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ABSTRACT

Promoting ACademic Excellence in Mathematics and Science for Workers of the 21st Century (PACE) was a consortium project made up of Indiana University Northwest, the Gary Community Schools, and the Merrillville Community Schools. The focus of this project was to prepare teachers and curricula for Tech Prep mathematics and science courses for the two school districts. The courses and course units prepared by the project are intended to promote the Core 40 Competencies of the Indiana Department of Education for High School courses. This document contains units for pre-algebra designed for students who have not mastered the necessary competencies which would enable them to enter into a first year algebra course. It is not a core 40 course, but does maintain the applied perspective. Units include: (1) Rational Number Foundations; (2) Exponents, Area, and Volume; (3) Data and Probability; and (4) Algebraic Foundations. (JRH)

Pre-Algebra
an Applied Approach

Clyde A. Wiles • Kenneth J. Schoon
Editors

PACE **Promoting Academic Excellence In Mathematics, Science & Technology for Workers of the 21st Century.**

Gary Community School Corporation
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Pre-Algebra
An Applied Approach

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PACE

Promoting ACademic Excellence in Mathematics and Science for Workers of the 21st Century

PACE was a consortium project of Indiana University Northwest, the Gary Community Schools, and the Merrillville Community Schools. It was supported by funds from the three institutions and by Eisenhower grants from the Indiana Higher Education Commission.

The focus of the project was to prepare teachers and curricula for Tech Prep mathematics and science courses for the two school districts. The effort took place over 1994 - 1996 and involved more than 70 teachers from seven High Schools. The Director of the project was Dr. Clyde A. Wiles, and the Associate Director, Dr. Kenneth J. Schoon, both of Indiana University Northwest.

Part of the effort was the developing of units and course outlines for use in the first two years of a High School Tech Prep program. Individual schools and faculty will be using these materials in a variety of ways from being a course guide to being a supplement to an already existing program.

We have taken the position that Tech Prep is not a program for the academically deficient. Rather it is an applied approach to curriculum that has the goal of promoting competencies recommended by the State of Indiana for non-remedial high school courses, and which does so in a learning environment that emphasizes applications. We would like students to find within these course materials and instructional approaches immediate and obvious responses to the questions: "What does this look like?" and, "Why would anyone want to know?"

These courses and course units then are intended to promote Core 40 Competencies of the Indiana Department of Education for High School courses. For mathematics, we viewed this as beginning with Algebra One and for science beginning with Biology. The Pre-Algebra course is not a Core 40 course, but does maintain the applied perspective.

Our efforts have had to accommodate to several factors. First there is an Indiana mandate that all high schools have a Tech Prep curriculum that targets the academic and school-to-work needs of the middle 50% of the high school student population. There are on the other hand, persistent beliefs of counselors, teachers, administrators, students, and parents that something called "tech" anything, is just another name for a program intended for "at risk" students who are not expected to acquire competencies at a level that would enable them to pursue post secondary schooling at the college or university level. These beliefs are often supported by admission policies at some universities. We have, therefore, attempted to position Tech Prep courses as courses that meet exactly the same Core 40 competencies (as defined by the Indiana Department of Education) as are to be met by college prep courses of the same name, but to do so in applications-based and problem-centered approaches.

Clyde Wiles, Director
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Division of Education
Indiana University Northwest
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PRE-ALGEBRA

An Applied Approach

Course description

This Pre-Algebra course is designed for students who have not mastered the necessary competencies which would enable them to enter into a first year algebra course. Students enrolled in this course would benefit from a two semester sequential study of mathematical and basic algebra skills. This sequential study is necessary to enable the students to receive a solid foundation in arithmetic skills, as well as the basic topics in the first semester of algebra, some geometry, and other mathematical topics. This curriculum is designed to help students become proficient in the competencies needed to enter into Algebra I.

This course will integrate the topics studied in the curriculum by a process of problem solving, connecting mathematics to every-day life and the world of work. The methodology of this course will include:

- problem solving
- small cooperative learning groups
- use of calculators
- computer programs
- individual and group tests, reports, and quizzes
- portfolios.

Students entering this course should have a basic proficiency in the following competencies:

- reading and writing whole numbers and decimals
- using estimation and approximation to answer questions on given data
- analyzing and graphing data
- performing basic operations involving rational numbers (integers, fractions, and decimals)
- converting between customary and metric units

Upon completion of this course, students should be able to use algebraic and mathematical skills to solve problems that reflect real world situations. This should be accomplished by selecting appropriate strategies and technologies. Students should be able to competently communicate through oral and written words their understanding of mathematical concepts, as well as read with understanding.

Course Goals

Students will:

1. Demonstrate computational proficiency with real numbers.
2. Develop and use strategies for solving problems.
3. Develop and practice effective communication using mathematical language
4. Translate data into mathematical language.
5. Use variable equations and inequalities to solve problems relating to real world applications.
6. Develop an understanding of relationships between representations of a function.
7. Apply properties of two-and-three dimensional figures to realistic problems.
8. Collect and organize real world data and distinguish among and use measures of central tendency and dispersion.

Pre-Algebra Course Divisions

Unit I Rational Number Foundations

Section 1. Fundamentals of Mathematics
Section 2. Fractions, Decimals, Percent, Ratios, and Proportions
Section 3. Measurement

Unit II Exponents, Area, and Volume

Unit III Data and Probability

Section 1. Statistics, Graphs and Data Analysis
Section 2. Probability

Unit IV Algebraic Foundations

Section 1. Algebraic Expressions and Equations
Section 2. Graphing Linear and Systems of Equations, and Inequalities

Competencies

1. Research and report on the systemic use of numbers.
2. Mentally compute exact and estimated sums and differences.
3. Perform basic operations using integers.
4. Apply basic operations using fractions.
5. Find the prime factorization of composite numbers.
6. Use order of operations to simplify numerical expressions.
7. Write ratios as fractions in lowest terms and to write unit rates.
8. Solve proportions.
9. Interpret and use scale drawings.
10. Use proportions to solve problems.
11. Use proportions to solve percent problems.
12. Measure items using standard and metric units.
13. Identify equivalent units of measure within the standard and metric systems.
14. Convert units from standard to metric and vice versa.
15. Estimate distances in standard and metric units.
16. Develop an awareness of the differences among triangles, squares, rectangles, and other polygons.
17. Use exponents instead of writing the same factor several times.
18. Recognize when to use the laws of exponents to simplify algebraic expressions.
19. Use exponents to solve area and volume problems.
20. Recognize the square root as the length of the side of a square.
21. Identify space figures.
22. Identify common shapes in two and three dimensions.
23. Solve real world problems using perimeter, area, and volume.
24. Collect and organize data from real world situations.
25. Construct bar and line graphs, and tables.
26. Interpret bar and line graphs, and tables.
27. Use statistical data for decision making.
28. Use statistical data for predicting outcomes.

29. Calculate the common measures of central tendency (mean, median, range, mode).
30. Find the probability of simple events.
31. Count the number of ways an event can happen.
32. Draw diagrams and charts to help find probabilities.
33. Use the calculator to find probabilities as decimals and percents.
34. Construct frequency tables, tables, graphs and histograms to represent real world data.
35. Compare/contrast a set of data using several methods (frequency tables, tables, graphs and histograms).
36. Choose the appropriate measure of central tendency.
37. Make and interpret stem-and-leaf plots, box-and-whiskers plots, scatter plots, and circle graphs.
38. Determine which form of plot or graph best represents the data.
39. Sketch the best fit curve for a set of real world data.
40. Use a best fit curve to make predictions.
41. Write a variable expression to represent mathematical situations.
42. Evaluate or simplify variable expressions by combining like terms and using properties of real numbers, (distributive, associative, and commutative.)
43. Solve simple equations by substitution or mental math.
44. Solve one step equations by using algebraic properties of equality.
45. Solve two step equations by algebraic properties of equality.
46. Represent situations by writing an inequality in one variable.
47. Given a rule or function that demonstrates a linear equation, generate ordered pairs and graph the equation.
48. Translate between tables and graphs of functions.
49. Given a linear inequality, determine whether a given value is in the solution set.
50. Given a linear inequality in two variables, graph the solution set.
51. Find the slope and intercepts of a line and write the equation of a line using the slope
52. Represent situations by writing a linear equation in one variable.
53. Represent situations by writing a linear inequality in one variable.
54. Graph the solution set to a one variable linear inequality on a number line.
55. Given a formula, make appropriate substitutions and solve for one variable.
56. Solve applications involving the Pythagorean Theorem.
57. Set up and solve proportion statements to find missing values.

Unit I

Rational Number Foundations

Unit I.1 FUNDAMENTALS OF MATHEMATICS

In this unit the learner will apply a planned process to solve problems. Students will integrate problem solving skills and strategies in the use of estimation and mental mathematics in real world applications leaning to the clusters that they shall be entering for Tech Prep. Also, each learner will demonstrate mastery of the different competencies before moving on through the use of authentic assessment.

Each student should work at her/his own pace to achieve maximum achievement, through meaningful activities, projects, written and oral quizzes, and written exams.

Section Goal and Objectives:

Goal: Learner will demonstrate computational proficiency with real numbers.

Objectives:

1. Students will perform basic operations with positive and negative numbers.
2. Students will compare two or more rational numbers including negatives.
3. Students will identify prime numbers and factor composite numbers.
4. Students will use the order of operations to simplify numerical expressions.
5. Students will research and report on the systemic use of numbers.

Instruction

Discuss the number system, i.e. whole numbers, integers, rational, and real numbers.

Activity 1

There are many places where the systematic use of numbers provides help in getting things done efficiently. The list below contains 26 applications of numbering systems.

Applications

- **A** Auto Identification
- **B** Banking and Checking
- **C** Credit Cards
- **D** Dewey Decimal System
- **E** License Plates
- **F** Airports
- **G** Geographic townships
- **H** Astronomy
- **I** Catalogs
- **J** Stock Indices
- **K** Latitude,
- **L** Library of Congress
- **M** Mental Measurements
- **N** Buildings and Rooms
- **O** Highways - State, County
- **P** Photography
- **Q** Neilsen Ratings
- **R** Radio, TV Frequencies
- **S** Stadium Setting
- **T** Telephone Area Code
- **U** Bar Codes
- **V** Systems of Currency
- **W** Highways-US/Interstate
- **X** Telephone Exchanges
- **Y** Time Zones
- **Z** Zip Codes

<u>Assignment:</u> One of the applications will be assigned to you. You are to research your topic by going to at least two references. Then you are to prepare a typewritten or word-processed one page report and a poster. The report and poster should include the following information:

1. The numerical scheme that has been employed. Use diagrams, examples, graphs, and charts as necessary.
2. References.

Be prepared to make a brief oral report of your findings. This report is to be submitted no later two weeks of the date assigned. Suggestion: This project can be worked collaboratively with an English teacher. Source: Collaborative Learning Workshop Valparaiso University, 1994 (CLWVU)

Activity 2

Career Poster

Interview someone who uses mathematics in his or her job. Ask the person to give you an example of a problem used in his or her work. Make a poster . The title of the poster should be the name of the career, such as "Engineer" or "Carpenter." On the poster you should include three things:

1. The mathematics problem given to you by the person interviewed.
2. A one-page paper explaining in general what the problem is about, how it is used, and general information about the career.
3. A picture to illustrate the job. It can be a photograph of the person you interview, or it can be a picture cut from a magazine of a person in that occupation.

On the back of the poster, you must put three things:

1. Your name and class.
2. The name of the person you interviewed and the name of the place she or he works.
3. The daytime telephone number of the person you interviewed, This number will enable me to verify any information, if necessary.

This assignment will be graded. The grade will be based on these things.

1. How well you followed instructions.
2. Neatness. Please print your title. You paper should be typed or processed on a computer.
3. Timeliness - must be turned in on or before the due date.

Activity 3

Play the Grocery Store Estimation Game. (p. 137 *Indiana Mathematics Proficiency Guide*).

Activity 4

In pairs, students will mentally solve prepared computation problems where one person reads the problem aloud and the other writes down the solution then vice versa. As a whole class discuss how each person solved his/her problem. E.g. 42 + 35 = 40 +30 + 7 =77. 42 is 8 less than 50 and 35 is five less than 40, so the answer is 13 less than 90 to give 77.

Activity 5

Discuss typical strategies for estimating a sum (reference *Arithmetic Teacher Volume 40*. No. 6, February, 1993. "Making Connections with Estimation" by Joanne E. Lobato)

$2.53 + $3.59 + $1.67 + $0.56 + $4.59 + $ 2.83

Rounding		Front End Estimation of 1st digit's sum	
$2.53 ---->	$3.00	$2.53 $3.59	> Approximately $1.00 more
$3.59 ---->	$4.00		
$1.67 ---->	$2.00	$1.67 $0.59	> Approximately $1.00 more
$0.56 ---->	$1.00		
$4.59 ---->	$5.00	$4.59 $2.83	> Approximately $1.00 more
$2.83 ---->	$3.00		
Estimate:	$18.00	Front end estimate :	$12.00, then look for adjustments
		Adjusted estimate:	$12.00 + $3.00 = $15.00

1. Suppose you walk up to the cash register in a grocery store with six items having these price tags. The clerk says, " That will be $18.37". Do you think that total is reasonable, or could the clerk have made a mistake? Support your answer with a written explanation. (Note: use the above price list).

2. The school cook looks at today's attendance at the middle school: Grade 6 has 656 students, grade 7 has 538 students and grade 8 has 557 students. The cook has enough food to serve 1500 people and estimates that about 75 of these students will bring lunches from home. Does the cook have enough food for today's lunch?

3. Your family is driving to your grandmother's house. The last road sign indicated that you have 185 miles left to go. The time is 1.55p.m. Will you get to your grandmother's house before 6.00 p.m., which is when dinner will be served?

4. Latasha swims laps in a pool in which thirty-six laps equals 1 mile. She has already swum ten laps. About what part of a mile has she swum?

5. The Last Ditch Deli charges $3.65 per pound for Swiss Cheese. Camilla only wants to spend about $2.00 on cheese. About how much of a pound should she ask the deli clerk to slice for her?

Activity 6

The Estimation Jar.

Have students estimate the number of M&Ms in a jar. Then, without revealing their original estimate, have students pair off and agree on an estimate. When the pairs have an estimate, have them merge with another pair and follow the same procedure, i.e. without revealing their estimate, the group will agree on an estimate. Continue to merge groups, following the same procedures until the whole class is together. If the class is unable to agree on an estimated amount, have each of the last two groups that merged select a student to go in the hall. These two will act as representatives for the class and they will make an executive decision for the class. After the class estimate is given, give the students the actual count. Have them compare their original estimate to the actual count. As a part of the discussion, have students identify any strategies they may have used throughout this activity. You may also ask them to share how they felt each time they merged and the new estimate varied from their original estimate. As an fun conclusion, the student that had the closest estimate can be awarded the jar of M&Ms.

This estimation activity can also be done using * Number of straws in a container * Number of seats in the auditorium * Number of books on a shelf in the library * Perimeter of playground or gymnasium (then do actual measurement as a class activity)* Guess height of the flagpole and then use similar triangles to solve. (p 137 *Indiana Mathematics Proficiency Guide*).

Estimation Jar: The following are suggested strategies for helping to guide students in their estimation of the number of objects in an estimation jar (CLWVU, 1994).

1. **MORE THAN Strategy.** Have students begin with smaller numbers or larger numbers as a reference point. It is more than 100? Is it less than 10,000? This will help students to at least be able to make more reasonable estimates.

2. **LAYER Strategy.** Many estimation jars can be counted by the number of layers times the amount in the layer. Both the amount in a layer and the number of layers need to be estimates also. You might advise students to do this process twice so they have a high and low estimate of the number of the entire jar.

3. **WEIGHT Strategy.** Students could take this suggested strategy in several directions. For example, if the jar contained marbles, a student could be given a handful, weigh the handful, and use this information to estimate the weight of the entire jar, and hence the total.

4. **SMALLER JAR Strategy** Show your students a smaller jar that contains the same type of objects as the larger jar the students are trying to estimate. Tell how many objects are in the smaller jar. This is extremely helpful to many students especially if you are trying to get them to refine their initial estimates.

5. **PARTS Strategy.** This is similar to the layer strategy but slightly different, Mentally divide the jar into any number of equal parts. Estimate the amount of objects in one of these parts and multiply by the number of parts.

6. **BLIND GUESS Strategy.** Unless the teacher guides a student in the estimation process or uses problem-solving skills, many students will never go beyond this level. This is not true estimation, but merely a guesstimation.

Activity 7

Students are to prepare a written report of solutions for the following four problems. Have students solve the problems in groups of 4, turning in all the papers. (Say to students ahead of time that you may look at any one of the papers in the group and that will be the grade assigned to each member.

WORKSHEET FOR ACTIVITY 7

1. Gloria is driving to Chicago. The last sign said it was 160 miles to Chicago. If she has driven 40 miles since she saw the sign, and is halfway to Chicago, how far is it from her home to Chicago?

2. Portia has 25 cents in her pocket. If you can tell her all the possible combinations of coins she could have that sum to 25 cents, she will give you the 25 cents.

3. Farmer Ben has only ducks and cows . He can't remember how many of each he has, but he really doesn't need to remember. He does know that he has 22 animals which is his age. He also remembers that those animals have a total of 56 legs, which is his father's age. Assuming that each animal is normal, how many cows and ducks does Farmer Ben have.?

4. Ken said that there are 1600 students in his high school. Kara said that there are 70 more girls than boys. How many girls attend their school. *(Please note that these questions and more like them can be found in MAPS I and II).*

Activity 8

MERGER GAME. This game demonstrates the use of negative and positive numbers and introduces students to the number line and to computation with integers, as well as the absolute value. More information will follow. (CLWVU, 1994)

In the world of business, some companies make a lot of money (profit) while other don't do so well (debt). Sometime these companies join together (merge) and when they do this, they also combine their profits and debts. In this game, you are your own company, and each member of your group is a different company. You will experiment merging with each of the other companies in your group.

Each group of 4 to 6 students receives a deck of cards. Use all cards Ace through nine plus a few face cards. Pick one dealer who gives each player one card. The card that you receive tells how much your company has in millions of dollars. A red card means your company is in debt (owes money) and a black card means your company is showing a profit (making money) . Example: A black 2 = $2 million profit, a red Ace = $ 1 million debt, a red Nine = $9 million debt. A face card denotes zero profit or loss.

To Play

1. Merge your company with at least three other companies. (Next to you, directly across from you, diagonal from you, etc.) Record your cards and the total debt or profit after the merger.

2. Within the group decide which merger is the best possible of all mergers. Record these cards and their results.

3. Within the group , decide which is the worst possible of all mergers. Record these cards and their results.

4. Repeat steps #1 - 3 two more times after thoroughly shuffling the cards.

5. When does a merger give a profit?

6. When does a merger give a loss/debt?

7. Repeat the game from the beginning. Do your answers to questions #5 and #6 hold true after a second try at the game?

8. Prepare a written summary of your experience with the Merger Game.

MERGER WORKSHEET

(to be prepared ahead of time)

Write down what the outcome of each merger would be.

Group A	2B +1B	3B +4B	4B + 2B	2B +5B
Group B	3R +2R	1R +3R	4R +1R	2R +4R
Group C	4R +2B	2B +5R	3R +1B	5B +7B
Group D	3B +1R	5R +8B	2B +1R	3R +6B

Please answer these questions based on the worksheet

1. Why are these problems grouped the way that they are?

2. Do you see a pattern?

3. Can you make a general prediction about this pattern.

Next have students in their groups place all of the cards on a number line. Lead a summary discussion of all activities.

Activity 9

MERGER GAME PART TWO. The purpose, of this game is to end up with the greatest possible profit (using the ideas from the Merger Game). Using all cards face through nine, shuffle the deck thoroughly and give each player one card ***face down*** Next give each player one card ***face up***. Each player may take a card until she/he has a total of five cards, although they may choose to stop taking cards at any point. The dealer begins by saying "Investment Call" to the first player. The player must decide if she/he wants 1, 2, or 3 cards, or if she/he will pass. To receive a card the player should say "invest', or otherwise "pass" to receive no cards. The dealer then goes to player 2, then player three, etc. This continues until the dealer has gone around the table three times, at which point no more investments can be made.

The table needs to decide which person has the best investment, even though they cannot see one of the person's cards. At this point, the game continues by all players turning over their cards and totaling up their investment. The player with the highest positive score wins and gets the number of points contained in her/his investment. (If player one wins with an investment totaling +4, she would get 4 points). All other players do not score on this round. Player one now becomes the dealer and the game continues until all players have been the dealer one time.

Present positive and negative number worksheets ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION from your text book. Discuss why negative numbers are needed. Talk about absolute value.

Find values for the variables (a, b, ... g) such that each row, column, and diagonal of 4 has the same sum.

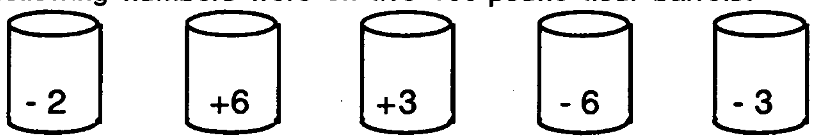
(Heath Pre-Algebra, Lowry, Ockenga, Rucker, p 97)

-16	a	b	-10
+6	c	-4	d
e	+2	+4	f
+8	g	-14	+14

Activity 10

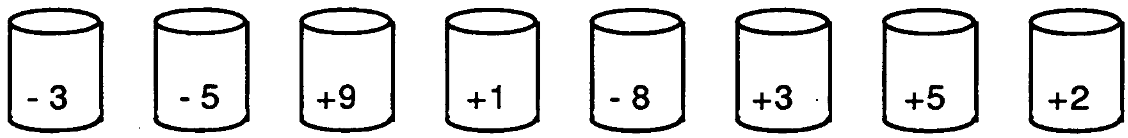
1. In the 15th Century, European flour merchants used positive and negative numbers. On a barrel, +5 meant that it was 5 pounds overweight, -5 meant that a barrel was 5 pounds underweight.

   a. The following numbers were on five 100-pound flour barrels:



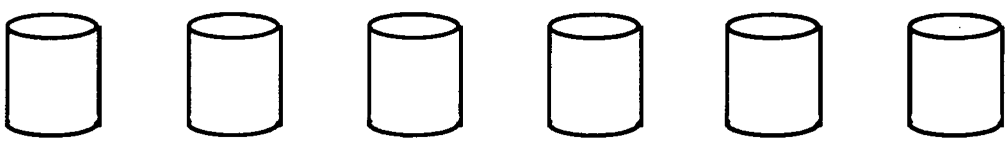
   Did the five barrels contain more or less than 500 pounds?
   How much more or less?
   How did you find this (explain your answers)?

   b. These numbers were on eight 100 pound barrels:



   Did the barrels contain more or less than 800 pounds?
   How much more or less?
   How did you find this (explain your answers)?

   c. The drawings below show six 100-pound barrels. The total weight in the barrels is 13 pounds less than 600 pounds. Suppose that each barrel is marked with a different number. Show a way that the barrels could be marked. (Keep your numbers between -10 and +10)



2. Tank is a football fullback. Sometimes he gains yardage (+5 means a five yard gain). Sometimes he looses yardage (-8 means an eight yard loss).

   a. Determine Tank's total yardage in each game.
   Game 1: +3, +5, +6, +13
   Game 2: - 2, -6, -1, -2, -4
   Game 3: - 4, +4, -3, +3, +10, -13
   Game 4: - 5, +3, +6, -2, +8, -6, +9, -1, K +8, K+3, +7, K -11, -6.
   Game 5: + 6, -7, -5, +10, -3, -10, +5, -8

   b. Discuss the variety of ways that students solved each total yardage problem.

Activity 11

A. Arrange the numbers in increasing order

1. +3, +8, -2, 0, -10, - 1.5
2. +4, -3, -39, +6.3, +1, -63

B. Arrange in decreasing order.

3. -9, +15, +3.5, +19, -6, -2.7
4. +1/3, -128, +5, +9, -7.2, +11

C. Relate these to a vertical and horizontal number line.

You could do the exploration of negative numbers using calculators as found on (pp 65,66, 67 in *Integrated Mathematics 1,* by Houghton Mifflin)

Activity 12

Materials: Two sets of integer cards: 1. Integers 1 to 9 (for larger problems, two sets may be needed) and 2. Integers -9 to -1. N.B You could use black cards as positive and red cards as negatives where face cards are zeros. Use only one face card in a double set.

Teacher Notes. The purpose of this activity is to encourage students to use number intuition to compute answers for various computational operations. The number cards can be used with whole number operations, or with rationals (Both positive and negative.) In this activity , one exercise is discussed at length to suggest possible questions the teacher may use to guide the discussion. Other activities are suggested in the extensions. Note that zero may be included as a digit. Some students may try to use 0 as a denominator. Discuss why division by 0 is undefined.

So if 0 is drawn, the only possible ratio is $\dfrac{0/a}{b/c} = 0$

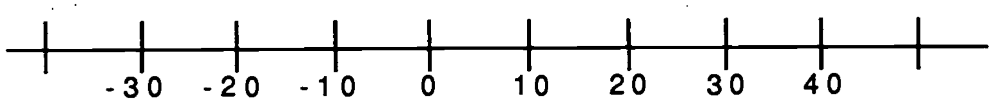
Exercise: Teacher should draw four cards from the 1 to 9 set. The four whole numbers should be used to make the largest quotient of two fractions. Suppose the cards 2, 3, 5, and 7 are drawn.

(1) Possible student quotients: 3/2 divided by 7/5 = 15/14, 2/3 divided by 5/7 = 14/15, 7/2 divided by 3/5 = 35/6, etc. Have students state answers as they are found and record on the chalkboard or overhead. Continue recording any answer that is larger than those answers already given. Verify the largest answer.

(2) Discuss strategies used to determine the largest quotient. (35/6).
Example: If you are trying to discover the largest quotient, would you want your first fraction (dividend) to represent a large or small number? (Large). Would you want your second fraction (divisor) to represent a large or small number? (small). Why? (By definition of division, a divide b means how many measures of b can be found in a: i.e., 12 divide 3 means how many 3's are in 12; or 1/2 divided by 1/6 means how may 1/6's are in 1/2.) To have a larger quotient, one would want a larger number to be divided by a smaller number.)

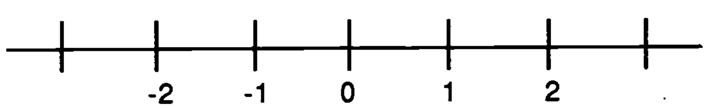
Assume any four of the digits 1 - 9 could be drawn.

1. Ask: "Where on the number line is the value of the largest possible quotient?" (36) Label a number line on graph paper and locate your quotient.



How did you get it? [Note: There are 4 different fraction ratios that yield the quotient 36.]

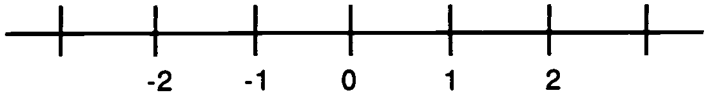
2. Have students find the smallest quotient. Where on the number line is the smallest quotient? (1/36, 0 if a face card is included in the set.)



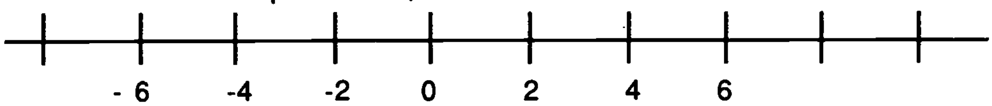
3. What relationships do you see between the smallest and largest quotients? (reciprocals for non-zero quotients)

4. Would you expect this relationship to be true for any four digits? Why or why not?

Suppose three of the numbers drawn were negative (using both sets of cards), for instance -2, -3, -5 and 7. What would the largest possible quotient be? (-6/35; Again 4 ratios will give this quotient). Where would the largest quotient be on the number line?



What would the smallest quotient be, and where would it be on the number line? (-35/6)



EXTENSIONS:

1. Draw a certain number of digits and find the largest whole number product, quotient, difference or sum, or find the smallest for each operation.
2. Use digits to determine inequalities with decimals, whole numbers, fractions, etc. Example: Decimals: Have each student enter each digit as it is drawn. Teacher may use two decks of 0 - 9 integer cards. Since this activity is partly chance, student may construct a false statement. If so, have them rearrange the numbers to make a true statement.
   Example: . ______ _______ < .____ _____ _____ < . ____ _____

   If digits drawn are 5, 7, 3, 6, 3, and then 1, students may write .43 < .567 <.13. Since they must enter digits as drawn. Change to .13 < .43 < .567 to make a true statement.
3. Use digits as exponents and write numbers in order.

   Example: 4 < 3^2 < 10 Draw: 4, 2, 3, 1, 0
4. Use digits as bases and exponents to make largest (or smallest) number.

   Example: Draw 3, 5, 9, 0.

   Largest: (530) to 9th power;
   Smallest: (935) to zero power since any number to zero power equals 1.

Activity 13

Math Content: Prime numbers, composite numbers, prime factorization, classifying integers according to the number of factors.

Numbers that have exactly two factors are known as "prime numbers". Except for the number 1, numbers that are not prime are called "composite numbers" and can always be broken down into a product that consists entirely of prime numbers. This product is known as the 'prime factorization" of the number. For example, the prime factorization of 6 is 2 X 3 and the prime factorization of 60 is 2 x 2 x 3 x 5.

Complete a table for the whole numbers 2 through 25 using the headings below.

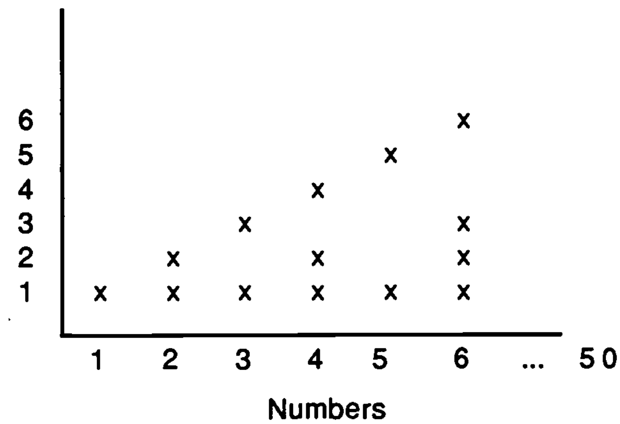
Number	List of all factors	Number of factors	"Prime" or Prime factorization
2	1, 2	2	2
3	1, 3	2	3
4	1, 2, 4	3	2 • 2

Activity 14

LOCKER PROBLEM: Students at an elementary school decided to try an experiment. When recess is over, students will walk into the school one at a time. The first student will open all of the first 100 locker doors. The second student will close all of the locker doors with even numbers. The third student will change all the locker doors with numbers that are multiples of 3. (Change means closing lockers that are open and opening lockers that are closed.) The fourth student will change the position of all locker doors numbered with multiples of four; the fifth student will change the position of the lockers that are multiples of five, and so on. After 100 students have entered the school, which locker doors will be open? (CLWVU)

Note that students could use a graph format as they search for patterns of factors labelled on the x axis 1 through 50 and called numbers; and on the y- axis labelled 1 through 50 and called factors of the numbers. Here is an example

Searching for Patterns of Factors



ACTIVITY Factor Feat

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Rules: Players alternate taking turns picking numbers and marking it (one player use a circle and the other use an x). When a player has marked a number, she/he also marks all the factors of that number that have not already been marked. The winner is the player with the **largest sum of marked numbers** when the game is over (when either time is called or all the numbers have been marked). (CLWVU)

Activity 15

Follow up on the previous day's activity, stressing the important idea of the prime numbers as the building blocks of the set of integers; the structure of the integers is built upon a prime number base. Discuss the game in your groups, then move on to looking at divisibility tests allowing students to discover when to tell a number is divisible by 2, 3, 4, 5, 8, 9, 10. This information should be in your current text.

Activity 16

Worksheet. Searching for Patterns of Factors. On a 50 by 50 grid, students will identify the factors and number of factors of the composite numbers between 1 and 50. The prime numbers between 1 and 50 will also be identified. Students are to place an **X** in the boxes of the factors of each number. The numbers are on the horizontal axis, and the factors are on the vertical axis. (See grid on previous page - Searching For Patterns of Factors)

Activity 17

Worksheet Classifying Numbers By Their Factors. (CLWVU)

1. What numbers, 50 or less, have exactly 2 factors? Exactly 3 factors? Exactly 4 factors? Exactly 5 factors? Exactly 6 factors? Exactly 7 factors? Exactly 8 factors? Exactly 9 factors? Exactly 10 factors? Exactly 1 factor?

2. How would you classify this information? Is there any pattern you have observed?

3. Try to predict a number greater than 50 with exactly 2 factors. Check it. Record all trials until a number is found.

4. Try to predict a number greater than 50 with exactly 3 factors. Check it. Record all trials until a number is found.

5. Find the prime factorizations of 110 and 204. Describe the process you used to find the prime factorizations.

Activity 18

Order of Operations Activity. (CLWVU)

1. Have students solve the following problem in as many different ways as possible. Have students place grouping symbols to provide as many different results as possible. Tell them keep track of the order in which they used to get each answer.

   25 + 2 * 16 / 4 - 2. [Note: * means multiply]

2. In pairs, have students compare and contrast their answers.

3. In groups of 4, students are to repeat #2. Then they are to compile a list of the various answers and the groupings used to get them.

   E.g. (25 + 2) * 16 / (4 - 2) = 216
   25 + ((2 * 16) / 4) - 2 = 31

4. As a part of group discussion, students should answer the following questions:
   A. Why did you have different answers?
   B. What can you do to consistently get the same answer?
   C. How can you get the greatest (least) answer?
5. Give each group another problem of your choice. This should be available in your current text.
6. Have each group record its answer and the order of their steps on the board.
7. Numbers 5 and 6 should be repeated until the class agrees on a standard procedure.
8. Refer to your text to compare the classes' decision to the standard order of operations rules.
9. Discuss the need to have things standardized. Have groups compile a list of things in the classroom and their homes that are standardized, i.e., time, lightbulbs, shoe sizes, weights and measures, etc..

Activity 19

Follow up to Activity 18. Select any order of operations problem. Write each number and operation symbol on a different card. Distinguish the answer by color. Students are to arrange the elements in an order that will produce the given answer. This is a small group activity. (CLWVU)

Assessment Plans

Methods of evaluation contained in this unit include:

- a typewritten or word-processed report.
- a poster graded on following instructions, neatness, and timeliness
- a group grade based on randomly selected papers from each group or individual grades with the group losing points for an individual's substantially low grade.
- a group question with an individual objective test or research on a project.
- an objective test.

Resources

Collaborative Learning Workshop, Valparaiso University (CLWVU). (1994).
Lobato, Joanne E.. (1993). "Making Connections with Estimation." *Arithmetic Teacher 40*, 6.
Heath *Pre-Algebra*, Lowry, Ockenga, Rucker, pp 95, 97.
Indiana Mathematics Proficiency Guide, p 137, 1991.
Integrated Mathematics 1, Houghton Mifflin , pp 65-67.
Mathematical Connections A bridge to Algebra and Geometry, Houghton Mifflin, pp 94 - 97.
Mathematics Teacher, September 1994.

I.2 FRACTIONS, DECIMALS, PERCENTS, RATIOS, AND PROPORTIONS

Fractions, decimals and percents have long been stumbling blocks for students because they are not given the opportunity to develop concepts as well as number sense. This unit is devoted to concept development and understanding the processes underlying the operations.

Section Goal and Objectives:

Goal: Students will solve simple problems involving fractions, percents, decimals, ratios, and proportions.

Objectives:

1. Students will write ratios as fractions in lowest terms and as unit rates.
2. Students will solve proportions.
3. Students will interpret and use scale drawings.
4. Students will use proportions to solve problems.
5. Students will use proportions to solve percent problems.

Instruction

Activity 1

This activity emphasizes the part-whole definition for fractions, where fractions represent a part of a whole. This concept will enable students to solve problem situations involving operations with fractions.

Set Model

Give the students a set of fifteen objects (i.e. marbles, cookies, toy cars, etc.), and ask them the following questions:

Can 15 objects be partitioned equally among 5 people?; 4 people?; 3 people?; 2 people?

Discuss and physically demonstrate why the 15 objects can or cannot be partitioned. With this understanding, children can now answer questions such as these:

What is one-fifth of 15?, two-fifths of 15?, three-fifths of 15?.

Extension: Complete this same activity using 4, 9, 12, and 20 objects.

Area Model

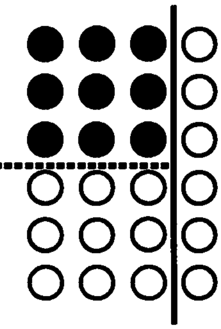
Give each child a fourth of a sheet of paper(plain or colored). Have each child fold the paper into thirds and shade two-thirds. Now fold the paper in half the other way. Ask how many parts and what kind of parts are represented by the folded sheet of paper [6, sixth]. Next, ask what part is shaded. Emphasize both 2/3 and 4/6. Tell the students that we call 2/3 and 4/6 equivalent fractions because they represent the same amount.

Extension: Complete this activity using different fractions. Use a clock face to see halves, thirds, fourths, etc..

Activity 2

Introduce operations with fractions using the set and area model, for example:

Multiplication (Set Model: * <=> "of")



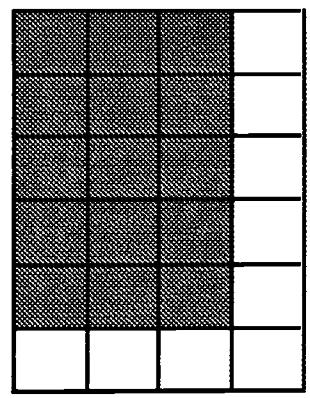
Problem : You have 3/4 of a case of 24 bottles. How many bottles do you have? If you have half of these, how many bottles do you have?

Solution: Have the students partition 24 bottles into 4 equal qroups and physically count 3 of the four groupings. [3/4 X 24=18]

3/4 of 24 is seen to be 18. 3/4 * 24 = 18

1/2 of 3/4 is seen be be 9. 1/2 * 3/4 = 9/24 = 3/8

Multiplication (Area Model)



Problem: If you own 3/4 of an acre of land and 5/6 of this is planted in trees, what part of the acre is planted in trees?

Solution: Consider first the acre partitioned into fourths and the amount you own, which is 3/4. Have students fold a sheet of paper into fourths and shade in 3 parts. Now partition the same "acre" into sixths on the other dimension and focus on the five-sixths that you have planted with trees. Students are to fold the same paper into six parts and mark 5 of each 6 with a tree. Since you are calculating area, 3/4 X 5/6= 15/24 = 5/8. Students should be able to count 15 squares out of 24 marked with trees.

Extension: For additional problems refer to *Helping Children Learn Mathematics* by Robert E. Reys, Marilyn N. Suydam and Mary M. Lindquist, fourth edition.

Activity 3 - Making Chicken Wings (Individual)

Objective: Students will convert measurements in a recipe.

Chicken Wings

Serving Size: 4
Preparation Time: 40 minutes

1 lb. chicken wings	2 tbsp. peanut oil	1 tbsp. soy sauce
1 1/2 tsp. ginger	2 1/4 tsp. garlic	1/2 tbsp. vinegar
1 cup tomatoes	1 tbsp. Worcestershire	1 tsp. sugar

1. Rewrite the recipes with amounts appropriate for 10, 15, and 20 people.
2. Suppose that when the recipe is increased to accomodate more people, the preparation time increases by 5% for each person
   How long would we expect it to take to prepare 6, 10, 15, or 20 servings?
   .05 X 40 = 2 minutes / servings
   So, the time for 6 persons = 40 minutes + (2 * 2) = 44 minutes.

Extension Activity: Have students find recipes at home or in a cookbook and convert the amounts in the recipe to feed everyone in the class.

(*Performance Assessment Task*, MacMillan/McGraw-Hill School Publishing Co., 1994.)

Activity 4

(FROM CLWVU)

Discuss fractions and review. This activity will help identify student misconceptions about the basic operations with fractions. Possible meanings of and responses to incorrect responses follow.

1. 1/3 = 2/6 = ?/12 = ?/15 Problem with fractions /equivalent fractions; needs to be reviewed.
2. 1/8 + 3/8 = Use visuals to demonstrate.
3. 3/4 + 1/2 = Needs help in finding common denominator. Use area models.
4. a. 1/3 + 2 1/6 b. 7/8 - 2/3 (same as #3)
5. 3/4 x 1/2 = Review by shading and use pieces of paper through folding.
6. 3 2/5 x 6 = Students may multiply two whole numbers then add fraction next to answer. Review changing to improper fractions or use distributive law.
   (3 + 2/5) * 6 = (3 * 6) + (2/5 * 6) = 18 + 12/5 = 20 2/5
7. 2/3 ÷ 5/6 = Demonstrate with integers first. Not a good understanding of division.
8. 3/5 ÷ 4 = May not know reciprocal. Review reciprocals.
9. 5 2/3 ÷ 3/4 = Same as #8
10. 7/11 + 3/11 = 10/22. Students will add both the numerators and denominators.
11. 7/11 + 3/11 = 11/10 Students will invert numerator and denominator
12. 7/11 + 3/11 = 18/14 Students will add numerator of each fraction to denominator.

Activity 5

In introducing decimals, this activity will link decimals to common fractions. Before introducing the decimal notation, remind your students that to partition a unit into tenths, there must be 10 equal parts. They should also be able to explain and demonstrate why 10 tenths equal 1 whole, 7 tenths is less than a whole, and 11 tenths is more than a whole. Now introduce the new symbol, for example .7 is a new symbol for 7/10. Stress that .7 is read just as 7/10, seven-tenths. *Use money and meter stick models to make this connection.*

Can You Beat the Toss?

Material: Paper and pencil for the students; a penny for the teacher.

Directions:
1. The teacher reads a number.
2. Each child writes the number in either decimal or fraction notation.
3. Each child receives a point if he or she writes the number correctly.
4. The teacher tosses a coin. If it's head, those who wrote a decimal receives one more point. If it is tails, those who wrote a fraction receives one more point.

Suggested Numbers: two-tenths; seven-tenths; one and three- tenths; thirty four and no tenths; five and seven-tenths; eleven and four-tenths, and sixteen - tenth.

Winner - the person with the most points after a set time or the teacher can set a winning score.

Resource: Reys, Suydam and Lindquist, *Helping Children Learn Mathematics*. 1995, p. 231-256.

Activity 6

In mathematics, a proportion is a statement of equality between two ratios. Early Greeks used proportions to describe the perfect human body, which they believed, was based on two values, H, a person's height from heel to head, and W, a person's waist height, measured from heel to navel (navel is sometimes called the "belly button"). The Greeks use the proportion $H / W = (H + W) / H$ as a test for what they considered the ideal human shape.

THE IDEAL HUMAN SHAPE

Divide students into groups of four and have them complete the following :

1. Each member of the group is asked to measure his height H and waist height W. Express each measurement in inches and record the measurement in the appropriate spot in the table below. (Note: This might be done with selected individuals so as not to embarrass anyone.)

GROUP MEMBER Name	HEIGHT H (inches)	Waist Height W (inches)	$\frac{H}{W}$	$\frac{H+W}{H}$

2. Compare the ratio H/W for each group member and enter it in the appropriate spot on the table.

3. Add H and W for each person, then compute the ratio $\frac{H+W}{H}$ for each group member and enter it in the appropriate spot on the table.

4. How do the two ratios $\frac{H}{W}$ anc $\frac{H+W}{H}$ compare for each member of your group?

   Are they very nearly equal or not? Describe in words.

5. How do the two ratios $\frac{H}{W}$ and $\frac{H+W}{H}$ compare among members of your group?

   Are they very nearly equal or not? Describe in words.

6. How close are your proportions , $\frac{H}{W}$ and $\frac{H+W}{H}$, to the ideal Greek body?

7. Suppose your height is 60 inches. To be "Greek ideal" what should "W" be? (37 inches)

Resource: Leidig, Julie; *The Competitive Edge: Sharpening Your Skills in the Workplace*; Texas Univ., Austin. Extension Instruction and Material Center, 1993 .

Time : 2 -3 days

Evaluation: Group participation and accuracy of responses.

Extension: Have students do a one page research paper on the Golden Ratio and how it relates to the body. This paper should be typed or processed on a Computer. If possible, schedule one class day for research in the school library, and allow seven (7) days for completion.

Activity 7

Bring in an actual model of a car, airplane, boat, or a blueprint for a house or other building. Discuss the scale used to make the item. Have students calculate the actual measurements of all parts of the actual item. Then have students make a scale drawing of their model, basketball court, classroom, or other real-life surface.

Resource: Mathematical Connection, Houghton Mifflin, 1992, p 386c

Evaluation: Objective evaluation of answers.

Extension: Get a copy of your school map. Students will calculate the size of a given room and compare the calculated amount to the actual measurement. Discuss any variations that seem to exist.

Activity 8

Assign students to go to a nearby department store and find 10 different advertised items (cannot all be blue jeans, etc.) that are shown at a percent discount. Have them determine if the price marked is the original price or the discount price. If it is the original price, they must calculate the discount price. If it is the discount price, then they must calculate the original price. Each problem must be accompanied by a paragraph describing the item and the exact amounts listed in the advertisement.

Resource: Sunday newspaper advertisements and local department stores or malls

Evaluation: Student presentation of problems and evaluation of the project.

Assessment Plans

Objective evaluation of real-life measurements of models and/or blueprints.
Student presentation of problems and evaluation of the project.

Unit I.3 MEASUREMENT

This chapter will give students the opportunity to use various measuring tools and to determine how and when these tools can or should be used. They will explore relationships between various units of measure and will understand their appropriate uses. Measurement should be used in all units, but this section focuses on relationships within and among measurement systems.

Section Goal and Objectives

Goal: Students will measure in English and metric units, and use measurement to solve problems.

Objectives:

1. Students will measure items using standard and metric units.
2. Students will identify equivalent units of measure within the standard and metric systems.
3. Students will identify similar standard and metric units.
4. Students will convert standard units to metric units and vice versa.
5. Students will identify the appropriate standard or metric unit of measure.

Instruction

Activity 1

A. Students will use rulers and meter sticks in this activity. Have students work in pairs and measure several items in the classroom in metric and standard units, i.e., desk for length and width, a book, the chalkboard, etc.. Discuss partial units of measure and how they are determined, i.e., 21 3/4 inches vs 22 inches. Check to see if students used partial units or if they rounded to the nearest whole unit.

Assessment: Verify student's ability to measure accurately.

B. **How Do I Measure Up?** Students will use rulers and meter sticks to measure various body parts, such as fingers, arms, feet, arm span and height. Students will also identify body parts which are close as possible to an inch, a foot, a centimeter, millimeter, and a decimeter.

Optional: Prior to part A., have students use a body part to measure something in the classroom. For example, use their foot to determine length and width of the classroom. List the outcomes on the board. Discuss why variations occurred and the need to have measurement standardized. This would also be a good time to discuss the origin of standard units of measure.

Activity 2 - Converting Units Within the Standard System

Start this activity with several easy examples of converting units from the smaller unit to a larger unit. Examples: 36 inches = ___ feet 4 pints = ___ quarts
6 feet = ___ yards 8 quarts = ___ gallons

In pairs, have students write equations with ratios to show how they got their answers.

For example: $\frac{?\text{ feet}}{36\text{ inches}} = \frac{1\text{ foot}}{12\text{ inches}}$

Then, have them state in general terms, which mathematical operation is used when converting from a smaller to a larger unit (division). Repeat these steps to convert from larger to smaller units.
Examples: 1 mile = ___ feet 3 gallons = ___ pints
6 yards = ___ inches 5 pounds = ___ ounces

Refer back to Activity 1. Have students write equations with ratios to convert the units they used to larger or smaller units.

Activity 3 - Converting Units Within the Metric System

Follow the same procedure in Activity 2 to convert from smaller to larger units and vice versa. Make sure students know that the prefixes of metric units determine the conversions

Units of length and their relationships to the meter

millimeter =	0.001 m	or	1/1000 m	1000 millimeters = 1 meter
centimeter =	0.01 m	or	1/100 m	100 centimeters = 1 meter
decimeter =	0.1 m	or	1/10 m	10 decimeters = 1 meter
meter				
dekameter =	10.0 m	or	10/1 m	10 meters = 1 decameter
hectometer =	100.0 m	or	100/1 m	100 meters = 1 hectometer
kilometer =	1000.0 m	or	1000/1 m	1000 meters = 1 kilometer

Grams and liters: Have students copy this chart and complete one like this for both liters and grams as well. (The redundancy that they will see is important.)

Activity 4 - Converting Between Standard and Metric Units

Use the following chart to help answer the following exercise.

1. Give your height in metric and English units.
2. How many gallons are in 2 (two) 3-liter bottles of cola?
3. Which will serve more: Six cans of soda or a 2-liter bottle?
4. To the nearest 1/10 of an inch, how thick would a stack of 200 sheets of paper be if one sheet is 0.1mm thick?
5. Chicago, Illinois, is approximately 30 miles west of Gary, Indiana. How far apart are they in kilometers?

Conversion Chart

1 inch = 2.54 centimeters	1 meter ≈ 39.37 inches
1 pint = 28.875 cubic inches	1 meter ≈ 1.094 years
1 pound = 453.59237 grams	1 kilometer ≈ 0.621 (5/8) miles
1 quart ≈ 0.946 liters	1 liter ≈ 61.025 cubic inches
1 kilogram ≈ 2.205 pounds	

(Note: that some conversions are exact while others are approximate)

Consult a standard reference for other exact or approximate values (e.g. *The World Almanac*)

Activity 5 - Distance

Students should work in small groups to complete this activity. Select a location in the United States that is currently in the news a lot, for example, Atlanta, Georgia for the 1996 Olympics. Have each group select a country and find out the distance from there to your chosen location in miles, kilometers, taking land , sea and air routes. Students will need a globe, atlas, or world map. If necessary, remind them that a kilometer is about 0.62 miles. For sea routes, students should figure the distance in nautical miles. A nautical mile is about 1.2 miles. Each group should create a chart to present their research results to the class.

Extension: Take students to the school track. Ask them to explain why runners are staggered at the beginning of a race. Have them measure the lanes in metric and standard units to support their responses.

Activity 6 - Research Project

Research the history of the metric or the standard system of measuring. Divide the class so that half of the students will research one of the two systems.

Resources

Applied Mathematics; A Contextual Approach to Integrated Mathematics: Unit 3; Measuring in English and Metric Units, The CORD Project Staff; CORD Communications Waco, Texas, 1993.

Pact '95: Hands-on Activities (with on-line connections to the World Wide Web). Scott Foresman, 1995.

Assessment Plans

Each activity should be assessed separately. Evaluate students ability to successfully complete the activities. A unit exam should be given upon completion of the activities. Quizzes may also be included to address mastery of the skills used in the activities.

Unit II

Exponents, Area, and Volume

Unit II EXPONENTS, AREA, AND VOLUME

This unit introduces students to the idea of powers and roots by showing that powers are exponents. Roots are introduced as the inverse of powers, and powers and roots can be used to calculate lengths, areas, and volumes of geometric figures.

Terminology, characteristics, and procedures for calculating perimeter, area, surface area, and volumes of various shapes are also presented in this unit. Activities will improve spatial visualization skills and critical thinking skills. This unit will provide a connection between geometry and algebra.

Unit Goals and Objectives

Goals: 1. The learner will demonstrate computational proficiency with exponents and roots.
2. The learner will integrate problem solving skills and strategies in the use of geometry in real world applications.

Objectives:

1. Students will distinguish between squares and rectangles which are not squares.
2. Students will use exponents to indicate the repetition of a factor.
3. Students will use laws of exponents to simplify algebraic expressions.
4. Students will use exponents in area and volume formulas.
5. Students will identify a square root as the length of the side of a square of given area (e.g. The square root of 2 is the length of the side of a square whose area is 2.).
6. Students will identify common space figures.
7. Students will solve problems involving both two and three dimensional shapes.
8. Students will solve applications that require the use of perimeter, area, and volume.

Instruction

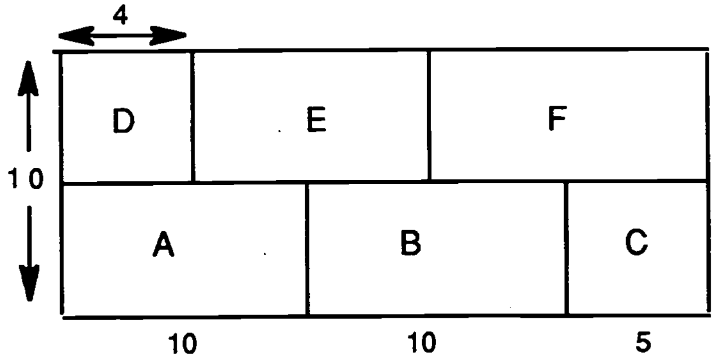
Activity 1

Students will receive several rectangular shapes of different color and sizes. Each student will measure the sides of each shape and classify each as a square or rectangle. Students will derive distinctions between a square and a rectangle based on equal sides.

Material: colored shapes and rulers

Evaluation: Students will be given a map with missing dimensions and asked to complete the map based on the definition of square and rectangle. This will be graded as plus, check, or minus.

Example:



Copy and find all the missing dimensions. Assume that C is a square and A and E are congruent (i.e. the same).

Activity 2

Students will be given a base 10 unit block and told to multiply it by 10 (repeat it 10 times) using base 10 units. Discuss and visualize the base 10 block as 10 x 10 = 100 (Have them count the individual ones units if they have difficulty.) Introduce the word base as a replacement for 10 unit and direct the student toward realization that $10 \cdot 10 = 10 \times$ itself, which can be written as 10^2. Reinforce the idea that the exponent tells how many times the base is multiplied by itself. Students will complete the above task using 2, 3, 4, 5, and 6 as their base.

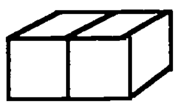
Activity 3

Divide the students into groups.

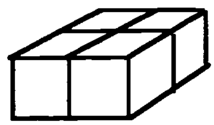
First using cubes, then graph paper, students are to draw pictures of 2^1, 2^2, and 2^3

Examples:

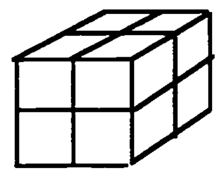
$2^1 = 2$



2 squared
$2^2 = 4$



2 cubed
$2^3 = 8$



Students should also draw pictures of the first 3 powers of two using isometric dot paper or drawing freehand.

The students will repeat these two exercises with 3 and then 4 as a base.

Students should then be led to disciver the rules for adding and subtracting exponents for multiplication and division respectively.

Activity 4a

Students will each receive a geoboard. Instruct them to mark a unit with a length and width of one using rubber bands on the geoboard. They will be directed to use their knowledge of squares on the geoboard to construct squares that are 2 units, 3 units, and 4 units on each side. The area of each will be found by counting the squares inside the rubber band.

Students will be asked to answer the following questions

1. If you wanted a string replica for each square made on the geoboard, how much string would be needed? [Note: This is a review of perimeter]
2. What mathematical term does the length of string for each square represent?
3. What shortcut or pattern can you use to calculate the string lengths? (4 x length of the side)
4. How much aluminum foil is needed to cover the inside of the string for each square? (area)

Use the above information to create friendship cards. Example:
Outside: Roses are red and violets are blue. The person I love is . . .
Inside: You (written under the mirror created by the aluminum foil and the string for the mirror frame.)
Material: geoboard, rubber bands, ruler, string, aluminum foil, graph paper, and cardboard

Activity 4b

Students will be given straws and string to construct squares with sides measuring 2", 4", and 6". They will be instructed to use additional straws to construct a box without a top. They will calculate the surface area in square units of each square and the total surface area of the box. Students will create 1" cubic boxes to aid in determining the space in the boxes created by the straws. (The cubic boxes can be made from graph paper and cardboard.) Then discuss how much space the box occupies leading into volume and the measure of volume in cubic units.

Evaluation: The straw activity will be assessed with a check, check plus, check minus based on measurement and accuracy. A quiz will be given (see below), and a test on solving area and volume equations using exponents.

Quiz: Tell whether the question involves perimeter, area of volume. If it involves area or volume of a square, write a formula using exponents.

1. How much sand is needed to fill a square sandbox?
2. How much sod is needed to cover a square shape lawn?
3. How much fence is needed to fence a square garden?
4. How much water is needed to fill a square swimming pool?
5. How much paper is needed to wrap a cube?

Activity 5

Students will cut out:

4 red circles, 4 green circles, and 4 blue circles, 4 green triangles, and 4 orange squares

The shapes and colors have the following values:
4 red circles = "2," 4 green circles = "3," 4 blue circles = "5"
4 green triangles = "11," orange squares = "7."

The colored shapes will be used to find the square roots of 16, 25, 36, 72, and 75. This will be done by finding pairs of colors and shapes used as factors of the numbers.

Example: Since the red circles are assigned a value of 2, and green circles are assigned a value of three, 72 is the product of 3 red circles and 2 green circles.

We might say: 72 = rrrgg = (rr)(gg)r. So the square root of 72 is represented by 2 x 3 x (the square root of 2).

In later formal algebra, we will write $\sqrt{72} = (2)(3)\sqrt{2} = 6\sqrt{2}.$

The geoboard and graph paper will be used to demonstrate the perfect squares 9, 16, 25, and 36. Investigation of the square root of 20 on the geoboard will show that it lies between the square roots of 16 and 25.

Resource: *Mathematics Teacher*, Vol. 85, No. 6.

Extension: Students will remodel their bedrooms, landscape their yards, etc. using the concepts of perimeter, area of a square, and volume of a cube. They are to create a model for this project and write a descriptive paper. They will be assessed based on actual measurements, creativity, use of appropriate formulas, and presentation of the descriptive papers. They will also be evaluated by means of an objective test.

Activity 6

The measure of a closed region of a plane is called its area. When you find how much paper it takes to cover a gift box, not counting the end flaps and folded parts, you are finding the surface area of the box. In this activity you are going to find the approximate surface area of your body. Students will need centimeter grid paper and pencils or crayons.

What Is Your Surface Area?

1. Place your hand on the centimeter grid paper.
2. Trace an outline of your hand with the pencil or crayon.
3. Put a check in each square within the outline, even if only part of the square is included.
4. Multiply the number of checks by 100. This number is the approximate surface area of the body.
   Example: The surface area of one's body is about 100 times the surface area of the hand.
   100 X 92 = 9,200 square centimeters.
5. What is the surface area of your hand? What is the approximate surface area of your body?
6. Repeat the activity using your other hand. Does each hand have about the same number of square centimeters?
7. Do you think that the way you found your surface area can be used to approximate the surface area of a baby or a large person? Why or why not? (Answers vary.)
8. Would the same procedure work to find the surface area covered by hair on a dog or cat? (Answers vary.)

Extension: Work with a friend. Use a roll of bathroom tissue. Have your friend wrap your body from head to toe with the bathroom tissue. (Do not tear the sheets apart.) Then have your partner gently unwrap the tissue. Count the number of sheets of tissue that have covered your body. What is your surface area is tissue squares?

Repeat the activity by wrapping your partner. Do you think this is an accurate way to find the surface area of your body? Why or why not?

Resource: Dyches; Griffith; Brown; and Neil, *Great Explorations in Mathematics*, Alpha Publishing Co. Annapolis , Maryland, 1994.

Evaluation: Students should be assessed based on class participation and discussion. This will be accomplished with a simple +, /, or - system.

Activity 7

Students will make a model of their bedroom, a popular landscape, a building such as McDonald's etc. using the concepts of perimeter, area of a square, and volume of a cube. They can use Leggos, popsicle sticks, cardboard or any sturdy material to create their models for this project. They must write a descriptive paper, which must include their unit identification (e.g. 1 unit = a leggo with 2 bumps, 1 unit = 1 stick, or 1 unit= 1 foot), and detailed descriptions of their calculations of total surface area (i.e. wall 1 = 3ft X 8ft = 24 sq. ft.). Encourage the students to have windows and doors in their models.

Evaluation: Students' projects will be assessed based on accuracy, creativity, use of appropriate formulas, and presentation of the descriptive papers. The objectives should also be evaluated by means of an objective test

Activity 8

A cardboard company deals with many product companies that want their products packaged in various shapes. How much material will it take to make the shapes?

Paper Folding

Paper folding and cutting duplicate patterns for a rectangular prism, cube, cylinder, rectangular pyramid, etc. Students will cut, fold, and paste (tape) to form each solid. They will also fill out an information sheet about each solid answering questions such as: How many polygons or other surfaces formed the figure? What was the shape of each? What was the area of each face? What was the total surface area? How would you tell someone to find the total surface area of each solid? Explain how to find the perimeter and area of each figure.

Resource: *Mathematical Connection*, Houghton Mifflin, 1992, p. 626c
Time: 2 weeks
Evaluation: Worksheets that contain answers to the above questions would be turned in.

Activity 9

In order to make a hollow plastic ball with a diameter of 123 inches, you will need to know how much plastic to use. What is the surface area of the ball?

Surface Area of a Ball

Slice an orange in half and draw four circles congruent to the great circle created by the cut. Ask students what the total area of the 4 circles is in terms of the radius. Peel the orange halves, breaking the peels into smaller pieces, and then fill in the four circles with the peels to check the reasonableness of the formula. Follow up by using the formula to find surface area of other figures.

Resource: *Mathematical Connections*, Houghton Mifflin, 1992, p. 626d

Evaluation: Quiz on surface area, which would include a paragraph analyzing the activity.

Activity 10

A museum has a display that needs to be painted. (You could substitute a display in your school) Find the amount of paint that needs to be purchased for the project. What information will you need to know before you purchase the paint? (Define a display for a museum, such as a triangular prism). Students must ask for the information they will need, and only the information asked for will be given. A model of the display should be made.

Evaluation: Quiz on surface area, which would include a paragraph analyzing the activity.
The model should be evaluated and the final answer should be evaluated.
Worksheets that contain answers to the above questions would be turned in.
The model should be evaluated (Correct type of figure?), and the final answer should be evaluated.

Resources *Mathematical Connection*, Houghton Mifflin, 1992, pp 626c-d

Activity 11

To compare the volume of hollow three-dimensional containers, you can fill each with the same kind of material. In this activity you will make cylindrical containers of various sizes, fill them with rice or beans, and then compare the volumes of the containers.

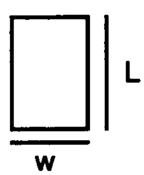
Same Volume = Same Capacity = Same Surface Area?

Materials needed for this activity are 9 X 12 construction paper (three or more sheets), tape, 2 cups of dried beans or rice, and a measuring cup.

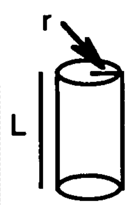
1. Work with a partner.
2. a. Cut one 9" X 12" sheet of construction paper in half.
   b. Make a cylinder by rolling one piece of construction paper the long way. Overlap the edges approximately 1/4 inch and tape the sides. Cut a circle of cardboard to cover one end of the tube. Tape securely with two long pieces of tape in an X over the bottom.
   c. Roll the other half of the sheet of construction paper the "short" way to make a shorter, fatter cylinder. Cut and tape a cardboard bottom for one end.
   d. Stand both of the cylinders up on a table or other smooth surface. Carefully hold one cylinder upright. Have your partner fill the container with beans and rice.
   e. Then carefully pour the beans or rice into the other cylindrical container. Do the containers hold the same amount?
3. a. Roll a 9" X 12" sheet of construction paper to make a long cylinder.
   Roll another 9" X 12" sheet of construction paper to make a fatter cylinder.
   b. Repeat the procedure as you did with the half sheet of paper. Do these two containers hold the same amount of rice or beans?
   c. Explain why the cylinders do or do not hold the same amount and formulate a generalization. (Explanations will vary. However, the short fat cylinder will always hold more than the tall thin one. Though students will not be this complex in their answers, consider:

The Proof:

Consider a rectangle (below) where w = width and L = length



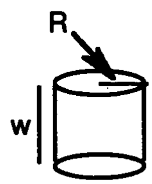
For the tall thin cylinder:



C (thin) = $2\pi r = w < L$

V (thin) = $\pi r^2 * L$
$= \pi (w/2\pi)^2 * L$
$= (w^2 * L) / 2\pi$

For the short fat cylinder:



C (fat) = $2\pi R = L > w$

V (fat) = $\pi R^2 * w$
$= \pi (L/2\pi)^2 * w$
$= (L^2 * w) / 2\pi$

Now, since $L > w$, $(L^2 * w) / 2\pi > (w^2 * L) / 2\pi$

Therefore V (fat) > V (thin)

Extension: Repeat the activity using two 8-inch squares of paper. Fold each in half and tape them together to make a rectangular container. Cut and tape a cardboard bottom for one end. Fill with beans or rice. Use a measuring cup to find how much the rectangular container will hold. Relate this to given formulas. (V = Area (base) * Height) You will need to relate cubic inches to cups to do this. 1 cup ≈ 14.4 cubic inches

Resource: Dyches;Griffith; Brown; and Neil, *Great Explorations in Mathematics*, Alpha Publishing Co., Annapolis , Maryland, 1994.

Evaluation: Students should be assessed based on class participation and discussion. This will be accomplished with a simple +, /, or - syystem.

Unit III

Data and Probability

Unit III.1 STATISTICS, GRAPHS AND DATA ANALYSIS

Statistics are represented by bar, line, circle graphs, stem-and-leaf plots, box-and-whiskers plots, scattergrams, histograms, and frequency polygons. Problem solving is strongly supported through the analysis of tables and graphs. Students read and interpret graphs as well as examine, justify, and explain why data are presented in a particular way.

Section Goal and Objectives

Goal: The student will integrate problem solving skills and strategies in the use of statistics and graphs in real world applications.

Objectives: The students will

1. Construct frequency tables, tables, graphs and histograms to represent real world data.
2. Compare/contrast several methods of representing a set of data (frequency tables, tables, graphs and histograms).
3. Choose measures of central tendency appropriate for the data.
4. Make and interpret stem-and-leaf plots, box-and-whiskers plots, scatter plots, and bar, circle, and line graphs.
5. Determine which form of plot or graph best represents the data for a given purpose.
6. Sketch a best fit curve for a set of real world data.
7. Use a best fit curve to make predictions.
8. Use graphing calculators to make graphs of tabular data.

Instruction

Activity 1

Define mean, median, mode and range. Select a set of data from data bases within your class. In small groups, compute the mean, median, mode, and range of the data. After computing these, discuss their definitions. Have students apply the definitions to real world situations. Groups can list situations where each term will best apply.

Suggested data: shoe sizes; height; brand of shoes; number of siblings or people in a household.

Resources: Each student will need a spiral bound notebook as a journal.
Time: 1 day for explanation and beginning. Carries on throughout the following activities.
Evaluation: Subjective evaluation of the definitions and notebook or develop a rubric.

Activity 2

Get one bar graph and one line graph from some published material and bring to class. After some discussion, ask students to write a paragraph stating the subject of the graph, where the graph came from (i. e., newspaper, magazine, etc.) , one piece of information given by looking at the graph, and the expected use of the graph.

Evaluation: Paragraph content. Rubric will be helpful in keeping grades objective.

Activity 3

Calculator Activity (if graphing calculators are available)

Students will use graphing calculators to graph data. Students love to discover and rediscover! Give them the manual and the calculator and let them review how to get a bar graph and a box-and-whisker plot. Watch them light up with delight at being the first to recall it and share their findings with their classmates.

Activity 4

Cooperative group assignment: Each group will design a survey for which data will be collected. The survey must specify a segment of the population to be surveyed, the method used to insure that the survey is random, and the time(s) when the survey will be taken.

Results of the survey will be shown by a bar graph and a line graph. An analysis of the results will include range, mean, median, mode, and a prediction concerning any future surveys. A brief discussion will be held to evaluate the prediction, followed by a paragraph telling why or why not the prediction has validity.

TOOLS: Aculine for making bar and line graphs and rulers.

Evaluation: Oral presentation of group. Use a rubric that includes evaluation of the oral presentation, graphs, and follow up paragraph.

Activity 5

Support for above activities. Practice exercises from your text.

Activity 6

In small groups, students will select a topic and question and collect data that addresses the question. Students will select the appropriate statistical measures to analyze their data. The data should be recorded on at least one plot and one graph, with an explanation for using that particular plot and graph. During this activity, students should discover and define related vocabulary words such as random selection, population, data dispersion, variance, standard deviation, sample size, biases, and unbiased. They should also review various plots and graphs. Your current text should make a good reference.

Upon completion of the survey analysis, each group should make a brief oral presentation to the class.

Resources: Media Center for research, aculine for drawing plots and/or graphs.
Time: 1 week
Evaluation: Oral presentation of group. Use a rubric that includes evaluation of the oral presentation, graphs, and follow up paragraph.

Resources

Mathematics Connection, Houghton Mifflin,
Statistics and the Graphing Calculator, 1995 NCTM Chicago
Regional Presentation, Donna L. Biggs, Burris Laboratory School, Ball State University, Muncie, Indiana
Activities for the Statistics Classroom, Pamela Clark, Mary Ann Hill, Gary Community Schools
Media Center for research, aculine for drawing plots and/or graphs.

Assessment

Subjective evaluation of the definitions.
Objective test.
Oral presentation of group. Use a rubric that includes evaluation of the oral presentation, graphs, and follow up paragraph.

Unit III . 2 PROBABILITY

In this unit students will develop an understanding of the basic concepts of probability and an ability to apply these concepts to making appropriate predictions. The learner will integrate problem solving skills and strategies in the use of probability in real world applications.

Goal and Objectives

Goal: The learner will find the probability of simple events.

Objectives: The student will
1. Find the probability of some simple events.
2. Count the number of ways an event can happen
3. Draw diagrams and charts to help find probability
4. Use the calculator to find probabilities as decimal numerals and as percents.

Instruction

Activity 1

Discussion of Vocabulary. Group Quiz (on the next page).

Vocabulary
Tally (a marking system to keep track of how many times something happens) 5 = ~~IIII~~
Event (an outcome of an experiment)
Outcomes (a possible result of a probability experiment)
Sample space (a list of all the possible outcomes for an experiment)
Random experiments (an experiment whose outcomes are equally likely)
Favorable outcomes (an outcome that produces a positive result)
Possible outcomes (the number of outcomes that could result from the probability experiment.)
Probability (the likelihood something will happen represented as a ratio of favorable outcomes to possible outcomes of an experiment)
Experimental probability (a ratio of tallies that occurred to possible outcomes)
Theoretical probability (a ratio of expected favorable outcomes to possible outcomes)
Prediction (a statement or educated guess of what will happen before the experiment is done)
Independent events (two or more events that do not affect each other)
Dependent events (events such that the result of the first event affects the next)
Sample group Or survey population (the size or amount of times an experiment is conducted)
Equally likely (the chances of each outcome of an experiment to happen is the same)

PROBABILITY VOCABULARY TEAM QUIZ

Team Name ____________________

Team Members ____________________

____

____

____

Directions: Select the most appropriate term from the word bank for each of the blanks below. No term is to be used more than twice.

Word Bank:	random experiment (s)	sample space (s)	probability (s)
	favorable outcome (s)	possible outcome (s)	event (s)
	independent event (s)	dependent event (s)	

1. Given a number cube, the set $\{1,2,3,4,5,6\}$ represents the ____________________
2. On the number cube, there are a total of six ____________________
3. Since each number is equally likely, rolling the number cube is a (n) ____________
4. If we wanted a 3 to show on the number cube, then rolling a 3 would be a(n) ____________________
5. A(n) ____________________ may be one or more possible outcomes. It may also include impossible outcomes.
6. The ____________for flipping a coin are heads, tails.
7. If I call heads and heads come up, heads is a (n) ____________________
8. Since tossing a head does not affect the probability of tossing a tail, flipping a coin is a(n) ____________________
9. The ____________________ that something other than heads or tails will come up is impossible
10. The ____________ of getting a tail is 1/2

The teacher may want to add his/her own questions.

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Activity 2

RANDOM EXPERIMENTS

In small groups, students will complete assigned random experiments and share the results with the class. Forms to complete this are at the end of this chapter.

Level A

Paper Cup toss
Toss one coin
Pull something from a bucket
(same amount of each)
Roll a single number cube
Spin a spinner (equally likely)

Level B

Paper cup toss with penny
Toss 3 coins
Pull something from a bucket
(different amounts of each)
Roll a pair of number cubes (find sum)
Spin a spinner (not same size amounts)

Example of follow up questions for random experiments:

1. Did your results match your prediction?

2. Which of the following happened?

   # of heads = # of tails.
   # of heads > # of tails.
   # of heads < # of tails.

3. If you redid the same experiment, do you think your results would be exactly the same, about the same, or completely different? Please explain why?

4. If you were told one coin was repeatedly tossed, and heads came up 17 times, how many times do you think the coin was tossed all together? Why?

5. Theoretical Probability : $P(\text{event}) = \dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

   Find P (H)   P (T)

6. Compare theoretical probability with experimental probability

	Theoretical	Experimental	Comparison
Heads			
Tails			

7. If you increased the size of the sample group for the experiment, how do you think this would affect the comparison between theoretical and experimental probability? Why?

Activity 3

IS THIS GAME FAIR?

A game for two participants. Each team will need a pair of number cubes.

RULES:
1. One of the participants is the Player and the other is the Opponent. Only the player rolls the number cubes.
2. Each participant starts out with ten points.
3. Each time the Player rolls a sum of 7, the Opponent must give up or transfer three points to the player. (i.e. The player gains 3.)
4. Each time the Player rolls any sum other than 7, she or he must give up one point to the opponent. (i.e. The player loses 1.)
5. Record the results of each roll on this game sheet. Record how many points the Player or the Opponent has at the end of each roll of the dice.
6. The student with the most points at the end of ten rolls is the winner. If one of the participants runs out of points before ten rolls, the other participant is the winner

BEFORE YOU BEGIN, DO YOU THINK THAT THIS GAME IS FAIR ?

WHY OR WHY NOT?

Play the game at least 3 times and reconsider the matter of "fairness" and this game. How would you adjust the points to make the game more fair?

START		Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6	Roll 7	Roll 8	Roll 9	Roll 10
SUM OF 7		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
		No	No	No	No	No	No	No	No	No	No
PLAYER	10										
OPPONENT	10										

Though the proof is beyond the scope of the students, they may intuitively determine the fair payoffs to be +5 and -1 for the "player."

Activity 4

PROBABILITY PROJECT

GROUP ASSIGNMENT

Create a random experiment that involves probability of independent events. The experiment may be similar but not identical to any experiment done in class.

Develop a form that includes the name and the directions on using / completing / playing the experiment. The form should contain areas for:

- a sample space
- a prediction
- data chart for the tally scores
- theoretical probabilities
- experimental probabilities

Make a trial run of your random experiment and record your results. A duplicate of the form you made should be used to fill in the results of your trial runs. A third sheet with 3 to 5 follow-up questions is also required.

Turn these three sheets in along with a cover sheet containing:
Your group's name, Name of your activity, Date, Class, Teacher.

Activity 5

PROBABILITY WITH AND WITHOUT REPLACEMENT (using M&Ms)

Objectives:
1. Students will use M&Ms to see probability with and without replacement.
2. Students will see the connection between equal probability and equivalent fractions.

Materials:
1. One bag of M&Ms per student or one bag per two students.
2. Paper towels or blank paper. (Paper towels work better because M&Ms do not slide as easily.)
3. Transparent tiles to represent M&Ms on overhead.

Teacher directions:

This is a teacher-directed group-discovery activity. Most students quickly understand that if you have one green M&M and two reds, the probability of selecting a green on the first try is 1/3 (written : P(G) = 1/3.) Some students understand probability with and without replacement, but very few understand the concept well enough to answer questions that require higher-level thinking skills. This activity begins with simple questions and becomes more challenging. The teacher should accept all answers given and demonstrate each answer individually. Students can understand a lot about probability through wrong answers.

Directions for Activity and Teacher Notes

Distribute one package of M&Ms per person or per pair of students. <u>DO NOT EAT M&Ms YET!!</u>

A. Probability with and without replacement. Have students place 2-G, 3-R, 1-Y, 2-0 M&Ms on the paper towel.
   1. If you put these M&Ms in a bag and selected one, what is P(R)? (3/8). Suppose you put the red back in the bag and selected again, what is P(R)? (3/8). If you did this five times, would the P(R) ever change? (No) Why? (you always are selecting from the same pile.)

2. Suppose you use these same M&Ms and again select red the first time. However, this time you do not put the red back in the bag; Instead, you leave it out and then select from the bag again. What is P(R)? (2/7) Why? (Two of the seven remaining M&Ms are red.)
3. Suppose you leave both the reds out and select again. What is P(R)? (1/6) Why? (only 1 red of 6 M&Ms)
4. On the fourth draw, if all three reds have been drawn and not replaced, then what is P(R)? 0/6 or 0 Why? (There are no reds to draw.)
5. Have students write their answer to the following question: We know that when we draw a red from the bag and replace the red. then P(R) does not change. However, what happens to the P(R) when we do not replace it? Do not just say "It changes", but say how it changes. (It decreases).
6. What happens to the probability of the other colors if we do not replace the red? (Probability increases for the other colors.)

B. Probability with equivalent fractions. For each problem, begin with 3 G, 2R, 1Y. Place M&Ms on paper towel. Use extra M&Ms to answer questions if necessary.
   1. What is P(G) (3/6 or 1/2)
   2. How many yellows must be added to the pile before P(Y) = 1/2 ? (4) Note: Most students will answer 2. Consider all answers and discuss the probability of each. Have students model each answer with their M&Ms.

      Solution: Add 1 => P(Y) = 2/7   Add 3 => P(Y) = 4/9
      Add 2 => P(Y) = 3/8   Add 4 => P(Y) = 5/10 = 1/2
   3. Could you add 5 yellows (for a total of 6Y) and have a P(Y) = 1/2? (Yes, if you added one of any other color: 5 yellow and 1 green would give P(Y) = 6/12 = 1/2)
   4. If 8 yellows are added, how many other non-yellow M&Ms would have to be added to give P(Y) = 1/2 ?

      (Note: 8Y added gives 9 yellow total; 9/? = 1/2; ? = 18.
      So we must have 18 total in the pile.)
   5. How many of other colors must be added if 9 reds were added and P(R) = 1/2? (9 reds added gives a total of 11 reds.) 11/x = 1/2, x = 22. There must be 22 total M&Ms.
   6. How many green must be added to make P(g) = 1/4 if 5 reds and 4 yellows have already been added?
   7. Eat the M&Ms and enjoy.

Resources

Cooperative Learning Workshp at Valparaiso University, 1994

Assessment Plans

Students create project
Team quizzes

TOSS A COIN

RANDOM EXPERIMENT

Group Name______________________________ Date__________________

DIRECTIONS: Each student in your group will toss a coin 10 times. Record your data. (Use either check marks or tally marks.)

Sample space ____

Number of possible outcomes ______________________________

Prediction ____

Event Outcome	Tally	Totals	Ratio	Decimal	Percent
HEADS					
TAILS					

Tally - Favorable outcomes

Ratio: $\dfrac{\text{Favorable outcomes}}{\text{Number of possible outcomes}}$

TOSS THREE COINS

RANDOM EXPERIMENT

Group Name______________________________ Date_________________

DIRECTIONS: Each student in your group will toss 3 coins 10 times. Record your data. (Use either check marks or tally marks.)

Sample space ____

Number of possible outcomes ______________________________

Prediction ____

Event	Tally	Totals	Ratio	Decimal
3 Heads				
3 Tails				
2 Heads 1 Tail				
2 Tails 1 Head				
Total trials		**10**		

Repeat and combine with other groups to get 100 trials and create a new table.

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TOSS A NUMBER CUBE

RANDOM EXPERIMENT

Group Name______________________________ Date_________________

DIRECTIONS: Each student in your group will toss a cube 10 times. Record your data. (Use either check marks or tally marks.)

Sample space ____

Number of possible outcomes ______________________________

Prediction ____

Event	Tally	Totals	Ratio	Decimal	Percent
•					
• •					
• • •					
• • • •					
• • • • •					
• • • • • •					

Combine your data with that of others in the class to get over 100 total trials. Recopy the total chart.

TOSS A CUP

RANDOM EXPERIMENT

Group Name______________________________ Date__________________

DIRECTIONS: Each student in your group will toss a cup 10 times. Record your data. (Use either check marks or tally marks.)

Sample space ____

Number of possible outcomes ______________________________

Prediction ____

Event	Tally	Totals	Ratio	Decimal	Percent
Top					
Bottom					
Side					

SPIN YOUR WHEELS

RANDOM EXPERIMENT

Group Name________________________ Date________________

DIRECTIONS: Each student in your group will spin 10 times to choose items. Record your data. (Use either check marks or tally marks.)

Sample space ________________________________

Number of possible outcomes ____________________

Prediction ________________________________

Event	Tally	Totals	Ratio	Decimal	Percent
Car					
Vacation					
Jewelry					
Sport Tickets					
Money					

Note: An appropriate spinner will have to be created. The sectors do not have to be equal.

Unit IV

Algebraic Foundations

Unit IV.1 ALGEBRAIC EXPRESSIONS AND EQUATIONS

This unit connects and extends the skills learned in the first units by having students learn to solve one-and-two step equations. This unit allows the sharpening of student skills involving order of operations, simplifying expressions, giving students an opportunity to work with integers and fractions, using the distributive property, all prior to being introduced to simple equations thereby connecting and extending the skills previously learned in the first units. This unit serves as an introduction to solving equations, a skill students will find is used extensively in algebra and more advanced mathematics.

Goal and Objectives

GOAL: The student will solve equations in one variable by using the properties of equality, (addition, subtraction, multiplication, division).

OBJECTIVES:

Upon completion of this unit, the student will be able to:

1. Write a variable expression to represent mathematical situations.
2. Evaluate or simplify variable expressions by combining like terms and using properties of real numbers, (distributive, associative, and commutative).
3. Solve simple equations by substitution or mental math.
4. Solve one step equations by using algebraic properties of equality.
5. Solve two step equations by algebraic properties of equality.
6. Represent situations by writing an equality or inequality in one variable.

Instruction

Activity 1

Students will do a group assignment on writing an algebraic expression for several given word expressions. The students will be allowed to select their groupmates for this particular activity.

Write an expression for each problem:

1. 3 more than a number
2. A number increased by 3
3. A number is subtracted from 13
4. A number divided by 3
5. $500 more than triple a number
6. 5 more than the sum of 6 and a number
7. One-third of a number decreased by 19
8. 6 times a number
9. 4 more than 5 times a number
10. One fifth of the sum of 5 and a number

Assessment: Students will receive two points for each correct answer.

Activity 2 - Model With Tiles

Algebra tiles will be used to model variable expressions. Expressions modeled will involve similar tiles and unlike tiles.

Suggested Assignment:

1. Given a set up of algebra tiles, describe in words the expression represented.
2. Show how to use tiles to represent each expression:
   $4n + n$, $n + 4$, $3 + 2n$
3. Use algebra tiles to represent : $3(n + 2)$ and $6(n + 4)$
4. With each tile representing a value or variable, represent
   $2n + 4n$ $\quad$ $4n - n$ $\quad$ $3n + 4 + 2n$
5. Students will do a textbook assignment on simplifying expressions by combining like terms and by using the distributive property.

Activity 3 - Guess and Check (A calculator activity)

1. Using the calculator solve the equation $2x + 3 = 11$ by completing the table. The solution you should find is 4, because value of $2x + 3$ in that row of the table is 11.

x	$y = 2x + 3$
0	3
1	______
2	______
3	______
4	______
5	______

2. In the same way, complete the table to solve the equation $1.35x + 2.58 = 6.63$.

x	$y = 1.35x + 2.58$
0	2.58
1	______
2	______
3	______
4	______
5	______

3. In Exercise 1, notice that the x-values increase by 1 in going from each row to the next. The y-values in Exercise 1 increase by 2 each time.
   1. In Exercise 2, what is the change in the x-value in going from one row to the next? ______
   2. What is the change in the y-values in going from one row to the next? ______
   3. Looking at your answers and the equations, what do you notice? ______________.

Activity 4 - Poster activity

Students will make a poster listing the rules for solving equations.
The poster will be put up in the classroom to remind the students how to solve equations.

Assessment:	Design	20 points
	Readability	20 points
	Accuracy in stating the rules	45 points
	Neatness	15 points

Activity 5

Groups will be selected by the teacher. Students working in groups will:

1. Choose a variable representing one unknown.
2. Write expressions representing the other unknown numbers.
3. Use the facts of the problem to write an equation.
4. Solve the equation and find the unknown numbers.

Each group will present its solution to the class:

PROBLEM: A parking garage charges $3 for the first hour and $2 for each additional hour. On a recent day, a motorist paid $17 to park a car in the garage. How many hours was the car parked in the garage?

Create similar problems, or select them from your text.

Activity 6

Begin with questions that you have prepared. Then have students do tasks #1 and #2 with each. Ask an application question such as #3 for each of your equations.

BEGINNING PROBLEM: The postal rate for first class mail can be defined by the equation,

$P = \$0.20W + \$0.05.$

Where P is the first class postage due, and W is the weight of the package rounded up to the nearest ounce, and not exceeding 12 ounces.

1. Identify the variables, coefficients, and constants in the equation.
2. What does a graph of this equation look like? Is it a line, a curve, or what?
3. Suppose you find client's package to weigh 8.8 ounces. According to the equation, what's the first class postage due?

Unit IV.2 Graphing Linear Systems of Equations and Inequalities

This unit gives students an opportunity to sharpen their skills in working with integers, to apply the distributive property, and to simplify expressions. It also serves as an introduction to solving equations and inequalities.

Equations and inequalities are solved algebraically. Graphing is used to illustrate the solutions and connect the algebraic representations to geometry. Connections are made with the areas of science, social studies, and carpentry.

Goals and Objectives

Goals: 1. The learner will use variable equations and inequalities to solve problems.
2. The learner will demonstrate an understanding of relationships between representations of a function.

Objectives: The student will

1. Represent situations by writing a linear equation in one variable.
2. Represent situations by writing a linear inequality in one variable.
3. Solve one-step or two-step linear equations in one variable by the algebraic method.
4. Solve one-step or two-step linear inequalities in one variable by algebraic methods.
5. Graph the solution set to a one variable linear inequality on a number line.
6. Given a formula, make appropriate substitutions and solve for one variable.
7. Solve applications involving the Pythagorean Theorem.
8. Set up and solve proportion statements to find missing values.
9. Given a rule or function that demonstrates a linear equation, generate ordered pairs and graph the equation.
10. Translate between tables and graphs of functions.
11. Given a linear inequality, determine if a given value is in the solution set.
12. Given a linear inequality in two variables, graph the solution set.
13. Find the slope and intercepts of a line and write the equation of a line using the slope and y-intercept.

Instruction

Activity 1

Assign students to groups of two. Each pair gets a blank five by five grid in order to play a game of battleship. Explain that you have previously determined the position of four different ships on a grid. The student must try to locate each ship by naming coordinate points. Teams will take turns calling out the coordinate points until all the ships have been located and identified. As point are called out, place an X where a ship or part of a ship is located and a circle around those points where there is no ship. The team that locates and identifies all the opponent's ships first is the winner.

Emphasize that graphs are ordered pairs, and the first number indicates a move left or right and the second number indicates a move up or down. Stress that a positive integer indicates a move to the right or up, and a negative integer indicates a move to the left or down.

Evaluation: Group Participation.
Resource: *The 4MAT System Course*, Excell, Inc., 1995.

Activity 2

Assign students to groups of two. Give each pair a map of Washington, D.C. (or other major city) marked with point of interest. Have each team search for the points of interest and identify their locations using the letters on the side of the map and the numbers at the bottom of the map. The first team to find five points of interest wins.

Evaluation : Group Participation and accuracy of coordinates.
Resource: *The 4MAT System Course*, Excell, Inc., 1995.

Activity 3

Introduce the following activity as a problem to be solved.

Examine the relationship between the height of a cylinder and its volume when the area of the base is constant. Pour water to a depth of about 1/2 inch in the 500 ml beaker. Measure and record the height of the water on a sheet of data paper. Use the 10 ml graduated cylinder to add 20 ml of water to the beaker and record the volume added (20 ml) on the data paper. Measure the new height -from the same reference line used to measure the original 1/2 inch - and record the height of the water level on the data paper. Repeat adding 20 ml of water to the beaker, recording the volume and height on the data paper until the water level is about 1 inch from the top of the beaker.

Plot the data on a coordinate plane. Determine the equation of the line in the form $y = mx+b$.

Resource: Cord Applied Mathematics, Unit 17, pp 36-37.
Time: 3-4 days
Evaluation: Evaluate graphs and equations

Activity 4

Individual practice: Combine working problems on paper with working problems on a graphing calculator.

Note: Emphasize that a linear graph of an equation shows all the solutions to an equation. This can be done by initially solving for 4 solutions, discussing the number of solutions that can be found for an equation (infinite), then assigning problems for which *all solutions* must be found. Generally, this inspires students to estimate that the task is impossible. Then discuss the linear graph which shows all the solutions.

Resource: Mathematical Connections, Houghton Mifflin, 1992, pp 580-623. Note: pp 602-610 may have been completed in a previous unit)
Time: 2 weeks
Evaluation: Objective testing

Activity 5

Present the class with the following practical problems. The first one could be shown at the beginning of the unit to promote discussion of skills already possessed or needed, and skills that will be learned in this unit. After developing those skills, both problem 1 and 2 should be solved and discussed. The first problem can be presented again later as two inequalities. The second problem shows a career problem. Keep returning to these problems as progress is made in solving equations.

Problem 1: Your family is planning a vacation and wish to rent a van for the trip. If you rent the van by the day, the cost will be $75 a day plus $.20 a mile for each mile driven. If you rent the van by the week, the cost will be $500 a week or any portion of a week. This is a 10 day vacation, and the destination is 300 miles away. Of course you'll be driving the van while you're at your destination, too. Which will be cheaper? Are there any circumstances that need to be considered to make this decision? What circumstances?

To solve this problem, equation solving abilities would be helpful. What skills do we already have that will help us simplify this equation: $\$75*10 + .20*(600+n) = 1000$. Finding the solution to this equation would help you to decide the most economical way to take your vacation.

Problem 2: A nurse or health practitioner might determine blood pressure. This is a measure of the pressure of the blood within the arteries. Systolic pressure is the highest level in the pressure cycle; diastolic pressure is the lowest level. The normal systolic blood pressure (P) for a person of a given age (a) is 110 more than the quotient when that person's age is divided by two. If a person has a normal systolic blood pressure of 150, how old might you expect that person to be?

Activity 6

Assign students to groups of two and give each group 18 cards. Remove all face cards. Have each pair divide the cards so that each student has 9 cards. Students will play a short version of "WAR." Each time students put down a card, they will graph the two numbers on a number line and write an inequality to represent the outcome. The student with the greater value wins that round. Red cards are negative and black cards are positive. The inequalities should be written in a chart so that they will not always list the greater value first. Example: Student A put down a 3 of diamond and student B puts down a 5 of hearts; graph this on the number line and the inequality should be $-3 > -5$. Next A put down a 4 of hearts and B a 2 of clubs ($-4 < 2$). Upon completion of the game, each pair should have 9 number lines and 9 inequalities. The person with the most cards at the end is the winner. As an added feature, have the students keep track of the cards they put down and have them add their cards. (A good review of integers.)

Activity 7

Have students graph equations and inequalities on a graphing calculator. Have them find the intersection of lines and regions, which is the solution to a system of equations and inequalities.

Activity 8

Assign problems dealing with formulas, proportions, and the Pythagorean Theorem to small groups. Each group should have copies of all the problems, but should be made responsible for presenting the methods for solving problems that meet only one description.

Resources: *Mathematical Connections*, Houghton Mifflin, 1992, pp 481-482, 391-395, 375
Evaluation: Group presentation. Rubrics chart.

Resources

4MAT COURSE SYSTEM; EXCEL, Inc.; 1995
Cord Applied Mathematics, Unit 17, pp 36-37.
Group Work from Collaborative Learning 1994 with Cindy Swihart
Mathematical Connections, Houghton Mifflin, 1992, pp 580-623.

Assessment

Group presentations
Rubric charts
Objective testing

PACE Pre-Algebra: An Applied Approach

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