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ABSTRACT

This essay describes the role of graphing calculators in mathematics reform. Among the topics discussed are the history of graphing calculators in mathematics education, recent technological innovations, and professional development opportunities. The case is made for a balanced approach between calculator use and paper-and-pencil techniques. (Contains 17 references.) (MM)

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The Role of Graphing Calculators in Mathematics Reform

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Introduction

The authors began working together over fifteen years ago at The Ohio State University with computer visualization (computer generated graphs) to enhance the teaching and learning of mathematics (algebra, analytic geometry, and calculus). We found early in our work that students at typical high schools rarely had regular access to a computer lab during their classes. In fact, most teachers we knew indicated they seldom used computers and when they did, it was for "demonstration" only. They reported it was almost impossible to schedule their mathematics classes in a computer lab because the labs were usually fully scheduled with non-mathematics classes. Software and its high cost presented additional problems. Some software required training that many teachers found difficult or inconvenient to obtain.

Graphing calculators, first introduced in 1986 by Casio, started a revolution in the teaching and learning of mathematics in the United States and in many other countries as well. Graphing calculators are really inexpensive hand-held computers with built-in numerical solvers and graphing software. Graphing calculators could be viewed as *computers available to all students* because of their low cost, ease of use, and portability [Demana and Waits, 1992]. The National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards* released in 1989 for grades 9 - 12 includes the assumption that graphing calculators will be available to *all* students at appropriate times because they are *personal* computers that fit in a pocket or purse.

Before graphing calculators, teachers had to rely exclusively on expensive computers (usually housed in a separate computer laboratory) to deliver computer enhanced visualization in mathematics teaching and learning. Only a few elite schools could provide such an experience to *all* mathematics students on a regular basis - graphing calculators changed that! *The pedagogical significance to the mathematics community of the small, inexpensive, hand-held graphing calculators should not be under estimated.*

Prior to 1996 graphing calculators provided only some of the important features of a computer algebra system (CAS) - all but the symbol manipulating software of a CAS - at far less cost and often in a more user friendly environment PC based computer algebra systems. In 1996, a remarkable hand-held "portable and affordable" complete CAS with CABRI computer dynamic geometry was introduced by Texas Instruments (the TI-92). The TI-92 uses the same computer CPU than the early Macintosh computers! Now other powerful hand-held CAS are available as well such as the new TI-89 and Casio CFX-9970.

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The internet and the new upgradable calculators from TI

Texas Instruments recently announced a family of *Flash Technology* based calculators (the TI-73 for middle and early high school students, TI-89 for advanced high school and college students, and the TI-92 PLUS module with advanced mathematics software for the existing TI-92s) **that will end calculator box obsolesce**. Do you remember what we did with our wonderful TI-81s when the "more wonderful" TI-82 was introduced (and then that was followed by the TI-83...)? These new *Flash Technology* based calculators can be easily renewed with NEW computer software via the internet and also will be able to use software applications from popular sources outside of TI that can be downloaded via the internet. For example, *Sunburst Communications* is making Some of their more popular software titles available to run on the TI-73. It is important to note that *Flash Technology* means that calculator functionality can expand as curriculum needs change. We are entering a new era of technology value added!

We believe that *students* must use computers on a regular basis for both in-class work and for homework outside of class if there are to be significant changes in the mathematics that students learn today. The introduction of

1. the first graphing calculator by Casio in 1986,
2. the first hand-held pocket complete CAS calculator by Texas Instruments in 1996, and
3. the first FLASH based upgradable calculators by Texas Instruments in 1998

are all extremely significant. Here for the first time in their eras were serious "portable and affordable" computers with pedagogical software designed to enhance the teaching and learning of mathematics.

The C^2PC and C^3E Projects

Our first computer graphing project (known as **the Computer and Calculator Pre-Calculus - C^2PC Project**) began in the mid eighties at Ohio State. The materials developed in that project evolved into a textbook, *Precalculus Mathematics, A Graphing Approach* (now is its fourth edition; Demana, Waits, Clemens, & Foley, 1997) and is now in use in many high schools in the US. The first edition of that textbook in 1988 is recognized as the first widely adopted high school or university textbook *to require* the fully integrated use of computer graphing or graphing calculator technology *by students* on a regular basis. A subsequent project which began in 1990 (known as the **Computer and Calculator enhanced Calculus - C^3E Project**) also resulted in a calculus textbook that fully integrates graphing calculators (now is its second edition; Finney, Demana, Waits, and Kennedy, 1999).

It is interesting to note that by 1989 we were almost a 100% graphing calculator based project. Today most algebra and calculus textbooks in the US fully integrate graphing calculators and assume their use by students. Many mathematics education research studies have been conducted as an outgrowth of the C^2PC project. Professor Penny Dunham nicely summarizes them in her paper that appeared in the *Proceedings of the Fourth Annual Conference on Technology in Collegiate Mathematics* (Dunham, 1992).

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What did we do in our graphing calculator projects that were different from past practices? It is important to note that the mathematics content of our projects was easily recognizable. We did not "replace" traditional mathematics content. We used "power of (graphing calculator) visualizations" to do many "new things" itemized as follows.

1. Approach and solve problems numerically using tables, etc. on graphing calculators.
2. Graphically SUPPORT the results of applying algebraic paper and pencil manipulations to solve equations and inequalities.
3. Solve equations and inequalities using graphing calculators and then CONFIRM the results using analytic algebraic paper and pencil methods.
4. Model, simulate and solve problem situations using graphing calculators and then confirm, when possible using analytic algebraic paper and pencil methods.
5. Use graphing calculator generated scenarios to illustrate mathematical concepts.
6. Use graphing calculator methods to solve equations and inequalities that *can not* be solved using analytic or algebraic methods.
7. Conduct mathematical experiments assisted by graphing calculators to make and test conjectures.
8. Use graphing calculators to study and classify the behavior of different classes of functions.
9. Use graphing calculators to foreshadow concepts that will be encountered in later courses (to build intuition)
10. Use graphing calculators to investigate and explore the various connections among different representations of a problem situation.

When graphing calculators are being used we have found that students are *actively* involved in problem solving and talk and read about the mathematics they are learning. We revisit important problem situations so that students can anchor new ideas in familiar context. Also student learning is facilitated by encountering many instances on which to make generalizations.

A Balanced Approach

We have for many years advocated a *balanced* approach to the use of graphing calculators in the teaching and learning of mathematics. We have long summarized our approach in the following way:

<u>Appropriate</u> use of graphing calculators in the teaching and learning of mathematics means **the student**:

1. Solves analytically using traditional paper and pencil algebraic methods, and then **<u>supports</u>** the results using a graphing calculator.
2. Solves using a graphing calculator, and then **<u>confirms</u>** analytically the result using traditional paper and pencil algebraic methods.
3. Solves using graphing calculator when appropriate (because traditional analytic paper and pencil methods are too tedious and/or time consuming or there is simply no other way!).

Prior to the advent of easy to use hand-held technology, we estimate about 80% of the mathematics algebra and calculus curriculum consisted of paper-and-pencil computation. We know, we taught mathematics at OSU for many years before calculators technology was first marketed by HP and TI in 1972. The computation involved the algebraic and analytic process of mathematics including the common symbolic manipulations of algebra and calculus (by paper and pencil). These computation involved very low order thinking skills. They often have been associated with the phrase "drill and kill mindless manipulations." In the pre hand-held technology curriculum, there were a few application examples which occurred as consequences of mathematics concepts developed algebraically or analytically. Little or no real proof occurred in the standard courses. There is growing evidence that paper and pencil manipulation skill does not lead to better understanding of mathematical concepts. Indeed, the use of hand-held technology can provide more classroom time for the development of better understanding of mathematical concepts by eliminating the time spent on "mindless paper and pencil manipulations."

Many mathematicians have become concerned with the perceived lack of attention to paper-and-paper skills in the new evolving mathematics reform curricula. Nothing could be further from the truth. Paper-and-pencil skills are and should continue to be an important part of the curriculum. However, the role of paper and pencil computation will change dramatically in the future because of hand-held technology. Technology simply provides a better "tool" (when compared to paper and pencil) for much of the "computation and algebraic manipulation" done today **We must recognize and accept this fact - our students already do!**

Our new challenge is to think about computation differently. Each paper-and-pencil algorithm should be analyzed to see if the procedure contributes any understanding to the process. If not, it should be removed and done with technology. For example, there is probably widespread agreement that the square root algorithm and finding trigonometric and logarithmic values from a table by interpolation are obsolete. The *concept* of interpolation is not obsolete as it is an important idea in mathematics. However, using interpolation to find values of trigonometric and logarithmic functions from a table *is now obsolete*. Hand-held computer symbolic algebra will soon make the many paper and pencil factoring algorithms obsolete *but not the process* of factoring (which is a key concept in the fundamental theorem of algebra). The same will be true for many of the paper and pencil symbolic procedures typically taught today (including solving equations, etc.)

We believe computation should be done in one of three ways today and in the future. By computation we mean the following manipulative procedures associated with "paper and pencil" arithmetic, algebra, and calculus:

Mental computation,
Paper-and-pencil computation,
Computation done with technology.

We repeat, all three methods are important today *and* will remain important in the future. We also believe we will see even more emphasis on mental skills as we move to doing less paper and pencil manipulation and more computer symbolic manipulation in the future.

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The *Teachers Teaching with Technology* (T^3) Professional Development Program

Another point from our many years of collective experience is the recognition that teacher inservice is essential and can not be done in several afternoon or one day workshops. **Teachers are our most valuable resource**, and we must provide for adequate training and follow-up. We cannot expect teachers to make fundamental change in their teaching methods without a good deal of help and ongoing support. *The Teachers Teaching with Technology* ($T^{3)}$ staff development program that we founded in 1988 is a model for such training. For more information, about the US T^3 program see http://www.ti.com/ or contact the authors via email at waitsb@math.ohio-state.edu

It is clear to us that ten or fifteen years *in the future* the mathematics curriculum of today will have changed considerably to take full advantage of just the technology that exists *today*. We believe pocket computer technology will be recognized as a major change agent. In the past ten years, graphing calculators have provided students with excellent learning experiences, have made the study of mathematics something to be *valued*, and can have provided a vehicle for *all* students to engage in doing real mathematics. And there is more to come, much more to come. It will be an exciting new century indeed!

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