Mathematics is at the apex in priorities pertaining to state-mandated testing of students. With 49 out of 50 states having mandated the testing of students, all of these have mathematics in the testing format. This paper discusses the modern school mathematics movement, recent approaches in improving the teaching of mathematics, and specific teaching suggestions for mathematics. (ASK)
Assessing Innovative Proposals in Mathematics

by

Marlow Ediger
ASSESSING INNOVATIVE PROPOSALS IN MATHEMATICS

Mathematics is at the apex in priorities pertaining to state mandated testing of students. The 3r's (reading, writing, and arithmetic) have a long standing of being considered as basics in the curriculum. Students need to achieve well in school and later at the work place in mathematics. With 49 out of 50 states having mandated testing of students, all of these have mathematics in the testing format. But, even science and social studies may not be included in state mandated tests. This shows the importance of mathematics in the curriculum. Selected educators have said, “What is tested, that is what will be taught in the classroom.”

The Modern School Mathematics Movement
(A Brief History)

The Modern School Mathematics movement of the late 1950s and early 1960s was well funded federally with the National Defense Education Act of 1958. Few teachers and professors are aware today of this Act and its impact, or lack thereof, upon mathematics teaching. Federally funded groups included the following: the Greater Cleveland Mathematics Project, the Madison Project, The School Mathematics Study Group, the University of Illinois Arithmetic Project, and The Minnesota Project. When the findings of the Modern School Mathematics movement were included in textbooks in the public schools beginning in 1961, there were numerous innovations mentioned and advocated such as the structure of mathematics. The following structural ideas in mathematics were strongly emphasized in these textbooks:

1. the commutative property of addition and multiplication.
2. the associative property of addition and multiplication.
3. the distributive property of multiplication over addition.
4. the inverse operations of subtraction and division.
5. the identity elements of addition and multiplication.

These, among other structural ideas always held true and were to be emphasized strongly by teachers in teaching mathematics(Ediger, 1975, Chapter Three). They are as valuable today as then. Professors of mathematics at leading universities were heavily involved in developing the modern school mathematics curriculum. Reasons given for the strong inclusion of mathematicians were the following:

1. highly knowledgeable academicians are in the best position to know what should be taught in mathematics, much more so than public school teachers.
2. a teacher proof mathematics curriculum might well then be in evidence.
3. mathematicians need to select key ideas for teachers to emphasize in the classroom. These key ideas then become objectives of instruction for teaching students in the public school classroom.

4. the key ideas, or structure of knowledge approach in teaching mathematics eliminate the unimportant being taught. Major concepts and generalizations are then in the offing for students to achieve.

5. methods of learning in the elementary, middle school, and high school levels should emphasize those of the professional mathematicians on the university level.

6. inductive procedures should be used by students to achieve the identified structural ideas in mathematics.

7. textbooks and accompanying workbooks were developed for teachers to use as teaching devices in helping students attain the key ideas, but always in greater depth.

8. with greater depth learning, the spiral mathematics curriculum was emphasized. Thus, the student would revisit each vital concept and generalization. For example, the commutative property of addition may be stressed in the first grade with single digit addends as well as all higher grade levels with addends of any size in terms of place value.

9. base values of two, five, six, and eight, in addition to base 10, provided students with a depth understanding of place value and base values of number systems.

10. trivia and the mundane needed to be eliminated from the mathematics curriculum (Ediger, 1974).

A teacher education textbook which related to the trends of the National Defense Education Act of 1958 was entitled The Process of Education (Bruner, 1960). Dr. Bruner listed his well known hypothesis, "any subject matter can be taught in some intellectually honest form to any child at any stage of development." This is a powerful statement today for teachers and school administrators. The key here is "in some honest intellectual form." Thus early primary grade pupils may study and achieve in geometry by

1. seeing and feeling squares, circles, and triangles. These can be made from different colors of construction paper.

2. noticing and discussing the geometric figures, in number one above for example, by identifying squares on the floor from a tile representation.

3. observing and commenting on squares, circles, and triangles from filmstrips, slides, transparencies, and video tape presentations.

4. analyzing squares, circles, and triangles from the basal textbook.

5. looking at geometric figures and reading the lines underneath each with teacher guidance. The teacher may also read aloud library books to students which show these geometric illustrations in the book as the oral reading continues.
The author considers these examples as stressing geometry in “some honest intellectual form” to young learners. The teacher may then build upon these key ideas to develop increased understanding of more complex ideas and in greater depth in the k-12 geometry curriculum.

Shepherd and Ragan (1982) wrote the following: Bruner made a powerful formation for the selection of content. Bruner utilized a concept he called “structure” to describe those aspects within a discipline that should be taught. Structure denoted those aspects of a discipline that simplified, clustered, and generated. Bruner seemed to perceive some kind of a unifying whole which operated within and through a field of study. His contribution -- structure -- had a tremendous and significant impact upon the organization of the curriculum during the 1960s, especially upon mathematics and the sciences.

What should be taught has always been a major consideration in education. Jerome Bruner’s contributions (1968), in many ways, are highly valid today in that he
1. stressed that which is important subject matter for students to learn in each academic discipline.
2. advocated a method for students to use in acquiring viable subject matter.
3. emphasized students achieve key ideas in an academic discipline, not trivia.
4. realized the importance of students making “revisits’ to previously studied concepts and generalizations. These revisits involved students acquiring subject matter at greater depth and at increasing complexity levels. These revisits can certainly minimize retention problems of students as well as having content presented developmentally for learners at different stages of achievement.
5. having students learn inductively to achieve structural ideas. Induction helps students to perceive sequence in learning since the teacher with appropriate learning opportunities involves students inductively in revealing what is and what is not understood. The teacher may then diagnose and remediate problem areas based on student responses.

Recent Approaches In Improving the Teaching of Mathematics

There are a plethora of attempts which have been made to improve the mathematics curriculum. The National Council Teachers of Mathematics (NCTM) after much deliberation, time, effort, and expense came out with a quality, comprehensive set of standards for teachers to follow in teaching and learning situations. Pertaining to NCTM standards that emphasize communication, reasoning, and connections, the
following, for example, are listed for kindergarten though grade four:

Standard 2; Mathematics as Communication. In grades k-4, the study of mathematics should include numerous opportunities for communication so that students can:
* relate physical materials, pictures, and diagrams to mathematical ideas.
* reflect on and clarify their thinking about mathematical ideas and situations.
* relate their everyday language to mathematical language and symbols.
* realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics.

Standard 3: Mathematics as Reasoning. In grades k-4, the study of mathematics should emphasize reasoning so that students can:
* draw logical conclusions about mathematics.
* use models, known facts, properties, and relationships to explain their thinking.
* use patterns and relationships to analyze mathematical situations.
* believe that mathematics makes sense.

Standard 4: Mathematical Connections. In grades k-4, the study of mathematics should include opportunities to make connections so that students can
* link conceptual and procedural knowledge.
* relate various representations of concepts and procedures to one another.
* recognize relationships among different topics in mathematics.
* use mathematics in other curriculum areas.
* use mathematics in their daily lives.

In the above named standards 2-4, as a summary, students are to:
1. communicate ideas in mathematics by relating the concrete and the semi-concrete to the abstract, think upon what has been learned, and use the language arts areas to use mathematics in ongoing experiences.
2. emphasize drawing logical conclusions, use of the concrete, semi-concrete, and abstract (including patterns and their relationship) to explain mathematical ideas.
3. making application of what has been learned.

Thus by studying and reflecting upon what is emphasized by the National Council Teachers of Mathematics, as an important data source,
the local school district may move from what is to what should be in teaching and learning situations. School principal assessment may be used to determine how well mathematics teachers are doing in making effective use of the NCTM standards to improve teaching and learning.

Problems involved in mathematics teachers using objectives developed by a national group are the following:
1. how to get the information into the hands of teachers.
2. how to obtain the development of workshops and inservice education programs in public schools in order to use the national standards.
3. how to motivate teachers to attach meaning to each standard.
4. how to apply using each standard in the classroom.
5. how to ensure that teachers continue to use these standards and continually work in the direction of improving mathematics instruction.

Mathematics teachers should always look for improved ways of teaching to increase learner achievement in the curriculum. Assessment here should be ongoing to develop the best mathematics curriculum possible.

Writers in professional journals and teacher education textbooks may well provide quality information for mathematics teachers to use in teaching and learning situations and thus improve the curriculum. (Ediger and Rao, 2000, Chapter Seventeen) list the following objectives for students to achieve in mathematics instruction:

1. students need to attach meaning to what is being learned in mathematics. Meaning theory stresses that students are able to comprehend that which is being achieved within an objective of instruction. Thus, a learner can state in his/her very own words what is being emphasized in a concept, generalization, or problem being solved in mathematics.

2. students attend to the ongoing lesson/unit of study due to being interested in the learning opportunity being pursued. Interest in the learning opportunity generates enthusiasm for achievement in the mathematics curriculum. Teacher enthusiasm for mathematics and mathematics teaching should inspire students to new heights of achievement.

3. student purpose in learning involves learners accepting reasons for attaining, growing, and developing in mathematics. Achieving purpose increases learner reasons for achievement of objectives of instruction.

4. three categories of objectives need attainment. These include knowledge, skills, and attitudes in the mathematics curriculum. Knowledge includes facts, concepts, and generalizations. Skills involve student using the acquired knowledge. Making much use of knowledge aids in retention so that what is learned may not be forgotten readily.
Quality attitudes should arise with useful knowledge and skills having been achieved.

5. student sequence is achieved whereby improved relationships are perceived between the background information possessed and the new content to be achieved. A more seamless mathematics curriculum is in the offing when the gaps between the old and the new learnings are perceived as one, not separate entities.

6. student achievement in mathematics is aided when the background information is scaffolded with the new objectives to be achieved (Vygotsky, 1978). The gaps between the old and the new learnings may be too great unless the mathematics teacher scaffolds the in between with small sequential steps of learning. Teachers need to be aware of and provide for the scaffolding when learners reveal the new objectives are too complex to achieve.

7. students need to achieve relevant subject matter and skills in ongoing lessons and units of study. Relevancy is an important idea in teaching mathematics. What is perceived as being irrelevant may turn a student off. Relevant knowledge and skills are vital in assisting optimal learner achievement.

8. students need to be motivated learners in the mathematics curriculum. The mathematics teacher then needs to think of and implement strategies of instruction which increase energy levels for learning. Encouraging, assisting students when needed, developmental instruction, and praising learners for quality performance serve as motivators for instruction. A caring, responsible teacher is desired in teaching mathematics.

9. students are at diverse levels of achievement in mathematics. Each student is unique and has much worth as an individual. The mathematics teacher then must provide for individual differences within any lesson and unit of study. Accept students with their present individual level of performance in mathematics and then work for continuous optimal progress for each.

10. student assessment of achievement must be shown unifying multiple data sources, not a single test only. With diverse methods and techniques of assessment, the student may then better reveal what has been learned and what is left to learn. Multiple Intelligences Theory indicates that there are a plethora of ways for students to indicate that which has been learned using the personal intelligence style of the involved learner.

Self and peer assessment may be used to evaluate how well a teacher is using each of the above named tenets from the psychology of learning. A five point Likert scale may be used in the assessment process. Feedback from the assessment may be used to improve the mathematics curriculum.

Gardner (1993) recommended the following intelligences, with a
brief explanation by the writer, which individual students possess and by which learnings acquired may be presented:

1. verbal/linguistic. Students indicate what has been learned through reading and writing such as in test taking skills.

2. visual/space. Here, the student may reveal learnings through art products developed within any curriculum area.

3. logic/mathematics. In addition to the mathematics curriculum, the student may also indicate learnings gained through reasoning and thinking logically across the curriculum.

4. intrapersonal intelligence. The student may reveal what has been learned on an individual basis in working by the self.

5. interpersonal intelligence. Here, students may indicate best what has been learned through cooperative endeavors or in groups, not as individuals.

6. bodily/kinesthetic. Athletic and physical prowess are used here to show what has been achieved.

7. musical/rhythmic. There are young children, for example, who love to develop songs based on the set of counting numbers. Or number chants may be created when jumping rope!

8. scientific. There are students who are very strong in being objective thinkers which science does emphasize. Objective thinking is valuable in any curriculum area. Prejudice, biases, and dogmas then are minimized or eliminated in one’s thinking. The methods of science then are important to use in any academic or vocational area.

Mathematics teachers need to be assessors of the intelligence(s) possessed by each student. They also need to use one or more intelligences from students to have the latter reveal what has been learned. Mathematics teachers need to discuss and assess each other's teaching in terms of how well multiple intelligences theory is used to provide for individual differences in the classroom and, particularly, in assessing student progress in the curriculum. Self assessment in terms of desired standards can be very illuminating for teachers. The results can make for a modified mathematics curriculum with optimal learner achievement as a continual objective of instruction. Standards used for self assessment may involve use of videotapes of lessons taught and specific instances pinpointed of where multiple intelligences was used in the ongoing presentation. Also, areas where multiple intelligences might have been emphasized may also be identified.

**Specific Teaching Suggestions in Mathematics**

Developing the mathematics curriculum is a complex process indeed. The ingredients that go into developing daily lesson plans and units of study are numerous and many. Critical and creative thinking as
well as problem solving are involved here. So far, the author has provided broad guidelines for curriculum developing in mathematics. What about elaborating upon the highly specific teaching suggestions of each learning opportunity to be stressed and in what sequence or order of presentation? (See Ediger, 2000, 29-34).

For each teaching suggestion, the mathematics teacher needs to develop a strategy for initiating an activity. Initiating activities provide readiness of students so that they may truly benefit from the ongoing experience. Thus, subject matter readiness is one factor. The student then needs to have the necessary background information to understand the new subject matter. Emotional readiness, as a second factor also needs to be in evidence. Emotional readiness pertains to the affective domain of the student. The student then needs to possess the prerequisite, necessary attitudes to benefit form the new subject matter and skills. Third, social readiness may be needed if the teaching suggestion emphasizes committee work. Teaching suggestions involving cooperative learning then have as a prerequisite in being able to work together for the good of each involved learner. Fourth, physical readiness stresses necessary biological maturation to benefit from the teaching suggestion. A student here needs to be able to use neuromuscular skills to manipulate items and objects in the project method of teaching mathematics. Making models, constructing things, drawing examples, graphing data, and developing/using transparencies requires the use of the gross and finer muscles of the individual student. Thus, readiness to benefit from a given teaching suggestion requires that a student possess certain characteristics which are already there or can be achieved through scaffolding. Scaffolding involves teaching/learning situations which take care of gaps between where the student is as compared to where he/she should be as indicated in a terminal objective (Vygotsky, 1978).

In addition to initiating a teaching suggestion, intensive teaching needs to be in evidence. Intensive teaching is opposite of survey methods of instruction. The mathematics teacher stresses depth instruction for students to benefit from any teaching suggestion. Intensive teaching emphasizes each student reflecting upon what has been taught. Each new learning activity in the teaching suggestion is related to previously achieved knowledge/skills of the learner. Seeing these connections is salient and vital. Several related learning activities are needed for students to perceive depth involving one concept or generalization. Application also needs to be made of acquired information in order for a student to perceive depth in achieving an objective in mathematics instruction. Critical and logical thinking would further advance the cause of depth teaching. Thinking about what is taught tends to emphasize depth teaching. Teacher assessment is necessary to appraise the quality of depth teaching and learning.
Teaching suggestions need initiation as well as depth instruction together with a third factor which is an evaluation of learner achievement as a result of implementing the teaching suggestion. A variety of procedures need to be used. Based on quality criteria, teacher observation, diagnosis, and remediation can be excellent ways to use in evaluating the effectiveness of using a teaching suggestion. Assessment here means that at a given, specific time, the mathematics teacher notices how sequential the assessment process is to notice depth teaching and readiness factors for learning.

Readiness, intensive teaching, and assessment refer to each of the following teaching suggestions in mathematics with the use of

1. software, computers, and technology. These devices should emphasize meaning, sequence, and success in learning for students.

2. basal mathematics textbooks. Careful selection of one or more basals to be used in the classroom is a must to meet objectives of the curriculum. The textbook is not a self-teaching device but requires a knowledgeable, creative, and skillful teacher to implement its use.

3. flannel boards to illustrate selected learnings. Flannel board use with its cutouts requires the portrayals to represent accurate figures. For example, the cutouts may pertain to a square and the student is learning how two fourths is equivalent to one half. Thus, the one half may be laid over the two fourths to notice equivalency.

4. markers. Sticks used as markers may be used to show fundamentals in addition, subtraction, multiplication, and division. A hands on approach may then be used for a student to understand the commutative property of addition whereby two sticks and three sticks equals three sticks and two sticks. These learnings provide the building blocks for a student to use the commutative property no matter how large the addends are in value.

5. place value charts. A neatly made place value chart will show the ones, tens, hundreds, and thousands columns. The number of columns indicated pertains to the developmental level of the student being taught. Congruent slips of construction paper may be used to show the value of any numeral. The columns may be extended to ten thousands, hundred thousands, and millions. Or to the right of the one's column to show tenths, hundredths, and so on.

6. transparencies and the overhead projector. Among other uses, for example, the teacher may show initial learnings in multiplication such as three sets of circles with four members in each set to indicate 3 x 4. Division as the inverse operation may also be shown with these same circles and the use of the overhead.

7. filmstrips and slides. Here, there is no motion or movement involved in the presentation from the projection on the screen. Thus, for example, the teacher may show the associative property of addition
through one or more of the frames on the filmstrip or on a slide such as
two squares plus three squares plus four squares equals nine squares or
2 +3 +4 =9. Rearranging the squares, the student may notice that any
arrangement of the squares or addends will still make for a total of nine,
e. g. 3 +4 +2 = 9.

8. graphs. A very practical bar graph may be made of the number
of students who have birthdays for each month of the calendar year. The
bars may be either horizontal or vertical. If three students, for example,
have birthdays in January, the length of the bar will be three units. If one
student has a birthday in February, then the length of the bar will be one
unit, and so on for the other months in the year. The graph should have a
title, be neatly drawn on the chalkboard, on a transparency, and/or using
computer software (spread sheets). In addition to bar graphs, line and
pie graphs may also be used to show data on different topics.

9. songs. There are a plethora of songs involving number which
students like to sing. The following is an example:
   One, two, buckle my shoe
   Three, four, shut the door.
   Five, six, pick up sticks
   Seven, eight, close the gate
   Nine, ten, a big fat hen.

   Rope jumping chants, writing poetry, and dramatizing are
   enjoyable ways of learning and experiencing number.Words in poetry
   may be set to music. Dramatizations may involve number concepts.

10. money. Money, real or in toy form, may be used for playing
   store. Empty food containers and empty vegetable cans may have prices
   attached. Money may then be used to “go shopping” for food containers
   on shelves with prices attached to each item. Here, students may learn to
   add food items purchased as well as subtract to obtain the correct
   amount of change from the currency given for the purchase.

11. geoboards. Geoboards may be purchased or made with a
   sheet of plywood used as the base. Nails, approximately one inch apart
   in both dimensions to make squares, should be nailed into the plywood.
   A student may then form a geometric shape such as a parallelogram by
   stretching a rubber band around the needed number of nails. Many
   geometric shapes may be made from rubber band use on the geoboards
   (Ediger, 1988, 130 -137).

The mathematics teacher needs to be creative in designing
teaching aids which will benefit students to achieve as well as possible.
Additional possibilities include making games for students to play to
learn mathematics concepts and generalizations, and student
development of art projects. Teaching suggestions then are used as
learning activities to assist learners to achieve objectives of instruction.
Assessment of teaching aids should be emphasized within a community of learners who are mathematics teachers. The informal discussion may discuss aids made to teach students and how they were used in ongoing lessons. Results of using these teaching aids provide feedback to teachers in the community of learners.

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