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ABSTRACT

This booklet is an addendum to the conference proceedings of the 20th annual meeting of the International Group for the Psychology of Mathematics Education (PME 20). It contains three reactions to the research forum: (1) "Mathematizing: The 'Real' Need for Representational Fluency" (R. Lesh); (2) "Mathematics Teacher Development: An Alternative Scenario" (N.F. Ellerton); and (3) "Improving Knowledge, Professional Growth and Monitoring the Development of Mathematics Teachers: A Necessary Integrating of Theoretical Frameworks" (S. Llinares). (ASK)

Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education

PME 20

July 8 - 12, 1996

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Addenda

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Edited by

Luis Puig
Angel Gutiérrez

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Universitat de València
Dept. de Didàctica de la Matemàtica

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TABLE OF CONTENTS

Reactions to the Research Fora

REACTIONS TO RESEARCH FORA

Mathematizing: The "Real" Need for Representational Fluency

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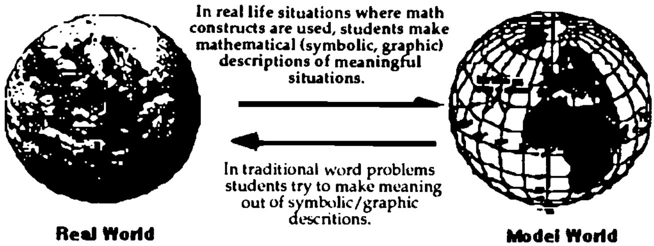
This paper is written as a reaction to Lois Rico's paper about "The Role of Representational Systems in the Learning of Numerical Structures." My paper cannot be called a critique because I find very little to be critical about in Rico's paper. Instead of criticizing, what I propose to do is to briefly highlight several of Rico's points that I consider to be especially important, and to describe another half of the picture that Rico did not have time or space to develop ... but that is critical for a balanced discussion of the role of representations play in mathematics learning and problem solving.

Rico describes the central focus on his research by saying "Our research team is interested in the difficulties young people find on managing numerical structures when they face advanced mathematical questions." Then, he goes on to describe interesting concrete or graphic representations that are useful to address the preceding difficulties.

Rico's research is in a strong tradition of others, including myself, who have investigated the roles that representations play in mathematical reasoning by focusing on how students make sense of word problems or symbolic computations. But, in this paper, I would like to focus on a class of mathematical activities that tends to emphasize almost exactly the opposite kind of processes than those needed to make sense of most to the symbolic computations or word problems (exercises) that occur in traditional textbooks, tests, and teaching. These activities involve mathematizing real life situations.



For traditional textbook word problems (or computation exercises), the problematic aspects tend to involve trying to make meaning out of symbolically stated questions; but, when attempts are made to use mathematics in real life situations, the processes that are needed usually emphasize the need to make symbolic descriptions of situations that are already meaningful. Therefore, to focus on activities in which representational abilities are salient, this chapter will focus on activities in which the results students produce consist largely of descriptions, interpretations, explanations, constructions, or justified predictions; and, because the products that students are expected to produce directly involve representations, representational fluency plays a central role in the understandings and abilities that are needed for success.

A Brief Overview of Some of Rico's Central Points:

Among the many important perspectives that are described in Rico's paper, I would like to emphasize the following.

1. Above all, mathematics is the study of structure. Contrary to conclusions that most people have formed based on school experiences, mathematics is NOT simply about doing what you're told, and mathematical knowledge is NOT simply a checklist of machine-like condition-action rules (definitions, facts, or skills) that need to be programmed in students' heads and executed flawlessly. Mathematics is about SEEING at least as much as it is about DOING. Or, alternatively, one could say that DOING mathematics involves more than simply manipulating mathematical symbols; it involves INTERPRETING situations mathematically; it involves MATHEMATIZING (e.g., quantifying, visualizing, or coordinatizing) structurally interesting systems; it involves using specialized language, symbols, graphs, graphics, concrete models, or other representational systems to develop mathematical descriptions, or explanations, or constructions that enable useful predictions to be made about such systems.

2. When mathematical systems (structures) are constructed or explored, it is the structural properties of these systems that are of interest; it is not the isolated elements within the system; and, it is not the isolated rules for operating on these elements. Consequently, when representations are generated to deal with these structures, it is the underlying patterns and regularities that must be highlighted.

3. The meaning of mathematical constructs tends to be distributed across several interacting representation systems, each of which emphasizes and de-emphasizes somewhat different characteristics of the underlying constructs.

Now, in this paper, I would like to add the following fact to this list. That is, the meanings of both mathematical constructs and representations tend to be interacting, unstable, and continually evolving; and, the interacting and unstable nature of mathematical constructs and representations are primary driving forces contributing to their evolution.

To accomplish the preceding goal, I will call attention to a type of problem solving episode that we refer to as *construct-eliciting activities* ... because the goal is not simply to produce brief answers to someone else's questions; instead, the goals involve developing constructions, descriptions, explanations, or justifications that explicitly reveal how the situation was interpreted. On the other hand, we also refer to these problem-solving episodes as *local conceptual development sessions* because the construct-development cycles that children go through to develop responses often turn out to be strikingly similar to the unfolding stages that developmental psychologists have observed over time periods of several years for the conceptual systems that underlie elementary-but-deep mathematical constructs such as ratios, rates, fractions, or proportional reasoning.

Representation: Simplifying External Systems or Externalizing Internal Systems.

When students use representations to mathematize problem solving situations, mathematical representations function as simplifications of external systems at least as much as they function as externalizations of internal systems. For an example where this later function tends to be emphasized, consider the *summer jobs problem* given on the following page (Katims, Lesh, et. al, 1994).

The *summer jobs problem* is an example of a *construct-eliciting activities*. The goal is to produce an operational definition that stipulates the problem solvers' notion of how to measure some construct (such as "productivity" at summer jobs, or "hovering ability" for paper airplanes). Therefore, descriptions, explanations, and justifications are not simply accompaniments to useful responses, they are the heart of useful responses. Also, because the *summer jobs problem* is intended to be addressed by three-person teams of average ability middle school students, it tends make heavy demands on communication capabilities and representational fluency for purposes such as: (i) analyzing problems, and planning solutions involving multiple steps and multiple resources and constraints, (ii) justifying and explaining suggested actions, and predicting their consequences, (iii) monitoring and assessing progress, (iv) integrating and communicating results in forms that are useful to others.

For activities like *summer jobs problem*, construct development is what solutions to such problems is all about; and, because this evolution tends to involve a series of modeling cycles in which progressively more sophisticated representations and ways of thinking are introduced, tested, and refined, the purpose of representations is not simply for students to communicate with one another, it is also for students to communicate with themselves and to externalize their own ways of thinking so they can be examined and improved. Therefore, the

The "Summer Jobs" Problem

Last summer Maya started a concession business at Wild Days Amusement Park. Her vendors carry popcorn and drinks around the park, selling wherever they can find customers. Maya needs your help deciding which workers to rehire next summer.

Last year Maya had nine vendors. This summer, she can have only six – three full-time and three half-time. She wants to rehire the vendors who will make the most money for her. But she doesn't know how to compare them because they worked different numbers of hours. Also, when they worked makes a big difference. After all, it is easier to sell more on a crowded Friday night than on a rainy afternoon.

Maya reviewed her records from last year. For each vendor, she totaled the number of hours worked and the money collected – when business in the park was busy (high attendance), steady, and slow(low attendance). (See the table.) Please evaluate how well the different vendors did last year for the business and decide which three she should rehire full-time and which three she should rehire half-time.

Write a letter to Maya giving your results. In your letter describe how you evaluated the vendors. Give details so Maya can check your work, and give a clear explanation so she can decide whether your method is a good one for her to use.

HOURS WORKED LAST SUMMER

	JUNE			JULY			AUGUST		
	Busy	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*
MARIA	12.5	15	9	10	14	17.5	12.5	33.5	35
KIM	5.5	22	15.5	53.5	40	15.5	50	14	23.5
TERRY	12	17	14.5	20	25	21.5	19.5	20.5	24.5
JOSE	19.5	30.5	34	20	31	14	22	19.5	36
CHAD	19.5	26	0	36	15.5	27	30	24	4.5
CHERI	13	4.5	12	33.5	37.5	6.5	16	24	16.5
ROBIN	26.5	43.5	27	67	26	3	41.5	58	5.5
TONY	7.5	16	25	16	45.5	51	7.5	42	84
WILLY	0	3	4.5	38	17.5	39	37	22	12

MONEY COLLECTED LAST SUMMER (IN DOLLARS)

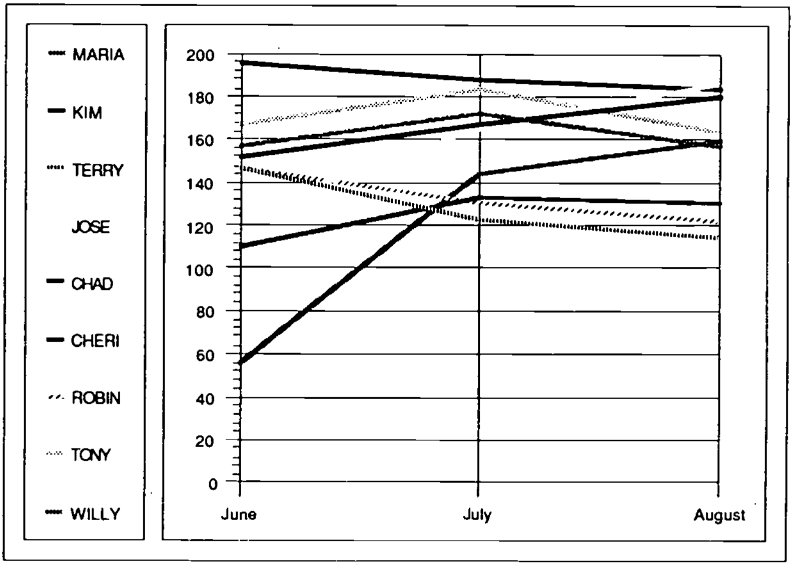
	JUNE			JULY			AUGUST		
	Busy	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*
MARIA	690	780	452	699	758	835	788	1732	1462
KIM	474	874	406	4612	2032	477	4500	834	712
TERRY	1047	667	284	1389	804	450	1062	806	491
JOSE	1263	1188	765	1584	1668	449	1822	1276	1358
CHAD	1264	1172	0	2477	681	548	1923	1130	89
CHERI	1115	278	574	2972	2399	231	1322	1594	577
ROBIN	2253	1702	610	4470	993	75	2754	2327	87
TONY	550	903	928	1296	2360	2610	615	2184	2518
WILLY	0	125	64	3073	767	768	3005	1253	253

Figures are given for times when park attendance was high (busy), medium (steady), and low (slow).

meanings and functions of representations that students use are not static, they are continually evolving; and, the same is true for the underlying constructs that they embody, as well as for the external systems that they describe.

Final results that students produce often go beyond "static" solutions to also involve conditional statements which include a variety of options or mechanisms for taking into account additional information. For example, the solution may enable the clients to assign different "weights" to reflect the client's views about the relative importance of information about different months or different periods of work. Or, it may enable the client to adjust suggested weights to suit their own preferences, it may use supplementary procedures, such as interviews, to take into account additional information, or it may consider new hiring possibilities that were not considered when the problem was posed (such as hiring more or fewer full-time or part-time employees). Also, rather than using only a single rule which is applied uniformly across all of the possible employees, the procedure that students use may involve a series of telescoping procedures. For example, to begin, one approach may be used to select employees that are in a "must hire" category; then, a different procedure may be suggested to select employees among the possibilities that remain. Or, instead of relying on sums or averages to simplify the information, graphs like the ones shown in next page may be used to focus on trends.

Trends from June to July to August



During the solutions of *construct-eliciting activities*, the general processes that students go through tend to be straightforward. When an initial description is produced, it may involve a combination of spoken words, written symbols, pictures or diagrams, or references to concrete models or real life experiences. But, in any case, the representation tends to organize and simplify the situation so that additional information can be noticed, or so that attention can be directed toward underlying patterns and regularities which may, in turn, force changes in conceptions. Then, this new information often creates the need for a more refined or more elaborate description; and, this new description again tends to make it possible for another round of additional information to be noticed. So, internal conceptual systems and external representational systems both tend to be unstable, interacting, and continually evolving; and, the general cycle of development repeats until the match between the model and the modeled is experienced as being sufficiently close and sufficiently powerful to produce the desired results without any further adaptations.

The preceding observations lead to a number of corollaries about the forms and functions of representations in mathematics learning and problem solving. For example, when one person perceives a system that another person has constructed, the perceived system is not necessarily identical to the constructed system. In fact, even when individuals observe systems that they themselves have constructed, the perceived system is not necessarily identical to the constructed system. For instance, when a student draws a diagram of a complex situation, or when the situation is described using spoken language or written symbols, the person who constructed the description often reads out more information or different information than they read in. Perhaps, before the representation was generated, attention was focused on details; but, after the representation became available, new patterns or regularities often become apparent. Students early conceptualizations may fail to recognize the proverbial forest because of the trees, or vice versa; or, when they focus on one type of detail, other details may be conceptually neglected. But, in these and other ways, the meanings of both constructs and representational systems tend to be unstable; and, this lack of stability, together with adaptations that are made which are aimed at increasing stability, are some of the most important driving forces behind construct development. ... For conceptual systems just as for other types of complex self-organizing systems, "survival of the stable" tends to the be the most relevant modern counterpart of Darwin's law of "survival of the fittest."

Construct-Eliciting Activities are Local Conceptual Development Sessions.

When *construct-eliciting activities* involve basic conceptual systems (e.g., involving fractions, ratios, rates, proportions, or other elementary mathematical constructs) that have been investigated by developmental psychologists or mathematics educators, the modeling cycles that students go through during sixty-minute problem solving sessions often appear to be local or situated versions of the stages that developmental psychologists have observed over time periods of several years. Furthermore, the processes and mechanisms that contribute to the development of these constructs tend to be the same as those that contribute to large scale conceptual development. In particular, cognitive conflict, or the need to develop increased conceptual stability, is a primary factor that creates the need for conceptual adaptation; and, representation systems facilitate the progressive differentiation and integration of relevant conceptual systems.

For *construct-eliciting activities,* where solutions typically involve a series of interpretation cycles, two of the most important phenomena that need to be explained are: *How is it that students perceive the need to develop beyond their first primitive conceptualizations of a problem situation, and how is it that they are able to develop toward interpretations that are less barren and distorted?*

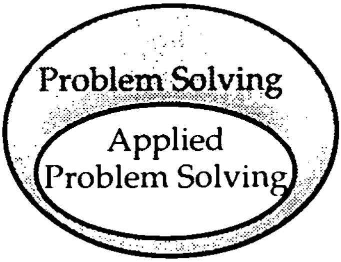
To answer the preceding question, theories generated by developmental psychologists have proven to be relevant because, when *construct-eliciting activities* are interpreted as *local conceptual development* (LCD) *activities*; : (i) mechanisms that contribute to general conceptual development can be used to help explain students' problem solving processes (Lesh & Zawojewski, 1987), and (ii) mechanisms that are important in *local conceptual development sessions* can be used to help explain the situated development of students' general reasoning capabilities (Lesh & Kaput, 1988).

The application of developmental perspectives to problem solving is a relatively new phenomenon in mathematics education research where, traditionally: (i) problem solving has been defined as *getting from givens to goals when the path is not immediately obvious or it is blocked;* and (ii) heuristics have been conceived to be answers the question: *What can you do when you are stuck?* When attention focuses on *construct-eliciting activities,* the essence of problem solving involves finding ways to interpret these situations mathematically. Therefore, in general, it is more important for students to find ways to adapt, modify, and refine ideas that they DO HAVE, rather than to try to find ways to be more effective when they are stuck (i.e., when they have no relevant ideas or when no substantive constructs appear relevant, as often happens in puzzles and games). Consequently, to develop useful responses to *construct-eliciting activities,* the kinds of heuristics and strategies that are most useful tend to be quite different than those that have been emphasized in traditional problems where the solutions involve only a single interpretation cycle ... and where fewer demands are made on students abilities to introduce, modify, and adapt useful representations.

Applied Problem Solving ≠ Construct-eliciting Activities

The Traditional View

Applied problem solving is treated as a special case of traditional problem solving.



If problem solving is thought of as *getting from givens to goals* then it makes sense to assume that:

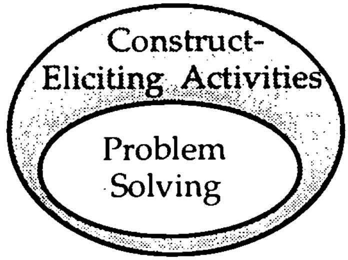
- applied problems (or construct-eliciting activities) are special cases that involve "messy" data, and "real world" knowledge in addition to the knowledge that is needed in more general situations.
- heuristics and other mechanisms that apply to "general" problem solving should particularize to be productive in applied problem solving situations.

Therefore, learning to solve "real life" problems should be expected to involve three steps:

- First, learn the prerequisite ideas and skills.
- Next, learn some general (content independent) problem solving processes and heuristics.
- Finally (if time permits) learn to use the preceding ideas, skills, & heuristics in situations where additional "real life" information also is required.

An Alternative View

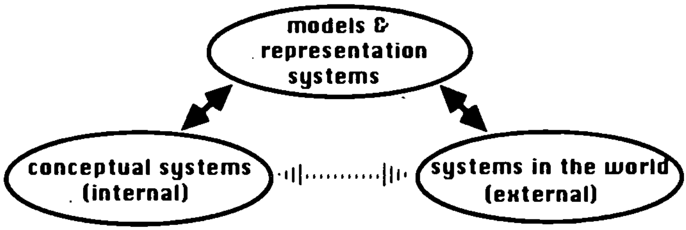
Traditional problem solving is treated as a special case of construct-eliciting activities.



If the essential characteristic of mathematical problem solving involves interpreting situations mathematically (*modeling*), if non-routine problems involve more than a single modeling cycle, and if multiple modeling cycles involve different ways of thinking about givens, goals, and/or solution paths, then it makes sense to assume that:

- traditional problem solving is a special cases where multiple modeling cycles aren't needed.
- solution processes involve much more than information processing using a single invariant model; they also involve model transformation ... because it is the model or interpretation itself that is being modified, extended or refined.
- model construction and refinement is the same thing as construct development; so, applied problem solving experiences are important on the way to learning the underlying constructs.

In *construct-eliciting activities*, and in other problem solving situations that emphasize the generation of interpretations, the language, symbols, graphs, and organizational schemes that students introduce tend to be partly descriptions (simplifications) of external systems. Yet, because these descriptions focus on hypothesized relationships, patterns, and regularities that are attributed to external systems, rather than being derived from them, the representations are also externalizations of internal systems. Consequently, solutions to such problems involve interactions among three types of systems: (i) (internal) conceptual systems that reside in students' minds, (ii) (external) systems that are given in nature, or that are constructed by humans (but which were constructed for their own sake rather than being created as representations for making sense of other systems, and (iii) (external) models or representational systems that function both as externalizations of internal conceptual systems and as internalizations of external systems.



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MATHEMATICS TEACHER DEVELOPMENT: AN ALTERNATIVE SCENARIO

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The paper by Becker and Pence has addressed the notion of teacher change, and has attempted to identify "critical aspects of staff development which translate into change in teachers' beliefs and classroom practices." This reaction paper puts forward the suggestion that much greater change in teacher beliefs and classroom practices may have been possible had the project incorporated action research as an integral part of its implementation.

Reaction or Reflection

The word *reflect* embodies several meanings, and is often used in a metaphorical way. Phrases such as "to throw back from a surface, as rays of light or heat," "to give back an image or likeness," "to be thrown back," "to think closely over," are typical phrases used (see, for example, Thomas Nelsons and Sons, 1952, p. 419). *Reflection* carries with it images of "the power of the mind (becoming) conscious of its own thoughts." On the other hand, *reaction* is "action which resists another action," "a backward movement," or a "response to suggestion" (p. 413).

The notion of "reflection" is linked closely with the theme of the paper by Becker and Pence—the term is in fact used by these authors several times. In this reaction paper, therefore, I will be attempting to *reflect* rather than to *react*, as it is through reflection that we "give back" an image but through reaction that we "move backwards." Clearly, our goal is to move discussion forwards rather than backwards.

Introducing the notion of reflection has another important role in this paper, as it provides an entré to *action research*. In particular, I believe that many of the features of the research carried out by Becker and Pence could, in fact, have been approached from an action research perspective. Further, I will suggest that, had the whole exercise involved genuine action research from the start, then the changes discussed at the end of the project may have been quite different in both quantitative and qualitative terms from the outcomes described in the paper under consideration.

A Reconsideration of the Project

In this paper, therefore, the main aspects of the project described by Becker and Pence will be identified and retained. A (hypothetical) "new" project which involves these key aspects will then be described, and, based on experiences from other, similar projects reported in the literature, the potential impact on the professional lives of the teachers involved in this "new" project (hereafter referred to as "Project N") will be presented.

Background to Project N

Project N involved two parts: one which aimed, initially, for a one-year involvement of teachers ("bridge-builders"), and another which it was hoped would be sustained for an indefinite period ("mathematics leadership"). Two tertiary researchers were involved in the project as facilitators; teachers from 20 schools, including schools which had a large Hispanic and African-American population, were invited to participate in the study.

Bridge-Builders Program

One of the key features of the Bridge-Builders Program in Project was to encourage teachers to identify the areas of their mathematics teaching which they felt they would like to address. Clusters of five schools were formed, and teachers from these "Cluster" schools met regularly. The two tertiary researchers each linked themselves with two Cluster Groups, and attended alternate meetings with each Group. These meetings were chaired by one of the teachers rather than by a researcher. In fact, the tertiary staff were conspicuous by their apparent lack of involvement in initial discussions.

The following questions were identified by the teachers in the initial meetings of the Bridge-Builders Program in Project N.

- As teachers, we would like to know more about how students learn mathematics, and this might help us to design and implement more student-centred instruction;
- Some teachers felt concerned that their expectations of student achievement were influencing the way they taught in the mathematics classroom;
- Most teachers expressed interest in learning new mathematical content, as an initial step in keeping up with the topics and strands introduced in the NCTM *Standards* document (NCTM, 1989).

An important feature of the Bridge-Builders Program was the collegiality which developed among teachers. Initially, the teachers asked (and expected) the tertiary facilitators to design appropriate professional development activities. The activities which emerged, however, were predominantly put forward by the teachers involved in the Program.

The provision of teacher release time is an integral part of the Program, and teachers can spend an average of two lessons per week observing/working in another classroom. Some teachers have used this opportunity to have a "critical friend" comment on their approach to teaching a particular mathematical concept, while others have team-taught in order to learn more about what is involved in working closely with other teachers.

Mathematics Leadership Program

In the Mathematics Leadership Program of Project N, teachers who had been given special responsibility within their schools and/or districts for coordinating and planning the teaching of mathematics began meeting regularly to discuss common problems and to plan effective strategies for the introduction of new curriculum initiatives in mathematics. The teachers from the 20 schools involved decided to work in small cluster groups, in a similar way to the teachers working in the Bridge-Builders' Program. Thus four Mathematics Leadership Groups were formed.

During the first year, the Mathematics Leadership Groups concentrated on a careful examination of innovative first year algebra materials. The teachers shared the responsibility of piloting different aspects of the materials in their mathematics classrooms. A regular feature of the monthly meetings held by each Group was the reporting back to other members of their Group about different aspects of these materials. In particular, a major concern of teachers has been how best to use technology to maximum effect in the mathematics classroom. Application of appropriate technology has therefore been incorporated into all of the pilot work with new materials.

The Mathematics Leadership Groups have held combined meetings twice a year, and the tertiary researchers involved in the project have been invited to attend. One of the aims of these meetings was to maintain contact between the Groups, and to help sharpen the focus of the broader shared perspectives of all involved. Major common concerns were identified at these larger meetings, including alternative forms of assessment, and ways of extending teachers' own mathematical knowledge, particularly as newer areas of mathematics were introduced into the secondary school curriculum. Each year, the combined Groups organise a Summer Institute which runs for 16 days. During the year, follow-up workshops are held, the organisation being rotated among the four smaller Mathematics Leadership Groups.

The Mathematics Leadership Program is now in its third year, and has chosen to focus on the development of curriculum at the school level. In the second year of the Program, the focus chosen was geometry.

The Role of Action Research

Project N, then, which incorporates both the Bridge-Builders Program and the Mathematics Leadership Program, can be described as a series of action research projects. According to Kemmis and McTaggart (1988), action research "is not research done on other people. Action research is research by particular people on their own work, to help them improve what they do, including how they work with and for others" (p. 22).

Traditional education research, on the other hand, is research which involves carefully designed quantitative and/or interpretative studies. It is generally carried out *by* researchers *on* other people. Those who are "researched" usually have little ownership of the project.

The Bridge-Builders Program and the Mathematics Leadership Program have placed teachers at the centre of their own professional development. They alone are best placed to decide which aspects of their teaching they would like to change. Teachers are the key people who can bring about change in the classroom. If teachers initiate the change, then change will take place much more efficiently and effectively. If change is imposed either hierarchically or externally, real change in the classroom will be a much longer process.

Having said this, however, change in any classroom or school is likely to be slow. Those who have been involved in the action research driven Project for Enhancing Effective Learning (PEEL) which began in Melbourne, Australia, would be the first to agree that change is a slow process (see, for example, Mitchell, 1992). In fact, PEEL was planned, initially, as a two-year project. Mitchell wrote:

> We regarded this time as sufficient to allow for much of the teacher change that we believed was a necessary precursor to the desired student change. ... Supporting teacher change over a long term has proved more complex than we imagined; important new lessons have been learnt in every year of the project. (p. 14)

PEEL is now in its twelfth year, and those involved continue to learn new lessons.

What is Action Research?

The following statement of the purpose of action research was put forward by Kemmis and McTaggart (1988):

> [Action research is] a form of collective self-enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own social or educational practices, as well as their understanding of these practices and the situations in which these practices are carried out. (p. 5)

Thus, action research is seen as much more than testing hypotheses and drawing implications from data (Carr & Kemmis, 1986).

The most common image of action research is the so-called action research "cycle" or "spiral" involving the repeated notions of plan —> act —> observe —> reflect.

The following six essential characteristics of any education action research project have been put forward to complement Kemmis and McTaggart's (1988) statement of the purpose of action research (see Ellerton & Clements, 1996):

> 1. *Action research involves teamwork.* Action research involves a team of committed practitioners/researchers working together to improve existing practice. All members of the team should contribute a "fair share" of the work.
>
> 2. *Membership.* Because the achievement of effective change in education does not come easily, an individual's decision to accept membership of an action research team represents a long-term commitment. Even so, any team member should feel free to be able to withdraw from the action research team at any time.
>
> Membership of an action research team should be on a voluntary basis, with all members regarded as being of equal status. However, after a project has begun no person should be invited to become a new member of the action research team unless this is agreed to by all existing team members.
>
> 3. *Team meetings.* Democratic team planning and reflection should take place at regularly held team meetings. Open and equal participation in discussion is essential, and the interests of "underdogs" within the team should be scrupulously protected. Decisions reached at meetings are to be mutually binding.
>
> 4. *The research agenda.* Research question(s) and corresponding methodologies should be agreed upon at team meetings. These should relate to an agreed theme of interest—the temptation to investigate a series of unrelated individual concerns, contributed by group members, is to be avoided.
>
> 5. *Documentation.* Any observations and reflections of team members should be shared at team meetings, and consensus should be reached on decisions for action. Observations, reflections and decisions for action should be documented in the minutes of the meetings.
>
> 6. *Data collection, analysis, and reporting.* Data should be collected, analysed, and reported, systematically, in a manner agreed to by the action research team. (pp. 117-118)

Thus membership of an action research team embodies a commitment to the team, and to the profession, which is likely to take on a different and more fragile form if external researchers come into the school setting and begin to ask questions.

The Role of Tertiary Researchers/Facilitators

Action researchers may choose to draw from traditional education research methodologies as they investigate and reflect on their own teaching. They may choose to invite tertiary researchers to work as partners in the research process.

The contrast with traditional education research approaches is, however, very simple. In action research, the members of the Group (the action research team) are the ones who decide what to do, how to do it and who to involve. If an education research approach were to be used in the Building Bridges and Mathematics Leadership Programs, then tertiary researchers would have first chosen the focus of each of the programs (possibly in consultation with the teachers), and would have used a range of "quantitative and qualitative approaches to ascertain the impact of the programs on teachers' beliefs and classroom practices" (Becker & Pence, 1996). In an action research approach, the impact of the program may still be observed by externally-based researchers, but deep understanding of the changes brought about at the classroom level can best be interpreted and reported by the teachers themselves.

As Kemmis and McTaggart (1988) have pointed out:

> Action research is not just about hypothesis-testing or about using data to come to conclusions. It adopts a view of social science which is distinct from a view based on the natural sciences (in which the objects of research may legitimately be treated as "things"); action research also concerns the 'subject' (the researcher) him or herself. Its view is distinct from the methods of the historical sciences because action research is concerned with changing situations, not just interpreting them. Action research is a systematically evolving, a living process changing both the researcher and the situations in which he or she acts; neither the natural sciences nor the historical sciences have this double aim. (p. 22)

Action research is *not* about bringing together practitioners and researchers so that they can undertake research which is of particular mutual interest. Rather, teachers become researchers in their own right, incorporating research approaches—conjecturing, designing, trialing and evaluating—into their day-to-day practice (Schon, 1987).

In the Building Bridges Program, and the Mathematics Leadership Program, the teachers involved designed the following questions that they felt they needed to answer if they were to begin to understand how best to bring about change in their own school. With the help of the tertiary facilitators, they designed appropriate questions which they planned to use to provide a framework for describing their involvement in their respective programs. The questions included the following:

1. As a participant teacher, which aspects of the Program have been most important to me?
2. Have my beliefs changed about the teaching and learning of mathematics as a result of my involvement in the Program?
3. Has my classroom practice changed because of my involvement in the Program? Why? Which came first, a change in beliefs or a change in practice?
4. What impediments to change have I encountered?

It is, perhaps, tempting to argue that the answers framed by participant teachers in response to these Group-designed questions are likely to be subjective at best. And how can teachers judge whether change has occurred in their own classrooms, or whether their own beliefs have changed?

I believe that there is a more fundamental question. Which is more valid: an "outsider" asking teachers questions, with the teachers wondering all the time about the agenda behind the questions, or the same teachers reflecting on the same questions and sharing the responses with colleagues who continue to see each other, professionally, on a day-to-day basis? In other words, the self-accountability of self-reporting, with monitoring taking place through the collegial support of the Group, makes reflection in the context of an action research group much more valid than responses to an "objectively" designed and administered instrument. Responses given in a Group context must be able to be defended in the light of contrary evidence; responses given to external researchers are unlikely ever to need defence.

The Role of the Principal

Can the Principal of a school take part in an action research study? Perhaps it would be more appropriate to rephrase the question in the following way: On what basis can the Principal of a school take part in an action research study? In line with the essential characteristics of action research summarised above, provided that Principals undertake "a fair share of the work," and provided that *all* members of the action research team have equal status, then a Principal should be able to take part in a full and productive way as a member of the action-research team.

The Building Bridges Program and the Mathematics Leadership Program included several Principals who chose to be involved. The following questions were designed by participants in these Programs, and helped to guide the reflections of those involved. They also provided a framework for presentations at the Seminar held to mark the end of the second year of the Mathematics Leadership Program.

- Can you identify any changes in your day-to-day life in the school community which you can link directly (or indirectly) to either of the two Programs?
- Were there any barriers or obstacles to the changes your Group planned on introducing? Were you able to overcome any constraints to change?
- Do mathematics classes in the school look any different today compared with mathematics classes here 2 years ago? Try to identify the differences, and the reasons for these changes.
- Describe the support (or lack of support) that you feel in your current professional role in the school?
- Has the time frame for change in the school community surprised you? Why?
- How have the students in your school responded to change? Have teachers' expectations of students changed over the past 2 years?
- What would you envisage as the next stage for these Programs? Should they be continued and in what format? Why?

The future of Project N is, of course, yet to be decided. It is my hope, though, that a decision about whether to proceed with Project N, and in what ways, can be made by examining examples provided by other, recent action research projects which illustrate the principles involved.

Some Observations from Other Projects

The following brief examples from other action research projects will be given in an attempt to illustrate the potential of Project N to change the professional lives of participating teachers.

1. Key Group Project. Key Group was a project initiated in the Australian state of Victoria. Robinson (1989) described how the project exemplified professional development through the empowerment paradigm. Groups of three teachers (Prep to Year 3) from each of a range of schools were invited to form teams, together with a mathematics consultant from the State Ministry of Education. Each group of four was called a "Key Group." A "planning" conference was held for 18 Key Groups at which each Group "reflected on its current practice, celebrated its successes, decided on some aspect of their mathematics teaching that they wanted to improve, and devised a plan of action for setting about it" (Robinson, 1989, p. 276). The role of the "outside agent" in each Group played a very important role in the success or otherwise of the Key Groups. Enthusiastic change agents could, in fact, undermine the success of a Group by becoming "evangelists." In contrast, Robinson described caring facilitators as those who "encourage each group member to recognise and take responsibility for the choices underlying that member's own behaviour. The caring helps to define choices; change is the participant's privilege" (p. 280).

2. Aga Khan University Project. A professional development program initiated by the Aga Khan University in Karachi, Pakistan aims at upgrading the quality of instruction in schools by assisting leading teachers to work in school-based action research teams. Each participating school is regarded as an entity in its own right. Bacchus (1995) described the action research project which began in 1994 in the following way:

> It was assumed that schools were in some ways like "total" institutions with cultures of their own, including a fairly closed network of interrelationships. Therefore upgrading one or two teachers and sending them back to their schools to bring about change would not, by itself, be very effective. It was therefore felt that we should work with schools as a whole—with the head teachers, with other classroom teachers, with school supervisors and others—to help create a culture that was supportive of the change which the professional development teachers were likely to initiate on returning to their schools. (p. 8)

After one of the sessions in the classroom, Bacchus (1995) described how "we sat around and praised ourselves for having been successful" (p. 12). However, one leading teacher recognised the difficulty of attempting to bring about change yet simultaneously resisting change. He noted that "we are talking the talk but not walking the walk" (p. 12). The project has been operating for two years now, and the action research teams, according to Bacchus, have begun to question their practice in fundamental yet healthy ways.

3. Project for Enhancing Effective Learning (PEEL). Initially involving a group of teachers from one school in 1985 (in Victoria, Australia), and consultants, PEEL focused on giving students training so that they would become "more willing and able to accept responsibility and control for their own learning"

(Baird & Mitchell, 1986, p. ii). There are now about 20 "PEEL" schools. Two books document the project (Baird & Mitchell, 1994; Baird & Northfield, 1995), and a monthly newsletter ("PEEL Seeds") helps to sustain contact and discussion between those involved, as well as to initiate the sharing of ideas and reports. Significant changes in the ways in which students and teachers work together during the teaching/learning process are now well recognised in PEEL.

4. Program for Innovation Excellence and Research (PIER). In Malaysia, one of the four sub-programs of the PIER project has focused on education research in general, and on establishing action research projects throughout the education system, in particular. Currently, about 500 action research have been initiated in the school system in Malaysia. Teachers speak of the changes that have been brought about because *their* ideas and work are seen to be valued, and because *they* can decide on what it is about their classrooms that most urgently need changing (Ellerton, Kim Phaik Lah, Madzniyah Md Jafaar & Norjiah Sulaiman, 1996).

Conclusion

Robinson (1989) contrasted the management paradigm with an empowerment paradigm. He stressed that,

> rather than seeing change in schools as a finite process with externally specified objectives, as the management paradigm does, the empowerment paradigm sees change as an on-going activity generated within the school by teachers, parents and students as part of an organic process of professional renewal. (p. 274)

The empowerment paradigm sees the source of knowledge, for example, as the teachers themselves, while the management paradigm relies on the expert and the theorist to provide fundamental knowledge. The focus of the management paradigm, according to Robinson (1989) is *change*, while that of the empowerment paradigm is *choice*.

The aim of Project N has been to try to maximise *change* by enabling the teachers involved in either of the Programs to have maximum *choice* in all respects. Commitment is enhanced by a sense of ownership; the opportunity for choice provided in a supportive atmosphere is likely to nurture this sense of ownership.

The project described by Becker and Pence (1996) is more closely aligned with a "management" paradigm than with an "empowerment" paradigm. That is not to say that the *intention* was not there to encourage teachers' empowerment. But the structure of the project involved externally planned and provided professional development sessions, and externally determined goals and questions, together with questions designed by the researchers and given to Principals to answer about the teachers and classrooms at their schools. Such a structure would work against empowering teachers regardless of how supportive the researchers involved in the project were, or how responsive the teachers were to the various activities. Even in the coaching sessions, "an attempt was made to ascertain those aspects of classroom behavior teachers were trying to improve," although teacher and "visitor" discussed classroom observation data and "collectively tried to determine ways to improve." In an action research setting, *teachers*, within the supportive network of an action research team, decide where change is needed, and in that setting (and possibly involving an outside "visitor") collectively plan ways of bringing about change.

There are important differences in the language used to talk about empowerment within an action research context, and the language used to talk about trying to change teachers' beliefs and classroom practices within a traditional education research setting. These differences are reflected in the paper by Becker and Pence and the alternative scenario presented here. Transition from a traditional professional development setting to an action research paradigm is not as simple as often assumed; it is not easy to change one's patterns of discourse, or to start to work collegially with peers. For example, Becker and Pence noted that teachers "did not feel they had the skills in group dynamics to communicate effectively with all of these constituencies [students, parents, administrators and colleagues] or counter the opposition [to change] that arose."

Action research is presented in this paper as providing a powerful approach for the professional development of teachers of mathematics. In presenting this alternative scenario to the project described by Becker and Pence, my aim has been to draw attention to (a) providing a valid model for professional development; (b) presenting a teacher-focused model of professional development rather than an "outsider-insider" model; and (c) avoiding a researcher-researched model of investigating the impact of professional development. The importance of finding appropriate ways of changing the beliefs and practices of teachers of mathematics cannot be over-emphasised.

I believe that, had the Seminar referred to in the discussion about "Project N" actually been held, then the papers presented would have testified to the changes in teachers' beliefs and classroom practices in far stronger terms than was evident in the paper by Becker and Pence (1996). The proposed action research structure for Project N would have established, like PEEL, the foundations for a long-term project. There would still be frustrations and disappointments along the way. However, one of the major differences between the Becker and Pence project and the alternative scenario presented in this paper is the sustainability of the latter. Action research is sustained by the commitment of those involved, even in the face of little or no funding; externally designed professional development programs coupled with education research projects are likely to be sustained only by the key players—the tertiary researchers—and then often only because funding is available to support the project.

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IMPROVING KNOWLEDGE, PROFESSIONAL GROWTH AND MONITORING THE DEVELOPMENT OF MATHEMATICS TEACHERS: A NECESSARY INTEGRATING OF THEORETICAL FRAMEWORKS.

(A response to Even, Tirosh and Markovits's paper)

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Mathematic Teacher Education is a practical subject but recently, it is being considered as a context for educational research: Teacher education as a research and development activity. Teacher education is oriented towards the development of beliefs, knowledge and practical competence. These objectives focus on the acquisition processes and on the factors that influence them.

Research agendas, now being developed, try to determine: (i) how research about teacher knowledge has implications in teacher education programs, (ii) how we can understand the changes in knowledge and beliefs of prospective teachers and teachers through the training program, and (iii) describe means to promote and manage the processes of change.

These research agendas have a common denominator. The research results have implications in program features, in the elaboration of materials and in course design (Blanco, 1994). That is to say, teacher education programs have a practical component as well as a research component. The theoretical frameworks adopted help design and analysis both practices and research.

However, the theoretical frameworks used, up until now, have focused on different aspects of the learning to teach process (Borko et a.. 1992; Eisenhart, et al. 1992; Simon, 1991, 1994; Wittman, 1984). This can be understood due to the complexity of the learning to teach mathematics process. The overlapping role of beliefs and knowledge (of mathematics, pedagogical content which is specific to mathematics and knowledge about students) in these processes can justify these differences in theoretical frameworks used. We will point out how theoretical frameworks can be complementary to design activities and to understand the learning process and the professional growth of mathematics teachers. This can give us a perspective that includes the context in which learning to teach develops.

Subject Matter Knowledge, Teacher's Pedagogical content knowledge and knowledge about students. Implications of theoretical orientation for the practical dimension of designing activities.

Even, Tirosh and Markovits' paper presents an example of the attempts that are being carried out to relate the practice in teacher education programs, research about teacher knowledge and theoretical reflection. Even, Tirosh and Markovits look for the origins of the presentation of teacher material (planning, answering questions and student observations): knowledge about the subject matter and knowledge about students. This approach is based on an analytic framework of necessary subject matter knowledge for teaching a specific mathematics topic (Even, 1990). Their research has illustrated as teachers' knowledge of mathematics influences their pedagogical decisions, how teacher knowledge of subject matter influences their ability to focus on the essence of student's questions. Two are the components considered: teacher's knowledge of mathematics and knowledge about students.

This report is an example of the integration of research within teacher education program design. Research about teacher knowledge is carried out in teacher education. Reciprocally, research results provide information that justifies the designing of program activities (Figure 1). The content of these tasks has emerged from cognitive research about how students learn, and from mathematics.

Even, Tirosh and Markovits, from a theoretical framework of neccesary teacher knowledge for teaching a specific mathematics topic, design activities in the teacher mathematics education programs. Nevertheless, there is no specific attention given to how prospective elementary teacher learning/development is conceptualized. Furthermore, if we consider "learning to teach" as a process that develops in different contexts, then we would complement learning activities in the program by designing teaching practices and analyzing the learning that is generated (student teaching at school).

Teacher Education Programs: I. Mathematics Classroom situations; II. Improving mathematics teaching; III. Teacher-Leader Preparation. Even, Tirosh and Markovits
I. Aimed at improving of pedagogical content knowledge of elementary school teachers ACTIVITIES • Analysis of mathematics classroom situations (actual, hypothetical); the teachers responds to a student's questions or ideas • Exposure to research on understanding how students learn; several studies and articles on learning and thinking in mathematics are presented and discussed with the teachers. • The teachers explore students' ways of thinking about mathematical situations and tachers' explanations by interviewing students. II. Focused on enahacing subjet matter knowledge and pedagogical content knowledge (developing local leaders and guiding local leaders in their work with teachers) III. Resource files for main curriculum and didactic tipics (e.g. algebra, analysis, heterogeneous classes) are being developed. Ex 'Algebra Resource File', its main components are: - Historical views on the development of algebra - Various (and sometiine confusing) meanings of letters in algebra - Students' conceptions of algebraic concepts, empahsizing and operatinal approach and a structural approach - 'Characteristics of a 'good problem' in school algebra and ways to design such problems and activities - Teacher role in algebra classes

* 4 models for meeting
- Model 1: Raising a research question. presentation of a relevant study, discussion of the results, and actual replication of the study.
- Model 2: Presentation of pedagogical-content question, working on a related task, crystallizing components for a framework for the question, closing the circle -re-discussion of the opening question.
- Model 3: Presentation of written and /or video documented teaching/learning events, analysis of events, conclusion.
- Model 4: Working on a task reflective discussion on the task, connecting to learning process

Figure 1.

<u>The conceptualizing of professional growth of teachers. Implications of theoretical orientation for the practical dimension of designing activities.</u> (Cooney and Shealy, 1994)

Cooney tries to identify ways in which we can conceptualize professional growth, considered as "the dynamic process of constructing belief." He uses the notion of "authority" as a growth indicator (from being internal to being external) to conceptualize the process by which a teacher becomes a reflective person. From his theoretical framework Cooney et al. (i) prepares case studies (research), and (ii) obtains practical implications to design activities in the teacher education program (practice) (Figure 2).

A key idea in the work of Cooney and his colleagues is:

"We need to recognize that the very notion of being reflective and its corollary of being adaptative is based on the ability of a person to see themselves operating in a particular context, that is, the ability to 'step outside of themselves' in order to reorient themselves [...] the ability to be reflective and adaptative <u>requires</u> that an individual has the capacity to see the world as contextual, that is, as world in which one tries to understand how others (e.g. students) come to know and believe as they do" pp. 226, 227.

In teacher education, the main focus of interest is integrating mathematics pedagogy and mathematical contentfrom which certain activities arise (Wilson, 1994).

Integrating Mathematics Pedagogy and Content in Pre-service Teacher Education (T. Cooney)

Aimed to help the teachers to become more reflective
ACTIVITIES (focused on mathematical notion of function):
* Teachers read and discuss a classroom vignette.
* Teachers develop an informal, intuitive understanding by considering dependent-independent variable and correlational relationships in the media and everyday languaje
* A classification activitiy based on Kelly's repertorire grid technique
* Give the students an oppotunity to challenge preconceived notions about functions by investigating common school mathematics topics from a functional approach
* The teachers reflect on these experiences through journal writing, and small group and class discusssion.
* Student-conduced interviews (is an important activity to help teachers better understand another persons' beliefs as well as reflect on their own understanding.

Figure 2

Briefly described the activities and materials prepared for both projects, it is easy to identify certain similarity. But, the different theoretical frameworks, used by Even et al. and in Cooney's project, are based on different key aspects of practice and research in mathematics teacher education: (i) explore sources of a main components of pedagogical content knowledge and development and study research-based teacher education program aimed at promoting teacher subject matter knowledge and pedagogical content knowledge, and (ii) conceptualized professional growth and find ways of promoting and monitoring this development in teacher education programs. The two projects also prepare research-based materials for use in teacher education. From that, if the materials and activities are similar (practice), Can the theoretical frameworks be complementary? (research).

Learning to teach: "Web" of knowledge, beliefs and context

Knowledge of mathematics and pedagogical content knowledge of specific mathematics topics play a major role in a teacher's pedagogical reasoning. Furthermore, they constitute a key aspect of content of learning to teach. But it is difficult to separate knowledge and beliefs during the process of learning what is necessary to teach (Llinares and Sánchez, 1996). The way in which prospective elementary teachers make sense of learning situations shows the relationship between knowledge, beliefs and context (Llinares, 1994 a; Simon, 1991, 1994).

In context of learning to teach, developing a new understanding of mathematic topics and adequate mathematical specific-pedagogical content knowledge can be related to the way in which beliefs develop. It is not clear if having correct knowledge of both mathematic topic and different representation methods in addition to knowledge of the way students learn will influence in the evolution of beliefs or, on the other hand, a specific belief will condition the improvement of understanding mathematical topics and will generate adequate pedagogical content knowledge specific to mathematics.

Our research is focused on prospective elementary teachers' beliefs and knowledge during their teacher education program, including their student teaching, has illustred some features of learning to teach as a "web" of subject matter knowledge, pedagogical conten knowledge, epistemological beliefs and context. In one of the parts of our research we worked with prospective elementary teachers in their student teaching and we generated several case studies (Llinares, 1989, Sánchez, 1989). Based on Green's ideas about belief systems (Green, 1971), symbolic interaccionism (Blumer, 1982) and Kelly's repertorire grid technique, and through semi-structured interview, class observation and artifacts and prospective elementary' journal, the studied cases illustrated the former "web". The description of beliefs and dilemas' substance and their sources pointed out the complexity of learning to teach. The cases of MC and N during their student teaching, two prospective elementary teachers at University of Sevilla, are summed up in the following table (Llinares, 1989).

	MC	N
Socialization Strategy	-interior adjustment -strategic submission	-interior adjustment
Beliefs about Mathematics (Meaning)	-Useful for life (Reasoning Development)	-Useful for life (Tools)
Teaching Implications (Interpretative process)	-Why are they useful? Problem solving content-reality connection	Model to be repeated
Dilemmas	-The nature of elementary mathematics and its teaching -Professional identity	-The nature of elementary mathematics

The origin of some of their beliefs of mathematics, teaching and their dilemmas generated during their practice teaching were related to the lack of knowledge of specific mathematics pedagogical content and their own weak understanding of the mathematics content they had to teach. The dilemmas were generated by the characteristics of the context where they carried out their practice teaching in contrast with their beliefs hold. From these cases, it was difficult to know "if the chicken came before the egg" (improving knowledge or promoting change of beliefs).

The relationships among knowledge, beliefs and context where student teacher carries out his/her activities (teacher education program at the university, student teaching at school and induction year at school) should be considered in the theoretical frameworks constructed to conceptualize the improving of knowledge and the professional growth of prospective teacher. Addition to, the complexity of learning to teach implies to have to take account different opportunities where prospective teacher can construct different aspects of necessary knowledge for teaching (Sánchez, 1994). So, it is necessary to take account a diversity of both the development activities and the contexts. (Cooney (1994) pointed "*One could argue that the means by which teachers learn such knowledge is one, if not the, defining point for teacher education and consequenly should be the focal point of research on teacher education*).

From that, different learning environments are being designed at initial primary teacher education program at University of Sevilla. The perspective adopted comes from the analysis of (i) professional knowledge of mathematics teacher and (ii) process of learning to teach considered as cognitive apprenticeship (situated learning). Learning environments are focused on

- mathematical activity (what mathematics is learned is interwined with how one learn mathematics) (García et al. 1994)
- knowledge base developed from cognitive research on learning of mathematics topics (through analysis of vignettes of learning/teaching situations, conduced interview, etc)(Llinares, 1994 a)
- mathematical representations to primary mathematics topic (Llinares and Sánchez, 1996)
- student teaching as a learning environment where beliefs, context and knowledge are interwiened (Sánchez and Llinares, 1996).

These activities are aimed to improving of mathematical knowledge and mathematics-specific pedagogical knowledge, taking account the prospective teachers' beliefs and context and considerating student teacher as a reflective being. The emphasis is placed on how the prospective teacher learn.

* Improving Subject Matter Knowledge and Pedagogical Content Knowledge and promoting professional growth of epistemological beliefs: A necessary integration of theoretical framework for the inquiry and design the practice.

Complementary to the theoretical frameworks used [(i)subject matter knoweldge, pedagogical content knowledge and pedagogical reasoning (Even, Tirosh and Markovits), (ii) epistemological beliefs and the conceptualizing of professional growth (Cooney, 1994)] to research process of learning to teach and the implications of these frameworks on the practical dimension of activity design in the programs can furnish better results for both Teacher Education and research.

The different aspects of theoretical frameworks that can be adopted (i) should generate characteristics in learning to teach that help prospective teacher to

- question his/her previous epistemological beliefs,
- improving his/her understanding of mathematics topics,
- develop his/her understanding of pedagogical conten knowledge,
- begin to generate cognitive skills and pedagogical reasoning process, and
- become more reflective,

and (ii) to be powerful ways to conceptualize professional growth (improve knowledge and evolution of belief systems).

One perspective that integrate knowledge, beliefs and context is looking at learning to teach mathematics as a situated learning. Recently it has started to be indicated that the activity "where" knowledge is constructed is part of what is learned (Brown, Collins and Duguid, 1989; Greeno, J.G. 1991; Leinhardt, G. 1988) From this perspective, knowledge is situated and it is a product of activity, context and cultura where is development and used. The metaphor, to describe that knowledge is situated and is development through activity, used by Brown et al. is considered knowledge like a "conceptual tools set" *"(conceptual knowledge is, in some ways, similar to a set of tools) ... can only be fully understood through use, and using them entails both changing the user's view of the world and adopting the belief system of the culture in which they are used" (Brown, collins and Duguid, 1989, p.33)*

Different contexts can be necessary to development and integrate the components of teacher's knowledge. The content and design of these learning environments should be based on the analysis of professional teacher knowledge (Llinares, 1995, 1994 b). On the other hand the evolution of teacher's beliefs should be link to improving his/her knowledge. This relationship should be one of the focuses of next research in mathematics teacher education. Even et al. 's proposal, focused on SMK and PCK, as well as Cooney's project, focused on ways of conceptualized of professional growth (including beliefs and knowledge, Wilson, 1994), can be understood from the perspective of situated cognition (Anderson, et al. 1996). The focus should be

- what is the content of program,
- how materials designed are a reflect of theoretical framework,
- how considered "activity" that prospective teacher have to generated, and
- going on the way of conceptualizing the professional growth.

To find out ways of to do operative the notion of situated knowledge and learning is, at the moment, one challenge.

Some final remarks.

The integration of knowledge, beliefs and context within a theoretical framework can also widen understanding obtained. This should causes research to begin to consider (i) the administrative conditions where the training programs are developed (Borko et al.1992; Eisenhart et al.1992; Llinares, 1996) and (ii) widen the research contexts (not only in the University but also when prospective teacher students are in the classroom teaching). It is possible that in the future, we should emphatize teacher cognitions and content even more.

The theoretical frameworks developed and the implications arising in the mathematics teacher education programs should consider the difference between prospective elementary teacher students, who in Spain have no specific mathematics training, and prospective secondary teacher students, who have five years of training mathematics before enter in mathematic education.

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