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ABSTRACT

This document features two articles concerning professional development in mathematics education. The first article, "Forging a Partnership: Intent, Decision Making, and Curricula," provides a vivid example of how teacher-written vignettes drawn from teachers' personal experiences in using the Investigations curriculum provided a springboard for exploration, reflection, and discussion, and helped teachers address the challenges posed by the curriculum. The second paper, "Elementary Mathematics Curricula as a Tool for Mathematics Education Reform: Challenges of Implementation and Implications for Professional Development," presents portraits of two teachers, both of whom initially struggled to enact new curricula as intended, and who became more successful at using the curricula as a result of their own experiences while exploring mathematical content and considering student thinking about mathematics. (Contains 38 references and 12 notes.) (ASK)

CENTER FOR THE DEVELOPMENT OF TEACHING
PAPER SERIES

Fostering a Stance of Inquiry Among Teachers:

Professional Development in Mathematics Education

Amy Morse

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December 2000

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This CDT Paper Series is intended as a vehicle for discussion of research on teaching and teacher development as they relate to education reform. Publications in this series will contribute to the community's understanding of the practice and profession of teaching, as well as to the process of change. It is our editorial policy to have papers reviewed by a panel of four readers—three named by the author and one chosen by CDT.

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Fostering a Stance of Inquiry Among Teachers:
Professional Development in Mathematics Education

Introduction

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The view of mathematics learning being promulgated by reform efforts (e.g., NCTM, 1989, 1991, 1995, 2000) is characterized by thinking, reasoning, and communicating in rich problem-solving situations where "mathematical thinking is not separated from mathematical concepts or skills" (NCTM, 1991, p. 25). Actualizing this view will require significant changes in both the nature of the mathematics that is taught and the manner in which it is taught.

New, recently developed elementary curricula (e.g., *Everyday Mathematics; Investigations in Number, Data, and Space*) offer considerable promise in meeting new goals for student learning, providing elementary teachers with challenging mathematical tasks based on important mathematical ideas. These curricula represent a dramatic departure from more traditional textbooks that have focused primarily on memorizing facts and applying procedures with little attention to the underlying meaning, concepts, and understanding. These new curricula, however, represent only one component of what is needed. New forms of instruction will become broadly available to students only if a substantial portion of the current teaching force transforms its current pedagogical practice. This transformation will require teacher professional development and support.

Little (1993) and others have argued that new forms of professional development for in-service teachers must differ in fundamental ways from traditional staff development, which has tended to treat teaching as routine and technical. They argue that the new approaches need to share many of the features of the subject-matter reforms they are designed to support. In particular, they need to build teachers' capacity for complex, nuanced judgments about the process of mathematics teaching and learning. For teachers in the elementary grades this is especially critical, since their current knowledge of mathematics may not be adequate to meet the new instructional goals (Ball, 1991; Ma 1999), and research suggests that challenging tasks are often enacted in waysthat reduce their demands (Stein, Grover, & Henningsen, 1996; Doyle, 1988).

One new approach to supporting teacher learning in a mathematics reform environment focuses on fostering a stance of critique and inquiry among teachers. According to Ball (1996), such a stance involves the consideration of new ideas and an openness to the insights and images of others. The papers in this set describe professional development for teachers who were using new curricula in which inquiry about mathematics teaching and learning was central. Classroom vignettes and tasks that embodied mathematical ideas central to elementary curricula provided teachers with opportunities for inquiry into their own as well as their children's mathematical understandings.

In "Forging a Partnership: Intent, Decision Making, and Curricula," Morse provides a vivid exemplar of how teacher-written vignettes, drawn from the teachers' personal experiences in using the *Investigations* curriculum, provided a springboard for exploration, reflection, and discussion and helped teachers deal with the challenges posed by the curriculum. In the second paper, "Elementary Mathematics Curricula as a Tool for Mathematics Education Reform: Challenges of Implementation and Implications for Professional Development," Davenport provides us with portraits of two teachers, both of whom initially struggled to enact new curricula as intended, and who became more successful at using the curricula as a result of their own experiences with exploring mathematical content and considering student thinking about mathematics. The professional development experiences described in these papers build on much of what we know about teacher learning (Ball, 1996) and effective professional development (Fullan, 1991; Loucks-Horsley, Hewson, Love, & Stiles, 1998) and provide us with new models of how to support teachers in their use of new curricula, with insights into the benefits of these approaches.

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Forging a Partnership: Intent, Decision Making, and Curricula[1]

Amy Morse

"No matter how well curriculum materials are tested and how many times they are revised, each school brings its own mix of resources and barriers; each classroom brings its own needs, styles, experiences, and interests on the part of both teacher and students; and each day in the classroom brings its own set of issues, catastrophes, and opportunities. We could test and revise endlessly; each new classroom test would result in new ideas we might incorporate, and would raise new questions about pedagogy or content. But at some point, we have to decide that the curriculum materials themselves are good enough—ready for teachers to use and revise in their own classrooms. Teacher decision making, therefore, is key, and the curriculum must be designed with this assumption in mind. The teacher's role is to connect the particulars of her classroom and students to investigations in the curriculum." (Russell, 1997, p. 251)

While "curriculum" is often taken to refer strictly to the textbook or curriculum materials, the enacted curriculum is actually jointly constructed by teachers, students, and materials in particular contexts. (Ball & Cohen, 1997, p. 7)

The implementation of a new mathematics curriculum—one that is reliant on a teacher's thoughtful interpretation and is designed within the context of the teacher as an active and discerning partner—calls on teachers to construct a new relationship to the curriculum and to students (Russell, 1997; Ball & Cohen, 1997). Teacher education and school culture has encouraged, if not required, elementary school teachers to view mathematics textbooks as hierarchically superior, yet inanimate. In this context, the textbook's job is to supply rules, strategies, and answer sheets, in addition to dictating the sequence and timing of mathematics content. At the core is the assumption that mathematics classes are simply a series of predictable events in which the teacher and students play narrowly defined—and even prescribed—roles of transmitter and receiver (Cohen, McLaughlin, & Talbert,

Morse, A. (2000). *Forging a Partnership: Intent, Decision Making, and Curricula*. Newton, MA: Center for the Development of Teaching, Education Development Center, Inc.

1993). The character of the relationship between the teacher and the curriculum in this scenario is neither interactive nor decision-filled; rather, it is one of two entirely separate entities. In fact, the traditional version of curriculum enactment is a sort of "race to the finish" scenario, with a tie viewed as a measure of success. The teacher strikes a bargain on the first day of school; when he or she turns the last page of the text, the students have learned a year's worth of mathematics.

Successful enactment of a new, reform-based curriculum requires a significantly different relationship between the curriculum and the teacher. He or she must immerse herself in the mathematics content in a way she has not before, as a learner and a seeker of sense-making. He or she must expect and listen for mathematical responses from his or her students, responses that may hold completely new ideas for her. He or she must develop skills for interpreting his or her students' ideas and conjectures in order to make sound decisions about where to go next. Finally, he or she must develop a wide repertoire of possible scenarios so that he or she can make choices even as he or she is facilitating mathematical discussion in the moment.

In order to have resources to draw on in the teaching event, the teacher must also have ways of understanding and interacting with the intentions of the curriculum. And to use this new curriculum well, a teacher must work to understand and fully engage in her new role as "discerning partner."

A curriculum written in alignment with the teaching standards of the National Council of Teachers of Mathematics (1991) calls on a teacher to examine and strengthen her notions of pedagogy. To teach for understanding requires a deeper knowledge of the elementary school mathematics concepts. To use the students' interactions with the mathematics activities as guideposts for instruction suggests that a teacher has developed highly sensitive listening and analyzing skills. To listen carefully to children's thinking, with an appreciation for the complexities of their developing ideas, requires not only content knowledge, but also a reflective stance toward the ways in which children come to understand that content. It is not possible to prepackage these elements of teaching mathematics in the new curriculum materials. Substantial shifts in standards and materials alone do not constitute a successful shift in classroom experiences (Ball & Cohen, 1997; Davenport, 1998; Schifter & Fosnot, 1993).

In districts across the country, teachers are putting aside traditional curricula in an effort to enhance the quality of their mathematics teaching and to deepen their students' mathematical understandings. A teacher who continues to engage in the traditional teacher/curriculum relationship will likely devote her attention to readying the physical materials for the next day's class. She may read over the agenda in order to plot out a time frame for class. She may read the rules of a math game or concentrate on the grouping or pairing of children. Although these activities are essential, they are not ones that will help her build the resources to implement the new curriculum in her classroom. She must now prepare for a significantly different event.

Where and how does a teacher begin to construct a working relationship with a new curriculum? How does a teacher prepare for facilitating a discussion based on children's mathematical ideas? How does she come to understand the mathematics of her curriculum deeply enough so that she is able to discern appropriate directions to take in class, ask probing questions of her students, and focus students on new and rich territories of work? Where might a teacher focus her attention in order to learn how to develop her practice?

The story of the professional development of 12 kindergarten through sixth grade teachers in a mid-size district may help to identify one landmark in the teacher's journey. The teachers in this story were participants in a National Science Foundation-funded teacher enhancement project, Mathematics for Tomorrow.[2] Teachers from four Boston-area districts participated in this project for two years; this story describes the work of the teachers in one district. While this paper focuses on the second year of the project, the teachers had developed some structures for learning together during the first year. In Year 1, they met twice monthly in a mathematics inquiry group. The group had two central goals: to strengthen participants' own content knowledge, and to work toward developing a more fine-tuned lens for students' mathematical understandings. At the end of the first year, the district selected a new elementary mathematics curriculum, *Investigations in Number, Data and Space* (Russell & Economopoulos, 1998). In Year 2, the inquiry group teachers continued to meet, adding a third goal and a core focus: to successfully implement the new curriculum.

Using Vignettes as a Mechanism for Collegiality

In the year preceding curriculum implementation, the teachers in the inquiry group used their own writing as a way to focus on students' mathematics work. In preparation for the inquiry group meetings, the teachers wrote vignettes, or short cases, describing children's mathematical thinking. The initial assignment for the vignettes was modeled after the writings and assignments found in *What's Happening in Math Class?* (Shifter, 1996). The teachers found the writing and analyzing of vignettes to be a very powerful focusing tool. The assignment—to listen carefully to students, so carefully that children's ideas and confusions might be recorded as a transcript or story for study—helped teachers recognize and hear the power in their own classroom communities. Writing in this manner about mathematics and classroom events helped teachers cultivate a deep curiosity about their students' thinking—a key component of the decision-making role of the good classroom teacher and a requisite disposition for the successful implementation of the new curriculum. In contrast to traditional models of professional development, vignette writing defined the teachers' collaboration as work that was embedded in inquiry (Sassi, 1998).

Sharing a vignette is to share the teaching experience through one's story. For the teachers in the group, the issue of "expert" or "novice" teacher became a non-issue. Examining vignettes, and the discussion that takes place in working to understand the case itself, calls on all of us to be colleagues in careful consideration of one another's ideas—to respect one another's questions and to move into a stance of curiosity and questioning. By shunning the "expert only" or transmission model of professional development and by encouraging inquiry, the vignettes offered new opportunities for learning and the teachers treated them as such. Ultimately, the vignettes helped to create a fundamentally different culture of learning for the teachers.

Vignettes also served as a resource for teachers' mathematics learning. Close examination of the interactions of students and math work described in the vignette prompted teachers to pay close attention to the ways they themselves understood the mathematics. This "peering in" on one idea at a time, holding still a collection of children's varied responses to a single problem, allowed teachers to use the vignettes as a magnifying glass on the mathematical ideas and a territory for investigation that held all else outside its barriers. Taking the stance of a math learner herself within the group, in turn, deepened a teacher's sense of trust in her colleagues as well as her respect for the learners in her classroom.

In Year 2, the year of curriculum implementation, the teachers continued writing vignettes as a way of coming to understand the new curriculum. Students' learning and ideas generated in the process of using this curriculum were the focus of each vignette. In that way, teachers could examine students' work and the specific ways in which the *Investigations* activities impacted student learning. Every elementary grade level was represented in the teachers' group. As a result, teachers were able to see how mathematical thinking develops over time and to understand how this learning process is articulated throughout the curriculum.

October Inquiry Group Session

Vignettes, and the mathematics discussions they generated, provided important insights for the group in the teachers' struggle to forge a new teacher/curriculum relationship. Over the course of the school year, the teachers began to focus on "getting a hold on" the intentions of the curriculum as an important new way of understanding their own relationship to the intended work. Identifying and articulating the strategy *to read the curriculum for intention* was a critical turning point for the teachers' work. The teachers began to ask themselves and one another, "What is the math here? What math concepts will the children encounter? What are they to learn?"

This effort to understand—to "get inside"—the intentions of the curriculum became a powerful component of each teacher's work toward being an informed and capable decision maker in her classroom. The inquiry group session described in this paper was a touchstone event for the teachers. It marked the beginning of a new way of interpreting their curriculum. They referred to this session frequently over the course of the year as a powerful shift in the group's thinking.

In the excerpt from the inquiry group session in October, we can see that the teachers began to pay a fine-tuned and energetic attention to the language in the new curriculum. In struggling with a new mathematical term, the teachers pushed to contrast the mathematical ideas in it to the mathematical ideas in the more traditional math language they had all shared. For example, examining the mathematical implications of the words "compare" and "combine" in this particular curriculum activity invited a more considered look at the words "subtraction" and "addition." In fact, the teachers in the group had never previously associated the word "compare" with the word "subtract" or "add." As is the case for many teachers, the words these teachers associated with subtraction were limited to "take away," which describes an action quite different from "compare." The question that arose—for the children and the teachers—in the case of "Compare and Combine" was, What is the mathematical meaning of "compare," and how is it related to the way we use the word in common speech? As the teachers began to appreciate the difference between thinking about the words "subtract" and "add" and the words "compare" and "combine," they also began to engage in a deeper inspection of the curriculum developers' intent. The following is a description of this inquiry group session, presented in three parts: the vignette, the teachers' responses to the same math lesson, and the whole-group discussion of the word "compare."

A teacher-written vignette about a third grade classroom using an Investigations *lesson, "Handfuls," from Mathematical Thinking in Grade 3*

> The students are asked to grab handfuls of an object. First, with one hand, then with the other. They are then asked to compare the amounts and show how they compared the numbers. The students were asked to combine the two amounts and record *how* they combined them.
>
> Well, the lesson started off great but then I realized that students did not understand what was really being asked of them. I had to stop the activity and go over the directions again. Students seemed to understand what they were supposed to do but they didn't understand how you compared the two amounts. The more I thought about it the more complicated it seemed to me because these students weren't quite sure what I was asking. I didn't want to tell them that they could tell me that one number was more than another number or less than another number. I felt that telling them would kind of give it to them. Somehow I wanted it to come from them. So I decided to see if someone could define the word "compare" for me. They were silent.

Finally, someone gave a response: "It's when you see how different things are." I asked how we could see how different two numbers were. They were silent again. I had a student show two amounts with cubes and then asked if anyone could tell me what was different about the numbers. Well, the answers were varied. Some students thought that one number had more red cubes. [Others said] one was bigger. [Still others said] one had 3 greens, 1 white. [Or] one could be odd, one could be even.

The independent part of the lesson went much better from that point on but some students still struggled with the "showing" part. That night their homework assignment was to grab handfuls of something at home and record the same information about their amounts as they did in class. . . Some students even started to show that when they compared the right hand grab to the left hand grab it was "3 less." I am not sure if that was because they understood the concept of comparing a little better or if a parent told them what to do. It did help to have a varied group of answers the next day because students were really listening to each others' answers.

—Third Grade Teacher Charlene C., October 15, 1996

As with any case, the case alone is not "the thing." It's in the discussion of a case that the significance or power of the learning is revealed. We read in this vignette that the teacher faces a number of teaching choices. She needs to make decisions—but based on what? She wrestles with what to say, what to reveal. As we read her story, we can begin to formulate questions: What is it that she wants her students to understand? What *do* they understand? How does she interpret the children's silence? What does she make of the varied answers the children offer? What is the mathematical meaning of "comparing"? These questions form the basis for an inquiry into teaching and the implementation of a curriculum.

While in the moment of teaching, the teacher must not only *hear* the children's ideas that "one number had more red cubes," one "had 3 greens and 1 white," and "one could be odd, one could be even"; she must also *use what she knows* about the mathematical significance of "comparing" to make grounded decisions about how to proceed. As with a kaleidoscope, the children's words become the "flick of the wrist" that causes the entire picture to suddenly change. What seemed in preparation a quite simple activity shifts to display a myriad of questions: What does it mean to "compare"? What is the intention of this curriculum investigation? On what grounds, principles, and/or knowledge should the teacher base her next moves?

Teachers respond to the same mathematics lesson[3]

In this inquiry group session (and what would subsequently become an inquiry group routine), the teachers had a chance to do the same math activity for themselves in addition to reading Charlene's vignette. In small groups or pairs, the teachers worked from the activity sheet, "Handfuls," from the unit Mathematical Thinking at Grade 3, which Charlene had brought along with copies of her vignette. In the discussion that followed the teachers' math exploration, they freely admitted their own confusions. In fact, the teachers discovered, quite similarly to the students in Charlene's class, that they had responded to the words "compare" and "combine" in a variety of ways. The teachers brought several meanings for the word "compare" and a collection of representations of those ideas on paper to the group's discussion.

As it turned out, some teachers—just like a number of third graders from Charlene's class—had simply assumed their task was to add. They had not bothered to read the directions carefully. A teacher discovered that she had combined amounts both times. She laughed and said that the words "combine" and "compare" looked so much alike—"After all, they both start with 'c'!"

Others wrote a mixture of mathematical equations and descriptive sentences. Interestingly, Kate combined mathematical symbols with a description of the handfuls. On her activity sheet, she wrote, "4 + 4 , 4 = 4, 4 – 4 = 0." The first "sentence" set the stage, in a literal way: 4 + 4 described the 4 cubes in the right hand and the 4 cubes in the left hand; 4 = 4 described the "sameness" of the two amounts; and 4 – 4 = 0 was a mathematical expression of the difference between the two handfuls.

Carla drew two rows of cubes. She simply displayed her method for finding a comparison, leaving interpretation to the viewer.

Another teacher wrote, "10 here, 9 here. 1 more than."

One teacher drew a single row of cubes. To signify the two handfuls, she drew black dots and empty circles. She then drew an arc from one of each of the black dots to one of each of the empty circles. Because there was one dot left with no "partner," had she really "compared" the two amounts?

Yet another teacher found a pattern in the comparison and wrote of the amounts, "6 and 3, doubled!"

The teachers were surprised by the variety of the group's responses. They came to see through the ensuing discussion that they had underestimated the nature of the mathematical task in this activity. Thinking through what students would be learning by engaging in the work themselves became an exciting and purposeful conversation.

Inquiry group discussion of the word "compare"[4]

The teachers gathered around the table, bringing math papers and cubes, for a whole-group discussion. Cubes spilled onto the long library table, and the teachers talked quietly together as the group settled in to discuss the responses to the math activity. In this discussion, the vignette and the teachers' own responses to the math melded together into a long discussion, which Lydia began. She laughed and shook her head. "I can't believe it. I just completely blew off the directions without even thinking and simply added amounts both times. I didn't recognize 'compare' and 'combine.' They start with 'c'; they didn't look different."

"In fact," Charlene said, "that's what a lot of kids did. They did the 'combining' up top [even though it says to compare amounts], and then they had nothing at the bottom. And they were stuck on the word 'compare.'" Waving her vignette in her hand, she continued, "The kids said, 'I don't know how to show it.' They didn't know how to draw it." She seemed to shift gears. Laying the vignette down on the table, Charlene looked at her colleagues and asked for their ideas. "How *do* you show 'comparing'?" she asked.

The facilitator[5] took a step back from Charlene's question to focus on the word itself. The mathematical significance of the word was new territory for the teachers. In sorting out its mathematical meaning, the teachers began to get close to the real task at hand—coming to understand that the words "compare" and "combine" in this curriculum activity call on students to consider the relationships between quantities and the actions on them. The facilitator posed a question to the group: "So . . . if you unpack the word 'compare,' what does it really . . . what does it really imply?"

Kyle suggested, "To . . . like, to look for similarities and differences."

Lin nodded in agreement. She and her partner had discussed this while working on the math. "That's what I said to Deanne."

But Sarah took the group in a slightly different direction to consider the word in terms of familiar mathematical notation. She stated confidently that, to her, "It's like an equation." She said, "When you compare, you put something on one side and something on the other side. It's either going to be equal to, or greater than, or less than . . . like you see they're two different sets of numbers." There was silence for a minute as people absorbed Sarah's image of "compare."

May described what she did with her handfuls of cubes. She used the experience to help explain the meaning of "compare." "I looked at, that I would have to put them one to one. You know, one-to-one correspondence . . . I'd put them in 'one-to-one correspondence' to see which one had 'leftovers.'"

The facilitator asked May to say more. "Are you saying now that there's a kind of . . . there's something about quantity that's implied in the word 'compare'?"

After hesitating for a minute, May responded. She spoke slowly and looked down at her cubes as she began to peel back the layers of the meanings of "compare."

"In math, yes. But I would go with. . . But in something else—different qualities . . . if you asked me to compare two books or two people, I don't think I'd look in terms of numbers. I would look at similarities and differences."

Here May connected one meaning of "compare" to the definition Lin and Kyle had offered. Yet, she was suggesting a distinction between a more general meaning for "compare" and a mathematical definition. Sarah offered a way to bridge the two meanings by suggesting that one interpretation might be a subset of the other. She said, "You could think of numbers as one of the attributes of the word."

The facilitator brought Charlene's vignette back to the discussion in an effort to incorporate the children's interpretations of the meaning of "compare." "If a student put down on paper . . . Let's say I'm *comparing* and I put the number 9 and the number 6 here as my answer . . . there's something 'compare-y' about that."

The group laughed at her language. The facilitator laughed too; there was some self-consciousness in exposing confusion about a word everyone had used so easily up until this discussion. She continued, "But isn't there an action implied in 'compare'? I think what you, Charlene, were saying earlier, [was that] it wasn't acceptable to have just the two numbers, 9 and 6. Even if, though, that's a picture of how they're similar or different in some sense . . . There's more than just a visual comparison . . . it seems to me. So, when Charlene saw kids writing . . . putting down the two numbers, it felt to you like there was something *they hadn't yet done*."

"Yeah," Charlene nodded, "there were some who just left it blank. And some others did . . . you know, just wrote the numbers 9 and 6. I wasn't sure if those students were getting the *comparing* aspect to it. We stopped and we discussed. But I didn't want to *tell* them what 'comparing' was. Like, I thought it should be coming from them. Like, they should be trying to figure out what 'comparing' means. And they ... *they* brought up the comparing thing."

Here, Charlene moved the discussion from the definition of the word to the implications for her as a teacher using it. She was sharing one of many on-the-spot decisions called up in response to her students' confusions. She told the teachers, "I thought it should be coming from them." It wasn't entirely clear that Charlene knew what "it" was. She knew she had multiple agendas. She expressed a desire for her students to struggle through to the meanings of "compare." However, while she wanted her students to see the issue as a mathematical one, it was not clear that Charlene, herself, had a sense of the mathematical task.

"First," she said, "there was silence when I asked them what it means to compare. Somebody finally said, 'It means when you find what's different about two things.' So then we got . . . you know . . . " She closed her eyes as she thought back to the events in the classroom. "They made two different numbers with cubes, and I asked them, 'How can you show me what's different about these?'" Charlene's voice became animated as she imitated her third graders: "'Oh, that one's got two oranges,' and 'That one's got a white one' and 'That one's got a pattern.'" She began to talk more slowly as she recalled thinking hard about where to go with these ideas while facilitating a discussion with a room full of third graders.

"Well, I was like . . . 'Well, let's not stay with color, let's think numbers. Well, that one's bigger and that one's smaller.' So then they started getting into that, which felt more right. But then when they went back and worked on their own, if they got an even amount [e.g., pulled 3 in each handful], then they said, 'Well, you can't *compare* those! They're the same!'"

Summary of Lessons Learned

The discussion continued as the teachers talked about the particular event in Charlene's class, their own experience of confronting "compare" in the inquiry group, and the broader context of enacting this curriculum investigation and others like it. Over time, the teachers recognized their work to define "compare" as more substantively contributing to their understanding of the intentions and mathematics of the curriculum, as well as to their skills in creating the rich mathematics classes in which the curriculum is enacted. Before her lesson, Charlene assumed that the activity was self-explanatory. In the class itself, she discovered that the children's sense of "compare" as a mathematical term was not clear. In the inquiry group, Charlene had a chance to work toward a much deeper understanding of the mathematical term and what it was she wanted her students to learn. She headed toward a new focus of curiosity about the children's ideas of "compare" and the operations they used to model "comparing" situations.

In Charlene's classroom story, it was in the particular moment of making sense of the students' varied responses and questions that she had to make a decision about where to go next. Taken by surprise, Charlene realized that she was not prepared. She had "prepared" in the traditional sense: All the papers were ready, and the students were working in assigned groups. And yet, as she faced a choice about where to go next in the midst of her students' discussion, the line of reasoning on which she should make her choice was not entirely clear. She herself wasn't sure of the mathematical purpose of the task.

This dilemma was a familiar one to the teachers in the group; it highlights the different nature of the decision-making role required by their "partner in curriculum" responsibilities. In fact, it is a role required of all teachers who use these new curricula. By putting the classroom moment on hold, by capturing the story on paper to discuss with her colleagues, Charlene had afforded herself and her colleagues the benefit of reflection and collective inquiry. Specifically, the teachers had a chance to work together to refine their notions of "compare." They moved in and out of the word's meaning in common speech and its meaning as a mathematical term—and, while they did so, they began to develop foci for interpreting their work of partnering with the curriculum.

It is through this work that the teachers developed new understandings about and enacted new relationships to the curriculum itself. By developing a more fine-tuned focus on the children's reasoning, ideas, and responses to the curriculum; by learning to hear the questions, assumptions, and connections their students were making; and by becoming more knowledgeable of the mathematics content in the curriculum through the writing about their classrooms and their students, the teachers enriched their teaching.

Conclusion

In implementing reform-based curricula, teachers confront an array of challenges. They struggle to make mathematical sense of new terminology and new content. They strive to competently assess and analyze children's interpretations of the tasks the curriculum assigns. Finally, teachers draw on their own understandings of the mathematics content and their students' mathematical thinking in order to make the "right" decisions as they teach.

Professional development that supports the goals of the new curricula can also support teachers by addressing the challenges new curricula pose. It can focus teachers' attention toward reading the curriculum for intention and to highlight the question, "What is the mathematics my students should be learning?" Professional development can point teachers toward listening carefully and with curiosity to the student's words and their work, and can help teachers deepen their understanding of the mathematics of their own classrooms.

The inquiry group described in this paper found vignette writing to be an effective mechanism for these efforts. By capturing a decision-making moment in one's classroom—when one is no longer inside the predictable confines of a traditional curriculum and traditional expectations of teacher/student roles—a teacher can use a vignette as a basis for study. She can use her writing as a tool for developing and strengthening her notions of how learning takes place in a classroom and her role in the process. She can use the vignette to carefully inspect the words and ideas that children bring to the mathematics she is offering. She can use her writing as a tool for uncovering her own mathematical knowledge and the impact of her understanding on the learning that takes place in her classroom. Finally, she can use the vignette as a tool for discerning the intentions of the curriculum and how her students interact with those intentions.

Such activity, done over time, enables the teacher to play her role as full partner with the curriculum and with the children in her class. It also provides an opportunity to reflect on the very acts of perception, intent, and decision making that she will need to exercise when she is teaching.

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[2] Mathematics for Tomorrow is funded by the National Science Foundation, Grant No. ESI - 9254479. Barbara Scott Nelson of Education Development Center, Inc., is the principal investigator.

[3] The data in this section are from field notes and audiotapes of the sessions.

[4] The inquiry group sessions were tape-recorded and transcribed. For this paper, I chose to reconstruct the session in narrative form. Facial expressions, silences, and tones of voice, lost in a transcript, are important factors in conveying these teachers' efforts to stay focused on what could be perceived as a simple task—giving the mathematical meaning of a commonly used word. The quotes in the paper are direct quotes from the transcript.

[5] The facilitator of the session is the author of this paper.

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Elementary Mathematics Curricula as a Tool for Mathematics Education Reform: Challenges of Implementation and Implications for Professional Development

Linda Ruiz Davenport

A number of standards-based elementary mathematics curricula have been created to serve as a tool for mathematics education reform. Although these curricula have much to offer teachers, they also pose serious challenges; in order to use these curricula as intended, teachers must shift how they think about mathematics, mathematics learning, and mathematics teaching. This paper provides two stories of teachers learning to work with an innovative elementary mathematics curriculum while they are participating in a year-long *Developing Mathematical Ideas* seminar. In the first story, a teacher using *Investigations in Number, Data, and Space* is working through the question of what her students should be learning; as she learns more mathematics herself, she finds that she is better able to articulate mathematics learning goals for her students. In the second story, a teacher using the *Everyday Mathematics* curriculum is developing a curiosity about her students' mathematical thinking; as she becomes more intrigued with the different ways her own students are thinking about the problems she is posing, she begins to make more space for their thinking in her classroom. An examination of these stories shows how professional development that engages teachers in thinking deeply about the mathematics content of the elementary mathematics curriculum, and exploring how students think about that mathematics content, can help prepare teachers to use standards-based curricula as a tool for reforming their practice.

Over the past 10 years, a number of standards-based elementary mathematics curricula have been developed and are now available to teachers. These curricula were designed to help teachers implement reform recommendations in their classrooms, and many districts are adopting these curricula in order to forward their reform agendas. But we know that reforming one's

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mathematics teaching practice is not as simple as exchanging a textbook for a new curriculum. For many teachers, learning to use a standards-based mathematics curriculum presents significant challenges.

This paper examines the stories of two elementary teachers learning to use an innovative mathematics curriculum. The first story allows us to consider the importance of developing a deep understanding of the complex mathematical ideas embedded in the elementary curriculum so that instructional decisions made by the teacher, both in the planning and the enacting of a lesson, do not compromise the curriculum's mathematical integrity. The second story focuses on the importance of learning to listen to students' mathematical thinking in ways that reflect an appreciation for the insights they bring to their work with mathematical ideas, so that students' ideas can be part of the mathematics discourse during a lesson. Together, the stories highlight two important challenges that teachers face as they work to implement standards-based mathematics curricula.

Because the teachers in these stories were part of a professional development project, they also provide a context for examining the kinds of supports that professional development can offer to teachers who are learning to implement these curricula. How can we help teachers more deeply understand the mathematical ideas in the curriculum so that, as they plan and enact a lesson, their sights are sharply focused on the lesson's important mathematics? How can teachers be helped to listen more carefully and thoughtfully to their students' mathematical ideas, so that these ideas have an appropriate place in the discourse of a lesson? Just as the two stories offer us images of the challenges teachers face as they work to implement their curricula, the stories also illustrate how a professional development experience helps these two teachers make important shifts in how they think about mathematics, mathematics learning, and mathematics teaching in ways that impact their use of the curriculum.

The paper concludes with a discussion of what sustained professional development that focuses on deep explorations of the complexities of the elementary mathematics content, examinations of how students across the elementary grade levels are thinking about that content, and the implications for standards-based mathematics teaching can offer teachers. In particular, it considers how these kinds of professional development experiences place teachers in a position of being better able to take advantage of what a standards-based mathematics curriculum offers, thus making these curricula more powerful tools for the mathematics education reform effort.

Mathematics Education Reform and Standards-Based Mathematics Curricula

The mathematics education reform recommendations call for a new kind of mathematics teaching practice (NCTM, 1989, 1991, 1995; NRC, 1989). They call for a practice that supports the development of mathematical understanding—one that engages students in explorations of and discussions about mathematical ideas, involves students in constructing procedures based on their understandings, and supports students in becoming more powerful mathematical thinkers. In order to create this kind of practice, teachers must foster classroom communities in which students can explain and discuss their reasoning, make and test conjectures, and dig deeply into the mathematical ideas at play in particular lessons.

A number of standards-based mathematics curricula are now readily available to serve as tools for constructing this new practice, which districts are adopting as a way to strengthen mathematics teaching and learning in their schools. These curricula reflect the reform recommendations in their focus on the development of mathematical understanding, their attention to helping students learn to explain their mathematical thinking, and their suggestions for ways that teachers might orchestrate classroom discussions about the students' ideas. They lay out a comprehensive set of activities that teachers can undertake with their students over the course of the school year, and they include assessment tools that can be used periodically to evaluate student learning. Ultimately, these curricula provide hope that larger numbers of teachers might successfully and more easily engage in the mathematics reform effort.

But these curricula are very different from most traditional texts in several ways. First, students are assumed to be sense-makers who have something to offer to the endeavor of mathematics learning. The curricula typically include mathematics investigations and activities that students explore in depth, with partners or small groups, devising and discussing their approaches to the mathematical problems and then sharing their approaches in a whole-class discussion, so that a variety of ways to think about mathematical problems are made public, and opportunities to think and reason about the problems as a group are available. Second, the mathematical content of the elementary curricula often goes beyond what is traditionally offered in mathematics texts. There are units on geometry, measurement, probability, data, and statistics. There are explicit discussions of problem solving and critical thinking. In addition, computation—traditionally the core of the elementary curriculum—is now addressed through an exploration of the number system so that computational procedures grow out of an understanding of our number system, and are not just a set of memorized rules. Third, teachers working with these curricula are expected to assume new roles in their classrooms as "facilitators of student learning." Teachers are no longer always at the front of the room offering explanations. Rather, they circulate frequently, asking students questions that help them make sense of the investigations and activities they are exploring and creating; during classroom discussions, teachers help students listen to and learn from one another's ideas.

What does it mean to construct a practice using these standards-based mathematics curricula? First, teachers must have a solid grounding in the important mathematical ideas so that they can fully understand the goals of each lesson. Teachers must also be able to listen carefully and critically to their students' ideas so they can be alert for the mathematical ideas present in what students are thinking, and use what they are learning about their students' thinking to inform their instructional decisions. Finally, teachers need to be able to move back and forth between the mathematical ideas that arise from their students and the mathematical ideas that are central to the lesson in ways that often cannot be anticipated, particularly for teachers new to this kind of practice.

Challenges of Implementing Standards-Based Mathematics Curricula

Teachers learning to use a standards-based mathematics curriculum as a tool to reform their mathematics teaching practice face many challenges; history has taught us that simply handing teachers a new curriculum is not sufficient (Cohen, 1990; Cohen and Hill, 1998; Cohen et al., 1990; Heaton, 1994). Their struggles have been well-documented (e.g., Heaton, 1994; Remillard, 1996; Schifter, 1996a, 1996b), even when teachers are working with a mathematics curriculum designed to support them in this new practice. By and large, teachers' traditional understanding of mathematics, mathematics learning, and mathematics teaching shapes how they make sense of their new curricula.

The new curricula embody a practice that seems foreign to the way that many teachers themselves learned mathematics. Indeed, most teachers' learning of mathematics focused on practicing rote procedures for arithmetic computation, with limited opportunities to explore mathematical ideas or discuss mathematical thinking. Consequently, many teachers using a standards-based curriculum to construct a new practice have a great many questions: What does it mean to "teach for mathematical understanding"? What does it mean to have students "talk about their thinking" and "construct their own procedures for solving mathematical problems"? What does it mean to facilitate a classroom discussion that simultaneously honors students' ideas and attends to the important mathematics? These are fundamental questions that teachers must consider and work through in order to use standards-based mathematics curricula as powerful tools to support the development of mathematical understanding. Otherwise, teachers will rely on their old visions of mathematics teaching and learning as they attempt to implement these new curricula, leading to, at best, a superficial or mechanical implementation.

On the whole, the recent spate of standards-based curricula take seriously the suggestion that the curriculum be a "site for teachers' learning" (Ball and Cohen, 1996, p. 8; Remillard, 1996). The curricula contain support materials that speak to teachers about how to construct this new practice. For example, there may be discussions about the mathematics content embedded in a set of lessons that highlight the important mathematical ideas. There may be information about the research on how children think about particular mathematical ideas, along with examples of how children are known to approach particular kinds of problems. There may be sample classroom dialogue that helps teachers anticipate the ideas that might arise in a classroom discussion. There might be guidelines for what to watch for in using assessments that are written into the curriculum. These are all useful supports for teachers.

But in order to use these innovative curricula as intended, even with the supports that are written into them, teachers must be able to shift how they are thinking about mathematics, mathematics learning, and mathematics teaching. Teachers must understand that a lesson's goals center on *discussions* of ideas rather than merely the practicing of procedures and the completion of tasks. Teachers must be able to engage students in discussions of the students' thinking about these ideas while considering what it is that students understand and what ideas students still need to work on. Teachers must also be able to consider the kinds of questions to ask or problems to pose that help students work through what it is that they are struggling to understand, while keeping an eye on the important mathematics of a lesson. These are no small challenges.

What is the role of professional development in helping teachers take advantage of what standards-based mathematics curricula can offer? How can professional development help teachers embrace the assumptions about mathematics, mathematics learning, and mathematics teaching that these curricula reflect? How can professional development prepare teachers for the new kind of teaching skills these curricula require? These are some of the questions this paper seeks to answer.

Two Stories of Learning to Use Standards-Based Curricula

This paper examines the stories of two teachers learning to work with a standards-based mathematics curriculum while participating in a year-long professional development project. Pat, the teacher in the first story, worked with *Investigations in Number, Data, and Space* (TERC, 1995). Helen, the teacher in the second story, worked with *Everyday Mathematics* (University of Chicago School Mathematics Project, 1998). These teachers were each observed and interviewed in the fall and the spring of one academic year.

The professional development project in which these teachers participated was a *Developing Mathematical Ideas* (Schifter et al., 1999a, 1999b, 1999c, 1999d) seminar that met for three hours approximately every other week for 16 sessions, beginning in the fall and ending in late spring.[1] The seminar was designed to help teachers think through major ideas of K–6 mathematics and examine how children develop those ideas. At the heart of the materials are sets of classroom episodes, or cases, illustrating student thinking as described by their teachers. In addition to case discussions, the curriculum offers teachers opportunities to explore mathematics in lessons led by facilitators; share and discuss the work of their own students; plan, conduct, and analyze mathematics interviews of their own students; view and discuss videotapes of mathematics classrooms and mathematics interviews; write their own classroom cases; analyze lessons taken from innovative elementary mathematics curricula; and read overviews of related research. Teachers complete a set of portfolio assignments as part of their participation in the seminar and also write a final evaluation of their seminar experience.

The seminar is designed to embody many of the reform recommendations regarding mathematics teaching and learning, with the teachers participating as "students" and the facilitator of the seminar acting as the "teacher."[2] The goals of the *Developing Mathematical Ideas* (DMI) seminar, which are communicated to DMI facilitators in the support materials through the journal of a semi-fictional facilitator named Maxine, reflect this commitment to the reform recommendations:

> [N]ow that I am about to meet a new group of teachers for a seminar that I haven't taught before, I'm thinking about what I want them to learn. What are my goals for this seminar? First, I want the teachers to come to see that mathematics is about thinking and that *they* have mathematical thoughts . . . Second, I want the teachers to recognize their students as mathematical thinkers. I want them to learn to listen to their students' mathematical ideas and to respond in ways that communicate that those ideas are valued . . . Third, I want the teachers to learn how to analyze their students' ideas. What is the logic in what this child is saying? Even though there is something incorrect in the idea, why does it make sense to the child? How do these ideas relate to the central mathematical themes of the elementary curriculum? . . . Fourth, I want the teachers to learn how to engage a whole class in analysis of student ideas . . . Finally, whatever processes I envision for mathematics classrooms, I also envision for this group of adults. I want them to learn to pose their own questions about mathematics and come up with ways of thinking about answers. I want them to become curious about children's mathematical ideas and learn how to listen carefully for those ideas. And I want them to think hard about what constitutes a teaching practice that supports children's development into powerful mathematical thinkers . . . (pp. 109–110).

These goals are met by a deep exploration of mathematics content—including the base 10 structure of our number system, the meaning of operations, and methods for calculating with multi-digit numbers and fractions—as well as analyzing children's thinking about that content.

We begin each teacher's story in the fall, looking at what they say and write about the challenges they face in their mathematics teaching and how these challenges are played out in a mathematics lesson. We then shift to the spring to see what the teachers are now writing and saying about what they are learning and how this new learning plays out in a mathematics lesson. We end each story with a discussion of the specific features of the professional development experience that supported the teachers' learning. Through their stories, we can learn more about the kinds of professional development experiences that are important to teachers as they learn to work with standards-based curricula.

Pat's Story: What Are My Students Supposed to Be Learning?

When mathematics lessons focus on mathematical ideas instead of facts and procedures to be memorized, identifying the important mathematical learning goals for students can be difficult. What are the important mathematical ideas to focus on in any given lesson? What do you want students to be thinking about as you engage them in the activities of the lesson? How do you know what your students are actually learning? These questions have implications for the kinds of discussions that teachers work to support during a lesson, the ideas and behaviors they watch for in their students, and the decisions they make about how long to stay with a particular mathematical idea.

Getting clear on the important mathematical ideas in a lesson can be difficult for many teachers (e.g., Heaton, 1990; Cohen, 1990). Even when the important mathematical ideas are explained and elaborated in the curriculum materials, they can be missed or misinterpreted, particularly since many elementary teachers have had few opportunities to explore the ideas for themselves. The result is that teachers use the curriculum without a clear regard for its mathematical purpose, and the lesson can become a set of activities to complete rather than opportunities to explore an idea. Pat's story allows us to examine the process of coming to understand what it means to have a clear mathematical focus for one's mathematics teaching and a set of mathematics learning goals for students.

Pat is an experienced third grade teacher in an urban school district. This was her third year using at least some portion of *Investigations in Number, Data, and Space* in her classroom. In her first year of working with the curriculum, she used only one unit—Mathematical Thinking in Grade 3 (Russell and Economopoulos, 1998)—and the rest of the year she relied on a traditional text. In her second year, she used the Mathematical Thinking unit again plus one other—Things that Come in Groups (Tierney, Berle-Carman, and Akers, 1998)—but again relied on her traditional text for the bulk of her mathematics instruction. This third year, *Investigations* was her primary curriculum.

In this story, we see Pat working through questions about what mathematics her students should be learning and how well they are learning it. In the fall, we hear Pat describe her struggles with this question, we see how her struggles are reflected in how she thinks about her students' work, and we witness how her uncertainty about the important mathematics is played out in the context of a particular lesson. In the spring, we hear Pat talking in a more grounded fashion about the mathematics learning goals she has for her students, and we can see this reflected in the way she is now able to analyze her students' work and in the way she conducts her mathematics lesson. The story ends with a discussion of what Pat learned in the DMI seminar and how her seminar experience helped her take better advantage of what the *Investigations* curriculum offered her and her students.

Pat in the fall

Although she believed that the *Investigations* curriculum had a lot to offer, Pat struggled in the fall with being able to determine what her students should be learning. Were they acquiring the skills they needed? Were they mastering the important ideas? What *were* the important mathematical ideas to be addressing at the third grade level? These were the questions she raised in her interview with me in early November. She described how the mathematical ideas of the curriculum were feeling "nebulous" to her, even though she spent many hours reading through the curriculum, including its support materials. Pat would often go back to the traditional text in order to check on whether students were learning the skills they needed. She would also check the scope and sequence of the traditional text to make sure the content listed there was being addressed in her lessons. She would assign homework from the traditional text to make sure that students could solve the problems it contained. She explained that the traditional text was her "grounding" as she worked to implement the new curriculum.

Pat also struggled with her pacing as she worked with the new curriculum. Even if you could identify the important mathematical ideas, she wondered, how do you judge when students have "mastered" them? How long do you need to stay with an activity, and how do you know when is it time to move on to the next idea?

Pat's uncertainty about what her students should be learning was reflected in a portfolio assignment she completed in late September for the DMI seminar. The assignment called for her to analyze three samples of student work, discussing what she found satisfying or unsatisfying about them, and to identify learning goals for those students based on her analysis of their work. She wrote:

> The assignment was a challenge for me . . . The most difficult part of the assignment for me was to set learning goals for the students . . . Setting learning goals is difficult for me because I don't think I really understand the deeper mathematical issues.

The difficulties Pat describes are borne out in what she was actually able to say in her analysis.

The student work Pat analyzed came from the investigation "What's a Hundred?" from Mathematical Thinking at Grade 3. Students were to count out 100 interlocking cubes, figure out ways to *prove* that they had 100 cubes, and record their strategies, using words, pictures, and numbers. The mathematical focus was on helping students appreciate that numbers can be "chunked" into groups, including groups of 10, and that these groups can be a helpful way to keep track of a quantity. The curriculum identified several things to watch for as students worked on this task, including how students were counting (e.g., by 1's, 5's, or 10's), how students were keeping track of their counts, whether students had ways of double-checking their counts, and whether students were accurate in their counts.

In writing about the two students who concerned her, Pat described how they often lost track of their totals as they attempted to count by 1's to 100, and how they seemed to not have a way to organize their tallies. In writing about a third student, whose work was strong, Pat described how this student also counted by 1's but then organized her cubes into groups of 4, and then groups of 16, to demonstrate that she had 100.[3] These observations are in line with what the curriculum suggests that Pat should be watching for as students work through this activity. However, although Pat was able to describe how these students approached the assigned task, her discussion was not focused on their mathematical ideas. She talked in generalities about their process but did not discuss what they seemed to understand (or not understand) about the idea of "chunking" or grouping numbers or grouping by 10's. For example, for the two students who had trouble counting 100 cubes, Pat wrote:

> For Jessie and Luisa, my goal is for them to focus on the mathematical purpose of the assignment. Also, they need exposure to more strategies and the opportunities to discuss the advantages of different strategies so they will make wiser decisions. They both need more experiences with counting and adding on.

For the student whose work she found strong, Pat wrote:

> For Manuela, my goals are to have her experience building and taking apart larger numbers and discovering number patterns within these numbers. Also, I'd like her to experiment with visually estimating quantities of objects and comparing larger and smaller numbers.

Pat did not comment specifically on what she hoped that students would come to understand about the idea that numbers can be grouped, say, by 5's or by 10's, in order to build 100. Rather, she identifies some general behaviors she would like to see the students improve. For Jessie and Luisa, what did Pat hope they would learn from their classmates' strategies? What would more experience with counting and adding on help them to better understand? What did it mean to help them focus on the mathematical purpose of the assignment? For Manuela, it is likely that more experience with building and taking apart numbers and looking for patterns within these numbers would be helpful in developing an appreciation for the number system, though Pat does not specify what might be learned through these actions.

The challenge of identifying learning goals for students that focused on mathematical ideas was also demonstrated in the lesson I observed Pat teaching in early November.

The lesson, "Playing 'Guess My Rule,'" was also from Mathematical Thinking in Grade 3 and addressed ideas about collecting and analyzing data. The Teacher Notes for the lesson explained that "Guess My Rule" was a classification game in which players try to figure out the common characteristic, or attribute, of a set of objects. To play the game, the rule maker (a teacher, student, or small group of students) decides on a secret rule for classifying a particular group of things. For example, a rule for classifying people might be "is wearing blue" or "has brown hair." The rule maker starts the game by giving some examples of people who fit the rule, and the guessers then try to find others. The support materials identify two guidelines that are important to stress throughout the lesson: (1) It is important to have two groups, one for individuals who fit the rule and one for individuals who do not fit the rule; and (2) it is important for students to give reasons why they think the classmate they have chosen fits the rule. These guidelines are highlighted in the support materials for the lesson:

> "Wrong" guesses are clues and just as important as "right" guesses. "No, Cesar doesn't fit, but that's important evidence. Think about how Mark is different from Kate, Ly Dinh, and Jeremy." This is a wonderful way to help students learn that errors can be important sources of information . . . When you think you know what the rule is, test your theory by giving another example, not by revealing the rule. "Midori, you look like you're sure you know what the rule is. We don't want to give it away yet, so let's test out your theory. Tell me someone who fits the rule." Requiring students to add new evidence, rather than making a guess, serves two purposes. It allows students to test their theories without revealing their guess to other students, and it provides more information and more time to think for students who do not yet have a theory. (p. 56)

The support materials include a sample of classroom discourse, which also highlights these guidelines.

Pat's enactment of the lesson included several important features of how the game was to be played and discussed. For instance, as she introduced "Guess My Rule" to her students, she let them know that it was important that they be able figure out what the rule was by trying to use it, that they were to think about who fit the rule as well as who did not fit the rule, and that she would ask students to explain what they were thinking when they made their choices. It was clear that she had read the curriculum materials carefully and was well-prepared for the lesson. However, her lesson, *as it was enacted*, missed several important features that limited what students were able to learn. For instance, during the game, Pat asked students who fit the rule to circulate so they could be very visible to their classmates. However, she did not create such visibility for the students who did *not* fit the rule. Instead, the students who did not fit the rule remained in their seats among those that had not yet been chosen. This made it difficult for students to use the information about who did not fit the rule to inform their choices. It was not until after the rule had been correctly named, at the end of the game, that students were asked to get into two groups—those who fit the rule and those who did not. This was not done in order to help students make conjectures about the rule, but rather as a way to demonstrate that the rule was correct.

In addition, although Pat encouraged students to think about the rule, there was actually little discussion of their conjectures, many of which were not stated as valid rules for classification.[4] Although Pat encouraged students to give their reasons for their conjectures ("What was going on in your head when you saw something that was the same [about the students that had been chosen]?" she asked when someone suggested that the rule was "hair"), and she often urged students to check their rules against the evidence, there was little follow-up on these efforts to have students articulate their thinking. In fact, students often responded to these invitations to share their thinking by changing their conjectures. Pat did not work toward helping students state their conjectures as rules, nor did she work toward helping them explain the rationale for their conjectures. In the end, the correct rule—"eyes"—was finally agreed on by the class, not through a discussion of the evidence, but because Pat named it as the correct rule, adding that someone had said this rule three times and the rest of the class had not been listening.[5]

Immediately following the lesson, Pat and I had an opportunity to talk. As Pat reflected on the mathematical goals of the lesson, she talked about how an important goal was to develop students' critical-thinking abilities—a much less focused learning goal than those identified in the curriculum materials. In her discussion of what her students were doing and thinking during the lesson, she said that several students "made no connection to the whole activity," while other students had "taken risks," and one, in particular, showed "good leadership" during the activity. However, she thought she was going to need to spend more time on this activity, even though it was already their second day on an activity intended for one class session.

While Pat's reflection on the lesson included some important observations, what was strikingly absent was any substantial discussion of the specific mathematical ideas that she hoped her students would be thinking about during the lesson, as well as *how* they were thinking about those ideas. Left with only a vague sense that there was more work to be done on this mathematical topic, her choice was to spend yet more time on "Guess My Rule"—but without any clear sense of what it was she hoped students might learn or how she might be able to focus her teaching on the specific ideas she wanted them to understand more deeply. A consequence of this and other similar decisions was that Pat spent almost half the year on the introductory unit of the curriculum—a unit designed to be explored over a period of three to four weeks—rather than moving on to the important content of the other units.

Pat in the spring

In her interview in late May, Pat talked very differently about how it felt to use the *Investigations* curriculum. She no longer seemed to be struggling, to the same extent, with the question of what students were learning and how well they were learning it, even though the content that Pat was teaching in the spring—fractions—was particularly challenging for her. In an April portfolio assignment for the DMI seminar, she wrote the following about her own explorations of fraction ideas:

> The experience with fractions has been bewildering. I feel myself floating in and out of understanding . . . I feel weary. I don't have the energy to hold my thinking . . . It's like my golf game. Sometimes my swing just disappears. I have to steady my mind and body and remind myself that I've hit the ball well before. It's in me. I can do it again. When working on fractions I go through pretty much the same process . . . My understanding of multiplication and division of fractions is still airborne.

Despite her struggles to understand the important ideas about fractions, in the spring Pat seemed alert for these ideas in her students.

An indication of Pat's focus on her students' ideas is reflected in how she approached the *Investigations* unit on fractions. As she explained in her interview, she began the unit by asking her students what they knew about fractions. This discussion, which she considered a preliminary assessment, helped her see where her students were with their ideas and formed the basis for her thinking about how their ideas were developing. As she explained:

> From that [preliminary assessment], I knew basically where they were beginning with me and then I could follow how they developed, considering their experiences. What do I feel I've exposed them to? What do they seem to be taking from that exposure?

Pat remained concerned about students' mastery of mathematical skills, but she was beginning to develop a richer understanding of what it meant to work on those skills and was recognizing the importance of working on the underlying ideas. She explained:

> I am still struggling with [the idea of mastery], but I am having more faith in what [some other teachers] are telling me, that things will reappear . . . Especially after using Mathematical Thinking in Grade 3 and Things that Come in Groups, you can see everything repeating, like the ideas that come up in fractions. I think I am having more faith in the time recommendations . . . What I am getting at is that I don't think I will strive for mastery within that unit, but I will say that by the end of the year, I would be looking for mastery of certain ideas or the development of those ideas.

Pat was beginning to develop an appreciation for how ideas developed over time.

This feeling of being more solidly grounded in the important mathematical ideas and how her students were thinking about them is reflected in the analysis of student work that Pat completed for her portfolio in May. In this assignment, she examined student responses to fraction problems from "More Brownies to Share" in the first investigation in the Fair Shares unit of *Investigations* (Tierney and Berle-Carman, 1998).[6] The problems included the following: How can 3 people share 4 brownies? How can 3 people share 5 brownies? How can 7 people share 4 brownies? How can 5 people share 4 brownies? The student worksheet asked students to cut up large brownie rectangles, glue the pieces onto the worksheet, show how they made fair shares, and tell what each person would get. The curriculum indicated that the mathematical focus was on helping students realize that fractional parts must be equal, becoming familiar with conventional fraction words and notations, and understanding how to group unit fractions (e.g., 1/4 + 1/4 = 2/4).

In the analysis itself, Pat began by identifying the mathematical topics the class had been working on, instead of beginning with a description of the activity and the task they had been working on as she had in the fall. She listed the following: "Making equal parts, grouping unit fractions (1/4 + 1/4 + 1/4 = 3/4), equivalent fractions, and mixed numbers." Then, as she had in the fall, Pat described the activity and the task, discussed the work of each of three students—one whose work she thought was strong and two whose work she thought was not so strong—and identified her learning goals for each student. Finally, she ended with some reflections about what she had learned about the process of analyzing student work.

In writing about one of the students whose work she thought was not so strong, Pat began with a discussion of what he *did* seem to understand. He was able to divide the brownies into fractional parts by folding and cutting, and he did seem to realize that "fractional parts must be equal." He was also able to distribute the fractional parts equally among the number of people sharing brownies. Pat's concern was that he did not label any of his work with fractional notation. "Did he understand the notation?" Pat wondered.

In her analysis of the second student whose work concerned her, Pat noted that this student drew her solutions with paper and pencil, rather than folding and cutting paper brownies into fractional parts, and that her drawings of the fractional parts did not always convey a sense of their relative sizes. That is, a drawing of one-third of a brownie was the same size as a drawing of a whole brownie. For one of the last problems, involving seven brownies shared among four people, the student attempted to fold and cut paper brownies into fourths, but the four pieces were unequal in size. Although this student was able to accurately identify what each person received, using fractional notation—thus suggesting that she had some knowledge of how to group unit fractions together (e.g., after cutting each of seven brownies into fourths and distributing them among four people, she wrote in Spanish that each person received 7/4)—Pat's concern was whether this student really understood that fractional parts needed to be equal and relative in size to the whole. Since this student had some coordination difficulties, both in her writing and in her folding and cutting, Pat could not be sure.

For the third student, whose work she thought was strong, Pat noted the following: Manuela carefully folded each whole "brownie" into fractional parts before cutting the paper, placed each part on top of a whole brownie as if to check their relative sizes, carefully distributed the portions, and then wrote on her paper (in Spanish), "I gave two-thirds to each person and one whole. That's 1 2/3." This work prompted Pat to conclude that this student seemed to have a good understanding of adding unit fractions. Pat also noted how Manuela approached a more complex problem involving five people and four brownies: Manuela cut the four brownies into fourths, doled out fourths to each person, and then wrote on her paper (in Spanish), "I had 1/4 extra so I drew lines and converted it to fifths. Each person gets 3/4 + 1/5." Based on *this* work, Pat concluded that Manuela was developing some important understandings of relationships among halves, thirds, and fourths, though "not enough to make the leap into understanding 1/5 of 1/4." What is particularly interesting to note in this analysis is that the student whose work Pat thought was strong did *not* necessarily always get an answer that was mathematically correct—that is, the answer is *not* 3/4 + 1/5. However, Pat was able to value the fact that this student was beginning to think about what it meant to find a fractional part of a fraction.

In her learning goals for these three students, Pat seemed much more clear on what she wanted to help them understand about fractions. For the first student whose work concerned her, Pat wanted to help him develop a stronger sense of fractional notation and what it meant to combine fractions. She hoped to achieve this by partnering him with other students who were already using fraction notation to label their work, and by encouraging him to say more about his mathematical thinking in small-group and whole-group discussions. For the second student whose work concerned her, Pat wanted to work with her more closely in order to learn whether coordination problems or conceptual confusions were interfering with her ability to divide a whole into fractional parts. Pat planned to provide this student with wholes that had already been divided into fractional parts to see if this student could identify various fractions. If the student knew her fractions, and if in fact the problem was one of coordination, Pat planned to work more closely with the student, helping her fold and cut fractional pieces for their various activities, thus freeing the student to work on mathematics issues without being held back by her lack of coordination. For the student whose work Pat thought was strong, she wanted to help the student continue to explore relationships among fractions, including what it meant to divide fractional parts into smaller fractional parts.

Pat's reflections on how her analyses of student work changed over the year are consistent with what we observed. She wrote:

> I realize I focus on their work habits as well as their math abilities. I observe how a student cooperates with team-mates, in what size group a child works best, and with which partners a child produces more. I consider if the child focuses on the mathematical purpose or is distracted by the artistry of his or her illustration, for instance. I continued to make those observations about work habits throughout the course . . . [But] I think my analysis of the children's work became more focused on their thinking process . . . I [now] assess my students differently . . . When I assess students' work, I tend now to analyze their work in terms of what do they understand, what don't they understand, how are they thinking about it.

Pat is now better able to pay more attention to the important mathematical ideas and how her students are thinking about them, in addition to the kinds of behaviors she attended to in the fall, such as their focus on the activity and how students worked together. Identifying the important mathematical ideas that students are to be learning is a complex process, particularly when the content is challenging. In the spring, we see that Pat is working hard to consider the mathematical ideas that are important for her students to be thinking about, what they *do* seem to understand about those mathematical ideas, and what they have yet to learn.

This feeling of being grounded in the important mathematical ideas and how her students were thinking about them is also reflected in the *Investigations* lesson I observed in her classroom in late May, a few days before the lesson Pat wrote about, above. The lesson was from "Making Fair Shares" in the first investigation of the Fair Shares unit. Students were to use sheets of paper to make a set of fraction cards that included 1/2, 1/3, 1/4, 1/6, and 1/8, as well as what remained of each sheet of paper after they cut out those cards (1/2, 2/3, 3/4, 5/6, and 7/8). Students were then to help the teacher display a set of the cards in order by size.

The curriculum suggested that the teacher conduct a whole-class discussion about the display of cards that helped them focus on the relationships among the cards:

> Give students a minute to look for patterns among these ordered fractions; then cover the pieces or turn them so the labels don't show and ask students what they noticed. You might ask a few questions: Which fractions are larger than 1/2? Which are smaller than 1/2? Which is larger, 5/6 or 3/4? Each is missing one piece—why aren't they the same size? (p. 11)

The Teacher Notes for this activity emphasize being able to compare the sizes of the fraction cards even when they have different shapes, and suggest that students may need to do some cutting and pasting to be sure of how sizes compare.

The students had made their fraction cards in an earlier lesson. Their charge in this lesson was to work in pairs to put the fraction cards in order by size. As Pat introduced the lesson, she stressed that their job was to be able to prove or explain the order of the fractions. With students working in pairs, Pat circulated around the room and asked each pair of students for their explanations. Some students explained their reasoning by placing the cards on top of one another and discussing their relative sizes. Others explained that one had to be larger than the other because of the way the paper had been folded and cut. Pat focused on the fractions that seemed the most problematic to compare: "Which is bigger, 7/8 or 5/6? How do you know? Can you prove it?" Pat seemed very comfortable with the mathematical focus of the lesson, and asked questions that addressed that mathematical focus.

Pat was also alert for mathematical tangents that seemed central to the point of the lesson. For instance, when one pair of students who had finished their ordering decided to make fraction cards out of circles, Pat spent time with them, discussing their ideas about how to divide the circle into equal parts; she then pointed out what these students were doing to the rest of the class. She suggested that others might like to try making this kind of fraction card set.

In her conversation with me about this lesson, Pat focused on the important mathematical ideas. She talked comfortably about the "mental checklist" she had constructed that helped her decide what to look for as students engaged with the content of the lesson:

> I looked at whether they put the fractions in the correct order, and, since we had just done equivalent fractions, I was looking at whether they were making connections to the idea of equivalent fractions. Then, more basically, could they identify the fractions? . . . I was also looking to see if they were . . . having to turn them over and upside down in order to compare them . . . So that was my checklist . . . I see [the items on my checklist] as sort of the developmental steps in understanding fractions.

Pat said that being able to construct these kinds of mental checklists helped her think about what she wanted to be watching for as her students engaged in a lesson. She did not create her checklist by consulting some outside source like the traditional text, as she did in the fall, but rather by reading the curriculum carefully—including all the support materials—and thinking deeply about the mathematics embedded in each of the lessons:

> Reading over the activity—and over and over sometimes!—but also reading the whole section of the unit . . . In reading over the section, I get to this point where I know . . . I am reading and rereading, trying to identify the concepts and figuring out what behaviors I would expect to see, what I would hear kids saying, what I would see them doing, and then I would look for that.

Although she had read the curriculum carefully in the fall, Pat was less able to use it to help her do this important piece of work.

What emerged most strongly in Pat's spring lesson was her mathematical focus, as she thought about what her students should be learning. Although she enacted the lesson slightly differently from the way it was described in the curriculum, as she did in the fall, in this instance she did not sacrifice the important mathematical ideas. Indeed, it might even be argued that by having students discuss the ordering of the fraction cards in pairs, and by circulating to ask questions of the students, Pat maximized the opportunities for students to engage with the important mathematical ideas.

What did Pat learn during the course of the year?

In this story, we see that various facets of Pat's practice with the *Investigations* curriculum changed between the fall and the spring. In her analysis of student work, she became much more focused on what students understood about the important mathematical ideas, and what ideas needed further development. In her lessons, Pat more carefully attended to the important mathematical ideas that are at the center of the lesson, and asked questions of her students that kept them focused on those ideas. In her planning of lessons and in her reflections about them, Pat became more attentive to what she should be watching for in her students and what this could tell her about their mathematical understandings. She seemed to be able to get much more from the support materials in the *Investigations* curriculum.

What contributed to the change in the way Pat was able to work with the *Investigations* curriculum? It is unlikely that a single factor was responsible for these changes. For instance, it may be that more experience with teaching from the curriculum, or other professional development activities, or even informal conversations with colleagues all worked to help Pat understand more about the mathematical ideas embedded in the curriculum and what it meant to engage her students in thinking about those ideas. While a number of factors may have supported Pat's work with the curriculum, we do not have a way to assess their contributions to her learning. But, through what Pat wrote in her portfolio and what she said in her interview, we *can* examine what the DMI seminar offered her.

In the seminar, Pat explored important mathematical ideas for herself. Activities designed to deepen teachers' understanding of the mathematical ideas in the cases were part of almost every seminar session. For instance, the sessions about the nature of our base 10 number system included explorations of teachers' mental or invented strategies for solving multi-digit addition, subtraction, multiplication, and division problems, with a focus on how teachers used their understanding of our number system to think about the problems; building models of multi-digit addition, subtraction, multiplication, and division problems that highlight how the structure of our number system comes into play in these operations; and explorations of models and methods for working with decimals. The later sessions, which explored the meaning of operations, included numerous opportunities to work through story problems involving whole numbers and fractions—and building models for these operations—in order to consider the relationships between the actions and situations represented by these kinds of problems and the operations themselves. In addition to the mathematics explored through the activities, the analysis of student work in the cases provided additional opportunities to work through one's own understanding of these mathematical ideas.

In her fall interview, as Pat was participating in the beginning sessions of the seminar, she said the following about her own mathematics learning:

> I have never been good in math. I have been afraid of it and afraid of teaching it. I used to wish for a book that would just tell me what to do. But this [doing mathematics in the seminar] I find so freeing. I had never looked at numbers and realized that you can take them apart. I hadn't approached them that way. So for me to go through all these experiences myself . . . well, I am beginning to think that I might actually become a mathematician at some point!! It really has been exciting.

Pat was beginning to appreciate the excitement that comes from exploring a mathematical idea, and she was also beginning to believe that she might indeed be capable of thinking about these ideas for herself, even though in the fall she still drew on her traditional textbook to help ground her practice. Pat was beginning to develop a more solid grounding in the ideas of our base 10 number system and what it means to decompose and recompose numbers—an idea very much associated with the student work that she struggled to analyze in the fall. By learning more about the mathematical ideas, Pat was beginning to feel more comfortable with the notion of engaging students in explorations of those ideas, as well as how to look for those ideas in her students.

But the connection between Pat's own mathematics learning and the teaching of mathematics to her students was not simply the fact that by knowing more mathematics she would be better able to teach it. She was also learning something important about the *process* of learning mathematics—something she was experiencing firsthand and could now use to think about the process of mathematics learning for her students. In her interview, she wrote about the power of this learning:

> That was probably the strongest [part] of the seminar. It helped me to feel like I was studying mathematics as my children were. I think having a chance to talk about feelings about it, and the learning process, I think it just gave me more of an appreciation of all the steps that the kids were really going through in order to learn something. So sort of by putting myself in their position . . . I learned through that experience.

One particular aspect of the process of learning mathematics that Pat was able to solidly connect with was the notion that mathematical ideas can be elusive, that you can move in and out of mathematical understandings, and that it is often necessary to revisit ideas and recreate our understandings of them. As she continued to explain in her interview:

> I was actually having to DO the math, and I really enjoyed that because it really challenged me, and it made me feel that this was what the kids must be going through. Sometimes when I would have an understanding and then lose the understanding—I had just done a problem and I had just explained it to my partner and then five minutes later someone would ask me a question and I wouldn't remember. For me to go through that so I could see it was really important because I have kids who can prove something and then can't remember . . . they would say "Hey, what did I do?" and they would have to re-prove it to me. So I became more aware of this in my kids.

The fact that mathematical ideas can be elusive and that they build over time was an exciting new insight for Pat, and one that she applied to her thinking about her students' mathematics learning, particularly in relation to her questions from the beginning of the year about what it meant to "master" a mathematical idea.

In the seminar, Pat learned to analyze students' mathematical thinking. The discussions of the cases helped Pat learn to examine what students understood about important mathematical ideas and what they had yet to learn about those ideas. An important part of this learning involved trying out the methods the students in the cases were using, and, from that vantage point, reflecting on their logic. We see this in her final reflection at the end of the seminar:

> I learned most about students' understanding of [topics] by applying the methods that students used in the cases. Doing the problems this way took me through their thought processes.

Pat was able to use what she was learning about the students' thinking in the cases to reflect on the mathematical thinking of her own students. As she wrote in her final reflection:

> I tried to compare my students' work with the students in the cases so I could see where my students were in their understanding.

By learning to analyze her students' thinking, and by considering where her students were in relation to the students in the cases, Pat was better able to think more deeply about what her students were learning.

In our May interview, Pat described how she was able to connect what she read in the cases to facets of her own mathematics teaching practice:

> The cases . . . were helpful because it was a little glimpse into other classrooms. I always enjoyed the teachers' analysis and their questions. I can manage the classroom, and the kids' behavior, but this helped me think about the mathematical understandings. It was good to have [the teachers'] insights and hear them talk about [the mathematics thinking of their students], and then talk about it in our seminar, relating it to our own classrooms. It was helpful to imitate what the kids had done and get into their heads and get at their understanding of the mathematical issues. I would pay particular attention to the third grade pieces where I could see my students following that pattern and going down that road.

We see Pat make connections between a particular case and her own mathematics teaching practice in a January portfolio assignment. In that portfolio assignment, she wrote about the case by Eleanor, who taught a combination of grades 3 and 4 and who seemed to have worked through some of the issues that had been questions for Pat. Pat wrote:

> This classroom interested me because I, too, teach grade 3 . . . One reason that the classroom interests me is the pacing of math concepts. The episode is dated October and the teacher, Eleanor, is introducing division . . . Eleanor's classroom is like mine [and] clearly has a wide range of mathematical understandings. She also has an understanding that division is a vehicle for children solidifying their understanding of numbers . . . I wonder if she paces her math program faster than I do because she thinks children's understandings will grow through all the topics? Does she strive for mastery for each student as I do? We tend to get bogged down . . . Eleanor's classroom and her analysis of their work in division is very interesting to me. I am trying to loosen my control on pacing. I'm also analyzing student work to see where they are . . .

Through her examination of this particular episode, Pat was able to reflect on Eleanor's idea that explorations of division ideas are also opportunities to think about "what numbers are made of," including the idea that numbers can be "chunked" into multiples of fives, tens, and hundreds. Pat was able to use Eleanor's reflections to think about how explorations of a range of mathematical topics include opportunities to continue to work on a set of mathematical ideas and that it is important to keep moving. This provided Pat with another way to think about the issue of pacing.

Finally, the seminar provided Pat with a model of what standards-based mathematics teaching might look like and feel like, firsthand. As she wrote in her final evaluation for the seminar:

> The seminar was connected to the *Investigations* curriculum through the structure and content. At each seminar session I actively struggled with the math activities, much as my students struggle in my classroom. I experienced a floating understanding of math concepts. I struggled to explain my way of solving a problem to my classmates, and I had to listen carefully to understand my classmates' solutions to problems. The seminar was a model for the *Investigations* curriculum.

Pat's experience of the DMI seminar was one that she built on as she worked to provide a similar experience for her students.

Pat's own experiences in the seminar—learning mathematics and exploring student thinking—are reflected in the way she thought about and used the *Investigations* curriculum in the spring. She became more concerned with the *development* of a mathematical idea *as well as* the mastery of mathematical skills. She now has a way to think about what to watch and listen for as students develop their ideas. She also has ways to attend to those ideas in the context of the lesson, enacting the lesson in a way that enables her and her students to focus on the important mathematical understandings and recognize the tangents that are mathematically significant. This is a very different orientation to the *Investigations* curriculum than she displayed in the fall, when she struggled to identify mathematical learning goals for her students, when the mathematical goals of the *Investigations* curriculum seemed nebulous to her, and when she was enacting a lesson in a way that did not contribute to its mathematical focus.

Helen's Story: What's Going On in My Students' Heads?

Listening to students' mathematical thinking and making instructional decisions that are informed by that thinking is at the core of the reform recommendations. This, too, is a challenge for many teachers. When the focus of mathematics teaching was the practicing of procedures, teachers explained and students practiced. What was there to talk about except the questions students had when they had trouble following the steps? The reform recommendations, on the other hand, call for a very different kind of conversation. Students are encouraged to talk about their mathematical ideas and are considered sense-makers who have something to offer to the endeavor of learning mathematics.

Attending to students' mathematical thinking can be a very challenging undertaking, even for teachers working with a standards-based curriculum (e.g., Cohen, 1990; Heaton, 1994; Schifter, 1996a, 1996b; Wilson, 1990). It requires an ability to listen carefully to what students have to say about their mathematical thinking—following their logic, recognizing what they do or do not yet understand about the mathematical ideas that are important, and being able to build on what students *do* understand about those mathematical ideas in a way that helps their understanding move forward. What happens when teachers begin listening to their students' mathematical ideas? Heaton (1994) noted her first genuine experience with this kind of listening as a turning point in her effort to teach for mathematical understanding—her first instance of "being in the moment" with her students, simultaneously attending to their ideas as well as the ideas of the lesson. We can look more closely at what is entailed in this shift by examining Helen's story.

Helen is an experienced fifth grade teacher who was working with *Everyday Mathematics* for the first time. Up until then, she had used a traditional textbook, except for piloting two units from the *Connected Mathematics Project* the previous year. Her students had used the *Everyday Mathematics* curriculum since the first grade. Although Helen used *Everyday Mathematics* as her primary curriculum, she often drew from other sources in her teaching, including the traditional text she had used earlier and activities from other resources, such as recent NCTM publications.

In this story, we see Helen developing a curiosity about and an appreciation for the mathematical thinking of her students. In the fall, we hear her frustrations with how little mathematics her students seem to know, we see her focus on neatness and completeness in her students' work without much attention to their thinking, and we observe her breaking her lesson down into small, tightly controlled chunks of information in the hope of helping students learn the material. In the spring, we hear her talk with interest and enthusiasm about the different ways her students have of making sense of the mathematics content. We see this reflected in how attentively she listens to her students' ideas in an interview, how hard she works to make sense of their ideas, and the space she makes for her students' ideas to emerge. This story ends with a discussion of what Helen learned in a DMI seminar, how her learning in the seminar helped her cultivate a curiosity about and an appreciation for her students' mathematical thinking, and how this began to influence her work with the *Everyday Mathematics* curriculum.

Helen in the fall

In the fall, Helen explained that she found *Everyday Mathematics* a very difficult curriculum to use. She felt that it moved very quickly—that where it said to review or revisit a topic, she was actually having to reteach it because her students didn't seem to "get it" the first time. Students were struggling with the classwork and the homework, and, for Helen, "repetition was the word of the day." She felt that she had to continually break things down for her students, going over and over ideas, moving slowly through the curriculum. Helen wanted to help her students get correct answers, but sometimes this meant that she focused on the mechanics of their work, rather than the mathematical ideas underlying it.

Helen's focus on the mechanical aspects of her students' work can be seen in the analysis of student work that she completed for her portfolio in early October. This assignment called for her to analyze three samples of student work, discussing what she found satisfying or unsatisfying about them, and identifying learning goals for those students based on her analysis of their work. The task Helen posed to students was adapted from a textbook that she used occasionally to supplement her work with *Everyday Mathematics*. It called for students to match each of three line graphs—one showing an increase, a second showing a steady rate and then a decrease, and a third showing a change from a decrease to an increase—with its title: *Population of Recently Discovered Animals Falling, Higby Toys Recover with New Product,* or *Tests Show Students Are Getting Smarter.* Helen extended the task by asking students to reproduce the graphs on their papers, label each of the axes, and write a paragraph that elaborated on the situation the graph represented.

Helen chose Alistair's work as strong because he correctly paired each graph with its title, reproduced the graphs accurately on his paper, labeled the axes correctly, and wrote stories about each that were "imaginative" and made sense. She considered Brent and Carol's work not so strong, even though they too correctly matched each graph with its title, and their paragraphs, like Alistair's, made sense; however, they did not correctly label the axes on their graphs, and they did not explicitly refer to numbers from their graphs in their paragraphs. She identified no learning goals for Alistair, who completed the task accurately and completely. Her learning goals for Brent and Carol involved working with them on "details" and "neatness," including being precise in recording numbers, using a ruler to make straight lines, and centering the work on the page. Nowhere in her analysis did she consider what students needed to understand about graphing and the representation of data in order to complete this task, what it was that Brent and Carol failed to understand, or how what Alistair understood was more powerful than what Brent and Carol seemed to understand. While she claimed, in her analysis, that these students showed promise in their "ability to write clearly about their understanding," what they wrote, in fact, was an elaboration of a story suggested by a title.

Helen's desire to help her students solve problems correctly by laying out methods that would take them to the answers is also reflected in a lesson that she wrote about for her portfolio in early December. Helen was working from an *Everyday Mathematics* lesson that focused on ideas about division. The support materials for the lesson emphasized using students' number sense and knowledge of multiplication to estimate reasonable answers to multi-digit division problems. It also emphasized using a division algorithm that builds on these understandings. In the lesson, Helen worked to help students use their number sense and multiplication knowledge to predict the numbers and the *size* ("number of places") of the numbers that were to be in the quotient. She wrote the following about how she handled one student's suggestion that 300 ÷ 5 = 600:

> I introduced the "long division box" method of division notation and the use of graph paper as aids to avoid [this student's] error. We discussed this new problem [written with the long division box] and where the first digit in the quotient should be written. Only one digit can go in each "place" (each box on the graph paper) . . . I send [students] home with an assignment asking them to rewrite several horizontal division problems in a "long division box" format.

What Helen wrote about her lesson suggests that, by December, while she may have allowed more of her students' thinking to emerge during discussions, she was quick to devise ways to fix what they seemed to be doing wrong. Her presentation of the "long division box" and the use of grid paper to avoid errors indicates that setting up students to get correct answers was still an important focus in her teaching.

Helen's focus on helping her students get correct answers by offering them techniques for avoiding errors, rather than by considering their understanding of the ideas, was also consistent with the *Everyday Mathematics* lesson I observed in early November involving the teaching of problem solving. The lesson included a discussion of strategies for solving story problems and involved a story problem about the amount of paint on the Eiffel Tower and the amount of copper on the Statue of Liberty. As Helen taught this lesson, she worked hard to help students learn the problem-solving strategies identified in the curriculum so they could apply these strategies to the problem. In doing so, she kept a tight rein on the classroom discourse, reviewing the information in the reading that was part of the lesson, making sure the students were "on track," and pulling students back on track when necessary.

There was a great deal of repetition in the lesson. For instance, students were first given time to read the description of the problem-solving strategies contained in their student workbooks; students then took turns reading different sections out loud. "This way, you will get it twice," Helen explained. As students read, Helen occasionally interrupted to emphasize a point or flag an important piece of information. Following this, Helen went over the content again, reviewing and elaborating on each of the problem-solving strategies. As Helen walked around the room, she checked to see if students were taking notes or underlining important words in the material.

Occasionally, Helen simplified the content of the lesson for students. As she went through the explanation of each of the problem-solving strategies, she created simpler problems on which they could practice each strategy. For instance, in the discussion of the strategy "describing the data," Helen facilitated the following discussion with her students during the lesson:

> "I have 5 pumpkins, and my friend brings me 5 more. What's the data I would need to solve the problem?" Students tried to give her the total number of pumpkins, but she reminded them, "The data is the information you need to know." One student offered that you needed to know 5 and 5. She responded by saying, "Right. You need to know that I have 5 pumpkins and 5 pumpkins." She continued, asking, "Do you need to know that a friend brought it over?" "No!" students responded. "Do you need to know how old she is?" "No!"

This was typical of much of the discussion that went on in the lesson. Helen asked highly structured questions about a somewhat simplified version of the content of the lesson and then coached students in a way that helped them provide correct answers.

Throughout the lesson, there were opportunities for students to talk about how they were thinking, but Helen did not make these opportunities available to her students. This was particularly noticeable as they talked about the problem posed in the lesson:

> The Eiffel Tower, which stands in Paris, France, was completed in 1889. It weighs about 7,000 tons. The paint on the Eiffel Tower weighs about 40 tons. The Eiffel Tower is about 984 feet tall, and you would have to climb 1,652 steps to get to the top. Once you reached the top, you might feel it sway up to 1/3 foot from side to side on a very windy day. The Statue of Liberty was unveiled on October 28, 1886, on Ellis Island in New York. It is not as tall as the Eiffel Tower—in fact, it is about 833 feet shorter, and it weighs about 6,775 tons less. The statue has an iron frame that is covered with copper sheeting. The copper weighs about 2 1/2 times as much as the paint that covers the Eiffel Tower. The Statue of Liberty was a gift from France. About how many more tons does the copper on the Statue of Liberty weigh than the paint on the Eiffel Tower? About _____ more tons. (p. 40, Student Workbook)

Helen asked her students if anyone had gone ahead and thought about the problem. Several students raised their hands. What was striking was that these students were *not* then invited to say what it was they had been thinking. Instead, Helen continued the discussion as follows:

> Helen asked, "What IS the problem? What IS the problem?" She wrote "problem" on the board and drew a cloud around it. A student restated the problem, explaining that it was asking how many more tons does the copper on the Statue of Liberty weigh than the paint on the Eiffel Tower. As he did this, she wrote and underlined the word "more," followed by "copper of SOL" and "paint of ET." Then she called on a student to repeat the question for her. The student repeated the question in his own words. Helen asked, "When I read the question, can I tell which one weighs more? Does it already tell me which one is heavier?" After a few moments, she prompted, "When it says 'How many more _____ than _____ . . .?' More than?!" She followed this up by asking, "What are the most important words to underline in that problem?" A student responded, "Than!" She asked, "What else?" Another student responded, "How many?" She asked, "Anybody else?" Yet another student responded, "More tons?"

The discourse was tightly controlled, with little space for student thinking. Making space for students' ideas in an open-ended fashion was not a part of Helen's agenda.

This kind of teaching is not unusual. In fact, it is similar to the way that many traditional mathematics lessons are taught. Even though Helen was working with a standards-based curriculum, much of her teaching practice remained the same. There was a good deal of repetition, content was broken down and simplified, students were coached through the various portions of the lesson, and there was little, if any, discussion of how her students were thinking. In fact, it probably did not even occur to Helen to be curious about their thinking.

Helen in the spring

In the spring, Helen seemed very interested in the different ways it was possible to think about mathematics problems. In her interview at the end of May, she brought out folders of her students' work and talked about how many of them had some interesting and powerful ways of thinking about the mathematics they were working on. She explained that she was now beginning to realize that the methods that *she* had in mind were not the only ones that could be used to solve particular problems. For example, in a problem that involved sharing three brownies among eight people, Helen's method would have been to divide each brownie into eighths and then distribute them, so that each person received one-eighth from the first brownie, one-eighth from the second brownie, and one-eighth from the third brownie. But one of her students, Alegra, solved this problem differently, dividing each brownie into thirds, distributing one-third to each person, and then working to divide up the remaining one-third. Helen reflected on Alegra's method as we looked at it together:

> Alegra . . . what is it? We are dividing into eighths? Three brownies and we are dividing it among eight people. So she took three brownies and divided them into thirds, and gave one to each person and there was one left over. Which is very interesting, because when I first looked at it, I said, this is all wrong!! Why didn't she divide it into eighths? That's what she should have done. But actually dividing it into thirds is not a bad idea. I wish I had her for another year because she is very interesting.

Helen expressed the wish that she could make more time to do mathematics with these students so there would be more time for students to talk about their mathematical ideas. It was as if a new and intriguing world of mathematical thinking was opening up for Helen.

Helen's developing curiosity about her students' thinking was also evident in a portfolio assignment she completed in March about how an understanding of student ideas might help in the work of teaching. For this assignment, she reflected on her students' understanding of the division ideas they had been exploring in the fall. Her interest in revisiting this topic was prompted by the arrival of a foreign student who used a different algorithm for long division. As she worked with him, she found herself struggling to understand how what he understood about division related to his use of this algorithm. What she learned from her conversation with this student led her to engage in similar conversations with her other students. She wrote:

> I decided to work with *other* students to see if there was a separation between understanding the concept and understanding the algorithm... All claimed to understand or derive meaning from the traditional notation I had taught them. None of them told me they did not understand the concept of division. I think they were aware that they were "supposed to" understand. They wanted to do and say the right thing. It took some digging, but I felt that there were some students who were only confused by the algorithm . . . [Some students] need to do the kind of repeated subtraction work or work with manipulatives that is described in the cases that deal with younger grades.

Here, we see Helen digging into the question of what her students understand or are confused about, and, for those who are confused, what might help them work through their confusions. This is very different from the way she approached her students' work on division ideas in the fall, where she focused on procedures and offered students techniques that would help them get correct answers.

Helen's interest in her students' thinking about division, and her focus on what they did and did not seem to understand, is reflected in the way she wrote about her students in this portfolio assignment. She wrote about how one of her students persisted in reversing the placement of the dividend and the divisor, whether writing a division problem horizontally or with the "long division box," and went on to describe a conversation that she and this student had about this way of writing a division problem:

> When she and I talked about her way of writing division problems, she felt that she does understand that the larger number is to be split into a certain number of groups, and that it really doesn't matter where she puts the numbers . . . She understands division but has neither learned the traditional algorithm nor developed an alternative method.

Helen also explored the mathematical thinking of the student who, in the fall, was having trouble knowing where to put the digits in the quotient. Last fall, at Helen's suggestion, this student had begun using graph paper in an effort to avoid errors. Now Helen realized that this had not turned out to be a useful suggestion. This student was still making errors, even with the graph paper. For instance, in calculating 42,000 divided by 6, this student got an answer of 2,7000. Helen wrote:

> Why was the 2 as a first digit? The 7 would have been correct, in its place, without the 2. Why the extra 0? Why did she put the

> comma in the wrong place? I wondered if teaching her [another student's] algorithm would be helpful but I didn't even try. I spent another session trying to understand what she knew about division.

By spending more time with this student, trying to learn more about what she did or did not understand, Helen acquired new insight into what was confusing for this student. Helen wrote:

> [Her] difficulty [knowing where to put the digits in the quotient] was not, as I thought in December, simply a visual-spatial difficulty. Working one-on-one with her, I now think that she does not understand what is meant by dividing, so she has no sense of her answer's reasonableness.

These examples of her work with students in the spring suggest that Helen was working hard to understand their thinking, was able to appreciate that there is a logic to *both* the mathematics and their thinking, and was curious enough about her students' understanding to set aside time to talk with them individually about their ideas. She was also beginning to realize the inadequacies of some of her earlier approaches.

Helen's portfolio included an extended example of a conversation with two students about their mathematical ideas, revealing just how hard she was working to listen to the mathematical thinking of her students. The example comes from an extra portfolio entry that Helen completed in April, which examines the mathematical thinking of two girls in her class, Abby and Tina, who seemed to be struggling. The two girls had turned in a worksheet that Helen used to supplement the *Everyday Mathematics* curriculum. It required students to make circle graphs showing, for instance, that one-sixth of the students in the class had blue eyes, one-sixth had brown eyes, one-half had hazel eyes, and one-sixth had green eyes. Helen explained that because these two girls had not done well on the worksheet, she had set up a meeting to "explore their thinking."

Helen wrote that she began her work with Abby and Tina by asking them what was hard or easy about the problems. Abby explained that the first problem was easy because it only had a mix of halves and sixths, but the others were harder because they had a wider range of pieces. Then Abby and Tina talked about how they figured out how to divide the circle into the specified portions. Helen included many of the details of their conversation as they worked on the problems together, as well as her own thoughts about what she was hearing and thinking—much like the cases she had been reading in the seminar.

An example of a piece of the conversation that Helen found particularly powerful concerns their work on the third problem on the worksheet, where the girls needed to divide a circle into 1/3, 1/6, 1/8, 1/8, 1/8, and 1/8, but were running into difficulties. The conversation ran as follows:

> We looked at the third problem together. I encouraged Abby to tackle dividing the circle into 1/3, 4/8, and 1/6. She did not want to try, and Tina reasoned that "you can't do 1/2 because it doesn't tell us to divide into a half." Abby listened and offered, "But I would still start by dividing it into halves, lightly, but that doesn't mean I'm going to use it. Here, I'll make thirds [using a portion of the lightly indicated line that divided the circle in half]. Then I split one of the thirds and I have 1/6 and 1/6 . . . I mark off 1/3 and 1/6. Then I have the other half, and I start on the eighths." Tina said, "I split the 1/6 in half and it's 1/8." Abby said, "No." Tina said, "Well, I know 1/4 from the clock. I could divide it in half. But my pieces are equal." Abby said, "But they could be . . . I have 1/4 that is still blank. I can divide it into 2/8." Tina said, "But we need 3/8. Can it be the same as 1/6 or—no, a little bigger?" Abby said, "You need to use your imagination. Imagine another quarter, ignore the 1/6 line, divide the imaginary quarter, and you have the third 1/8, with a little piece left blank!"

What is particularly striking is the extent to which Helen hung back from this conversation, allowing the girls to work through their ideas, even when confusions arose and there was a good deal to work through. Helen described her reflections on their interchange, remarking on the difficulty of following their ideas, considering the reasonableness of their solution, and thinking about how to build more opportunities for what happened in their conversation to also happen in her classroom. She wrote:

> I am overwhelmed. Does Abby have a good solution? I find her stop-and-start thinking hard to follow, but her solution looks reasonable to me... I think it is important that they be able to visualize 1/4, 1/3, etc. It is important to understand that a sixth of a whole is larger than an eighth of the whole. It is important to understand that the whole can be divided into thirds or eighths. I think they are still working to make these concepts clear to themselves . . . They need to ask more questions and verbalize their thinking more often, which are risk-taking behaviors they are unaccustomed to. I need to do a lot more small-group work to encourage them to feel secure about what they *do* know.

What is also striking here is that Helen not only values her students' thinking for herself, but is also considering how students' articulation of their thinking can help them and others work through their ideas. Student thinking has moved to the center of *her* thinking about her mathematics teaching practice.

At the end of May, Helen was asked to select a piece of her portfolio that revealed the most about her growth; the piece that she selected was her writing about her work with Abby and Tina. She felt that this marked a place where she learned to really listen to what her students understood so that she would be in a better position to build from what they knew. In her writing about why she selected this portfolio piece, she explained:

> In earlier pieces of writing for this seminar, I have been clear about what I am trying to teach and what I expect students to do and learn. Sometimes I have written about what students do and say when they are confused or when they understand very well. At this meeting [with Abby and Tina], I was really trying to "go backwards" to find a point where the students' understanding was clear and see where I could help them move forward in their thinking. I was focused on the questions we keep asking ourselves [in this seminar]: What are the mathematical underpinnings of the task? What do they need to know about fractions to be able to divide this circle into fractional parts? What do they need to know about division and its relation to fractional parts? Sticking to these questions helped me see Abby and Tina's different confusions and I was able to help them . . .

We, as well as Helen, can see that this interaction with her students around their mathematical ideas was very different from the kinds of interactions that were typical in the fall.

This attention to and respect for her students' mathematical thinking was also reflected in the *Everyday Mathematics* lesson I observed in the spring. The focus of the lesson was mixed numbers and improper fractions. Throughout the lesson, Helen encouraged students to talk about their ideas and seemed genuinely curious about their thinking. For example, in the discussion of homework from the night before, Helen asked a student about the second problem on the homework, 19/4. The dialogue went as follows:

> "How would you solve 19/4?" Helen asked. Alegra responded, "What I did was I looked for the closest multiple of 4—so that was 16—so 4 would be the whole number with 3 left over." Helen responded, "Good strategy. I like that strategy."

Next, Helen asked Jonathan to share his strategy. He explained, "I counted up by 4's and then figured out how many groups of 4's I had." Helen engaged students in a discussion about how these two strategies compared with each other and what ideas they built on. She then went on to elicit strategies from several other students, moving through the content of the homework and then moving on to the problems that were part of the lesson for the day. Helen's interest in her students' mathematical thinking went beyond just exploring their strategies for converting a mixed number to an improper fraction or vice versa. In fact, she often inquired about how they had done their actual calculations. For instance, in the discussion about 56/9, Helen asked them for their strategies for getting multiples of 9. The students talked about knowing their 9's facts, or being able to figure them out by adding 9 to the last fact they knew. One student offered that you could also add 10 and then subtract 1.

This is not to say that Helen no longer explained things to her students. When the problems they were working on shifted from converting improper fractions to mixed numbers and began to involve converting mixed numbers to improper fractions, she explained that this involved trying to apply their strategies "backward." Using the problem 7 2/3 = 23/3, she explained that you multiplied 3 x 7 and then added 2 to get 23, so you had 23/3. But she went beyond just explaining the strategy to her students. She worked to connect it to the ideas that had come up in their earlier discussion of 23/3 and the process of thinking about how many groups of 3 there were in 23 and how many were left over.

What emerges most strongly in Helen's lesson is her interest in the different strategies that students used to think about their work with fractions and decimals, her efforts to follow their mathematical thinking and understand where the ideas were solid or not so solid, and her realization that discussions about the ideas can help further students' understandings. This is very different from the lesson in the fall, where the focus was on presenting information to the students and prompting them so they could get correct answers. Indeed, in the spring, Helen seemed to believe that students have good ideas they could bring to their discussions of the problems they were attempting to solve and that it was important to make room for these ideas in her instruction.

What did Helen learn during the course of the year?

In Helen's story, we can see that various facets of her practice changed between the fall and the spring. In her work with individual students, she became much more focused on understanding their thinking, listening carefully to their ideas, considering where their ideas were solid and where they needed more work, and appreciating the fact that students might have powerful ways of thinking about problems that differed from her own. She worked to explore what they *did* understand about the important mathematical ideas so that she could try to build on these understandings in her teaching, in order to help move their thinking forward. She also recognized the value of students articulating their thinking and exploring their ideas together. In her teaching, Helen made more space in the classroom discourse for her students' ideas, and expressed an appreciation for their thinking and a curiosity about their approaches. She seemed to now believe that her students were quite capable of having valuable mathematical ideas.

We do not know exactly what opened this door to students' mathematical thinking for Helen. Was it a matter of getting to know her students better? Was it an issue of being on more comfortable mathematical terrain? Was it a matter of other professional development activities that were made available to Helen during the year? We cannot be sure. However, we do know that during the year she participated in a DMI seminar. What did Helen have to say about what she learned in the seminar, and how did her seminar experience connect to the changes she made in her teaching practice and her work with the *Everyday Mathematics* curriculum?

In the seminar, Helen found herself getting excited about the exploration of mathematical *ideas*. In her interview at the end of May, she eagerly brought out the portfolio of her seminar work and took me through a number of the mathematics activities she had worked on in the seminar: "The Ben and Jerry ice cream one, remember that?[7] These are my own diagrams from dealing with it." She showed me her work with several other problems having to do with division with fractions, and talked about how much she had learned about important mathematical ideas, while also acknowledging that she still had much to learn. "I need to take the seminar again!!" she admitted with enthusiasm.

While Helen was learning to appreciate the power of her own mathematical thinking, she was also learning that there were often multiple approaches to a problem and that the approaches that differed from hers often had merit. As she explained in her spring interview:

> I just feel like my mind has opened up to more possibilities, or possible ways, as solutions... so it doesn't have to be, "No, that's not correct, do it this way." It's hard to get away from that.

These explorations of her own approaches to the mathematics activities and her interest in her seminar colleagues' approaches piqued her curiosity about how her own students might approach those same activities, and she often used many of the mathematics activities from the seminar in her work with her students.

But her posing of the mathematics activities to her students was not only driven by her curiosity about their thinking. She was also beginning to recognize that these kinds of discussions helped support her students' learning. She explained in her interview:

> I think it is very valuable [to have these kinds of discussions using these kinds of problems]. I think some of the discussions we have had [in the seminar] make the most sense to me out of all the years I've taught math. Rather than just saying that you multiply or you divide by the denominator, then you multiply by the numerator. From that kind of a rule to being able to see some of this as making sense.

She wanted her own students to have similar opportunities to make sense of the mathematical ideas, just as she had in the seminar.

Her growing enthusiasm for explorations of mathematical ideas, and her increasing appreciation for the learning that those explorations supported, played an important role in helping her reconsider her focus on helping students get the correct answers. She explained in her spring interview:

> The seminar has been really helpful to me in sorting out what is really important. I feel better . . . at least LCM and GCF as terms, I don't think are so important, as terms . . . but the idea of looking at a denominator and thinking that I need to know what the multiples are, making a connection between a fraction and a division problem, being able to draw a diagram that has something to do with the problem, those skills are more important to me now.

In fact, the mathematics activities and the classroom cases helped Helen realize that even the students who got the correct answers or correctly applied procedures actually might not understand the important mathematical ideas. In a portfolio assignment in May, during her work with fractions, Helen wrote about the following insight:

> Fractions are on my mind a lot this past month, and the classroom cases and math activities have helped me sort out my own confusions and given me insight into my students' thinking... Students who are able to manipulate the numbers to find equivalent fractions or common denominators, for example, but who do not really understand why they are doing the computations concern me more as a result of this seminar . . . The seminar has helped me think more clearly about "what is needed" in order to perform the calculations. I feel that in some ways I . . . want to slow [my teaching] down to unravel and discuss (deconstruct?) what is being taught.

Her final evaluation of the seminar contained a similar insight about the difference between knowing the long division algorithm and understanding the ideas of division:

> I understand now that the traditional long division algorithm does not inherently have meaning for students; that students who understand the "idea" of dividing can use the algorithm as a tool, but learning the method will not contribute to an understanding of the idea of division.

Helen was beginning to think very seriously about what it meant to understand a mathematical idea, how knowing procedures did not mean that you understood the ideas, and what it meant to craft a practice that centered on ideas.

But the seminar experience offered her more than just the opportunity to learn to appreciate the importance and necessity of making student thinking about mathematical ideas a focus in her classroom. It also helped her learn to analyze that thinking in order to better assess what her students understood or did not yet understand. We can see this in the way that she used students in Valerie's fifth grade case about 12 ÷ 4, and Melinda's second grade case about 6 children sharing 45 pumpkin seeds, to think about a student in her classroom. In her March portfolio assignment, Helen wrote the following about moving back and forth between these cases and her classroom:

> [My student] Denise seems to be at a level of understanding similar to many of the students in Valerie's case. She is willing to compute, using multiplication and division facts she has memorized, if told what to divide into what and where to write an answer. But she is not making sense of the numbers in the problems . . . Denise is not using repeated subtraction as a strategy to solve problems, as are some of the second graders [in Melinda's case], nor is she drawing pictures as a few of the fifth graders [in Valerie's case] did. I think she uses facts she has learned but shows a lack of number sense . . . and she is unable to hold onto the ideas that the numbers represent.

The cases provided Helen with some specific examples of students' mathematical thinking about a topic that she was exploring with her own students. Moving back and forth between the students in the cases and her own students, she began to learn to be more analytical about her students' thinking.

As a consequence of her experiences in the seminar, Helen reevaluated her mathematics teaching practice and made important shifts in how she interacted with her students. This is not to say that her new practice was completely figured out. In fact, she realized that she had a great deal to learn. But in a May portfolio assignment, she acknowledged that her seminar experience had resulted in important learning and had helped her make important shifts in her practice:

> Too many of us have moved through math classrooms knowing where to put the next digit without really understanding what we are doing. This seminar has taught me to sort out these ideas and demand a clarity of teaching from myself as well as my students. I am still confused about fractions, I admit, but I think I've been more successful as a math teacher this year as a result of focusing on the "big ideas" in the seminar. I have become more thoughtful about what is important to spend class time on, about looking carefully at students' errors, and listening to students explain their thinking. I do listen more and talk less . . . This seminar has been a wonderful support group.

This learning is reflected in many facets of Helen's practice in the spring—her discussions with students about their ideas about division, her work with Abby and Tina as they struggled to make sense of the fractions in their pie graph, and her lesson on converting improper fractions to mixed numbers and vice versa. She is now better prepared to use the *Everyday Mathematics* curriculum to explore her students' ideas and support their mathematics learning.

Conclusion

Although the standards-based mathematics curricula have much to offer teachers, they also pose serious challenges. These curricula promote a focus on developing mathematical understanding, helping students learn to think and reason mathematically, and appreciating students' abilities to construct procedures for solving mathematical problems. For many teachers, this is a very new way of thinking about mathematics teaching and learning.

In order to use the innovative curricula as intended, many teachers must shift how they are thinking about mathematics, mathematics learning, and mathematics teaching. They must understand the mathematics content when mathematical ideas are the focus of what is to be learned. They must learn to listen carefully to their students' mathematical thinking so they can attend to the ideas that students are bringing to the mathematics content. Finally, they must use their understanding of the mathematics, and their appreciation for the subtleties and complexities of how students think about that content, as the foundation on which to build their mathematics teaching. While supports for this learning are often embedded in the curricula themselves, there is still much that teachers must learn.

The two stories in this paper offer images of the challenges teachers face as they learn to implement new curricula. In Pat's story, we observe a teacher struggling with the question of what her students should be learning when the focus is on mathematical ideas, particularly when those ideas feel "nebulous" to her, and we watch her become more solidly grounded in the important mathematical ideas in her curriculum. In Helen's story, we consider a teacher who is becoming intrigued with the mathematical thinking of her students, particularly as she comes to recognize that she *and* her students have powerful ways of thinking; we see her attending more carefully to her students' ideas in her one-on-one work with them and in classroom discussions. Through these stories, we have opportunities to think about how Pat and Helen's professional development experience in the year-long *Developing Mathematical Ideas* seminar helped them learn to more effectively use their new mathematics curriculum.

How was the seminar able to help these teachers shift their thinking about mathematics, mathematics learning, and mathematics teaching, and consequently use their mathematics curriculum in more powerful ways? For Pat and Helen, the seminar provided an opportunity to reexamine and explore a set of complex mathematical ideas that underlie much of the elementary school mathematics curriculum. They became mathematics learners themselves as they explored how our number system is organized; what it meant to add, subtract, multiply, and divide with different kinds of numbers; and how the situations and actions associated with each of the operations are interconnected. The experience of thinking deeply about these mathematical ideas, inventing their own ways of thinking, and listening to how their colleagues were thinking, helped these teachers understand the mathematics content more deeply, get in touch with the process of building mathematical understandings for themselves, and learn that there are diverse and powerful ways to think about and work through the mathematics content. For these teachers, the mathematics content—despite the fact that it was from the elementary mathematics curriculum—was a challenging and rewarding mathematical arena in which to work.

The seminar also offered Pat and Helen the opportunity to learn to look more thoughtfully at students' mathematical thinking. The classroom cases allowed them to learn to look carefully and critically at what students did and did not yet understand about a set of mathematical ideas, often by trying out student strategies with other sets of numbers and exploring the underlying logic that students were using. They were often taken by surprise at the mathematical power that was embedded in students' invented and often unfamiliar methods. Through this sustained examination of students' mathematical thinking, they were able to (1) develop an appreciation of and an ear for the powerful mathematical thinking that students are able to bring to bear on the investigation of a mathematical problem; (2) learn to follow students' logical thinking and analyze their confusion in order to determine where the ideas were solid and where they needed further work; (3) consider how students build their mathematical thinking over time and across the grade levels; and (4) connect the students' process of learning mathematics to their own process of learning mathematics. The seminar assignments, which included analyzing student work, conducting clinical interviews, and writing cases about the mathematical thinking that was emerging in their *own* classrooms, allowed these teachers to connect the thinking of the students in the casebook to the students in their own classrooms.

Finally, the seminar allowed Pat and Helen to explore images of other classrooms where teachers were working with standards-based curricula as a tool to support mathematics learning. They described moving back and forth between the cases they were reading and the classrooms they were teaching. Sometimes it was a matter of seeing how the teachers in the cases were working with their students to explore an idea—reading and thinking about the questions that were posed to the students, paying attention to how classroom discussions were facilitated, and thinking about how the ideas that emerged were examined and built upon. Sometimes it was a matter of hearing the teachers in the cases reflect on their learning goals for all of their students. Sometimes it involved being able to recognize that the teachers in the cases shared some of their questions about their teaching practice, and had some helpful ways of thinking about those questions. Being able to read and reflect on the classrooms depicted in these cases, and sometimes to see classrooms on video clips, helped Pat and Helen imagine new possibilities for their own practice.

Professional development designed to support teachers' implementation of a standards-based curriculum often consists of acquainting them with the curriculum itself and taking them through some of its activities. This kind of professional development no doubt meets many of the needs of teachers embarking on this path to reform. However, professional development that engages teachers in thinking deeply about the mathematics content of the elementary curriculum, explores how students think about that content, and provides vivid images of classrooms that embody many of the reform recommendations can provide a valuable, perhaps even essential, experience for teachers as well.

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Notes

1 For more information about the *Developing Mathematical Ideas* currriculum, see the discussion of its use with teachers in *What an Innovative Curriculum for Teachers Reveals About Supporting Teachers' Professional Learning* (Geist and Remillard, in press) and the discussion of its use with parents in "Learning to Listen: Lessons from a Mathematics Seminar for Parents" (Morse and Wagner, 1998).

2 In fact, there are often at least two "teachers" or facilitators who share responsibility for each seminar and cofacilitate each seminar session.

3 This student demonstrated that she had 100 cubes by stacking each group of 16 while she counted by 16's, using the work she had done on her paper with diagrams and numbers to help her. However, Pat did not explain what this student did when she reached 96, when a final group of 4, rather than another group of 16, was needed to bring the total to 100.

4 For example, students offered "hair" instead of "has brown hair" or "eyes" instead of "has blue eyes."

5 In fact, the correct rule was "has blue eyes."

6 The portfolio assignment was the same as in the fall. However, an additional question was inserted for this final analysis: "You have done

this assignment twice before. How does your response now compare with your previous ones? What has remained consistent in your thinking and what has changed?"

[7] The "Ben and Jerry ice cream problem" is as follows: *You are giving a party for your birthday. From Ben and Jerry's ice-cream factory, you order 6 pints of ice cream. If you serve 3/4 of a pint of ice cream to each guest, how many guests can be served?* The task was to solve the problem with a diagram, write an arithmetic statement that matched the situation, and discuss what connections you saw between the diagram and the arithmetic. It was one of several problems posed to explore the idea of division with fractions.

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