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ABSTRACT

This investigation uses classroom discourse in an undergraduate mathematics course to challenge preservice secondary mathematics teachers' notions about mathematical discourse, what it should resemble in the classroom, and how it can be cultivated by classroom teachers. The research setting was an upper level geometry course. Eleven preservice mathematics teachers participated. Results indicated a transition in participants' image of discourse as an active process by which students use the collective knowledge of their peers to build mathematical understanding and a development in students' ability to participate in such discourse. This awareness, along with participants' analyses of their own habits of discourse as classroom teachers, prompted shifts in their pedagogical image of the role of discourse in their future practices of teaching. Results further suggested that the undergraduate mathematics classroom (as opposed to the methods classroom) offers a powerful and unique forum in which preservice secondary teachers can practice, articulate, and collectively reflect on reform-minded ways of teaching. (Contains 27 references.) (Author/SM)

Running Head: CHALLENGING PRE-SERVICE TEACHERS' DISCOURSE

Using a Subject Area Course to Challenge Secondary Pre-service Teachers' Models of Teaching: A Teacher Educator's Experience

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Abstract

This investigation uses classroom discourse in an undergraduate mathematics course to challenge pre-service secondary mathematics teachers' notions about mathematical discourse, what it should resemble in the classroom, and how it can be cultivated by classroom teachers. The research setting was an upper level geometry course taught by the author. Eleven pre-service mathematics teachers participated. Results indicated a transition in participants' image of discourse as an active process by which students use the collective knowledge of their peers to build mathematical understanding and a development in students' ability to participate in such discourse. This awareness, along with participants' analyses of their own habits of discourse as classroom teachers, prompted shifts in their projected image of the role of discourse in their future practices of teaching. Results further suggested that the undergraduate mathematics classroom (as opposed to the methods classroom) offers a powerful and unique forum in which pre-service secondary teachers can practice, articulate, and collectively reflect on reform-minded ways of teaching.

Developing a cadre of classroom mathematics teachers whose practices support our current ways of knowing rests in part on how pre-service teachers, as students, experience mathematics. Such a claim is theoretically grounded in the sociocultural argument that the nature of intermental functioning is subsequently reflected in an individual's intramental functioning (Vygotsky, 1978/1934; Wertsch & Toma, 1995). That is, the way that teachers and students interact with mathematical ideas in the social context of the classroom, whether passively or actively, structures students' thinking about mathematics. From this, we can additionally infer that classrooms rooted in traditional models of teaching portend the nature of future instruction. In other words, the power of pre-service teachers' mathematical experiences extends to their pedagogical thinking as well. Lortie (1975) maintains that one's internalized models of teaching are a legacy of the 'apprenticeship of observation' realized through years of schooling. In effect, as students of mathematics, pre-service teachers acquire what might be described as <u>incidental pedagogies</u>. By this, I mean models of teaching that are necessarily "intuitive and imitative rather than explicit and analytical" (Lortie, 1975, p. 62) because they derive from an orientation (the student's) that is intrinsically barred from the internal complexities of teaching. Moreover, in the case of pre-service teachers, such models are likely to reflect the most recent (hence, more accessible) memories of the undergraduate classroom (Grossman, 1990).

Exploring an Alternative Context for Mathematics Pre-Service Teacher Education

This latter point concerning models of teaching which pre-service teachers appropriate from their undergraduate experiences imposes an obvious dilemma for mathematics teacher educators. That is, as educators we are compelled to craft undergraduate mathematical experiences that are organized around reform-minded ways of teaching, yet we have (understandably) limited access to advanced courses in mathematics or the faculty who teach them. Elementary mathematics education is arguably an exception to this. Mathematics educators in this area seem to enjoy greater access to subject-matter courses and have consequently used these to pioneer considerable reforms in teacher education (see e.g., Ball, 1993; Simon & Blume, 1996; Swafford, Otto, & Lubinski, 1999; Wood, Cobb, Yackel, & Dillon, 1993). These efforts, together with an awareness of and commitment to systemic reforms emerging within the undergraduate mathematics community on behalf of all students (Kaput & Dubinsky, 1994; Schoenfeld, 1990; Steen, 1992), should make it easier to carve a niche within mathematics that addresses the peculiar domain of pre-service secondary teachers. To this end, I would argue along with others (e.g., Zeichner, 1996) for subject-matter courses in which pre-service secondary teachers seriously explore content in parallel with pedagogy.

My own thinking about the need for mathematics courses that also address pre-service teachers' pedagogical thinking crystallized recently while teaching a one-semester undergraduate geometry course for pre-service secondary[1] teachers. Prior to this course, I had decided to explore the singular question of how to design a course that preserved its intended mathematical integrity while challenging pre-service teachers' incipient notions of practice –in essence, a course that successfully integrated content and pedagogy. (I distinguish such a course from traditional mathematics methods courses because of the inherent emphasis on subject matter in a content course.) The geometry course seemed an opportune research setting. All eleven students participating in the course were pre-service secondary mathematics (or science and mathematics) teachers in their final academic year of a four-year teacher preparation program. Additionally, the course had a diverse enrollment that included six female and two minority students.

Discourse as a Pedagogical Focus in the Mathematics Classroom

While 'notions of practice' might invoke an expanse of pedagogical arrangements as objects of study (e.g., technology-based instruction, or cooperative learning environments), I was interested in the particular experience of classroom discourse[2] as it contributes to one's thinking about mathematics and, consequently, teaching mathematics. That is, how could (or could) an undergraduate mathematics course be used to heighten students' awareness of and ability to engage in and cultivate the kinds of discourse that promote a conceptual understanding of mathematics? The earlier premise that the nature of pre-service teachers' mathematical experiences shapes how they ultimately teach mathematics brought such a question to the fore in my own thinking. Thus, I specifically targeted students' thinking about classroom discourse, what it should resemble, and how it could be cultivated. To this end, the intent of this study was to extend current research about the role of discourse in teaching and learning mathematics to a setting that contributes to pre-service teachers' notions of mathematical discourse (i.e., the undergraduate mathematics classroom), but is less understood for this purpose.

Although there are variant terminologies about discourse in the literature that could have served this focus, I found Lotman's (1988) characterization of text succinctly appealing in framing our class discussions about mathematics, about discourse, and consequently this study on discourse. Lotman's notion of text, which he defines inclusively as a "semiotic space in which languages interact, interfere, and organize themselves hierarchically" (p. 37), includes verbal text such as classroom discourse (see e.g., Peressini & Knuth, 1998). He argues that text dualistically functions as either a "passive link in conveying some constant information between input (sender) and output (receiver)" (p. 36) or as a "thinking device" that generates new meaning when a participant actively interprets the text by questioning, validating, or even rejecting it. In the former case, text is viewed as information to be received, encoded, and stored, and its goal is the alignment of codes, or languages, between the speaker and listener. Furthermore, any discrepancy between what is transmitted by the speaker and received by the listener is attributed to a defect in communication. In contrast, the latter case describes text which serves as a starting point for making sense of an idea or constructing new ideas (see also Wertsch & Toma, 1995). (Hereafter, I will use the respective terms univocal and dialogic, ascribed by Wertsch and Toma, when referring to these functions.) This characterization of text as univocal or dialogic was integrated into the course through a progression of events. Initially, students were assigned readings from current literature that applied or delineated this framework. From this, we had class discussions about the meanings of this characterization and used it as a tool for ongoing informal analysis of our mathematical classroom discourse as it occurred. Students subsequently used it to describe their own classroom teaching experiences in the course.

The theoretical justification for a focus on discourse was further drawn from Vygotsky's (1978/1934, 1986/1934) sociocultural approach, which espouses the primacy of language in an individual's development. In particular, Vygotsky maintained that "higher voluntary forms of human behavior have their roots in social interaction, in the individual's participation in social behaviors that are mediated by speech [italics added]" (Minick, 1996, p. 33). He believed that psychological tools, such as language, serve to control human behavior by "transforming the natural human abilities and skills into higher mental functions" (p. xxv, 1986). In this, he argued that language is the primary medium through which thought develops and that it is a "manifestation of the transition between social speech on the inter-psychological plane (between individuals) and inner speech on the intra-psychological plane (within the individual)" (Wertsch, 1988, p. 86).

Integrating Vygtosky's perspectives on language with Lotman's characterization of text, Wertsch and Toma (1995) argue not only for the existence and necessity of classroom discourse, but that its very form (that is, whether it is univocal or dialogic in its function) will be internalized by and reflected in the individual's inner speech. Thus, for instance, if the purpose of classroom discourse is to make sense of ideas and to use those ideas to generate new thinking, then it can reasonably be expected that students will interpret utterances as thinking devices, "taking an active stance toward them by questioning and extending them [and] by incorporating them into their own external and internal utterances" (p. 171).

The ongoing emphasis on classroom discourse in mathematics education is rooted philosophically in extant reform agendas such as the National Council of Teachers of Mathematics (NCTM) Principles and standards for school mathematics (2000) and Professional standards for teaching mathematics (1991). Current research reported in these documents argues that "learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills (Hanna and Yackel, in press, as cited in Principals and standards, 2000) and that "classroom discourse… can be used to promote the recognition of connections among ideas and the reorganization of knowledge (Lampert, 1986, as cited in Principals and standards, 2000).

Beyond this, classroom discourse draws its significance from research perspectives that explore linkages between social interactions and the development of teaching and learning mathematics. In an analysis of elementary students' small group mathematical activity, Cobb, Yackel, and Wood (1992) found that students learned mathematics as they "participated in the interactive constitution of the situations in which they learned" (p. 119). In this, they attributed the development of group consensus to interactions within the group, in which individuals negotiated incongruities between their own and others' mathematical activity. This suggests that discourse among students is a central characteristic of learning mathematics.

Research on teaching K-12 mathematics indicates that classroom discourse analyses are an informative way to understand a developing practice of teaching and to identify the social and cognitive aspects of the learning environment. In Wood's (1995) interactional analysis of a classroom teacher's practice, discourse provided a picture of the shifts in that teacher's practice from traditional to inquiry-based teaching by documenting patterns of interaction between teacher and students as they negotiated their roles in the classroom. Elsewhere, Peressini and Knuth (1998) used Lotman's characterization of text as a framework for analyzing an experienced secondary mathematics teacher's verbal participation in a discrete mathematics course for in-service teachers and the course's subsequent effect on his ability to cultivate dialogic discourse in his own practice. They found that modeling dialogic discourse in a professional development setting was not sufficient to produce change in the teacher's ability to foster dialogic discourse. They concluded that professional development should explicitly address univocal and dialogic discourse and, particularly, instruct how to create dialogic discourse. Additionally, Blanton, Berenson, and Norwood (in press) found that the discursive act (or lack thereof) of students reasoning about mathematics and sharing their ideas helped to shape a student teacher's practice of teaching. In this case, the teacher's pedagogy was not merely a function of influences external to her classroom, but was deeply connected to this setting by virtue of the discourse that occurred. They also argue that this connection underscores the need to explicitly address classroom discourse in undergraduate settings prior to student teaching.

These theoretical and empirical results inform us of (a) the necessity for creating contexts in which students (including teachers as students) build their mathematical thinking through social, discursive interactions; and (b) the value of classroom discourse as both an object of study for in-service and pre-service teachers and as a tool for understanding teacher learning. Specifically, these results informed my own pedagogical thinking and actions about a focus on classroom discourse. First, students were encouraged to participate in discourse about mathematics and teaching mathematics, providing them in Vygotskian terms with an essential (social) precursor to their internalization of mathematical and pedagogical thought as inner speech. Moreover, my pedagogical intent was to create a discourse that reflected the balance of univocal and dialogic functioning argued by Lotman (1988) so that students had the potential for internalizing an appropriate form of discourse. Finally, the act of social speech that reflected a dualistic balance in its function was publicly valued and examined within the classroom setting as an appropriate pedagogy for these students to cultivate in their own classrooms as future teachers.

Designing a Mathematics Course to Challenge Pre-service Teachers' Notions of Discourse

The stated goals of the course were to integrate a study of content and pedagogy so that students would (a) develop an understanding of geometries (emphasis on the plural); (b) develop logical thinking skills through an emphasis on constructing proofs; and (c) develop an understanding of meaningful mathematical discourse. The two emphases for the course, pedagogy and mathematics, are elaborated in the remainder of this section.

The Pedagogical Focus of the Course

The geometry course was structured to challenge students' thinking about mathematical discourse on three levels: (a) that of student, as a participant in discourse; (b) that of pre-service teacher, as a student in the pedagogy of discourse; and (c) that of teacher, as an architect of discourse. Although the levels comprising this organizing triad are recorded here as disjoint events, they were in fact intricately connected within the classroom, where mathematical discourse naturally merged into reflections about the nature of that discourse and its implications for future teaching practices. But how might this triad have challenged these pre-service teachers' notions about discourse? First, it is in the mathematics classroom that pre-service teachers, as student participants, internalize culturally-sanctioned rules of (mathematical) discourse. As such, what more compelling forum do we have in which to engage their habits of discourse than an authentic mathematical setting? Moreover, in this setting, pre-service teachers can dissect a discursive mathematical event as they create it, allowing them to apprentice powerful techniques of discourse in situ. By comparison, the same task in other undergraduate contexts (e.g., methods courses) seems at best academic since mathematics is not the primary focus. Finally, situating pre-service teachers as architects of an object for critique (such as discourse) within the mathematics classroom shifts their challenge to an arena in which they can personally and actively construct knowledge about teaching mathematics with the continuous guidance of a more knowing other, the classroom instructor.

In this sense, it bears emphasizing an apparent feature of the organizing triad: a mathematics educator. By virtue of our discipline, mathematics educators should be more prepared to extract the pedagogy of the mathematics classroom as an object of reflection for pre-service teachers. It is our business to scrutinize the nuances of the classroom for pedagogical soundness and to be aware of and model reform-minded ways of teaching. This is not to say that content faculty cannot or do not contribute in a similar way. It is instead to point out that our research as educators requires us to grapple with issues that can become a rich part of the mathematics classroom comprised, in part or whole, of pre-service teachers. (Conversely, content faculty can make a unique contribution from their research perspectives as well.) While I had the advantage of being the sole instructor for this course, other arrangements based on partnerships with content faculty are suggested in the literature as well (see e.g., Fallon & Murray, 1991).

Students as participants in mathematical discourse. According to the Principals and standards (2000), "the act of formulating ideas to share information or arguments to convince others is an important part of learning. When ideas are exchanged and subjected to thoughtful critiques, they are often refined and improved" (p. 348). From this perspective, challenging pre-service teachers at the level of student participant meant treating mathematics as an aggregate of ideas to be established within the intellectual parameters of the class, not distributed 'ready-made' by the teacher. Thus, assuming that students were already well versed in the syntax of univocal discourse (Wertsch and Toma (1995) claim that classroom discourse in American schools is 80 percent univocal), I set out to craft classroom experiences that encouraged dialogic discourse. That is, it was my task to orchestrate conversation through which students could build their knowledge as they "participated in the interactive constitution of the situations in which they learned" (Cobb, et al. 1992, p. 119). This meant taking a secondary role in the conversation while occasionally posing questions that clarified students' thinking or that pushed them further in their thinking. This pedagogical choice was intended to place students (not the teacher) in a position of justification and argumentation. As it has been argued elsewhere,

> conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. Students who are involved in discussions in which they justify solutions – especially in the face of disagreement – will gain better mathematical understanding as they work to convince their peers about differing points of view (Principles and standards, 2000, p. 60).

Students in the pedagogy of discourse. From the mathematical discourse, I engineered class discussions about the nature of our dialoguing in order to challenge pre-service teachers as students in the pedagogy of discourse. By the 'pedagogy of discourse', I mean the function of discourse as described by Lotman, and its resultant implications for classroom instruction. Questions such as 'What was the nature of our discourse?' and 'What should it be?' were considered. My own conflicts of balancing the teacher's role in discourse became an artifact for discussion, thereby inviting pre-service teachers to see and analyze a mathematics teacher's dilemmas as they occurred. To support our discussions, students were asked to provide written reflections on the nature of our discourse and to read selections from current literature related to univocal and dialogic discourse (Blanton, 1998; Wertsch & Toma, 1995) and patterns of classroom interactions (Wood, 1995). In retrospect, the reading activities emerged as an important part of our dialogue because it cast students, informally, as participants in a research community. More importantly, it gave them a language for expressing their increasingly refined notions about discourse.

Students as architects of discourse. Challenging pre-service teachers as classroom teachers took the form of a Discourse Analysis Project (DAP). As part of this project, pairs of students selected one lesson in Euclidean geometry from the course syllabus to prepare to teach to the class. After teaching an approximately one-hour lesson (as a dyad or individually), each student completed an out-of-class assignment involving a transcription of the videotaping of the lesson and an analysis of these data according to the univocal or dialogic function of classroom discourse. As part of this assignment, each student provided a written analysis to support a particular designation of the function of discourse as univocal or dialogic. The analysis also included reflections on perceived strengths and weaknesses in the discourse and the benefit of the DAP to the student's professional development. In the final stage of the DAP, I conducted a structured, 30-minute clinical interview with each student to discuss the results of his or her analysis.

I wish to differentiate teaching in a mathematics classroom as experienced by these pre-service teachers from a practice that may be described as micro-teaching. In the latter case, pre-service teachers posing as K-12 'students' in a 'classroom' taught by a peer is intended to simulate teaching mathematics in a K-12 classroom. Although this certainly has merit, micro-teaching typically occurs in a methods course. Moreover, the mathematics typically constitutes a review for the 'students', which additionally detracts from the feeling of a 'real' mathematics classroom. And while one could argue that teaching in an undergraduate classroom differs greatly from teaching in a K-12 classroom (hence, the former experience is itself somewhat contrived), it does have the advantage of an authentic mathematical setting. I will conjecture here (and later revisit this conjecture with supportive data) that the DAP provided these pre-service teachers with a substantive experience in their preparation for teaching mathematics.

The Mathematical Focus of the Course

The dominant mathematical themes of the course were proof and justification, in particular, building formal and informal arguments for topics in Euclidean, non-Euclidean, and finite geometries. While the course addressed traditional Euclidean topics, including congruency and similarity relationships in polygons, special properties of triangles, the Parallel Postulate and transversals, geometric properties of circles (e.g., tangents, chords, arc-angle relationships), area of polygonal and circular regions, and Platonic solids, we also explored Reimannian (spherical) geometry and finite (three-, four-, and seven-point) geometries. Our study of finite and non-Euclidean geometries, which represented about twenty-five percent of the course material, began with an exploration of axioms as well as the development of a model consistent with those axioms. For finite geometries, we then constructed arguments, both collectively (in class) and individually (out of class), for theorems that arose within these axiomatic systems.

Investigations using physical manipulatives (e.g., *Lenart Spheres*®, Platonic solids) and *Geometer's Sketchpad*® technology were also incorporated into in-class activities as a means for both strengthening students' mathematical understanding and exposing them to concrete, grade-appropriate ways they could promote their students' mathematical understanding. Including a pedagogical focus in the course necessarily precluded the study of some geometric concepts. Even so, mathematics was the predominant focus of the course, requiring about eighty-five percent of in-class activity and discussion. In-class mathematical discussions and activities centered on the development of arguments for conjectures posed by students or included in the course content, as well as hands-on investigations of geometric concepts.

In addition to in-class mathematical problems geared toward peer argumentation and the development of dialogic discourse, students were asked to complete approximately weekly out-of-class geometry assignments that emphasized the construction of written arguments to traditional theorems and non-traditional problems. None of the geometry problems assigned were rote or repetitive in nature, but were instead non-routine, problem-solving activities. A typical assignment included three to five such geometry problems. (See Table 1 for sample problems.) Assignments also included alternative problems such as reflective writings (e.g., Use metaphors to describe your understanding of mathematics and teaching mathematics.; How is this course similar to or different from other math courses?; Reflect on the nature of our class discussions about finite geometry.) or designing secondary -level investigative activities using geometric concepts from class. The alternative assignments were given in addition to, not at the expense of, geometry assignments, with geometry assignments comprising approximately seventy-five percent of the total problems assigned. Students were provided feedback on all out-of-class assignments.

[INSERT TABLE 1 ABOUT HERE.]

Table 1. Sample geometry problems assigned to students.

Because of its rich axiomatic structure, geometry lends itself well to a focus on argumentation. Moreover, its concepts are not as sequentially bound as those in other mathematics courses and so the instructor has more flexibility to include a wide range of challenging but accessible problems for upper level undergraduates. For example, our study of finite geometry did not require an extensive build-up of mathematical structure before we could engage in fairly sophisticated conversations. Even so, other upper level, undergraduate mathematics courses could easily be tailored to include a particular pedagogical focus.

Data for the Study

The data corpus for this study consisted of selected video recordings of the geometry class, students' discourse analyses of their teaching (the DAP), a clinical interview with each student about his or her discourse analysis, students' reflective writings about their notions of mathematical discourse and the pedagogical and mathematical structure of the geometry course, and selections of students' in-class work.

Findings and Interpretations

An analysis of students' reflective writings confirmed that the course's structure, organized by the expectation that students mutually negotiate mathematical ideas, emphasized (a) teacher-student and student-student discourse; (b) understanding how to teach geometry in secondary grades; (c) collaborative, hands-on explorations during class; (d) understanding, rather than memorizing, mathematics; (e) collective thinking rather than copious note taking; and (f) alternative forms of assessment. Students also reported that the level of mathematical difficulty was comparable to that of their other undergraduate mathematics courses.

Emerging Mathematical Discourse

Confronting students' self-reported experiences with (predominantly) traditional pedagogy meant prompting students, throughout the semester, to engage their peers (and the teacher) in mathematical conversations. Yet, changing students' notions about discourse required more than their participation in dialogue or my efforts to model appropriate pedagogy (see also Peressini & Knuth, 1998); it also required a focused attention on and analysis of the nature of our conversations. Eventually, the discourse seemed to mature towards that dualistic balance set forth by Lotman (1988) in which utterances are, as appropriate, questioned for the purpose of generating new thinking (dialogic) or clarified for clear communication (univocal). The following excerpt, which occurred over half way through the semester and after students had completed the DAP, was selected to convey the nature of dialoguing about mathematics which had emerged in our class. In this episode, the mathematical task was to develop a model, or representation, for three-point geometry (see Table 2).

[INSERT TABLE 2 ABOUT HERE]

Table 2. Axioms for three-point geometry (Smart, 1994).

One of the students, Jia, had drawn her group's representation on the board as a point for discussion (see Figure 1). As the excerpt opens, students are trying to verify that the axiom "two distinct points are on exactly one line" is true for this representation. In particular, Brad seems to be confusing the task of showing that any two distinct points are on exactly one line with showing that any line contains two points. While the excerpt is lengthy, it is difficult to convey the essence of the discussion otherwise. Even so, this represents only a portion of the class discussion, which extended for most of an hour. (All names are pseudonyms.)

[INSERT FIGURE 1 ABOUT HERE]

Figure 1. Representation proposed for three-point geometry by Jia's group.

1❋ Teacher: OK, find two distinct points on that model.

2❋ Brad: Well they're not distinct … because those are two points [on this line], but there are also two points for that [line], so they're not distinct.

Brad seems to take the notion of "distinct" to mean that only one line can have exactly 2 points. As he argues here, there is at least a second line that also has exactly 2 points which, for him, contradicts the notion of distinctness.

3❋ Teacher: Well, you circled 2 [points] over there. Are you saying they're not distinct or they are distinct?

My goal here was not to explain my own thinking (and hence the error in Brad's), but to push Brad's thinking further and get him to articulate his position.

4 Brad: According, no, they're not.

5 Laura: They're distinct, they're distinct.

At this point, students begin to simultaneously argue their interpretations of 'distinct'. Brad's question (15) overrides the other comments.

6 Brad: What is distinct?

7❋ Teacher: What <u>is</u> distinct? Good question.

At this point, I made the choice to turn the question back to students, rather than giving my own clarification of 'distinctness'. This pedagogical decision was intended to confront their readiness to lean on the teacher's knowledge instead of their own or that of their peers. As it turned out, Laura and Jia were able to clarify this notion for Brad (8[3], 17).

8❋ Laura: Not the same points. They're two different points.

9❋ Brad: But on that line, well (pause)….

10❋Teacher: Do we agree that what Jia has drawn up there (see Figure 1)...[are] three distinct points, A, B, and C? (Students indicate agreement.) Now, I want you to show me that 2 distinct points are on exactly one line.

11 Beth: Shouldn't we define what a line is?

12 Teacher: What is a line?

Again, the class spontaneously erupts in an intense (but untranscribable) discussion about what is meant by 'line', thus calling into question a notion they had heretofore taken for granted.

13 Laura: What I don't understand, if it's saying there are two distinct points on exactly one line, does that mean a point can only be on one line and not another line?

14 Brad: That's what I thought...

15 Laura: That's what I thought when it said exactly one line, that only these two points can make one line and they can't be on another one.

16 Brad: That means [this is] wrong (Brad indicates the representation on the board).

At this point, I asked students from Jia's group, who had until now been silent, to respond.

17 Jia: As far as the 2 distinct points are on exactly one line, so that means A and B are on the line that connects them, and they are 2 different points, and A and C are 2 different points that are just on one line. Same for B and C.

18 Andrea: [Our group] looked at [the points] as a set: {A, B}, {A, C}, and {B, C}.

Andrea and Jia were two members of the triad whose representation (see Figure 1) was being argued.

19 Teacher: So let's go back to what you were saying Jia. You pointed out {A, B}, for example, as being 2 distinct points that are on this line connecting points A and B (indicating the respective line in the representation).

20 Jia: Right.

21 Teacher: And are A and B, those two distinct points, on another line, an additional line?

22 Laura: Yes.

23 Teacher: Which one?

24 Beth: Wait, wait.

25 Laura: Oh, they're not on [another line] <u>together</u>.

26 Beth: Couldn't you have a third [point] on that line though? It says two distinct points are on exactly one line, so what about the other third point? Could that also be on the line?

27 Jia: But not all three points in that geometry are on the same line. (Here, Jia is referring to one of the axioms of three-point geometry.)

28 Teacher: That's good. Now, let's ... make sure everybody's clear. We've got two distinct points A and B. They are on the line connecting A and B. Are they on a different line?

29 Laura: All right, when you say those two distinct points, are you saying those two distinct points together, or just those with another one?

30 Teacher: No, A and B, two distinct points collectively.

31 Laura: No (indicating they aren't on a different line). (Other students register their agreement.)

At this point, after a fair amount of discussion, we seemed to have established the idea that "two distinct points are on exactly one line" for the representation posed by Jia's group. Later in the discussion, Beth proposed another model (see Figure 2) in which she questioned if lines had to be "straight". As students began to confront their long-held Euclidean notions of a line, a debate ensued over how a line might be represented in three-point geometry.

[INSERT FIGURE 2 ABOUT HERE]

Figure 2. Beth's proposed model.

32 Teacher: Is that a model (referring to Beth's representation – see Figure 2)?

33 Laura: Because on that one you don't have.... I think it would be. Because, you have to define a line...

34 Beth: Yeah.

35 Laura: ...is a line straight or is it curved? Earlier we just defined a line as connecting 2 distinct points.

36 Teacher: Worth? (He goes to the board).

37 Worth: I think this (he indicates the model drawn by Jia – see Figure 1) is closer to correct where it has to terminate there (i.e., at the points) because what about this example where this

line can curve. It can do all sorts of things. What if it did that? (He extends one of the lines in a curved manner so that it intersects a third point [see Figure 3]). In Euclidean geometry, it never would do that. But if we allow it to go here (i.e., to the third point), and we say it can do whatever it wants, and it hits that third line, then it (the model) doesn't pass those axioms anymore.

[INSERT FIGURE 3 ABOUT HERE]

Figure 3. Worth's counterexample of a three-point geometry model.

38 Teacher: It picks up another point.

39 Worth: It picks up another point. So I think it has to terminate at the [point].

Worth is arguing against a model for which the 'lines' extend beyond a point of intersection (as in lines BC and AC, which extend beyond point C in Figure 2). Although such a representation (i.e., Figure 2) is mathematically correct, his argument is that if a line is allowed to extend beyond a point of intersection, or vertex, then it might "eventually" intersect with a third point.

40 Laura: But when it terminates, [the line] can still terminate at [a] point and not be straight.

41 Worth: In between the 2 points it can curve, but it can't go through another point.

42 Teacher: Why can't it go through another point?

43 Laura: Because then it would be like...

44 Worth: [It would contradict] axiom 3.

As our discussion continued, we were able to establish that representations such as those given in Figure 2 and Figure 4 were models for three-point geometry and that Worth's argument was legitimate and should be (and was) considered in generating a model. That is, it was agreed that a line could extend beyond a point of intersection as long as it did not intersect the third point of the geometry.

[INSERT FIGURE 4 ABOUT HERE]

Figure 4. Model included by the instructor in the discussion.

Meanwhile, Laura amended Worth's model to consist of two "straight" lines (AB and BC) and one "curved" line (AC) and proposed it as yet another representation (see Figure 5).

[INSERT FIGURE 5 ABOUT HERE]

Figure 5. Model proposed by Laura as an extension of Worth's representation.

45 Teacher: How is Laura's model (see Figure 5) different from [the model proposed by Jia's group] (see Figure 1)?

46 Beth: It's not.

47 Brad: It sure looks different.

48 Jia: It isn't different though.

49 Beth: Just like if you drew ... a circle and [divided the circle into three parts and] you considered each part as like one line. That's why I was asking if you could curve [the line].

50 Katy: The model [drawn by Jia's group] (see Figure 1) is exactly like the model that Laura just drew (Figure 5) because you could say that ABCA is a line, whereas the teepee (Figure 4) has 3 distinct lines.

Katy seems to be arguing that lines are distinct when their 'endpoints' do not adjoin another line. In this sense, she argues that Jia's and Laura's models (see Figure 1 and Figure 5, respectively) are alike in that they each represent one line containing 3 points.

51 Sheila: So what is a line?

52 Katy: So that one triangle (Figure 1), that whole thing, is a line.

Again, the class spontaneously erupts in (untranscribable) argument over Katy's claim, invoking the axioms recorded on the board (see Table 2) to support their positions. Finally, Beth's objection (53) to students' arguments that a line cannot contain points A, B, and C is distinguishable above the rest.

53 Beth: But you can say that ABC is one line.

54 Katy: If you did say that, if you saw it as one line, then axiom 3 would not be satisfied. That's what I'm saying. That model. But with the one below it (referring to Figure 4), it is satisfied.

From (50, 52, and 54), it seems that Katy is arguing that one could interpret a line as containing three points when there is no visual notion of distinct lines. However, she notes that while the representation may be characterized as a line (perhaps in some geometry), it cannot be a model for three-point geometry because the line contains three points and this contradicts the axioms of three-point geometry. Students agreed with Katy's reasoning.

What can we infer from this excerpt about these students' mathematical discourse? Since the function of discourse (i.e., whether univocal or dialogic) is based on both the speaker's intent and the respondent's passive or active interpretation of a speaker's utterance, identifying it required an analysis of each speaker's utterance and the response to it. Moreover, interpreting the function of discourse is a subjective process and ultimately requires one to identify an utterance as being predominantly, not exclusively, univocal or dialogic. As Peressini and Knuth (1998) describe, any social interaction requires that each participant decipher text and generate his/her own meaning, thus, all discourse contains a measure of both dialogic and univocal functioning.

Perhaps most significantly, an analysis of the function of discourse in this excerpt suggests that these students were able to use the collective knowledge of the class to generate new understanding, the essence of dialogic discourse. Consider the interaction between Laura and Brad (13-16). The speaker (Laura) articulated her point of confusion (13, 15) and Brad responded by making sense of her utterance, matching it to his own conception. We can infer this from his response (14) in which he validated her position by identifying it as his own. He subsequently treated this interaction as a springboard for assessing Jia's proposed representation (16). Later in the discussion, Beth questioned if a third point could also be on a line (26). Again, the respondent (Jia) took a questioning stance towards the speaker's utterance and ultimately rejected it based on the axioms of the geometry (27). As Beth's belief that a line was characterized by its constitutive segments, not the number of points it contained, persisted (49, 53), Katy actively addressed this conception by assuming it to be true, then building an argument to it that simultaneously allowed for a line to contain three points, but not to serve as a model for this geometry. In this exchange, Katy seemed to treat Beth's utterance as a starting point for arguing her own position. In particular, she began by validating Beth's perspective that the symbol designated as line ABC could be a line ("If you did say that, if you saw it as one line…"), then pursuing the implications for this assumption ("then axiom 3 would not be satisfied. That's what I'm saying…. But with the one below it (referring to Figure 4), it is satisfied.")

There were also points in the dialogue that functioned univocally in that information was given or requested for clarification. For instance, in (19) I restated to Jia what I thought represented her prior comment (17) in order to confirm that our codes were aligned. At another point, Laura asked for clarification about what I meant by 'two distinct points' (29). Interpreting her utterance univocally, I responded by clarifying the intended meaning (30). Finally, in (17-18) Jia and Andrea provided information about their representation (Figure 1). I characterize this as univocal because they seemed to be responding to my request for information, not actively addressing Brad's conclusion about their representation (16). In other words, their comments did not seem intended to build on Brad's notion, but instead were a form of information we could use to asses how our ideas about the representation coordinated with theirs. It is important to note that discourse (or text, more generally) should include univocal functioning. Indeed, it must occur in order for communication to be clear (Lotman, 1988).

Although I have described instances of univocal and dialogic functioning here as disjoint events, it seemed that they were tightly connected in actual dialogue. That is, throughout this episode, there was a natural, ongoing transition between these two functions that seemed to speak to Lotman's claim that they should be dualistically balanced in practice. For example, I characterized (19-20) as univocal in its function, yet this interaction quickly merged into dialogic discourse when Laura and Beth joined the exchange. In particular, rather than interpreting Laura's incorrect response (22) univocally by assuming a fault in Jia's or my communication about the meaning of two distinct points on a line (17; 19), I questioned Laura (23) to try to engage her active participation. Doing this required Laura to create her own sense (25) rather than respond to an explanation I might have given. I would argue that this induced an active rather than passive stance in how Laura participated in the discourse, thus creating a dialogic context that seemed to engage her as a learner. Moreover, Beth drew on this exchange to conjecture about the possibility of a third point on a line (26), and as noted earlier, Jia seemed to interpret Beth's utterance dialogically in that she questioned and ultimately rejected the conjecture (27). Finally, the conversation merged back into discourse that functioned univocally (28-30), as I questioned students to confirm that our codes were aligned (28) and was consequently asked to explain the meaning of 'two distinct points' (29). My clarifying comment (30) suggests that I interpreted Laura's question univocally.

This type of transitioning is reflected in Figure 6, which provides an approximate representation of how univocal and dialogic functioning occurred in a portion of the discourse excerpted here. It is approximate in that the nature of a conversation dynamic makes it difficult in practice to tease apart these functions as discrete events and in that identifying the function of discourse is interpretive and thus cannot be an exact process. This graphic is not intended to imply that all discourse should have approximately equal instances of univocal and dialogic functioning. The balance depends on the purpose of the dialogue (e.g., giving someone directions to a location would be predominantly univocal). I do suggest that this representation does reflect the existence of an appropriate balance in our conversational purpose.

[INSERT FIGURE 6 ABOUT HERE]

Figure 6. Representation of the occurrence of univocal and dialogic functioning in a selection of classroom discourse.

In essence, (1-54) could be characterized as an activity of eliminating perceived incongruities between one's own and others' mathematical activity (Cobb, Yackel, & Wood, 1992) through a process of mutual sense making which included both univocal and dialogic discourse. That is, what seems clear is that students were engaged in an activity of collective negotiation in which a speaker's utterances were questioned and validated or rejected in order to generate new understanding, or clarified so that clear communication could occur.

At a minimum, this excerpt offers an existence proof that students can interrupt the inertia of passive listening and create and sustain meaningful dialogue. In particular, it seems that there was a shift in how they, as students, were able to participate in mathematical discourse, a shift away from their self-reported experiences as passive listeners toward a level of participation that reflected a more dualistic balance between univocal and dialogic discourse. Through this process, these students were able to identify and articulate their points of confusion (e.g., 13, 15), form conjectures (e.g., 26), and successfully argue the validity of conjectures posed by teacher or student (e.g., 17, 27, 45-54). As one student observed about this discussion, "Each student helped the next to change or refine his ideas", such as in (32-44) and (45-54), where students negotiated a more rigorous notion about what constituted a line in this geometry. The Principles and standards (2000, p. 348) argues that through this process of students refining each others' ideas, they are able to "sharpen their skills in critiquing and following others' logic" and consequently, "as students develop clearer and more coherent communication (using verbal explanations and appropriate mathematical notation and representations), they will become better mathematical thinkers."

Finally, it is significant that students' perceptions about the three-point geometry discussion seemed consistent with this analysis. Their written reflections about this discussion contained the following observations:

(a) "What happened in class was a perfect example of dialogic function. The students kept building on each other's thoughts, making it a collaborative process and a real learning process.";

(b) "Students posed their own interpretation of the rules which the other students evaluated and either supported or rejected.... Had the teacher introduced three-point geometry by discussing its rules, giving her own examples, and the model for the geometry, students would have inevitably been bored.";

(c) "Finite geometry was completely new to me.... Thus, [the teacher] could have just given us the facts... or let us explore on our own. Luckily, [the teacher] chose the latter approach. Thoughts were flying all over the room about the basics of this geometry as we slowly figured it out on our own."

It seems that the advantage for these students as participants in the discourse was that it exposed and prioritized their thinking (not mine), it allowed them to make sense of what ideas needed to be established and to form and argue conjectures, it required them to justify their positions in establishing the validity of a conjecture, and it enabled them to develop precision with the language of mathematics (e.g., notions of 'distinct'). I would argue that, because of this process, students developed a more complex understanding of the topics. For example, by proposing and articulating a justification for their representation of three-point geometry, students were able to openly confront their strongly-held notions from Euclidean geometry (e.g., the representation of a line), pick apart the subtleties of meaning packed in a statement such as "two distinct points are on exactly one line", and refine their notions of when a mathematical claim has been established and what it takes to do so (a recurring issue which students reported as unresolved from their previous course work). In short, these students seemed to be learning how to think and argue mathematically by the sheer act of doing it. They also seemed to be learning that students need those types of discursive experiences in order to become critical thinkers. As one student observed, "I am beginning to realize now that it is going to be difficult to expect my own students to think critically when I have never been expected to do so."

Emerging Models of Teaching

The discussion excerpted above (1-54) tells us about these students as participants in mathematical discourse. The question remains as to how the organizing triad outlined earlier challenged their internalized models of teaching with respect to the function and role of discourse in instructional practice. In this section, I draw from an in-class pedagogical discussion and students' DAPs to examine shifts in this aspect of students' thinking.

Analyzing mathematical discourse in situ. By design, mathematical discourse became an artifact for our in-class analysis, the purpose being to treat an authentic mathematical event (for example, a mathematical discussion that occurs as part of a mathematics course as opposed to a methods course) as an object by which to expand students' thinking about the discourse that constituted that event. As such, our conversations about discourse were not based on hypothetical referents (which could arguably be the case in a traditional methods course), but mathematical discourse that was intensely students' own, immediate experience. I conjecture that having this type of pedagogical discussion (see 55-82) occur as a natural progression of an experience with that pedagogy in an authentic mathematical setting (e.g., 1-54) fundamentally altered the ways these students thought about discourse. The frequency and length of these pedagogical discussions varied throughout the semester, occupying about fifteen percent of in-class time, or, on average, about fifteen minutes of a 75-minute class. While this seems like a brief amount of time, I still found that this, in conjunction with out-of-class activities, increased students' sensitivity to classroom discourse. The selection below, which chronicles part of our collective reflection on the mathematical discussion about representations for three-point geometry (1-54), is included to illustrate this type of discussion and to document students' emergent thinking about the pedagogy of discourse.

55 Teacher: Sometimes teachers will think that in this type of interaction (i.e., 1-54) there's a lot of confusion, which there seems to be, which is good, I think. It's very good, and [you might think] that you don't get stuff done, it's not efficient, you don't "cover the curriculum". But if you get from Chapter 1 to Chapter 50 and your students have no idea what's going on, I don't know that you've been successful.

56 Brad: Yeah, but it does get frustrating after a certain period of time. Can you imagine middle school? I mean they would enjoy it, but there's going to be some kid that's going, "What in the heck?".

57 Sheila: That's why you give them notes.

58 Worth: You do this [type of discussion] to a point, but then you've got to tell them what the right answer is.

59 Brad: Yeah.

60 Laura: You need a nice combination of both (i.e., discussion and leading to the "right answer").

61 Brad: I was going to say that the reason we think like that, the reason we get frustrated is we're used to getting right answers.

62 Teacher: Absolutely.

63 Brad: If you start kids doing this at the beginning, I guess they would be used to it and they wouldn't care about right or wrong answers. But we're so concerned about right or wrong answers, that's why we get frustrated. (Students expressed their agreement.)

64 Teacher: It's not that we're not concerned with what the right answer is. I want to make that clear.... It is that there's [sometimes] more than [getting] the right answer. Students need to experience what it means to argue mathematics and think about ideas because when they leave class, whether or not they can do it (by "do it", I mean do mathematics in a deep, conceptual way, not follow rote procedures) is going to depend on whether or not they can argue with themselves and think through theorems, and so forth.

65 Lori: But can you do this with finite math? Do you think you can have this much dialogic conversation with finite math?

66 Brad: I don't know.

67 Teacher: Do you mean, like, Euclidean geometry?

68 Laura: Yeah.

69 Teacher: I certainly do.

70 Laura: Like $x + 6 = 10$.

71 Jan: I think you can because, like in those articles you gave us,... the teacher would ask the kids how they came to a conclusion on an answer and I had to really think about some of those answers because I was like, "How did they get that?". Then I [realized what they meant] and I thought, "It does make sense!"

72 Laura: It's just like [the article] where [the teacher] was doing algebraic functions. She broke them up into groups and she was like, "How did your group get that?". (Here, Laura is referring to an instance in which a teacher prompted students to justify their results). It's not as much dialogic [conversation] as we give, but at least it's a little bit.

73 Katy: The thing that I get from [the readings] is that it is good to hear different ways to arrive at the right solution, but I don't understand why it's helpful to explore how a student got the wrong solution. I mean, for the teacher, yeah, because you can learn where they're messing up, but for the rest of the class, it seems like.... I think in one of the articles a teacher explored why a kid got [an incorrect answer] and I didn't understand why it was necessary....

74 Teacher: Why a teacher would want to do that?

75 Katy: Yeah.

76 Brad: Well, there are probably others who are doing that (incorrect procedure), first of all.

77 Andrea: And if you can recognize an incorrect procedure, that helps you. It's like, even though your model (referring to a representation proposed by Katy during the three-point geometry discussion) wasn't correct, it helped us all understand a correct model.

78 Teacher: It's a counterexample.

79 Brad: Yeah and we see where they messed up.

80 Andrea: So in a way, you did help us [by sharing an incorrect model for the 3-point geometry] because we had to think about, "Is this right?".

81 Brad: We thought it was and then we [realized it wasn't] and that reinforced these things (pointing to the axioms on the board).

82 Jia: And we can figure out why [the model] was wrong.

What can we glean from this conversation about the development of these students' notions about discourse? First, it seems that this <u>pedagogical</u> discourse was structurally similar to our mathematical discourse in that it reflects a balance of univocal and dialogic functioning, with students using their peers' utterances to generate new understanding (e.g., 73-82). For instance, when Katy questioned why it might be useful to explore a student's incorrect response (73), Andrea took a previous mathematical episode in which Katy had proposed an incorrect representation for three-point geometry and, building on Brad's comment (76), argued that an incorrect response allows students to better understand a correct one (77). Brad and Jia supported Andrea's argument by noting that exploring Katy's incorrect model had allowed them to determine how and why the model was incorrect (79, 82). Furthermore, Katy's question had stemmed from her effort to make sense of the assigned reading and thus illustrates her active stance toward that material. Taken together, these events seemed to generate new understanding for Katy about the instructive power of exploring students' incorrect solutions in classroom discourse. In this sense, I would describe the interaction (73-82) as essentially dialogic in its function. I take students' application of such discursive practices to a context other than mathematics as further evidence for their internalization of the ways they think about and participate in discourse. Additionally, I take the following as an indication that these students were beginning to critically reflect on the pedagogy of discourse:

(a) Brad's metacognitive activity, through which he identified characteristics of classroom discourse as an attribute of school culture (56, 61, 63);

(b) reflections and observations by Jan (71), Laura (72), and Katy (73) that built on their knowledge of current research about discourse;

(c) Laura's effort (65) to extend her knowledge about dialogic discourse to areas of mathematics she would soon teach (e.g., algebra);

(d) Andrea's ability to connect her mathematical experience in the class with a peer's question about a particular discursive practice (i.e., exploring a student's incorrect response) (73; 77; 80); and

(e) students' increasingly sophisticated language about discourse, in particular, their use of the constructs 'univocal' and 'dialogic' as a framework by which they talked about discourse (65, 72). (This was confirmed by informal data as well: Students reported using univocal and dialogic functioning as a framework to analyze discourse in other classes.)

Students as architects of discourse: The DAP. The DAPs provided a snapshot of students' teaching practices, and more significantly, students' reflections on and analyses of the kinds of discourse they created and their notions about what discourse should resemble. In addition, the experience itself of doing a DAP seemed to be a powerful mechanism for these pre-service teachers' professional development. Analysis of the DAPs showed that individual results were highly comparable (this was not surprising, given the academic homogeneity of the class). These results are organized in the remainder of this section according to (a) students' reflections on their teaching experience in the DAP, in particular, the function of and reasons for the discourse they promoted in their teaching experience; (b) the development of students' understanding about discourse; (c) shifts in students' thinking about the role of discourse in their practice of teaching; and (d) perceived benefits of the DAP to students' professional development. (These categories are neither unique nor mutually exclusive.)

Students reported in their DAPs and in clinical interviews that it was far more difficult to create dialogic discourse than they had anticipated. They recognized constraints imposed by time as well as the patience it takes to allow students to lead the discourse. In analyzing his classroom transcripts, one student observed that

> "there were many times ... I could see places that univocal functioning took place when dialogic functioning should have, ... times when I jumped too quickly to answer questions and give the right answer when I should have allowed the students to discuss the matters and resolve them among themselves."

Students noted that, if at all, dialogic conversation occurred during group or paired activities, or when the instructor was confused (because, as one student noted, he then "could not take the position of authority"). In the latter case, that confusion essentially placed the pre-service teacher in the role of learner; hence, the fact that dialogic discourse grew out of this dynamic is consonant with findings by Peressini and Knuth (1998). It suggests that teachers need to be especially sensitive that they don't cultivate predominantly univocal discourse when their mathematical understanding seems most clear.

Without exception, students each found that the function of discourse constituting their respective classroom teaching experiences was essentially univocal, although their perceptions about their teaching prior to the DAP was that it would be characterized by dialogic discourse. In her DAP, Andrea noted that "univocal text dominated the classroom dialogue. Students [were] not allowed to develop their own methods for [constructing a] proof and [were] instead led to [my] proof solution step by step. It is clear that no dialogic conversation [took] place because the students [did] not use the teacher's or each other's utterances as a basis for re-interpretation."

Katy found that she was challenged by her beliefs about what constitutes good teaching and her actual practice of teaching:

"Students heard mainly what I planned for them to hear. If the discourse started moving away from my script, I fought to get it back where I wanted it. I always believed in a teacher as a facilitator, [someone who] encourages thinking and does not "spoon feed" ideas to students. Yet, I did not think about how to facilitate thinking."

As a result of their findings, students stated their intent (and wish) to cultivate more dialogic discourse in their practice, with one student noting how "ineffective and mundane univocal dialogue can be".

Reflecting on reasons for the type of discourse they promoted in their teaching, some students suggested that discourse was essentially univocal because of a need for structure and control, or because of a perception about the nature of mathematics: Jia wrote, "I believe univocal functioning [dominated] my lesson because of my need for a direct, concise and organized presentation", while Katy observed that "with univocal discourse I do not have to risk confusion in the classroom". Sheila wrote "my view of mathematics… as a series of steps… contributed to how I [conducted] the lesson in a univocal style".

As a result of the DAP, students not only developed a more accurate perception of their practice of teaching, but also a more complete understanding of the pedagogy of discourse. Katy's observation documents a clarification in her own thinking:

"Before [the DAP], I thought two-way communication existed if the teacher and students alternated dialogue. Well, I was not the only person speaking during my lesson. Alternating dialogue took place. Still, the discourse was mostly univocal."

In addition, students reported that understanding the dualistic function of discourse (Lotman, 1988) would better equip them to create active learning situations for their students because that knowledge gave them a tangible sense of what classroom discourse could and should resemble.

Students' analyses of the discourse in their teaching made them aware of the type of practice they might likely have. It seemed that this realization, coupled with the evolution in their understanding about discourse and its appropriate forms, converged to promote shifts in their thinking about what should characterize their practice of teaching. For example, recognizing their habit of "giving answers" seemed to underscore students' felt need, as teachers, to instead ask thought-provoking questions and allow their students to be the problem solvers – in essence, to cultivate a different kind of discourse (dialogic) than that which characterized their teaching experience in the DAP (univocal). Brad wrote:

> "The fact that I believe [that the time students use struggling with the mathematics] is lost is a misconception that many teachers have. This time is actually a time when students are attempting to generate new meaning and understand the material. This is an area in which I must become more comfortable … and must learn not to…give out answers."

Evan's thinking reflected a similar shift:

> "Students will learn more if they are allowed to share and explore their own thoughts. Also, students need to be able to interact with one another as well as with the teacher. It is very easy for the teacher to fall into the trap of spoon feeding material to the students without realizing that it is happening. I now realize that it is okay if students struggle with problems some or come up with different ways to solve problems. The teacher must share the classroom and the position of authority with the students."

Katy recognized that she needed to be listening rather than telling: "I must hear and understand my students' thinking in order to help them learn. They must raise questions and share ideas different from my own to learn from each other".

Worth, a middle school teacher seeking certification, articulated a perspective about the role of dialogic discourse that was more conservative than that of his peers, yet consistent with his ongoing (and not uncommon) concern that dialogic discourse required more class time than he felt he could afford. In his DAP, he wrote

> "I was open to dialogic discourse when it did occur.... If the discourse shifted away from my questions and game plan, I was patient and encouraged the discussion until I could get back to my planned lesson.... This is not always easy to do when the clock is slowly creeping towards the end of class and we are only half way through the lesson."

In Worth's thinking, it seemed that the purpose of dialogic discourse was at best a type of mental field trip for students, at worst an interruption. It was not a critical path by which students could construct an understanding of mathematics; it was an activity he allowed them to participate in, even though it was not the real purpose of his agenda. In spite of this uncertainty, he acknowledged that the "biggest weakness of the discourse [during his teaching experience] was the lack of extensive dialogic function and class discussion". Moreover, he described his intent to cultivate a different type of teaching practice, one that included dialogic discourse:

> "I (until now) looked at teaching very much in terms of teaching a lesson and asking if there are any questions. If not, then I move on. I might occasionally ask a question designed to provoke thinking but never as a means to teach the lesson. Based on [the DAP] and our discussions in class, I am going to attempt to change my way of thinking and be more open to discussion, even if it means falling behind in the curriculum."

With a perspective more cautious than that of his peers, it still seems that even Worth's models of teaching, particularly his notions of discourse, were challenged. In fact, it seems implicit that the DAP was a useful catalyst for promoting change in these pre-service teachers' thinking about discourse. Students argued this explicitly as well:

"I think doing this [DAP]...opened my eyes in a lot of ways. Applying what I had learned to my own experience (i.e., teaching the class) allowed me to understand where I am and where I would like to be by the time I begin teaching.";

"[This DAP] has pointed out to me the importance of student input.";

"I can't say enough how beneficial this [DAP] has been.... It showed me how important dialogic discourse is in a classroom".

Conclusions

This study explored an approach for challenging pre-service secondary mathematics teachers' fundamental notions about discourse by using the undergraduate mathematics classroom to engage their thinking as participants in discourse, as students in the pedagogy of discourse, and as architects of discourse. Results suggest that the three components of this organizing triad converged to (a) shift pre-service teachers' thinking to include an image of discourse as an active process in which students use the collective knowledge of a group to build understanding (i.e., dialogic discourse); (b) strengthen pre-service teachers' ability to participate, as students of mathematics, in discourse that reflects a balance of univocal and dialogic functioning; and (c) through an analysis of their own practice, reveal their habits of discourse as classroom teachers and subsequent implications for their own professional development. Given these shifts in pre-service teachers' thinking in such a limited time frame (one semester), it is feasible that a focus on classroom discourse which extends over a longer period of time and which includes secondary classrooms as a teaching site for pre-service teachers would intensify their development.

The results of this study offer a compelling argument for designing pedagogically challenging undergraduate mathematics courses for pre-service secondary teachers. Indeed, it suggests that, more than the traditional methods classroom, the mathematics classroom is a powerful and unique forum in which pre-service teachers can practice, articulate, and reflect collectively on reform-minded teaching. The litmus test for this study will be how these pre-service teachers create discourse as instructors in their own classrooms. At this point, I can only attest to their intent to have a teaching practice that balances classroom discourse in the manner argued by Lotman (1988) and report that they have an early understanding of and experience with what that means. It is in this sense that this study provides a fruitful approach for challenging pre-service teachers' notions of classroom discourse as participants in and architects of that discourse.

Notes

[1] I use the term "secondary" here to include middle and upper grades (i.e., grades 6-12).

[2] Since the notion of discourse may connote a variety of meanings, it is specified here to denote talk, or utterances, made about mathematics or teaching mathematics by teacher and students in the classroom.

[3] Numbers indicate lines in the protocol.

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References

Ball, D. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. Elementary School Journal, 93(4), 373-397.

Blanton, M. (1998). Prospective teachers' emerging pedagogical content knowledge during the professional semester: A Vygotskian perspective on teacher development. Unpublished doctoral dissertation, North Carolina State University.

Blanton, M., Berenson, S., & Norwood, K. Using classroom discourse to understand a prospective mathematics teacher's developing practice. Teaching and Teacher Education (to appear).

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. Educational Studies in Mathematics, 23(1), 99-122.

Fallon, D., & Murray, F. (1991). Project 30: Year two report. Newark: School of Education, University of Delaware.

Grossman, P. (1990). The making of a teacher: Teacher knowledge and teacher education. New York: Teachers College.

Hanna, G., & Yackel, E. Reasoning and proof. In A research companion to NCTM's Standards, edited by Jeremy Kilpatrick, W. Gary Martin, and Deborah Shifter. Reston, Va.: National Council of Teachers of Mathematics (to appear).

Kaput, J. & Dubinsky, E. (Eds.). (1994). Research issues in undergraduate mathematics learning: Preliminary analyses and results. Washington, D.C.: Mathematical Association of America.

Lampert, M. (1986). Teaching multiplication. Journal of Mathematical Behavior 5 (3), 241-80.

Lortie, D. (1975). Schoolteacher: A sociological study. Chicago: University of Chicago.

Lotman, Y. (1988). Text within a text. Soviet Psychology, 26(3), 32-51.

Minick, N. (1996). The development of Vygotsky's thought. In H. Daniels, (Ed.), An introduction to Vygotsky (pp. 28-52). London: Routledge.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: Author.

Peressini, D., & Knuth, E. (1998). Why are you talking when you could be listening? The role of discourse and reflection in the professional development of a secondary mathematics teacher. Teaching and Teacher Education, 14 (1), 107-125.

Schoenfeld, A. (Ed.). (1990). A source book for college mathematics teaching: A report from the MAA Committee on the Teaching of Undergraduate Mathematics. Washington, D.C.: Mathematical Association of America.

Simon, M., & Blume, G. (1996). Justifcation in the mathematics classroom: A study of prospective elementary teachers. Journal of Mathematical Behavior, 15, 3-31.

Smart, J. R. (1994). Modern geometries (4th ed.). Belmont, CA: Brooks/Cole.

Steen, L. A. (Ed.). (1992). Heeding the call for change: Suggestions for curricular change. Washington, D.C.: Mathematical Association of America.

Swafford, J., Otto, A., & Lubinski, C. (in press). Content preparation for teaching early algebra. To appear in J. Kaput (Ed.), Employing children's natural powers to build algebraic reasoning in the content of elementary mathematics.

Vygotsky, L. (1978). Mind in society. (M. Cole, S. Scribner, V. John-Steiner, & E. Souberman, Trans.). Cambridge, MA: Harvard University. (Original work published in 1934.).

Vygotsky, L. (1986). Thought and language. (A. Kozulin, Trans.). Cambridge, MA: Massachusetts Institute of Technology. (Original work published in 1934.).

Wertsch, J. (1988). L.S. Vygotsky's "new" theory of mind. The American Scholar, 57, 81-89.

Wertsch, J., & Toma, C. (1995). Discourse and learning in the classroom: A sociocultural approach. In L. Steffe & J. Gale (Eds.), Constructivism in education (pp. 159-174). Hillsdale, NJ: Lawrence Erlbaum.

Wood, T. (1995). An emerging practice of teaching. In P. Cobb & H. Bauersfeld (Eds.), The emergence of mathematical meaning: Interaction in classroom cultures (pp. 203-227). Hillsdale, NJ: Lawrence Erlbaum.

Wood, T., Cobb, P., Yackel, E., & Dillon, D. (Eds.). (1993). Rethinking elementary school mathematics: Insights and issues [Monograph No. 6]. Journal for Research in Mathematics Education.

Zeichner, K. (1996). Designing educative practicum experiences for prospective teachers. In K. Zeichner, S. Melnick, & M. L. Gomez (Eds.), Currents of reform in preservice teacher education (pp. 215-233) New York: Teachers College.

Table 1

1. The six executives of Company X have a business charter that requires the following:

(1) every pair of executives are on exactly one committee together;
(2) each committee consists of at least 3 executives;
(3) there must be 3 executives who are not all on the same committee.

What do you think and why?
(Hint: Use Fano's Geometry)

2. PROVE: In any right triangle, the altitude to the hypotenuse forms two right triangles that are similar to each other and to the original triangle.

Figure 1

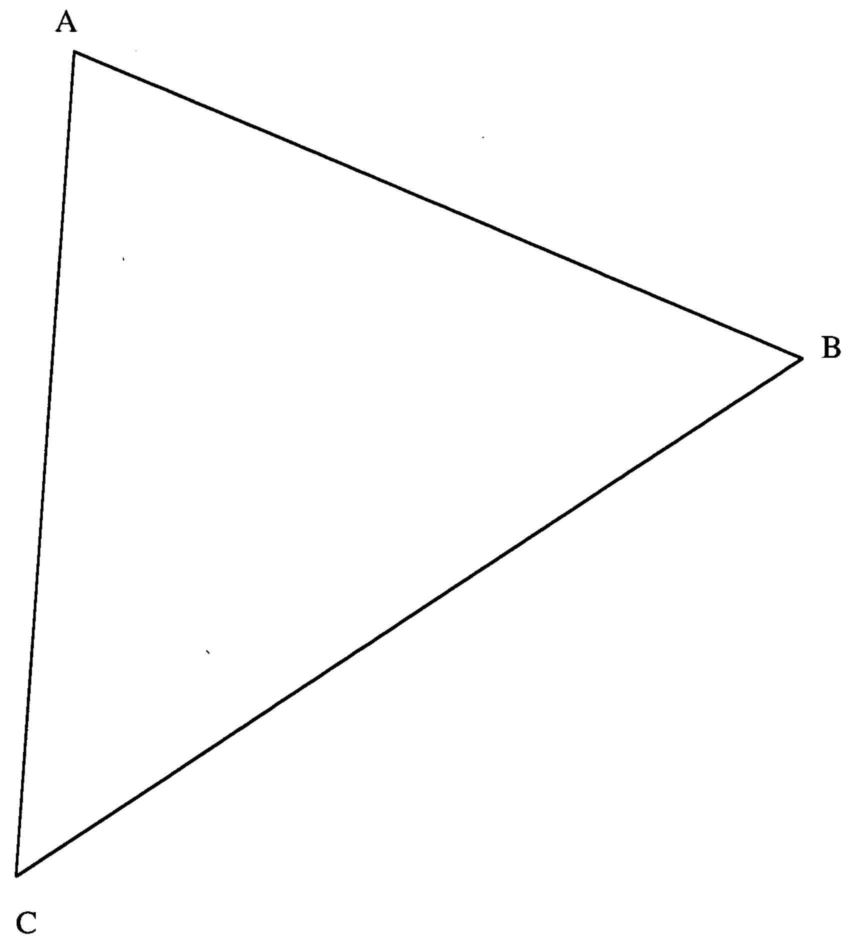


Figure 2

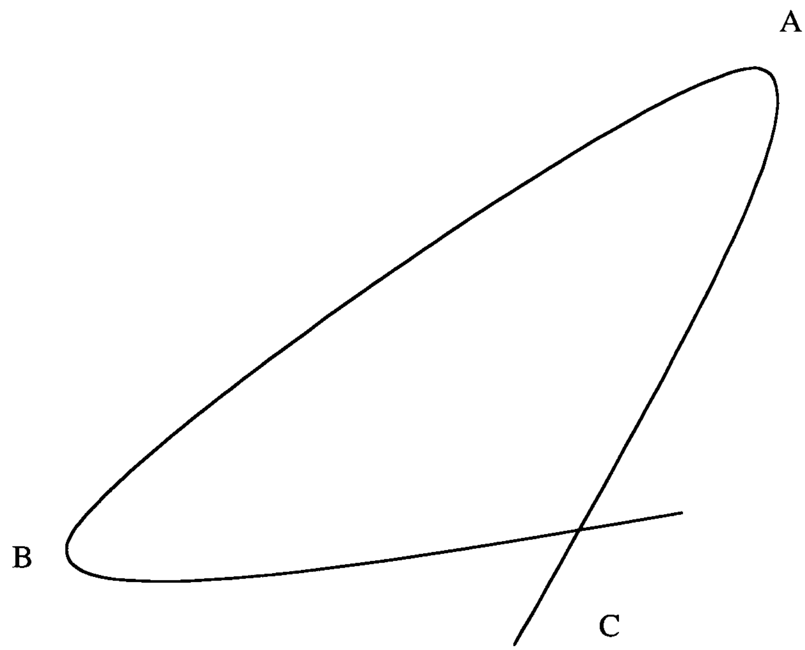


Figure 3

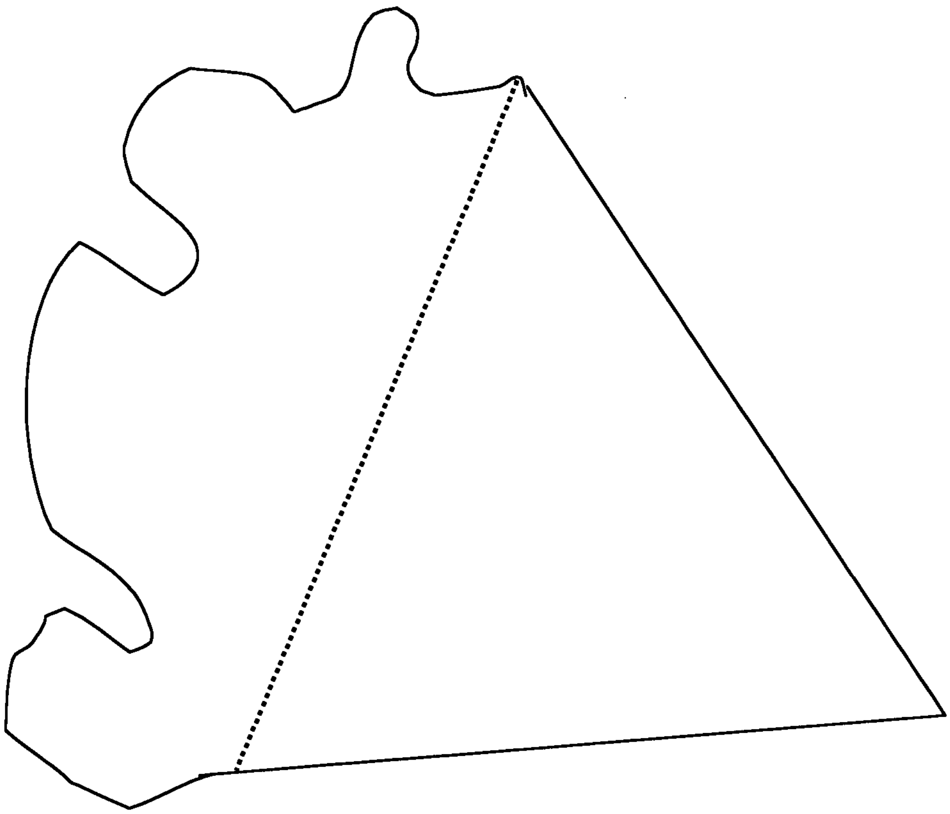


Figure 4

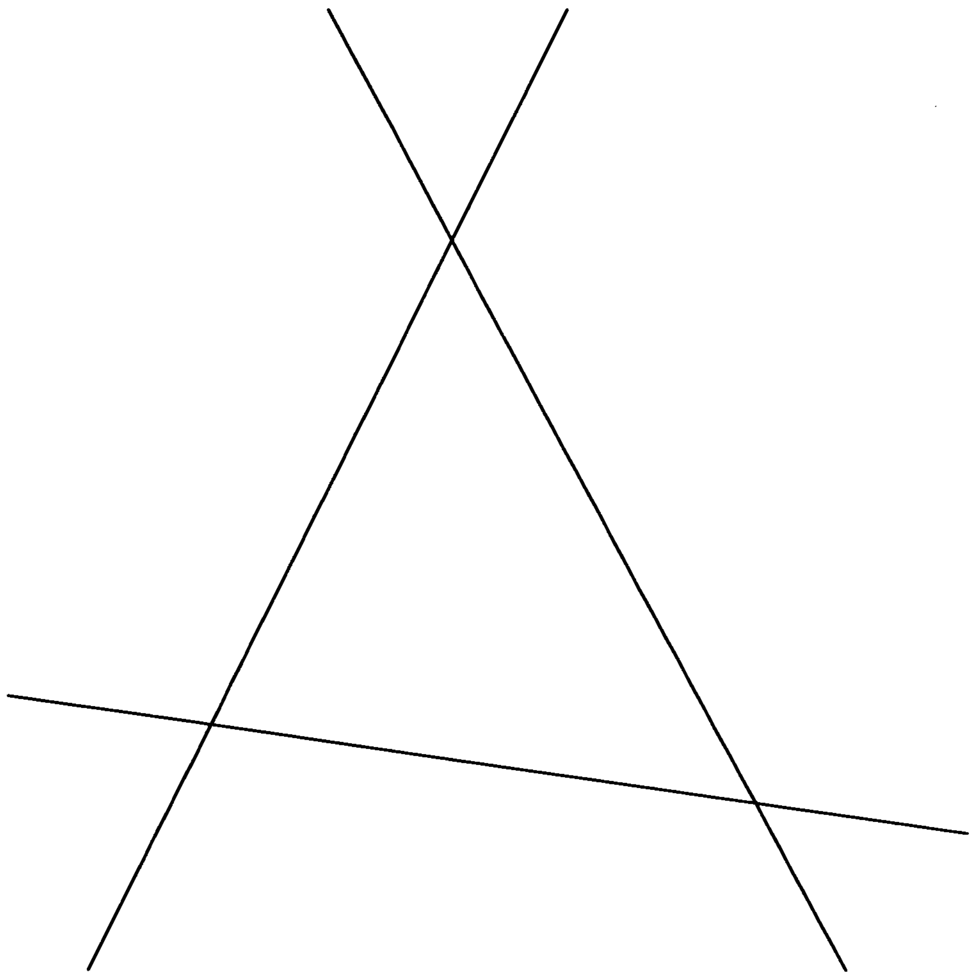
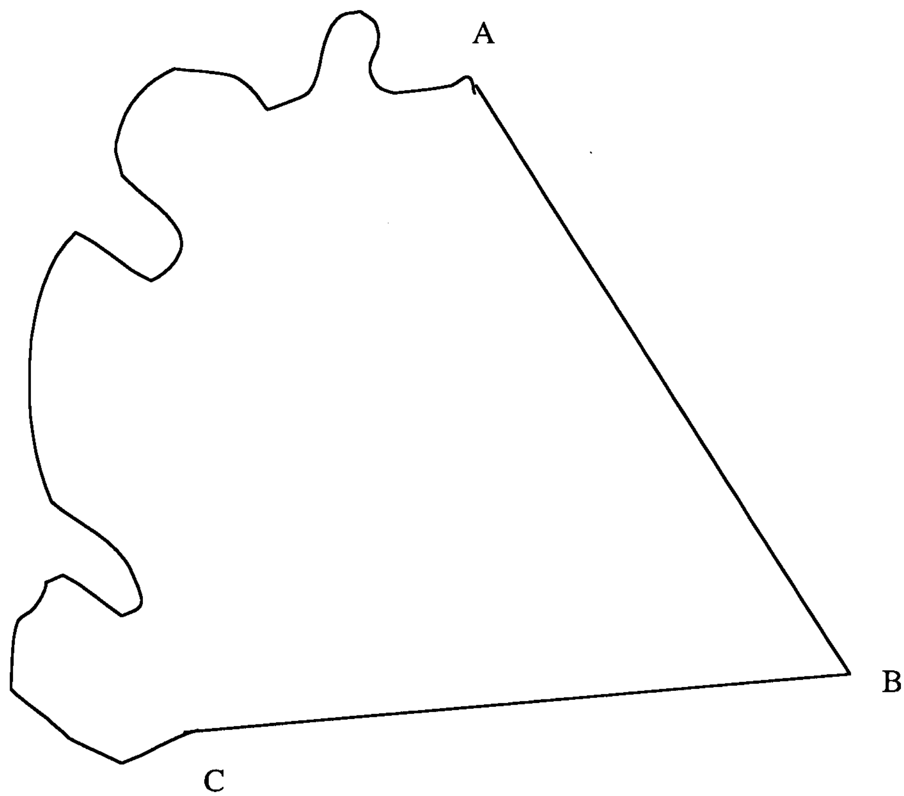


Figure 5



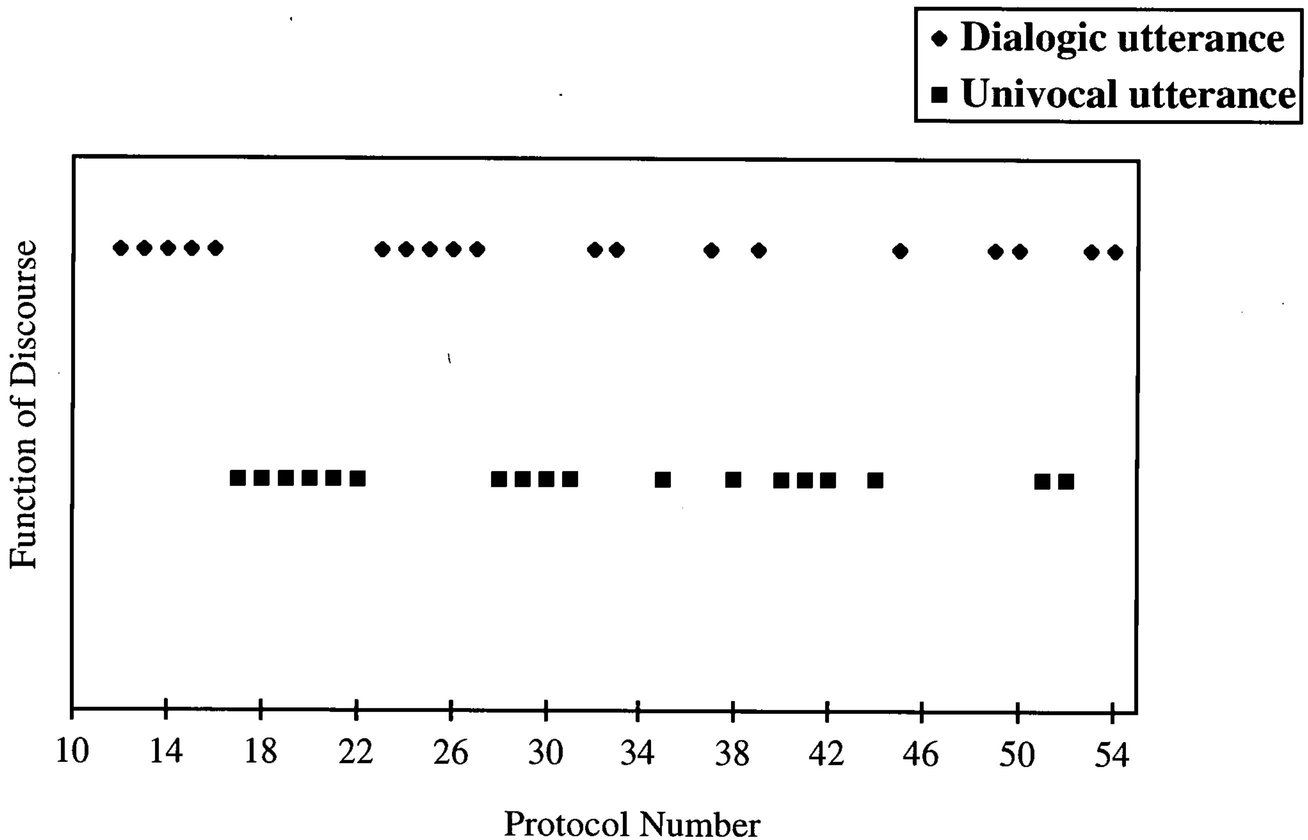

• Dialogic utterance
■ Univocal utterance
Function of Discourse
10 14 18 22 26 30 34 38 42 46 50 54
Protocol Number

Figure 6

NOTICE

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