor integers into primes and find the greatest common factor of several integers. (2)
Find a common monomial factor in a polynomial. (7.9)
Factor the difference of two squares. (7.10)
Factor a simple quadratic trinomial. (7.11)
Factor a polynomial by grouping. (7)
Use factoring techniques to solve a polynomial equation. (7)

Unit Eight

Divide two monomials. (7.7)
Divide a polynomial by a monomial. (7.8)
Divide two polynomials. (7)
Simplify ratios involving algebraic expressions. (7)

Unit Nine

Simplify ratios involving algebraic expressions. (8.1)
Solve proportions. (9.2)
Use ratios and proportions to solve real world and "word" problems. (8.3)
Solve real world and "word" problems involving percents. (8.4)

Unit Ten

Use a graph to find the solution of a pair of linear equations in two variables. (6.1)
Graph the system of linear inequalities in two variables and find the solution set. (6.2)
Use graphing technology to solve systems of linear equations. (6.3)
Use the substitution method to find the solution of a pair of linear equations in two variables. (6.4)
Use the addition and subtraction method to find the solution of a pair of linear equations in two variables. (6.5)
Use multiplication with the addition or subtraction method to solve systems of linear equations. (6.5)
Use systems of linear equations to solve real world and "word" problems. (6.6)

Unit Eleven

Estimate the square root of any non-negative number. (2.7)
Find the product of two square roots and simplify the result. (2)

Find the quotient of two square roots and simplify the result. (2)
Simplify two radicals and find their sum. (2)
Simplify two radicals and find their difference. (2)
Use the Pythagorean Theorem to determine the hypotenuse of a right triangle. (4)
Solve an equation containing a radical (3.10)

Unit Twelve

Graph a quadratic equation. (9.1)
Use graphing technology to find the solution to a quadratic equation. (9.2)
Solve a quadratic equation when one member is in factored form and the other member is zero. (9.3)
Solve a simple second-degree equation by factoring. (9.4)
Use graphing technology to relate the solutions of quadratic equations and the x-intercepts. (9.5)
Understand that the vertex provides the maximum or minimum value of the quadratic equation. (9.6)
Solve a quadratic equation in which a perfect square equals a constant. (9.7)
Solve a quadratic equation by using the quadratic formula. (9.8)
Determine if a set of data represents an exponential function. (9.9)
Use formulas, calculators, and graphing technology to explore and solve problems involving exponentials. (9.10)

Unit 1

Measurement

Unit 1: Measurement

Suggested Time Frame: 3 weeks

This unit is intended to take a student with basic proficiency in measurement with both the US Customary and International Systems, and advance him or her to the proficiency level required to form a concrete basis from which to apply Algebra One concepts. Through the use of the most fundamental concepts and the history of measurement, the successful student will make important connections to the real world throughout the other units in this course. With that goal in mind, activities in this unit will be used to create a database of measurements physically gathered by the students that can be used when the teacher introduces the ideas of perimeter, area, volume, and even the Pythagorean Theorem later in the course. We feel this would give more meaning to each of these ideas, better than a group of artificially contrived numbers from a book that just happen to always work out evenly.

Unit Objectives

Students will:

1. Measure length with both the US Customary and International Systems.
2. Draw a line segment of a given length using both the US Customary and International Systems.
3. Read and write numerals using scientific notation.
4. Convert common fractions to decimals (and vice-versa) using a calculator.
5. Apply the correct number of significant figures to the answer of any given problem.

Workplace Relationships

1. Carpenters, builders, welders, landscapers, and grounds keepers all have to measure and draw lines of specific lengths.
2. Automotive workers have to adjust micrometers to decimal fractions of an inch.
3. Quality control inspectors must know how accurate manufacturing parts have to be made to pass inspection.

Learning Activities/Teaching Strategies

This unit employs cooperative learning techniques and activities to accomplish its tasks.

The following materials are necessary for the instruction of this unit:

10 metric micrometers	10 metric tape measures	Globe
10 meter stick	Roll of adding machine tape	String
10 yard sticks		

The following is a day-by-day breakdown of an effective sample unit on measurement.

Week One

Day One: (Initiatory Activity): Lead students in a discussion on the topic, "What is mathematics without measurement?" Challenge students to invent a use for mathematics or even numbers that does not involve counting or measuring. When (hopefully) the students admit defeat, introduce an activity that requires the students to add five and three under different circumstances that do require measurement. For example, "five minutes plus three minutes makes eight minutes," eventually leading to length. From there, segue into how to measure length. For example, ask how to compare the heights of two students in the classroom without tools. Most likely, the students will suggest that they stand together, which is fine. Then, choose two students more close in height and have them stand in opposite corners of the room. Lead the students into using a hand, or arm as the unit of measure.

Relate how this was also the way that people measured very long ago, but point out there was a problem with this. Ask what that problem might have been. When the question is answered, allow the students to use their first artifact for measurement: a pre-cut strip of paper.

When the closeness of the measurement is too much for the piece of paper, demonstrate for the students how to divide the paper by folding it in the obvious way. Fold it a second time, and make sure the students can still identify the original folds. By the time the fourth fold comes, the students may have trouble identifying the original folds. When that occurs, suggest that you start again, only this time marking the folds as you proceed. Lead the students to the idea that for every fold, the sections of the paper are only half as large, so perhaps the marks for each should only be half as long.

When the sections are identified as sixteenths, begin to show how some of the sections have other ways of being named, like eighths, fourths, or even a half. Then show how the divisions of an inch work on a ruler. From there, try to measure several line segments, and then draw line segments of a given length less than a foot.

Day Two: Try to involve the students in measuring again by asking them how long and wide the classroom is. When their one foot rulers don't exactly measure up to the task, suggest they use another tool, either a yard stick or a tape measure. Then ask them what they would do to measure even greater distances, like the distance between Gary and Chicago. Eventually, a student may suggest an odometer, which measures in miles. How close do you have to be? From Gary to Chicago may only require measuring to the nearest ten miles. Then, how many tens is it? Ask if there are other greater distances which might need to be measured, like the distance between Gary and San Francisco, and determine how close you need to be then. Perhaps you only have to be accurate to the nearest hundred miles. If so, figure out how many hundreds it is. Then, to really throw a wrench in the works, look up the distance between the earth and the moon, or even the distance between the earth and the sun. After you have determined how close you need to be, show that the numbers you're working with are a little large to work with, and perhaps there should be another way to write them. Introduce the concept of scientific notation, and teach an appropriate lesson, focusing on writing numerals in scientific notation, and writing scientific notation numbers in numeral form.

Day Three: Now that the students' universes are significantly larger, concentrate on small numbers. Using a micrometer, begin to show how to measure the gaps on spark plug leads. Allow the students to experiment with this as well, measuring the widths of several small objects, eventually getting to the point where the numbers simply get too small to work with easily. Then pull the switch and show how you can measure in multiples of tenths, hundredths, thousandths, or smaller, and surprise! That also can be accomplished with scientific notation. Assign homework.

Day Four: Issue eight inch strips of paper to the students and ask them to fold the strips into eighths. Since the sections should be an inch long, have the students measure the first section to the best of their ability. Ask if the sections were exactly an inch long, and if they were not, why not? Show that since there were eight sections made, the students should be able to find the length of the paper by multiplying the length of the section by eight. Ask if it came out to eight inches. The result should be even farther off than the measure of the first section. Lead the students to the idea that their measurement inaccuracy was made worse by the fact that it was multiplied, and perhaps at some point you have to take the exactness of a physical measure with a grain of salt, and at that point, we need to control that sort of error, and to do that, we use significant digits. Show how to use significant digits, and assign homework. Also ask the students to find an example of real world exaggeration of accuracy, and bring it to the class the next day. An example might be the gas pumps that claim to measure to the nearest thousandth of a gallon.

Day Five: Share the students' examples of inaccuracy that they were asked to bring in. Review, reflect, and connect. Give a short quiz for weekly assessment.

Week Two

Day One: Show the students that occasionally they will be forced to add, subtract, multiply, and divide measures. Ask the students if they would rather work with fractions or decimals. Almost invariably the students will choose decimals. That's fine. Show them how to measure another way that uses decimals. Introduce them to the metric system. With meter sticks, the students should be able to measure distances to the nearest meter fairly quickly, and with some reminders of how micrometers worked, they can adjust to measuring to the nearest hundredth of a meter, and even to the nearest thousandth. If there are any questions about the similarity between the meter and a yard, hold a meter stick next to a yard stick to show that they are different. Do not use the 39.37 inches conversion yet. Save that for later when you're making connections. Then introduce them to the terms centimeter and millimeter. Please avoid jumping across charts with multiplication and division to show how to change from one unit to another. This technique has no meaning for the naive learner, and a conceptual approach will be more effective.

When the student makes the rule, it's more likely to be retained and followed. Therefore, through discovery, lead the students to make their own conclusion about the connection between meters, centimeters, and millimeters. Have them measure the same objects in centimeters, then millimeters, and look at the results. They may be able to make the connection there by themselves. Once that is accomplished, have them practice drawing line segments and assign homework.

Day Two: Take the students outdoors if feasible, and measure the length of the football field, which most students should know is one hundred yards long. Using a metric tape measure, measure off the football field, and compare. Inform the students that Canadian football fields have fields that are one hundred meters long, and that Canada is not the only country that uses the system. Compound that fact by explaining that the US is the only major industrialized country that does not use the metric system, and so if we want to remain a major industrialized country, Tech Prep students must be well versed in the metric system. Show that every measure used by the US Customary System can also be made using the metric system and that adding and subtracting parts of centimeters is a much more simple thing than using fractions of inches. At this point, we're not interested in converting; we simply want to allow the two systems to co-exist. Assign homework.

Day Three: Because the US and metric systems do co-exist in the United States, it is important that the students become intimately familiar with both fractional and decimal ways of naming parts of a whole. Familiarize students with simple fractions and their corresponding decimal values. One way to do this is to relate similarly structured fractions together. For example, if you demonstrate any number that can be expressed in hundredths, students can quickly relate that to money. Just be aware that the concept of .6 being the same as .60 is sometimes misleading. Eighths, while expressed in thousandths, can still be expressed as a part of fourths, and so on. Thirds, sixths, and ninths are also related, and can be expressed as multiples of each other. Assign appropriate homework. (55 minutes)

Day Four: Lead the students in a discussion about getting away from school. Since it is the beginning of the year, that shouldn't be too difficult. Ask them if they were to get as far from Gary as possible and still be on Earth, where would they be? Entertain conjectures for a short time, dodging the inevitable, yet intuitive suggestion that China is on the other side of the planet. Once the guesses are exhausted, bring out a basketball with a globe painted on it (guaranteed to capture the boys' attention), and pass it around, suggesting that it might help. If the students look for a tool, and try to use a ruler, suggest a more flexible item, like a string. Allow the students to try and err, until they arrive at a successful conclusion.

Day Five: Now that the students are thinking on a larger scale, take them into space. If feasible, take the students to the football field for the role-playing aspect of this activity. But first, use an almanac to construct a scale model of the solar system, where one student, holding a sphere is to take the position of the sun, and other students are to take the positions of the planets, and possibly the moons around the planets. Probably the hardest part of the activity is to choose the scale. This will encompass everything learned in the unit, from scientific notation to significant digits. When the scale

is chosen and determined to fit on a football field, and the roles are assigned, take the students to the field and simulate the motion of the solar system.

Week Three

Day One: Now that the students are versed in both the US and metric systems, it's time to put the knowledge to work. In order to provide a database of actual lengths to work with in subsequent chapters, like the ones dealing with perimeter, area, the Pythagorean Theorem, scale drawings, and so forth, the class should take some actual measurements, starting with the room. Using a prepared spreadsheet, break the students into groups of four, and have them measure the room's length, width, height, floor and ceiling diagonals, and wall diagonals in the US System. If possible, measure the windows and doors the same way. If this does not provide sufficient information, go to the hallway, and measure there. If your neighbor is cooperative and has a differently sized classroom, why not trade rooms for more data?

Day Two: Continue measuring the classroom and its surroundings using the metric system. Tabulate data.

Day Three: Lay eight students head to toe on a flat surface, and have students roll out an adding machine tape from the first student's head to the eighth student's toe. To establish an "average," take the sum measure of the students and fold the paper into eight equal sections (connection!) and compare the length of a section to the height of each student who participated. Taking a look at how well the average represents the height of the typical student, and then look at the likely disparate measurements that each group of students made. Suggest that perhaps the same process could be used for the different results, and that way a uniform number could be used for the entire class. When the averages are computed (this could be used as an assessment tool) poll the students to see which system was easier to use to find the mean. This should further reinforce the ease of the metric system.

Day Four: Reflect, Review, Reinforce

Day Five: Test

Evaluation

1. 20%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 35%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

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Unit 2

Rational Numbers with Integers

Unit 2: Rational Numbers (with Integers)

In this unit the students will be introduced to rational numbers. Focus will include decimal, common fraction, and integer form. The successful student will complete the unit with the ability to apply fractions, decimals, and integers to real world problems and situations.

Unit Objectives

Students will:

1. Evaluate algebraic expressions. (1.1)
2. Use the associative, commutative, and distributive properties. (1.4)
3. Simplify real number expressions. (2.1)
4. Determine the additive and multiplicative inverses of a number. (2.2)
5. Determine the absolute value of expressions. (2.3)
6. Use exponents with real numbers. (2.4)
7. Distinguish between rational and irrational numbers. (2.6)
8. Compare real number expressions. (2.8)

Workplace Relationships

1. Air conditioning/refrigeration & furnace/heating repair: Monitoring temperature variance in buildings, coolers.
2. Stock broker/banker: Calculating capital gains/losses from daily stock reports.
3. Electrician/electronics technician: Finding amplitudes of radio frequency signals charted on an oscilloscope.
4. Tactical officer, US military: Determining firing solutions for opposing aircraft, vessels, or ground forces.
5. Diver, oceanographer, geographer: Determining mean depth or altitude over non-uniform surfaces above and/or below sea level.
6. Quality control specialist (steel mill): Setting and monitoring precision instruments that determine width and thickness of steel rolls.
7. NFL statistician: Recording and calculating net yardage.

Learning Activities/Teaching Strategies

This unit employs cooperative learning techniques and activities to accomplish its tasks.

The following materials are necessary for the instruction of this unit:

350 blue poker chips	350 white poker chips	100 "ring" magnets
35 pay stubs (decimal)	35 debt lists (decimal)	35 pay stubs (fractional)
35 debt lists (fractional)	10 walking odometers	10 meter sticks
10 copies of USA Today each day for one week		10 postal scales

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Week One

Day One: This lesson will require a bit of patience and creativity on the part of the teacher. Using a modern VCR with indexing capabilities, the teacher should first record a football game from television, marking all the places where a particular running back makes a gain or a loss. After the game, the teacher should use a second VCR or a camcorder to edit together all the rushes made by that player.

In class, the videotape can be viewed by the students with the goal of determining the net yardage gained by the running back. To model the situation, break the students into groups of four. Each group is to be given several blue chips and an equal number of white chips. Explain that the blue chips are "losses" and the white chips are "gains." Further explain that when a white chip and a blue chip are placed together, they make "no gain." Show the video tape to the class, demonstrating how to measure gains and losses. Make certain that some of the viewed rushes require the addition of additive inverses, by showing how the team has merely returned to the line of scrimmage. Have the students work within their group to arrive at the proper sum. This total can be confirmed with the newspaper coverage of the game. This activity should take no more time than the first day of the unit.

Day Two: After the initial activity, explain that this concept of negatives and positives is evident in the physics world as well. Supply each group of students with 10 "ring" magnets and 1 pencil. (Each magnet should have an N and an S marked on each side to designate polarity.) Each group should assign a person to be the group record keeper. Have one of the students (not the record keeper) hold the pencil vertically while another student (again, not the record keeper) places a magnet over the top and rests it on the hand of the pencil holder with the N facing up. The third group member should take another magnet and place it over the top of the pencil with N facing up. The record keeper should record the result. Then, one of the students takes the top magnet off of the pencil and puts it back on, this time with the S facing up. The result is again recorded. Have the students experiment with several magnets at the same time. Allow the groups to "do their own thing", but make certain that one member is recording the results of every experiment (20 minutes).

At the end of the activity, have a class discussion of each group's findings. Detail any inconsistencies in results found by different groups. Highlight and record important conceptual facts discovered by the groups. Focus on what happened when "opposites" were combined. Explain the idea of additive inverses. (15-20 minutes)

Review the ideas discussed on Day One. Explain the concept of "charge boxes", drawing an analogy to the magnet exercise of the previous day as well as the football example from day one. Expand the idea of negative and positive numbers to include decimal numbers. Put the class into groups of four, designated as "families". Assign one of the group members to be the head of the "family" and assign another group member to be the "keeper of the checkbook". Give each student a "salary" and a list of bills for which they are responsible. (Dollar amounts should be in decimal notation.) It is up to the group to determine which bills must be paid. The "family" must also determine how much extra money they would need to put into their checkbook to cover all of their bills. (30 minutes)

Discuss the findings of each group. Verify correct answers and assist with any difficulties that group members may have with decimal numeration.(15 minutes)

Assign appropriate homework to reinforce student learning. Students may begin working in class if time allows.

Day Three: Subtraction can be demonstrated by removing manipulative chips from sets that are already in place. The difficulty here is, if there is not enough of a particular kind of chip to remove, then one has to bring in neutral sets, made up of a positive and negative chip, until there are enough chips of the right kind to remove. Using the same groups as before, lead the students though several problems and question them until they begin to make the connection that shows that addition and subtraction really aren't the separate things they appear to be.

Day Four: Using the poker chips as manipulatives, the teacher should begin to formalize the addition process, leading the students through a small number of exercises, increasing the integers involved until the manipulatives become inefficient, and a generalization becomes desired. Through discovery, the students can begin to form their own rules for addition (20-25 minutes).

By using the poker chips once again, the teacher can lead the students through the process of learning how to add quickly several equally sized groups of both positive and negative integers, until

once again a generalization is more desirable than the use of manipulatives. When this generalization is discovered, then a table can be constructed leading students to identify a pattern and extrapolate a method for multiplying negative integers by negative integers. Since division is basically multiplication done in reverse to students of this age, the concept of the rules applying to division shouldn't be too much of a stretch (30-35 minutes).

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the material. If no one asks, remind the students to think about the things they learned in their poker chip and magnet experiments, as well as their "family checkbook" activities. (10-15 minutes)

Administer a quiz. (20-25 minutes)

After all of the quizzes have been turned in, give each student a copy of the correct answers for immediate feedback. Discuss the correct answers, and answer any questions to clear up any confusion that the students may have. Do not assign any homework. (10-15 minutes)

Week Two

Day One: The teacher will give a short lecture to reinforce the relationship between decimals and fractions. The focus should include conversion from fractions to decimals and back. Also, attention should be paid to addition and subtraction of fractions, with detailed emphasis on common denominators. (15-25 minutes)

Use stock market information from the USA Today to demonstrate examples of how to calculate gains and losses on given stocks over a two day period. Give the students a couple of problems on the board to make sure that they have retained this information. (10-15 minutes)

Break students into groups of four and have each group peruse the Stock Market report in the USA Today. Each student should select one stock that they think has the potential to rise in value during the next week. Suggesting Nike or other shoe companies is often a good motivator for this activity. Each group will then have a stock portfolio of 4 companies to follow for one week. It will be necessary for each student in the group to record the opening value for each stock on a clean piece of notebook paper and keep track of each stock's progress daily for a week. Students will need to spend time daily, both in and out of class, updating the most current information on their portfolio. On Fridays, the students will each turn in a paper detailing the rise and/or fall of their stock portfolio in fraction and decimal form, along with a short explanation of which stock they chose for their group and why. (30-45 minutes)

If time permits, discuss any questions that students may still have regarding addition and subtraction of decimals or fractions, and any questions that still remain concerning conversion between these two methods of notation.

Day Two: Give students a short period of time to gather necessary stock information from the newspaper for their stock project. (10 minutes)

Break the students into groups of four and give each group a walking/jogging odometer. Under close supervision, have each group take turns walking a pre-designated route through the school halls. One of the students should record the starting mileage and ending mileage on the odometer for each group. (Make certain that the distance walked is enough to move the odometer significantly. Five hundred yards is sufficient.) After each group has completed their walk, disconnect the odometers and take the entire class to the end of the route. Reconnect the odometers and have the students walk back to the classroom. Each group should record the results of the return walk.
(15-20 minutes)

When everyone has returned to the classroom, discuss the results with the students. Make note of the fact that the students walked a specific distance from point A to point B and back. Record both sets of distances on the board for each group. Wonder aloud why the first and second distances are both

positive and the same. (Why wasn't the second distance the "opposite" of the first distance?) Use student interaction and discussion to introduce the idea of absolute value. (15 minutes)

Reinforce the order of operations. In groups, assign a worksheet which includes examples such as: |-3|, 2|-5.34| - 9|-4.63|, and so on. The worksheet should challenge the students to use order of operations and decimal, fractional, and integer notation, as well as the newly introduced idea of absolute value. (20 minutes)

Assign appropriate homework to reinforce student learning. Students may begin working in class if time allows.

Day Three: In purchasing, a buyer will often compare prices based on per unit cost. Show the class by using advertisements in the newspaper how the price for a can of soda can differ by bringing it from home rather than buying it in a machine at school. For example, if the soda can be bought for $5.99 per 24 can case, then each can costs approximately 25 cents. Buying the soda from a machine can cost anywhere from 35 to 65 cents, even up to $1.25 in some machines in Chicago. Ask why the price is higher for the machines. Sometimes students will say that it's not cool to bring food or soda from home. This is all right. Simply explain that the high cost of the soda in the machines is the cost of convenience, and that if they're willing to throw that money away for convenience, that's their business. But then ask them how much they're willing to pay to be cool.

Use the cost of a can of soda in the building as an example of how much this could cost over one year. Ask the students to compare the cost of buying one can of soda per day for an entire 181 day school year in the building, and doing the same thing bringing the soda from home. When they see how much money they could save, ask them what they could buy with the money they saved. This usually will motivate them. Then ask them what else they ordinarily buy every day. If no one volunteers anything, suggest potato chips. This time, show how the calculations can be made per ounce, or better yet, per gram. For homework, ask the students to calculate ten unit prices from newspaper advertisements. (50-60 minutes)

Day Four: Explain the Distributive Property. Demonstrate examples such as 72 x 3. Break the 72 into 70 + 2, and explain that the 3 is being "distributed" over these two numbers. Supply several examples for the students to work individually. Check for accuracy. (10 minutes)

Discuss the idea of variables. Explain that sometimes a problem may be assigned that will tell the student the exact value of a variable. Write the algebraic expression $3(x - y)$, $x = 4$, $y = -5$ on the chalkboard. Ask for student feedback on how they might solve this problem. When the correct method has been explained, have that student go to the board and complete the problem. Ask if anyone has another idea how to solve the problem. Wait for someone to suggest the Distributive Property. Use subtle prompting if necessary. When someone suggests the Distributive Property, have them go to the board and complete the problem using this method. (10-15 minutes)

Break the students into groups of four. Reveal several algebraic expressions on the board without values for the variables. Have each group select values-both positive and negative-for each of the variables. One student in the group should keep track of the values that were assigned to each variable. Each group should solve the expressions using their own assigned values. The groups should then report what values were used for each problem and what their answer is. Verify the correct answers and locate errors in the wrong answers. Inform the class that there will be a quiz the next day. Assign appropriate homework to reinforce student learning. Students may begin working in class if time allows. (20 minutes)

Day Five: Give students a short period of time to gather necessary stock information from the newspaper for their stock project. (10 minutes)

Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the material. (10-15 minutes)

Administer a quiz. (20-25 minutes)

After all of the quizzes have been turned in, give each student a copy of the correct answers for immediate feedback. Discuss the correct answers, and answer any questions to clear up any confusion that the students may have. Do not assign any homework. (10-15 minutes)

Week Three

Day One: Take the class on a field trip to LTV Steel. Students will be given a guided tour of the mill which will focus on every aspect of steel making. This will include-but not be limited to-computer operations, on-line maintenance, furnace operation and maintenance, and quality control, with numerous examples of fractional and decimal measurements being employed. At the end of the tour, students will participate in a question and answer session that will include information about various jobs in a steel mill and prerequisite skills for each job. Students should be given an information packet at the end of the field trip that details the highlights of the trip and includes all information regarding employment by LTV Steel.

Inform students that there will be a quiz the next day that will include all material learned during the week, as well as an essay question detailing what they learned on the field trip.

Day Two: Give a brief quiz over the field trip to see if they were paying attention, and ask them to write a three paragraph essay on how the trip related mathematics to the real world. (20 minutes)

Give students a short period of time to gather necessary stock information from the newspaper for their stock project. (10 minutes)

Discuss the meaning of the word "irrational". Explain that someone who is irrational is kind of mixed up. Use this definition to demonstrate the meaning of an irrational number in algebra. If a number is irrational, it is "mixed up". That is, it cannot be expressed as a terminating or repeating decimal. The decimal representation of an irrational number keeps going forever with no repetition. Further explain that an irrational number cannot be expressed as the ratio of two integers. Spend several minutes looking at examples such as the decimal representations of pi and e, as well as radicals such as the square root of 2. Make sure that all students are comfortable with the idea of irrational numbers. Then, briefly explain the Pythagorean Theorem to the class. Do not dwell on the concept. Simply tell them how to use it. (20 minutes)

Optional: Break the students into groups of 4 and supply each group with a 25-foot tape measure. Take the students out to the baseball field and have them measure the distance from first base to second base and from second base to third base. Have each group use this information-along with the Pythagorean Theorem-to find the throwing distance from first to third base. The students should express their answers in radical form and decimal form. Have the students write their decimal answer to eight decimal places so they can observe that the digits do not yet repeat. You may want to discuss this at greater length with some students. (20 minutes)

Discuss the results with the class. Answer any questions that may arise. Do not assign homework. (5-10 minutes)

Day Three: Give students a short period of time to gather necessary stock information from the newspaper for their stock project. (10 minutes)

Demonstrate to the class how to use a postal scale. Break the students into groups of four and supply each group with a postal scale (a digital scale would be ideal). Have each group begin weighing random items that they have on their person (bracelets, pencils, earrings, wallets, combs, brushes, and so forth). Have one student record each item and how much it weighs. Each group should weigh at least 10 items. After all of the items have been weighed and recorded, the groups should write the items down in order from lightest to heaviest. (15 minutes)

Discuss the idea of comparing real numbers with the class. Explain that ranking things in order according to a specific criteria is a way of comparing numbers. Write several real number expressions on the board. Have the students work in their groups to evaluate and compare these expressions. (15 minutes)

Assign appropriate homework to reinforce student learning. Students may begin working in class if time allows.

Day Four: Give students a short period of time to gather necessary stock information from the newspaper for their stock project. (10-15 minutes)

Review the highlights of the previous 3 weeks. Discuss and reinforce any ideas that students may still be unclear about. Answer any questions that students may have.
(15-30 minutes)

Administer a short "practice test" to prepare the students for the real test the next day. (10 minutes) Answer any other questions that the students may have. (5-10 minutes)

Day Five: Give students a short period of time to gather necessary stock information from the newspaper and complete the data collection for their stock projects. Students may turn in their stock projects at the end of this time or at the beginning of the next day's class. (20 minutes)

Administer the test. (30-40 minutes) After all of the tests have been turned in, give each student a copy of the correct answers for immediate feedback.

Evaluation

1. 20%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 35%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method, Book One*; Houghton-Mifflin Company, Boston.

Indiana Core 40: Algebra First Course. (1994). Indiana Department of Education, Indianapolis.

Musser, G.; and Burger, W. (1991). *Mathematics for Elementary Teachers*, 3rd edition; Macmillan Publishing Company, New York.

Unit 3

Algebra in a Plane

Unit 3: Algebra in a Plane

The manner in which points and lines are graphed today was created by a French soldier named Rene` DesCartes over 350 years ago. His idea for picturing points and lines worked so well that the system was named after him: The Cartesian Coordinate System. This unit focuses on the usefulness of the coordinate system and its application to real world situations. The students will explore concepts of the coordinate system which are used in the world of employment.

Unit Objectives

Students will:

1. Graph and locate sets of real numbers on a number line. (4.1)
2. Graph ordered pairs. (4.2)
3. Find distance between two points on a number line. (4.3)
4. Graph a relation on the coordinate plane. (4.4)
5. Distinguish between a relation and a function. (4.5)
6. Graph a relation given an equation and a domain. (4.6)
7. Sketch a reasonable graph for a given relation. (4.7)
8. Interpret a graph representing a real world situation. (4.8)
9. Use graphing technology to explore the graph of functions. (4.9)
10. Determine if data are behaving in a linear fashion. (5.1)
11. Graph a linear equation in two variables. (5.3)
12. Graph a line given the y-intercept. (5.4)
13. Find the slope of a non-vertical line given the graph of a line or an equation of the line or two points on the line. (5.5)
14. Describe the slope in a real world linear relationship using real world terms. (5.6)
15. Write the slope of an equation of a line. Find the slope and y-intercept of a line. (5.7)
16. Write the equation of a line given the slope and one point or two points on the line. (5.8)
17. Write an equation of a line which models a set of real data. (5.9)
18. Use the line which models real data to make predictions. (5.10)
19. Foreshadow inequalities by describing the location of an ordered pair in a relation to a line. (5.11)
20. Graph a linear inequality in two variables. (5.12)

Workplace Relationships

1. Nurse's Assistant: Charting a patient's progress by charting.
2. City Planning Dept. Worker: Expanding/modifying a city's limits.
3. General Contractor: Designing functional stair systems, plumbing and electrical systems: determining the appropriate pitch of a roof.
4. Drafting: CAD Applications
5. Manufacturing: Setting up and operating a milling machine.
6. Aerospace Industry: Navigation and piloting simulations.
7. Economist: Using supply and demand models.
8. Computer Programmer: Developing software that creates artwork and animation.

Learning Activities/Teaching Strategies

This unit employs cooperative learning techniques and activities to accomplish its tasks.

The following materials are necessary for the instruction of this unit:

10 TI-82 graphing calculators	10 tennis balls
10 4-foot long pvc pipes	10 stop watches

Week One

Day One: Break the students into groups of four. Take the students on a walk in order to gather locational information about important buildings, parks, and so forth from the neighborhood around the school. (It may be necessary to obtain permission from the school administrator to take the class on a "neighborhood tour".) Designate the school as "Ground Zero" (the origin). (45-55 minutes)

Day Two: Using the information gathered from the previous day, each group should make a map of the neighborhood in which the school is located. Each map should include a key, a scale and a directional compass. (45-55 minutes)

Day Three: Begin the class period with a short lecture on the Cartesian plane. Focus should include naming the quadrants, axes, and origin, and an introduction into graphing points in the plane. (15 minutes)

Now that the students have the knowledge of the Cartesian plane, each group should label the various locations on their map with a coordinate. (10-15 minutes)

Students should determine the distance in units between various locations on their map that lie on the same north-south and east-west streets. Encourage them to use locations that lie on either side of the school. Have each group find at least 10 distances. Each group will have to report how they found these distances. (20 minutes)

Select a set of points at random [for example, (2,1),(1,-3),(-4,-2),(0,5)] and have the students graph these points on a sheet of graph paper. After this has been done, the groups should report what lies at those locations around the school. (10 minutes)

Day Four: The graphing/mapping activities of the previous three days should be tied together with appropriate textbook work and lecture to explain the ideas behind the real-world map-making concepts that have been utilized. Explanation of a relation should be given on this day. (55 minutes)

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz. (20-25 minutes)

After all of the quizzes have been turned in, give each student a copy of the correct answers for immediate feedback. Discuss the correct answers, and answer any questions to clear up any confusion that the students may have. Do not assign any homework.
(10-15 minutes)

Week Two

Day One: Begin the class with a short lecture on relations and functions. Focus on the differences between the two. This would be a good time to introduce the vertical line test and the idea of domain. (20 minutes)

Break the class into groups of four. Give each group a TI-82 graphing calculator to use. Explain that graphing calculators are designed to graph functions. Write the equation $x = y2$ on the board and challenge the groups to find a way to graph it. After 3-4 minutes explain that this is not a function, so in order to graph it, to separate equations need to be used. Demonstrate to each group how to graph this relation using the equations $y = \sqrt{x}$ and $y = -\sqrt{x}$. (10 minutes)

Write several other relations-including some functions-on the board for the groups to work with. Each group should record whether the relation is a function or not, and how it was graphed. This exercise should give all students in each group an opportunity to use the graphing calculators. (30 minutes)

Day Two: Break the class into groups of four. Give each group a 4-foot long pvc tube, 8-10 textbooks of comparable thickness, a tennis ball, and a stopwatch. Have the students take the pvc pipe and lean it against a stack of 5 books. One student should hold the pipe in place, while a second student should drop the tennis ball in the top of the pipe. A third student should use the stopwatch to time how long it takes for the ball to come out the bottom of the pipe, and the fourth student should record the time and the number of books used for the test. Each group should repeat this experiment several (6-7) times using stacks of different heights. After all of the trials have been completed, the group should develop a theory about the relationship between the slant of the pipe and the time for the ball to exit the tube. (15-20 minutes)

Using the pvc tube experiment as a reference, introduce the idea of slope. Discuss what happens when a slope gets very steep and very level. At this point, it will be necessary to introduce the slope formula. Give the class several sets of points, and explain how they can find the slope of the line that connects them simply by plugging in the values to evaluate the expression. (20 minutes)

Have the students select 2 locations on their neighborhood map-one that is on the same north-south street as the school (y-axis) and another that is not on an "axis". Have the students find the slope of the line that goes through both locations-first by evaluating the slope formula, and then by counting "up and over". Use this opportunity to introduce the slope-intercept form of a line. (20-25 minutes)

Day Three: Break the class into groups of four, and supply each group with a TI-82 graphing calculator. Using information from a real-life model, have the students plot a series of points on the calculator. (Stat Plot will need to be used.) After the information has been entered and graphed, the students should use the CALC menu in the STAT mode to find the curve with the "best fit" (As the teacher it is your responsibility to make sure that the curve with the "best fit" in this activity is a line). Students should experiment with the various forms of regression EVEN IF THEY DON'T KNOW WHAT THEY MEAN! After completing this exercise, the groups should interpret what will occur with the graph if other values for x are included. Each group member should be able to describe the action of the slope and determine its value in easy-to-understand terms. (20-30 minutes)

Using the slope and one of the points from the above graph, Have the students evaluate the expression $y - y1 = m(x - x1)$. Explain that the x and y values for the coordinate should be substituted for the x1 and y1. (Don't dig too deeply into theory here. Let the students evaluate the expression and ask any questions that they might have.) After they have finished this step, assist the students in solving the equation for y. When this is finished, point out that the result is an equation in point-slope form (Refer to the previous day's introduction). (15 minutes)

Have the students repeat the exercise with the same slope and a different point. When they have completed this procedure a second time, discuss with the class why the result was the same with a different point. Assign appropriate homework to reinforce learning objectives. (10-15 minutes)

Day Four: Break the class into groups of four, and supply each group with a TI-82 graphing calculator. Write several sets of points and slopes on the board along with the equation for the point slope form of a line. Using the skills learned on the previous day, have the groups find the slope-intercept form of the lines with the given points and slopes and graph them one at a time on the calculator. Point out to the class that when they graph these equations, there are 2 variables involved. Explain that it is necessary to have both variables in order to have a point. (20-25 minutes)

After the students have completed this activity, assign a group worksheet that involves 3-4 real-world applications which model linear relationships. Have the students work in their groups to find the appropriate line for each problem. (30 minutes)

If time remains, discuss the solutions aloud with the class. Answer any questions that individuals may have. (5-10 minutes)

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz. (20-25 minutes)

After all of the quizzes have been turned in, give each student a copy of the correct answers for immediate feedback. Discuss the correct answers, and answer any questions to clear up any confusion that the students may have. Do not assign any homework.
(10-15 minutes)

Week Three

Day One: Break the class into groups of four, and supply each group with a TI-82 graphing calculator. Have the students graph the following equation: $y = 2x - 4$; on their calculators. Ask several students to locate points on the line. Examples will likely include (0,-4),(2,0), and so on. Ask the students about the points (0,5),(-7,3), and (4,6). Upon investigation, the groups will report that these points lie above the line. Have each group locate coordinates of 5 more points that lie above the line. (10-15 minutes)

Explain that there needs to be a method to describe the relationship between lines and points in the plane that are not on the line. Discuss the idea that points on the line are not a part of the line so they cannot "equal" a part of the line. Ask what word means "not equal". When the term inequality is finally mentioned, discuss a way to write the relationship between the points above the line and the line itself. Eventually, lead the students to the inequality: $y > 2x - 4$. Point out that since the points being discussed are higher than the line, a "greater than" symbol is used. Discuss points below the line and points on the line. Explain $<$, $>$, and $>$ in the plane, using solid and broken lines to demonstrate inclusion or non-inclusion of the line in the inequality. Assign appropriate homework for reinforcement. (35-45 minutes)

Day Two: Break the class into groups of four, and supply each group with a TI-82 graphing calculator. Further reinforce the method of graphing linear inequalities with 4-5 real-world problems drawn from the text. (30-40 minutes)

Discuss the students' findings. Give each group an opportunity to report their findings. (20-30 minutes)

Day Three: Explain that in real-world situations, sometimes it is necessary to use a series of linear inequalities to find the "overall" solution to a problem. Write a linear programming model on the board that involves maximizing profit or minimizing cost. By this time, the groups should be comfortable with the line graphing, but they will need help interpreting the meaning of all of the linear intersections. After all of the groups have graphed their lines, assist them in determining the point of maximum (or minimum) value. Use this point in all of the lines to show that it truly is the maximum (or minimum) value. (20 minutes)

Give each group 2 different linear programming problems to work. Assist with any questions that the groups might have. (20-30 minutes)

Discuss the findings of each group. Hand out photocopies of the correct answers to each group member. Assign one linear programming application as homework. (10 minutes)

Day Four: Review the important learning objectives from the previous four days. Answer any questions that the students may have. (20-30 minutes)

Administer a practice worksheet for the students to complete and turn in at the end of the period. Allow students to use textbooks and notes. (30 minutes)

Day Five: Administer the test. (45-55 minutes)

After all of the tests have been turned in, give each student a copy of the correct answers for immediate feedback. If there is time, discuss the correct answers, and answer any questions to clear up any confusion that the students may have. Do not assign any homework.

Evaluation

1. 20%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 35%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

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Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method, Book One*; Houghton-Mifflin Company, Boston.

CORD, *Applied Mathematics, A contextual approach to integrated mathematics*; Unit 17-Graphing Data; (1991); Cord Communications; Waco, Texas.

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Musser, G.; and Burger, W. (1991). *Mathematics for Elementary Teachers*, 3rd edition; Macmillan Publishing Company, New York.

Unit 4

Solving Equations

Unit 4: Solving Equations

Suggested Time Frame: 3 weeks

In this unit the students will learn to solve one-step, two-step, and literal equations. The principle focus will be on solving equations that apply to real-life situations. The successful student will complete the unit with the ability to read, interpret, and solve applied equations.

Unit Objectives

Students will:

1. Use formulas to solve problems. (1.2)
2. Solve an equation using the addition property of equality and the idea of additive inverse. (3.1)
3. Solve an equation using the multiplication property of equality and the idea of multiplicative inverse. (3.2)
4. Solve an equation graphically and by using more than one property of equality. (3.3)
5. Solve an equation that contains like terms. (3.4)
6. Solve an equation which has the variable in both members. (3.5)
7. Solve an equation in which the numerical coefficients and constant terms are fractions or decimals. (3.6)
8. Solve a formula for one of its variables or find the value of a variable when values of the other variables are given. (3.7)
9. Use problem solving skills to solve real world and "word" problems which involve a linear equation or formula. (3.8)

Workplace Relationships

1. Fence Construction: Determining the correct amount of fencing necessary to enclose an area to the customer's specifications.
2. Entrepreneur: Finding the break-even point on an item to find out how much product should be made to generate a profit.
3. Pharmacist's Assistant: Using mixture equations to find the correct amount of two medicines to mix together.
4. General Contractor: Using a work equation to determine how long it will take to complete a job if more than one man works on the job.
5. Cement Contractor: Determining how high a concrete slab will buckle when it is subjected to a rise in temperature.
6. Car Rental Employee: Determining the cost of renting a vehicle to a customer for a specific period of days and miles.
7. Navigator: Using the Pythagorean theorem to determine distance from a fixed point.

Learning Activities/Teaching Strategies

Week One

Day One: Break the class into groups of four and give each group a balance scale, six 1-1/2 pound blue blocks, and twenty 8-ounce yellow blocks. Have the students position one blue block and four yellow blocks on one side of the scale, and 7 yellow blocks on the other side of the scale. Students should remove like pairs of blocks from each side of the scale simultaneously until there are no more like pairs. At this point, have the students discuss within their groups and come up with a hypothesis about the relationship that exists between the blocks that remain on the scale.

Experiment with adding to both sides, then abstract it into multiplication and division. Emphasize inverses such as adding versus subtracting and multiplication versus division.

Day Two: The activity from Day One is to be expressed in written form. First have the students describe and summarize the previous day's activity and complete a few new examples. Then incorporate the concepts into equations using decimals and fractions.

Day Three: Demonstrate how to model variable expressions using algebra tiles. Then demonstrate how to combine like terms using the tiles. Have the students successfully complete several problems using algebra tiles. The students may need to model their work or paper for their own use or to turn in to you.

Day Four: Compare combining like terms using algebra tiles to the written work for the same problems. Separate students into groups and let the students individually choose whether to complete the work with the tiles or in written form or both.

Day Five: Reflect on the activities and concepts from this week. Answer any questions the students may have. Review methods to solve problems.

Administer a quiz covering this week's concepts.

Week Two

Day One: Review the activity done during week 1 with the algebra tiles. Introduce solving equations by use of algebra tiles. Today concentrate on solving equations by adding or subtracting from both sides of the equation or separating both sides into equal groups (dividing). Once this has been mastered, continue with variables on both sides of the equation.

Day Two: Continue with the algebra tiles today by letting the students show how equations with like terms can be solved by using the tiles. After a few students have demonstrated how the equations are solved, have the students work individually to complete the solutions to several equations of different types as covered during Day One or Two.

Day Three: Compare using tiles to solving the same problems using the standard written form. Have the students compare and contrast the two methods verbally until they seem confident using the written method. Assign a few problems for them to present tomorrow to the class.

Day Four: Choose a few of the students from yesterday to present the solutions of the equations to the class. Have them demonstrate both the written and the modeled form (algebra tiles) of the solution. Separate the students into groups and have them work several equations of different types. Encourage them to concentrate on the written form of the solution at this time.

Day Five: Review, connect, reinforce, reflect. Administer a quiz.

Week Three

Day One: Separate the students into groups. Each group is to find a variety of objects of varying sizes to measure in inches and in centimeters as accurately as possible and record their data. Then they will construct a coordinate plane and graph their data. After selecting a best fit line they must determine if the graph is linear, and if it is, the equation of the graph. To check their equation, an object not measured by anyone in the group must be measured in one unit (inches or centimeters) and the equation used to convert the measurement to the other unit. The object can then be measured to determine the reliability of the equation.

Day Two: Formulas (equations) are used to convert from one unit of measurement to another as in yesterday's activity. Another formula that is frequently used is the one for converting from Fahrenheit

to Celsius. Using this particular formula, give the students several Celsius temperatures and have them determine if the day is suitable for swimming by converting the temperature to Fahrenheit. The object is to get them to solve the same equation over and over until they decide to find a better way - converting the formula to its inverse, then using the new formula for computation instead of continually resolving the same equation. Suggest that there are other times when a formula may need to be converted to benefit the user. Have them try converting their own formula from yesterday and check out the reliability of the new formula.

Day Three: Present several formulas in which the substitution for all but one of the variables has been made. (Example: 48=6w) Have the students solve for the variable. Sequentially have the students take the original formula and solve for the same variable. (Example: A=LW solve for W). Have the students verbalize how doing it the first way can help them convert the formulas. Give them several formulas to convert.

Day Four: Tie together the work done the past 3 weeks with solving equations and converting formulas. Review the process and answer any questions that the students still have. Provide a practice worksheet for the students to use in preparing for the following test.

Day Five: Administer the test.

Alternate Learning/Laboratory Activities

(These activities are intended to supplement your own materials for this unit.)

Break the students into groups of four. Give each group 3 1000 ml beakers; one filled with 500 ml of an 11% vinegar solution, one filled with 500 ml of a 6% vinegar solution, and one that is empty. Each group will also need an instrument to test acidity levels. The task is for the groups to determine how many ml of each type of vinegar are needed to make 600 ml of an 8% vinegar solution. Remind the students that they do not want to wildly dump solutions together. If the volume and the acidity level of the mixed solution both exceed the maximum, it will be necessary to begin the experiment again. Solutions should be slowly mixed and constantly tested.

Break the students into groups of four. Give each group a one-gallon basin, 3 different sized funnels with attached tubes, and 2 gallons of water in separate milk jugs. Pour one gallon of water through funnel #1 and into the tub, recording the length of time it takes for the funnel to fill the basin. Drain the basin and repeat this exercise for each of the other two funnels. After the individual funnels have been timed, repeat the exercise using funnels #1 and #2, #2 and #3, #1 and #3, and #1, #2, and #3. Record these times and develop a theory about this work relationship.

Break the students into groups of four. Give each student a thermometer that measures both Celsius and Fahrenheit. Have each group read the thermometer and record "temperature pairs" (such as: 50o F and 10o C, or 32o F and 0o C). By trial and error, each group should develop a theory regarding the relationship between Celsius and Fahrenheit.

Give each student in the class a 3x5 index card that has a short list (2 or 3) of "superhuman powers" that each student possesses. Divide the class in half. A student on each team who possesses the skill of leadership will be in charge of his/her team. Each leader, with the assistance of the team members, will proceed to find out how many of each type of superhuman power their team has. After both sides have determined their skills, the teams will square off against each other with the following goal: Evenly balance the skills so each side has exactly the same abilities. The only thing that will prevent the two sides from starting a war is if they have equal strength. The teams must trade personnel to achieve a balance. (Hint: Treat each ability as a variable. For example, if one team has three people who can fly and two people who can breath underwater, they have "3 flyers + 2 water breathers".) Let the class work to balance the sides correctly. Only interrupt if they are about to make a fundamental algebraic error.

Evaluation

1. 20%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 35%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

Indiana Core 40: Algebra First Course. (1994). Indiana Department of Education, Indianapolis.

Musser, G.; and Burger, W. (1991). *Mathematics for Elementary Teachers*, 3rd edition; Macmillan Publishing Company, New York.

Unit 5

Solving Inequalities

Unit 5: Solving Inequalities

Suggested Time Frame: 3 weeks

In this unit, techniques for solving inequalities will be developed in a manner similar to that of solving equations. Methods for solving simple inequalities are extended to include conjunctions, disjunctions, and absolute value. Applications that are best determined using inequalities are solved.

Unit Objectives

Students will:

1. Find the solution set for a linear inequality when replacement values are given for the variable. (3.11)
2. Solve a linear inequality by using properties of order. (3.12)
3. Use problem solving skills to solve real world or "word" problems which involve inequalities. (3.13)
4. Find the solution set of combined inequalities. (3.14)
5. Graph a linear inequality in two variables. (5.12)
6. Solve an inequality involving absolute value.
7. Graph an inequality on a number line.

Workplace Relationships

1. Quality Control Expert: Determining the target thickness (for example) of a sheet of steel within a certain range of tolerance.
2. Computer Programmer: Using linear inequalities to minimize or maximize application-based scenarios.
3. Economic Consultant: Analyzing supply and demand applications.
4. Bank Loan Officer: Using inequalities to determine loan eligibility.
5. Project Supervisor: Estimating labor, material costs.

Learning Activities/Teaching Strategies

The following materials are necessary for the instruction of this unit:

20 meter sticks
200 identically sized bolts
10 graphing calculators

Week One

Day One: (Initiatory Activity) Have the students divide into pairs and supply each pair of students with a meter stick. Have the students measure each other's heights to the nearest centimeter. Record all of the heights on the board. Next draw a number line on the board and have the students do the same on a separate sheet of paper. (The range of the number line should be a bit lower than the shortest student and a bit higher than the tallest student.) Mark, by name, the location of each student on the number line. After all of the names have been placed on the number line, introduce the four inequality relationships ($<$, $\leq$, $>$, $\geq$), and use them to demonstrate the relationship between the students (for example, Jim > Jon). Have the students make up lists of their own inequalities using the heights of themselves and their classmates. Give each student a chance to share one or two of their inequalities with the entire class.

Day Two: Ask students to expand on the previous day's activities by exploring a larger domain: the world. By asking how many people on the number line are perhaps shorter than Jim, and asking what that graph would look like, students should begin to think not in dots, but in lines. Care will have to be taken to ensure the students do not assume there are people of negative or even zero height, however. Expand this idea to include a different set of values that can be sufficiently negative, like the national debts of several countries.

Day Three: Use the heights from the graphs of the initiatory activity to demonstrate the properties of order. This is especially effective with live models (if Jim > Jon, and Jon > Violet, then Jim > Violet, for example).

Day Four: Demonstrate how to solve linear inequalities and graph them. One useful application of this skill is the ability to calculate one's own weighted grades. Students can solve to discover what they must achieve on a final exam to attain a desired average score for a semester.

Day Five: Review, connect, reinforce, reflect. Administer a quiz.

Week Two

Day One: Have students graph $y = 6x$ on quartile graph paper over the domain $\{-2, 0, 2, 6\}$ and report their results. When they discover that they cannot graph (6, 36) because of size constraints, ask them to figure out the maximum domain and range that can be graphed given said constraints. When this is done, ask if there was a way to model this dilemma using an inequality. When that is accomplished, finish by asking the students to solve for a minimum domain and range for the other side of the function.

Day Two: Lecture and demonstrate how to graph linear inequalities in two variables manually. Using an appropriate graphing calculator, the class will be assigned a linear programming application in which their objective is to minimize cost or maximize profit. The students will be given a linear equation and several "constraints"- linear inequalities which are meant to confine the problem to a certain set of guidelines.

Day Three: Solve systems of linear inequalities on the graphing calculator.

Day Four: Solve systems of linear inequalities manually. Compare and contrast.

Day Five: Review, connect, reinforce, reflect. Administer a quiz.

Week Three

Day One: Each class will engage in a fund-raising activity to raise money for the Math Club. After setting a goal (purchasing a class set of graphing calculators, for example), each student will determine the number of units of each item (candy, fruit, cookies, and so forth) that need to be sold in order to raise the minimum funds necessary. Each student will need to express this application as an inequality, and solve it for the number of different items being sold.

Day Two: Break the students into groups of four. In cooperation with the U.S. History class, arrange for each group to design and present a timeline over a 50-year time frame. Each group will select a period of time and research the events that shaped the United States during those years. The timeline should include-but not be limited to-foreign and domestic politics, sports, entertainment, and cultural and scientific achievement. Each timeline should include at least 50 events.

Day Three: Break the students into groups of four. Give each group 20 "identical" bolts. Each group will use a micrometer caliper to measure the overall length of each bolt. Each student will take turns measuring the bolts, and one student will record their lengths. After all of the lengths have been recorded, an inequality range will be constructed by each group. Have a spokesman for each group present their findings. After all of the groups have reported, use all of the results to determine a "class inequality". Repeat this exercise using the width of the heads of the bolts.

Day Four: Review

Day Five: Test

Evaluation

1. 15%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 10%: Each student should include a copy of their group's timeline in their own portfolio. This timeline will constitute 10% of this unit's grade.
4. 30%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
5. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method*, Book One; Houghton-Mifflin Company, Boston.

CORD, Applied Mathematics, *A contextual approach to integrated mathematics; Unit 27-Inequalities;* (1991); Cord Communications; Waco, Texas.

Indiana Core 40: Algebra First Course. (1994). Indiana Department of Education, Indianapolis.

Musser, G.; and Burger, W. (1991). *Mathematics for Elementary Teachers*, 3rd edition; Macmillan Publishing Company, New York.

Unit 6

Polynomials

(Part One)

Unit 6: Polynomials (Part One)

Suggested Time Frame: 3 weeks

Exponents and their rules are expanded upon in this unit. Operational rules for exponents are further developed and used in conjunction with additional exponential rules. This unit also focuses on further development of problem solving skills such as uniform motion and area applications.

Unit Objectives

Students will:

1. Write and simplify expressions involving exponents.
2. Add and subtract polynomials. (7.1)
3. Multiply monomials. (7.2)
4. Find an indicated power of a monomial. (7.3)
5. Multiply a polynomial by a monomial. (7.4)
6. Find the product of two binomials. (7.5)
7. Multiply two polynomials. (7.6)
8. Solve application problems.
9. Recognize problems that do not have solutions.

Workplace Relationships

1. Construction Contractor: Manipulating the dimensions of rectangles to determine the effects on perimeters and areas in a blueprint or floor plan.
2. Packing/Shipping Clerk: Determining the number of specific objects of various shapes and sizes that can be packaged into rectangular cartons for shipping.

Learning Activities/Teaching Strategies

This unit employs cooperative learning techniques and activities to accomplish its tasks. The following materials are necessary for the instruction of this unit:

TI-82 calculators Algebra tiles

Week One

Day One: Break the students into groups of four. Give each group a TI-82 graphing calculator and a list of four polynomial functions for them to graph. (Make sure that each student gets a chance to use the calculator!) Explain to the class that polynomials are not just a confusing string of numbers and letters thrown together, but rather a code that tells us how to draw a specific and unique picture. Give the class some time to "play" with the calculators by themselves before directing them to try four new polynomials that you will have written on the board for them. Have the students make a list of patterns that they notice about the polynomials, i.e. opens up, opens down, number of "loops", y-intercepts, x-intercepts, etc. These patterns and equations need to be recorded and saved for futher review in unit 12. A formal written summary of this work might be useful in a portfolio as well.

Day Two: Use algebra tiles to present combining like terms, adding and subtracting polynomials. Use the TI-82s with the skills developed on Day One to have them graph both parts of the problem as well as the answer. Have the students describe what it means graphically to add and subtract polynomials. Then have the students do several problems using the algebra tiles.

Day Three: Compare the answer when adding like terms of a polynomial with what happens when those terms, taken as monomials, are multiplied. Use algebra tiles to demonstrate this. Review the distributive property, again using tiles, then demonstrate multiplying a polynomial by a monomial. Assign several polynomials to be added and subtracted and several monomials to be multiplied.

Day Four: Compare using tiles with the written method of adding polynomials and multiplying by a monomial. Extrapolate from this multiplying $x^2 \cdot x = x^3$ and $x^2 \cdot x^2 = x^4$, etc.

Assign written problems as homework.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Two

Day One: Break the students into groups of four. Take everyone to the gymnasium and have each group measure the length and width of the basketball court. Return to the classroom and assign the following problems to the class: (1) What is the total area (in square feet) of the basketball court?, (2) If the length of the court is decreased by 5 feet and the width is increased by 8 feet, what is the new area?, (3) If the length of the court is increased by x feet and the width is increased by 12 feet, what is the new area?, and, (4) If the length is decreased by y feet and the width is increased by y feet, what is the new area?

Day Two: Reflect on yesterday's problem and point out that equations for manipulating area are not always in the form (l-y)(w+y). They can be presented in a standard form ax2+bx+c. Discuss how the equation from yesterday could be changed into this form.
Using algebra tiles demonstrate finding the product of two binomials. Graph both the equation from Day One and the equation in today's form to show they are the same.

Day Three: Make the conversion from using tiles to multiplying binomials in the traditional written form. Assign several binomials for the students to do. Have them explain in words how they proceeded to do the problems. Compare this with multiplication by FOIL.

Day Four: Compare the problem with the basketball court with finding the volume of a swimming pool. If the swimming pool is (50+x)(100-x) in area and (30-x) in depth, what is the volume of the pool. Adapt the methods used from Day Three to multiplication of polynomials. Assign problems that include concepts from each day this week.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Three

Day One: Two cars are following each other on a long distance trip. They have been averaging 65 mph on the expressway, when one car signals that they are stopping for gasoline. The second car continues, but reduces speed to 55 mph to allow the first car to overtake it. If the first car takes 15 minutes to refuel and get back on the expressway, how long will it take them at 70 mph to overtake the other car?

After the students have solved the above problem jointly, discuss methods that will help them approach solving problems in an orderly manner. Give them several problems for experimentation.

Day Two: Looking at the ideas the students used yesterday for solving problems, compare them with the following GUESS method. G = list all the given, U=list the unknown (sometimes both of the previous can be included in a sketch or chart.), E = formalize the equation, S = solve the equation, S= give the answer as a complete sentence which restates the problem. How is this the same or different from the methods used in Day One?
Assign more problems for the students to solve using a formalized approach.

Day Three: Suppose that the cars from Day One are 2.5 hours from their destination when the first car stops for gas and takes 20 minutes. Suppose the second car continues traveling at 57 mph and the first car resumes traveling at 65 mph. When will the first car overtake the other car? Can there be problems without solutions? How would the graph of these problems appear?

Day Four: Review the important learning objectives from the previous three weeks. Answer any questions that the students may have. (20-30 minutes)

Administer a practice worksheet for the students to complete and turn in at the end of the period. (Or give a review assignment from your current text.)

Day Five: Administer the test.

Evaluation

1. 15%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 10%: Each student should include a copy of their group's timeline in their own portfolio. This timeline will constitute 10% of this unit's grade.
4. 30%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
5. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method*, Book One; Houghton-Mifflin Company, Boston.

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Unit 7

Polynomials

(Part Two)

Unit 7: Polynomials (Part Two)

Suggested Time Frame: 3 weeks

Much of this unit is devoted to factorization of polynomials. Factoring out the Greatest Common Factor is stressed. Special attention is paid to special types of factorizations, such as: differences of squares, sums and differences of cubes, perfect square trinomials, and factoring by grouping.

Unit Objectives

Students will:

1. Factor integers into primes and find the greatest common factor of several integers.
2. Find a common monomial factor in a polynomial. (7.9)
3. Factor the difference of two squares. (7.10)
4. Factor a simple quadratic trinomial. (7.11)
5. Factor a polynomial by grouping.

Workplace Relationships

1. Construction Contractor: Manipulating the dimensions of rectangles to determine the effects on perimeters and areas in a blueprint or floor plan.
2. Packing/Shipping Clerk: Determining the number of specific objects of various shapes and sizes that can be packaged into rectangular cartons for shipping.
3. Military Supply Clerk: Using inventorying techniques to determine how much of each type of item will be assigned to personnel.

Learning Activities/Teaching Strategies

This unit employs cooperative learning techniques and activities to accomplish its tasks.

The following materials are necessary for the instruction of this unit:

algebra tiles	jelly beans	boxes
paper clips	pencils	index cards

Week One

Day One: Take only the "1" squares from a set of algebra tiles. Have the students represent the numbers from 1 to 20 in as many rectangular ways as possible. For instance, the number 3 can only be represented as a 1x3 rectangle or a 3x1 rectangle, so it is prime with the factors of 1 and 3. The number 6 can be represented by 1x6, 6x1, 2x3, and 3x2 rectangles, so it is composite. It's factors are 1,6,2 and 3. To find the greatest common factor of 2 integers such as 24 and 32, you must recognize that 24 can be represented by an 8x3 rectangle and 32 can be represented by an 8x4 rectangle. There are no 2 rectangles with a side greater than 8 to represent the two numbers, so 8 is the greatest common factor. Lead the discussion from factors and prime numbers to representing integers as a product of prime factors. Show how the greatest common factor can be determined from this product.

Day Two: Break the students into groups of four. Give each group four small boxes that have five or six different items in them (for example, paper clips, #2 pencils, tacks, index cards). There should be several, but not the same number, of each item. Have each student inventory and record the contents of their individual boxes. The group should then discuss how each student within the group can pull out exactly the same number of items from their own boxes. View each box as a term of a polynomial and each item as a factor of the terms. Compare this with the GCF from the area in Day One. This will introduce the concept of greatest common factor in ways separated from integers.

Day Three: By reflecting on the activity from the previous days, show how to factor out common factors of a polynomial. Break each term into its "prime" factors and "take out" the largest number of each type (numbers must be represented in prime factorization form). Have the students attempt several polynomials on their own, in groups, or at the board.

Day Four: Have the students analyze the work done yesterday with a view to a shortcut of that method. If necessary, suggest that they especially compare their actions, and the answers, with the exponents used by each variable. Again, have the students attempt several polynomials on their own, in groups, or at the board.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Two

Day One: Using algebra tiles, represent the differences of two squares as rectangles. You will have to use opposites to fill out the square. Show how the sides of the rectangles tell the factors of the binomial. This can be related to Day One of last week with area and to products of binomials in unit 6. The students need to do several examples on their own to become assured that they can do this. Work can be modeled on paper if needed.

Day Two: Using algebra tiles, represent trinomials as rectangles. Adding opposites as necessary to fill out the rectangle. Show how the sides of the rectangles show the factors of the trinomial. This relates directly to the work done yesterday as well as the work last week. Again, have the students try several different types on their own, or in groups to develop their skills. Their work can be modeled on paper as needed.

Day Three: Review the problems done on Day One and Two. Compare this with doing the problems in written form. Compare this with the reverse of FOIL as discussed in the previous unit. Assign several problems of each type from both days to be done in written form.

Day Four: Extrapolate from work done on this unit and the last unit to show how to factor polynomials in which the common factor should be factored out first, then factor the rest into binomials. Present the expression 2x2 -2x-12 to be factored as before, tiles may be used if desired. When the students come up with (2x+4)(x-3) or (x+2)(2x-6), suggest that the polynomial has not been factored completely. Allow the students to suggest what needs to be done. The assignment for today should include types from each day this week as there will be a quiz tomorrow.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Three

Day One: Break the students into groups of four. Give each group a handful of jelly beans (or m&ms) with all of the green ones removed. Since mixing blue and yellow colors will make green, tell the students that 1 blue jelly bean + 1 yellow jelly bean = 1 green jelly bean. Have each student separate all the "green" combinations from their pile. Then "factor out" the same number of "green" combinations from each pile in the group. After this is done, the students can compare the number of jelly beans they have left. Compare this to factoring out factors that have been grouped such as (x+2) from the polynomial x(x+2) + 3(x+2).

Day Two: Present the polynomial x2 + 2x + 3x + 6. Show how x can be factored from the first 2 terms and 3 can be factored from the last two terms (using the distributive property). The polynomial

now looks like x(x+2) + 3(x+2). Compare this with the work done yesterday and with 1 blue + 1 yellow jelly bean. Have the students do several problems of this type.

Day Three: Divide the students into groups. You will need the same number of groups as students in each group if possible. Assign each group one of the objectives to review. Have them make up problems with solutions that reflect that objective. They will need one problem for each member in the group. When the students have done this, rearrange the groups so that the new groups contain one member from each of the previous groups.
The students must then review their groups on the objective they've worked with and have them try the problems they developed. Each member can do a different problem or each member can do all of the problems. This activity should prepare the students for the closing activities of the next two days.

Day Four: Continue with the activity from yesterday and/or review the important learning objectives from the previous three weeks. Answer any questions that the students may have. (20-30 minutes)

Administer a practice worksheet for the students to complete and turn in at the end of the period. (Or give a review assignment from your current text.) The questions developed by the students can be used in the practice worksheet, in the test tomorrow, or in some combination of both.

Day Five: Administer the test.

Alternate Learning/Laboratory Activities

Break the students into groups of four, and send the groups on a controlled treasure hunt. The idea is for each group to be assigned a specific set of items to find. They may bring back as many of each item as they can. (For example, a group may be assigned to find a pencil with the school name on it. They may find 10 or 100.) Make certain that the groups have some overlap in what they are seeking. After all of the groups have returned, inventory each group's "finds" on the board. After all groups are inventoried, "factor out" any common items. The group with the most items left at the end of the "factoring phase" is the winner. (Awarding 5 points of extra credit will help to stimulate interest.)

Break the students into groups of four. Give each group a box filled with mixed-up pieces to several model airplanes, and four sets of instructions. (IMPORTANT: This should be a small, snap together model. Airplane glue should NOT be used!) It is important that all of the pieces for at least one or two of the planes be present, but not all four. Have the groups assemble as many planes as possible with the parts provided. After all groups are done, point out that the number of completed models includes the GCF of each individual piece. Use this as a springboard to deeper discussion of the concept of GCF.

Evaluation

1. 20%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
2. 60%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
3. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method*, Book One; Houghton-Mifflin Company, Boston.

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Indiana Core 40: Algebra First Course. (1994). Indiana Department of Education, Indianapolis.

Musser, G.; and Burger, W. (1991). *Mathematics for Elementary Teachers*, 3rd edition; Macmillan Publishing Company, New York.

Unit 8

Algebraic Fractions

Unit 8: Algebraic Fractions

Suggested Time Frame: 2 weeks

This unit is designed as a preparation for unit nine, which focuses on ratios and proportions.

Unit Objectives

Students will:

1. Divide two monomials. (7.7)
2. Divide a polynomial by a monomial. (7.8)
3. Divide two polynomials.
4. Simplify ratios involving algebraic expressions.

Workplace Relationships

1. Electrical Contractor: Using resistance formula to determine the total resistance of a circuit. (1/R = 1/R1 + 1/R2)
2. Rental Car Employee: Using the formula for average speed of a round trip to fill out rental car return forms. (r = 2v1v2 / (v1 + v2)

Learning Activities/Teaching Strategies

(Most workplace relationships that utilize algebraic fractions involve solving algebraic equations. This unit on algebraic fractions is a stepping stone to solving proportions.)

The following materials are necessary for the instruction of this unit:

10 circuit boards	30 resistors, various resistance
10 9-volt batteries with leads	10 soldering irons, with solder
10 ohmmeters	20 stopwatches

Week One

Day One (Initiatory Activity): The Sieve of Eratosthenes. Give each student a list of the natural numbers from 1 to 100, inclusive. Ask them to identify the first number greater than one that is only divisible by one and itself. Inform them that such a number is called a prime number. Ask, then, if four qualifies. When the students eliminate four because two is a factor, ask them to eliminate it and any other number for which two is a factor. Continue this procedure for three and its multiples, five and its multiples, and continue until a prime number does not have any multiples on the list. Compare to a physical sieve, and be sure to name the numbers, other than one, that were eliminated as composite numbers.

Day Two: Demonstrate for the students how any number can be broken into prime factors in several different ways and how to reduce simple fractions using this method. Also lead students to suggest how exponents can be used to abbreviate the cancellation process by using very large multiples of two to make the process less tedious.

Day Three: Ask students to theorize how algebraic expressions involving exponents can be broken down even though their factors are not "prime." Ask them to hypothesize how they can simplify complex fractions by duplicating the process developed on day two. Introduce the idea of GCF and LCM (least common multiple).

Day Four: Use the skills (and we emphasize skills) taught the previous days to add and subtract algebraic fractions.

Day Five: Review, connect, reinforce, reflect. Administer a quiz.

Week Two

Day One: Break the students into groups of four. Assign each group a set of five or six polynomials to factor. After everyone has finished factoring, verify correct answers. Next, give each group several more polynomials to factor, but write the polynomials in fraction form. Make certain that the numerators and denominators have at least one common term. Have the students use the rules for factoring polynomials as well as the rules for fractions to simplify the algebraic fractions. Continue this activity until all students are comfortable with simplifying algebraic fractions. Note that the values of x that make the denominators zero are not allowed.

Day Two: Break the class into groups of four. Under close supervision of the teacher, each group will build their own triple resistor, electrical circuit-breaker board. After construction is finished, each student will use an ohmmeter to measure and record the electrical resistance of each resistor. After measuring resistance, each student will use the resistance formula to calculate the total expected resistance. Each group member will share his/her answer with the other group members. Then each group will use the ohmmeter to find the exact resistance. Each student will keep his/her own record book which will contain information on individual resistance, expected total resistance, and actual resistance.

Day Three: Take the class out to the track around the football field. Break the class into pairs and give each pair a stopwatch. Instruct the students to walk (or jog) around the track, starting their stopwatch when they begin and stopping the watch when they arrive back at the starting point. (Most high school tracks are 440 yards.) When they return, record the time elapsed on their watches and send them back around the track in the opposite direction. Encourage the students to try to go either faster or slower than their first trip. When the students return from their second trip, again record their elapsed time. After everyone has completed both of their circuits, return to the classroom and assist the students in converting their times and distances into miles per hour or some other useful unit of measure. Once everyone has determined their two rates of speed, they should use the formula above to determine their average rate of speed for the two trips combined.

Day Four: Review

Day Five: Test

Evaluation

1. 10%: Evaluations will be completed using a checklist of student performance and proficiency as demonstrated on the graphing calculator.
2. 10%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 15%: Students will be graded based on the functionality of their circuit board from exercise one. Their recorded information from the resistance experiment should be in their portfolio.
4. 50%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
5. 15%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method*, Book One; Houghton-Mifflin Company, Boston.

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Unit 9

Ratios, Proportions, and Percents

Unit 9: Ratios, Proportions, and Percents

Suggested Time Frame: 3 weeks

Ratios, proportions, and percents are critical to everyday activities. From increasing the size of a recipe to calculating the amount of power needed to run large electrical devices, proportions prove critical to the success of many projects.

Percents are used daily in the sales industry to indicate price reductions. This unit will focus on the different ways to solve these applications.

Unit Objectives

Students will:

1. Simplify ratios involving algebraic expressions. (8.1)
2. Solve proportions. (9.2)
3. Use ratios and proportions to solve real world and "word" problems. (8.3)
4. Solve real world and "word" problems involving percents. (8.4)

Workplace Relationships

1. Chef: Using proportions to calculate the amount of ingredients added to (or subtracted from) the original recipe in order to increase or decrease the size of a food dish without changing the recipe itself.
2. Sales Manager: Using percents to calculate discounts and premiums.
3. Sales Person: Using percents to calculate earned commission.
4. Electrician: Using ratio and proportion to calculate Ohm's Law.
5. Surveyor: Using ratio and proportion to calculate landscape distances.
6. Operations Analyst: Using ratio and proportion to determine the maximum output of inventory produced, sold, or distributed.

Learning Activities/Teaching Strategies

The following materials are necessary for the instruction of this unit:

2 action figures of about the same height, representing characters of different heights

10 maps of Indiana	1 class set of rulers
200 small bolts	10 stopwatches
circuit boards created in Unit 7	10 ammeters
10 sets of various weights	10 boards with springs attached

Week One

Day One (Initiatory Activity): Bring in two commonly available 5" superhero action figures that represent characters of extremely disparate height, like Spider-Man (5' 10") and the Incredible Hulk (7' 6"). Ask the students what is wrong with the two figures and what could be done to correct the error. Draw them into a discussion about scale and lead them to a method of discovering what adjustment should be made to increase the height of the Hulk figure, or to decrease the height of the Spider-Man figure.

Day Two: Break the students into groups of four. Give each group a map of the state of Indiana and four rulers. Have the students calculate the distance in inches between Indianapolis and the following cities: Evansville, Gary, Merrillville, Vincennes, Ft. Wayne, Terre Haute, Michigan City, South Bend, Muncie, and Bloomington. Point out that these cities are obviously not mere inches apart. Discuss the scale on the map and how it is used. Demonstrate how to set up a proportion using the distance measured in inches and the map's scale.

Day Three: Since the properties of numbers apply to variables as well, demonstrate how to use the cross product property with variables, followed by variable expressions.

Day Four: Write the word PER CENT on the board in large letters. Leave a small gap between the "R" and the "C". Explain the word percent is actually a combination of the two smaller words "per" and "cent". Use class discussion to determine the meaning of the word "per". (If necessary, point out that "per" is used in miles per hour and miles per gallon.) After the class has agreed that "per" means "for every" or "for each", discuss words that share the root word "cent". (Good examples include: century, centipede, centimeter, and centurion.) When the class agrees that "cent" means "hundred", put the two words together and point out that percent means "for each hundred". (It is crucial that everyone understands this idea.) Work with several examples of proportions to show how to find percents.

Day Five: Review, connect, reinforce, reflect. Administer a quiz.

Week Two

Day One: Have each student bring in a sales receipt from an item purchased at a discount. Using the marked down price and the discount, ask the students to calculate the original price. If the students are completely lost, show them how the discounted price was derived, and ask if they can work backward given that information. Then discuss markup pricing, and make connections, noting similarities to discount problems.

Day Two: Using small numbers, (e.g. teacher salaries) ask the students to calculate a beginning teacher's new salary, given the original salary and the negotiated raise in pay for the current year (if any). Then ask them to compare the increase to the estimated cost of living increase for the year, and to make suggestions as to what a teacher must do to maintain his or her standard of living. Discuss how to calculate percent change in stock prices as well, being sure to compare and contrast changes in vastly differently priced stocks.

Day Three: Ask a member of the faculty who has experience in real estate (there's almost always one) to discuss commission with the class, citing real examples of deals that resulted in high or low profits for the realtor. Then pose a question to the class asking them to calculate the required price of a property at 7% commission that one would have to sell to make a desired amount of money. Compare and contrast salaried employment vs. pure commission employment.

Day Four: Place a fixed number of bolts on a student's desk and ask that student to pick them up with one hand as fast as he or she can, while timing that student with a stopwatch. Then repeat the procedure with the same bolts for another student. Once the two times are compared, ask the students to theorize how long it would take for both of the students to pick up the bolts working together. When the actual time is found, discuss how to solve work problems. Use the data from the experiment to show why the time resulted as it did, and then have the students (in groups of four) reproduce the experiment for themselves using varying numbers of bolts. Have the students who are not picking up bolts predict the time required to perform the task.

Day Five: Review, connect, reinforce, reflect. Administer a quiz.

Week Three

Day One: Break the class into groups of four. Using the circuit boards created as a part of unit seven, have each group measure the power supply voltage and the resistance of R1 (One student in each group should record and keep track of the data). Close the circuit and measure the current flow through the circuit using an ammeter. (Be careful that the students do not put too much power through the circuit or the ammeter may be damaged.) Have the students break the circuit by disconnecting the resistor from the power source. Adjust the resistor to a different resistance level, and reconnect the circuit. The groups should re-measure the current and record the new information. Repeat this three or four times. After enough data has been gathered, discuss the findings with the class. Have the students work to come up with a formula that expresses the relationship between voltage, resistance, and current. (The relationship involves an inverse proportion.) This could also be done as a class demonstation using apparatus from the high school physics lab.

Day Two: Develop a math game in which each student earns a "commission" for answering the questions correctly. Break the class into groups of four. Each question should be worth a different number of extra credit points, to be divided evenly between the group members, with the following exception: The student who answers correctly will receive a "point commission" off the top, and the remaining points will be divided four ways. For example, the class is asked to solve the equation $2x - 5 = 17$. The equation is worth 15 points with a 40% commission. "Susie" raises her hand and answers the question and calculates her commission correctly. She earns 6 points (40% of 15) off the top, leaving 9 points to be divided between the four students in "Susie's" group. Each student gets 2 points, with one left over. Throw any "left over" points into a Grand Prize Pot for the game's last question. The questions for this game can be from any area of mathematics that has already been covered during the year.

Day Three: Break the students into groups of four. Give each group a set of weights, varying in mass from a few grams up to a kilogram, and a board with a spring attached. Have the students take turns placing a weight on the spring and measuring how far the spring is stretched. Each group should record the mass of the weight and the distance stretched. After all of the weights are tested, the students should develop a theory as to the relationship between the mass of the weight and the distance that the spring stretched. (The relationship involves a direct proportion.)

Day Four: Review

Day Five: Test

Evaluation

1. 15%: Evaluations will be completed using a checklist of student performance in group activity and productivity.
2. 15%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 50%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method*, Book One; Houghton-Mifflin Company, Boston.

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Unit 10

Solving Linear Systems

Unit 10: Solving Linear Systems

Suggested Time Frame: 3 weeks

In this unit the students will be introduced to the idea of solving systems of both linear equations and inequalities in two variables. The successful student will come away with the ability to apply variable equations to real world settings and his/her confidence in solving these systems will be reinforced with graphing technology.

Unit Objectives

Students will:

1. Use a graph to find the solution of a pair of linear equations in two variables. (6.1)
2. Graph the system of linear inequalities in two variables and find the solution set. (6.2)
3. Use graphing technology to solve systems of linear equations. (6.3)
4. Use the substitution method to find the solution of a pair of linear equations in two variables. (6.4)
5. Use the addition and subtraction method to find the solution of a pair of linear equations in two variables. (6.5)
6. Use multiplication with the addition or subtraction method to solve systems of linear equations. (6.5)
7. Use systems of linear equations to solve real world and "word" problems. (6.6)

Workplace Relationships

1. Corporate Buyer: Using a system of linear equations to determine from which company to purchase supplies.
2. Dietitian/Nutritionist: Determining amounts of protein and carbohydrate consumption depending on a patient's weight.
3. Car Salesman: Determining depreciation of two different automobiles based on factors such as mileage and wear.
4. Airline Pilot: Using equations to make mid-flight fuel consumption computations for a round-trip, compensating for headwinds and tailwinds.

Learning Activities/Teaching Strategies

This unit employs cooperative learning techniques and activities to accomplish its tasks.

The following materials are necessary for the instruction of this unit:

TI-82 calculators	algebra tiles	box of uniform nuts and bolts
paper cups	scales	

Week One

Day One: Break the class into groups of four. Give each group a TI-82 graphing calculator. Write four two-equation systems on the board for the students to graph. Each student should have an opportunity to use the calculator. Have the students record the equations and their intersections, if any. Discuss any patterns that the students notice such as; y-intercepts, slopes, etc.

Day Two: Have the students graph by hand several sets of equations and list the intercepts. They should check both by use of the graphing calculator and by substituting the intercept into the equation.

Day Three: Use the TI-82 to graph equations in 2 variables. Use the shading capabilities to convert the graph to the graph of an inequality. Have the students describe in words or mathematical expressions the area that is shaded. Lead this into expressing it as a set of inequalities.

Day Four: Review the concepts presented this week. Give the students several equations and inequalities to graph and shade to show the solution sets. Have the students select a point in each solution set as a check point. Use this to determine if the answer is correct.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Two

Day One: Present to the class a box of uniform nuts and bolts. They are to determine the number of nuts and bolts in the box without counting. First have the students determine the information needed to solve the problem. They will need the average weight of each nut and each bolt, and the total weight of all the nuts and bolts. This will give you one equation. Save this equation for the end. To get a set of equations, put a random number of nuts and bolts in a small container. Count the number in the container and weigh them. Use these pieces of information to obtain your 2 equations. Solve these last two equations and use your solution with the very first equation to find the number of nuts and the number of bolts in the large box.

Day Two: Discuss yesterday's presentation and methods that could be used to solve the equations. Use algebra tiles to demonstrate the substitution method of solving equations. Let the students solve a few problems on their own using the tiles. Have them model the problem and answer on paper. Have them solve day one's equations using substitution. Assign problems for them to do on their own from the current text or from a worksheet.

Day Three: Use algebra tiles to demonstrate the addition/subtraction method of solving simultaneous equations. Let the students solve a few problems on their own using the tiles. Have them model the problem and answer on paper. Have them solve day one's problem using addition if possible. If this doesn't work well, have them think of ways in which the equations could be "adjusted" so that it would work. (The "adjustments" will be discussed next week.) Assign problems for them to do on their own from the current text or from a worksheet.

Day Four: Review the methods presented the last two days comparing the modeling with the written method for solving the equations. Give the students time to ask questions and practice both methods with the written form of solution. Give the students a review assignment to prepare them for tomorrow's quiz.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Three

Day One: Discuss the adjustments that they thought of last week with the addition/subtraction method that would have allowed for the solution of the problem from last week. Make sure that the possibility of using multiplication on one or both of the equations is discussed. Algebra tiles may be used to demonstrate the multiplication-addition method for solving pairs of equations, however, one or two examples should be enough as this can be cumbersome. Show how some of the problems from the previous two weeks could be solved this way. Have the students practice this method.

Day Two: Break the class into groups of four. Give each group a shopping list of food items that they need to "purchase", and a dollar amount that they can spend. (The list should not state the exact number of items to be purchased, but for the of items to be purchased. For example, group one needs to buy x pounds of bananas at $0.40 per pound and y pounds of apples at $0.30 per pound. The group spends $4.40 to buy 24 pounds of fruit. How many pounds of each did group one buy?) Give each group several pairs of items to buy. They must set up the system and demonstrate which method they used to solved it. Graphing calculators may be used.

Day Three: Present some real world problems for the students to solve.

Example 1. The student has an opportunity to work on commissions. He can make $2 per hour plus 20% commission on everything he sells, or he can make $3 per hour and 15% commission on everything he sells. He decides to try each method for one week and then decide. He worked the same number of hours each week and had the same dollar amount of sales. The first week (20%) he earned $180, and the second week (15%) he earned $195. How many hours did he work and how much did he sell? Would he make more working at $4.25 per hour?

Example 2. A telephone company offers two plans. The first plan charges you a $25 monthly fee, allowing you 30 minutes free call time, and charges $.50 per minute for the rest. The second plan charges you a $50 monthly fee, and $.20 per minute for calls, but no free minutes. Determine the better plan.

Day Four: Review the important learning objectives from the previous three weeks. Answer any questions that the students may have. (20-30 minutes)

Administer a practice worksheet for the students to complete and turn in at the end of the period. (Or give a review assignment from your current text.)

Day Five: Administer the test.

Evaluation

1. 15%: Evaluations will be completed using a checklist of student performance in group activity and productivity.
2. 15%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 50%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Brown, R.; Dolciani, M.; Sorgenfrey, R.; and Cole, W. (1992). *Algebra One-Structure and Method*, Book One; Houghton-Mifflin Company, Boston.

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Unit 11

Radical Expressions

Unit 11: Radical Expressions

Suggested Time Frame: 3 weeks

Square roots are used daily by professionals in many areas of science and industry. It is necessary for all students to become proficient with radical operations, square root estimation, and the Pythagorean theorem. The successful student will complete the unit with the ability to read, interpret, and solve applied equations and expressions using radicals.

Unit Objectives

Students will:

1. Estimate the square root of any non-negative number. (2.7)
2. Find the product of two square roots and simplify the result.
3. Find the quotient of two square roots and simplify the result.
4. Simplify two radicals and find their sum.
5. Simplify two radicals and find their difference.
6. Use the Pythagorean Theorem to determine the hypotenuse of a right triangle.

*7. Solve an equation containing a radical (3.10)

*Optional

Workplace Relationships

1. Carpentry/Construction: Determining the correct length to cut a piece of wood.
2. Navigation: Using radicals to plot a course.
3. Air Traffic Controllers: Using radicals to recommend a course correction.

Learning Activities/Teaching Strategies

The following materials are necessary for the instruction of this unit:

10 US measurement tape measures
A class set of geoboards
10 meter sticks
30 pieces of string, each of a different length
10 stopwatches
20 or so colored disks
10 tennis balls
10 metric or US rulers
10 lead fishing weights

Week One

Day One (Initiatory Activity): Take the students out to the baseball field, confirming with them that it forms a 90 foot x 90 foot square. Ask, then, how far the catcher has to throw from home plate to second base to catch a runner stealing. When they measure the distance and it comes out unevenly, take them to the softball field, a 60 foot by 60 foot square. Repeat the question and the procedure. Take the students to the basketball court and confirm that the diagonal of the rectangle does not come out evenly. Then create a rectangle that forms a Pythagorean triple. Represent it on a large geoboard or on graph paper. Lead the students to find the relationship that gives them the Pythagorean Theorem, and confirm that it works for the measurements that they took at the fields and court.

Day Two: Begin discussing the various methods of estimating square roots. Demonstrate how to find them using tables and calculators. Discuss accuracy, and the issue of exactness. (e.g. If you measure two digits, your computation should have two significant digits.)

Day Three: Begin applying the Pythagorean Theorem to the length and width measurements of the classroom taken in Unit 1 to confirm the accuracy of the diagonal measurements. Review significant digits as well.

Day Four: Use the Theorem again to find a measurement not yet found in the classroom: The diagonal from a floor corner to the opposite ceiling corner. Then try solving right triangles with a leg and the hypotenuse given.

Day Five: Reinforce, connect, review, and reflect. Administer a quiz.

Week Two

Day One: Push all desks to the sides of the classroom and leave a large area in the middle of the room. Draw two perpendicular lines that intersect close to the middle of the class. The room is now divided into a large Cartesian Plane. Place colored disks on various floor tiles around the room. Ask the students to guess the distance in "tiles" that the disks are apart. Take several opinions before any measurement is done. After a few minutes, take a meter stick and measure the length of one tile in centimeters. Then measure the distance between the two disks in centimeters. Translate the answer into "tiles". Repeat this exercise two or three more times. Use this experiment as a way to introduce the distance formula, deriving it from the Pythagorean Theorem.

Day Two: Using geoboards, have each student construct a square inscribed in a square at the midpoints. By now, the students should be able to quickly find the hypotenuse of each triangle formed. The hypotenuses become the sides of a new square. Then ask the students to find the area of the inside square. They should discover that squaring is the inverse function of the square root. Have them repeat the experiment with rectangles, and keep the data for the next day.

Day Three: Take the measurements from the previous day and use them to find the perimeters of the inscribed squares and rectangles. This should put addition of radicals into sufficient context to begin a lecture.

Day Four: Formalize the ideas discussed during the week, and include a brief discussion about rationalizing denominators.

Day Five: Reinforce, connect, review, and reflect. Administer a quiz.

Week Three

Day One: Break the students into groups of four. Give each group a tennis ball and a meter stick. Have each group drop the tennis ball from various heights and record the length of time in seconds that it takes for the ball to hit the floor. The students should use the meter stick to accurately record the height that the ball is dropped from each time. At least ten trials should be completed. After all of the groups have completed their trials, write the following formula on the board:

$t = \sqrt{2\,s / g}$ where t = time in seconds, s = height in meters, and g = acceleration due to gravity (g = 9.81 meters/second squared).

Have the groups plug in all of their numbers for each trial and determine how accurate their experiment was. For extra credit, have them determine their percentage of error.

Day Two: (A day before this activity, assign every student to bring in 2 cylindrical cans of food.) Break the students into groups of four. Give each group a set of rulers. Have the students measure the height of the cans and the radius of the base (1/2 the diameter). One student should record all of the information. After all of the measurements have been taken, write the following formula on the board:

$r = \sqrt{V / \pi h}$, where r = radius of the base, $\pi = 3.14$, and h = height of the can.

Have the students plug in all of the figures for each set of measurements and solve the equation for V, the volume of the can. The students should compare their calculations to the volume by weight on the side of the can and determine the percentage of error, if any.

Day Three: Break the students into groups of four. Give each group four pieces of string of varying lengths, a weight to tie to the end of the string, a stopwatch, and a meter stick. The students should measure each of the pieces of string after the weight has been tied to it. Holding the string by the free end results in a pendulum. (Ideally, the motion of a pendulum is periodic.) Have the students swing the pendulum through one period, using the stopwatch to determine and record the time for one complete period. The time, T, for one period of a simple pendulum is determined by the following formula:

$T = 2 \pi \sqrt{L / g}$, where $\pi = 3.14$, L = length of the string, and g = acceleration due to gravity (9.81 meters/second squared).

Have the students attempt this experiment using all four pieces of string. The groups should compare their stopwatch times to the time suggested by the formula. Each group should find the margin of error (in percent), if any.

Day Four: Review, reflect, reinforce

Day Five: Test

Evaluation

1. 20%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 35%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

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Unit 12

Statistics, Linear, and Non-Linear Functions

Unit 12: Statistics, Linear, and Nonlinear Functions

Suggested Time Frame: 3 weeks

Linear and nonlinear functions provide ways to answer questions in a practical and visual manner. The graphs of functions provide insight into data and trends and allow alternatives to be analyzed to aid decision-making processes. The students will explore, graph, and interpret nonlinear equations and will define functions and/or inequalities from a graph to obtain specific data and analyze alternatives.

Unit Objectives

Students will:

1. Graph a quadratic equation. (9.1)
2. Use graphing technology to find the solution to a quadratic equation. (9.2)
3. Solve a quadratic equation when one member is in factored form and the other member is zero. (9.3)
4. Solve a simple second-degree equation by factoring. (9.4)
5. Use graphing technology to relate the solutions of quadratic equations and the x-intercepts. (9.5)
6. Understand that the vertex provides the maximum or minimum value of the quadratic equation. (9.6)
7. Solve a quadratic equation in which a perfect square equals a constant. (9.7)
8. Solve a quadratic equation by using the quadratic formula. (9.8)
9. Determine if a set of data represents an exponential function. (9.9)
10. Use formulas, calculators, and graphing technology to explore and solve problems involving exponentials. (9.10)

Workplace Relationships

1. Nurse's Aide: Comparing time and temperature on a patient's chart.
2. Medical Technician: Monitoring an EKG machine to determine the heart rate of a patient.
3. Entrepreneur: Charting production vs. cost levels to determine maximum level of cost-effectiveness.
4. Carpenter: Using quadratic equation to determine the maximum area that can be enclosed using a specific length of fence.

Learning Activities/Teaching Strategies

This unit employs cooperative learning techniques and activities to accomplish its tasks.

The following materials are necessary for the instruction of this unit:

TI-82 calculators	tennis ball	stop watch
hot water	thermometers	cups

Week One

Day One: Break the class into groups of four. Give each group a tennis ball, a meter stick, and a stopwatch. Have the students drop the ball from various heights and record how long it takes for the ball to stop bouncing. (At least 10 trials should be completed-20 is preferable.) After all data has been gathered, students should graph the points on a graphing calculator to determine whether this relationship is best represented by a linear or nonlinear function. Students should then predict how long it will take for a ball to stop bouncing if it is dropped from heights greater than the heights in their original experiment.

Day Two: Relate the results of yesterday's experiment to quadratic equations. Relate quadratic equations to the work done in unit 6, multiplication of binomials. Show an equation such as (x-6)(x+5)=0 to the students. Take two pieces of paper. Without letting the students see what is written, write 0 on one piece and a number such as 437 on the other. Fold the papers and give one to each of two students. Instruct them not to look at what is written on the papers. Challenge the class to guess at least one of the numbers. Inform them that the numbers can be positive, negative, fractions, decimals, or any real number. Allow a few guesses, then suggest that maybe they need a hint. Tell them that if you multiply the two numbers together, you will get zero. The students should come up with one of the two numbers quickly. Then have the two students unfold the papers and read the answers. Relate this to the equation above. Solve this equation by setting each of the two "numbers" (factors) equal to zero. Continue with several more examples.

Day Three: Refer to the equation (x-6)(x+5)=0 from yesterday. Point out that not all equations come in this form. Suppose the original equation was x^2-x-30=0. How would one approach this equation? Suggest several other factorable equations. Have the students factor and solve them. Have them verify that the answers are correct by substitution. Discuss ways to solve equations such as x^2-x-25 = 5. Include a mixture of equations based on the past 3 days to be solved.

Day Four: The area of a square is 169 square meters. Is it possible to decrease the length and width of each side by "x" and have the area become 100 square meters. The students should first find the actual length of each side of the square (13). Then propose an equation for the new square (13-x)(13-x) = 100. Have them suggest several ways to solve this equation before confirming that the easiest method may be taking the square root of each side. Give the students a mixture of problems reflecting the methods derived this week to solve.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Two

Day One: Break the class into groups of four. Give each group a TI-82 graphing calculator. Write a series of quadratic functions on the board, some with a > 0, and some with a < 0. Have the students take turns graphing the functions and recording a sketch of what the graph looks like. After everyone has had a turn, discuss the shapes of the various parabolas. Conclude that when a > 0, the parabola opens upward, and when a < 0, it opens downward.

Have the students graph on the calculator the problems presented last week in the different forms. Find the answers to the problems on the graph. Where are the answers located? Give the students a new equation such as y=x2-5x+6. If y = 0, what would be the solution to this equation. Use the graphing calculator to find the answers. Could there be equations with no solutions?
Try: $y = x^2 - 2x + 2$.

Day Two: The path of a ball thrown upward can be expressed by h=-4.9t2 +at, where a is based on speed and angle of the throw. Will the graph of this open up or down? What is the highest the ball can go? Have the students graph this on the calculators using several values for "a".

Compare this with the equation $e = .7x^2 - 28x + 387$ which is a quadratic model of energy used by parakeets based on speed. Is there an optimum speed for the birds? Are there x and y intercepts for the graph?

Compare the vertices of the two graphs. What answers were located at these points? Point out that these points have names, maximum and minimum.

Day Three: On the previous days, quadratic equations were graphed using the graphing calculators. Discuss what information was gotten from the graph that we could get without the calculator (x-

intercepts, y- intercept and (x,y) coordinates that would make the equation true). Use the equations used previously and graph them without the graphing calculator. Give the students several easy quadratics to graph.

Day Four: Reflect on this week's work, answer questions and give a review assignment.

Day Five: Review the important learning objectives from the previous four days. Answer any questions that students may have over any of the materials. (10-15 minutes)

Administer the quiz.

Week Three

Day One: Break the class into groups of four. Give each group a cup of hot (not boiling) water and a thermometer. The students should use the thermometer to determine the initial temperature of the water. Every two minutes, the temperature of the water should be rechecked and recorded. This process should be repeated until the temperature of the water stabilizes at room temperature. Then the groups should chart their results and use TI-82 graphing calculators to determine an appropriate function. After completing this process, have the students look up Newton's Law of Cooling and compare the results. Check out several pieces of data with the formula to see if it gives the same results.

Day Two: Reflect on the problem from yesterday. Notice that some equations are not quadratic. Have the students recall the work from the beginning of unit 6 where they experimented with graphing polynomials. Have them repeat the experimentation, noticing some of the same patterns they observed before. Have the students graph several equations of various degrees to see if they can determine solutions for x-intercepts, or local maximum or local minimum.

Day Three: Have the students take a new look at quadratic equations in the form $ax2 + bx + c - 0$. After they graph the equations and pick out the solutions by looking at the x - intercepts, have them substitute a, b, and c into the quadratic formula and discuss the results. Review simplifying radicals from unit 11. Then give the students quadratic equations that are not factorable, to solve without the graphing calculator. Discuss how using the calculator gives approximate answers, where the answers derived from the formula gives exact answers.

Optional: Program the calculator with the quadratic formula so students only have to enter a, b, and c in order to get approximate answers.

Day Four: Review the important learning objectives from the previous three weeks. Answer any questions that the students may have. (20-30 minutes)

Administer a practice worksheet for the students to complete and turn in at the end of the period. (Or give a review assignment from your current text.)

Day Five: Administer the test.

Evaluation

1. 20%: Evaluations will be completed using a checklist of student performance in group participation and productivity.
2. 25%: Assessment of individual student portfolios. Students will keep a "file" of their work, which will be turned in for teacher's assessment at the end of the unit.
3. 35%: Standard Friday quizzes and tests, with immediate feedback, will reinforce the applications taught.
4. 20%: Student behavior, attendance, class participation, and assigned homework tasks will be evaluated to determine whether student has achieved required learning objectives.

Resources

Aufmann, R.; Barker V.; and Nation, R. (1993). *College Algebra and Trigonometry*, 2nd edition; Houghton-Mifflin Company, Boston.

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Indiana Core 40: Algebra First Course. (1994). Indiana Department of Education, Indianapolis.

Musser, G.; and Burger, W. (1991). *Mathematics for Elementary Teachers*, 3rd edition; Macmillan Publishing Company, New York.

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