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ABSTRACT

This one-year qualitative study analyzed how nine middle school English language learners (ELLs) of Mexican descent constructed meaning on think-aloud protocols of Spanish and English word problems. Strategies used by these students to process information from English to their native language included translating to Spanish, reading the problem at least twice, inferring meaning, understanding mathematical symbols, and ignoring words that were irrelevant to the solution. The students failed to construct meaning in both languages when the word problems mixed mathematical with natural language. The findings indicate the importance of using students' sociocultural and linguistic experiences to make mathematical connections between natural language and the language that is specific to mathematics. (Contains 39 references.) (Author/ASK)

Running Head: CONSTRUCTING MEANING: ELLS AND MATHEMATICS

Constructing Meaning: Think-Aloud Protocols of ELLs on English and Spanish Word Problems

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Abstract

This one-year qualitative study analyzed how nine middle school English language learners (ELLs) of Mexican descent constructed meaning on think-aloud protocols of Spanish and English word problems. Strategies used by these students to process information from English to their native language included translating to Spanish, reading the problem at least twice, inferring meaning, understanding mathematical symbols, and ignoring words that were irrelevant to the solution. The students failed to construct meaning in both languages when the word problems mixed mathematical with natural language. The findings indicate the importance of using students' sociocultural and linguistic experiences to make mathematical connections between natural language and the language that is specific to mathematics.

Constructing Meaning: Think-Aloud Protocols of ELLs on English and Spanish Word Problems

There is a paucity of studies that specifically address issues regarding the learning of mathematics by English language learners (ELLs)[1], especially at the middle to secondary level. Literature suggests that first and second language factors affect mathematics learning (Aiken, 1971; Cuevas, 1984; Mestre, 1988; Secada, 1991; Khisty, 1995; Ron, 1999). According to Ron (1999), there are two misconceptions about the role of language in mathematics. First, the language of mathematics is considered to be as easily acquired as the natural language, or the everyday language. Second, if a person is bilingual, then the assumption is that "…he or she automatically knows the language of mathematics in both languages" (p. 23). While it is true that some mathematical symbols are shared in many languages, this does not imply that mathematics has no linguistic concepts.

Given the increase in the number of Spanish-speaking students (U.S. Department of Education, 1994) and the current emphasis on learning through word problems (NCTM, 1989, 2000), questions remain on how ELLs use language to negotiate meaning. Even though researchers have recognized the important role that language plays in mathematics performance (Aiken, 1971), they have not always acknowledged its equally crucial role in the process of acquiring mathematical concepts and skills in a second language. To better understand the successes and challenges ELLs encounter as they solve word problems during the initial stages of second language acquisition, this study addresses the following questions: 1) How do ELLs use language to negotiate mathematical meaning given five English and five Spanish word problems? 2) What problem-solving strategies (i.e., translation, reading the problem at least twice, inferring, etc.) do ELLs use?

To address these questions, I present specific examples from word problems in which language interacts with problem-solving performance among nine students of Mexican descent. First, theories in learning a second language, including implications for mathematics, are discussed. Second, the mathematical problem-solving process is presented. Next, I describe the mathematics register to address difficulties language can create for ELLs when solving word problems. A detailed analysis of the findings follows from the English and Spanish think-aloud protocols to discuss how particular language used in word problems helped (or not) students in solving them. I end with a discussion of conclusions and implications of this study.

Background

Learning a Second Language

English language learners face the dual task of learning new linguistic structures in the second language and learning academic content. The process of learning this new linguistic system and the academic content does not occur automatically. In fact, to attain the same performance as native English speakers in the content areas, ELLs may take five to seven years or more to develop academic proficiency in a second language (Collier, 1995; Cummins, 1986). Cummins (1992) distinguished between two levels of language proficiency—basic interpersonal communication skills (BICS), i.e., social language, and cognitive/academic language proficiency (CALP). If students have not developed the language used for academic tasks in their first language (L1), they may experience difficulty with CALP in their second language (L2).

Cummins' description of the need for cognitive/academic language proficiency (CALP) in L1 before L2 has two implications. One is that if the bilingual student has CALP for mathematics in L1, then mathematical concepts (i.e., addition, subtraction, division, multiplication) attained in L1 will transfer to L2. What changes for students are the lexical items (vocabulary) attached to these concepts. However, reading word problems will be significantly complex for two reasons. One involves the mental time it takes to process linguistic structures from one language to another. There may be words that do not make sense to the reader. These two can lead the reader to misinterpret text. Second, mathematics word problems are not straightforward either (Pimm, 1987; Ron, 1999). To encourage CALP in L2, students need mathematical activities that address language in natural contexts as well as mathematical contexts. "Initial reading should focus on meaning" (Miramontes, Nadeau, & Commins, 1997, p. 133) so that students learn to differentiate between natural language and mathematical language. Secondly, according to Cummins, if the student does not have CALP in L1, the student will have considerable difficulty developing CALP in L2. The use of L1 by teacher and the student may be very necessary in order to clarify confusions that result because of differences between English and Spanish mathematical terms (Cuevas, 1984) in mathematical problem solving.

Mathematical Problem Solving

Using Mayer's model (1987) of Problem Solving, Cardelle-Elawar (1995) defined four types of processes required to solve mathematics word problems. The first process involves translating, which requires two types of knowledge. The first type of knowledge is linguistic knowledge, which allows students to understand English sentences and translate from words to numbers or symbols. Using schema theory to understand different ways ESL students may interpret (or misinterpret) texts (Alexander, Schallert, & Hare, 1991; Anderson & Pearson, 1984; Schallert, 1991) is critical in the first process of problem solving. The second type of knowledge is factual knowledge, which requires that students know certain facts in order to make a mental representation of a problem. For example, factual knowledge needed in order to convert 60 centimeters to 0.6 meters would involve knowing that 100 centimeters equals one meter.

A second process defined by Cardelle-Elawar (1995) involves integration, in which students are required to use schematic knowledge in order to recognize different problem types. The third process involves planning and monitoring. The students are required to create a plan to solve the problem by breaking the problem into subproblems and establishing a sequence for the solution. Finally, the fourth process is solution execution. Students are required to use procedural knowledge to apply the rules of arithmetic accurately and efficiently while carrying out the calculations.

Overall, Mayer's (1987) model has implications for the processes that are involved in problem solving, especially for non-native English speakers. The first step, translation, is a very critical step since students cannot proceed with the problem solving process unless they can use their own words to understand the problem. Some of the difficulties experienced by ELLs during the process of solving word problems are discussed next.

Mathematics Register

A register, as defined by Romaine (1994), "is typically concerned with variation in language conditioned by uses rather than users and involves consideration of the situation or context of use, the purpose, subject-matter, and content of the message, and the relationship between participants" (p. 20). Hence, a register is what you are speaking, i.e., determined by what you are doing (nature of activity in which language is functioning) (Halliday & Hasan, 1989). Registers tend to differ in semantics and, therefore, in grammar and vocabulary.

In addition, as Khisty (1995) explained, the mathematics register involves the nature of the language used to communicate mathematical ideas. However, the mathematics register should not be limited to the special terminology used in mathematics. Rather, the mathematics register extends to the use of natural language in a way that is particular to a role or function (Halliday, 1978, as cited in Khisty, 1995, p. 282). There are many ways a mathematics register can be developed, including reinterpreting existing words in natural language such as "point", "reduce", "carry", "set", "power", and "root". Thus, there are many words in the mathematics register which have different meanings from what children initially expect. In fact, Cazden (1979) stated that mathematics tends to be a restricted language, "limited in both size and meaning" (p. 135), by which she meant that mathematical terms are rarely encountered in non-mathematical contexts. This has important implications for ESL students learning mathematics. Researchers (Carrasquillo et al., 1996; Kang, 1995; Kessler, 1986, 1987; Khisty, 1992, 1995; Olivares, 1996) identified areas that create difficulty for ESL students when engaged in problem solving. Overall, these areas include vocabulary, syntax, and discourse.

Vocabulary.

The role vocabulary plays in the learning of mathematics by second language learners cannot be underestimated. As explained above, there are words used in natural language that take on new meaning in mathematics. For example, words such as "equal," "rational," "irrational," "column," and "table" would need to be explained to ELLs in the mathematical and non-mathematical contexts. These two different explanations are needed for two important reasons. One is that ELLs tend to acquire the basic communication skills during the first two years, thus some may already know the everyday usage of the vocabulary but not the mathematical meaning. Secondly, explaining the mathematical meaning increases students' opportunities to acquire CALP, the language needed to perform in the content areas. In addition, words that indicate the same mathematical meaning—i.e., "add," "plus," "combine," "and," "sum," and "increased by"—also need to be emphasized to students by using them repeatedly in different word problems.

Research has shown, however, that teachers do not focus much on vocabulary when teaching second language learners (SLLs) (Khisty, 1992). The instruction that is given is not thorough in its explanation of basic concepts. Therefore, it is essential that mathematics educators place emphasis on new vocabulary when introducing a new concept. If possible, the walls should be covered with charts that stress important words (Buchanan & Helman, 1993).

Syntax.

When reading word problems, second language learners may experience difficulties with syntax, which involves using correct language. Some examples include comparatives such as "is less than," "is greater than," and "x times as much." For example, consider the following problem:

> Write an equation using the variables *S* and *P* to represent the following statement: 'There are 6 times as many students as professors at this university. Use *S* for the number of students and *P* for the number of professors.' (Mestre, 1988, p. 208)

In this word problem, 35% of nonminority engineering undergraduates provided an answer of "$6S=P$," reversing the order of the variables (Clement, Lochhead, & Monk, 1981, as cited in Mestre, 1988). In comparison, a population of Hispanic engineering students made the same error with a frequency of 54% (Mestre, Gerace, & Lochhead, 1982, as cited in Mestre, 1988). During clinical interviews, the authors noted that students were aware that there were more students than professors. However, two explanations were given why students made this mistake. One was that S and P were treated as labels for "students" and "professors," instead of treating them as variables to represent the *number* of students and the *number* of professors. Secondly, students used a sequential left-to-right translation of the word problem. Thus, "six times as many students" was translated as $6S$, and this was equated to P, the number of professors.

In addition, Castellanos (1980) illustrated examples that lack one-to-one correspondence between mathematical symbols and the words they represent. One example is division: "23/7" is often read as "7 goes into 23." In many Spanish-speaking countries, however, the two numbers are transposed, and the problem is referred to by saying, "*Vamos a dividir 23 entre 7*," which literally translated means, "Let's divide 23 into 7." Thus, this can lead to a misinterpretation. A second example involves substitution. If in English we say, "Let's substitute 8 for x," we mean that in place of x we are going to use 8. If this is translated literally into Spanish, it will read, "*Vamos a sustituir 8 por x*," and will mean that in place of 8 we are going to use x, which is exactly the opposite. In algebra, a typical mistake involves mixing the order of the following: The number a is five less than the number b. This translates into symbols correctly as "$a = b - 5$," not "$a = 5 - b$."

Syntactic variation on a single semantic notion also creates difficulty for ELLs. Examples of such variations include "*How many* were there in all?," "*How many* were there then?," "*How many* more were there?," "*How many* were left?," and "*How many* fewer were there?" In addition, Dawe (1983) indicated that logical connectives are critical to the development of mathematical reasoning in English as a second language. In the English language, there is a group of words and phrases that serve to link propositions in reasoned argument. Some examples of logical connectives are "suppose", "so that", "but", "either. . .or", "if. . .then", "neither. . .nor," and so on.

Mathematics Discourse.

To illustrate an example of mathematics discourse difficulties, Kessler (1986) conducted a study with a class of fourth-grade ESL children. The students were asked to do a multiplication problem using symbols only. The students scored 51% accuracy. However, when the same problem was given as a word problem, the accuracy level dropped to 6%. The discrepancies in these findings, Kessler explained, were due to word problems requiring processing of lexical and syntactic structures and appropriate interpretations of language functions.

This review of the literature suggests that mathematics educators must be aware of the difficulties that a second language learner may experience with word problems. I will next discuss how this study was designed to explore this topic.

Method

Site

This one-year qualitative study was conducted in an urban public middle school located in Central Texas. The ESL/mathematics classroom was self-contained, meaning there were 22 students who were in the 6th, 7th, and 8th grades in the same classroom. The total number of students in the class varied throughout the semester because students were transitioned into regular or honors courses. However, the nine students who participated remained in the class during the length of this study. The classroom's ethnic distribution was all Latino(a), and most students' home country was Mexico, El Salvador, Guatemala, or Honduras. The students who participated in this study were all of Mexican descent.

Participants

Nine middle school students who were identified as ELLs by their ESL/Mathematics teacher and by the scores on the Language Assessment Battery (Texas Education Agency, 1996) participated in this study. The sample of students represented a range of academic skills and proficiency levels of English. For example, César was the only student who was the closest to attaining CALP in English (see Table 1). Most students had developed BICS in English to communicate their daily needs and with their peers, but they had not yet attained CALP. The students' native language was Spanish.

Of the students chosen, three students were considered to be successful (Juany, Yolanda, and Oscar); four students were considered to be average (Roberto, Leticia, Betty, and César), and two students were characterized as having difficulties (Ismael and Criselda)[2]. The students were identified in the aforementioned categories based on the teacher's recommendation. Students who had a grade average of 85 or above in the class were considered to be successful; those with grades between 70 and 85 were considered to be average, and students with an average below 70 were considered to have difficulties.

To better comprehend the schooling experiences of ELLs in relation to language and mathematical background, I administered a survey at the beginning of the study. The results from the survey on background information for the nine students are summarized in Table 1. All students had attended public schools in Mexico.

The students were taught in English and in Spanish. Mrs. Brown, the teacher, used Spanish about 90% in the beginning of the school year, and her goal was to use 50% English and 50% Spanish toward the end of the year. At the time of this study, Mrs. Brown had taught ELLs five years, and she considered English and mathematics a priority because she knew students needed to transition into these courses to continue their middle and high school education.

Insert Table 1 here.

Think-Aloud Protocols

The think-aloud protocols had two major purposes: (1) to describe how ELLs used language to negotiate mathematical meaning given five English and five Spanish word problems and (2) to explore problem-solving strategies (such as translation) used by ELLs. To investigate how these nine ESL students used language, I selected word problems from two textbooks used in the course during the 1996-97 school year (Eicholz, Young, O'Daffer, Barnett, Charles, Fleenor, Clemens, Reeves, Thornton, & Westley, 1991). The textbook was at a 6th grade level, but all students used this textbook. The word problems included in the think-aloud protocols were chosen because they had a variety of linguistic structures that are challenging for ELLs (Mestre, 1988). Initially, I included five English word problems as part of the think-aloud protocols then added the Spanish version of those five problems. The final version of the protocols included two phases similar to those employed by Chamot, Dale, O'Malley, and Spanos (1992).

Phase I.

The first part of the think-aloud consisted of a warm-up activity. Students were asked to solve basic computation problems, for example 65×4, and were asked to explain what they were thinking. If they needed more practice, students were asked to compute $58 - 29$. The time provided was two minutes, but most students completed the task within a minute.

Phase II.

After the warm-up activity, students solved five word problems written in English. Students were not given a time limit, but they were encouraged to finish within their advisory period, which was 30 minutes long. Students were allowed to use a calculator. Each student had an individual think-aloud session that was audio-taped and later transcribed.

I interviewed all students using the think-aloud protocols for the English word problems before the Christmas vacation. When the students returned after three weeks, I implemented the first and second phase using the Spanish version of the same word problems. Similar to the survey, most students preferred to use Spanish during their think-aloud protocols with the Spanish and English word problems because that was the language they felt most comfortable using. The first set of problems was written in English; these are shown below.

- Problem #1: On Saturday, 203 children came to the swimming pool. On Sunday, 128 children came. How many more children came to the pool on Saturday than on Sunday?

- Problem #2: Admission to Hall's planetarium costs $4.50 for adults and $2.75 for students. Mr. Emery took his class of 27 students to a show. How much did Mr. Emery and his class spend for tickets?
- Problem #3: *Meteor* magazine has 16 pages. 3 1/2 pages are poems and 7 1/2 pages are short stories. How many pages are not poems or stories?
- Problem #4: The number of known asteroids is about 1,600. Astronomers believe that about 20 times that many exist. How many asteroids do astronomers think exist?
- Problem #5: A number 300 can holds 13 7/8 ounces. A number 2 1/2 can holds 28 ounces. How many more ounces does a 2 1/2 can hold than a number 300 can?

The second set of word problems was obtained from the Spanish version of the textbook (Eicholz et al., 1991b). These problems are shown below:

- Problema #1: El sábado fueron 203 niños a la piscina. El domingo fueron 128 niños. ¿Cuántos niños más fueron el sábado que el domingo?
- Problema #2: La entrada al planetario cuesta $4.50 para los adultos y $2.75 para los estudiantes. El señor Emery llevó a su clase de 27 alumnos. ¿Cuánto pagaron el señor Emery y su clase por las entradas?
- Problema #3: La revista *Meteor* tiene 16 páginas. 3 1/2 de las páginas son de poemas y 7 1/2 de cuentos cortos. ¿Cuántas páginas no contienen ni poemas ni cuentos?
- Problema #4: Se conocen aproximadamente unos 1,600 asteroides. Los astrónomos creen que existen alrededor de 20 veces más. ¿Cuántos asteroides creen los astrónomos que existen?

- Problema #5: Una lata del número 300 contiene 13 7/8 onzas. Una lata del número 2 1/2 contiene 28 onzas. ¿Cuántas onzas más contiene una del 2 1/2 que una del 300?

Findings and Discussion from the English Think-Aloud Protocols

In analyzing the findings, I generated Table 2 to summarize the nine students' think-aloud protocols. I included words which students reported as not understanding; these are represented in italics. Part of the think-aloud protocol involved observing what strategies were used to solve these problems. These strategies included the following: Reading the problem twice, translating to Spanish, inferring (or lack of), understanding mathematical symbols, and ignoring irrelevant words. The number in parenthesis indicates the students that had difficulties with certain parts of the problem. The difficulties students encountered are discussed in detail in the next section.

Insert Table 2 here.

<u>Three Patterns in Problem Solving</u>

As illustrated in Table 2, some students were successful on the first three problems but most had difficulties with the last two problems. Three patterns identified from the students' responses were the following: (1) some students were successful in solving the problem without language difficulties; (2) some students were able to solve the problem even though they encountered a few difficulties with language, and (3) some students were not able to solve the problem because they had difficulties with the English language and/or because they did not know how to approach the problem mathematically.

To solve the first problem, students needed to know that "How many more?" required a comparison of the number of children who went to the pool on Saturday and

Sunday. Students tended to follow the first pattern with Problem #1. Three students, including Criselda, who was considered to be a weak student by Mrs. Brown, were able to solve this problem. Betty and Criselda focused on comparing the number of children who had gone to the pool on Saturday and on Sunday. Betty, César, and Criselda began the task by reading the English problem aloud, then immediately translating key phrases into Spanish. For example, Betty translated the problem to Spanish as follows: **Bueno, aquí dice que doscientos tres niños vinieron el Saturday...y que veintiocho...ciento veintiocho más...menos niños vinieron el Sunday** *{Well, here it states that 203 children came on Saturday...and that 28...128 more...fewer came on Sunday.}*. In this example, Betty is using Spanish as a tool to understand the question by focusing on key phrases and stating the information in her own words. The use of students' first language in understanding the problem is encouraged by Cardelle-Elawar (1995).

The second pattern identified is illustrated in the second and third problem. Betty and César (Table 2) were able to solve the second and third problem, despite the fact that there were some words they did not comprehend. For example, Betty was not familiar with the words "admission" and "planetarium," and César did not understand "planetarium" in the second problem. Furthermore, César was able to solve the third problem even though he did not understand the word "Meteor." These three students reported ignoring the words they did not understand and focusing on the phrases that were more important in solving the problem. Implementing these problem-solving strategies can be attributed to the teacher modeling for students which information in the problem was useful or not.

The third pattern identified is very special, when students were not able to solve the problem because they encountered difficulties with the English language in the word problem and/or because they did not know how to approach the problem mathematically. Criselda was able to translate the word problem into Spanish, but she was not sure how to solve the problem (see Table 2, Problem #3). Criselda's first attempt at solving the problem included subtracting 7 1/2 and 3 1/2. After I probed to find out how she would subtract the numbers, Criselda was not sure of her approach.

My analysis showed that the fourth problem was complex for most students because they were not able to infer information needed to solve the problem, and they were not familiar with lexical items (vocabulary) (see Table 2). The phrase "about 20 times that many exist" created difficulties for these students because they were not able to infer that "about 20 times that many exist" referred to "asteroids." These findings concur with those of Spanos, Rhodes, Corasaniti Dale, and Crandall (1988). In addition, students stated they did not know words such as "asteroids" and "astronomers". These findings are consistent with Secada's (1991) work on Hispanic bilingual students learning mathematics. Specifically, he found that second language learners might encounter difficulties with word problems when their personal experiences do not match the linguistic expressions presented in the word problems.

The last problem created the most difficulties for all students in both languages because there was a mixing of natural language with mathematics. Students had to compare the number of ounces to compute the solution. Betty, César, and Criselda translated "can holds" to Spanish as **puede sostener**, translating "can" as an auxiliary verb, not as its noun form. The word "can" has two functions, an auxiliary verb and a noun. Words that have multiple functions may cause difficulties for second language learners (as well as native English speakers) because they must be able to distinguish their proper functions. Another important observation in the last problem is the use of the numbers to label the cans. Betty, César, and Criselda stated that in this problem they had more numbers to consider; students were not aware that the numbers were being used to label cans. When students attempted to solve the problem, they considered all numbers presented in the problem. In other words, they were not able to ignore the numbers used to label the cans.

Homophones

In addition to the three patterns aforementioned, some students occasionally used homophones as they solved the problems. Ismael (Table 2) translated *"than"* to **entonces**, which translated to English is *"then."* Thus, Ismael was not able to solve the first problem because the comparative meaning of "than" was crucial in solving the problem. Like Ismael, Leticia thought of "many" as "money" (Table 2, Problem #1). Understanding the meaning of the word "many" was important in determining the number of children. Leticia encountered the same difficulties with the second and fourth problems. In the second problem, Leticia thought of "took" as "talk;" and in the fourth problem, she thought of "known" as "now" and of "exist" as **salidas**, or *exits*. Similarly, Roberto (Table 2, Problem #5) thought of "holds" as "holes." In other words, there was a mismatch between what Ismael, Leticia, and Roberto thought and the information needed to solve the problem.

Symbols

Although symbols are sometimes thought of as being the universal language of mathematics, there are instances in which symbols are presented in text form that may distort their meaning. For example, in the third and fifth problem the "/" is used with fractions, rather than the horizontal bar. Criselda, Juany, Oscar, and Roberto (see Table 2) read the third problem in the following way:

> Meteor magazine has sixteen pages. Thirty-one slash two pages are poems and seventy-one slash two pages are short stories. How many pages are not poems or short stories? (Think-aloud Protocol, fall 1997).

When I rewrote the mixed numbers, 3 1/2 and 7 1/2, with a horizontal bar, all four were able to read the numbers correctly. Three out of four students were able to solve the problem when I explained that the "/" and the horizontal bar were used interchangeably.

Making sense of symbols was not an easy task for these students. On average, students took about five minutes or longer to solve each problem. The students needed extra time to read the problem at least twice, to translate to Spanish, to figure out meanings of symbols and process words, and to plan how to solve the problem.

Problem-Solving Process

Overall, these nine students followed a pattern when solving the English word problems. At the end of each individual think-aloud session, students stated that reading the problem at least twice and translating words into Spanish were important in the beginning stages of understanding the problem. By translating the problems into Spanish, some students were successful in solving the problems, while others struggled in their search for what they thought was the correct translation.

Some of the students who were successful in solving a few of the five problems stated that ignoring irrelevant words in the problem was a strategy they used to focus on phrases that provided more useful information. For example, Juany circled important information in the problems and underlined the question immediately after reading the problem. After underlining important information, Juany developed a plan, such as what operation (i.e., addition, subtraction, multiplication, or division) to use to reach a solution. The students who were less successful tended to focus their efforts on understanding words that were difficult for them.

Findings and Discussion from the Spanish Think-Aloud Protocols

The findings of the think-aloud protocols in Spanish are reported in a similar way as the English think-aloud protocols. I created Table 3 to summarize the results of the nine students' think-aloud protocols in Spanish. Bold words are used to represent Spanish words.

Insert Table 3 here.

Problem #1

Similar to the English version, language did not have a significant effect on the Spanish version of this problem. However, for a few students the comparative **cuántos más** created difficulties. For example, Betty stated that she understood all words in the problem, but she was confused by the question. Her first response was "Saturday" because Betty thought she was being asked on what day more children went to the pool. After I rephrased the question to "Exactly how many more came on Saturday?" she was able to answer correctly.

Like Betty, Leticia was confused by the question the first time she read the problem. Then, she restated the question asking herself in Spanish: **¿Cuántos niños faltaron el domingo?** *{How many children did not go to the pool on Sunday?}*. By doing this, Leticia compensated or attempted to make both numbers equal. This finding concurs with Lo Cicero, Fuson, and Allxsaht-Snider (1999) who found that Spanish-speaking children tend to ask "How many are missing?" instead of "How many more?" Yolanda mentioned that the word **más** helped her solve the problem, indicating that she needed to compare both days. Ismael (Table 3) added the numbers for Saturday and Sunday because of the word **más** which, translated to English, means *"more."* In Ismael's case, the word **más** did not help him decide how to solve the problem.

César did not encounter difficulties with the words. He decided that he needed to subtract because the question asked to compare the number of children who went to the pool each day. Like César, Criselda had no difficulties with the language, but she did not know how to approach the problem mathematically.

When Juany made a plan to solve the problem, she mentioned dividing, but she had a minus sign written as she carried out the plan. There was a mismatch in the way she planned to solve the problem and the way she actually solved it. However, she still got the correct answer. Like Juany, Oscar also had a discrepancy in the way he planned and actually solved the problem. Oscar had stated he would add, but when he carried out the plan, he subtracted. In other words, there was a discrepancy in both cases between the conceptual and the procedural knowledge (Hiebert & Carpenter, 1992).

Problem #2

The students who solved the problem correctly focused on inferring correct meaning from **señor Emery** to calculate the cost of the adult ticket. The approach these students implemented was to multiply 27 students by the cost of the ticket, $2.75. Then, they added $4.50, the cost of the teacher's ticket. All students (Table 3), except Criselda and Ismael, solved this problem. Criselda took the correct approach, but she forgot to add the teacher's ticket. Ismael was not sure how he would solve the problem. He added 27 and $2.75 and multiplied $4.50 by 1, understanding that there was one adult ticket but adding the number of student tickets instead of multiplying them. Thereafter, he did not know how to proceed to reach a solution.

Leticia and Oscar both had discrepancies on what they proposed to do and what they actually did to get an answer. For example, in the beginning Leticia said she would add to solve the problem, but she wrote the multiplication symbol as she executed her plan. Oscar said he would subtract, but he multiplied at the end to reach a solution.

Problem #3

Logical connectives played an important role in solving this problem because students needed to know that the **ni...ni** *{"neither...nor"}* construction asked them to exclude the number of pages for poems and short stories. Criselda and Ismael did not solve the problem. Both understood the information given about the number of pages, but they were confused by the **ni...ni** construction in the question. Dawe (1983) found that logical connectives sometimes create difficulties for second language learners. In this case, however, Criselda encountered difficulties in her native language as well as her second language.

Similar to the first and second problem, there were three students who were inconsistent with what was said and what was written as an approach to solve the problem. Leticia and Oscar (Table 3) said they would subtract, but then added when they executed their plan. Roberto mentioned multiplying 3 1/2 and 7 1/2, but he added these two numbers as part of the plan.

Problem #4

There is a consistent pattern in this problem with the students who succeeded in obtaining an answer. Whereas the phrase *"about 20 times that many"* created some difficulties for students in the English version of the problem, it was evident that **20 veces más** helped students answer the question in the Spanish version (see Table 3). The words **astrónomos** and **asteroides** were words that students knew in the Spanish version because only one student had difficulty with the vocabulary. Although these words are cognates (words that are similar in spelling and meaning in two languages), the students may have had more difficulty with the English version for two possible reasons. One is that students had the extra task of inferring meaning from the phrase *"about 20 times that many."* Another possibility is that students may have learned the words by the time they took the Spanish version.

Problem #5

Similar to the English version, all students had the most difficulty with this problem in the Spanish version because there was a mixing of natural language with mathematics language. Although students were able to distinguish that **lata** meant "can" and that it was being used as a noun, they were still confused by the use of the numbers to label the cans. Some students also mentioned that working with fractions was difficult for them. Betty, Roberto, and Yolanda successfully solved the problem.

Juany (Table 3) knew exactly what numbers to subtract. In other words, she was able to distinguish which numbers were being used to label the cans and which numbers were being used to represent the number of ounces, but there were inconsistencies with the procedural knowledge. Instead of getting 14 1/8, she got 15 1/8 because she had counted up from 13 7/8 using tallies.

Conclusions and Implications

Based on the think-aloud protocols, students performed about the same on the first three problems regardless of whether the problem was given in Spanish or English. This finding indicates that the language did not have a significant effect on the word problems. On the contrary, students performed poorly on the fourth and fifth problems regardless of what language was used. This finding shows that the language had a significant effect on the word problems because, as was the case with the fifth problem, the mathematical language was being used in non-mathematical contexts.

The English and Spanish think-aloud protocols provided information that was valuable in understanding problem-solving processes undertaken by these nine second language learners. These students implemented a process that was influenced by their interpretation of the words introduced in the word problems. My purpose was to analyze that process and the words that helped or hindered the students in reaching a solution to the problems.

From the students' responses, giving the word problems to students in Spanish made a difference. Overall, students improved by one or two problems when they were given the word problems in Spanish, with the exception of Criselda and Ismael. Leticia improved from no correct solutions on the English version to four problems solved correctly on the Spanish version. This finding points to the critical need of making both languages accessible to second language learners during problem solving.

The think-aloud results show the need to address conceptual and procedural knowledge in the mathematics classroom with second language learners. These nine ESL students discussed the way they thought about approaching the problem, but in some instances there was a mismatch between what they conceptualized and how they executed their plan to solve the problem. The reason for this mismatch was that the students did not know the correct vocabulary for different mathematical operations in English (i.e., addition, subtraction, division, and multiplication), but they knew the concepts represented by the symbols. This also indicates that the transfer from the first to the second language is not automatic. In the classroom, educators should provide opportunities for second language learners to express their ideas about solving a problem and allow students to show their work in writing (i.e., on the board, on paper, on small eraser boards, etc.) so that these inconsistencies can be addressed.

Furthermore, the results show that students' sociocultural and linguistic experiences should be used to make mathematical connections between the everyday use of English and the language that is specific to mathematics. Knowledge of the mathematics register is needed so that mathematics and bilingual educators can better serve the needs of second language learners. Mathematics educators should seek help from bilingual/ESL specialists and bilingual/ESL educators should seek help from mathematics educators so that both can benefit from each other's areas of expertise.

Given the reform movement to implement standards-based curriculum, future studies focusing on second language learners in mathematics classrooms should investigate how language affects mathematics learning. Because there is often much reading and writing involved in the textbooks used in standards-based curriculum, it would be important to study how students make sense of text. How ELLs construct meaning would help mathematics educators better understand this process as well as how to integrate natural language and mathematical language into teaching. Another study could investigate how ELLs who have attained CALP at the middle to secondary level construct meaning of similar word problems. This population would probably consist of students who have already been mainstreamed into regular or honors mathematics courses.

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Endnotes

[1] In this study, English as a second language (ESL) or second language learner (SLL) is used interchangeably.

[2] The names used for all participants are pseudonyms.

Table 1: Students' Profiles

Student	Sex	Age	Grade Level	# years of schooling in native country	# years of schooling in U.S.
Betty	F	14	8th	6.5*	3.5
César	M	14	8th	5*	4
Criselda	F	12	7th	5	3
Ismael	M	12	6th	3	2.5
Juany	F	10	6th	6*	2
Leticia	F	13	8th	9**	2
Oscar	M	13	8th	6	2
Roberto	M	14	8th	6*	2.5
Yolanda	F	13	7th	4	3

* Includes kindergarten schooling
** Includes 3 years of preschool and kindergarten schooling

Table 2: Summary of English Think-Aloud Protocols

Problem #1	Problem #2	Problem #3	Problem #4	Problem #5
(6)* successfully completed the problem; (3) difficulties with language used in the word problem.	(4) successfully completed the problem; (3) difficulties with the language; (2) successfully completed the problem but with some difficulties.	(6) successfully completed the problem; (3) difficulties with the language used in the word problem.	(8) difficulties with the language in the word problem; (1) successfully completed the problem.	(8) difficulties with the language used in the word problem; (1) completed the problem but with some difficulties.
Difficulties: (2) *than* (interpreted as **entonces**- *then*) (2) *How many more* (Students wanted to add; the phrase helped one student in her decision to subtract.) (1) *pool* (1) *Sunday* (1) *many* (interpreted as "money")	Difficulties: (9) *Hall's planetarium* (4) *admission* (3) *took* (1 thought it was "talk.") (2) *spend* (1 interpreted *spend* as **perdio**- "lost") (2) *show* (1) *Mr. Emery* (1) *did*	Difficulties: (4) "/" symbol (2) *Meteor Magazine* (2) understood all words but did not know what to do mathematically (1) "not"	Difficulties: (8) *asteroids* (7) *astronomers* (7) *20 times that many* (5) *believe* (4) *known* (1 interpreted as "now.") (2) *exist* (1 thought of as **salidas**- "exits"; another thought of as **éxitos**- "successes.") (2) had difficulty with numbers in the problem; they wanted to add. (1) interpreted "many" as **menos**- "less."	Difficulties: (8) *can holds* (interpreted *can* as **puede**, the auxiliary verb, rather than *can*, the noun form) (4) were confused by the use of numbers to label cans (1) interpreted *holds* as *holes*.

*Numbers in parenthesis indicate the number of students.

Table 3: Summary of Spanish Think-Aloud Protocols

Problem #1	Problem #2	Problem #3	Problem #4	Problem #5
(1)* solved the problem with some difficulties; (2) successfully completed the problem; (2) difficulties with the language used in the word problem.	(7) successfully completed the problem; (2) difficulties with the language used in the word problem.	(6) successfully completed the word problem; (3) difficulties with the language used in the word problem.	(7) successfully completed the word problem; (2) difficulties with the language used in the word problem.	(1) solved the problem but with some difficulties; (6) difficulties with the language used in the word problem; (2) successfully solved the problem.
Difficulties: (1) did not know whether to add or multiply. (1) added numbers for Saturday and Sunday because of the word **más**- "more".	Difficulties: (1) did not understand **planetario** (1) included only the cost of the Ss and forgot to add the cost of the adult (1) did not know what to do mathematically	Difficulties: (2) understood all words but were confused by **ni...ni** *neither...nor* construction of the question. (1) obtained 15 as an answer instead of 5. (1) read 1/2 as 12.	Difficulties: (1) did not understand **asteroides** and **astrónomos**. (2) did not know whether to add or multiply.	Difficulties: (4) were confused by the use of numbers to label cans. (1) did not know how to solve problem. (1) made a mistake in final calculation.

*Number in parenthesis indicates the number of students.

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