

DOCUMENT RESUME

ED 455 120

SE 065 091

AUTHOR Schoenberger, Kathleen M.; Liming, Lori Ann
TITLE Improving Students' Mathematical Thinking Skills through Improved Use of Mathematics Vocabulary and Numerical Operations.
PUB DATE 2001-05-00
NOTE 68p.; Master of Arts Action Research Project, Saint Xavier University and Skylight Professional Development.
PUB TYPE Dissertations/Theses (040) -- Tests/Questionnaires (160)
EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Grade 6; Grade 9; High Schools; *Mathematical Vocabulary; Mathematics Education; Middle Schools; *Number Concepts; Special Education; *Thinking Skills

ABSTRACT

This report describes a program for improving students' mathematical thinking skills through improved use of mathematics vocabulary and numerical operations. The targeted population includes sixth grade general education mathematics students and ninth grade special education mathematics students. The students' inabilities to effectively solve multi-step problems involving mathematical vocabulary and higher-order numerical operations were documented by story problem, vocabulary, and cue-word assessments. There are two schools in this study. Site A, a middle school, is located within a middle-class suburban community. Site B, a high school, is located within a middle-class urban community. Both sites are near a large mid-western city. Analysis of probable cause data indicated that a significant percentage of students had weaknesses in their thinking skills as related to their use of mathematics vocabulary and numerical operations. Review of the research literature suggested that some of the causes for this problem included: an emphasis on repetition and rules, inadequate language skills, a lack of prior knowledge of mathematics concepts, the amount of personal risk students will invest, and the students' decline in engagement with mathematics activities. A review of the solution strategies suggested by the researchers in the field of education combined with an analysis of the problem setting resulted in the development of a program for improving students' thinking skills as related to their use of mathematics vocabulary and numerical operations. This program involved the use of story problems that included elements of self-monitoring, pair-share strategies, direct instruction, and student-made glossaries of mathematical vocabulary. Post-intervention data indicated an increase in students' abilities to correctly use vocabulary words in literal and abstract sentences and identify the parts of mathematical equations, identify the correct cue words in story problems and operation needed to solve the problems, and an increase in students' abilities to accurately complete word problems and label their answers. (Contains 26 references.) (Author)

IMPROVING STUDENTS' MATHEMATICAL THINKING SKILLS THROUGH IMPROVED USE OF MATHEMATICS VOCABULARY AND NUMERICAL OPERATIONS

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

K. Schoenberger

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

Kathleen M. Schoenberger
Lori Ann Liming

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

This document has been reproduced as received from the person or organization originating it.

Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

An Action Research Project Submitted to the Graduate Faculty of the
School of Education in Partial Fulfillment of the
Requirements for the Degree of Master of Arts in Teaching and Leadership

Saint Xavier University & Skylight Professional Development

Field-Based Master's Program

Chicago, Illinois

May 2001

BEST COPY AVAILABLE

SIGNATURE PAGE

This project was approved by

Dr. Susan L. Mason

Advisor

Arnold Barbareck

Advisor

Beverly Colley

Dean, School of Education

ABSTRACT

This report describes a program for improving students' mathematical thinking skills through improved use of mathematics vocabulary and numerical operations. The targeted population includes sixth grade general education mathematics students and ninth grade special education mathematics students. The students' inability to effectively solve multi-step problems involving mathematical vocabulary and higher-order numerical operations were documented by story problem, vocabulary, and cue-word assessments. There are two schools in this study. Site A, a middle school, is located within a middle-class suburban community. Site B, a high school, is located within a middle-class urban community. Both sites are near a large mid-western city.

Analysis of probable cause data indicated that a significant percentage of students had weaknesses in their thinking skills as related to their use of mathematics vocabulary and numerical operations. Review of the research literature suggested that some of the causes for this problem included: an emphasis on repetition and rules, inadequate language skills, a lack of prior knowledge of mathematics concepts, the amount of personal risk students will invest, and the students' decline in engagement with mathematics activities.

A review of the solution strategies suggested by the researchers in the field of education combined with an analysis of the problem setting resulted in the development of a program for improving students' thinking skills as related to their use of mathematics vocabulary and numerical operations. This program involved the use of story problems that included elements of self-monitoring, pair-share strategies, direct instruction, and student-made glossaries of mathematical vocabulary.

Post-intervention data indicated an increase in students' abilities to correctly use vocabulary words in literal and abstract sentences and identify the parts of mathematical equations, identify the correct cue words in story problems and operation needed to solve the problems, and an increase in students' abilities to accurately complete word problems and label their answers.

TABLE OF CONTENTS

CHAPTER 1 – PROBLEM STATEMENT AND CONTEXT.....	1
General Statement of the Problem.....	1
Immediate Problem Context.....	1
The Surrounding Community.....	8
National Context of the Problem.....	9
CHAPTER 2 – PROBLEM DOCUMENTATION.....	11
Problem Evidence.....	11
Probable Causes.....	15
Probable Causes Site Based.....	18
CHAPTER 3 – THE SOLUTION STRATEGY.....	20
Literature Review.....	20
Project Objectives and Processes.....	31
Project Action Plan.....	31
Methods of Assessment.....	32
CHAPTER 4 – PROJECT RESULTS.....	34
Historical Description of the Intervention.....	34
Presentation and Analysis of Results.....	35
Conclusions and Recommendations.....	39
REFERENCES.....	41
APPENDICES.....	44

CHAPTER 1

PROBLEM STATEMENT AND CONTEXT

General Statement of the Problem

The students of the targeted sixth grade general education mathematics class and the ninth grade special education mathematics class exhibit a deficiency in thinking skills that inhibit their abilities to effectively solve multi-step problems involving mathematical vocabulary and/or higher-order numerical operations. Evidence for existence of the problem includes anecdotal records, teacher survey, and student thinking-skills inventory.

Immediate Problem Context

Sites A and B have contrasting community and student characteristics. Site A, a middle school, is located within a suburban community. Site B, a high school, is located within an urban community. The percentages of low-income students at Sites A and B are 6.0 % and 35.2%, respectively. The percentages of limited-English proficient students who qualified for a bilingual program at Sites A and B are 5.1% and 11.3%, respectively. Site A exhibits higher attendance rates, lower student mobility, and lower chronic truancy than state averages. In contrast, Site B exhibits a lower attendance rate than state average, but higher mobility and chronic truancy ratings (School Report Card, 1998 and 1999).

The student body of Site A is predominately white, with less than 25% comprised of black, Hispanic, and other minority groups. In contrast, the student body of Site B is

predominately Hispanic with white, black, and other racial groups comprising less than 50% of the student population. Site A has a total enrollment of 1,678 students and Site B has a total enrollment 3,437 (School Report Card, 1998 and 1999). The differences between Sites A and B are summarized in Tables 1 and 2.

Table 1

Racial/Ethnic Background of Students at Site A and B

Location	White	Black	Hispanic	Asian/Pac. Islander	Native American	Total Enrollment
Site A	77.8%	5.1%	8.4%	8.6%	0.1%	1,678
Site B	21.9%	28.7%	45.7%	3.6%	0.0%	3,437

Table 2

Attendance, Mobility, and Chronic Truancy at Site A and B

Location	Attendance	Student Mobility	Chronic Truancy
Site A	95.2%	7.9%	0.1%
Site B	90.3%	35.8%	15.5%
State	93.6%	18.1%	2.3%

The districts of Sites A and B have differences in their teacher characteristics. Of the teachers in the district of Site A, 13.6% are male and 86.4% are female. Of the teachers in the district of Site B, 26.6% are male and 73.4% are female. Teachers in the districts of Site A and B are predominantly white. Over 39% of the teachers in both districts have Master's degrees or above. The pupil-teacher ratio in the districts of Site A and B is 17.8:1 and 23.1:1, respectively

(School Report Card, 1998 and 1999). Tables 3 and 4 describe the characteristics of the teachers in the districts of Sites A and B.

Table 3

Teachers by Racial/Ethnic Background in the Districts of Site A and B

Location	White	Black	Hispanic	Asian/Pacific Islander	Total Number
Site A	98.6%	0.0%	0.5%	0.8%	367
Site B	77.2%	8.6%	13.7%	0.5%	755

Table 4

Teacher Characteristics at Site A and B

Location	Average Teaching Experience	Teachers with Bachelor's Degree	Teachers with Master's & Above	Pupil-Teacher Ratio
Site A	8.8 years	60.2%	39.8%	17.8:1
Site B	13.2 years	53.2%	46.8%	23.1:1

The districts of Sites A and B have similar instructional expenditures for students and teacher and administrator salaries. The instructional and operating expenditures per pupil for both districts are below the average state expenditure. The average teacher salary for both districts is significantly below the state average. The average administrator salary exhibits the only noteworthy difference between the districts of Sites A and B. The average administrator salary in the district of Site A is above the state average, while the average administrator salary in the district of Site B is considerably below the state average. Tables 5 and 6 include the specific amounts for instructional expenditures for students and teacher and administrator salaries.

Table 5

Instructional Expenditure per Pupil

Location	Instructional Expenditure per Pupil	Operating Expenditure per Pupil
Site A	\$3,013	\$5,938
Site B	\$3,370	\$5,888
State	\$3,990	\$6,682

Table 6

Teacher/Administrator Salaries

Location	Average Teacher Salary	Average Administrator Salary
Site A	\$36,485	\$80,033
Site B	\$38,929	\$66,109
State	\$45,337	\$76,917

The districts of Sites A and B have similar administrative structures. The district of Site A has a superintendent, public information specialist, technology coordinator, human resource specialist, supervisor of special education/pupil and personnel services, and directors of curriculum and instruction, pupil services, and operations. The district of Site B has a superintendent, deputy superintendent, and associate superintendents for curriculum and instruction and business services. Each district has a board of education and a teachers' union. Administrators and board members in both districts have worked with the community to have referendums for school improvement passed.

Site A, a middle school with a new building that opened in August 1997, serves grades 6 through 8. The three story structure has 80 regular classrooms, four gymnasiums on the first level, two additional gymnasiums on the second level, two computer labs, a technology lab, and exploratory classes including: applied practical arts, health, foreign language, and life skills. Gifted programs are available for identified students in the areas of language arts and mathematics. The administrative, health service, support service, gymnasium, library, and cafeteria areas are designed to service up to 3,000 students in anticipation of future enrollment.

Site A is organized following the “school-within-a-school” concept. Faculty and students are divided into “houses” and “teams.” Houses are identified by color. Each house is composed of at least one team per grade level. The students move within their own team area for the core subjects of math, science, language arts, and social studies. Site A provides students with many activities to participate in after school. Activities include team sports, intramural sports, and a variety of clubs.

The sixth grade regular education mathematics’ curriculum at Site A is divided into units. The units are as follows: Whole Numbers, Properties, Graphing, Decimals, Perimeter and Area, Fractions I, Fractions II, Fractions III, Geometry, Ratio and Proportion, and Percent. Problem solving is incorporated into each unit. Each unit is pre-tested and post-tested. If students test out of a unit, they are given alternate enrichment worksheets. Site A does not practice leveling or tracking. Sixth grade math students are not allowed to use calculators during units focusing on operations with whole numbers or decimals.

Sixth grade teachers at Site A emphasize identifying trends or the absence of trends during units including graphing and functions. In relation to problem solving, teachers emphasize knowing whether enough information is given, identifying relevant information, and

using trial and error as a problem solving procedure. Students are taught to formulate reasonable questions from given information, and to identify information used in arriving at a conclusion. Students are taught to check whether the answers to arithmetic problems are correct. The primary focus of Site A “is to encourage all teachers to appropriately challenge all students every day and to be able to measure student achievement throughout the learning process” (School Report Card, 1999, p. 9).

Site B is a high school with two campuses. All ninth-grade students attend the ninth-grade campus while the tenth-, eleventh-, and twelfth-grade students attend the upper-grade campus. The ninth-grade campus is the original high school building for the community. When a second high school was built in the community, the students were split between the two campuses. In 1991, all students were moved to the new building and the bottom two floors of the original high school were converted to a middle school. In 1998, the community passed a \$33 million referendum to build a new middle school and renovate the original high school so it could be used as a ninth-grade campus. The ninth-grade campus opened in 1999 and now occupies three floors of the building. The building includes one main gymnasium and four auxiliary gymnasiums. The pool is currently being renovated into an indoor rifle range for the World Champion JROTC rifle team. Students have access to three computer labs, a state-of-the-art industrial technology lab, and a library with word processing lab and Internet access. Part of the third floor of the building is used as an alternative setting for students with severe behavior disorders. The ninth-grade campus also includes a second building that houses an auditorium and an alternative-setting high school.

The ninth-grade campus utilizes a team concept. Students are part of teams of four core teachers – English, math, social studies, and science. The only exception to this organization are

the teams for students needing special education or bilingual services and the teachers of encore subjects – health, physical education, art, music, foreign language, business, and drama. The teachers of these teams belong to the special education department, bilingual services department, or encore departments, respectively. One of the building's teams, CSP (college studies program), is designated for college-bound students. All students are encouraged to participate in extra-curricular, sport, and school-spirit activities. The ninth-grade campus was created to provide a transition to high school. It was also created to reduce overcrowding at the upper-grade campus.

The ninth grade special education mathematics' curriculum at Site B is divided into chapters. The chapters are as follows: Adding and Subtracting Whole Numbers and Decimals, Multiplying Whole Numbers and Decimals, Dividing Whole Numbers and Decimals, Introducing Fractions and Mixed Numbers, Adding and Subtracting Fractions and Mixed Numbers, Multiplying and Dividing Fractions and Mixed Numbers, Percents, Ratio and Proportion, Probability, Statistics, Graphs, Customary Measurement, Metric Measurement, Pre-Algebra: Expressions and Sentences, Pre-Algebra: Integers and Equations, Geometry: Perimeter and Area, and Geometry: Surface Area and Volume. Each chapter focuses on a particular skill and includes word problems. Students are encouraged to use calculators, multiplication tables, and adaptive strategies to assist them in solving all mathematics problems. The teacher emphasizes identifying pertinent and non-pertinent information, choosing the appropriate operation, using trial and error, and checking that the answer matches the question asked in solving word problems.

The special education mathematics teachers at Site B sequence and deliver the chapters based on the needs of the students in the classes. The material is covered with an emphasis on

quality, or depth, rather than quantity, or breadth. Covering all the material is not a requirement of the curriculum. There are three mathematics courses in the special education program. Each reviews and builds on the previous. Therefore, there is an overlap of material. The emphasis on state tests and graduation exit exams has made special education curriculums become more accountable than in previous years. New state and federal laws are beginning to require that special education classes align themselves with the regular education curriculum and state goals and objectives. Many curriculums are being re-written to accomplish this. By doing this, the school district is attempting to better prepare the students to work in the community.

The Surrounding Community

Site A is the only school servicing grades six through eight in its district. The district includes four grade centers: early childhood through grade 1, grades 2 and 3, grades 4 and 5, and grades 6 through 8. Site B is the only school servicing grades 9 through 12 in its district. The district includes an early childhood center, 15 elementary schools (kindergarten through grade 5), five middle schools (grades 6 through 8), and one high school. The high school is divided into two campuses – a ninth grade campus, which includes an alternative-setting school, and an upper grade campus. Because the students from each district’s feeder schools attend Sites A or B exclusively, the districts’ demographics match those of the schools.

Site A is located in a village with a population of just over 25,000. Site B is located in a city with a population near 75,000. Both the village of Site A and city of Site B have park districts that offer a wide-range of programs for all ages. The village of Site A enjoys a strong tax base, and is home to a large theme park, regional mall, and numerous other businesses and corporations located within its six business and industrial parks. The village of Site A has a median household income of \$52,000 annually. The average cost of a single family home is

\$160,000. The city of Site B is a commercial, trading, and industrial community. It is also home to four community colleges and the county seat. The city of Site B has a median household income of \$39,665 annually. A three-bedroom house averages \$112,000. A major interstate runs through the corporate limits of the village of Site A and the city of Site B. Both the village and city are located within 45 miles of a major city and within 35 miles of a major international airport. Because the village and city are suburban and urban communities near a major city, the authors feel they are representative of our national community and problems.

National Context of the Problem

In the national educational system, educators are concerned with students' limitations in conceptual thinking, reasoning, and mathematical problem-solving skills (Henningesen & Stein, 1997). According to Greeno (1997), "One of the persisting aims in mathematics education is that students become more able in mathematical thinking" (p. 85). Often students cannot identify the specific mathematical operation(s) necessary to solve a problem when the problem involves more than simple computation (Cornell, 1999). Students with disabilities or difficulties with mathematics struggle with identifying and disregarding extraneous information in word problems, accurately completing each part of a multi-step problem, and basic fact computation. Students who have co-existing problems in reading and comprehension may find word problems even more difficult (Jitendra, et al., 1998). This lack of thinking skills inhibits students' abilities to effectively solve multi-step problems involving mathematical vocabulary and/or higher-order numerical operations.

The severity of this problem has prompted national concern. The National Council of Teachers of Mathematics published mathematics reforms that address curriculum, evaluation, and instructional practices (Henningesen & Stein, 1997). In addition to these council

recommendations, researchers have performed in-depth studies of students' error patterns when solving word problems in an attempt to better understand students' errors and provide correctional feedback (Jitendra & Kameenui, 1996). "...[W]ord problems in mathematics present difficulties for students of all ages and ability levels" (National Assessment of Educational Progress as cited in Jitendra, et al., 1998, p. 345). Some of these problems are amplified for students with disabilities. Current findings indicate that some students with disabilities are functioning seven to ten years below their chronological age on applied math problems (Cawley & Miller as cited in Jitendra, et al., 1998). To alleviate these problems related to mathematics problem solving, investigations are being done on ways to encourage students to take more active roles in acquiring, questioning, and utilizing mathematical procedures and rules (Lawson & Chinnappan, 2000).

CHAPTER 2

PROBLEM DOCUMENTATION

Problem Evidence

In order to document students' inability to effectively solve multi-step problems involving mathematical vocabulary and higher-order numerical operations, three tools were designed by the researchers. These tools are assessments to collect baseline and growth data before and after the researchers' intervention, respectively. These assessments focus on recognizing mathematics cue words and operations in written problems, vocabulary assessments for basic mathematics words, and story problem completion. These tools are included in Appendices A, B, C, and D, respectively.

The story problem assessment is a two-page worksheet that includes ten story problems. The problems vary in difficulty from simple single-operation problems with one answer to multi-step problems requiring two-part answers. The assessment includes instructions, empty spaces, and answer boxes. The problems are in a random order, and are not enumerated.

The assessment was given to the students prior to the researchers' intervention to assess the students' current ability to complete story problems accurately. Table 1 shows the results of the cue word pre-assessment. It appears that the majority of students from Site A chose the correct operation for the story problems, except for problem 7, and were able to identify the correct cue word. The students seemed to pick more words as cue words on the story problems

that included division. The students of Site B also generally picked the correct operation for the story problems, except problems 5 and 7. However, the students of Site B were generally unable to identify the correct cue word for that operation.

Table 1

Cue Word Pre-Assessment

Problem Number (Number of words)	Site A			Site B		
	Operation identified correctly	Word most frequently chosen	Number of words chosen	Operation identified correctly	Word most frequently chosen	Number of words chosen
1 (17)	14	more	4	8	how many	14
2 (27)	11	leftover	14	7	can he fill	21
3 (14)	14	gained	4	9	how much	11
4 (18)	13	less	3	8	less, how much	13
5 (18)	14	each, much	11	4	how much	15
6 (18)	13	took	7	9	took away	13
7 (21)	5	per, many	13	3	how many	6
8 (28)	12	how many	16	7	how many	18
9 (18)	13	fewer	4	9	how many	13

A second assessment used by the researchers was a cue word assessment. This assessment includes nine single-operation story problems printed on one page. The directions indicate that students are to circle the cue word(s) in each problem that tell them which operation should be performed to find the answer to the problem. The students indicated what operation they would use to solve each problem by writing “add,” “subtract,” “multiply,” or “divide” on the line provided. Table 2 shows the results of the story problem assessment. The students from

Site A were generally able to compute the correct answer to the story problem and label the answer with the correct unit. The students of Site B were generally unable to correctly solve the problem or did not write an answer. Most errors seemed to be related to logic and students did not generally put a unit on their answers.

Table 2

Story Problem

Item		Site A (182)	Site B (143)
Correct answer	Number correct	105	23
	No answer	2	16
Incorrect answer	Logic error	28	65
	Computational error	13	0
	No work shown	34	14
Unit	Correct	143	10
	Incorrect	10	11
	Missing	29	67

The researchers developed two assessments for mathematical vocabulary. One assessment is two pages and evaluates the students' ability to appropriately use mathematical vocabulary in eleven definition-style sentences and the students' ability to correctly identify the parts of five mathematics number sentences. The other vocabulary assessment is one page and evaluates the students' ability to utilize vocabulary in twelve application- and higher-order-thinking-skill sentences. Each assessment used a fill-in-the-blank format and has a separate word bank that students used for prompts. Tables 3 and 4 reveal the results of this assessment. Most of the students from Site A are able to correctly answer the questions from the Vocabulary I assessment while most of the students from Site B were not. Table 5 also shows that the students from Site A were generally able to correctly complete the higher-order vocabulary sentences while the students from Site B were not.

Table 3

Vocabulary I – Side 1

Word	Site A		Site B	
	Correctly chosen ^a (14)	Incorrectly chosen (140)	Correctly chosen (11)	Incorrectly chosen (110)
Addend	11	3	3	11
Difference	10	2	1	6
Dividend	7	7	2	8
Divisor	7	7	4	6
Factors	11	5	2	7
Operations	12	0	6	4
Perimeter	7	0	6	3
Product	9	5	0	6
Quotient	10	4	2	7
Remainder	14	0	8	1
Sum	13	3	1	8

Table 4

Vocabulary I – Side 2

Word (Possible: N)	Site A		Site B	
	Correctly chosen (Possible: N)	Incorrectly chosen (Possible: N)	Correctly chosen (Possible: N)	Incorrectly chosen (Possible: N)
Addend (2)	22 (28)	5 (154)	15 (22)	1 (121)
Difference (0)	0 (0)	0 (0)	0 (0)	6 (0)
Dividend (2)	14 (28)	16 (154)	4 (22)	7 (121)
Divisor (2)	14 (28)	15 (154)	11 (22)	8 (121)
Factors (2)	20 (28)	6 (154)	7 (22)	6 (121)
Operations (0)	0 (0)	0 (0)	0 (0)	4 (0)
Perimeter (0)	0 (0)	0 (0)	0 (0)	7 (0)
Product (1)	10 (14)	2 (168)	3 (11)	7 (132)
Quotient (2)	20 (28)	5 (154)	11 (22)	4 (121)
Remainder (2)	18 (28)	0 (154)	13 (22)	5 (121)
Sum (1)	13 (14)	2 (168)	6 (11)	6 (132)

Table 5

Vocabulary II

Word	Site A		Site B	
	Correctly chosen (14)	Incorrectly chosen (168)	Correctly chosen (11)	Incorrectly chosen (132)
Addends	6	6	2	8
Difference	14	0	3	3
Dividend	7	7	4	5
Divisor	8	6	5	5
Factors	5	9	2	8
Length	13 (28)	1 (154)	14 (22)	5 (121)
Operation	11	2	3	6
Perimeter	12	1	5	7
Product	9	5	2	7
Quotient	8	8	0	9
Remainder	14	0	9	2
Sum	14	1	4	5
Width	13 (28)	1 (154)	14 (22)	3 (121)

Probable Causes

Developmental researchers are attempting to identify similarities in the thinking patterns of children with a goal of characterizing their predominant ways of thinking at different ages (Alibali, 1999). One of the goals of mathematics education is for students to become better mathematical thinkers (Greeno, 1997). The literature suggests several underlying causes for students' inability to effectively solve multi-step problems involving mathematical vocabulary and higher-order numerical operations. Some of these causes include: an emphasis on repetition and rules, inadequate language skills, a lack of prior knowledge of mathematics concepts, the amount of personal risk students will invest, and the students' decline in engagement with mathematics activities.

A potential cause for students' inability to solve these multi-step problems is an overemphasis on repetition and rules. "...[E]xtensive experience with traditional arithmetic

word problems induces in pupils a strong tendency to approach word problems in a mindless, superficial, routine-based way in their attempts to identify *the* correct arithmetic operation needed to solve the word problem” (Vershaffel, De Corte, & Vierstraete, 1999, p. 265). An emphasis on accuracy, speed, repetition and rules may have usurped the understanding of concepts (Doyle as cited in Henningsen & Stein, 1997; Neiss, 1998). Some teachers may misuse repetition by assigning too many problems or too many of the same kinds of problems for homework. This could lead to increased student frustration and math anxiety (Cornell, 1999). Although accuracy, speed, repetition, and rules can be used appropriately, children must be challenged to utilize concept-based thinking as well. Schools may expect students to accumulate basic skills and may not necessarily emphasize applying these skills to solve complex problems (Greeno, 1997).

Mathematics problems have their own unique language. An incomplete understanding of the concepts behind new mathematical language can make learning the new language difficult for some students. Students can have difficulty with the new language if they have not had the experiences described by the language or if unique words are not explained (Cornell, 1999; Steele, 1999). This lack of experience may contribute to student explanations that “reveal either a total lack of understanding or perceptions that contradict accepted mathematical practice” (Campbell, 1997, p. 108). Some students may also lack the pre-requisite language skills necessary to identify the cue-words provided in a problem or to discern relevant from irrelevant information (Jitendra & Kameenui, 1996; Lawson & Chinnappan, 2000).

A lack of knowledge that is strongly connected and relevant to the problem may also hinder students’ ability to solve multi-step word problems (Lawson & Chinnappan, 2000). Students may have difficulty completing tasks that are not aligned with their own prior

knowledge (Henningsen & Stein, 1997). Sometimes, mathematics instruction is presented in isolation and students are unable to draw connections between the instruction and the application in the real world. When this happens, students may store knowledge of mathematical operations and later be unable to connect this stored knowledge to a need (Cornell, 1999). Some mathematics teachers may assume too much about what their students do or should know. "...[M]ath teachers acted as if computational procedures and processes were simple and self-explanatory; even worse, students said, their teachers had little sympathy for students who did not understand the concepts" (Cornell, 1999, p. 226). Thus, some teachers may not realize or accept the need to teach or re-teach prerequisite skills that are fundamental to solving more difficult problems. If instruction does not build upon students' existing ideas, students may not be able to construct a progressively more advanced understanding of mathematics (Campbell, 1997).

"Students' learning is seen as the process of acquiring a 'mathematical disposition' or a 'mathematical point of view'" (Schoenfeld, 1992, 1994 as cited in Henningsen & Stein, 1997, p. 525). If instruction is not connected to students' prior-knowledge, students may experience fear and failure in mathematics (Cornell, 1999). Fear and failure can manifest themselves in a lack of personal risk executed by the students. The tasks students are asked to complete can influence their thinking, also, by structuring their views of the material (Henningsen & Stein, 1997). Low self-efficacy or lack of persistence to find a solution may cause students to not access all information needed to solve problems (Lawson & Chinnappan, 2000). More ambiguity is involved when students engage in higher-level reasoning and understanding of mathematics. Therefore, they are executing more personal risk than they would use while completing more routine activities (Henningsen & Stein, 1997).

Mathematics teachers may try to engage their students in activities that are more complex or cognitively demanding. Aside from the students' lack of prior knowledge and personal risk, these activities may not be successful because they do not align with students' interests and motivations, there may not be accountability for high-level products or process, and there may not be suitably specific task expectations (Henningsen & Stein, 1997). "Tasks that are set up to engage students in cognitively demanding activities often evolve into less demanding forms of cognitive activity" (Doyle 1983, 1986, 1988 as cited in Henningsen & Stein, 1997, p. 525). These high-level tasks can take more time in class to complete than regular classroom activities (Henningsen & Stein, 1997). Student frustration can be the result of trying to keep up with the pace of the class when completing these tasks. Because math courses usually depend on students building their knowledge on previously learned material, the necessity to keep up is critical. Once a student fails behind, catching up can be very difficult (Cornell, 1999).

Probable Causes Site Based

Site A

The students at Site A come from a variety of ethnic and socioeconomic backgrounds. Some of these students are learning English as a second language. This may play a role in students' lack of understanding of mathematical language. Site A is located in an area where many of the students have lived in a variety of different school districts. A lack of alignment in curricula from district to district may contribute to some students' lack of prerequisite skills.

Site B

The students at Site B may exhibit an inability to effectively solve multi-step problems involving mathematical language or higher-order numerical operations for several reasons. As cited in the literature, these students may have inadequate language skills, a lack of prior

knowledge, low levels of personal risk, and a lack of interest in more cognitively demanding mathematics activities. Language skills may be a cause for the students' low functioning in mathematics since most of the students at Site B are bilingual and speak primarily Spanish at home or may be learning English as a second language. These students may also have difficulty in mathematics activities because of their low socio-economic status, mobility between school districts, lack of parent involvement in school activities, poor attendance, a low value of education, and participation in illegal or potentially dangerous activities (i.e., gangs, drugs, alcohol, violence). There are many reasons for students' lack of mathematical skills operating at both Site A and B. Fortunately, there are solutions to these problems.

CHAPTER 3

THE SOLUTION STRATEGY

Literature Review

The following topics for discussion have been found to be solutions for students who lack mathematical thinking skills: constructivist-style instruction, conceptual thinking, increase interest, hands-on and real world experiences, access pre-existing knowledge, improve vocabulary, cooperative learning, and self-monitoring. The development of mathematical thinking skills will assist students' success throughout life. The classroom setting affords the perfect opportunity to learn and transfer these skills.

Constructivist-style Instruction

Using a constructivist-style of instruction, including ongoing assessment, working with peers, and a teacher as facilitator, can enhance student problem-solving skills. Ongoing assessment is critical to the constructivist theory of learning. "Assessment is integrated in instruction and focuses on what students understand and can do instead of what they do not know or cannot do" (Manouchehri, 1998). This means that teachers must continually assess and implement assignments that will enable students to bridge what they know with the concepts that the teacher wants them to learn. "[T]eachers must know their students well in order to make intelligent choices regarding the motivational appeal, difficulty level, and degree of task explicitness needed to move students into the right cognitive and affective space so that high

level thinking can occur and progress can be made on the task” (Henningsen & Stein, 1997). Ongoing assessment also helps teachers identify needed remediation for students in an authentic setting. “Instruction should incorporate an element of diagnosis and remediation that detects and addresses student errors as they occur. This aids the students, while informing the instructor of unlearned concepts or operations that need re-teaching” (Cornell, 1999). Teachers should use continuous evaluation in order for students to address and correct errors while still in the process of learning the concept (Cornell, 1999). The instruction is given when the students need it for an application in progress, not after they have failed.

Working with peers can enhance learning in a constructivist classroom. Students can internalize the constructivist theory of learning, and thereby expand their own learning. More concretely, “children who construct explanations, which clarify processes to help classmates arrive at their own solutions, learn more than children who simply tell classmates answers” (Fuchs, Fuchs, Hamlett, & Karns, 1998). Students can also use scaffolding when working with peers. “Scaffolding occurs when a student cannot work through a task on his or her own and a teacher or more capable peer provides assistance that enables the student to complete the task alone, but that does not reduce the overall complexity or cognitive demands of the task” (Henningsen & Stein, 1997). Students also learn through observations of others. “Observing other students solving a problem may help learners internalize either the cognitive functions they are attempting to master or those that are within their zone of proximal development” (Mevarech, 1999). These observations need to be coordinated by the teacher so the observed student is not functioning at a level that is much higher than the level of the observing student. If the level of function observed is too high, the student observer may not be able to internalize the process.

The role of the teacher in a constructivist classroom is different than in the traditional classroom. “The teacher acts as facilitator of learning instead of imparter of information, asking questions, probing student understanding, and encouraging active learning. Students are introduced to computational procedures as they need them” (Manouchehri, 1998). The teacher challenges the students to access their own knowledge and apply it in varied situations. Teachers must be careful not to make the tasks too simple for students. They must press their students to think about concepts and figure out how to apply them. “When teachers create a high press for conceptual thinking, mathematics drives not only the activities but the students’ explanations as well. As a result, student achievement in problem solving and conceptual understanding increases” (Kazemi, 1998). When students are challenged at an appropriate level, and explain their thinking, they understand better.

Conceptual Thinking

Increasing conceptual thinking in the classroom can enhance student problem-solving skills. One way to increase conceptual thinking is to teach students how to solve the same problems using arithmetic methods as well as algebraic methods. According to the Division of Mathematics of People’s Education Press (as cited in Cai, 1998):

The objectives of teaching students to solve both arithmetically and algebraically are: (1) to help students attain an in-depth understanding of quantitative relationships by representing them both arithmetically and algebraically, (2) to guide students to discover the similarities and differences between arithmetic and algebraic approaches, and (3) to develop students’ thinking skills and flexibility of using appropriate approaches to solve problems. (p. 227)

This use of algebra in mathematics classrooms should be introduced early on and incorporated into mathematics lessons throughout student educational careers. This will help students connect mathematical thinking with real life experiences. “The mathematics curriculum, even in elementary and middle schools, should include explorations of algebraic ideas and processes so that students can use algebraic thinking to solve a variety of real-world problems” (Cai, 1998). Learning algebra at an early age can help students organize their mathematical knowledge so they can more readily apply what they know (Prawat as cited in Jitendra & Kameenui, 1996). Students that can organize their knowledge logically around a focal set of ideas are better able to solve mathematical problems.

Increase Interest

Using a variety of techniques to hold student interest is another way to improve student problem solving skills. In order to hold student interest, the teacher must consider teaching techniques, appropriate amount of time given for tasks, student confidence, and ability level of the student. “Textbook exercises, workbooks, and worksheets are rarely stimulating. They typically focus upon calculations in isolation and in the absence of meaningful context, and they can contribute to students’ perceptions of math as being irrelevant to their lives” (Cornell, 1999). These types of exercises can be de-emphasized and replaced with other activities. Manouchehri suggests that mathematical work should be meaningful to students and have a purpose. Students should play an active role in deciding what to do and how to do it (1998). According to Hildebrandt (1998):

Children enjoy the challenge of inventing strategies and procedures that they can use to solve problems that are meaningful to them. This challenge can be achieved in the context of a game or in the context of other projects involving mathematics. (p. 193)

“Incorporating projects (large or small) provides meaning and purpose, adds interest, and helps develop intrinsic motivation and discipline” (Cornell, 1999). Teachers can incorporate groupwork to hold student interest as well. “[C]ollaborative groupwork on academic tasks can facilitate children’s learning...” (Fuchs, Fuchs, Hamlett, & Karns, 1998). Teachers should be careful to not get carried away with making math fun and straying from the content that needs to be taught. The teacher’s goal should be to help students learn lessons in an engaging format. “The key idea is that students learn to think by participating in activities where they actively engage in thinking” (Greeno, 1997).

Teachers must also carefully consider the amount of time given for students to complete tasks. If students are given too little time, they can become rushed or overwhelmed. If they are given too much time, they can become bored and stray off-task. “[I]t appears as though planning for appropriate amounts of time and being flexible with timing decisions as the task implementation phase unfolds are extremely important in order to avoid declines of all types” (Henningsen & Stein, 1997). Teachers must watch their students as they work on activities and adjust time allotments accordingly.

Students must be comfortable in the learning environment in order to increase learning. “Personal ridicule, direct or implied, has no place in math instruction” (Cornell, 1999). Teachers must “foster an atmosphere of caring and mutual respect” (Hildebrandt, 1998). Students need to be encouraged to see themselves as good problem solvers. “[S]uccessful problem solvers tend to view themselves a[s] good problem solvers...” (Wedman, Wedman, & Folger, 1999).

The difficulty of a task contributes to how long it will hold student interest. If a task is too difficult for students, they may become frustrated. If the task is too easy, they may become bored. “[T]he complexity of any problem should match, and even stretch a bit, the abilities of

children...” (Waite-Stupiansky & Stupiansky, 1998). Tasks must also be related to how society expects children to utilize mathematics. “The challenge is to listen to children’s thinking and to consider both the children’s existing understanding and society’s mathematical expectations” (Campbell, 1997). Teachers must use techniques in the classroom that bridge what students know and how they think with what children are expected to know and be able to apply.

Hands-on and Real World Experiences

Another way to promote better problem solving skills in students is to reduce rote memory exercises and introduce problems in a hands-on format or using real world experiences. Cornell states that dependence upon rote memory is often a student initiated compensation for a lack of true understanding of material and that rote memorization exercises should be de-emphasized (1999). Rote memorization exercises should be replaced with exercises that help students form meaningful connections to previous knowledge. One way to do this is to explain information to others. “...[I]f new information is to be retained and meaningfully related to previously acquired knowledge, the learner must elaborate or generate connections between that information and representations in memory” (Fuchs, Fuchs, Hamlett, & Karns, 1998). In addition to students explaining concepts to others, “instruction must include discussion of the real-world applications of the operations, calculations, and processes being taught” (Cornell, 1999). This helps students understand how to apply what has been learned. When students are given a real world example to connect with a problem solving strategy, they can more readily access that problem solving strategy in another real world example. In addition to giving students a real life example to apply knowledge to, new instruction should also be related to previously learned information. “Instruction must build on children’s existing ideas, so that the children will construct progressively more advanced understanding and simultaneously perceive

mathematics as ‘making sense’” (Campbell, 1997). When each mathematical concept is taught individually, rather than in a progressive manner, concepts seem disjointed to students and they have difficulty making sense of them. “In order to develop students’ capacities to ‘do mathematics’ classrooms must become environments in which students are able to engage in rich, worthwhile mathematical activity” (Henningsen & Stein, 1997).

Access Pre-existing Knowledge

Most teachers realize the importance of accessing students’ previous knowledge and linking new knowledge to old knowledge. However, most teachers find this challenging. Teachers must develop instruction that continually spirals. Students should be able to simultaneously access previous knowledge and link new knowledge in order to see how math makes sense. Teachers should not proceed with mathematics instruction until they have considered the students’ current understandings and how that links to the new material (Campbell, 1997). If teachers access this previous knowledge, the “students’ intuitive understandings can be used as resources for helping them build their understanding, rather than as impediments that need to be overcome” (Greeno, 1997).

If a student is not a successful problem solver, usually this is because he or she fails to access his or her previous knowledge and apply it to the new problem. However, that student can usually access this knowledge if prompted to do so. Successful problem solvers are able to access and utilize their previous knowledge without prompting (Lawson & Chinnappan, 2000). If a student experiences a significant delay in being able to answer a problem, the student may not have created a strong relationships within the previously learned problem-specific knowledge and problem solving knowledge (Anderson as cited in Lawson & Chinnappan, 2000). Teachers should train students to be able to access their prior knowledge. “...[W]hen teachers helped

students build on their thinking, student achievement in problem solving and conceptual understanding increased” (Kazemi, 1998).

Improve Vocabulary

Mathematics, as well as other core subjects, has a very specific vocabulary. Often times, mathematical vocabulary is assumed and not explained. If teachers would explain the vocabulary, future confusion and misunderstanding by the students could be avoided (Cornell, 1999). Mathematics vocabulary often plays a key role in students being able to understand problems. Students should be able to use and understand vocabulary in order to think about and discuss mathematical situations. Research supports “the importance of mathematical communication to build students’ capacity for mathematical thinking and reasoning (Stein, Grover, & Henningsen, 1996).” Vygotsky reports that, “as children talk, they internalize the meanings of words that they say” (as cited in Steele, 1999).

“Language and meaning develop together only when new vocabulary is presented in a meaningful context...” (Steele, 1999). If students engage in mathematical conversations, with themselves or others, this may help them analyze problems more in-depth and develop a deeper understanding of the problems (Wedman, Wedman, & Folger, 1999). “...[S]ocial interaction and communication are essential components in this conceptualization process” (Vygotsky as cited in Steele, 1999). Communication about math problems helps students create links between their informal and intuitive understandings of math to the more abstract language and symbols of mathematics (National Council of Teachers of Mathematics as cited in Steele, 1999). Proper use of mathematical vocabulary in communication also “plays a key role in helping children make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas” (NCTM as cited in Steele, 1999).

Cooperative Learning

Cooperative learning puts children in a situation where they are working with others to find a solution to a problem or accomplish a task. “Children need active, physical experiences with tools or objects as they are exposed to the concepts for which the language will be used” (Straker, 1993). Individual students can achieve more when working with others who can help them organize their thinking and move into new areas of understanding (Vygotsky as cited in Steele, 1999). “As students interact cooperatively, they explain strategies and mathematical ideas in their own words, thus helping one another to process complex cognitive activity” (Schoenfeld, 1985). Even just observing others in a cooperative situation can help students internalize the cognitive or conceptual knowledge being demonstrated (Vygotsky, 1978). “Without engaging in such active process[es] during classroom instruction, students cannot be expected to develop the capacity to think, reason, and problem solve in mathematically appropriate and powerful ways” (Henningsen & Stein, 1997).

When students are placed in groups, they will need to be taught conflict resolution skills. Students will learn more in the group if they are able to solve problems within the group and without teacher intervention (Fuchs, Fuchs, Hamlett, & Karns, 1998). Teachers should create a classroom environment that lays fertile ground for problem solving. Students will have a more valuable educational experience if they have plenty of access to problems that need to be solved (Waite-Stupiansky & Stupiansky, 1998).

Teachers should create cooperative groups deliberately. Groups can be heterogeneous or homogeneous. Both types of grouping have advantages and disadvantages. When heterogeneous groups are created, low-ability students seem to benefit the most, as long as they are actively engaged in the collaborative activity (Fuchs, Fuchs, Hamlett, & Karns, 1998). High

achievers can also benefit from heterogeneous groups. In the same heterogeneous group, high achievers would be challenged to construct explanations and deepen their understanding of the material (Webb as cited in Fuchs, Fuchs, Hamlett, & Karns, 1998). However, high achievers may also benefit from homogeneous groups. According to Phelps and Damon, “collaboration in homogeneous pairs was a more effective learning environment for reasoning than for rote learning or copying tasks” (as cited in Fuchs, Fuchs, Hamlett, & Karns, 1998). “In homogeneous or narrowly ranging ability groups, medium-ability students not only are more active but also learn more” (Webb as cited in Fuchs, Fuchs, Hamlett, & Karns, 1998). Cooperative groups engage students in mathematical thinking and conversation that cannot be duplicated by individual activities.

Self-monitoring

One way to help students view themselves as good problem solvers is to teach them to monitor themselves. “Self-monitoring can increase students’ feelings of competence and control, and in turn, their motivation to remain engaged with a task at a high level” (Henningesen & Stein, 1997). Self-monitoring techniques such as think-aloud, explaining processes to other students, and reflection increase students’ problem solving skills and application of the concept (Wedman, Wedman, & Folger, 1999; Fuchs, Fuchs, Hamlett, & Karns, 1998). When students reflect on their mathematical problem solving, they should do so orally and in writing. Students should ask themselves how and why questions so they analyze what was done and why it was done (Manouchehri, 1998). According to Henningesen and Stein, “self-monitoring and regulation of students’ own thought processes, are a hallmark of high-level thinking” (1997). Teachers can support students high-level thinking processes and self-monitoring by teaching

these skills explicitly. Teachers may model the strategies themselves or have other students demonstrate (Anderson, 1989).

Teachers need to facilitate students' thinking. In order to help students go from concrete thinking to abstract thinking, teachers should start with the students' abstract ideas. This will help the students see the advantages of abstract thinking in mathematics (Cai, 1998).

“[C]lassroom instruction time should be allocated to display and discussion of the schemas that students develop for topics within their mathematics programs” (Lawson & Chinnappan, 2000). Teachers should expect higher-order thinking and self-monitoring from their students. When teachers focus on the thinking skills, “student achievement in problem solving and conceptual understanding increases” (Kazemi, 1998). Teachers need to give students time to solve problems. According to Waite-Stupiansky and Stupiansky, good problems should challenge students thinking and could take several days to solve. Students should have time to collect information, brainstorm, argue their results, and choose an answer (1998). “All children must be expected to contribute and to explain” (Campbell, 1997).

Students can use “elaborative interrogation,” Socratic questioning, self-talk, or inductive thinking to monitor their progress and thinking. When students are taught to monitor their thinking, they are more successful (Mastropieri, Scruggs, & Butcher, 1997; Perkins & Grotzer, 1997). Students should also use self-monitoring to make sure they are focusing on relevant information. They also need to make the new information relevant to themselves and relate it to their previous knowledge (James as cited in Greeno, 1997). Self-monitoring can help students to not make irrelevant or inefficient relationships between their previous and new knowledge (Thorndike as cited in Greeno, 1997). Students should regularly confront problems so that they have a variety of opportunities to explore, discuss, and reflect on various ways of solving

problems (Verschaffel, De Corte, & Vierstraete, 1999). Students need to be encouraged to utilize self-monitoring. Teachers should remind students to use self-monitoring throughout their problem or task (Henningesen & Stein, 1997). Any or all of these solutions can be taught to students to improve their mathematical thinking skills.

Project Objective and Process

As a result of teaching students a mathematical problem-solving strategy and vocabulary during the period of September 2000 to January 2001, the sixth grade regular education and ninth grade special education students from the targeted classes will increase their mathematical problem solving and vocabulary skills. This will be measured by cue word, vocabulary, and problem solving assessments. In order to increase mathematical problem solving and vocabulary skills, the following processes are necessary:

1. A problem-solving strategy will be developed by the researchers and utilized by students.
2. A list of key vocabulary terms and definitions will be developed by the researchers and taught to the students through direct instruction, student glossary of vocabulary, and quizzes.

Project Action Plan

This action plan describes a program for improving mathematical thinking skills and vocabulary. The targeted populations include students in a sixth grade regular education math class and a ninth grade special education math class in a middle and high school, respectively.

Table 6

Action Plan

DATE	ACTION
August 28 - September 1, 2000	Send out parent letter and collect permission forms Standard Curriculum with story problem/thinking skill enrichment
September 5 - September 8	Standard Curriculum with story problem/thinking skill enrichment Cue-Word, Vocabulary, and Story Problem assessments (for baseline)
September 11 – September 22	Standard Curriculum with story problem/thinking skill enrichment Vocabulary – begin A (glossary)
September 25 - September 29	Worksheet A, ODDE system, Error log
October 2 – October 6	Vocabulary – review A, begin B (glossary) Standard Curriculum with story problem/thinking skill enrichment
October 10 – November 3	Vocabulary – review A & B, begin C (glossary) Standard Curriculum with story problem/thinking skill enrichment
November 6 – November 10	Worksheet B, ODDE system, Error log
November 13 – December 8	Vocabulary – review A & B, begin C (glossary) Standard Curriculum with story problem/thinking skill enrichment
December 11 – December 15	Worksheet C, ODDE system, Error log
January 8 – January 12, 2001	Standard Curriculum with story problem/thinking skill enrichment Cue-Word, Vocabulary, and Story Problem assessments (for growth)

Methods of Assessment

In order to assess the effects of the intervention, students will retake the cue word, vocabulary, and problem solving assessments. The results will be tallied and compared to the

previous assessments. The researchers will monitor development of students' thinking skills using problem-solving worksheets throughout the intervention.

CHAPTER 4

PROJECT RESULTS

Historical Description of the Intervention

The objective of this project was to increase the mathematical problem solving and vocabulary skills of the sixth grade regular education and ninth grade special education students from the targeted classes. The implementation of a problem-solving strategy, the direct instruction of key vocabulary terms and definitions, and the use of student glossaries were selected to effect the desired changes.

The action plan called for the implementation of the ODDE (Own words, Draw, Do work, Explain) problem-solving system three times throughout the research period. The ODDE system includes completing four phases: a 10-question story problem worksheet (three were used during the intervention), ODDE prep worksheet, ODDE worksheet, and ODDE partner-check worksheet. These worksheets can be found in Appendices E, F, G, H, I, and J, respectively. Students used the ODDE worksheets as a metacognitive tool to examine how they solved one self-chosen word problem from each of the three story problem worksheets. The researchers provided direct instruction for the first implementation of the ODDE system. Subsequent ODDE systems were completed by the students with facilitation from the researchers.

The following steps were used for each implementation of the ODDE system. First, students independently completed a 10-question story problem worksheet. After the worksheets

were completed, students and researchers modeled the solutions. Next, each student completed an ODDE prep worksheet for one problem of his or her choice. The prep worksheet included questions and a graphic organizer to guide students in thinking about the problem and its solution. The researchers reviewed the prep worksheets. Upon approval, each student used the prep worksheet as a guide to complete an ODDE worksheet. The ODDE worksheet is a graphic organizer of the four ODDE parts. Students were required to write the problem in their own words, draw a picture or diagram that helped to solve the problem, show their mathematical work, and write a paragraph explaining and justifying their problem-solving process. After each student completed an ODDE worksheet, he or she traded ODDE worksheets with a partner. Each student independently completed an ODDE partner-check worksheet to evaluate his or her partner's ODDE worksheet. After completing the partner-check worksheets, the pair of students then worked together to improve and correct their ODDE worksheets. Finally, the students' corrected ODDE worksheets were reviewed by the researchers.

In addition to the ODDE system, the action plan called for the direct instruction of mathematical vocabulary terms included in Appendices B and C. Students were required to build a glossary of these terms as they were introduced. The researchers reviewed previously learned terms with the students each time new terms were introduced. The students' knowledge of these terms was monitored through a variety of classroom activities.

Presentation and Analysis of Results

In order to assess the effects of the ODDE system on student ability to identify the correct operation in a story problem and the cue word(s) for that operation, the researchers re-administered the cue word assessment given prior to the intervention. The data from this assessment is presented in Table 7.

Table 7

Cue Word Post-Assessment

Problem Number (Number of words)	Site A			Site B		
	Operation identified correctly	Word most frequently chosen	Number of words chosen	Operation identified correctly	Word most frequently chosen	Number of words chosen
1 (17)	14	more	2	8	5 more, many	11
2 (27)	13	leftover	5	4	25 glasses	16
3 (14)	14	gained	3	8	gained 10	11
4 (18)	13	less	3	8	less	13
5 (18)	14	each	4	7	much	14
6 (18)	13	took	4	7	took away	11
7 (21)	9	per	3	5	38, how many, need	14
8 (28)	12	(first) each	7	4	(second) each	21
9 (18)	14	fewer	4	7	how many	15

The intervention appears to have had a positive effect on correct operation identification for students at Site A. There is not much improvement in this area for the students at Site B. However, students at both sites appear to have improved their ability to choose words pertinent to the problem and reduced the number of words chosen as cue words.

In order to assess the effects of the ODDE system on student ability to correctly solve story problems, the researchers re-administered the story problem assessment given prior to the intervention. The data from this assessment is presented in Table 8. At both sites, the intervention appears to have had a positive effect on the students' ability to solve story problems. Students at both sites decreased the number of errors due to logic and the number of problems on

which they did not show work. Their abilities to use units on their answers as well as utilizing the correct unit improved. However, students at both sites increased their computational errors.

Table 8

Story Problem

Item		Site A (182)	Site B (143)
Correct answer	Number correct	154	42
	No answer	0	9
Incorrect answer	Logic error	8	54
	Computational error	16	26
	No work shown	4	12
Unit	Correct	156	62
	Incorrect	14	40
	Missing	12	32

In order to assess the effects of the direct instruction of mathematical vocabulary terms and student glossary building, the researchers re-administered the vocabulary assessments given prior to the intervention. The data from these assessments are presented in Tables 9, 10, and 11.

Table 9

Vocabulary I – Side 1

Word	Site A		Site B	
	Correctly chosen (14)	Incorrectly chosen (140)	Correctly chosen (11)	Incorrectly chosen (110)
Addend	14	0	10	9
Difference	14	0	3	5
Dividend	12	2	4	8
Divisor	12	2	5	8
Factors	14	0	4	4
Operations	14	0	9	3
Perimeter	14	0	7	1
Product	14	0	3	5
Quotient	14	0	3	5
Remainder	14	0	10	2
Sum	14	0	4	7

Table 10

Vocabulary I – Side 2

Word (Possible: N)	Site A		Site B	
	Correctly chosen (Possible: N)	Incorrectly chosen (Possible: N)	Correctly chosen (Possible: N)	Incorrectly chosen (Possible: N)
Addend (2)	28 (28)	0 (154)	12 (22)	3 (121)
Difference (0)	0 (0)	0 (0)	0 (0)	5 (0)
Dividend (2)	24 (28)	4 (154)	6 (22)	16 (121)
Divisor (2)	24 (28)	4 (154)	5 (22)	15 (121)
Factors (2)	28 (28)	0 (154)	2 (22)	2 (121)
Operations (0)	0 (0)	0 (0)	0 (0)	4 (0)
Perimeter (0)	0 (0)	0 (0)	0 (0)	4 (0)
Product (1)	14 (14)	0 (168)	2 (11)	6 (132)
Quotient (2)	28 (28)	0 (154)	13 (22)	7 (121)
Remainder (2)	14 (28)	0 (154)	13 (22)	2 (121)
Sum (1)	14 (14)	0 (168)	7 (11)	2 (132)

The first side of the Vocabulary I assessment shows that the intervention appears to have had a positive effect on the students' abilities to complete literal sentences involving the vocabulary terms. The students at both sites increased the number of correct answers on these items. The results from data from the second side of the Vocabulary I assessment reveals that the intervention appears to have had an overall positive effect on the abilities of the students of Site A while having an uneven effect on the abilities of the students of Site B to identify the parts of mathematical problem statements. The students of Site B showed improvement on four items, maintenance on one item, and regression on four items.

The Vocabulary II assessment required students to demonstrate their ability to apply their understanding of the mathematical vocabulary terms. The intervention appears to have had a positive effect on the number of correctly chosen items from the students at both sites. The number of incorrectly items chosen decreased at Site A while, for most items, they increased at Site B.

Table 11

Vocabulary II

Word	Site A		Site B	
	Correctly chosen (14)	Incorrectly chosen (168)	Correctly chosen (11)	Incorrectly chosen (132)
Addends	10	4	4	7
Difference	14	0	6	5
Dividend	12	2	5	7
Divisor	12	2	6	8
Factors	10	2	3	8
Length	14 (28)	0 (154)	10 (22)	2 (121)
Operation	14	0	3	7
Perimeter	14	0	6	5
Product	12	4	2	8
Quotient	10	3	4	6
Remainder	14	0	8	4
Sum	14	1	5	6
Width	14 (28)	0 (154)	8 (22)	1 (121)

Conclusions and Recommendations

Based on the presentation and analysis of the data on mathematical thinking skills and vocabulary understanding, the students appear to show an overall improvement. The sixth grade regular education students of Site A showed consistent improvements on assessment items. They appear to have improved their overall mathematical thinking skills and have increased their use of the mathematical vocabulary terms in class discussions. The ninth grade special education students of Site B answered more assessment items after the intervention than before the intervention. This increase in the total number of items answered may have contributed to the increase in incorrect answers. If the students had completed as many items before the intervention as after, the assessments may have shown growth more similar to that of the students of Site A.

The researchers endorse the use of the ODDE system as an intervention to improve students' mathematical thinking skills. Teachers must be willing to dedicate approximately one

week of class time to direct instruction and modeling of the ODDE system. After students have mastered the process of completing the ODDE system, teachers should be prepared to allot two to three class periods (44-55 minutes each) for students to complete the system. The ODDE system provides a forum for students to practice their metacognitive, communicative, and social skills by simultaneously involving students in mathematical and language-based activities.

Both researchers also endorse the use of a student-built glossary for vocabulary improvement. Students should include definitions and examples for the vocabulary terms in their glossaries. The researcher at Site A also used a word wall and student-made posters of vocabulary terms to enrich the vocabulary development. Both researchers encourage teachers to use a variety of vocabulary-building activities as this appears to positively effect vocabulary growth.

Future research should be conducted on students' abilities to identify cue words in story problems and how this affects their abilities to solve those problems. At the same time, the researcher(s) should investigate how cue word identification is related to the individual student's mathematical vocabulary knowledge base. Future research should also focus more specifically on an increase in vocabulary instruction and how this affects students' computation and application skills.

References

- Alibali, M. W. (1999). How children change their minds: Strategy change can be gradual or abrupt. Developmental Psychology, *35* (1), 127-145.
- Anderson, L. M. (1989). Classroom Instruction. In M.C. Reynolds (Ed.), Knowledge base for the beginning teacher (pp. 101-115). Washington, DC: American Association of Colleges for Teacher Education.
- Cai, J. (1998, December). Developing algebraic reasoning in the elementary grades. Teaching Children Mathematics, *5* (4), 225-229.
- Campbell, P. F. (1997, October). Connecting instructional practice to student thinking. Teaching Children Mathematics, *4* (2), 106-110.
- Cornell, C. (1999, Summer). I hate math! I couldn't learn it, and I can't teach it! Childhood Education, *75* (4), 225-230.
- Fuchs, L. S., Fuchs, D., Hamlett, C. L., & Karns, K. (1998). High-achieving students' interactions and performance on complex mathematical tasks as a function of homogenous and heterogeneous pairings. American Educational Research Journal, *35* (2), 227-267.
- Greeno, J. G. (1997, November). Theories and practice of thinking and learning to think. American Journal of Education, *106* (1), 85-126.
- Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. Journal for Research in Mathematical Education, *28* (5), 524-549.
- Hildebrandt, C. (1998, November). Developing mathematical understanding through invented games. Teaching Children Mathematics, *5* (3), 191-195.
- Jitendra, A. K., & Kameenui, E. J. (1996). Experts' and novices' error patterns in solving part-whole mathematical word problems. The Journal of Educational Research, *90* (1), 42-51.
- Jitendra, A. K., Griffin, C. G., McGoey, K., Gardill, M. C., Bhat, P., & Riley, T. (1998). Effects of mathematical word problem solving by students at risk or with mild disabilities. The Journal of Educational Research, *91* (6), 345-355.
- Kazemi, E. (1998, March). Discourse that promotes conceptual understanding. Teaching Children Mathematics, *4* (7), 410-414.
- Lawson, M. J., & Chinnappan, M. (2000). Knowledge connectedness in geometry problem solving. Journal for Research in Mathematics Education, *31* (1), 26-43.

- Manouchehri, A. (1998, September-October). Mathematics curriculum reform and teachers: What are the dilemmas? Journal of Teacher Education, 49 (4), 276-286.
- Mastropieri, M. A., Scruggs, T. E., & Butcher, K. (1997). How effective is inquiry learning for students with mild disabilities? The Journal of Special Education, 31 (2), 199-211.
- Mevarech, Z. R. (1999). Effects of metacognitive training embedded in cooperative settings on mathematical problem solving. The Journal of Educational Research, 92 (4), 195-205.
- Niess, M. L. (1998, November). Using computer spreadsheets to solve equations. Learning & Leading with Technology, 26 (3), 22-27.
- Perkins, D. N., & Grotzer, T. A. (1997). Teaching intelligence. American Psychologist, 52(10), 1125-1133.
- Schoenfeld, A. H. (1985). Mathematical problem solving. New York: Academic Press.
- Steele, D. (1999, September). Learning mathematical language in the zone of proximal development. Teaching Children Mathematics, 6 (1), 38-42.
- Stein, M. K., Grover, B. W., & Henningsen, M. (1996) Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33, 455-488.
- Straker, A. (1993). Talking points in mathematics. Cambridge, MA: Cambridge University Press.
- Verschaffel, L., De Corte, E., & Vierstraete, H. (1999). Upper elementary school pupils' difficulties in modeling and solving nonstandard additive word problems involving ordinal numbers. Journal for Research in Mathematics Education, 30 (3), 265-285.
- Vygotsky, L. S. (1978). Mind in society. Cambridge, MA: Harvard University Press.
- Waite-Stupiansky, S., & Stupiansky, N. G. (1998, August). Math in action: Create a climate for problem solving. Instructor, 108 (1), 54.
- Wedman, J. F., Wedman, J. M., & Folger, T. (1999). Thought processes in analogical problem solving: A preliminary inquiry. Journal of Research and Development in Education, 32 (3), 160-167.

APPENDICES

Appendix A

Math Cue Word Assessment

MATH CUE WORDS

Directions: Identify the **operation** being used in the following problems. Circle the cue word(s) and write the operation on the line. *Do not solve the problem.*

1. Karen has 5 more apples than John. If John has 12 apples, how many does Karen have?

2. Jose has 25 glasses to put into boxes. If a box holds 6 glasses, how many boxes can he fill? How many glasses will he have leftover?

3. Maria weighs 113 pounds. How much would she weigh if she gained 10 pounds?

4. Shaun has \$7 less than David does. If David has \$52, how much money does Shaun have?

5. Dashalla bought 20 shirts for her soccer team. If each shirt cost \$16, how much did she spend?

6. Andre has 15 PlayStation games. If his mom took away 7 of them, how many does he have?

7. Alicia is making 16 bracelets for a fundraiser. If she needs 38 beads per bracelet, how many beads does she need?

8. Elizabeth has 210 stickers. Her sticker book has 30 pages. If she puts the same number of stickers on each page, how many stickers are on each page?

9. Denise has 3 fewer brothers than Carlos. If Carlos has 5 brothers, how many brothers does Denise have?

Appendix B
Math Vocabulary Assessment I

MATH VOCABULARY WORKSHEET I

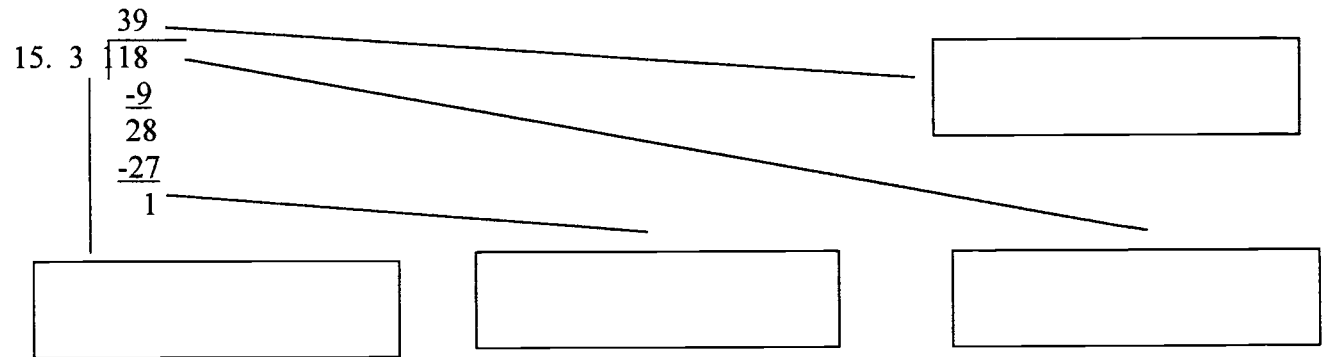
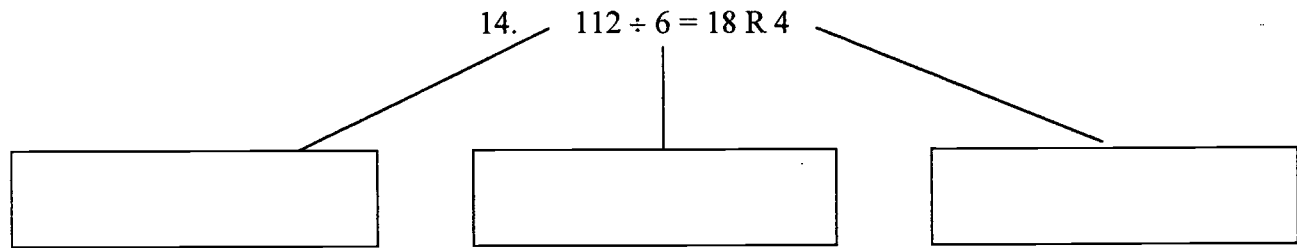
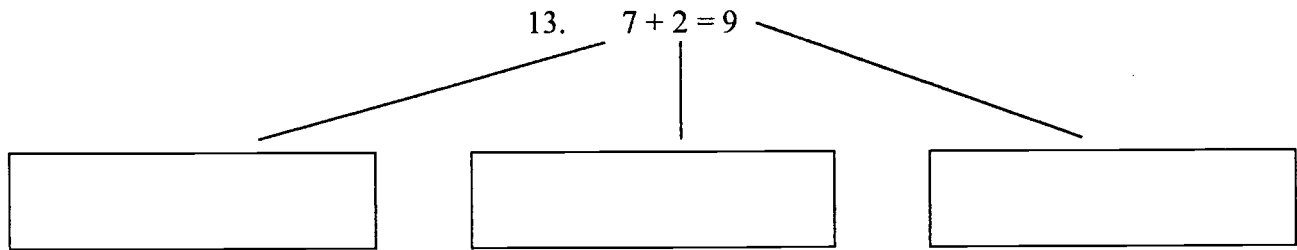
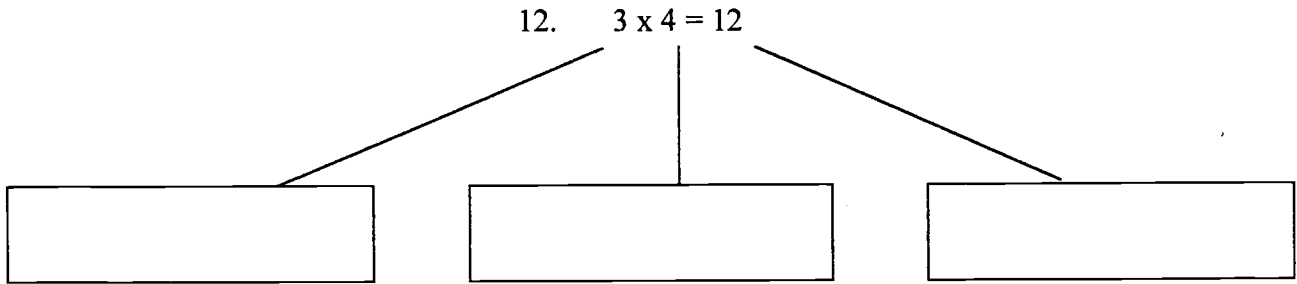
Directions: Fill in the blanks using the words from the word bank.

1. The answer to a subtraction problem is the _____.
2. The answer to an addition problem is the _____.
3. The numbers being multiplied are called _____.
4. The number you are dividing by is called the _____.
5. The numbers being added are the _____.
6. The number being divided is the _____.
7. The answer to a multiplication problem is the _____.
8. The answer to a division problem is the _____.
9. The _____ is the length of all sides of a rectangle added together.
10. The amount leftover in a division problem is the _____.
11. Add, subtract, multiply, and divide are mathematical _____.

<< Turn Page Over >>

Appendix B continued

Directions: Fill in the boxes with the words from the word bank to identify the parts of the problems.



Appendix B continued

MATH VOCABULARY WORKSHEET I – WORD BANK

addend

factors

quotient

difference

operations

remainder

dividend

perimeter

sum

divisor

product

addends

Appendix C
Math Vocabulary Assessment II

MATH VOCABULARY WORKSHEET II

Directions: Fill in the blanks using the words from the word bank.

10. The _____ between nine and eight is one.
11. The _____ of four and three is seven.
12. The _____ of three are three and one.
13. In the problem $24 \div 4 = 6$, the four is the _____.
14. Eight and five are _____ of thirteen.
15. In the problem $5 \overline{)60}$, the sixty is the _____.
16. The _____ of two and six is twelve.
17. The _____ of twelve and three is four.
18. The _____ of a rectangle is 18m if two sides are 6m each and two sides are 3m each.
19. In the problem $5 \overline{)59}$, the four is the _____.
- $$\begin{array}{r} 11 \\ -5 \\ \hline 09 \\ -5 \\ \hline 4 \end{array}$$
20. The _____ in the number sentence $3 \times 6 = 24$ is multiplication.
21. The sides of a rectangle are called the _____ and the _____.

Appendix C continued

MATH VOCABULARY WORKSHEET II – WORD BANK

addends	divisor	operation	quotient
width	difference	factors	perimeter
remainder	dividend	length	product
sum			

Appendix D
Story Problem Assessment

Story Problems

Name _____

Directions: Read each problem carefully.
Solve each problem.
Show all of your work.
Write your answer(s) in the box(es).

There are 10 boxes of jumbo shrimp. Each box contains 10 bags. Each bag contains 10 shrimp. How many jumbo shrimp are there?

The total number of coins in two jars is 42. The difference in the number of coins in the jars is 6. How many coins are in each jar?

Last year, 23 busloads and 152 carloads of tourists visited a local park. If each bus held 34 people and each car held 5 people, what is the total number of tourists?

In the Philippines, coconuts are lashed together and transported by raft. If a raft contains 400 coconuts, how many rafts are needed to transport 16,000 coconuts?

Carlos wants to use the entire length of a 120-foot roll of wire fencing to make a rectangular enclosure for his dog. What will the length and width of the enclosure be if the width is 20 feet less than the length?

Appendix D continued

Page 2

Name _____

Directions: Read each problem carefully.
Solve each problem.
Show all of your work.
Write your answer(s) in the box(es).

There are 135 cars waiting to cross the Columbia River on a ferry. If the ferry holds 16 cars, how many trips will it have to make to get all of the cars across?

Sandy bicycles 13 miles to school. After school, she bicycles 5 miles to work and then another 12 miles home from work. If she does this every weekday, how many miles does she bicycle each week?

Tony bakes 59 cookies and puts them into bags. Each bag holds 8 cookies. How many bags does he fill? How many cookies are left?

Tickets for the school play are on sale now for \$5 each. On the night of the play, tickets will cost \$6 each. How much would a family save by purchasing 4 tickets now?

In September, Maria bought a camera for \$84. She made four equal payments to pay for the camera. How much was each payment?

Appendix E
Story Problem Worksheet A

Story Problems Worksheet A

Name _____

Directions: Read each problem carefully.

Solve each problem.

Show all of your work.

Write your answer(s) in the box(es).

If each student in a class of 34 sells 2 candles for \$5 each, how much money will be collected?

There are 36 cats and dogs at the kennel. If there are 4 more dogs than cats, how many animals of each type are at the kennel?

Samantha runs 3 miles a day during the weekdays and 4 miles a day on Saturdays and Sundays. How far does she run in a week?

Christiane had 48 cuttings from her plants that she was going to give to 8 of her friends. If she gave the same number to each, how many cuttings did each receive?

Maria wants to use the entire length of a 28 inch long ribbon to edge a pillow. If the pillow will be 2 inches longer than it is wide, what is the length and width of the pillow?

Appendix E continued

Worksheet A Page 2

Name _____

Directions: Read each problem carefully.
Solve each problem.
Show all of your work.
Write your answer(s) in the box(es).

Kevin is a waiter. He must pay taxes every three months on his tips. This year, he expects to owe a total of \$300 in taxes. How much will each of his payments be?

Alveta is making a poster for a school competition. She chose 17 colors of pastels. If each pastel crayon costs \$1 and the tax is \$2, what is the total cost of the crayons including tax?

Shizuko is giving a party. She expects 45 people to come. She has ordered 15 large pizzas, which are cut into 8 slices each. How many slices of pizza can each person have if all are to get the same number? How many pieces will there be left over?

Rod has a 685 page book. If he reads 85 pages each night, how many pages will he have left to read after six nights?

Wanda is buying a radio for \$42. If she makes six equal payments, how much is each payment?

Appendix F
Story Problem Worksheet B

Story Problems Worksheet B

Name _____

Directions: Read each problem carefully.
Solve each problem.
Show all of your work.
Write your answer(s) in the box(es).

Each of 12 children has 2 bags of candy. Each bag of candy contains 16 pieces. How many pieces of candy are there in all?

There are Oreo cookies and chocolate chip cookies in the cookie jar. There are 27 cookies in the cookie jar. If there are 8 more chocolate chip cookies than Oreo cookies, how many of each cookie are there?

Coach Murphy told each player to bring 12 golf balls and 6 tees to the first practice. What is the total number of golf balls and tees if there are 8 golfers?

Margaret brought her dogs to work one day. Altogether 38 feet walked into the building. How many dogs does Margaret have?

Tom wants to use the entire length of a 34-foot roll of border fencing to make a rectangular flower garden. If the width is 3 feet less than the length, what is the length and width of the flower garden?

Appendix F continued

Worksheet B Page 2

Name _____

Directions: Read each problem carefully.
Solve each problem.
Show all of your work.
Write your answer(s) in the box(es).

A company must ship 1,432 packages. Each truck can hold 175 packages. How many trucks will be needed to transport the packages?

A squad of 16 cheerleaders needs new uniforms. Uniforms at All-Pro cost \$33. Uniforms at Sports-Plus cost \$36. How much money will the cheerleaders save in all if they buy from All-Pro?

If \$56,324 was donated to South High to buy computers. A computer costs \$958. How many computers can be purchased for the school? How much money will be left over?

Latisha bought 5 shirts for \$12 each. Sonya bought 8 shirts for \$11 each. How much more did Sonya spend?

Juan borrowed \$1,200 from his father to pay for a new engine for his car. If he plans to pay his father back in one year, how much should each equal monthly payment be?

Appendix G
Story Problem Worksheet C

Story Problem Worksheet C

Name _____

Directions: Read each problem carefully.
Solve each problem.
Show all of your work.
Write your answer(s) in the box(es).

There are 40 school buses owned by the school district. If each bus holds 52 students, how many students can all the buses hold?

There are a total of 983 students enrolled in school. If there are 271 more girls than boys, how many girls and boys are there?

A store received 26 boxes of medium shirts and 40 boxes of large shirts for shipment. If there are 20 medium shirts and 12 large shirts in each box, respectively, how many shirts are there total?

Shannon is packaging cookies. If each tin holds 24 cookies, how many tins will she need to package 1,008 cookies?

Shaun has 54 inches of wood trim to use for his project. If he makes a box, what will the length and width of the box be if he wants it to be 3 inches longer than it is wide?

Appendix G continued

Story Problem Worksheet C - Page 2

Directions: Read each problem carefully.

Solve each problem.

Show all of your work.

Write your answer(s) in the box(es).

A train car carries 9 cars at a time. If 106 cars need to be transported, how many train cars are needed?

Yoshi works 3 hours each day after school. She works 6 hours each day on Saturday and Sunday. How many total hours does she work in two weeks?

Tabitha bought 6 packages of invitations to her birthday party. Each package contains 8 invitations. If she invites 38 people, how many packages would she open and how many cards would be left?

Tickets for a concert are \$22 in advance. The tickets are \$45 at the door the night of the concert. How much will four friends save if they buy the tickets in advance?

Sheletha wants to buy a new computer. If the computer costs \$1399, how much will she need to save each month for the next year to be able to pay for the computer?

Appendix H
ODDE Prep Worksheet

ODDE PREP WORKSHEET

Remember ODDE: Own words, Draw, Do work, Explain

Directions: Read each section carefully. Answer each question in the space provided.

Own words

1. What is the problem asking you to find?
2. What are the important elements of the problem? (for example: cue words, vocabulary, numbers)

Draw

1. What type of illustration (for example: drawing, table, graph, chart) would be helpful in solving this problem?
2. What are the important elements of this type of illustration (for example: labels, titles, legend/key)?
3. Sketch your illustration in the space below.

Appendix H continued

ODDE PREP WORKSHEET – PAGE 2

Directions: This table will help you complete the Do work and Explain portions of your ODDE Worksheet.

- The Do work column may only contain one operation per box.
- For each Do work step, you must Explain WHAT you did and WHY you did it in the spaces provided.
- Fill in the boxes across the chart for each step as you go. In other words, each box for your first step must be completed before you move on to your second step.
- If you used fewer than five steps to solve your problem, you will not need to complete the entire chart.
 - If you used more than five steps, you will need to use an additional chart to finish.

Do work	Explain	
	WHAT	WHY
STEPS		
[1]	[1]	[1]
[2]	[2]	[2]
[3]	[3]	[3]
[4]	[4]	[4]
[5]	[5]	[5]

SOLUTION: _____

Go back and check your work and explanation. Add appropriate vocabulary words where possible.

Appendix H continued

D o work	E xplain	
STEPS	WHAT	WHY

Appendix I
ODDE Worksheet

ODDE WORKSHEET

Remember ODDE: Own words, Draw, Do work, Explain

O wn words	
D raw	D o work <div data-bbox="1062 1247 1435 1365" style="border: 1px solid black; padding: 5px; width: fit-content; margin-left: auto;">Answer:</div>
E xplain	

If you need more space, continue writing on a separate sheet of paper.

Appendix J
ODDE Partner-check Worksheet

ODDE PARTNER-CHECK WORKSHEET
Remember ODDE: Own words, Draw, Do work, Explain

Directions: Answer these questions using your partner's ODDE Worksheet.

- You may NOT talk while you complete the Partner-Check Worksheet.
- When you and your partner are both finished with the Partner-Check Worksheets, discuss your answers and how to improve your ODDE Worksheets.

Own words

1. Is the question clearly restated?
2. Are any of the important elements (for example: cue-words, vocabulary, numbers) missing?
3. Is any unnecessary information given?

Draw

1. Is the illustration (for example: drawing, table, graph, chart) appropriate for the problem? If NOT, what illustration would be more appropriate?
2. Are all the important elements (for example: labels, titles, legend/key) included in the illustration? If NOT, what is missing?
3. Are all the important elements clearly drawn and labeled? If NOT, what would you improve?

Appendix J continued

ODDE PARTNER-CHECK WORKSHEET – PAGE 2

Remember ODDE: Own words, Draw, Do work, Explain

Do work

1. Are there any computational errors? If YES, what is/are the error/s?
2. Are any steps missing or in the wrong order? If YES, what would you add or change?
3. Is the answer correct and labeled?

Explain

1. Are ALL sentences complete (including: subject, verb, capitals, and punctuation)?
2. Is EACH step explained with a WHAT and WHY? If NOT, what would you change or improve?
3. Are the steps written in the correct order? If NOT, what would you change or improve?
4. Are ALL vocabulary words used appropriately? If NOT, what would you change or improve?

When you and your partner are both finished with the Partner-Check Worksheets, discuss your answers and how to improve your ODDE Worksheets.



U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)



REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: <i>Improving Students' Mathematical Thinking Skills Through Improved Use of Mathematics Vocabulary and Numerical Operations</i>	
Author(s): <i>Liming, Lori Ann; Schoenberger, Kathleen M.</i>	
Corporate Source: <i>Saint Xavier University</i>	Publication Date: <i>ASAP</i>

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education (RIE)*, are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

The sample sticker shown below will be affixed to all Level 1 documents

The sample sticker shown below will be affixed to all Level 2A documents

The sample sticker shown below will be affixed to all Level 2B documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE, AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY, HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2A

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY

Sample

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

2B

Level 1

↑

Level 2A

↑

Level 2B

↑

Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g., electronic) and paper copy.

Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only

Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits.
If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Sign here →
please

Signature: <i>Kathleen M. Schoenberger</i>	Printed Name/Position/Title: <i>Student/s FBMP</i>	
Organization/Address: <i>Saint Xavier University E. Mosak 3700 W. 103rd St. Chgo, IL 60655</i>	Telephone: <i>708-802-6214</i>	FAX: <i>708-802-6208</i>
	E-Mail Address: <i>mosak@sxu.edu</i>	Date:



(over)

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:	ERIC/REC 2805 E. Tenth Street Smith Research Center, 150 Indiana University Bloomington, IN 47408
---	--