This study describes a program designed to alter instructional and assessment processes in mathematics in order to increase comprehension levels and engagement in mathematics. The targeted population consists of high school students in a community located in the western suburbs of a large metropolitan area. The socio-economic status of the community in which the school is located is middle to upper class. Evidence for the existence of the problem includes teacher observations, assessment scores, and student journal entries. Possible causes for the inadequate depth of involvement in mathematical activities and understanding of mathematical concepts can be divided into two categories. The student category includes attitudes and behaviors that contribute to the lack of engagement and understanding in mathematics. The instructional category includes limited teaching strategies, assessment techniques, and awareness of the importance of transfer. A review of the solution strategies suggested by professional literature, combined with an analysis of the setting of the problem, resulted in the selection of an interrelated three-part approach. The three-part approach incorporates cooperative learning, a portfolio, and math journals to improve engagement in mathematical activities and understanding of mathematical concepts. Post intervention data indicated an increase in the depth of involvement and engagement in mathematics as well as an increase in the understanding of mathematical concepts. These improvements in student achievement are reflected in students' grades and in student journal entries. (Contains 32 references.) (Author/ASK)
INCREASING THE ENGAGEMENT AND UNDERSTANDING OF CONCEPTS IN MATHEMATICS

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An Action Research Project Submitted to the Graduate Faculty of the School of Education in Partial Fulfillment of the Requirements for the Degree of Master of Arts in Teaching and Leadership

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Abstract

This study describes a program designed to alter instructional and assessment processes in mathematics in order to increase comprehension levels and engagement in mathematics. The targeted population consists of high school students in a community located in the western suburbs of a large metropolitan area. The socio-economic status of the community in which the school is located is middle to upper class. Evidence for the existence of the problem includes teacher observations, and assessment scores, and student journal entries.

Possible causes for the inadequate depth of involvement in mathematical activities and understanding of mathematical concepts can be divided into two categories. The student category includes attitudes and behaviors that contribute to the lack of engagement and understanding in mathematics. The instructional category includes limited teaching strategies, assessment techniques, and awareness of importance of transfer.

A review of the solution strategies suggested by professional literature, combined with an analysis of the setting of the problem, resulted in the selection of an interrelated three-part approach. The three-part approach incorporates cooperative learning, a portfolio, and math journals to improve engagement in mathematical activities and understanding of mathematical concepts.

Post intervention data indicated an increase in the depth of involvement and engagement in mathematics as well as an increase in the understanding of concepts in mathematics. These improvements in student achievement are reflected in students' grades and in student journal entries.
DEDICATION

To my family, the most important people in my life who I can always count on, I want to dedicate this research. This work is also dedicated to those who educate and take the time to know their students inside and outside of the classroom, those who value relationships.
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CHAPTER 1
PROBLEM STATEMENT AND CONTEXT

General Statement of the Problem

Students in the 10th, 11th, and 12th grades at the school being researched display an inadequate depth of involvement in mathematics and understanding of mathematical concepts. Evidence of the existence of the problem includes teacher observations, assessment scores, and student journal entries.

Immediate Problem Context

The school is one of two high schools in a unit district that has an enrollment of 1,960 students in grades 9 through 12. Based on the 1999 school report card, the teacher to pupil ratio is 20.3:1. The attendance rate is 93%. The dropout rate is 1.6%, lower than the district rate of 1.7%. The mobility rate is 21.2%, higher than the district rate of 13%. The ethnic make-up reported for the school is 83.7% White, 5.8% Asian, 5.4% Black, 4.9% Hispanic, and 0.2% Native American. There are 4.6% non-English speaking students who can participate in bilingual and English as a Second Language (ESL) programs. Low-income families make up 12.0% of the school population that is considerably higher than the district average of 8.3% (School Report Card, 1999).

The school is located at the most northern part of the community and draws students from the community as well as a portion of a neighboring town. Single family dwellings surround the school, which allows many students to walk to school. The 37-
year-old building is air conditioned and newly expanded due to continual increase in enrollment. Included in the expansion is a new science wing, complete with laboratory equipment and computers, two new computer labs with 30 computers each, an entirely new gymnasium with a dance facility, training room, fitness center, and expanded office space for each department. The school also has a courtyard, allowing students to use it during passing periods and at lunch. The library is modern with common computer search engines and training programs for students to learn how to use them.

The school day is seven classes each 50-minute with 5 minute passing periods between classes. Total graduation requirements include 21 hours of academic credits. Courses are available in remedial, intermediate, honors, and advanced placement (AP) levels.

General math and applied math are the remedial level courses offered in the math department. The algebra curriculum is offered in two different ways. There is a one-year algebra curriculum and a two-year algebra curriculum, which are both considered intermediate level. Basic geometry and geometry are the next courses offered, which are also both intermediate level courses. There are opportunities for advanced and intermediate level classes in geometry, intermediate math, and math analysis. After math analysis, there is an option for seniors to take math seminar or an AP calculus course.

Students are required to take 2 years of math, 2 years of science, 4 years of English, 4 years of physical education, 2 years of social sciences including U.S. government, and one course in consumer education. One course in health must be completed, which counts toward a semester of physical education. Other credits
available are from courses in the following departments: fine and performing arts, business and applied technology, and foreign language. One study hall per semester is an option that students may use to study, receive help from one of the departmental study labs, or become a teaching assistant. Another available option for students is Quest 2000 that allows students to have internships with members of the community. There are over 50 extracurricular activities available for students ranging from clubs to athletic teams.

The 197 faculty and staff members include classroom teachers, administrative and departmental secretarial staff, a guidance and social work staff of seven, two certified nurses, a police liaison, three deans, two assistant principals, and one principal. The faculty and staff consist of 113 females and 84 males. The support services for special education include six classroom teachers, five aides, two self-contained classroom teachers and three inclusion facilitators. The average teacher salary is $40,372 while the administrator average salary is $85,107. The average teacher experience is 17.9 years. The average number of teachers holding a bachelor's degree is 41.7% and 58.3% of the teachers have a master's degree or higher. The spending per pupil correlates with the district amount of $3,921. This is below the state average of $3,990 (School Report Card, 1999).

The school district encompasses 2 high schools, 4 middle schools, and 13 grade schools. The superintendent was recently hired last spring, 1999. The superintendent salary is $130,000 with a bonus of a housing allowance in the community (Local Newspaper, 1999).
The ethnic profile for the district is 85.9% White, 5.0% Asian, 4.7% Black, 4.4% Hispanic, and 0.1% Native American. The rate of students considered low-income is 8.3%. The attendance rate is 95.8%. The dropout rate is 1.7% with a mobility rate of 13%. The district to pupil ratio is 20.3:1. The per pupil expenditure is $3,921. The average teacher salary is $50,254 and the average administrator salary is $73,357. The average teacher experience is 14.0 years with 58.3% of the teachers holding a master’s degree or higher (School Report Card, 1999). Approximately 50% of the faculty and staff reside in the surrounding community.

The Surrounding Community

The community in which the school is located is a western suburb of a major midwestern city settled in 1839. It covers an area of 12 square miles and has a population of 54,442. The median home value is $196,867 and the average family income is $73,672. The average age of a member in the community is 35.6 with an average of 15.5 years of formal education. The community has 6.2% of its members listed as low-income, with an average household income below $15,000. The low-income households are distinctly placed in the northern section of the community, which can be seen by comparing the two high schools. The school in the southern part of the community has only 3.5% of its students from low-income households, while the northern school has 12% of its students from low-income households. The ethnic breakdown of the community follows the same pattern with 91.5% White, 3.8% Asian, 2.7% Black, and 2.0% Hispanic. The majority of the minority populations (85%) reside on the north side of town (Census Report, 1990).
The employment rate for the community is 70.3%, with 28.0% not in the workforce, leaving 1.7% of the community listed as unemployed. Of the employed, 61% have managerial, professional or administrative positions. The remaining 39% hold positions in sales, factories, crafts, agriculture, transportation, service occupations, or as technicians. There is a large community center, library, and courthouse in the town. There are 40 places of worship and 14 public childcare facilities (Census Report, 1990). These support services are available to students, yet they lack motivation to use the services.

**National Context of the Problem**

A problem identified by educators is American students are not achieving in mathematics. This is apparent when the results of the Sixth Mathematics Assessment of the National Assessment of Educational Progress, NAEP, are analyzed. The overall levels of student performance at grades 4, 8, and 12 are considerably lower than what math educators would hope to find (Kenney & Silver, 1997). The portrayal of mathematics teaching and learning is not the reality in a majority of classrooms, schools, and school districts (NCTM, 2000).

The evidence supports that the students are not learning the mathematics that they need to learn or what they are expected to learn (Kenney & Silver, 1997). The reasons for the deficiency include the following: students have not had the opportunity to learn important topics, curriculum did not engage them, quality of the mathematics instruction is highly variable and lack of commitment to learning (NCTM, 2000).

In high school mathematics, educators are concerned with the lack of intrinsic motivation displayed by their students. Intrinsic motivation is the desire to engage in an
activity for its own sake because of the satisfaction and enjoyment it provides (Kohn, 1999). Math students who are intrinsically motivated focus on learning goals such as understanding and mastery of mathematical concepts (Ames & Archer, 1988). Other qualities of intrinsically motivated students are spending more time on a given task, persisting in the face of failure, selecting more challenging tasks, utilizing more efficient learning and performance strategies, applying creativity, and taking risks (Lepper, 1988).

Math students tend to give up when concepts and tasks become more challenging. Instead of trying different strategies and working through different possibilities, students do not put forth the effort necessary. There is the perception in the United States that mathematics is a special domain in which smart students succeed and other students get by or fail. Successes and failures in math are attributed to ability and the effort put forth by the student rarely affects the progress and success he or she experiences in math (Kloosterman & Gorman, 1990).
CHAPTER 2

PROBLEM DOCUMENTATION

Problem Evidence

In order to document the lack of necessary involvement in mathematics and the shallow depth of understanding of concepts, teacher observations, assessment scores reflected as semester grades, and student journal entries were noted.

The teacher observations included the use of math lab, nonmathematical talk during cooperative learning activities, and student responses in class. Math lab is a resource for students who need extra help or tutoring during the school day, in particular for use during the students' study hall. It is offered to students almost every class period. Of the 30 students researched, one student attended math lab regularly 3 - 5 days a week. Students declined to use this resource and continued to come to class unprepared.

Nonmathematical talk during cooperative learning activities was also recorded. After 3 different days of observation, the students needed direction back on task on average 3 - 4 times in 40 minutes while the teacher was circulating. This may seem minimal, but over the course of a day with five different classes, a portion of the time is lost that could be spent on mathematics.
Student responses during class were also documented from 3 different days. Almost daily the class begins with warm-ups that review the material from the day before and preview the new material for the day. And after the introduction and teaching of new material, the students typically have 5 - 10 minutes of class time to begin homework. These are the times when students say the following: “I don’t want to do this,” “I don’t get this,” and “This is too hard.” More time is lost in mathematics because the students fail to look in their book or back over their notes to find methods to solve problems on their homework.

The course prior to first semester, Algebra II/Trigonometry, is second semester Geometry. Second semester Geometry includes topics such as similarity, right triangles, circles, planar measurement, and space measurements. The teaching and applications of these concepts are algebra based. Summaries of the second semester Geometry grades for the 30 students researched are presented in Table 1. Semester grades are determined by 80% of the semester work and by 20% of the final exam.

Table 1

<table>
<thead>
<tr>
<th>Grades</th>
<th>Number of students</th>
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<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
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<tr>
<td>D</td>
<td>9</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
</tr>
</tbody>
</table>
Eleven of the 30 students, approximately 37%, were not achieving in their math class. The reasons for such a high number of students not achieving in mathematics may be related to teacher attitudes and instructional strategies and student attitudes and understanding of concepts.

A content analysis of student journals indicated that only one third of a total number of 30 students liked math and could see the relevance of life applications. See student examples in Appendix A. Among the remaining 20 students, 80% indicated difficulties due to the students’ perceptions of poor teaching strategies, especially during their middle school years. Such negative strategies include math competitions and tracking of low-level students. Additionally, students indicated that some teachers made negative comments directly to them that included, “When will you learn?” and “You are dumb.” See student examples in Appendix B. These are the student attitudes and feelings that are carried over into high school mathematics.

The 30 students were also able to identify their difficulties and weaknesses they have in mathematics. A content analysis of student journals regarding weaknesses indicated that 30% of students identified test taking; feelings of nervousness and not having enough time to finish seemed to exacerbate feelings of anxiety. Forty percent of students identified understanding the mathematics as their weakness. Students indicated they had difficulties solving problems, memorizing formulas and properties, solving word problems that are multi-step, visualizing figures, and understanding proofs in mathematics.

The remaining 30% of the students identified weaknesses that are related to their attitudes and perceptions. These students mentioned they were never and will never be
good math students. Others mentioned they do not have any interest in mathematics. Achievement in mathematics is nearly impossible for these students because they do not understand mathematics taught and perceive themselves as never improving. These high percentages of students fail to see and relate the relevance of mathematics.

Probable Causes

The literature suggests several underlying causes for an inadequate depth of involvement in mathematical activities and understanding of mathematical concepts. Students' attitudes and beliefs regarding the mathematics and their abilities to understand the material inhibit their achievement level. Also, the traditional teaching method of mathematics creates an environment that does not allow students to communicate mathematically and fails to provide the necessary connections to student lives. The teaching strategies commonly used are not effective for students and their engagement in mathematics.

Poor Student Attitudes and Beliefs

Students' perceptions about their mathematical abilities directly relate to their depth of involvement and understanding of concepts. According to Kloosterman and Gorman (1990), students perceive math as a special domain where smart students succeed and other students get by or fail. Students believe that success is due to high math ability and failure is due to low math ability and effort rarely results in a significant change in their success patterns (Kloosterman & Gorman, 1990). This thought process is prevalent among most students. Many students will say, “I was never good at math” or “I am not a math person.” This perception reduces the amount of time and effort
students will apply towards their mathematics class. When students do not spend the time on their homework or other class work, then there is a decrease in their grade.

When students begin to think that they are poor math students then they develop negative attitudes and beliefs about their abilities. This is where the cycle of low achievement begins. Students do not understand new mathematical material the first time they see it and then they think they will never understand it. Therefore some students develop the “I know I can’t do it so there’s no point in trying” syndrome (Cornell, 1999). This syndrome translates into no time being spent on the homework or in preparation for tests and quizzes, which is then reflected in the student’s grade. Middleton and Spanias (1999), suggest that the decline in positive attitudes of students is due to a lack of teacher supportiveness. This lack of teacher supportiveness can occur in the elementary, middle, and high school level.

**Poor Teaching Methods and Lack of a Variety of Instructional Strategies**

Students start to develop their attitudes and beliefs about mathematics in their elementary school years. At this level, students already begin to perceive themselves as good math students or as bad math students. In the traditional method of teaching math, teachers tend to emphasize correct answers, high grades, and skipping problems that are too difficult (Stipek, Givvin, & Salmon, 1998a). Instead of crediting students for the effort to try a problem, teachers tend to mark the problem wrong with a red pen. This traditional teaching strategy is enforcing the concept that effort is not a factor and that the reality is there are high ability math student or low ability math students. This teaching method also enforces the thought that effort has an insignificant effect on the learning and understanding of mathematics.
This traditional method also creates an environment of fear for some students. Because teachers tend to want only the correct answers, students develop a fear of asking questions because they do not want to appear ignorant (Stipek, Givvin, & Salmon, 1998b). Students at any level are concerned about their peers and their thoughts. If students have to make a choice regarding asking a question and appearing ignorant, then most students would not ask a question and have a gap in their understanding. Then students also develop a fear of trying a problem that is more complex because they do not understand and they do not want to do the problem wrong.

Students are not interested in math because there is a lack of a variety in teaching strategies (Panasuk & Greenleaf, 1998). In mathematics, the daily routine tends to be the same. The class opens with a review of the material from the day before, then the correcting of the homework from the previous day, and finally the new lesson. During the lesson, teachers typically show how to solve a certain problem and then the class mimics the pattern for solving other similar problems. According to Alper, Gendel, Fraser, and Resek (1996), in a traditional classroom, teachers give students specific procedures for a particular sample problem and then expect them to use the same procedure on future problems. When students follow specific procedures to solve problems they are not learning and understanding the concepts involved. Students entering the work force will not have a set of blueprints to always solve a problem. They must learn problem-solving skills along with analytical skills to help transfer and heighten their knowledge of the subject material.
Traditional methods of teaching math fail to show transfer of material to life and relevance to the student. Transfer of learning occurs when students learn a concept in the classroom and apply it in a significantly different context (Perkins & Swartz, 1992). In some traditional classrooms, students are taught material so they can perform well on quizzes and various tests. This type of instruction does not transfer learning or relevance for the student. Students frequently ask the question, “When are we ever going to have use this?” in math class. Students, because of the way in which the material is presented to them, ask this question so frequently. Learning cannot take place if students fail to see the relevance of mathematics to their world (Lan, 1998). When there is a failure to see the relevance of a subject to an individual’s world, there cannot be any depth of involvement or understanding of the key concepts.

Issues in the Math Curriculum

The biggest problems in the math curriculum are too many topics to be covered in the given school year and these topics are not covered in depth (Stigler & Hiebert, 1999). Therefore, the content that is covered tends to be less advanced and is presented in a more piecemeal and perceptive way (Stigler & Hiebert, 1999). During the school year, students do not have enough time to foster learning the content at a mastery level because they are always moving into another topic. The fragmented curriculum only makes this problem more evident because the students are not transferring the material from previous units or even the previous school year. An example of this is spending the first 6 weeks of the year reviewing Algebra 1 concepts in Algebra 2.
The breath of the content covered is very large however the depth of the content is very shallow. Students do not get to see the relationships between various units and concepts because the content is not covered in a lot of depth. Again the fragmented curriculum makes it difficult for the students to see the transfer of mathematics to their lives, especially when mathematics is taught in isolation. This type of instruction assumes students will learn mathematical operations, “store” them for future use, and then identify and recall the appropriate operations when needed (Cornell, 1999). In reality, many students are not successful at this method. And even students who are successful at this method are not demonstrating understanding or mastery of the concepts involved.

Finally, the standards of curriculum are beginning to align with having interdisciplinary activities. Transfer of content will be increased by these activities as well as making the students more engaged because they will be learning more about their world and how it relates to math and their other core subjects. The teacher's role will be one of a facilitator and not of a lecturer who will develop student responsibility and ownership in the transfer of learning (Bellanca & Fogarty, 1991). Interdisciplinary units will provide students with higher order thinking activities to challenge and create an environment that is encouraging for the mastery of skills in the classroom.
CHAPTER 3
THE SOLUTION STRATEGY

Literature Review

A variety of instructional methods and assessment practices need to be incorporated in the classroom in order to increase the involvement in mathematical activities and understanding of mathematical concepts. A review of the literature shows that the inadequate depth of involvement and understanding in math comes from negative student attitudes and perceptions concerning mathematical ability and the current instructional and assessment practices used by teachers. An outlook of the alignment on the curriculum standards according to the National Council of the Teachers of Mathematics (NCTM, 2000) will be addressed concerning the curriculum standards at the school.

A review of the literature has also revealed several solution strategies. Reform in mathematics education has called for an increased emphasis on meaningful experiences in mathematics and a decreased emphasis on the repeated practice of computational algorithms (NCTM, 1989). This reform supports a variety of teaching approaches away from traditional methods of instruction to a learning environment that supports conceptual understanding of mathematics. Classrooms must be environments in which students have frequent opportunities to engage in dynamic mathematical
activities that are grounded in rich, worthwhile mathematical tasks (NCTM, 1991). The mathematical curriculum, instructional materials, and instructional lessons should clearly show connections between the different topics in math, such as the interconnections of Algebra and Geometry. A coherent curriculum effectively organizes and integrates important mathematical ideas so students can see how the ideas build on and connect with other ideas which enables students to develop understanding and skills (NCTM, 2000). After considering the need of the targeted students the following categories of major interventions were selected: cooperative learning, portfolio assessment, and journal writing. Various strategies within each category will be used.

**Cooperative Learning and Classroom Climate**

Cooperative learning has become an integral part of most teacher education programs because of its continued success in the classroom. Cooperative learning leads students to feel more positive about themselves, about each other, and about the subject they are studying. Students also learn more effectively when they can learn with each other instead of against each other or apart from each other (Kohn, 1999). Cooperative learning differs from group work because students learn the course content by working in a collaborative setting, performing assigned roles, assisting others in learning, and sharing responsibility for a group task. Each member has a role and responsibilities that contributes to the group's success. “Working together to achieve a common goal produces higher achievement and greater productivity than does working alone” (Johnson & Johnson, 1992, p. 173).

Cooperative learning also allows for students to approach a task from differing points of view. Students will be engaged in mathematical dialogue in order to
communicate and listen to each other's ideas. By using cooperative learning in the classroom students are more active in their learning, both physically and mentally (Gordan, 1998). This strategy also empowers students. Students become responsible for their own learning in mathematics. They are communicating mathematically, sharing resources, and developing a deeper understanding of mathematics. According to Barb and Quinn (1997):

If students work collaboratively in sharing ideas, developing explanations, and making sense of the ideas of others, they can be convinced that problems can be solved in more than one way. Research has shown that classrooms in which discussion of multiple solutions are allowed have higher-achieving students. (p. 538)

**Portfolios and Alternative Assessments**

After creating productive classroom environments where students have used cooperative learning to investigate challenging and meaningful mathematical situations, the students' learning is then typically assessed through a standard in-class test. A standard in-class test will have students working independently on problems that may not be as challenging or demanding as the previous class work (Crowley, 1997). This type of assessment is not consistent with the instructional method.

A goal is to help students develop an understanding of concepts, develop mathematical thinking, and engage them in mathematical activities. The assessments used should reflect these visions. One type of assessment that includes the goal above and encompasses individual learning and cooperative learning is a portfolio. Portfolios blend instruction and assessment to reflect students' mastery of the material. Portfolios
allow students to work meaningfully with the concepts and content and heightens their ability to incorporate that material with prior knowledge (Gilman, Andrew, & Rafferty 1995). Portfolios evaluate both the product and the process used to synthesize the product. Portfolios encourage students to take charge of their own learning (Gilman, Andrew, & Rafferty 1995). Students make key decisions about what to include in their final portfolios. They review, revise and make evaluative judgments about their work (Robinson, 1998). Students develop their own criteria for quality work instead of relying on the teacher to tell them what is good work and poor work.

Portfolios are examples of students' work in a context that is relevant and understandable (Gilman, Andrew, & Rafferty 1995). The use of portfolios in mathematics has many positive attributes. It is a method of evaluation that allows students to demonstrate their strengths rather than their weaknesses. Through portfolio use in the mathematics classroom, students are actively engaged in the role of a mathematician (Lambdin & Walker, 1994). Students are encouraged to investigate the mathematics beyond just getting the correct answers as well as to communicate their understandings of the mathematics. Finally, portfolios promote self-assessment about students' confidence in mathematics (Lambdin & Walker, 1994).

The Assessment Standards for School Mathematics (NCTM, 1995) supports activities that involve students in evaluating their own papers. According to the Assessment Standards, student self-assessment can be used to improve students' confidence in their ability to solve mathematical problems and allow students to become more independent in their learning of mathematics. Student self-assessment is defined
as the process in which the student determines whether the type of error made in his or her work is procedural or conceptual in nature (Stallings & Tascione, 1996).

This practice of self-assessment is commonly used by teachers for mistakes made on tests or quizzes. Written corrections should consist of an “in words” description of the error, an “in words” description of the correct procedure that should have been used, and the corrected answer to the problem. This 3 step method allows students to distinguish types of errors made in terms of a failure to understand versus an arithmetic or simple algebra mistake made while solving the equation (Stallings & Tascione, 1996).

Journal Writing

Self-assessment practices and self-evaluation practices are often incorporated in student journal writing. Student self-evaluation is the student’s reflections about his or her general understanding of the mathematics explored up to a given point (Stallings & Tascione, 1996). It is important to teach students to think about their thinking, their metacognitive processes. Reflection on the student’s metacognitive processes such as planning and monitoring effectiveness of attempted actions and outcomes of action, as well as testing, revising, and evaluating learning strategies has a strong effect on student learning (Wang, Haertel, & Walberg, 1993/1994).

Effective learners recognize the importance of reflecting on their thinking and learning from their mistakes (NCTM, 2000). This process of reflection and reflection writing creates an active learner. Writing allows students to reflect on mathematical experiences, which leads to reflecting on their ideas and procedures critically (Powell,
Also according to Powell (1997):

By providing students with opportunities to work with mathematical ideas in their own language and on their own terms, writing helps them to develop confidence in their understanding of mathematics and become more thoroughly engaged with mathematics. (p. 22)

There is evidence that supports journal writing as part of mathematics instruction because journals provide cognitive benefits to students. Cognitive benefits to students include increased procedural knowledge, conceptual understanding, and mathematical communication (Jurdak & Zein, 1998). Journal writing not only provides cognitive benefits to students but also affective benefits to students. Journal writing is a strategy for learning that allows for student self-expression. Through student self-expression, a line of communication is established between teacher and student (Stallings & Tascione, 1996). The teacher and student relationship is strengthened and trust develops. When this occurs there is a sense of community and safety in the classroom.

Incorporating these new strategies will allow the teacher and student to apply the concepts of mathematics to practical uses. Transfer of the content area is lacking and the reasons for studying math are lost. Traditional math classes are set in rigor that does not allow the students the ability to see the relationships of what they are studying in math to how it relates to the other core subjects. Students need to be made explicitly aware of the mathematical connections at every level (NCTM, 2000). New standards in math are leading to a curriculum that is centered on learning objectives dealing with the transfer of information from K-12. By emphasizing mathematical connections, students will not only learn mathematics but they will also learn the utility of mathematics (NCTM,
Students will be able to see and understand mathematics not as a collection of separate ideas and procedures but as a domain that is relevant and meaningful to many aspects of life.

Student involvement will increase because of the new strategies that are implemented. Team and cooperative strategies will help to foster the effort by students because they will have a peer group to aid in supplemental instruction. Alternative assessments of the groups will be used to identify how the group is progressing with their knowledge and skills. Fostering these peer relationships will help make the students feel safer and less likely to withdraw from active participation. This should provide the students with varied learning strategies to help them transfer information from the classroom to the world.

The curriculum will foster positive relationships of the students and the teachers by designing units where they are working together for group projects.

Project Objectives and Processes

As a result of altered instructional and assessment processes during the period of September 2000 to January 2001, the 10th, 11th, and 12th grade students will increase their comprehension levels and engagement in mathematics as measured by teacher-made tests, review of a portfolio assessment, analysis of teacher observations, and analysis of student journals.

In order to accomplish the project objective, the following processes are necessary:

1. Cooperative learning activities for instructional units will be developed.
2. Student portfolios will be used to assess and document student learning.

3. Students will use math journals to demonstrate and reflect on their learning.

Project Action Plan

I. Cooperative Learning
   A. Establish and build base groups in the first 2 weeks of semester.
   B. Teach and build cooperative learning skills in the first 2 weeks of semester.
   C. Base groups will meet and be used for problem solving activities, problem-based activities and projects, and reviews for tests.

II. Portfolios
   A. Define and explain portfolios the fourth week of the semester, Chapter 3 of the text.
   B. Explain the expectations of the portfolio process in the fourth week of the semester.
   C. Portfolios will be located in the classroom in a visible and accessible place for students daily.
   D. Portfolios will be turned in for assessment at the end of Chapters 3 and 5.
   E. Base groups will meet the day before the due date to review each other’s portfolios.

III. Math Journals
   A. Students will write in journals every Thursday for 10 - 15 minutes, alternating affective and mathematical topics.
B. Journals will be collected every other Friday for teacher response to topics.

C. The students will set goals for the class during the first week, sixth week, and the twelfth week.

D. Self-assessment or self-reflective activities will be used in addition to the student math journals.

Methods of Assessment

In order to assess the effects of the intervention, teacher-made tests covering the mathematical concepts will be developed. In addition, a portfolio of student work will be developed and kept for the specified chapters. Student journals will also be evaluated for understanding of concepts and involvement in mathematical activities. Scoring rubrics will be developed for both portfolios and student journals.
CHAPTER 4

PROJECT RESULTS

Historical Description of the Intervention

The objective of this project was to increase involvement in mathematics and deepen understanding of mathematical concepts by students. The implementation of cooperative learning structures, alternative assessment, and journal writing were selected as strategies to effect the desired changes.

Cooperative learning was used as an instructional strategy when new material was introduced and as a method of building an effective classroom community. Base groups were established during the second week of the school year and were maintained throughout the intervention. Basic cooperative learning skills were taught while the students were in base groups. The skills chosen for base group work included: listening, explaining, encouraging, teaching, cooperating, and sharing. Base groups met weekly for a variety of tasks. Base groups meetings were used for classroom activities such as problem solving activities, problem-based activities, introduction of new material, homework sessions, and review sessions for tests and quizzes. See Appendix F, Appendix G, and Appendix H for cooperative learning structures that were implemented.
A portfolio was also implemented as an instructional strategy. The portfolio was used as an instructional strategy for chapter 3 of the student text. A portfolio was incorporated for chapter 3, a unit on solving systems of linear equations and inequalities, as a teaching and assessment method rather than a chapter test at the end of the unit. A portfolio was defined and explained in the sixth week of the intervention, in the middle of chapter 3. The following week, the expectations of the portfolio process were clarified. Students were given the expectations and the scoring rubric (see Appendix C). The portfolios were either kept by the student or were left in a central area of the classroom. Portfolios were due at the end of chapter 3. The day before the portfolios were due, base groups met and students reviewed each other portfolios and made suggestions. Original plans called for two portfolio assessments but there was only time enough for one to be planned and completed.

Journal writing was the third strategy used to increase involvement in mathematics and deepen understanding of mathematical concepts by students. Students were given scoring rubrics for their journal writing the first week of the intervention so they were aware of the different aspects I was looking for in their writing (see Appendix D). Students were given a topic and wrote in their journals every Thursday for 10 - 15 minutes. I selected the journal topic for the students to write on. The topics selected alternated weekly between affective and mathematical to provide balance for the students. Samples of student journal topics are located in Appendix E. Journals were collected in pairs, one affective and one mathematical, every other Friday for teacher response to the student journals.
Students wrote on a wide variety of topics including a math autobiography of themselves highlighting their elementary, middle, and high school learning experiences. Students also identified their strengths and weaknesses in math as well as set goals for their achievement. Individual goal setting was also a significant part of students' response journals. Short term and long term goals were set the fifth week of the intervention and were revisited in the last week of the semester. Self-assessment and self-reflective activities were also used in addition to the student's journals. Students evaluated themselves in three different aspects at the end of the intervention. They graded themselves based on time and effort spent on in and out of class activities, their level of participation, and their level of understanding of concepts learned. These strategies were also used for such activities as test or quiz corrections and as preparation for a test or quiz.

Presentation and Analysis of Results

In order to assess the effects of implementing cooperative learning, alternative assessments, and journal writing on increasing student engagement and understanding of mathematics, semester grades were evaluated at the end of the intervention. The post Algebra II/Trigonometry semester grades are presented as a comparison to the previous Geometry semester grades in Table 1.
Table 1

Pre and Post Semester Math Scores

<table>
<thead>
<tr>
<th>Grades</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
</tr>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
</tr>
<tr>
<td>D</td>
<td>9</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
</tr>
</tbody>
</table>

The intervention had a positive effect on student achievement and understanding. The number of achieving students earning an A, B, or C increased from 19 students to 26 students, a 36% increase. Also as a result of the intervention, students who did not achieve and earned a D or an F in the prior semester, 11, decreased to only 4 students who did not achieve this semester in mathematics. Providing students with alternative instructional and assessment strategies allows students to achieve in a variety of domains. Achievement in domains of the mathematics classroom such as daily work, participation, teacher-made tests, journal writing, and alternative assessments. Overall, 87% of the students achieved in mathematics this semester.

It is also important to assess the effects of alternative assessments on student understanding and achievement in mathematics. A portfolio was used in place of a teacher-made test for the unit on solving systems of linear equations and inequalities this semester. Students were given the expectations and scoring rubric for their
portfolios as well as class time to prepare and consult with peers. Student results from the scoring rubric for the portfolio are presented in Table 2.

Table 2

Alternative Assessment - Portfolio

<table>
<thead>
<tr>
<th>Grades</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
</tr>
</tbody>
</table>

The use of a portfolio also had a positive effect on student understanding of mathematical concepts. The number of achieving students earning an A, B, or C on their portfolio was 27 out of a total of 30 students. The portfolio was incorporated into the intervention and replaced a teacher-made test as an assessment of a unit. Ninety percent of students achieved on this type of assessment. This parallels the end of the intervention semester grades where 87% of students achieved.

The final component of the intervention was journal writing. The journal writing process was intense for all, the students and the teacher researcher. Many honest and open feelings were expressed on both parts, regarding mathematics and other affective topics. In order to assess the effects of alternative assessments such as journal writing on increasing student engagement and understanding of concepts in mathematics, a
content analysis of student journal entries regarding their perceptions of the semester are provided.

Students were asked to evaluate themselves in their journal writing for the semester, the length of the intervention, based on three main aspects: time and effort spent on the mathematics, level of participation during class time, and level of understanding of the concepts taught. See samples of student self-evaluations in Appendix I. Students discussed their feelings about the three aspects and graded themselves. Many students openly shared that they understood concepts taught but did not spend a lot of time doing homework or studying for tests or quizzes because they did not think they needed to. Others spent a great deal of time working on their homework and studying and felt they understood the material but when it came time to take the test, they blanked out. Yet, there were some students who felt all aspects were good except for their in class participation, they could have done more. There was a wide variety of responses to time and effort spent on the mathematics, level of participation during class time, and level of understanding of the concepts taught.

I found some of the students' reasoning for their self-evaluations to be insightful and significant. Students evaluated themselves in math this semester and attributed a grade of achievement to more than just their ability and scoring high on tests. Students began to attribute their achievement to the amount of in class participation, amount of effort on homework, amount of time studying, number of times they came in for extra help, and amount of questions they asked for understanding in class. Test scores and the number of problems correct on a homework assignment were not the focal point. Students wrote about trying every problem and not leaving any problems blank.
Students even referred to self-motivation and self-competition. Students wrote about how they participated more in their math class than any other class or they spent more time working on math this semester than any other class or any other math class they have had. The focus shifted from just getting a good grade or just getting by in math to different types of accomplishments in mathematics. Accomplishments that the students were able to recognize in themselves and be proud of.

Conclusions and Recommendations

Based on the presentation and analysis of the data on increasing the engagement and understanding of concepts in mathematics, students showed an increase in achievement as measured by semester grades and journal entries. I was concerned about what prevents high school students from learning mathematics. There are many factors involved.

Students indicated disliking math notably as they got older. Some even remembered specific teachers and incidences that had a negative impact on their perceptions of math. Students' perceptions of math are directly related to their learning and comprehension of concepts. The teacher is a critical component of the classroom, and his or her classroom behavior and attitude impacts student learning. This suggests that mathematics teachers might consider further research that addresses teacher reform.

In conclusion, the following insights were garnered:

1. Collaborative environments provided a comfortable atmosphere that was not intimidating to students, and did not cause anxiety due to singling out of
students. Students were willing to ask each other questions as opposed to just sitting there and saying, "I don't know how to do this."

2. Students liked the opportunity to show their abilities in a variety of ways. Art work and technology were used throughout student portfolios. This, too, appeared to reduce student anxiety. Students commented they were less likely to forget the subject matter involved in the math unit, whereas this was not the case for traditional testing procedures.

3. Journal writing provided students the opportunity to reflect on their strengths and weaknesses in the mathematics classroom. Additionally, journals fostered a positive relationship among students and the teacher. Journaling helped students recognize that achievement is related to effort, time, and seeking help from more competent others versus their traditional thinking that "it was in the genes."

I cannot overemphasize the value of learning about your students and understanding their thought processes and their frames of reference. I strongly advise readers to read and reflect on Appendices A, B, and I. These appendices enlightened me regarding the impact teachers and schools have on their students.

Future researchers might consider expanding on areas related to emotions and learning in the mathematics classroom. Student journals clearly indicated that students felt more ownership and responsibility for their learning. Students who feel comfortable in the math classroom are more likely to seek help and ask questions for understanding. If students feel as if the teacher cares about their input and concerns, then they are
willing to put forth the effort to take risks in the classroom. Students recognize when a teacher is focused on student learning and achievement. Students appreciate and know when the teacher provides new opportunities for learning that are different from the traditional mathematics classroom. This is reason enough to continue to pursue research in the affective domain in the mathematics classroom.
References Cited


Local Newspaper. (1999).


The following is a journal entry from a student who liked math and could see the relevance and application of mathematics to his world. He wrote:

"I like math cause it could apply to my everyday life. Like, counting money, cards, people, etc. In middle school, math gradually became harder. The worst part was most of the time the skills I was learning I couldn't apply to my life so I didn't like math. This period of my math career was my down time.

Then I moved into high school where the math became even more difficult. But there was a difference because with algebra I could apply that to everyday things. What made me like math was when I knew if I learned this I would be able to use it in other places than just the classroom. I think this went for a lot of my classes."
APPENDIX B
Student's Journal Entries

The following student journal responses reflect the students' feelings and attitudes towards mathematics based on poor experiences with teachers and poor instructional strategies. Student responses included:

"I also remember learning the greater and less than, and also how to count money in 2nd or 3rd grade. One of those years I had a teacher and I went up to get help and she showed me. I sat down to do the problems, then went back up there to show her. She marked every single problem wrong, then looked at me and said, "______, when will you learn?" That broke my heart."

"My first recollection of math was 4th grade. We did "mad minutes" where we had to finish all of the problems in a minute before moving to the next level. I hated that! I always stayed on the first level! I started to dislike math when I realized that I wasn't motivated by my teachers to do it. They never helped me. Just told me I was dumb. My middle school math experiences were horrible also."
"When I first started learning math, I hated it. The first memories I can recall is a game we played in second grade called Math Baseball. There were baseballs with math problems and it was a race between two people in single file lines who could answer the problem first, that the teacher pointed to. I was never very fast, so I thought of it as an embarrassing and humiliating experience because I was always the first one out."

"I really liked math up until 8th grade. My teacher was so hard and just didn't help me. In middle school, I was an average math student. I could have done better but I didn't really apply myself. In 8th grade I didn't really like my teacher so I gave up on math."

"When I came to elementary school, my first subject that I liked was math. I don't know why. I don't know when, but I remember when I was six, I knew all the multiplication problems.

In middle school I was doing great. I had 102% for the semester. I was my math teacher’s pet and I’d do anything for a good grade. I loved math, and then in high school it began to change.

When I was a freshman I had 100% on my quizzes, until one time when one teacher put me down in front of the class for not doing my homework. I was so pissed that I couldn’t even think of math anymore. My sophomore year I had the worst geometry teacher you could ever imagine, she was the devil. She always made fun of me because of the way I was writing in my journal. She used
to say, "this student right here doesn't have any imagination." I hated her. Now my junior year, I have the best teacher who understands me and my problems. I started to do homework again, well most of the time. So I am a pretty complicated guy."
Chapter 3 Systems of Linear Equations and Inequalities

Portfolio Requirements

Instead of taking a Chapter 3 Test, we are going to create a portfolio of what was learned in Chapter 3 as an assessment. Each piece in the portfolio will represent the concept learned in that section. Also in your portfolio there will be a table of contents and two reflection pieces. The final portfolio will be due on Thursday, October 26, 2000 and will be graded based on the rubric. We will have in class portfolio workdays on October 23, 25, and 26. Do not procrastinate, the in class time will not be enough time to finish, outside work time will be necessary. You are welcome to ask me any questions during the in class time or schedule a time to meet with me before school, after school, or during lunch time to discuss your progress on the portfolio.

The following items need to be in your portfolio:

I. Section 3.1 Solving Linear Systems by Graphing
   A. Create 3 examples and their graphs of the three different types of solutions that can exist when solving a linear system of equations by graphing.

II. Section 3.2 Solving Linear Systems Algebraically
   A. Explain which method you like best and why for solving linear systems of equations, substitution or linear combination. Also, create an example and solve it using your preferred method.

III. Section 3.3 Problem Solving Using Linear Systems
   A. Circle the following three types of apes hidden in the puzzle and find the equations of the lines formed by each: gibbon, gorilla, and orangutan.
   B. Find the name of the fourth type of ape and the equation of its line hidden in the puzzle that intersects two of the lines already formed.
IV. Section 3.4 Solving Systems of Linear Inequalities  
A. The diagram shows a cross section of a roped-off swimming area at a beach. Write a system of inequalities that describes the cross section. Each unit in the coordinate system represents 1 foot.

V. Section 3.5 Linear Programming  
A. Explain linear programming, optimization, objective quantity, constraints, and how to solve the following:

\[ C = 4x + 3y \]  
Constraints:
\[ x \geq 0 \]
\[ y \geq 0 \]
\[ 2x + 3y \geq 6 \]
\[ 3x - 2y \leq 9 \]
\[ x + 5y \leq 20 \]

VI. Section 3.6 Solving Systems in Three Variables  
A. Solve the following using three equations: For breakfast you have one glass of orange juice, one cup of oatmeal, and one cup of milk for a total of 375 calories. Your brother has two cups of oatmeal, two cups of milk, and one cup of orange juice for a total of 640 calories. Your mom has one cup of oatmeal and one cup of orange juice for a total of 255 calories. How many calories are on one cup of oatmeal, one cup of milk, and one cup of orange juice?

VII. Personal Logo Design  
A. Design a logo for yourself and put it on a coordinate plane. Then write a system of inequalities that describe the region formed by the logo.

VIII. Reflection of the Piece You are the Most Proud Of

IX. Reflection on the Portfolio Process

X. Table of Contents
## Rubric for Chapter 3 Portfolio Assessment

<table>
<thead>
<tr>
<th>Section</th>
<th>Included all Requirements</th>
<th>Included all Requirements and was Accurate</th>
<th>Included all requirements and was Accurate, Creative, and Organized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table of Contents</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section 3.1</td>
<td></td>
<td></td>
<td></td>
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<td>Section 3.2</td>
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<td>Section 3.5</td>
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</tr>
<tr>
<td>Section 3.6</td>
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<td></td>
</tr>
<tr>
<td>Personal Logo Design</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflection on the Piece You are Most Proud Of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflection on the Portfolio Process</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

**Grade:** 49
III. Section 3.3 Problem Solving Using Linear Systems
IV. Section 3.4 Solving Systems of Linear Inequalities
# APPENDIX D
Rubric for Journal Writing

## Rubric for Journal Writing

<table>
<thead>
<tr>
<th>Category</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name, Date, Journal Question</td>
<td>2 pts</td>
</tr>
<tr>
<td>Neatness</td>
<td>3 pts</td>
</tr>
<tr>
<td>Vocabulary and Grammar</td>
<td>3 pts</td>
</tr>
<tr>
<td>Appropriate Length (one page)</td>
<td>2 pts</td>
</tr>
<tr>
<td>Mathematical Content</td>
<td>10 pts</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>20 pts</strong></td>
</tr>
</tbody>
</table>

Total: 52
One thing I wish we’d do in math class is....

For me to be a better math student, I....

How would you spend $1,000,000? Keep a record of money spent and what it was spent on.

Create two math bumper stickers, one funny and one serious.

Suppose you have a triangle and you know all side lengths. Explain how you would find the angle measures. Then create an example and solve using your steps.

Suppose we had a new student enter our class. What would you tell and explain to them about logarithms?

Write about a meaningful teaching experience you have had. Who did you teach and what did you teach them?

If a number is doubled, what happens to its square root?
If a number is tripled, what happens to its square root?
If a number is increased 10 times, what happens to its square root?

The thing I am proud of myself for learning is...

Self-Evaluation of Semester: Consider a grade for yourself that reflects the time and effort you spent on this class, your level of participation, and your level of understanding and explain your reasoning.

What was the most difficult decision you have made? What was so difficult about that decision?

What is the difference between a relation and a function? Create an example of a relation that is a function and represents that function three different ways.

What is one thing you want to do before graduating from high school? Why do you want to do this?

Explain how to multiply two matrices. Be sure to include any restrictions and explain why matrix multiplication is not commutative.

What are the three most important qualities for a teacher to have and why?
- What are the three most important qualities for a student to have and why?
- Write a math history or a math autobiography of yourself and highlight elementary, middle, and high school experiences, including your likes and dislikes.
- Brainstorm all you know about slope. How would you explain slope to an algebra student who is learning it for the first time?
- Describe and define what a “fair” test is. Also, describe how you prepare or should prepare for a math test.
- How many hours must an employee work in one week at $5.75 per hour to earn what a $6.50 per hour worker makes in a 40-hour workweek?
- What are your strengths as a math student?
- What are your weaknesses as a math student?
- How would you design the ideal math class?
Problem 1:

Jim is shopping in a large department store with many floors. He enters the store on the middle floor from a skyway, and immediately goes to the credit department. After making sure his credit is good, he goes up three floors to the housewares department. Then he goes down five floors to the children's department. Then he goes up six floors to the TV department. Finally, he goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street. How many floors does the department store have? After you have solved this problem, identify the problem solving strategy used.

Problem 2:

Find the next four values of each of these sequences. Then, write a sentence or two explaining the pattern used. Again after you have solved this problem, identify the problem solving strategy used.

1) 2, 2, 4, 6, 10, 16, 26, ___ , ___, ___, ___

2) 1, 3, 4, 7, 11, 18, 29, ___ , ___, ___, ___

3) 3, 1, 4, 5, 9, 14, ___ , ___, ___, ___

4) 1, 2, 3, 6, 11, 20, 37, ___ , ___, ___, ___

5) ___ , ___, ___ , 16, 25, 41, 66, 107

Problem 3:

Andrea is two years less than four times as old as Mike. Andrea is also one year more than three times as old as Mike. How old is each? Again after you have solved this problem, identify the problem solving strategy used.
APPENDIX G
Cooperative Learning – Application Problems

Intermediate Math
Matrix Applications

1) Write the following system as a matrix equation and solve for \( x, y, \) and \( z \).

\[
\begin{align*}
\mathbf{x} - y + z &= -2 \\
2x + 3z &= 4 \\
3y - z &= 7
\end{align*}
\]

2) Explain how to use an inverse matrix to solve the following system.

\[
\begin{align*}
-3x + y + 5z &= 2 \\
x - z &= 1 \\
2y + 3z &= 2
\end{align*}
\]

3) You have $30,000 to invest in the three stocks shown below. You want an average annual return of 9%. Because stock X is a high-risk stock, you want the combined investment in stock Y and stock Z to be 4 times the amount invested in stock X. How much should you invest in each type of stock?

<table>
<thead>
<tr>
<th>Stocks</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock X</td>
<td>12%</td>
</tr>
<tr>
<td>Stock Y</td>
<td>9%</td>
</tr>
<tr>
<td>Stock Z</td>
<td>8%</td>
</tr>
</tbody>
</table>

4) Gold jewelry is seldom made of pure gold because pure gold is soft and expensive. Instead, gold is mixed with other metals to produce a harder, less expensive gold alloy. The amount of gold (by weight) in an alloy is measured in karats. Anything made of 24-karat gold is 100% gold. An 18-karat gold mixture is 75% gold and so on. Three gold alloys contain the percents of gold, copper, and silver shown in the matrix. You have 20,144 grams of gold, 766 grams of copper, and 1,990 grams of silver. How much of each alloy can you make?

<table>
<thead>
<tr>
<th>Percent by Weight</th>
<th>Alloy X</th>
<th>Alloy Y</th>
<th>Alloy Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>94%</td>
<td>92%</td>
<td>80%</td>
</tr>
<tr>
<td>Copper</td>
<td>4%</td>
<td>2%</td>
<td>4%</td>
</tr>
<tr>
<td>Silver</td>
<td>2%</td>
<td>6%</td>
<td>16%</td>
</tr>
</tbody>
</table>
APPENDIX H
Cooperative Learning – Review Session

Intermediate Math
Sec 5.2 Parabolas: Graphs
of Quadratic Equations

Name: Date:

I. Define the Following Terms

A. Quadratic Equation:

B. Parabola:

C. Leading Coefficient:

D. Vertex:

E. Axis of Symmetry:

F. X - Coordinate of Vertex:

II. Analyze the Following Equations

A. \( y = x^2 + 4x - 12 \)

B. \( y = -x^2 + 6x + 7 \)

C. \( y = -2x^2 + 4x - 3 \)

1) Shape:

2) Direction Graph Opens:

3) Axis of Symmetry:

4) Vertex:

5) Y - Intercept:

6) X - Intercepts:
APPENDIX I
Student Self-Evaluations

When students were asked to evaluate themselves for the semester based on time and effort spent on mathematics, level of participation in class, and level of understanding of concepts taught, the students responded openly and honestly and wrote the following:

"If I were to evaluate myself for some specific things in math I would, and I'd write it in my journal. Much time was put into this math class this semester by myself. Usually I suck at math and I don't get it, but this semester I made extra efforts to learn what was going on. I probably made over 30 trips to the math office seeking help and spent as much time as was necessary to complete all class and homework. Out of all of the assignments for homework, I believe I didn't do like 3 at the most. I really didn't participate that much because it's first period (sleepy) and I really don't like to get all wound up at 7:30 a.m. Plus I wouldn't want to look stupid. Some of the stuff we learned was much easier to understand then other stuff. But then again if I didn't understand something, I was down in the math office come sixth period. If I were to give myself a grade based on my effort and time, I'd definitely give myself an A, but since my grade is compiled of other stuff overall. I'd say I earned a B- or so. But seeing as how I've never gotten a B in high school math, I guess I can expect a C or D."
"I think that I worked really hard this semester with math - actually more so then I have ever in the past. I really tried my best to understand all of the material by completing my homework and projects that were done in the past. The one thing I believe I did not do this semester was participation. I participated once in awhile, but not all the time like I should have. For this I give myself a "B." I think that participation is somewhat an important part of learning so I can't give myself an "A." Overall I had an excellent math semester that not only makes me proud of what I did but also my parents."

"I would give myself a B this semester in Algebra II/Trig. I put a lot of time and effort into the portfolio, into studying for tests and I never missed a homework assignment. I would say that my participation was moderate. I went up to the board many times and I asked a lot of questions when I didn't know how to do something. I think that my level of understanding is great. I have enjoyed this class because I have understood it."

"I feel that I have worked hard this semester and spent much time and effort devoted to understanding and being able to explain thoroughly the material we have studied in math. I have spent much time working hard to be able to perform the same problems/equations we work on as a class on my own as an individual. I feel that it is most important to understand the material before worrying about tests, quizzes and exams, and therefore I have put in the effort to be able to do this. In class, I feel that I have equally contributed in helping others
to understand, volunteering, and raising questions and comments. By performing techniques used in class on my own, I have learned not only to understand through devoted efforts but to enjoy and continually learn from a class in which I would not naturally excel without work. For these reasons, I feel that I have strived to successfully understand and contributed much through hard work, time and effort and feel that I deserve an A."

"In this class I think I deserve an A. I put a consistent amount of time into this class. We had homework almost every day so I would spend about 20 minutes a night on math homework. I didn’t have to spend anymore time because my level of understanding was high. I got A’s on almost all of my tests. That shows that I understand what I was taught. Also, I participated in class. I didn’t participate as much as some people did but I did participate a good deal more in this class than I did in some of my other classes."

"This semester my time and effort in class have been great. Everyday I’ve taken lots of notes, asked questions, and not drifted off. Outside of class, my time and effort have been good, but not perfect. As for participation, I’m there. As I said before I’ve been an active student in class. My notes, comments and questions prove that. Participation has been my best point. As for my level of understanding it’s pretty good, excluding the last unit. Sometimes my tests don’t show that, because many times I get nervous and I draw blanks lots of times on tests. The unique thing that demonstrates my level of understanding is my ability
to say why or how and not just the formulas. As I believe I, ironically, said in my first journal of the year, to really learn for me means knowing why. It's something that I feel is important and have tried to pay attention to this year."

"Through this semester I have put a lot of effort into my portfolio, my participation, and homework. I have put forth a good amount of time for math in homework and the portfolio, etc. Normally, I never really cared about homework in math, but this year I have because I have been getting a pretty good grade. In geometry, everything was so unclear to me and it made everything difficult, but this math class I can grasp more. I feel that I have participated in putting homework and warmups on the board. I have also answered questions that were asked and I helped erase and put up work. My level of understanding in this class has probably been better than any of my other high school math courses. I think that everything is explained better and it is easier to come and ask questions. I do think that some concepts were unclear to me, but I think overall my test scores were average. But, basically I would give myself a B because I make an effort in the portfolio, homework, study, participation, and I came in for help when I didn't understand a concept. So those are my reasons for deserving a B."

"In my 1st semester math class I thought I put a lot of time and effort in. I always had my homework done. I very rarely if ever miss a homework assignment. I came in and got extra help out of class whether it was in my study
hall or other times. I always tried a problem even though I might not have understood it. If it was on a homework assignment or on a worksheet I would at least try the problem and not leave the problem blank. I though my participation was very good in class. I always asked questions when I did not understand a problem. I always did problems on the board when we had homework. I thought I participated a lot especially when Miss P was looking for someone to answer a question or do a question on the board. My level of understanding was pretty good throughout the semester. Miss P always does a good job on explaining how to do the problem we are doing. I usually take a little longer than usual to learn the things but she does a good job. I think I deserve a final grade of an A-. I am getting a 92% right know which is an A- and I am not a very good test taker so sometimes I do bad on finals and it kills my grade. I worked all semester to get a good grade and one test can kill your grade. I hate that. I really need an A and if I do bad I will go down to a B."
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