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AUTHOR	Yoshiwara, Bruce; Yoshiwara, Kathy
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ABSTRACT

This collection of activities is intended to enhance the teaching of college algebra through the use of modeling. The problems use real data and involve the representation and interpretation of the data. The concepts addressed include rates of change, linear and quadratic regression, and functions. The collection consists of eight problems, four of which have solutions, and suggestions for the writing of further problems. (Contains 14 references.) (MM)

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Bruce and Kathy Yoshiwara
LA Pierce College

AMATYC Annual Conference
November 11, 2000

Bicycles, Birds, Bats and Balloons

New Applications for Algebra Classes

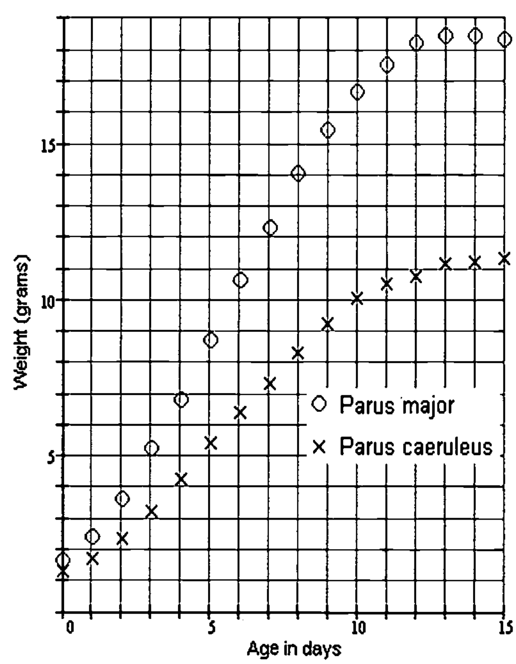
1. A cyclist works against both air resistance (drag) and rolling resistance. The magnitude of drag is proportional to the square of the cyclist's velocity, and the rolling resistance is a constant. The total force that the cyclist battles at velocity v is therefore given by $F = av^2 > b$. The table shows the average force F encountered by several professional cyclists while riding at velocity v. (Pugh, 1974)

velocity v (m/s)	5.4	6.6	7.4	7.8	8.7	9.5	9.9	10.0	10.5	10.7	11.4	11.8	11.9
force F (N)	12.7	15.6	19.0	22.4	23.4	24.4	30.7	27.3	32.2	30.2	36.1	37.6	35.6

a. Make a table of (v^2, F) values, and plot your data.
b. Draw a regression line, and estimate the values of a and b.
c. Predict the force the cyclist encounters at a velocity of 10.5 meters per second.
d. At what velocity will the cyclist meet a force of 25 newtons?
e. Use your values of a and b to graph $F = av^2 > b$ on a plot of the original data.

2. The figure shows the typical weight of two species of birds each day after hatching. (Perrins, 1979)

a. Describe the rate of growth for each species over the first 15 days of life. How are the growth rates for the two species similar, and how are they different?
b. Complete the tables below showing the weight and the daily rate of growth for each species.
c. For each species, plot the rate of growth against weight in grams. What type of curve does the growth rate graph appear to be?
d. For each species, at what weight did the maximum growth rate occur? Locate the corresponding point on each original curve.
e. Find a quadratic regression equation for the growth rate of *Parus major* in terms of its weight. Do the same for *Parus caeruleus*.



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Parus major

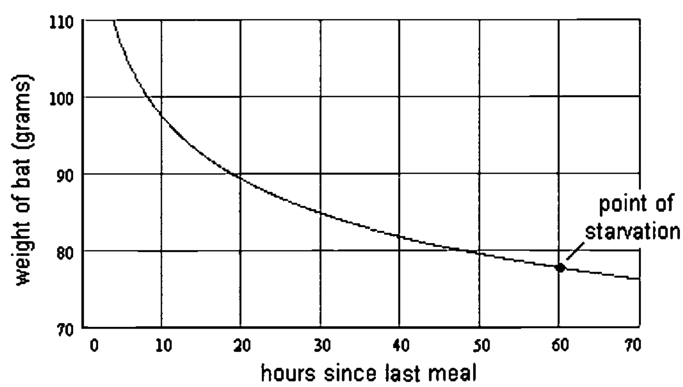
Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Weight																
Growth rate	–															

Parus caeruleus

Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Weight																
Growth rate	–															

f. For each species, graph the regression curve on the plot of growth rate versus weight.
g. Find the vertex of the graph of each regression equation. How do these estimates for the maximum growth rates compare with your estimates in part (d)?

3. Small animals like bats cannot survive for long without eating. The figure shows how the weight of a typical vampire bat decreases over time until its next meal. The curve is the graph of the equation $W = 130.25h^{>0.126}$. (Wilkinson, 1984)



a. What variables are displayed on the horizontal and vertical axes?
b. How long can the bat survive after eating until its next meal? What is the bat's weight at the point of starvation?
c. When the bat's weight has dropped to 90 grams, how long can it survive before eating again?
d. Fill in the table with the number of hours since eating at various weights. Label each of these points on the graph.

Weight (grams)	97.5	92.5	85	80
Hours since Eating				
Point on Graph	*A*	*B*	*C*	*D*

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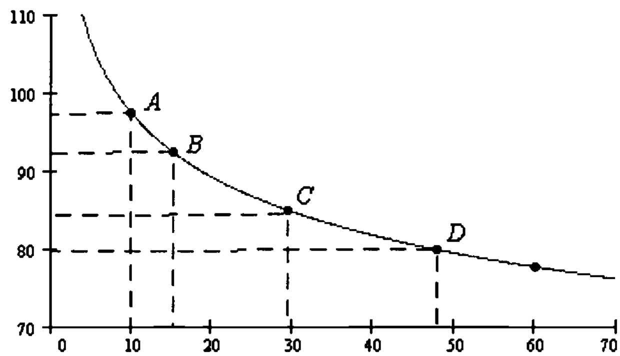

e. Compute the slope from point A to point B, and from point C to point D.
f. Vampire bats sometimes donate blood (through regurgitation) to other bats in dire need. Suppose a bat at point A on the curve donates 5 grams of blood to a bat at point D. Explain why this strategy is effective for the survival of the bat community.

4. If you blow air into a balloon, what do you think will happen to the air pressure inside the balloon as it expands? Here is what two physics books have to say:

"The greater the pressure inside, the greater the balloon's volume."

Boleman, Jay, *Physics, a Window on Our World*

"Contrary to the process of blowing up a toy balloon, the pressure required to force air into a bubble decreases with bubble size."

Sears, Francis, *Mechanics, Heat, and Sound*

a. Based on these two quotes and your own intuition, sketch a graph of pressure as a function of the diameter of the balloon. Describe your graph: is it increasing or decreasing? is it concave up (bending upwards) or concave down (bending downwards)?
b. In 1998, two high school students, April Leonardo and Tolu Noah, decided to see for themselves how the pressure inside a balloon changes as the balloon expands. Using a column of water to measure pressure, they collected the following data while blowing up a balloon. Graph their data on the grid.
c. Describe the graph of April and Tolu's data. Does the graph confirm the predictions of the physics books?

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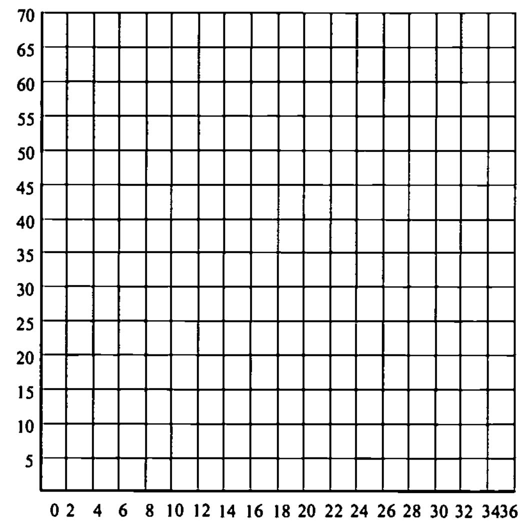
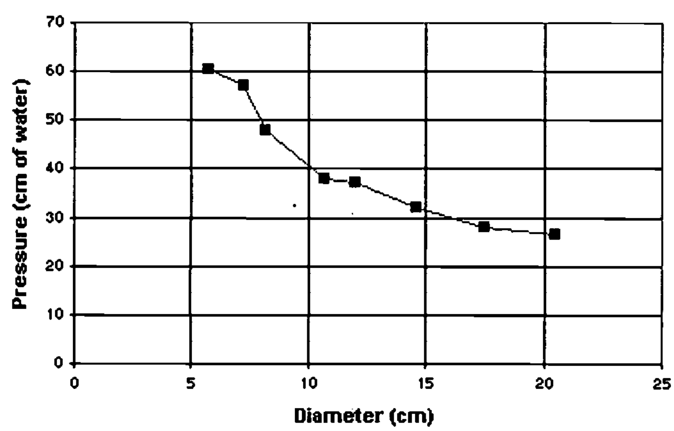
Diameter (cm)	Pressure (cm H_2O)
5.7	60.6
7.3	57.2
8.2	47.9
10.7	38.1
12	37.1
14.6	31.9
17.5	28.1
20.5	26.4
23.5	28
25.2	31.4
26.1	34
27.5	37.2
28.4	37.9
29	40.7
30	43.3
30.6	46.6
31.3	50
32.2	61.9



The figure shows a plot of the first half of April and Tolu's balloon data. As the diameter of the balloon increases from 5 cm to 20 cm, the pressure inside decreases. Can we find a function that describes this portion of the graph?



d. Pressure is the force per unit area exerted by the balloon on the air inside, or $P = \frac{F}{A}$. Because the balloon is spherical, its surface area, A, is given by $A = \pi d^2$. The force exerted by the balloon is not so easy to express in terms of d, but because the force increases as the balloon expands, we'll try an expression of the form $F = k\,d$, where k is a

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constant, and see if this fits the data. Use these relationships to express P as a power function of d.

e. Make a table of values for $x = \frac{1}{d}$ and P, and plot the points (x, P). Find a line of best fit by eye, or use your calculator's linear regression feature.

f. Use your regression equation to write a formula for P as a function of d. Do your calculations support the hypothesis that P a power function of d?

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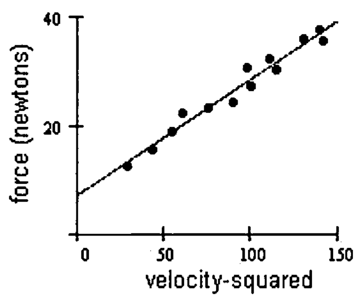
Answers

1. a.

v^2	29.16	43.56	54.76	60.84	75.69	90.25	98.01
F	*12.7*	*15.6*	*19.0*	*22.4*	*23.4*	*24.4*	*30.7*

v^2	*100*	110.25	114.49	129.96	139.24	141.61
F	*27.3*	*32.2*	*30.2*	*36.1*	*37.6*	*35.6*



b. $a \approx 0.21,\ b \approx 7.18$

c. *30.7 newtons*

d. *9.13 meters per second*

e. *[Graph]*

2. a. *The growth rate of each species increases until the fifth or sixth day after hatching, then begins to decrease, declining to zero around the fourteenth day. The growth rate of* Parus major *appears to be slightly negative around day fifteen.*

b. *Parus major*

Day	0	1	2	3	4	5	6	7
Weight (gms)	1.6	2.4	3.6	5.2	6.8	8.7	10.6	12.3
Growth rate (gms/day)	—	0.8	1.2	1.6	1.6	1.9	1.9	1.7

Day	8	9	10	11	12	13	14	15
Weight (gms)	14	15.4	16.6	17.5	18.2	18.4	18.4	18.3
Growth rate (gms/day)	1.7	1.4	1.2	0.9	0.7	0.2	0	> 0.1

Parus caeruleus

Day	0	1	2	3	4	5	6	7
Weight (gms)	1.3	1.7	2.3	3.2	4.2	5.4	6.4	7.3
Growth rate (gms/day)	—	0.4	0.6	0.9	1.0	1.2	1.0	0.9

Day	8	9	10	11	12	13	14	15
Weight (gms)	8.3	9.2	10	10.5	10.7	11.1	11.2	11.3
Growth rate (gms/day)	1.0	0.9	0.8	0.5	0.2	0.4	0.1	0.2

c. *Parabola.*

d. *Maximum growth rate occurs approximately at weight 9.6 grams for* Parus major*, and at weight 5.4 grams for* Parus caeruleus *.*

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e. *Parus major:* $y = > 0.023x^2 > 0.44x > 0.157$ *Parus caeruleus:*
$y = > 0.035x^2 > 0.428x > 0.191$

g. *Parus major:* (9.6, 1.99). *The estimate of 9.6 grams for the weight at which maximum growth occurs is the same as the previous estimate.*
Parus caeruleus: (6.1, 1.12). *The estimate of 6.1 grams for the weight at which maximum growth occurs is larger than the previous estimate of 5.4 grams.*

3. a. *On the horizontal axis, number of hours since eating. On the vertical axis, weight in grams.*
b. *60 hours; about 78 grams*
c. *About 42 hours*
d.

Weight (grams)	97.5	92.5	85	80
Hours since Eating	*10.0*	*15.1*	*29.6*	*47.9*
Point on Graph	*A*	*B*	*C*	*D*

e. *>0.98 grams per hour; >0.27 grams per hour*
h. *For the same amount of food, the recipient gains 18.3 hours of life, while the donor loses only 5.1 hours. The slope between the points A and B is steeper than the slope between C and D, so a vertical change of 5 grams at point D corresponds to a larger time interval than a change of 5 grams at point A.*

4. a. $P = \frac{k}{\pi d}$, *or just* $P = \frac{K}{d}$.
b. $P = 293x > 12$
c. $P = \frac{293}{d} > 12$. *No.*

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Audience Participation

1. Figure 1 gives data about snowfall, air temperature, and number of avalanches on the Mikka glacier in Sarek, Lapland in 1957. (Leopold, 1992)
   a. During June and July avalanches occurred over three separate time intervals. What were they?
   b. Over what three time intervals did snow fall?
   c. When was the temperature above freezing (0° C)?
   d. According to the data, under what conditions do avalanches occur?
   a. June 21 to June 24, June 29 to July 3, and July 8 to July 14
   b. June 17 to June 21, June 25 to June 29, and July 4 to July 7
   c. June 22 to June 24, June 27, June 30 to July 4, and July 8 to July 14
   d. Avalanches occur when there is a thaw after a snowfall.

2. The graph in Figure 2 shows the average profile of air temperature above the earth's surface. (Ahrens, 1998)
   a. Is temperature a decreasing function of altitude?
   b. The lapse rate is the rate at which the temperature changes with altitude. In which regions of the atmosphere is the lapse rate positive?
   c. The region where the lapse rate is zero is called the isothermal zone. Give an interval of altitudes that describes the isothermal zone.
   d. What is the lapse rate in the mesosphere?
   e. Describe the temperature for altitudes greater than 90 kilometers.
   a. No
   b. The stratosphere and the thermosphere
   c. $11 \leq h \leq 20$
   d. About 2.6° C per kilometer
   e. The temperature increases rapidly up to 100 kilometers, then more and more gradually up to the top of the thermosphere at about 500 kilometers. The temperature levels off at around 700° C.

3. Related species living in the same area often evolve in different sizes to minimize competition for food and habitat. Here are the masses of eight species of fruit pigeon found in New Guinea, ranked from smallest to largest. (Burton, 1998, Diamond, 1973)

Size rank	1	2	3	4	5	6	7	8
Mass (grams)	49	76	123	163	245	414	592	802

   a. Plot the masses of the pigeons against their order of increasing size. What kind of function might fit the data?
   b. Compute the ratios of the masses successive sizes of fruit pigeons. Are the ratios approximately constant? What does this information tell you about your answer to part (a)?

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c. Compute the average ratio to two decimal places. Using this ratio, estimate the mass of a hypothetical fruit pigeon of size rank 0.

d. Using your answers to part (c), write an exponential function that approximates the data. Graph this function on top of the data and evaluate the fit.

a. Exponential.

b. 1.55, 1.62, 1.33, 1.50, 1.69, 1.43, 1.35. *Yes. This supports the hypothesis of an exponential function.*

c. 1.50, 33 *grams*

d. $M = 33(1.5)^n$. *The fit is good.*

4. The order of a stream or river is a measure of its relative size. A first-order stream is the smallest, one that has no tributaries. Second-order streams have only first-order streams as tributaries. Third-order streams may have first- and second-order streams as tributaries, and so on. The Mississippi River is an example of a tenth-order stream, and the Columbia River is ninth order. Both the number of streams of a given order and the average length of a stream are exponential functions of their order. In this problem we consider all streams in the United States. (Leopold, 1992)

Order	*Number*	*Average Length*	*Total Length*
1	1,600,000	1	
2	339,200	2.3	
3			
4			
5			
6			
7			
8			
9			
10			

a. Using the given values, find a function $N(x)=ab^{x>1}$ for the number of streams of a given order.

b. Complete the column for number of streams of each order. (Round to the nearest whole number of streams for each order.)

c. Find a function $L(x)=ab^{x>1}$ for the average length of streams of a given order, and complete that column.

d. Find the total length of all streams of each order, and hence estimate the total length of all stream channels in the United States.

a. $N(x)=1{,}600{,}000(0.212)^{x>1}$

b. $L(x)=2.3^{x>1}$

c. About 3,120,000 *miles*

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Background Information for Write-Your-Own Application Problems

1. Avalanches *(Leopold, 1992)*

"The importance to geomorphic processes of a coincidence in time of meteorologic conditions is also illustrated by the relation between temperature, precipitation, and frequency of rockfalls in Norway above 60° latitude.... An additional example from this same region is shown in Figure 3-29. Here avalanches occur following thawing periods of snowfall. Rainfall apparently contributes to dirty avalanches (mixture of snow and debris). The relation of active movement to thawing is striking."

2. Air temperature *(Ahrens, 1998)*

"We have seen that both air pressure and density decrease with height above the earth -- rapidly at first, and then more slowly. Air temperature, however, has a more complicated vertical profile.... The rate at which the air temperature decreases with height is called the temperature lapse rate."

3. Fruit Pigeons *(Burton, 1998, Diamond, 1973)*

(See excerpt from the secondary source, Burton, and the corresponding primary source, Diamond.)

4. Stream Order *(Leopold, 1992)*

"... stream order is a measure of the position of a stream in the hierarchy of tributaries. ... the first-order streams are those which have no tributaries. The second-order streams are those which have as tributaries only first-order channels. ... The third-order stream receives as tributaries only first- and second-order channels.... Where the number of streams of various orders in a drainage basin are counted, their lengths from mouth to drainage divide are measured and averaged. The relation of stream order to number of streams and average length may be plotted.... Horton showed that stream order is related to number of streams, channel length, and drainage area by simple geometriv relationships; that is, stream order plots against these variables as straight lines on semilogarithmic paper."

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Signature: Katherine Yoshiwara Bruce Yoshiwara

Printed Name/Position/Title: Professor of Mathematics Katherine Yoshiwara Bruce Yoshiwara

Organization/Address: Los Angeles Pierce College 6201 Winnetka Ave Woodland Hills, CA 91371

Telephone: (818) 710-4159 FAX: (818) 710-9844

E-Mail Address: kyoshiwara@hotmail.com Date: 11/14/01

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