InterMath, a five-year effort funded through the National Science Foundation, is a statewide Internet-based project with the goal of designing and implementing a series of workshops and ongoing support programs that feature contemporary applications of technology and mathematics pedagogy in the middle grades. Technology is used to deliver the curriculum through Web-based materials and to explore the mathematics using cognitive tools such as dynamic geometry software, spreadsheets, and graphing calculators. Objectives of InterMath include: strengthening the middle school teacher's knowledge and understanding of mathematics; providing a support structure (online and in-school) to aid teachers in implementing and integrating technology tools for doing mathematics; and providing a structured in-service curriculum that follows Georgia's Quality Core Curriculum objectives as well as reform efforts expressed in publications by the National Council of Teachers of Mathematics. InterMath is a collaborative effort among the University of Georgia, Georgia Institute of Technology, and nine regional technology centers in the state of Georgia. (Contains 39 references.) (Author/AEF)
INTERMATH\textsuperscript{14}--PROFESSIONAL AND COGNITIVE DEVELOPMENT THROUGH PROBLEM SOLVING WITH TECHNOLOGY

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How can teachers teach a mathematics that they never have learned, in ways that they never experienced?

Cohen and Ball, 1990

Abstract

InterMath is a statewide Internet-based (http://www.intermath-uga.gatech.edu/) project with the goal of designing and implementing a series of workshops and ongoing support programs that feature contemporary applications of technology and mathematics pedagogy in the middle-grades. Technology is used to deliver the curriculum through web-based materials and to explore the mathematics using cognitive tools such as dynamic geometry software, spreadsheets, and graphing calculators. Objectives of InterMath include

- strengthening the middle school teacher’s knowledge and understanding of mathematics,
- providing a support structure (on-line & in-school) to aid teachers in implementing and integrating technology tools for doing mathematics, and
- providing a structured inservice curriculum that follows Georgia’s Quality Core Curriculum objectives as well as reform efforts expressed in publications by the National Council of Teachers of Mathematics.

InterMath is a collaborative effort among the University of Georgia, Georgia Institute of Technology, and nine regional technology centers in the state of Georgia. InterMath, a five-year effort to design and implement a series of field-based workshops and ongoing support programs to assist both teachers and administrators in effecting mathematics reform, is funded through the National Science Foundation.

Rationale

A Vision for School Mathematics

The pedagogical shifts embodied in a series of documents published by the National Council of Teachers of Mathematics (NCTM) emphasize vastly different approaches to mathematics teaching and learning than are typical in today's classrooms (NCTM, 1989, 1991, 1995, 2000). Rather than static knowledge and skills detached from both other domains and everyday events, mathematics is viewed as problem solving, reasoning, and communicating so that students are empowered to confidently "explore, conjecture, and reason logically [about the world around them]" (NCTM, 1989, p.5). This change in learning philosophy reflects a need for mathematics that is based in an information-rich and technology-based society. Learning goals should incorporate values that reflect mathematics for life, mathematics as a part of cultural heritage, mathematics for the workplace, and mathematics for the scientific and technical community (NCTM, 2000).

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NCTM (2000) suggests that the direction of mathematics education should involve six core principles: equity, curriculum, teaching, learning, assessment, and technology. The equity principle stresses the need for reasonable expectations, opportunities, resources, and support for all students in learning mathematics. Students should have access to different forms of technology that will help them generate ideas and support their thinking. The curriculum principle focuses on the need to develop a clear, coherent plan to promote important mathematics. Concepts in the curriculum should relate to other mathematical ideas and be used to promote mathematical thinking and reasoning. The use of technology encourages these mathematical connections by allowing students to understand, visualize, and conjecture about new or unfamiliar concepts.

In reform-based mathematics classrooms, teachers are not merely keepers and transmitters of mathematical knowledge; they facilitate student engagement by posing relevant problems that encourage deep mathematical thinking involving analysis, problem finding and problem solving, that result in a rich conceptual understanding. Thus, the teaching principle emphasizes that teachers need to be well-versed in mathematics and pedagogy, including how students learn mathematics and the most effective learning environments, in order to fulfill this role. Similarly, the learning principle emphasizes the need for understanding mathematical concepts. According to the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), conceptual understanding "enables children to acquire clear and stable concepts by constructing meanings in the context of physical situations and allows mathematical abstractions to emerge from empirical experience" (p. 17).

The assessment principle identifies assessment as a tool for enhancing learning and informing instructional decisions. Assessment should support continual and reflective learning based on values, multiple sources of information, and feedback, so that learners take responsibility for their ideas. Technology use not only influences how and what mathematics is taught, but it also gives students an opportunity to construct and express their mathematical ideas through their own creations and interpretations. However by itself, "technology is not a panacea" because any teaching tool can be used poorly (NCTM, 2000, p. 25).

Teachers should be provided extended opportunities to experience and do mathematics in an environment supported by diverse technologies (Dreyfus & Eisenberg, 1996). The development of mathematical understanding occurs when technology is used as a cognitive tool that supports thinking, reasoning, and problem solving (Jonassen and Reeves, 1996). The use of cognitive tools such as dynamic geometry, graphing calculators, spreadsheets, and symbolic processors, can provide opportunities and experiences for exploration, developing understanding, interpreting and communicating about mathematics (see Bransford, et al, 1996; Schoenfeld, 1982, 1989, 1992; Silver, 1987). Our approach in the InterMath project focuses on developing mathematical power—understanding, using, and appreciating mathematics.

Barriers to and Proponents of Reform

Reform, however, does not occur simply because new standards or approaches emerge. Several barriers have hampered reform efforts. One barrier appears to be linked to resilient and pervasive beliefs among in-service and inservice teachers as to what constitutes mathematics (Ball, 1988; Dossey, 1992; Thompson, 1984; 1992). Even and Lappan (1994) identified several widely held teacher beliefs: (1) computational proficiency is the major mathematics curriculum goal; (2) mathematical knowledge is rule bound and unconnected; (3) teaching is telling and learning is memorizing (p. 129). Howson, Keitel, and Kilpatrick (1981) noted that many curriculum projects fail because teachers tend to proceduralize methods in ways that are often inconsistent with the curriculum's underlying epistemological and pedagogical assumptions. This has been particularly evident in the use of widely available drill-and-practice programs that could be used to support emerging pedagogies, but rarely are. Cohen (1990), for example, documents the activities of a well-intentioned teacher who, based on lectures about reform mathematics, believed her methods were consistent with the current reform movement. However, she never actually experienced "doing mathematics" or learning mathematics in these new ways herself. While her intent and motives were admirable, the lack of experience in participating as a learner inherently limited her understanding and insight in implementing the approaches. In order to promote conceptual change, teachers must themselves experience mathematics as we want our students to: as conjecturing, reasoning, communicating, and
problem solving. Such experiences should prompt teachers to examine their fundamental beliefs about such questions as, "What is mathematics?," "What does it mean to know mathematics?," "How do students learn mathematics?," and "What is the role of the teacher in the mathematics classroom?"

Research in mathematics education suggests that a teacher's conception of mathematics has a strong impact on how mathematics is approached in the classroom (Cooney, 1985; Thompson, 1984; 1992). Furthermore, the nature of the classroom environment in which mathematics is done strongly affects how students view the subject and how it should be taught and learned. A common theme found throughout the reform documents is "What students learn is fundamentally tied to how they learn it" (NCTM, 1989, p. 5; NCTM, 1991, p. 21). Thus, if we want our students to view mathematics not as a static body of rules and procedures, but as a meaningful and dynamic, yet connected body of knowledge, we must make an impact on their teachers' views of mathematics.

In short, if we want our teachers to meaningfully teach mathematics, they must experience meaningful mathematics. In the words of Cohen and Ball (1990), "How can teachers teach a mathematics that they never have learned, in ways that they never experienced?" We cannot expect teachers to teach in a manner consistent with reform advocates simply because they have been told what to do or how to do it. To help our teachers meaningfully teach and model mathematical thinking, they must experience relevant mathematics as learners, benefiting from both the discovery processes as well as guidance from and modeling of capable peers. To break the cycle of stagnant curriculum and pedagogy, better teacher models are needed at all levels, K-12 through university.

**Project Overview**

**Description and Goals**

InterMath (http://www.intermath-uga.gatech.edu/) is a statewide Internet-based project with the goal of designing and implementing a series of workshops and ongoing support programs that feature contemporary applications of technology and mathematics pedagogy in the middle-grades.

InterMath has two primary teacher components:

- workshops comprised of in-class portions and a "follow-along" component in which participants create curriculum for use in their own classrooms.
- an ongoing system to support teachers beyond the initial laboratory/workshop.

Intensive support will be provided throughout the workshops under the close tutelage of InterMath facilitators distributed throughout the state. The site-based component will focus heavily on scaffolding in-school reform efforts. As participants near completion of the laboratory portion, they will transition to the ongoing support system—a peer community to ensure continuity beyond the laboratory.

The ongoing support system is supported by the Learning and Performance Support Laboratory (University of Georgia) and the Center for Education Integrating Science, Mathematics, and Computing (Georgia Institute of Technology) where shared resources and communication tools are provided; customization of support will be ensured through distributed implementation sites. Three INTECH (INtegrating TECHnology in the student-centered classroom) centers at the University of Georgia, Valdosta State University, and Kennesaw State University will be initially certified as InterMath sites. They will then mentor both subsequent InterMath INTECH centers as well as serve as regional support base for participants. During the project, we will establish a geographically distributed community of educators, K-12 through universities, who are committed to sustaining technology-enhanced middle-grades mathematics teaching and learning reforms. This community will be connected and supported through shared web-based resources, e-mail, and listservs.

Project goals and objectives reflect multiple targets aimed at involving teachers and administrators in technology-enhanced mathematics reform. They link the epistemological, pedagogical, and logistical activities designed to support OCC and NCTM standards.
Goal 1: Promote innovative practices in the tools uses of technology in middle-grades mathematics teaching and learning.

- To use technology tools to model and demonstrate standards-referenced mathematics content and pedagogy for the middle school.
- To enable teachers to experience mathematics using various technologies so that they can explore real world applications, engage in problem solving, and communicate about their investigations.
- To use technology to understand the distinction between demonstration and proof in mathematics and to emphasize the value of each in the understanding of mathematics.
- To use technology to engage in mathematics explorations, to form mathematics ideas, and to solve mathematics problems.
- To use technology tools to construct new and personally meaningful ideas of mathematics.
- To use general tools such as word processing, paint programs, spreadsheets to facilitate mathematics investigations and communication.

Goal 2: Revitalize middle-grades mathematics teaching and learning by modeling, then applying, innovative technology-enhanced approaches.

- To develop effective mathematics demonstrations using appropriate technology tools.
- To engage in independent investigations of mathematics topics from the middle school curriculum or from mathematics appropriate for that level.
- To communicate mathematics ideas arising from technology-enhanced investigations.


- To enable middle grade mathematics teachers to develop and adapt materials and goals from standards-based curriculum through the use of technology.
- To model and explore collaborative instructional strategies.
- To develop mechanisms and expectations of sharing instructional ideas, materials, and information among middle school mathematics teachers.
- To support comprehensive standards-based middle school mathematics curricula and the implementation of Quality Core Curriculum and NCTM goals.
- To utilize technology tools in the implementation of alternative assessment strategies.

Goal 4: Establish the human and technological infrastructure needed to sustain meaningful reform of middle grade mathematics instruction.

- To develop confidence in technology use as teachers explore, practice, reflect, and become adept in technology-enhanced teaching and learning of mathematics.
- To enable and encourage middle school mathematics teachers to collaborate by using technology support.
- To support professional development opportunities for middle school mathematics teachers and other key personnel through a network of peer teachers.

**Professional Development**

**Workshop Procedures**

The workshops are intended to immerse teachers in active problem solving with technology. Participants will explore different concepts each class meeting by working through various InterMath investigations and writing about one in-depth. Each participant will build a personal web page using artifacts and productions from the workshops to compile an electronic portfolio. Write-ups and projects, reflecting participants’ synthesis and reflection about their explorations, will be submitted electronically for workshop credit. The purpose and focus of a write-up is to communicate and synthesize investigations involving exploration, solving a problem, or working with an application. The key elements of a write-up consist of the learner's synthesis, communication, mathematical ideas, interpretation, and utility of an investigation. Final projects, focusing on a technology-enhanced mathematics investigation of the
individual participant's determination, will be submitted and discussed at the end of the workshop/laboratory. Participant productions will be placed on the web page for public sharing.

The laboratory leader will present demonstrations and explanations, clarify problems, and demonstrate alternative solutions using a projected image from the leader's workstation. In a typical session, a leader might allocate one-third of the time in whole-group mode, and during the balance of the meeting provide direct support for participants working on their projects or units, either individually or in groups. The InterMath web site (http://www.intermath-usa.gatech.edu/) will enable participants to work at their home or school sites.

In addition to the 45-hour workshop, the 55-hour "follow-along" course will promote the use of technology to enhance mathematics teaching in their home school and to extend each participant's expertise. This additional component to the workshop promotes reflective practice among the participants, emphasizing realistic applications of technology in middle school teaching. Each participant's web page contributions will include conceptual work, projects, activities for their classroom, and links to related teaching-learning resources in order to establish a highly connected framework of resources.

**Participant Selection and Credit**

Participation will be open to all middle school educators in Georgia, but teachers from historically underserved schools will receive top priority. Applications will be solicited from schools located within the service areas of the participating sites (First year: University of Georgia, Kennesaw State University, and Valdosta State University).

It is expected that a team of selected teachers will participate in the workshops and at least one administrator will participate in a minimum of 20 hours of professional development as part of the team. It is also imperative that each teacher member has classroom Internet access and there is e-mail access for all team members. Teams with a minimum of one teacher of mathematics from each grade level (4-8) represented in the school will receive priority in the selection process.

Credit could be in the form of graduate hours or staff development units depending on the institution offering the workshop. InterMath participants need to check with the site at which they will be participating to determine what type of credit they will receive. If graduate credit will be offered, participants will likely have to apply and be admitted to the graduate school at that particular institution.

**Administrative Support**

Administrative support and leadership are key in both promoting and sustaining school innovation. Several authorities advocate models that tie professional development to a particular school and are explicitly linked to reform activities that the individual school is undertaking (Darling-Hammond, 1995; Davis & Padilla, 1991; Lieberman, 1995). Therefore, administrative support in school improvement plans influences teacher and student use of technology. The target administrators for InterMath are building level personnel with primary responsibility for instructional leadership, i.e., principals, assistant principals, instructional specialists. However, central office administrators with responsibility for curriculum design, professional development, student assessment and the support of instructional technology are also encouraged to participate in the InterMath program.

In the InterMath workshops, administrators will engage in hands-on activities using the Internet to support teachers in their efforts. This participation will ensure that administrators better understand the power and potential of the learning activities, the technical needs of the teacher, and the classroom management techniques that complement technology-enhanced learning experiences. Administrators will be encouraged to share their ideas, problems, solutions, and successes for supporting their teachers. Thus, InterMath workshops can provide a forum to reduce administrator isolation and support administrators in follow-up activities including instructional leadership and teacher evaluation.
Our Use of Technology

The InterMath workshop intends to illustrate how and when technology can be used appropriately in the mathematics classroom. The literature describes two distinctly different approaches in the use of technology in classrooms: using the computer as a tool for exploration or problem solving and using the computer as a tutor that delivers instruction and provides feedback. Research on the use of computers in mathematics as a tutor and a tutee are usually not situated in problem solving environments. Most tutor-based technologies are in the form of drill and practice software, which tend to rely on lower ordered skills, and are often negatively related with student achievement (Jonassen & Reeves, 1996; Wenglinsky, 1998). Jonassen and Reeves (1996) argued that higher-order thinking occurs in environments where the student is learning with, and not from, the computer. It is this approach that InterMath promotes and intends to develop among its participants.

Many studies investigating technology-enhanced environments include an emphasis on conceptual development situations. For example, when calculators and computer software perform calculations and simplifications, teachers have more time to emphasize why something is happening, instead of focusing on algorithms (Grassl & Mingus, 1997; Heid, 1988; Maury, 1987; Palmiter, 1991). Moreover, the imperfections in calculator graphs and computations also provide opportunities for conceptual development. For example, Dion (1990) found cases where the graphing calculators' resolution caused certain functions to appear differently than they are supposed to. In addition, Goldenberg (1998) found that the graphing calculator window can provoke critical inquiry because different functions can appear to look the same if they are on different domain and range windows. Finally, Burrill (1992) noticed that the calculator has difficulty simplifying computations with extremely large and small numbers, consequently producing an incorrect answer. Used appropriately, these situations expose misconceptions and help students develop a richer understanding of the mathematics being studied.

Technology Applications and Facilities

The technologies used in the InterMath program range from low-end, hand-held calculators through high-end multimedia workstations. Computer software applications including spreadsheets, graphing tools, dynamic geometry, web editing, and Internet will be used regularly throughout the workshops. Technology will be available and supported both at INTECH sites and in the participants' schools. All INTECH labs have high-speed Internet access to support individual workstation, local network, and web-based mathematics activities and applications. The laboratory also affords ready access to non-computer technologies, including graphing calculators and manipulative materials.

Cognitive Development

Rationale of Workshop Activities

In designing the workshops, we have kept in mind the work of Malone and Lepper (1987) concerning the design of instructional environments that are intrinsically motivating. They have identified four sources of intrinsic motivation in learning activities: (1) gives an appropriate level of challenge, (2) appeals to the sense of curiosity, (3) provides the learner with a sense of control, and (4) encourages the learner to be involved in a world of fantasy in which learners can experience vicariously rewards and satisfactions that might not be available to them otherwise. While a workshop leader may not be able to incorporate all of these sources of intrinsic motivation into every learning activity, incorporating at least one appears to increase the likelihood that the activity will be intrinsically motivating.

Pertaining to the first source of intrinsic motivation, we have included a variety of problems on a continuum of difficulty levels. By posing challenging problems within a familiar context, teachers will develop confidence in problem solving and thus will more likely engage in the activities. The context of the problems enables teachers to safely sample and reflect on their own approaches to problem solving. The second source of intrinsic motivation is appealing to the sense of curiosity. Activities can stimulate curiosity by introducing ideas that are surprising or discrepant from the learner's existing beliefs and ideas. While the mathematical problems posed in the laboratories will center on middle-school curriculum, they
are more open-ended and generative than is typically seen in a traditional middle-school curriculum. Problems can be used as a springboard for ideas and investigations that participants find personally intriguing. Furthermore, teachers will be able to choose among several activities in which to actually engage. They can choose activities that are most applicable to their classroom needs and relevant to their mathematical understanding. Since participants can choose activities based on their preferences, the third source of intrinsic motivation (providing the learner with a sense of control) will be reflected throughout the laboratory.

The fourth source of intrinsic motivation is encouraging engagement through fantasy. As an example of a task using fantasy, consider the following problem requiring the use of the Pythagorean theorem:

The learner needs to calculate the distance from point a to point b in order to inform Captain James T. Kirk about how to set the transformer beam on the Federation Starship Enterprise so they can pick up the necessary dilithium crystals directly below on the planet's surface. Kirk only knows the distances of the ship and the crystals from a third point where his scouting party has stopped (Lepper & Hodell, 1980).

Fantasies are more intrinsically motivating when they employ characters and situations with which the learner can identify. Faced with either this fantasy-like problem or a series of abstract problems in which learners are asked to find the length of one side of a triangle, one can imagine which type of problem learners would prefer.

The philosophy permeating InterMath is that teachers must relearn mathematics in a more open-ended, generative manner so they may come to understand what reform documents intend by "meaningful learning." Furthermore, by encouraging teachers to create and modify their own curriculum units, InterMath attempts to avoid what Howson, et al. (1981) warn may be a cause for failed reform -- teachers failing to assume ownership of reform.

Workshop Content

The mathematics content and concepts of InterMath reflect curriculum that would enhance a teacher's understanding of middle-grades mathematics. The laboratory centers on the middle-school mathematics curriculum per Georgia's Quality Core Curriculum (QCC) and the NCTM Standards (1989, 2000). The InterMath curriculum is meant to engage teachers and is intended to deepen teachers' understanding of mathematical concepts related to the middle school curriculum. Thus, the investigations would likely need to be modified for use with middle school students.

There are 14 units that can be used for InterMath workshops. Thirteen units are called Fraction and Decimals, Integers; Ratios, Proportions & Percents; Quadrilaterals, Triangles, Polygons, Probability, Statistics, Solids, Circles, Graphs, Patterns, Functions & Equations. The fourteenth unit is comprised of over 200 problems adapted from Teaching Mathematics in the Middle School (NCTM). The following criteria have been used to highlight recommended investigations for teacher exploration:

- Multiple cases can be investigated using technology.
- Pre-Algebra students can rely on technology to investigate the situation.
- The investigation promotes generalizability or can be used as a springboard for further exploration.
- Multiple methods can be used to explore the situation.
- Multiple solutions are possible.
- The investigation, based on middle school mathematics, is easy to start exploring.
- The investigation can be modified for use in a middle school classroom.

The following investigation exemplifies these principles:
Choose two numbers. Add them together and form a Fibonacci-like sequence, starting with your first two numbers, and ending with a total of ten numbers. For example, if your first two numbers are 3 and 5, then your third number is 8 (3+5), your fourth number is 13 (8+5), your fifth number is 21 (8+13), and so on. Determine a relationship using the seventh term and sum of the terms of your sequence. Is this true for every sequence of this nature? Explain.

- Multiple cases can be investigated using technology.
  The use of formulas in a spreadsheet allows teachers to change the initial two numbers and instantly view calculations of the remaining 8 terms and the sum of the sequence. As multiple cases are tested, a formula relating the seventh term and sum of the sequence can be hypothesized, tested, and modified.

- Pre-Algebra students can rely on technology to investigate the situation.
  While students can conceivably make a reasonable conjecture about this investigation using a few cases, they will need an algebraic proof to verify that their conjecture is true. The use of technology in this case amplifies the confidence in their conjecture because multiple cases can be tested.

- The investigation promotes generalizability or can be used as a springboard for further exploration.
  Following the experimentation process with technology, teachers are encouraged to question why a particular pattern develops and then investigate a proof. Furthermore, after answering the initial question, teachers may develop further questions, such as:
  1. Will this hypothesis be true for negative numbers?
  2. Will this hypothesis be true for decimals and fractions?
  3. Are there relationships between the sum and other terms in the sequence?

- Multiple methods can be used to explore the situation.
  A spreadsheet can be used in a variety of different ways to investigate this situation. For example, students can construct one table that continually changes, or a grid with multiple tables. In addition to spreadsheets, symbolic manipulation can be used to investigate this problem.

- Multiple solutions are possible.
  Many people will propose that 11 times the 7th term will equal the sum of the sequence. In addition, multiple linear combinations are also acceptable, such as six times the 7th term plus 2 times the 9th term minus the 4th term will equal the sum of the sequence.

- The investigation, based on middle school mathematics, is easy to start exploring.
  Only basic arithmetic operations are used in this investigation. Most people begin using positive integers in their exploration, and then later broaden their scope to different types of numbers such as negative integers, fractions, decimals, and irrational numbers.

- The investigation can be modified for use in a middle school classroom.
  This investigation can be immediately adopted in the middle school classroom if the intention is to teach pattern recognition and creation of formulas from data. However, a shorter sequence might be used in the classroom if the teacher intends to illustrate adding like terms, using the distributive property, and creating linear combinations with variable expressions.

**Next Steps**

InterMath is in its second year of a five year project. Over the past year, the web-based InterMath materials have been developed and tested with various teachers in the state of Georgia. This year the project will run workshops in the spring and summer semesters to build a community of teachers that will develop technology-enhanced materials for their classrooms. In addition, we intend to develop an ongoing support system that will encourage a sustained effort among teachers in the InterMath program. The goal
at the end of the five year project is to have a self-sustaining system of resources, tools, and people with a common goal of enhancing mathematics education using technology as a catalyst for change.
References


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