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ABSTRACT

The purpose of this study was to investigate the effects of the Casio 9850 and the TI-85 graphing calculators on college students' procedural skills and conceptual understanding in two different developmental mathematics courses. The courses used in this study were Elementary Algebra and Intermediate Algebra. Both the non-graphing calculator group and graphing calculator group were taught with the same goals. The students in the graphing calculator group were allowed to use the graphing calculator in place of performing paper and pencil procedures in class and on all tests. Students in the control group classes did not have a graphing calculator in class or on the tests, although all students were required to have access to a scientific calculator. The tests used in the study were two researcher-generated tests for Elementary Algebra and Intermediate Algebra. The procedural questions on the tests were similar to the Compass Placement Test (CPT), and the conceptual questions on the test were similar to the Core Learning Goals Test (CLGT). The test questions were reviewed and critiqued by a panel of five experts in the field of mathematics education who are familiar with the distinction between procedural skills, conceptual understanding, and the content of these courses. The tests were scored with an analytical scoring rubric. A Cronbach's Alpha test showed that the tests were reliable. Results of the pre-test to post-test analysis showed that there were statistically significant gains in procedural skills and conceptual understanding for both the Elementary and Intermediate Algebra graphing calculator sections ($p<.0001$). Significant results were also found in comparing the amount of gain in procedural skills and conceptual understanding for the non-graphing calculator and graphing calculator sections. There were significant increases in gains of procedural skills and conceptual understanding for the graphing calculator group versus the non-graphing calculator group enrolled in Elementary and Intermediate Algebra ($p<.005$). Suggestions for teaching using graphing calculators and recommendations for further study conclude the dissertation. Copies of the tests administered to the subjects, a graphing calculator supplement, and course syllabi are also attached. (Contains 97 references.) (ASK)

The Effect of Graphing Calculators on College Students' Ability to Solve Procedural and Conceptual Problems in Developmental Algebra

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ABSTRACT

The Effect of Graphing Calculators on College Students' Ability to Solve Procedural and Conceptual Problems in Developmental Algebra

By Mark A. Shore

The purpose of this study was to investigate the effect of the graphing calculator (namely the Casio 9850 or the TI-85) on college students' procedural skills and conceptual understanding in two different developmental mathematics courses. The developmental mathematics college courses that were used in this study were Elementary Algebra and Intermediate Algebra. For this study, both the non-graphing calculator group and the graphing calculator group were taught with the same goals. The students in the graphing calculator groups were allowed to use the graphing calculator in place of performing paper and pencil procedures in class and on all tests. Students in the control group classes did not have a graphing calculator in the class or on the tests, although all students were required to have access to a scientific calculator.

The tests used in the study were two researcher-generated tests for Elementary Algebra and Intermediate Algebra. The procedural questions on the tests were similar to the Compass Placement Test and the conceptual questions on the tests were similar to the Core Learning Goals Test. The test questions were reviewed and critiqued by a panel of five experts in the field of mathematics education who are familiar with the distinction between procedural skills, conceptual understanding, and the content of these courses. The tests were scored with an analytical scoring rubric. A Cronbach's Alpha test showed that the tests were reliable.

Results of the pre-test to post-test analysis showed that there were statistically significant gains in procedural skills and conceptual understanding for both the Elementary and Intermediate Algebra graphing calculator sections ($p<.0001$). Significant results were also found in comparing the amount of gain in procedural skills and conceptual understanding for the non-graphing calculator and graphing calculator sections. It was found that there were significant increases in gains of procedural skills and conceptual understanding for the graphing calculator group versus the non-graphing calculator group enrolled in Elementary and Intermediate Algebra ($p<.005$).

Suggestions for teaching using graphing calculators and recommendations for further study conclude the dissertation. Also attached are copies of the tests administered to the subjects, a graphing calculator supplement, and course syllabi.

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CHAPTER I

Introduction

"America's educational system is at a crossroad. Down one path can be found the many successful schools that have emerged from the crisis of the 1980s to become shining examples of educational excellence. Down the other path are schools that are mired in failure or that have implemented erroneous reforms, succeeding only in worsening their already dismal performances. At the intersection of these two paths are the vast majority of America's schools – stagnating in mediocrity – at the crossroads of excellence and failure" (Mullis et al, 1998).

- Average 1996 NAEP scores among 17-year-olds are lower than they were in 1984, a year after *A Nation at Risk* was released;
- U.S. 12th graders outperformed only two out of 21 nations in mathematics on the Third International Mathematics and Science Study (TIMSS);
- American students fall farther behind students from other countries the longer they are in school;
- Public institutions of higher education annually spend $1 billion on remedial education (Mullis et al, 1998).

The factors behind stagnant scores and declining international performance must be addressed to ensure that U.S. students are competitive in a global marketplace when they graduate. The scores of America's best and brightest in math and science do not improve this dismal picture. The performance of U.S. twelfth-grade advanced mathematics students was

among the lowest of the 16 TIMSS nations. Eleven nations outperformed the United States, while our scores were not significantly different from those of four other nations.

Interestingly, students from the U.S. do not fare so poorly in the earlier years, and only begin to fall behind toward their middle-school years. The factors behind the declining international performance of U.S. students must be addressed. In today's global economy, such results suggest that our students are competitive in the earlier grades, but when they enter the job market, they are at a serious competitive disadvantage.

One of the factors behind the declining international performance of U.S. students may be the curriculum to which they are exposed. Assessments of mathematics achievement of students in the United States reveal that they perform better on tasks requiring symbol manipulation than on tasks requiring representation skills, such as drawing conclusions from word problems (Dossey, Mullis, Lindquist & Chambers, 1988). The lower performance on tasks requiring representation skills may derive from the students' lack of opportunity to explore the structure of new kinds of problems before they are asked to apply new symbol manipulation skills. Thus students are being asked to solve problems before they really understand them. Attempts to teach problem representation strategies for mathematical problem solving have focused on teaching students ways to translate the words of a problem into other modes of representation using diagrams, pictures, concrete objects, the problem solver's own words, equations, number sentences, computer programs, verbal summaries, and embedding learning within a familiar context (Brenner, 1995). Research on expertise indicates that strategies for building mental representations and metacognition are best learned within the context of specific situations rather than as general principles (Brenner, 1995).

The Curriculum and Evaluation Standards (1989) reflect a shift in the importance that the world outside the schools increasingly places on thinking and problem solving. Procedural skills alone do not prepare students for that world. Therefore, students deserve a curriculum that develops their mathematical power and an assessment system that enables them to show it. The National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1991) states that students should engage in solving realistic problems using information and the technological tools available in real-life. Moreover, skills, procedural knowledge, factual knowledge, and conceptual knowledge are assessed as part of the doing of mathematics. In fact, these skills are best assessed in the same way they are used, as tools for performing mathematically significant tasks (NCTM Standards, 1991).

Researchers in the field of mathematical learning define two levels of mathematical understanding. Hiebert and LeFevre (1986) divide mathematical knowledge into procedural skills and conceptual understanding. Procedural skills include the familiarity with symbol manipulation, formulas, rules, algorithms, and procedures, while conceptual understanding is a connected web of knowledge, a network in which students are able to apply and link mathematical relationships to a variety of problem situations. For Hiebert and LeFevre, instruction should foster conceptual understanding.

The National Council of Teachers of Mathematics in its 1989 publication *Curriculum and Evaluation Standards for School Mathematics* fosters conceptual mathematical learning and relates it to multiple representations by making use of the graphics calculator. The grades 9-12 standards call for a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures to one that emphasizes conceptual understanding. The standards also call for a change in the curriculum from proficiency with paper and pencil skills to an increase

emphasis in multiple representations of functions, mathematical modeling, and mathematical problem solving. The integration of ideas from algebra and geometry is particularly strong, with graphical representation playing a connecting role. Thus, frequent reference to graphing utilities will be found throughout these standards (NCTM standards, 1989). In fact, the NCTM standards for grades 9- 12 state that "Scientific calculators with graphing capabilities will be available to all students at all times" (p. 124). At the collegiate level, research has investigated the effects of graphing calculators and computer graphing utilities in courses ranging from college algebra to calculus (Porzio, 1997).

Background of Study

For several years, mathematics educators have been intrigued with how powerful computer software might alter how mathematics is taught and learned. Graphic plotters, computer assisted instruction (CAI), hypermedia assisted instruction (HAI), computer algebra systems, and statistical analysis packages are some broad classes of software which have held promise for use in the mathematics classroom. The problem of access has delayed fulfillment of that promise. Few schools can boast of the economic resources which would allow every mathematics student to have full access to a personal computer for both class work and homework (Dick, 1992). The question of access is rapidly being rendered moot. Affordable hand-held calculators with the capabilities to graph functions and relations, manipulate symbolic expressions including symbolic differentiation and integration, compute with matrices and vectors, and perform high precision numerical integration and root finding of functions will provide the reality of mathematics classrooms where every student has tools rarely available on mainframe computers 20 years ago (Dick, 1992). The graphing calculator, which is often referred to as a hand-held

computer, shows promise in recent research as a tool to assist the learner construct conceptual knowledge in mathematics in the areas of algebra and functions (Dick, 1994). Graphing calculators have an advantage over computers in mathematics classrooms in their lower cost and smaller size for portability. This portability allows the graphing calculator to be used in a variety of courses and classroom settings where computers are unavailable. Also, with the graphing calculator, students can take the calculator home for homework, rather than being forced to do their homework at a computer lab. Even with many of today's students having computers at home, very few students have software that can calculate algebraic solutions or graphs, thus requiring the student to buy an expensive computer algebra system. Because of these advantages, the graphing calculator has gained widespread acceptance as a powerful tool for mathematics classrooms (Dick, 1992; Wilson & Krapfl, 1994).

Along with calls for changes in subject matter and pedagogy, there is a strong chorus urging changes in the way mathematics achievement is evaluated (Howe, 1998). It is important that educators view assessment techniques in light of the tools used in the classroom. For example, if the graphing calculator is used to help students gain a conceptual understanding of the function concept, the assessment instrument should test conceptual knowledge. Mwerinde and Ebert (1995) found that students in cooperative groups engaged in the type of mathematical discourse to enable them to form connections between graphical and algebraic representations scored significantly higher on labs testing this connection. However, they also found that the same set of students scored significantly lower on the test not emphasizing these connections.

Purpose of the Study

The purpose of this study was to investigate the effect of the graphing calculator (namely the Casio 9850 or the TI-85) on college students' procedural skills and conceptual understanding in two different developmental mathematics courses. The developmental mathematics college courses that were used in this study were Elementary Algebra (Math 90) and Intermediate Algebra (Math 93). Comparing common test scores of students receiving graphing calculator based instruction to those of students receiving traditional instruction yields some information, but this process is much like comparing apples and oranges if the course goals are different (Dunham and Dick, 1994). For this study, both the control group (non-graphing calculator group) and the experimental group (graphing calculator group) were taught with the same goals. The goals of these courses are that students perform better on both procedural and conceptual algebra problems. The specific goals of the course are similar to the hybrid algebra course described by Slavit (1996) where students engage in problem solving activities, yet also perform procedural skills. However, these second-generation graphing calculators, such as the Casio 9850 and the TI-85, use numerical techniques to approximate algebraic solutions. For this study, the students in the graphing calculator groups were allowed to use the graphing calculator in place of performing paper and pencil procedures in class and on all tests. Students in the control group classes did not have a graphing calculator in the class or on the tests, although all students were required to have access to a scientific calculator.

The procedural skill section of the test for the Elementary Algebra classes consists of:

1. Creating tables of values for linear equations.
2. Solving linear equations.
3. Graphing linear equations.
4. Finding x and y intercepts for linear equations, finding the equations of lines given two points, and finding the slope of a line that is in the form $ax + by = c$.
5. Solving systems of linear equations with two unknowns that have a unique solution both algebraically and graphically.

The procedural skill section of the test for the Intermediate Algebra classes consists of:

1. Creating tables of values for linear and quadratic equations.
2. Solving linear and quadratic equations.
3. Graphing linear and quadratic equations.
4. Finding x and y intercepts for linear and quadratic equations, finding the equations of lines given two points, finding the slope of a line that is in the form $ax + by = c$, and vertex points of parabolas.
5. Solving systems of linear equations with two unknowns that have a unique solution both algebraically and graphically and three unknowns that have a unique solution algebraically.

The conceptual understanding section of the test for both the Elementary Algebra (Math 90) and the Intermediate Algebra (Math 93) consists of applications that involve the conceptual understanding of 1-5 above. Many of the applications include multiple representations of the function concept. For example, students needed to find solutions to real-life situations numerically, algebraically, graphically, and orally (by describing what the solution means in the context of the problem situation). Students were allowed to give the decimal approximation for any procedural or conceptual problem. Students in the graphing calculator classes were allowed to use the graphing calculator on any procedural or conceptual problem. Students in the non-graphing calculator classes were allowed to use a scientific calculator on any procedural or conceptual problem.

The Elementary Algebra (Math 90) test consisted of 11 questions. Questions number 1, 2, 5, 6, 8, 9, and 10 were procedural questions and questions 3, 4, 7, and 11 were conceptual questions. The point value of both the procedural and the conceptual sections of the Elementary Algebra Test were 14 points for each section. The Intermediate Algebra (Math 93) test consisted of the same first 11 questions as the Elementary Algebra Test, but included 7 additional questions. Questions 12, 14 and 15 were procedural questions, and questions 13, 16, 17, and 18 were conceptual questions. The point value of both the procedural and the conceptual sections of the Intermediate Algebra Test were 22 points for each section. A copy of the tests and the point value for each question is in Appendix A.

This study was conducted to examine the following questions:

1. Is there a significant increase in procedural skills for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Elementary Algebra?
2. Is there a significant increase in procedural skills for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Intermediate Algebra?
3. Is there a significant increase in the conceptual understanding and the ability to apply algebraic skills to real-life situations for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Elementary Algebra?
4. Is there a significant increase in the conceptual understanding and the ability to apply algebraic skills to real-life situations for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Intermediate Algebra?
5. Is there a significant increase in gains of procedural skills for the graphing calculator group versus the non-graphing calculator group for students enrolled in Elementary Algebra?
6. Is there a significant increase in gains of procedural skills for the graphing calculator group versus the non-graphing calculator group for students enrolled in Intermediate Algebra?

7. Is there a significant increase in gains of conceptual understanding and the ability to apply algebraic skills to real life situations for the graphing calculator group versus the non-graphing calculator group for students enrolled in Elementary Algebra?

8. Is there a significant increase in gains of conceptual understanding and the ability to apply algebraic skills to real life situations for the graphing calculator group versus the non-graphing calculator group for students enrolled in Intermediate Algebra?

9. Is there a significant difference between the non-graphing calculator classes and the graphing calculator classes in the proportion of students that successfully complete Elementary Algebra with a grade of A, B, or C?

10. Is there a significant difference between the non-graphing calculator classes and the graphing calculator classes in the proportion of students that successfully complete Intermediate Algebra with a grade of A, B, or C?

Rationale for the Study

If assessment is to be aligned with the curriculum, as suggested by Cain and Kenny (1992), then one must design assessment that reflects the content of the curriculum. Computers and calculators can play a significant role in the teaching and learning of every mathematical topic (Adams, 1997). These tools can have a great impact in the mathematics classroom (Leitzel, 1989). While the National Council of Teachers of Mathematics (1989, 1991) suggested that mathematics is problem solving and that the learning environment should reflect this position, many mathematics placement tests used for entrance into college math courses, such as *Compass*, *American Placement Test* (*ACT*), and *Acuplacer*, are mostly procedural. This places high school teachers in a quandary: should they follow NCTM guidelines and focus on conceptual understanding using graphing calculators, or should they focus more on procedural skills, so their students can pass placement tests? The controversy is even more perplexing in the Maryland School System, where teachers are informed that they must teach the "Core Learning Goals," (a set of learning goals for Mathematics that are more conceptual than procedural). Yet many Maryland students go to two-year colleges, whose mathematics placement test is a procedural test (either *Compass*, *ACT*, or *Acuplacer*). The division between procedural skills and conceptual understanding, how much of each should be taught, how they should be taught, and how and when to use technology to aid in students' procedural skills and conceptual understanding is the cornerstone of the dilemma in mathematics education.

While some teachers have embraced the use of technology and real world problems, others have continued teaching without technology and avoiding meaningful applications. The National Council of Teachers of Mathematics has long advocated the use of calculators at all levels of mathematics instruction, and graphing calculators are no exception. Indeed, the

Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) makes the following underlying assumption for grades 9-12 (p. 124): Scientific calculators with graphing capabilities will be available to all students at all times. However, this is in contradiction with many nationally standardized mathematics placement tests in which students are not allowed to use any type of calculator. This study determined if gains in procedural skills and/or conceptual understanding for college students enrolled in developmental algebra courses were significantly greater with the aid of graphing calculators than for students who did not have graphing calculators.

Limitations of the Study

The students in this study enrolled in one of two college developmental mathematics courses. These students have been placed into these courses based upon their score on the Compass Placement Test. The Compass Placement Test states that calculators are not allowed; however, for the past two years the mathematics department of the college and the policy of the Maryland Mathematics Statewide Standards Committee has permitted students to use any type of calculator on the placement tests at two-year colleges in Maryland. The results of this study can be generalized only to college students enrolled in a developmental algebra course. The results of the graphing calculator classes can only be generalized to developmental algebra courses which allow students to use either a Casio 9850G, Casio 9850GA Plus, or TI-85 graphing calculator in class and on all tests. The teachers for the graphing calculator sections were well trained on the operation of these two calculators; therefore the results should not be expected to be replicated unless teachers are also proficient in the use and operation of graphing calculators.

Also, due to the limited number of sections and teachers, the sample size and variety of teachers and sections will be limited.

All students in the graphing calculator sections were told that they must buy a Casio 9850G, Casio 9850GA Plus, or a TI-85 for the course. All students in the graphing calculator sections did buy one of these three calculators. All students in the non-graphing calculator sections were told that they would not be allowed to use a graphing calculator for the course or for any tests in the course. If students in the non-graphing calculator classes already owned a graphing calculator, they were required by the instructor to show all their paper and pencil steps in order to solve a problem. Students in the non-graphing calculator classes were not shown any of the features of how to use the graphing calculator in class, except for those features available on a scientific calculator. Students were also questioned on the pre-test and post-test of their specific graphing calculator abilities and were asked to write how to perform certain operations on the graphing calculator. A copy of the graphing calculator questionnaire is in Appendix A.

From student responses to these pre-test and post-test questions, it was found that two students in the non-graphing calculator Intermediate Algebra group knew how to perform some operations on a graphing calculator that are not available on a scientific calculator. However, these two students were not used in the results of the study. It was also found that all students in the graphing calculator sections had purchased a graphing calculator. However, information from the post-test questionnaire showed that some students in these sections did not know how to use any of the operations on the graphing calculator beyond that of a scientific calculator. Even though some students in the graphing calculator classes did not know how to use any of the features beyond that of a scientific calculator, these students were still included in the results for the graphing calculator groups. The reason these students were included in the results was that

they had purchased a graphing calculator and were enrolled in the sections in which a graphing calculator was required and instructed on how to use the calculator in class and were allowed to use the graphing calculator on all tests. Not to include a student in the results of the graphing calculator section, even though the student does not know how to use a graphing calculator, would be analogous to not including a student in the non-graphing calculator section that does not know how to perform paper and pencil algebra procedures.

There were different levels of importance placed on either procedural skills or conceptual understanding by the instructors. Instructors were free to instruct students using any teaching method they were comfortable with; therefore, some of the instructors may have used cooperative groups or lecture based instruction more often than other instructors. However, from post interviews with the instructors, it was found that all the instructors used mostly lecture based instruction. Also, in the graphing calculator sections there were different levels of use of the graphing calculator depending upon the teacher's belief. For example, some instructors that taught using the graphing calculator supplemented the students' paper and pencil skills with the graphing calculator, while others supplanted the students' paper and pencil skills with the graphing calculator. The level of use of the graphing calculator also depended upon the topic that was being taught. The amount of time spent solving problems and graphing with and without a graphing calculator varied with each instructor in the graphing calculator groups.

Since the tests are assessing a narrow scope of procedural skills and conceptual understanding, no standardized test could be found that only tests these specific skills. It was therefore necessary for the researcher to construct the tests for the study. There was a panel of five experts in the field of mathematics education, who are familiar with the distinction between procedural skills, conceptual understanding, and the content of these courses to critique the tests.

Three of the members of the panel were familiar with graphing calculators and two were not. Once the researcher obtained the critiques of the tests, revisions were made to the assessments to insure that the tests were valid measures of assessing procedural skills and conceptual understanding. The tests were scored with a analytical scoring rubric, so that students that did not get the question completely correct, would still obtain one point for reaching a particular point in answering some questions. A copy of the scoring rubric is in Appendix A. The same panel reviewed the grading of the tests for any inconsistencies or errors in scoring. Students took the tests in the college's testing lab. Students were requested to show a photo identification to take tests in the testing lab. This insured that the correct students were being tested. Similar to the Compass Placement Test, the tests were untimed. Students were not allowed to use any notes on the tests or to take any notes with them in the testing lab. To reduce the affect of test fatigue on either the procedural or conceptual sections of the test, procedural and conceptual questions were randomly placed throughout the test. Usually a procedural question was followed by a conceptual question of the same category. To insure the reliability of the tests, a Cronbach Alpha Test for reliability and internal consistency was performed on the tests with Elementary and Intermediate Algebra students with and without graphing calculators.

The procedural skill questions on the tests are similar to the Compass Placement Test that relate to the procedural objectives of the specific course. The conceptual understanding questions on the tests are questions similar to the Core Learning Goals Test that relate to the conceptual objectives of the specific course. A graphing calculator is not needed to answer any of the questions on the tests. All of the questions can be solved by typical algebraic procedures without a graphing calculator. Any significant results from this specific course content may not necessarily be generalized to other algebra levels or objectives.

Assumptions of the Study

The developmental algebra students that participated in this study represent typical college students enrolled at small two or four year colleges in a developmental algebra course. The instructors for the graphing calculator and non-graphing calculator classes are equally competent instructors. The students in the non-graphing calculator and graphing calculator sections have approximately the same motivation to successfully complete the course.

Definition of Key Terms

Developmental Algebra – Math courses offered at colleges that are usually non-credit, which students must pass in order to register for a college level mathematics course or the next developmental math course.

Math 90 – A three credit hour Elementary Algebra Course.

Math 93 – A three credit hour Intermediate Algebra Course.

Procedural Skills – The ability to solve a problem which requires only the manipulation of symbols with paper and pencil or with the use of a scientific or graphing calculator.

Conceptual Understanding – The ability to apply mathematical concepts to a variety of situations and translate between verbal statements and mathematical expressions.

Casio 9850G, Casio 9850GA Plus, and TI85 graphing calculators – The Casio 9850G, Casio 9850GA Plus, and TI-85 graphing calculators have a variety of built in solvers that are capable of performing most of the calculations and graphing needed in algebra, calculus, or

engineering. Graphing calculators with solvers can either solve or approximate solutions to linear equations to many significant digits. The TI-85 and Casio 9850 also can solve or approximate the solutions to systems of equations and polynomial equations (up to third degree), if the equations are entered into the calculator in standard form. Graphing features of these second-generation calculators include zooming features, range, trace, *x* and *y* intercepts, points of intersection, and relative maximums and minimums at the push of a button. However, for the calculator to find a solution graphically, the graph must be on the calculator's viewing window. Both the TI-85 and the Casio 9850 have a statistics menu that allows students to enter points and find equations of lines or curves. If two points are given, the calculator will find the equation of the line that goes through the two points. If more than two points are given, the calculator will find the line of best fit. The calculator can not find the equation of a vertical line or a horizontal line. While graphing calculators can perform calculations, they do not show how the calculations were performed.

CHAPTER II

Review of Literature

Technology in Mathematics

> On the most fundamental level, technology requires rethinking not only of the "how" but of the "what" we teach in mathematics. It is pretty clear that in the future no one is going to get a job based on the ability to add long columns of numbers accurately. Recently we have seen the appearance of calculators and computer software that can perform much of the repertoire of undergraduate mathematics and beyond. Even if everything had been fine with U.S. math education, we would have to pay attention now to how the availability of sophisticated calculational tools change what is important to teach. The automation of computation challenges the notion that mastery of computational technique should be the main criterion of mathematical success. The relation between computational expertise and conceptual understanding, and how each supports the other, is complex and requires careful study and thought. (Howe, 1998)

Since 1965 computers have been used as an instructional aid to promote learning in mathematics (Saunders & Bell, 1980). With the introduction of computer algebra systems and graphing calculators, the choices for type of instructional technology are more diverse. Saunders and Bell (1980) found that the use of computer-enhanced resources throughout an entire algebra course had no significant effect on algebra achievement, attitudes toward mathematics, and attitudes toward the instructional setting. Ganguli (1990) found that using the computer as a demonstration tool for instruction in a college algebra course had no significant effect on student performance on post-test scores. Tilidetzke (1992) did not find any significant differences between students enrolled in a college algebra course using computer-assisted instruction (CAI) with those using traditional methods of instruction.

Some studies did show significant gains through the inclusion of technology. Palmiter (1991) showed that knowledge of calculus concepts for students using *Macsyma*, a computer algebra system, was significantly higher than for students taught using traditional instruction. Judson (1990) investigated the effect of *Maple*, a computer algebra system, on students' understanding of the concepts in a business calculus course and found no statistical differences in

achievement between the control and experimental groups. However, motivation, interest, and class participation were markedly higher in the experimental group than in the control group. Shore (1997) found that students in college algebra using hypermedia assisted instruction (HAI) performed significantly better than students using only a graphing calculator, or a computer algebra system.

With society's growing need for critical thinking, the National Council for Teachers of Mathematics (1989) places strong emphasis on problem solving. In order to incorporate computer algebra systems or graphing calculators into problem solving, teachers need to be versatile by offering students a method of analyzing data and performing higher level critical thinking skills. The American Mathematical Association of Two-year Colleges (1995) has recognized that technology has reduced the importance of many paper and pencil algorithms. Simultaneously, an increasingly technological world is demanding graduates possess problem-solving skills similar to those used while solving algebraic word problems.

Studies conducted by Heid (1988) and Trout (1993) suggest that instruction that integrates computer algebra systems can lead to improved student problem solving ability. However, several other studies suggest that, in order for computer algebra systems to be effective, they need to be available to each student in the classroom as well as on homework (Cunningham, 1991; Smith, 1994). The computer has not had the impact on the teaching and learning of mathematics that had been predicted (Barrett and Goebel, 1990) since many schools do not have a computer in each mathematics classroom and since many educators have trouble defining the role of the computer in the classroom. Most algebra teachers indicate that they use computers for demonstration purposes only (Demana and Waits, 1992). The computer has had a great impact on mathematics, but mathematics is still being taught in most college courses just as it was 30

years ago as a paper-and-pencil discipline (National Research Council, 1991). Demana and Waits (1992) are convinced that, if teachers and students rely solely on desktop personal computers, no meaningful reform will occur in mathematics education in the 1990s; students must use computers on a regular basis for both in-class work and for homework if significant changes are to be made in the mathematics that students learn in the 1990s. Demana and Waits advocate the use of inexpensive pocket computers (graphing calculators) in mathematics education.

Research on graphing calculators

The graphing calculator can affect the nature of the instructional environment and the avenues for content delivery, but, perhaps more importantly, it can also affect the nature of the mathematics being discussed (Slavit, 1996). For example, many traditional textbooks simply ask the student to graph a function. A graphing calculator makes it possible to view the graph of a function as a first step instead of a last one. Dick (1992) suggests three avenues of teaching mathematics that are opened by graphing technology to include: (1) graphing as an exploratory activity; (2) graphing as a problem solving activity; (3) graphing as monitoring device. However, it is clear that instructional reform initiatives are running well ahead of the data, and instructional changes are often based more on theoretical than on empirical support (Hiebert & Wearne, 1993).

Rich (1990) found inconclusive evidence of the value of the graphing calculator for precalculus students' achievements, attitudes, and problem solving. However, Rich (1990) found students using graphing calculators are better able to relate graphs to their equations, to understand global features of functions, and to find an algebraic representation for a graph.

Students also better understand connections among graphical, numerical, and algebraic representations (Dick, 1994). Some studies indicate that women are not disadvantaged by the integration of graphing calculators and in some instances outperform males (Cassity, 1997; Dunham, 1990; Ruthven, 1990). Cassity (1997) also found that the variables of spatial visualization and mathematical confidence are related to conceptual mathematical performance when graphing calculators are utilized as a tool. The mere presence of graphing technology may not solely account for these results. Rather, the combination of technology, changes in curriculum and instruction, and the teacher's belief must be examined.

Although these research results are extremely encouraging, not all results have been positive. Becker (1992) found that graphing calculator use did not improve students' understanding of the concept of function in a college precalculus course. Shore (1997) assessed the problem solving ability of college algebra students using a variety of instructional software. A computer algebra system (*Derive*), a teacher written hypermedia program, a graphing calculator (TI–85), and the combination of both the computer algebra system and the hypermedia program were used to teach and to solve distance and mixture problems. A pre-test that included distance and mixture problems showed no significant differences between the groups; however, post-test results showed significant differences between the groups at a variety of levels. Students using the hypermedia program or the combination of both the hypermedia program and the computer algebra system scored significantly higher than the control group, the graphing calculator group, and the computer algebra system group. There were no significant differences between the control group, the graphing calculator group, and the computer algebra system group on posttest scores. Although current graphing calculator technology was used in the study, the procedural skills needed to solve mixture and distance problems do not require a graphing

calculator with a solver for most college algebra students. This may account for the lack of any significant differences among the control group, the graphing calculator group, and the computer algebra system group. It is therefore important to test the effect of the graphing calculator for specific objectives and student populations that may benefit from the use of a graphing calculator.

Runde (1997) found that College Algebra students using the TI-92 graphing calculator scored significantly higher than the control group (non-calculator group) on word problems involving mixture, distance, work, and geometric applications of the quadratic formula. It is important to note, however, that the control group for the study had to do all calculations by hand. Also, the TI-92 group had a higher dropout rate, which could have made the measured effect of the graphing calculator artificially high.

Leitzel (1993) noted the explosive growth of graphing calculators in secondary schools and urged college mathematics faculty to take advantage of students' ability with this technology. The availability of graphing calculators has motivated many mathematics educators to reexamine what and how we teach mathematics (Dunham & Dick 1994). Dunham (1992) examined the pre-test to post-test scores on a calculus readiness test and found that students receiving graphing calculator instruction attained sufficient calculus placement scores at nearly twice the rate of those receiving traditional instruction. In his review of research, Dick (1992) found significant differences in favor of graphing calculator groups compared with non-graphing calculator groups. In other studies, he found no difference in overall precalculus achievement between the experimental and control groups. However, graphing calculators were allowed on tests administered in some studies, but not on other studies.

Therein lies the paradox in attempting to perform an experimental group and control group study of achievement and graphing calculator use. On the one hand, those critics who worry specifically that students will rely on the graphing calculator as a crutch will certainly cry foul if the experimental group has the advantage of graphing calculators. On the other hand, Ruthven (1990) argues that not allowing students to use graphing technology when they have become accustomed to it forces them to do mathematics "under unduly artificial conditions" (p. 438).

Opposition to the use of symbolic graphical calculators in secondary and even college classrooms will inevitably arise out of fears for students' development of basic algebraic and graphing skills (Dick, 1992). Previously, the subject of algebra "has been dominated by its syntactical aspects, viewed as a series of manipulations to be learned often by drill and practice, that enable one to change the form of an expression or to solve an equation" (Kaput, 1992, p. 542). "Indeed many institutions maintain that it is crucial to stay this course in the beginning levels of the study of algebra so that the subsequent study of functions and graphing using the graphing calculator in later courses will be well grounded" (McCollum, 1997, p. 2). In fact, Schoenfeld, Smith and Arcavi (1993) have found evidence that student understanding of some graphing related concepts remain relatively unstable unless it is also cognitively anchored to symbolic representations. Just as numerical calculators have called into question the teaching of some numerical paper and pencil skills, so too the capabilities of symbolic calculators will naturally call into question the teaching of some symbolic manipulation skills.

Dunham and Dick's review of the research (1994) shows that graphing calculators have significantly changed the climate of the classroom. Farrell (1990) noted that students became more active in classrooms in which graphing technology was being used with more group work, investigations, and explorations.

Today's calculators provide a level of computational support for problem solving that was unimagined even when the 1989 NCTM *Curriculum and Evaluation Standards* were released. With calculators, students can have access to a wider range of complex problems, and they can address these problems earlier in their school experience. Students at all levels should have access to calculators and other technology to use as they solve problems (NCTM standards 2000 draft, 1998).

Research on procedural skills and conceptual knowledge

Clearly, research on the impact of graphing calculators is needed at the beginning algebra level where students are first introduced to the concept of a graphic representation of an algebraic function. Kaput (1992) identified two crucial questions in this area: (1) How does the graphing calculator "affect the relation between procedural and conceptual knowledge, especially when the exercise of procedural knowledge is supplanted by (rather than supplemented by) machines?" (p. 549) and (2) "How does one integrate multiple representations of mathematical knowledge, including the use of linked concrete and abstract notations and actions versus display notations?" (p. 550).

Kieran (1993) and Hollar (1996) both cite Sfard's Process-Object Model of mathematical conceptual development as a means of interpreting student understanding of the function concept. Sfard hypothesizes that students acquire a procedural conception first but then have a great deal of difficulty in making the transition to an object conception. Many mathematical educators surmise that the ease with which multiple representations of functions can be explored and examined using the graphing calculator will facilitate this transition.

Students need to learn different procedural skills in courses that use graphing calculators. Students are often unsure when parentheses are necessary and are confused about how to enter

complex computations. Students will need to learn what feature of the graphing calculator to use to perform certain operations. They will also need to learn what form an equation needs to be in to use a specific function on the graphing calculator. For example, the student will need to learn that, in order to solve linear equations, he/she will need to navigate to the calculator's solver and, depending on what calculator the student is using, the form of the equation differs. Also, if the student uses the calculator's solver for polynomial equations, the calculator will only give one solution. Therefore, the student must navigate to the calculator's polynomial solver. To use the calculator's polynomial solver, the polynomial must be set equal to zero and be in standard form. (The features of both the Casio and the TI-85 and directions on how to use these calculators are in Appendix B.)

Students will need to learn the navigational skills to position the viewing window for the best view of a graph. Awareness of the importance of scaling on the axis and the range of visible coordinate values demand more attention when a graphing calculator is being used. Once the student has the important feature of the graph on the calculator's viewing window, the student can find intercepts, intersection points, relative maximums, relative minimums, and x and y values at the push of a few buttons. However, the student must still have a conceptual understanding of what the problem situation is asking, and what realistic values the function might have for the independent and dependent variable in order to set the range on the calculator's viewing window. If not, the student may never see the graph of the function on the viewing window. Williams (1993) found the area in which students have the most widespread difficulty is dealing with the issue of domain, ranges, and scales of axis to see the important features of the graph of a function.

Adams (1993) found that the use of graphing calculators significantly affected students' concept of function regarding domain and range. Browning (1989), Harvey (1993), Rich (1990), and Ruthven (1990) cited significant results in terms of improved graphical understanding of functions. Browning (1989), Harvey (1993), Hollar (1996), Ottinger (1993), and Slavit (1994) all reported better overall understanding of the function concept. Only two of these studies addressed the question of whether or not procedural skills are impacted by the use of graphing calculators. Both Hollar (1996) and Ottinger (1993) found no significant differences in the level of procedural skills between the treatment and control groups.

In Cassity's (1997) review of the research, she found that research has often found little or no difference in overall performance for mathematics students using graphing calculators compared to traditional (non-graphing calculator) classes. However, when performance is divided into procedural and conceptual levels, significant differences appear at the conceptual level. Estes (1990) conducted a study to investigate the effects of implementing graphing calculators as teaching tools in Applied Calculus. Students in the control group were allowed to use graphing calculators, but did not receive any special attention to effective uses of the graphing calculator to facilitate conceptual learning. On end-of-semester tests, the experimental group scored significantly higher than the control group on conceptual measures. No significant difference was found between the groups on procedural measures. Caldwell (1995) conducted a study to determine the effect of the TI-81 Graphing Calculator as a learning tool on college algebra students' understanding of concepts and performance of procedures involving functions and graphs. At the conclusion of the functions and graphs unit, a concepts test and a procedures test were administered. The treatment group scored significantly higher than the control group on the procedures test. There was no significant difference between the two groups on the

concepts test. It is important to note that the calculators used in these studies were the Casio 7000g and the TI-81 (first generation graphing calculators), which do not have any solver, and the only graphing feature is the ability to trace a function.

Graphing calculators provide opportunities for students to connect graphical images, symbolic expressions, and sets of related numerical values (Caldwell, 1995). With graphing calculators such as the Casio 9850 and TI-85, students can represent and solve functions symbolically as algebraic representations, graphically as plots of input-output points, and with the Casio 9850, students can represent functions numerically as tables of input-output pairs. Students can use these calculators to translate across these multiple representations of functions and connect these representations to physical and social contexts. In terms of Polya's (1957) problem solving steps, it takes people to understand a problem, devise a plan for solving it, and interpret and evaluate that solution. A powerful calculator or computer can only help us carry out that plan.

The cost of graphing calculators is of some concern. Although the cost of the TI-92 is approximately $200, the cost of the TI-85 is approximately $100, and the cost of the Casio 9850G, Casio 9850GA Plus, and Casio 9970G is approximately $70. The added features of the TI-92, such as 3-D graphing, geometry software, and calculus features, are unnecessary for college algebra students.

The concept of function

A function can be described by a verbal or written statement, by an algebraic formula, as a table of input-output values, or as a graph (NCTM, 1989). Numerical, algebraic and graphical representations are used to jointly construct and define the mathematical concept of function;

consequently, functions and graphs cannot be treated as isolated concepts (Leinhardt et al., 1990). It may be argued that the function concept is the single most important concept from kindergarten to graduate school (Harel and Dubinsky, 1992). The function concept is a central one in mathematics, which grows in importance as one progresses in the depth and breadth of one's understanding of mathematics (Yerushalmy & Schwartz, 1993). Yerushalmy and Schwartz (1993) believe that the function concept is the fundamental object of algebra. The American Mathematical Association of Two-Year Colleges (AMATYC, 1995) in its Standards document includes function as one of its Standards of Content: "Students will demonstrate understanding of the concept of function by several means (verbally, numerically, graphically, and symbolically) and incorporate it as a central theme into their use of mathematics." The concept of function is a fundamental and unifying theme of mathematics (NCTM, 1989). The introduction of functions and graphs is a critical moment in mathematics education since it presents a setting with the opportunity for powerful learning to take place and since the concepts of functions and graphs are fundamental to more sophisticated parts of mathematics (Caldwell, 1996).

A major contention of the mathematics reform movement is that Elementary and Intermediate algebra courses focus mainly on symbol manipulation skills, such as how to solve equations, but do not emphasize the underlying problem representation skills, such as understanding what a word problem means. This lack of expertise in problem representation creates difficulties as students attempt the transition from elementary to college algebra. For example, elementary algebra students have difficulty in linking graphical and tabular forms of representation to algebraic forms of representation (McCoy, 1994). Many researchers have worried about students performing symbolic manipulations of algebraic expressions without reflecting on their meaning (Hiebert, 1992). In an analysis of the content of mathematics

textbooks used in the United States, Demana, Schoen & Waits (1993) found that less than 3% of the page space is devoted to graphical representations. Therefore, most elementary algebra students have not been expected to learn how to construct or understand the meaning of a graph. In her review of the research, Brenner (1995) found that, in existing textbooks, graphs are not integrated with other topics and graphical representations of functions played a minor role in the algebra curriculum. Brenner (1995) also found that students' misconceptions lead to a tendency to view all functions as linear, misunderstand the role of scale, think that a variable stands for a single number, and view graphs as a set of discrete points rather than a continuous relationship. Hart (1981) revealed that translating a functional relationship from data pairs into algebraic symbols was one of the most difficult of representation tasks for students. Based on such evidence, Kieran (1993) argues for teaching the notion of a function as a dependency relation in a practical situation rather than the more formal definition of a function as a correspondence between two sets. Swan (1982) has shown that mathematics textbooks emphasize skills such as tabulating, plotting, and reading values from graphs in abstract contexts. Kieran (1993) notes that the consequences of emphasizing exclusively these skills are that students lose sight of the meaning of the task, rarely see graphs other than those of straight lines, and get little practice at interpreting graphs in terms of realistic situations.

In contrast to the lack of attention paid to concrete representations of functions in the past, the National Council of Teachers of Mathematics state that, "The study of functions should begin with a sampling of those that exist in the students' world and students should have the opportunity to appreciate the pervasiveness of functions through such activities as describing real world relationships that can be depicted by graphs." Williams (1993) summarizes the major theme of research on functions as "the importance of being able to move comfortably between

and among the three different representations of function: algebraic, graphical, and tabular." To this list Brenner (1995) includes verbal representations as another primary way in which students should be expected to understand functions. She states, "This means being able to identify functional relationships as encoded in word problems as well as being able to explain in words the functional relationships represented in other representations."

The graph of a function

Students experience a major leap in their mathematical development when they are introduced to the concept of the graph of a function in two variables (Herscovics, 1989). Results from the Second Mathematics Assessment of the National Assessment of Educational Progress (Carpenter, Corbitt, Kepner, Linquist, and Reys, 1981) indicate that the majority of students do not manage this leap. Carpenter et al. (1981) found that students could graph ordered pairs of numbers in the Cartesian plane but that most did not understand the relationship between equations and their graphs. Results from the Fourth National Assessment of Educational Progress (Brown et al., 1988; Silver et al., 1988) indicated that U.S. students had a limited understanding of function concepts and of graphing.

Kenelly (1986) contends that calculus students experience difficulty with function concepts. He surmises that beginning algebra students fail to form a conceptual understanding of variables; as a result, for many students variables are simply symbols used in manipulative practice exercises and functions are "ordered pairs of these things." Kenelly states that "Students miss the idea that functions capture the spirit and essence of connections and interdependent relationships, and they fail to see that functions embrace the elements of input and output, control and observation, and cause and effect."

The use of the graphing calculator on the concept of function

The impact of the graphing calculator on students' understanding of the function concept depends on the instructor's belief of what students should know about functions. For example, if the teacher believes that the most important feature of the function concept is the manipulation of symbols, then the graphing calculator will be of little use. However, if the teacher believes the major focus of the function concept is on dependency relationships, and the teacher utilizes real data to illustrate these relationships, then the graphing calculator can have an impact on learning. Slavit (1996) found from teacher observances that pencil and paper algebraic techniques were introduced first to discourage the sole use of graphical methods. Even though student suggestions for problem solving strategies were consistently in terms of graphing representations, the teacher still insisted on solving the problem first by algebraic pencil and paper means. On several occasions when the teacher began to explore a problem symbolically with paper and pencil, students would remark, "Can't you just graph it?" Slavit (1996) also found that higher levels of discourse resulted from the explorations taking place within a multi-representational function environment, and the graphing calculator allowed the instructor to investigate problem situations from graphical and numerical perspectives while relating these to their symbolic form. The vision of making a "multiple representation approach" a central belief in the philosophy of a mathematics curriculum is impossible unless students and instructors actually have the tools to use numeric and graphic strategies in addition to the traditional paper and pencil algebraic techniques (Dunham & Dick, 1994).

Graphing calculators in developmental mathematics

The mathematics reform movement has the goal of enabling virtually all students to take more advanced mathematics courses. This means that even lower achieving students will be expected to take algebra and other courses that have traditionally been restricted to college preparatory students. While there is evidence to show that lower achieving students perform better if problems are represented in multiple formats anchored in a meaningful context, and discuss problem solving processes in cooperative groups (Brenner, 1995), there is virtually no research on the effect of graphing calculators on college students in developmental algebra courses. Symbol manipulation skills and word problem representation skills are cognitive prerequisites for success in algebra, yet traditional instruction may focus on symbol manipulation skills at the expense of representation skills. Problem representation skills in developmental algebra courses may prepare students better for future study of College Algebra because some colleges have adapted their College Algebra courses to a curriculum suggested by the NCTM and AMATYC standards. Since these standards place a greater emphasis upon the multiple representation of functions, it would make sense to focus on these topics in developmental algebra. This would provide better continuity for students from Elementary Algebra to College Algebra.

Anchored Instruction

Anchored Instruction is an approach which stresses the importance of placing learning within a meaningful, problem-solving context. When a learner is able to generate his or her own solution to a problem rather than simply being told the solution, this learning is often more easily applied to other less similar problems. Computer and calculator-based graphing has the potential

to change the way mathematics is taught and learned (Waits and Demana, 1988) since this technology enables students to handle more complicated, realistic, and noncontrived applications. The graphing calculator can transform the mathematics classroom into a laboratory environment where students use technology to investigate, conjecture, and verify their findings (NCTM, 1989). One of the basic principles of the AMATYC Standards is that mathematics must be taught as a laboratory discipline (AMATYC, 1995). There must be an emphasis on effective mathematics instruction involving active student participation and in-depth projects employing genuine data to promote student learning through guided hands-on investigations (Caldwell, 1996). "The graphing calculator promotes this type of a learning environment." (Caldwell, 1996)

Current trends in mathematics education include increased emphasis on developing learners' abilities to represent real-life situations with various methods and analyze functional relationships (NCTM, 1989). These learning outcomes have not traditionally been achieved by mainstream secondary school mathematics students (Verzoni, 1995). Instead, emphasis on procedures for manipulation of symbols and solving equations has been associated with weaknesses in students' abilities to connect algebraic representations with real world situations (McCoy, 1994). Several (Verzoni, 1994; McCoy, 1994) have supported changes in algebra instruction to facilitate students' abilities to translate between multiple representations of functions and map between algebraic equations and life situated relationships. Slavit (1996) found that the graphing calculator was an aid to the instructor in modifying or creating problems, which were relevant to the lives of the students. Graphing calculators furnish the tools to do mathematical modeling using real life data.

Problem solving

A problem has been defined as a situation in which a person wants something and does not know immediately what sort of actions to take in order to achieve the sought after goal (Henry, 1994). Incorporated into that definition is the fact that, for there to be a problem, the person must want to solve it (Nolan, 1984). Campione, Brown, and Connell (1989) have found that skills for solving problems are traditionally taught, but the reasons for learning to solve problems are not usually presented. They also found that the emphasis on skill training and not problem solving is even more exaggerated for low-achieving students. The National Council for Teachers of Mathematics (1989) in its *Curriculum and Evaluation Standards for School Mathematics* advocates mathematics teaching through activities that encourage students to explore mathematics, to gather evidence and make conjectures, and to reason and communicate mathematically. Recent literature in math education reflects the emphasis placed on problem solving skills by the NCTM (Henry, 1994).

The amount of time required to teach the basic algebraic manipulations has reduced the attention teachers can allot to problem solving (Campione, Brown, & Connell, 1989). As a result of the lack of time on problem solving skills, students are unable to develop the repertoire of strategies needed to solve word problems. The fast pace of exposure to new concepts in most college mathematics courses (including developmental algebra courses) does not allow students the time required to develop problem solving skills. Porter (1989) found that 70% of the topics covered in math classes received less than 30 minutes of instruction. While instruction of some topics in algebra do not require much time, effective instruction of problem solving requires time for students to develop metacognitive skills.

Dick (1992) cites three ways that graphing calculators can lead to improved problem solving: (1) calculators free more time for instruction by reducing attention to algebraic manipulations; (2) calculators apply more tools for problem solving, especially for students who have weaker algebraic skills; and (3) students perceive problem solving differently when they are freed from the burden of numerical and algebraic computation to concentrate on setting up the problem and analyzing the solution. Problem solving has been described as a "more difficult and demanding intellectual task than is most school work" (Nolan, 1984, p. 7). More than obtaining answers, problem solving entails strategies, thinking skills, and is partly cognitive and partly affective in nature (Meiring, 1980).

Personalizing word problems by having thematic strands in a set of problems is a helpful strategy for students (Giordano, 1990). Visualizing or constructing a nonverbal representation of the problem is a strategy that appears often in the literature (Bransford et al, 1988; Caldwell & Goldin, 1987). Other strategies used to solve word problems include using a checklist, mnemonic devices, or identifying key words.

It is a common belief among mathematics educators that if students "try hard enough" they will succeed. If this belief is true, then one way of getting students to try harder is through motivation. Increasing students' stamina or, in other words, their reluctance to give up on a problem can be accomplished with the use of real world problems. These problems need to be of interest to the students, so that they want to find the solution and seek to find alternative methods to solve a problem, if at first they do not succeed. Dunham's review of the research (1992) supports the fact that students who use graphing calculators were more willing to engage in problem solving, stayed with a problem longer, had more flexible approaches to problem solving, and solved non-routine problems inaccessible by algebra techniques. The reason why

students stayed with a problem longer may be due to the use of realistic problems more than the use of graphing calculators. However, without the use of graphing calculators, these applied problems, with their messy coefficients, may be very difficult, if not impossible to solve by hand. Therefore, it would seem that student motivation, the use of realistic problems, the use of graphing calculators, and more student interaction, are interrelated.

Garofalo (1987) suggests that metacognition also plays an important part in successful or unsuccessful learning activities. For instance, when students engage in mathematical problem solving they need to know when, how, and why they should explore a problem. However, metacognitive and other higher-order thinking can be undermined if students believe that mathematics is nothing more than computation and the memorization of algorithms and formulas (Garofalo, 1987). Schoenfeld (1988) gathered observational data indicating that students who believe that all problems can be solved in ten minutes or less will simply quit working on a problem after a few minutes. Therefore, mathematical stamina appears to be a function of the students' attitude, and the students' attitude is a function of how anchored the problem is to a real life situation. Graphical calculators remove the constraints which have required teachers and textbooks to concentrate on artificially nice examples and exercises.

Assessment

Teachers are faced with the task of fitting technology that specializes in connecting symbolic, numerical, and graphical representations into a curriculum that is driven by textbooks and tests made up of limited use of graphical and numerical situations (Slavit, 1996). The multiple representation approach to function has appeared as a central feature in many calculus reform projects; in turn, the assessment of student achievement is reflecting these changes

(Dunham & Dick, 1994). For example, starting with the 1995 Advanced Placement calculus examination, the College Board requires the use of a graphing calculator capable of at least numeric differentiation, numeric integration, and root finding. By analyzing the specific content of assessment items and students' responses, and by probing students' conceptual understanding, researchers can paint a more detailed picture of the effects of graphing calculator based instruction on students' learning (Dunham & Dick, 1994).

An important issue raised by the *Curriculum and Evaluation Standards* (NCTM, 1991) is to "align assessment methods with what is taught and how it is taught" (p. 110). Lehman (1992) employed several different teaching strategies to enhance students' understanding of mathematics, such as working in cooperative learning groups, giving presentations based on problems that were solved, and writing about solutions that were found. This innovative approach to teaching mathematics resulted in questioning whether traditional methods of assessment were sufficient for students who had been exposed to this teaching method and led to the development of a performance assessment as a final exam for a high school Algebra II class. Results of this innovative assessment indicated that some students performed surprisingly well and were able to explain concepts in detail. In contrast to drawing a blank on traditional tests, if the question was re-phrased students were able to do very well explaining a problem. He also found that students that usually did well on traditional tests did not do as well as expected, demonstrating an inability to explain concepts, but, instead, relying on memorized facts and simple computation. This is in agreement with the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), which states, "It is not enough for students to write the answer to an exercise or even to show all their steps. It is equally important that students be able to describe how they reached an answer" (p. 140). As stated in *Everybody Counts*, "In the long

run, it is not the memorization of mathematical skills that is particularly important – without constant use, skills fade rapidly – but the confidence that one knows how to find and use mathematical tools whenever they become necessary" (National Research Council, 1989, p. 60).

Instructional Design

"The purpose of instructional design is to assist the teacher in achieving the desired ends of instruction" (Henry, 1994, p. 42). In order to make an informed decision about the use of a computer algebra system or a graphing calculator, a systematic approach to instructional design should be in place, and it should have as its driving force the needs and goals of a particular learning situation. Cooperative learning is one of the most extensively studied and researched methods of instruction. Although research has shown that cooperative learning can produce positive results on achievement and attitude (Rysavy & Sales, 1991; Slavin, 1991), there does not seem to be agreement on the conditions that facilitate these results. Some factors that may affect results are reward structures (Klein, Erchul & Pridemore, 1993), gender differences (Rysavy & Sales, 1991), or age (Tudge, 1991). While results are not conclusive, homogeneous grouping is effective for high-ability students but not for low-ability students (Johnson & Johnson, 1986). Hooper (1992) suggests grouping based on learner characteristics.

Porzio (1997) found that having students solve problems designed to help them make connections between different representations of a concept, rather than having the connections pointed out to students by an instructor, appears to be a key component in the development of students' understanding of these concepts. This goes along with a concept that mathematics is not something you learn, but something you do (Clark & Lopez, 1994). Slavit (1996) found that the use of the graphing calculator was associated with higher levels of discourse in the

classroom, including higher level questioning by the instructor and more active learning behaviors by the students. Dunham and Dick in their review of research (1994) found a shift to fewer lectures by teachers and more investigations by students in graphing calculator classrooms.

Cooperative learning strategies have been credited with the promotion of critical thinking, higher level thinking, and improved problem solving ability of students. Current research that examines behaviors that occur during group problem solving sessions seem to indicate groups engage in monitoring their own thoughts, the thoughts of their peers, and the status of the problem solving process (Schoenfeld, 1987). Researchers who have studied cooperative learning at the college-level generally have found that students learn just as well as in more traditional classes and often develop improved attitudes toward each other and toward mathematics (Slavin, 1995). Mwerinde & Ebert (1995) studied the problem-solving behaviors of college students enrolled in a one semester college algebra and statistics course. Four instructional units of this course were chosen, two in which the students were assigned to cooperative learning groups and two in which the students worked independently. The findings suggest that students who worked in cooperative learning groups clearly exhibit important problem solving behaviors such as persistence and a willingness to explore alternative solutions; however, they still experience difficulty explicating the connections between mathematical actions and/or processes and the related mathematical concepts.

One reason why cooperative learning groups may increase student performance more than traditional methods is that cooperative learning groups sometimes cause situations where conflict is born between the students. Smith, Johnson, and Johnson (1981) report on a study in which they suggest that higher results on achievement and retention of the students in the "controversy group" may be attributed to the "cognitive rehearsal of their position and the attempts to

understand the opponent's position" (Smith, Johnson, & Johnson, 1981, p. 652). Hooper (1992) found activities that required mental effort, such as generating solutions to questions embedded in the lesson and explaining how a solution was determined, were positively and significantly correlated with success. Results of his study indicate that students who completed the instruction in pairs scored significantly higher on the post-test than students who completed the instruction alone. Furthermore, the relationship was consistent across ability groups.

Kanevsky (1985) found that competitive strategies show negative effects on student performance, because students are more worried about group rewards rather than helping a fellow student. Their attitudes toward working with others in social or academic settings reflected an increased interest in winning at the cost of cooperative behaviors. Results of Mevarechora's (1991) research showed that students who used computer assisted instruction (CAI) for drill and practice in pairs performed better than students who used the same program individually. In addition, the study showed that the individualistic and cooperative CAI methods equally affected students' mathematical self-concept, but the cooperative computer assisted instruction treatment alleviated math anxiety of low ability students more than the individual computer assisted instruction treatment.

CHAPTER III

Research Design and Procedures

Introduction of the study

The purpose of this study was to investigate the effect of the graphing calculator (namely the Casio 9850, Casio 9850GA Plus, or the TI-85) on college students' procedural skills and conceptual understanding that were enrolled in one of two different developmental mathematics courses. The developmental courses that were used in this study were Elementary Algebra (Math 90), and Intermediate Algebra (Math 93). This chapter includes a discussion about the setting, the instructors, the subjects, the instrumentation, and the way the data were analyzed.

The setting

A 15-week study was conducted at a small two-year college in Western Maryland over the course of the spring semester of 1999. The college has an enrollment of approximately 2,500 students. The student population consists of 66% female and 34% male. The average age of students at the college is 26, with the average age of females being 26 and the average age of males being 25. Approximately 50% of the students at the college are between the ages of 18 to 24. Ninety percent of the students are either first generation college students or low income students. Forty percent of the students come from low-income families. Approximately seventy percent of the students receive some sort of financial aid. Only three percent of the students are minority, and only three percent of the students are disabled. Eighty percent of the 18 to 24 year old students are in one of the developmental mathematics courses and ninety-five percent of the non-traditional students (age 25 or older) are in one of the developmental mathematics courses.

Placement into developmental mathematics courses is determined by the state cutoffs on the Compass Placement Test.

The college serves transfer students and students in career programs. Thirty-five percent of the students are transfer students, eleven percent of the students are transient students, and fifty-four percent of the students are enrolled in one of the college's two-year career programs. The college offers two-year degrees in a variety of fields including auto-tech, business, computer technology, criminal justice, culinary arts, dental hygiene, forest technology, hospitality management, human service, a variety of health fields, massage therapy, and office technology.

Authorization of the study

Authorization was sought and received from both the test-college and West Virginia University (WVU). The study was authorized at the test-college by the mathematics department chair and the president of the college. A copy of the written approval is on file with the researcher.

Approval for the study was also received from the WVU Institutional Review Board for the Protection of Human Subjects. The researcher typed a memo for each instructor to read to his/her class describing the purpose of the study and the fact that neither grades nor athletic standing would be influenced by refusing to participate in the study. The memo also stated that the results of the study would be kept confidential, and that the results of the study would not affect the student's grade nor athletic standing. (A copy of the memo is in Appendix D.)

The classes and the instructors

The math classes used in this study consisted of all the daytime Elementary and Intermediate Algebra classes offered at the main campus of the college. The researcher met with all the instructors prior to the semester to discuss if they would be using a graphing calculator or not. This decision was based on the instructor's knowledge of graphing calculators and their belief for the use of graphing calculators in Elementary and Intermediate Algebra. From these discussions, the researcher was able to find two instructors at the Elementary algebra level that were not going to allow students to use graphing calculators and three instructors that would require students to use graphing calculators. The researcher was also able to find two instructors at the Intermediate Algebra level that were not going to allow students to use graphing calculators and three instructors that would require students to use graphing calculators. Two of the instructors for the graphing calculator sections were instructors for both the Elementary and Intermediate algebra classes used in this study.

The instructors for the graphing calculator sections were well trained through workshops on the operation of the TI-85, the Casio 9850G, and the Casio 9850GA Plus. All the instructors of the graphing calculator classes were given a TI-85 and a Casio 9850G or a Casio 9850GA Plus, and the graphing calculator supplement written by the researcher. A copy of the graphing calculator supplement is in Appendix B. The instructors for the graphing calculator sections also required their students to buy one of these graphing calculators for their class.

If a student enrolled in one of these sections owned a different calculator other than the TI-85 or the Casio 9850 series, then that student was told that calculator would be either sufficient or insufficient for the course. For example, at least one student in the graphing calculator section previously owned a TI-82. Since this calculator does not have a polynomial solver nor a

simultaneous solver, these students were told that they would need to purchase a different calculator. None of the students in the graphing calculator sections previously owned a graphing calculator (such as a TI-86 or a TI-92) that was sufficient for the course other than the TI-85 or the Casio 9850 series. From post interviews with the instructors, and the post-test graphing calculator questionnaire, it was found that all the students in these courses did buy either the TI-85 or the Casio 9850G or Casio 9850GA Plus within the first two weeks of class.

The instructors for the non-graphing calculator sections were not familiar with the operation of graphing calculators and were not given one to use in their classes. Some of the non-graphing calculator instructors required students to show all pencil and paper steps that could not be performed on a scientific calculator and did not require their students to purchase a graphing calculator. However, all students in the non-graphing calculator sections were required to purchase a scientific calculator.

Students in the non-graphing calculator classes who bought a graphing calculator before the beginning of the semester were still required by the non-graphing calculator instructors to show all the pencil and paper steps required to solve a problem on the tests in the class. However, students in these classes were allowed to use their graphing calculator on the pre-test and post-test of this study. The students in the non-graphing calculator classes were not given any instruction in the class on the operation of the graphing calculator. It was found from the post-test graphing calculator questionnaire that only two of the students in the non-graphing calculator classes knew how to perform any operations on the graphing calculator except for those available on a scientific calculator. These two students were enrolled in one of the non-graphing calculator Intermediate Algebra sections, but had a teacher in Elementary Algebra who showed them how to use the Casio 9850G graphing calculator. Since these two students showed

that they knew how to use a graphing calculator on both the pre-test and the post-test, they were not included in the sample for either group.

The instructors for the courses for both the non-graphing calculator and graphing calculator groups have many years of successful teaching experience. Due to a difference in beliefs, some instructors involved in the study used graphing calculators regularly and required students to purchase a graphing calculator (either the TI-85, the Casio 9850G, or the Casio 9850GA Plus), while other instructors involved in the study did not utilize the graphing calculator and did not require students to purchase a graphing calculator. Each instructor was free to deliver the material in lecture format, cooperative learning groups, or the combination of various instructional designs. However, it was found from post interviews with the instructors that all the instructors generally used a lecture format with time for practice problems in class. Also, each instructor was free to emphasize procedural skills or conceptual understanding to the degree he or she desired. In addition, each instructor determined the amount of emphasis to place on applications. Each instructor was given a copy of the tests for this study prior to the start of the semester.

All instructors of Elementary Algebra used the textbook, *Elementary Algebra fifth edition* by Jerome Kaufmann. This textbook is very traditional and does not show the student any graphing calculator functions. In fact, the textbook does not even utilize the features of a scientific calculator such as the fraction key. All instructors of Intermediate Algebra used the textbook, *Intermediate Algebra for College Students fourth edition* by Allen Angel. This textbook is also very traditional and only instructs the student on features of graphing calculators that are not available on a scientific calculator on three pages at the end of three sections in the graphing unit and systems of equations unit. The text does not instruct the student on how to use

a graphing calculator to graph or to do any algebra procedure, merely that there are graphing calculators that can trace and zoom in on a graph. There is also a standardized course syllabus for both Elementary and Intermediate Algebra. (A copy of the course syllabi is in Appendix C.)

Subjects

All daytime Elementary and Intermediate Algebra sections at the main campus of the college were used in this study. The non-graphing calculator Elementary Algebra group was composed of students from three classes taught by two different instructors. The graphing calculator Elementary Algebra group was composed of students from three classes taught by three different instructors. The non-graphing calculator Intermediate Algebra group was composed of students from three classes taught by two different instructors. The graphing calculator Intermediate Algebra group was composed of students from three classes taught by three different instructors. There were a total of four non-graphing calculator instructors and four graphing calculator instructors.

The class sizes were supposed to be restricted by the college to a maximum of 20 students. However, some class sections had more than 20 students. The maximum number of students enrolled in a section was 24 and the minimum was 13. Both of these sections were in the non-graphing calculator group for Elementary Algebra. Table 1 shows the class size at the beginning of the semester for each of the sections.

Table 1

Class size at the beginning of the semester per section

Group	Section 1	Section 2	Section 3
Non-Graphing Calculator Elementary Algebra	13	19	24
Graphing Calculator Elementary Algebra	23	19	20
Non-Graphing Calculator Intermediate Algebra	18	17	16
Graphing Calculator Intermediate Algebra	15	18	15

The number of students in each of the sections had reduced by the end of the semester due to withdrawals. Table 2 shows the class size at the end of the semester for each of the sections.

Table 2

Class size at the end of the semester per section

Group	Section 1	Section 2	Section 3
Non-Graphing Calculator Elementary Algebra	9	15	20
Graphing Calculator Elementary Algebra	17	18	19
Non-Graphing Calculator Intermediate Algebra	18	17	16
Graphing Calculator Intermediate Algebra	15	12	13

A student's score was used in the study only if the student took the pre-test and post-test at the designated testing times and location. The subjects in this study took the pre-test before the end of the second week of classes and took the post-test during the last week of class. All the subjects took the pre-test and the post-test in the testing lab. Due to students not showing up for class after the withdraw period, lack of attendance, and lack of interest, not all students enrolled in the courses took both the pre-test and the post-test. Also, some instructors reminded students of taking the pre-test and post-test while others did not. Also, because students needed to take the post-test during the last week of class, when many instructors were giving final exams, caused the number of post-test scores to decrease. Table 3 shows the number of students from Elementary and Intermediate Algebra in the graphing calculator and non-graphing calculator sections that took both the pre-test and the post-test.

Table 3

Number of students used per section that took both the pre-test and the post-test

Group	Section 1	Section 2	Section 3	Total
Non-Graphing Calculator Elementary Algebra	5	10	11	26
Graphing Calculator Elementary Algebra	9	18	10	37
Non-Graphing Calculator Intermediate Algebra	10	7	2	19
Graphing Calculator Intermediate Algebra	6	4	7	17

Note. Two of the 19 students in the non-graphing calculator Intermediate Algebra group were not included in the study due to their graphing calculator ability.

Instrumentation

The instruments used to determine procedural skills and conceptual understanding were two researcher-generated tests for Elementary Algebra and Intermediate Algebra. The procedural questions on the tests were similar to the Compass Placement Test that were within the domain of this study. The conceptual questions on the tests were similar to the Core Learning Goals Test that were within the domain of this study. The test questions were reviewed and critiqued by a panel of five experts in the field of mathematics education who are familiar with the distinction between procedural skills, conceptual understanding, and the content of these courses. Three of the members of the panel were familiar with the features of these graphing calculators while two were not familiar with graphing calculators. After the researcher obtained the critiques of the tests, revisions were made to the assessments to insure that the tests were valid measures of assessing procedural skills, conceptual understanding, and unbias toward either graphing calculator users or non-users.

Neither the students nor the instructors had their pre-tests returned. Since the students did not have their pre-tests returned, and the time between pre-test to post-test was be more than 10 weeks, the pre-tests and the post-tests were nearly the same test.

Since the tests assessed a narrow scope of procedural skills and conceptual understanding, no standardized test could be found that only tests these specific skills. It was therefore necessary for the researcher to construct the tests for the study. The tests were scored with an analytical scoring rubric, so that students that did not get certain questions completely correct, would still obtain partial points for reaching a particular point in answering some of the

questions. A copy of the scoring rubric is in Appendix A. The same panel reviewed the grading of the tests for any inconsistencies or errors in scoring.

Students took the tests in the college's testing lab. Students presented photo identification to take tests in the testing lab to insure that the students were being tested. Similar to the Compass Placement Test, the tests were untimed. To reduce the affect of test fatigue, procedural and conceptual questions were dispersed throughout the tests. Typically, a conceptual question would follow a procedural question of the same nature. The Elementary Algebra (Math 90) test consisted of 11 questions. Questions number 1, 2, 5, 6, 8, 9, and 10 were procedural questions and questions 3, 4, 7, and 11 were conceptual questions. The point value of both the procedural and the conceptual sections of the Elementary Algebra Test were 14 points for each section. The Intermediate Algebra (Math 93) test consisted of the same first 11 questions as the Elementary Algebra Test, but included 7 additional questions. Questions 12, 14 and 15 were procedural questions, and questions 13, 16, 17, and 18 were conceptual questions. The point value of both the procedural and the conceptual sections of the Intermediate Algebra Test were 22 points for each section. (A copy of the tests and the point value for each question is in Appendix A.)

To insure the reliability of the tests, a Cronbach's Alpha test for reliability and internal consistency was performed on the procedural questions and the conceptual questions of the tests with Elementary and Intermediate Algebra students with and without graphing calculators. Alpha can range between 0 and 1. "If the measurement results are to be used for making a decision about a group or even for research purposes, a lower reliability coefficient (in the range of .30 to .50) might be acceptable" (Ary; Jacobs & Razavieh, 1985). If a scale has an alpha above .60, it is usually considered to be internally consistent (Mitchell, 1998). Table 4 shows the alpha level for each section of the tests for Elementary and Intermediate Algebra

Table 4

Chronbach alpha values for both sections of the test in Elementary and Intermediate Algebra

Group and section of test	Chronbach's Alpha	Standardized Alpha	Guttman Split-Half	Unequal length Spearman-Brown
Elementary Algebra Procedural	.7482	.7300	.6451	.6784
Elementary Algebra Conceptual	.8449	.8398	.8474	.8634
Intermediate Algebra Procedural	.8749	.8616	.9160	.9203
Intermediate Algebra Conceptual	.9245	.9134	.9390	.9451

Analysis of data

To answer the ten specific questions asked during the study, the following data analyses were done:

- Question 1: Is there a significant increase in procedural skills for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Elementary Algebra? Two paired *t*-tests were conducted comparing pre-test and post-test scores for the procedural questions of the tests to determine if there was statistically significant procedural skills gained for either the graphing calculator group and/or the non-graphing calculator group.
- Question 2: Is there a significant increase in procedural skills for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Intermediate Algebra? Two paired *t*-tests were conducted comparing pre-test and post-test scores for the procedural questions of

the tests to determine if there were statistically significant procedural skills gained for either the graphing calculator group and/or the non-graphing calculator group.

- Question 3: Is there a significant increase in the conceptual understanding and the ability to apply algebraic skills to real-life situations for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Elementary Algebra? Two paired *t*-tests were conducted comparing pre-test and post-test scores for the conceptual questions of the tests to determine if there was statistically significant conceptual understanding gained for either the graphing calculator group and/or the non-graphing calculator group.
- Question 4: Is there a significant increase in the conceptual understanding and the ability to apply algebraic skills to real-life situations for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Intermediate Algebra? Two paired *t*-tests were conducted comparing pre-test and post-test scores for the conceptual questions of the tests to determine if there was statistically significant conceptual understanding gained for either the graphing calculator group and/or the non-graphing calculator group.
- Question 5: Is there a significant increase in gains of procedural skills for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Elementary Algebra? An unpaired *t*-test was conducted first on the procedural questions from the pre-test scores for students that only took the pre-test and the post-test, to determine if there was any significant difference between the graphing calculator group and the non-graphing calculator group in procedural skills. An unpaired *t*-test was conducted on procedural questions from the post-test scores for students that only took the pre-test and the post-test. An analysis of covariance was also conducted to determine if there was statistically a significant increase in gains of procedural skills for the graphing calculator

classes versus the non-graphing calculator classes for students enrolled in Elementary Algebra.

- Question 6: Is there a significant increase in gains of procedural skills for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Intermediate Algebra? An unpaired *t*-test was conducted first on the procedural questions from the pre-test scores for students that only took the pre-test and the post-test, to determine if there was any significant difference between the graphing calculator group and the non-graphing calculator group in procedural skills. An unpaired *t*-test was conducted on procedural questions from the post-test scores for students that only took the pre-test and the post-test. An analysis of covariance was also conducted to determine if there was statistically a significant increase in gains of procedural skills for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Intermediate Algebra.
- Question 7: Is there a significant increase in gains of conceptual understanding and the ability to apply algebraic skills to real life situations for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Elementary Algebra? An unpaired *t*-test was conducted first on the conceptual questions from the pre-test scores for students that only took the pre-test and the post-test, to determine if there was any significant difference between the graphing calculator group and the non-graphing calculator group in conceptual understanding. An unpaired *t*-test was conducted on conceptual questions from the post-test scores for students that only took the pre-test and the post-test. An analysis of covariance was also conducted to determine if there was statistically a significant increase in gains of conceptual understanding for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Elementary Algebra.

- Question 8: Is there a significant increase in gains of conceptual understanding and the ability to apply algebraic skills to real life situations for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Intermediate Algebra? An unpaired *t*-test was conducted first on the conceptual questions from the pre-test scores for students that only took the pre-test and the post-test to determine if there was any significant difference between the graphing calculator group and the non-graphing calculator group in conceptual understanding. An unpaired *t*-test was conducted on conceptual questions from the post-test scores for students that only took the pre-test and the post-test. An analysis of covariance was also conducted to determine if there was statistically a significant increase in gains of conceptual understanding for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Intermediate Algebra.
- Question 9: Is there a significant difference between the non-graphing calculator classes and the graphing calculator classes in the proportion of students that successfully complete the Elementary Algebra with a grade of A, B, or C? A hypothesis test for the difference of two proportions was conducted.
- Question 10: Is there a significant difference between the non-graphing calculator classes and the graphing calculator classes in the proportion of students that successfully complete the Intermediate Algebra with a grade of A, B, or C? A hypothesis test for the difference of two proportions was conducted.

CHAPTER IV

Analysis of data

Introduction

This chapter covers the analysis of pertinent data that was collected during the project. The first section looks at the ten specific research questions. The second section looks at the results of the graphing calculator questionnaire.

Analysis of research questions

Question 1: Is there a significant increase in procedural skills for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Elementary Algebra?

It was found from the pre-test and the graphing calculator questionnaire that at the time of the pre-test, no students in either the non-graphing calculator or graphing calculator groups enrolled in Elementary Algebra knew how to perform any graphing calculator functions. Two paired t-tests (one-tailed) were conducted comparing pre-test and post-test scores for the procedural questions of the tests to determine if there were statistically significant procedural skills gained for either the graphing calculator group and/or the non-graphing calculator group. Table 5 shows the results from the pre-test and post-test for the Elementary Algebra procedural skill questions for the graphing calculator and non-graphing calculator groups.

Table 5

Elementary Algebra Procedural Pre-test to Post-test Results

Procedural skills in Elementary Algebra	Sample Size	Pre-test mean	Post-test mean	Difference mean	P-Value
Non-graphing calculator	26	1.6923	1.9231	-0.2308	.3190
Graphing Calculator	37	1.4594	6.5676	-5.108	<.0001

Therefore, there were significant gains in procedural skills in Elementary Algebra for the graphing calculator group but not for the non-graphing calculator group. Another noteworthy statistic is that the mode for both the non-graphing calculator and graphing calculator group on the procedural questions of the pre-test was a score of 2. The questions answered correctly most often were on solving a linear equation and completing a table of values.

Question 2: Is there a significant increase in procedural skills for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Intermediate Algebra?

It was found from the pre-test and the graphing calculator questionnaire that at the time of the pre-test, two of the students enrolled in one of the non-graphing calculator sections of Intermediate Algebra knew how to perform some of the graphing calculator functions namely the solver and the table menu. These students also showed that they retained knowledge on how to perform these graphing calculator functions on the post-test, even though they had not been shown how to perform these graphing calculator functions in class. Therefore, these two students were not used in the study.

Two paired *t*-tests (one-tailed) were conducted comparing pre-test and post-test scores for the procedural questions of the tests to determine if there were statistically significant procedural

skills gained for either the graphing calculator group and/or the non-graphing calculator group. Table 6 shows the results from the pre-test and post-test for the Intermediate Algebra procedural questions for the graphing calculator and non-graphing calculator groups.

Table 6

Intermediate Algebra Procedural Pre-test to Post-test Results

Procedural skills in Intermediate Algebra	Sample Size	Pre-test mean	Post-test mean	Difference mean	P-Value
Non-graphing calculator	17	4.4118	5.3529	-0.9412	.1601
Graphing Calculator	17	2.4706	10.7059	-8.235	<.0001

Therefore, there were significant gains in procedural skills in Intermediate Algebra for the graphing calculator group but not for the non-graphing calculator group. Another noteworthy statistic is that the mode for both the non-graphing calculator and graphing calculator group on the procedural questions of the pre-test was a score of 2. The questions answered correctly most often were on solving a linear equation and completing a table of values.

Question 3: Is there a significant increase in the conceptual understanding and the ability to apply algebraic skills to real-life situations for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Elementary Algebra?

Two paired *t*-tests (one-tailed) were conducted comparing pre-test and post-test scores for the conceptual questions of the tests to determine if there was statistically significant conceptual understanding gained for either the graphing calculator group and/or the non-graphing calculator group. Table 7 shows the results from the pre-test and post-test for the Elementary Algebra conceptual questions for the graphing calculator and non-graphing calculator groups.

Table 7

Elementary Algebra Conceptual Understanding Pre-test to Post-test Results

Conceptual Understanding in Elementary Algebra	Sample Size	Pre-test mean	Post-test mean	Difference mean	P-Value
Non-graphing calculator	26	1.2308	2.1154	-0.8846	.0575
Graphing Calculator	37	1.2973	6.4324	-5.135	<.0001

Therefore, there were significant gains in conceptual understanding in Elementary Algebra for the graphing calculator group at the .0001 α level and for the non-graphing calculator group at the .1 α level. Another noteworthy statistic is that the mode for both the non-graphing calculator and graphing calculator group on the conceptual questions of the pre-test was a score of 0. The questions answered correctly most often were questions 7A and 7B on applications of evaluating an expression and solving a linear equation.

Question 4: Is there a significant increase in the conceptual understanding and the ability to apply algebraic skills to real-life situations for the non-graphing calculator classes and/or the graphing calculator classes enrolled in Intermediate Algebra?

Two paired *t*-tests (one-tailed) were conducted comparing pre-test and post-test scores for the conceptual questions of the tests to determine if there was statistically significant conceptual understanding gained for either the graphing calculator group and/or the non-graphing calculator group. Table 8 shows the results from the pre-test and post-test for the Intermediate Algebra conceptual questions for the graphing calculator and non-graphing calculator groups.

Table 8

Intermediate Algebra Conceptual Understanding Pre-test to Post-test Results

Conceptual Understanding in Intermediate Algebra	Sample Size	Pre-test mean	Post-test mean	Difference mean	P-Value
Non-graphing calculator	17	4.4471	4.3529	.1176	.5424
Graphing Calculator	17	2.7647	12.5882	-10.05	<.0001

Therefore there were significant gains in conceptual understanding in Intermediate Algebra for the graphing calculator group but not for the non-graphing calculator group. The questions answered correctly most often were questions 7A and 7B on applications of evaluating an expression and solving a linear equation.

Question 5: Is there a significant increase in gains of procedural skills for the graphing calculator group versus the non-graphing calculator group for student enrolled in Elementary Algebra?

An unpaired *t*-test (two-tailed) was conducted first on the procedural questions from the pre-test scores only for students that took the pre-test and the post-test, to determine if there was any significant difference between the graphing calculator group and the non-graphing calculator group in procedural skills. Results of the pre-test scores for procedural skills in Elementary Algebra are in table 9.

Table 9

Pre-test score analysis for Procedural Skills in Elementary Algebra

Elementary Algebra Procedural	Sample Size	Mean	Standard Deviation	Minimum Score	Maximum Score	P-Value
Non-Graphing Calculator	26	1.6923	1.5689	0	4	.5426
Graphing Calculator	37	1.4595	1.4258	0	5	

Since there were no significant differences in pre-test scores, an unpaired *t*-test (one-tailed) was conducted on the post-test procedural skills in Elementary Algebra only for students that took both the pre-test and the post-test. Results of the post-test scores for procedural skills in Elementary Algebra are in table 10.

Table 10

Post-test score analysis for Procedural Skills in Elementary Algebra

Elementary Algebra Procedural	Sample Size	Mean	Standard Deviation	Minimum Score	Maximum Score	P-Value
Non-Graphing Calculator	26	1.9231	2.0961	0	6	<.0001
Graphing Calculator	37	6.5676	3.8912	0	14	

An analysis of covariance was also conducted to determine if there was statistically a significant increase in gains of procedural skills for the graphing calculator classes versus the

non-graphing calculator classes for students enrolled in Elementary Algebra. Results of the analysis of covariance gave a computed *F*-statistic of $F(1,62) = 34.45$, ($p < .005$).

There were no significant difference in pre-test scores in procedural skills in Elementary Algebra, yet the graphing calculator group scored significantly higher than the non-graphing calculator group in post-test procedural skills and the analysis of covariance showed significant results. Therefore, there is a significant increase in gains of procedural skills for the graphing calculator group versus the non-graphing calculator group enrolled in Elementary Algebra.

Question 6: Is there a significant increase in gains of procedural skills for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Intermediate Algebra?

An unpaired *t*-test (two-tailed) was conducted first on the procedural questions from the pre-test scores for students that only took the pre-test and the post-test to determine if there was any significant difference between the graphing calculator group and the non-graphing calculator group in procedural skills. Results of the pre-test scores for procedural skills in Intermediate Algebra are in table 11.

Table 11

Pre-test score analysis for Intermediate Algebra

Intermediate Algebra Procedural	Sample Size	Mean	Standard Deviation	Minimum Score	Maximum Score	P-Value
Non-Graphing Calculator	17	4.4118	2.5263	0	9	.0109
Graphing Calculator	17	2.4706	1.5459	0	6	

On further analysis a one tailed unpaired *t*-test was run on the pre-test scores to determine if the non-graphing calculator group scored significantly higher than the graphing calculator group. The results gave a p-value of .0055. Therefore, the non-graphing calculator Intermediate Algebra group scored significantly higher than the graphing calculator group on the pre-test in procedural skills at the .01 level. However, an unpaired *t*-test (one-tailed) was conducted on the post-test scores for procedural skills in Intermediate Algebra only for students that took the pre-test to determine if the graphing calculator group scored significantly higher than the non-graphing calculator group. Results of the post-test scores for procedural skills in Intermediate Algebra are in table 12.

Table 12

Post-test score analysis for Intermediate Algebra

Intermediate Algebra Procedural	Sample Size	Mean	Standard Deviation	Minimum Score	Maximum Score	P-Value
Non-Graphing Calculator	17	5.2359	4.4432	0	14	.0012
Graphing Calculator	17	10.7059	4.9720	4	22	

The non-graphing calculator group scored significantly higher than the graphing calculator group in pre-test scores in procedural skills in Intermediate Algebra, yet the graphing calculator group scored significantly higher t

Table 16

Post-test score analysis for Intermediate Algebra

Intermediate Algebra Conceptual	Sample Size	Mean	Standard Deviation	Minimum Score	Maximum Score	P-Value
Non-Graphing Calculator	17	4.3529	3.8233	0	14	<.0001
Graphing Calculator	17	12.5882	5.2328	3	21	

The non-graphing calculator group scored significantly higher than the graphing calculator group in pre-test scores in conceptual understanding in Intermediate Algebra, yet the graphing calculator group scored significantly higher than the non-graphing calculator group in post-test conceptual questions. An analysis of covariance was also conducted to determine if there was statistically a significant increase in gains of conceptual understanding for the graphing calculator classes versus the non-graphing calculator classes for students enrolled in Intermediate Algebra. Results of the analysis of covariance gave a computed F-statistic of $F(1,33) = 26.66$, ($p < .005$). Therefore, there is a significant increase in gains of conceptual understanding for the graphing calculator group versus the non-graphing calculator group enrolled in Intermediate Algebra.

Question 9: Is there a significant difference between the non-graphing calculator classes and the graphing calculator classes in the proportion of students that successfully complete Elementary Algebra with a grade of A, B, or C?

A hypothesis test for the difference of two proportions was conducted. Table 17 shows the results of the percentage of students that successfully completed Elementary Algebra.

Table 17

Successful completion rate for Elementary Algebra

Elementary Algebra	Sample Size	Number of success	Sample Percentage	P-Value
Non-Graphing Calculator	56	34	66.714%	
Graphing Calculator	62	42	67.741%	.4259

Question 10: Is there a significant difference between the non-graphing calculator classes and the graphing calculator classes in the proportion of students that successfully complete Intermediate Algebra with a grade of A, B, or C?

A hypothesis test for the difference of two proportions was conducted. Table 18 shows the results of the percentage of students that successfully completed Intermediate Algebra.

Table 18

Successful completion rate for Intermediate Algebra

Intermediate Algebra	Sample Size	Number of success	Sample Percentage	P-Value
Non-Graphing Calculator	51	34	66.667%	
Graphing Calculator	48	27	56.25%	.2868

Therefore, there is no significant difference between the non-graphing calculator classes and the graphing calculator classes in the proportion of students that successfully completed either Elementary or Intermediate Algebra with a grade of A, B, or C.

Results of the graphing calculator questionnaire

The interesting results from the graphing calculator questionnaire came from questions 6 and 7. Question 6 asked, "Do you feel you are better doing algebra with a graphing calculator or without a graphing calculator?" Question 7 asked, "Do you believe that students should use graphing calculators in math 90 and math 93?"

The results of the post-test graphing calculator questionnaire showed some interesting results. Most of the students that answered question 7 of the graphing calculator questionnaire in the non-graphing calculator Elementary Algebra group did not think that students in Elementary Algebra or Intermediate Algebra should be allowed to use graphing calculators. One student wrote, "To some degree I feel it is more beneficial to learn these methods without a graphing calculator so as not to be entirely dependent on a machine. The more difficult equations though would require a graphing calculator." Another student wrote, "Lower math teaches basics. You shouldn't be dependent on a calculator." Some of the students in the non-graphing calculator Elementary Algebra sections did write that they did think that students should be allowed to use graphing calculators. One student wrote, "Yes, students should be allowed to use graphing calculators so they can know how to use it down the road." Another student wrote, "Yes, it is another way of getting the answer. Isn't that going to be important in real life?"

Interestingly, every student that answered question 7 of the graphing calculator questionnaire in the non-graphing calculator Intermediate Algebra group answered that they believed that students should use graphing calculators in Elementary and Intermediate Algebra. One student wrote, "Yes, because I think that they should become familiar with it. Teachers should also so that they can help us." Many of the students commented that students should learn math first and then learn how to use the calculator.

In the Elementary Algebra graphing calculator group, all the students who answered question 7 of the graphing calculator questionnaire said that they did believe students should use graphing calculators in Elementary and Intermediate Algebra. Most of the students' comments were that it made the material easier to do and understand. One student wrote, "Yes, it makes it easier than doing the long way on paper. It also goes faster and you get further in the book." Another student wrote, " It is easier to plug in and less human errors. I also can go back and see where I made the mistake. In the higher level math classes they assume that you already know how to use the calculator. In these (developmental) classes you are taught how." Another wrote, "It is too complicated to learn so many sections in such a short time without a graphing calculator." Another student wrote, "I think you learn faster and it takes less time to do the math on the calculator."

Some of the comments from the graphing calculator Elementary Algebra group were not as positive for the graphing calculator. One student wrote, "Sometimes it is easier to do on paper and some problems you can't do with the Casio 9850G". Another student answered that she is much better at doing math with a graphing calculator since, "I can't do simple math problems without one." However, this student also showed in her post-test result that she was also unable to do math with a graphing calculator.

In the Intermediate Algebra graphing calculator group, all the students that answered problem 7 of the graphing calculator questionnaire said that they did believe that students should use graphing calculators in Elementary and Intermediate Algebra. Again the typical comments from students were that they made the math easier to do and understand, that it cut down on the memorization, and that they were able to cover more material in less time. A student wrote, "A graphing calculator simplifies the math so you can focus on formulas, rules, etc." Another

student wrote, "It helps you understand the problems, however, you should only be allowed to use the graphing calculator on the difficult problems." Some students wrote that students should learn how to do math both ways. Again, students wrote about their dependence on the graphing calculator. One student wrote, "I am better at doing math with a graphing calculator, because without it my algebra stinks."

CHAPTER V

Summary, Conclusions, and Recommendations

Introduction

This chapter first summarizes the entire study and states conclusions and implications related to the individual questions asked and the graphing calculator questionnaire. The second section makes suggestions for teaching with a graphing calculator. The chapter concludes with recommendations from this study and recommendations for further study.

Summary

This study compared the gains in procedural skills and conceptual understanding for students in non-graphing calculator and graphing calculator classes enrolled in developmental algebra. The developmental algebra courses used in the study were Elementary Algebra and Intermediate Algebra. The study took place over a 15-week semester at a small two-year college. All students in the graphing calculator sections were required to buy either the Casio 9850G, Casio 9850GA Plus, or the TI-85 graphing calculator. Both the control group (non-graphing calculator group) and the experimental group (graphing calculator group) were taught with the same goals. The goals of these courses are that students perform better on both procedural and conceptual algebra problems. The graphing calculators used in this study use numerical techniques to approximate algebraic solutions. The students in the graphing calculator groups were allowed to use the graphing calculator in place of performing paper and pencil procedures in class and on all tests. Students in the control group classes did not have a graphing calculator in the class or on the tests, although all students were required to have access to a scientific calculator.

All the instructors of the graphing calculator sections were well trained in the operation of these graphing calculators. All daytime Elementary and Intermediate Algebra sections at the main campus of the college were used in this study. The non-graphing calculator Elementary Algebra group was composed of students from three classes taught by two different instructors. The graphing calculator Elementary Algebra group was composed of students from three classes taught by three different instructors. The non-graphing calculator Intermediate Algebra group was composed of students from three classes taught by two different instructors. The graphing calculator Intermediate Algebra group was composed of students from three classes taught by three different instructors. There was a total of four non-graphing calculator instructors and four graphing calculator instructors.

Results of the pre-test to post-test analysis showed that there were statistically significant gains in procedural skills and conceptual understanding for both the Elementary and Intermediate Algebra graphing calculator sections ($p < .0001$). However, the only significant gains for any of the non-graphing calculator groups were in conceptual understanding for the Elementary Algebra group ($p < .1$).

Significant results were also found in comparing the amount of gain in procedural skills and conceptual understanding for the non-graphing calculator and graphing calculator sections. It was found that there were significant increases in gains of procedural skills and conceptual understanding for the graphing calculator group compared to the non-graphing calculator group enrolled in Elementary and Intermediate Algebra ($p<.005$). There were no significant differences in the success ratio for the non-graphing calculator and graphing calculator groups in either Elementary or Intermediate Algebra. This would therefore seem to imply that, with the graphing calculator, students can have a higher level of conceptual understanding without any

significant difference in the percentage of students that successfully complete the course. Also, students using graphing calculators can have a higher level of procedural skills than students not using graphing calculators if students are allowed to use the graphing calculator on tests.

Other noteworthy statistics should be reported. The mode for the graphing calculator and non-graphing calculator groups in Elementary and Intermediate Algebra on procedural skills was a score of 2. Also, the mode for the graphing calculator and non-graphing calculator groups in Elementary and Intermediate Algebra on conceptual understanding was a score of 0. This means that, even though many of these students had taken algebra in high school, most could only remember very little procedural skills and had basically no conceptual understanding in algebra. All of the students in the Elementary Algebra groups, both non-graphing calculator and graphing calculator, reported they had never used a graphing calculator before.

The results of the two students in the non-graphing calculator Intermediate Algebra group that previously knew how to use a graphing calculator were mixed. One student scored an eleven on the procedural questions of the pre-test and a nine on the conceptual pre-test questions. This student maintained the score of eleven on the procedural questions on the post-test, but the student's conceptual score fell to a five on the post-test. The other student's score rose on the post-test on both procedural skills and conceptual understanding.

Another interesting statistic is that the graphing calculator Elementary Algebra group's mean on both the procedural and the conceptual questions was higher than the Intermediate Algebra non-graphing calculator group. This was not a significant difference in procedural skills. However, the graphing calculator Elementary Algebra group did score significantly higher than the non-graphing calculator Intermediate Algebra group on the conceptual questions ($p < .1$). These results would question which group is more prepared for College Algebra.

The results of the graphing calculator questionnaire seem to point out that students that used the graphing calculator thought that students should be allowed to use the graphing calculator, and even students that did not use the graphing calculator thought that students should be allowed to use a graphing calculator in developmental algebra. Students gave reasons for the use of graphing calculators that many researchers have commented on. Students commented that graphing calculators made the math easier to do and understand, that it cut down on the memorization, and that they were able to cover more material in less time. Since it takes much less time to learn how to do the procedures on a graphing calculator, much more time may be spent on understanding the concepts.

However, students did comment on their dependence on the graphing calculator. This calls into question whether the graphing calculator should be used to supplement students' procedural skills by having students learn both paper and pencil methods along with graphing calculator methods, or should the graphing calculator be used to supplant students' procedural skills, by only showing students how to perform procedures on a graphing calculator. This question could not have been analyzed during this study, since the graphing calculators used could not perform certain procedures (solving literal or absolute value equations, solving inequalities, factoring, simplifying expressions and exponents, or finding exact algebraic solutions to quadratic equations). Therefore, all the students in the graphing calculator groups for this study needed to learn some paper and pencil skills. With the availability of low-cost algebraic manipulating calculators, such as the Casio 9970G, Casio FX2 and the TI-89, the question of where to supplant or where to supplement procedural skills should be studied.

Suggestions for teaching with graphing calculators

Since some studies have shown significant results using graphing calculators and others have not, the differences may be in the utilization of the graphing calculator and the methods used to teach with the graphing calculator. The following suggestions are for successful teaching in order for students to learn how to utilize a graphing calculator for both procedural skills and conceptual understanding.

1. The graphing calculator must be used in nearly every class session by the instructor, the students, and required in the class.
2. The instructor must require students during the first week of class to do problems on quizzes and or homework in which students are able to understand the importance and power the graphing calculator has on learning. This may include getting the equation of a line that best fits three or more datapoints, or having the students complete a long table of values for an equation with non-integer coefficients.
3. The college catalog and course description must say that a graphing calculator is required for the course, with reference to what specific calculators will be required.
4. The cost of the graphing calculator should be kept to a minimum. Therefore the TI-92 should not be used. However, algebra manipulating graphing calculators such as the Casio9970G, The Casio FX2, and the TI-89 are affordable.
5. Since it is clear from this study that, unless a teacher knows how to use a graphing calculator, the students will not bother with the features of a graphing calculator even if they own one, it is therefore necessary that the instructor knows how to use every graphing calculator that the students are allowed to use.

6. Since the manuals of graphing calculators show many features that students do not need to know at the developmental algebra level, teachers should make copies available of instructions for the graphing calculators that students are allowed to use. A copy of the graphing calculator supplement is in appendix B. The graphing calculator appendix is also available over the Internet at http://www.ac.cc.md.us/~marks and in the books *Elementary and Intermediate Algebra Concept and Models* by Mark A. Shore, and *College Algebra Concepts and Models* by Mark A. Shore. Also teachers should use books that have problems that require the use of a graphing calculator throughout the text.
7. The graphing calculators that are used must be at least at the level of a TI-82 or Casio 9850G. In order to have a polynomial solver or a simultaneous solver, a minimum of a TI-85 must be required of a *Texas Instruments* calculator. In order not to have to set linear equations equal to zero and to reduce the syntax, a minimum of a Casio 9850GA Plus must be required from a Casio calculator
8. The number of different calculators that the teacher will instruct the students on using must be kept to a minimum so the instructor does not have to show every procedure on many different calculators. Since the TI-85 does not have a method of producing tables of values, does not link the statistics program to the graphing program, finds the equation of a line in the form $y = bx + a$ and an exponential equation in the form $y = ab^x$, and operates very differently than a Casio, a multi-purpose program was written by Steve Fairegrieve, a part-time instructor at the college, at the end of the study to take care of these problems.
9. The class should focus on concepts rather than procedures. For example, instead of showing how to solve a system of equations by substitution, elimination, matrices, determinants, graphically by hand and on a graphing calculator, and using the simultaneous solver on the

graphing calculator, the instructor may want to show only one method of solving a system of equations by hand, and how to solve the system graphically and with the simultaneous solver.

10. The mathematics department of the college will need to decide if certain procedural skills should be de-emphasized or even eliminated, such as factoring, simplifying radicals and rational expressions. The instructors must also decide where to supplement a student's paper and pencil skills with a graphing calculator and where to supplant paper and pencil skills with the graphing calculator. When should students use a graphing calculator to solve an equation, after they know how to solve the equation by hand, or before? Perhaps the question should be, do students need to learn how to solve an equation by hand?

Recommendations from the study

The obvious recommendation from the study is that graphing calculators should be required in Elementary Algebra and Intermediate Algebra. Even with the statistical backing of this paper and numerous other research articles showing the usefulness of graphing calculators, there will still be some faculty, which for some reason disagree with the use of graphing calculators in developmental algebra. The reason may be due to their unwillingness to change, their belief that students should not rely on technology, the cost of the graphing calculator, or how the use of the graphing calculator in developmental algebra will affect students' ability in College Algebra and higher level math.

Certainly there will be faculty who are unwilling to use graphing calculators due to the fact that they are merely unwilling to change from a paper pencil driven curriculum. I have personally talked to some faculty that are unwilling to use a graphing calculator because they do

not know how to use one, and are unwilling to spend the time to learn how to use a graphing calculator, or attend workshops on graphing calculators. Another reason why some faculty are unwilling to incorporate the graphing calculator is that they see algebra as the procedures. For these teachers, if a calculator does the procedures for the student, than there is nothing for the student to learn. Clearly some teachers who believe this have avoided applications of algebra which anchor students' conceptual understanding within real-life situations. Results from the pre-test of this study show that students in Elementary Algebra retained nearly nothing that they were taught in high school algebra without a graphing calculator. However, the two students that did learn how to use a graphing calculator in Elementary Algebra and went on to take Intermediate Algebra in a non-graphing calculator section, retained their knowledge of how to do algebra on a graphing calculator.

A further reason why some faculty are unwilling to change, is due to the fact that without a graphing calculator they can maintain their dominance over the subject material and they do not have to worry about students approaching a problem from a different point of view (either graphical or numerical). Since without a graphing calculator, every student is solving an equation in basically the same manner, they therefore do not need to adapt their teaching from a pure lecture mode. Some faculty may also be unwilling to change due to their success rates without a graphing calculator. This is the area in which the faculty and administration need to be careful with how they define success. If a student is successful in developmental algebra yet is not successful in College Algebra, some faculty and administrators count that as being a success for the teacher in the developmental algebra course and a failure for the teacher in the College Algebra course.

But is this the case, or has the developmental algebra instructor passed a student, yet fail to prepare that student for College Algebra? McGowen (1999) examined the ability of students in a graphing calculator group and non-graphing calculator group to maintain or improve their grades in the subsequent course after completing a developmental algebra course. She found that the percent of students in the graphing calculator group who successfully maintained or improved their grade in a subsequent course was significantly greater than the percent of students that maintained or improved their grades in the non-graphing calculator group. The study indicates that 58 percent of the graphing calculator students maintained or improved their grades, while only 36 percent of the non-graphing calculator students maintained or improved their grade ($p < .0005$). McGowen (1999) found the most significant differences were with students that received a C in Beginning Algebra. She found that 72 percent of the students that received a C in the graphing calculator Beginning Algebra group went on to maintain or improve their grade in Intermediate algebra, where as 45 percent of those students in the non-graphing calculator group that received a C went on to maintain or improve their grade in Intermediate Algebra ($p < .01$).

If students are learning more in developmental algebra courses where graphing calculators are required, why aren't there significant differences in the success rates for developmental algebra classes that use graphing calculators compared to those that do not use graphing calculators? I believe that the difference is in what is being tested in these developmental courses. Since one of the reasons some faculty do not like using graphing calculators in developmental algebra is that they do the procedures for the students, many of the questions on the tests from these developmental classes are procedure oriented. Also, since the faculty that use graphing calculators realize that the graphing calculator does do the procedures,

many of the questions on their tests are on applications and conceptual understanding. I think both sets of teachers need to be careful of some difficulties that may incur. Teachers not using graphing calculators in developmental algebra, need to be aware of the research that has shown that students can be more successful in subsequent math courses if they use a graphing calculator in developmental algebra. Teachers using graphing calculators in developmental algebra, need to be aware of the research that shows conceptual understanding and problem solving skills take a longer time to develop than simply the procedural skills that they may have been use to teaching. They may therefore need to slow their teaching down and allow more time for students to try to solve problems themselves before being shown. If both non-graphing calculator classes and graphing calculator classes were given the same tests that counted toward their grade in the course, as the results from this study shows, the graphing calculator group would have a significantly higher success rate than the non-graphing calculator classes.

The fear that students will rely heavily on technology should not be a fear since it is already a fact. For several years some students that come to college have been relying on a scientific calculator with a fraction key anytime they need to add fractions or decimals. Students today accept the fact that technology does some of the grunt work. As I sit here at my computer dictating my dissertation to a computer with voice recognition, spell checking, and grammar checking, so I can FTP the file to the Internet in Pdf format, I wonder how people were able to do this without these technologies. We have come to accept technology in the fact that we probably used some form of technology to get to work or light a room, so why should mathematics be taught as if technology does not exist?

With many students on financial aid, the cost of a graphing calculator could still be considered as a problem. However, if a college has the graphing calculator as a requirement in

their college catalog, the student may use financial aid money to buy the graphing calculator at the college bookstore. Usually though, the price will be inflated from that at a local Wal-Mart, and some students may use up their financial aid before they buy their calculator. To alleviate these financial fears, faculty and administrators need to realize that a few students sell their graphing calculators. If teachers would post these used calculators on bulletin boards students would be able to get a used graphing calculator for a reduced price. However, when students see the power and the many uses of a graphing calculator, many are unwilling to sell the calculator, even if they never plan to take another math or science course.

Texas Instruments has also set up a system in which students with financial problems can get a free TI-85 for the semester by calling 1-800-TI-CARES. Also the teacher or the instructional assistance center could sign calculators out to students that can not afford one. Some students simply borrow a graphing calculator from a student that had the course, but is unwilling to sell their graphing calculator. Since the cost of a graphing calculator is a burden to some students, the college needs to have this requirement in the college catalog so students can start saving for the calculator before the first day of class. Whereas the cost of a textbook is usually for one semester, the cost of the calculator can be thought as covering several math and science courses. It is interesting to note that most students sell their math books, yet very few ever sell their graphing calculators.

Finally the question of how the use of the graphing calculator in developmental algebra will affect students' ability in College Algebra and higher level math. As discussed earlier, McGowen (1999) found that the percent of students in the graphing calculator group who successfully maintained or improved their grade in a subsequent course was significantly greater than the percent of students that maintained or improved their grades in the non-graphing

calculator group. But what about students that go from Beginning Algebra to a non-algebraic college level math? Results from this study show that the success rates for students in the non-graphing calculator Elementary Algebra group were 66.7 percent, while the success rates for students in the graphing calculator Elementary Algebra group was 67.7 percent. Therefore, the graphing calculator did not reduce the success rates for students in Elementary Algebra. The graphing calculator in Elementary Algebra affects more than the algebra skills or the success rates of students in these courses. The graphing calculator causes a shift in instructor toward facilitator and a shift in student from passive to active involvement in the classroom. McGowen (1999) and many other researchers cited in this dissertation have found significant changes in attitudes and significant shifts in students' self evaluation of their abilities to do mathematics in favor of students that use graphing calculators in Elementary Algebra. Even if students go on to a non-algebraic college level math course, the increase emphasis on problem solving skills and the added time that the graphing calculator allows an instructor to spend on problem solving will help in any mathematics course.

In fact some universities have argued just the opposite. Some believe that it is fine for students to use graphing calculators if they are not going on to Calculus, but if students are going on to Calculus, they should not be allowed to use a graphing calculator in College Algebra. However, Palmiter (1991) showed that knowledge of calculus concepts for students using *Macsyma*, a computer algebra system similar to the Casio 9970 or TI-89, was significantly higher than for students taught using traditional instruction. Judson (1990) investigated the effect of *Maple*, a computer algebra system, on students' understanding of the concepts in a business calculus course and found no statistical differences in achievement between the control and experimental groups. However, motivation, interest, and class participation were markedly

higher in the experimental group than in the control group. Dunham (1992) examined the pre-test to post-test scores on a calculus readiness test and found that students receiving graphing calculator instruction attained sufficient calculus placement scores at nearly twice the rate of those receiving traditional instruction. The multiple representation approach to function has appeared as a central feature in many calculus reform projects; in turn, the assessment of student achievement is reflecting these changes (Dunham & Dick, 1994). For example, starting with the 1995 Advanced Placement calculus examination, the College Board requires the use of a graphing calculator capable of at least numeric differentiation, numeric integration, and root finding.

As the pre-test results showed from this study most students retained essentially nothing from high school algebra in which a graphing calculator was not used. Perhaps this statistic can be reversed, and both procedural skills and conceptual understanding can be learned and retained from high school with the aid of graphing calculators in high school. Currently, many high schools are using graphing calculators in some math classes. However, often these are the higher-level math classes, such as Trigonometry and Calculus. I believe that this study also shows the need for students in lower level math classes, such as Algebra I, to use graphing calculators.

Recommendations for further study

The following questions are topics that could be considered for further study.

1. This study centered on the use of graphing calculators. Are there similar results when using technologies other than graphing calculators, for example, spreadsheets or on-line computer aided instruction?

2. This study was conducted over a 15-week period, with students taking the post-test at the completion of the course. Would the results be different if the post-test was given months after the completion of the course?
3. This study was with college students in developmental algebra. Would the results be different if the students were high school students in either Algebra I or Algebra II?
4. For this study students used either the Casio 9850G, Casio 9850GA Plus, or the TI-85, and therefore needed to learn some pencil and paper skills. Would the results be different if the students used either the Casio 9970G, Casio FX2, or the TI-89 algebra manipulating calculators and were not taught any paper and pencil skills?
5. Although this study found a significant increase in gains of procedural skills and conceptual understanding for both the graphing calculator Elementary Algebra and Intermediate Algebra groups compared to the non-graphing calculator groups, it is not known what specific procedural skills and conceptual abilities were enhanced the most by the graphing calculator?
6. This study focused on students at only one college. Would results be the same for students at another college?
7. This study used teachers in the graphing calculator classes that knew how to use a graphing calculator and believed in letting students use graphing calculators. Would the results be the same for teachers that are required to show students how to use graphing calculators, yet the instructor does not believe in their use?
8. Would the elimination of certain topics from the algebra curriculum, such as factoring, simplifying radicals, rational expressions, and exponents, affect the problem solving ability of the student?

9. Are there significant differences between students that were taught developmental algebra using a graphing calculator compared with students not using a graphing calculator in their performance on procedural skills and conceptual understanding in College Algebra, Trigonometry, Calculus, and higher level math?
10. How does the use of graphing calculators affect student performance on standardized placement tests and problem solving tests?
11. Are the teaching methods of an instructor changed with the use of graphing calculators, and does the teacher's belief of what should be taught in an algebra course change with the use of graphing calculators?
12. Do the results of this study hold true for students of different socio-economic background, race, gender, or math ability?
13. This study did not consider hemispheric (right mode/left mode) preference. A future study might include those preferences as well as learning styles and strengths.
14. This study did not compare students that used a Casio compared with students that used a *Texas Instruments* Calculator. Does the type of graphing calculator make a difference in either procedural skills or conceptual understanding?

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Appendix A

Elementary Algebra Test

Intermediate Algebra Test

Graphing Calculator Questionnaire

Scoring Rubric

Elementary Algebra test

You are allowed to use your calculator on the test.

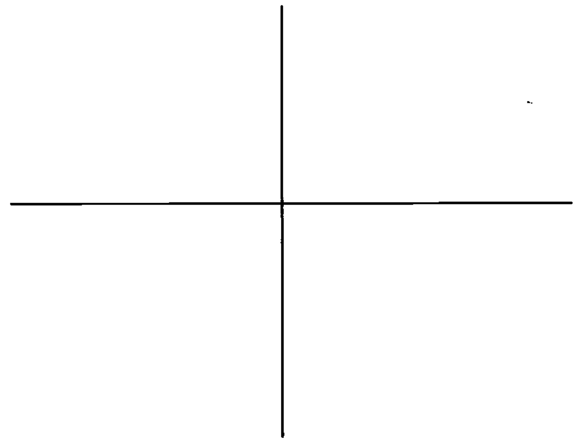
1. Solve the equation $C = .01x - 1.3$ if $C = 2.4$. (2 points procedural)

2. Find the equation of the line that goes through the points (-2,-3) and (0,5). (2 points procedural)

3. The value of a computer when it was put on the market was $2,200. Three months later, the value of the computer had decreased to $1,975. If we assume the decrease in value is linear, find the linear equation that describes the value of the computer in terms of the number of months the computer has been on the market. (2 points conceptual)

4. The cost of the TI-85 the year it was first put on the market was $150. Every year since then, the cost of the TI-85 calculator has dropped $5. Write the linear equation that gives the cost of the TI-85 calculator in terms of the number of years it has been on the market. (2 points conceptual)

5. Graph the equation $4x - 3y = 12$ and label the coordinates of the x and y intercept.
(2 points procedural)



6. What is the slope of the line whose equation is $4x - 3y = 12$?
(2 points procedural)

7. The value of a Pentium computer is dependent upon the number of months it has been on the market. The value of the computer can be determined by the linear equation $v = -50t + 2400$ where v is the value of the computer in units of dollars, and t is the number of months the computer has been on the market. (1 point each conceptual)

A. According to the equation $v = -50t + 2400$ what is the value of the computer after it has been on the market for 6 months?

B. According to the equation $v = -50t + 2400$ when will the computer have a value of \$2,000?

C. According to the equation $v = -50t + 2400$ when will the computer have no value?

D. For the equation $v = -50t + 2400$ what does the -50 tell you about the value of the computer in relation to the number of months the computer has been on the market?

E. For the equation $v = -50t + 2400$ what was the value of the computer when it was initially put on the market?

F. Graph the equation $v = -50t + 2400$ and on your graph indicate the points that answer questions 7C and 7E.

8. Complete the table for the equation $y = -4x + 7$ (2 points procedural)

x	y
-1	
0	
1	

9. Solve the system of equations (2 points procedural)
$4x - 3y = 18$
$y = 4x - 14$

10. Graph the equations $y = 2x + 6$ and $y = -4x$ and give the coordinate of their intersection point. (2 points procedural)

11. The value of a Pentium computer can be determined by the equation $v = -50t + 2400$ and the value of a AMD computer can be determined by the equation $v = -40t + 2000$. For both equations v is the value of the computer in units of dollars, and t is the number of months the computer has been on the market.

A. How many months must the two computers be on the market until they have the same value? What is that value? Explain how you determined your answer.
(2 points conceptual)

B. Graph both of the equations and show the point that answers problem 4A.
(2 points conceptual)

12. Did you use a graphing calculator on this test?

If so, circle the graphing calculator functions you used on this test.

1. Equation Solver
2. Polynomial Solver
3. Simultaneous Solver
4. Statistics menu
5. Table menu
6. Graphing features (specify the graphing features)______________________________
7. Any other graphing calculator function (specify)______________________________

Intermediate Algebra test
You are allowed to use your calculator on the test.

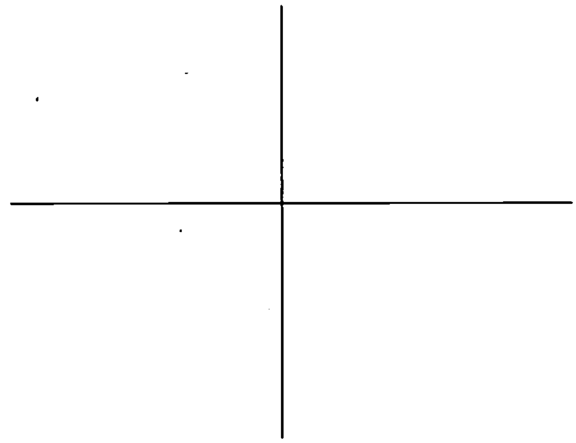
1. Solve the equation $C = .01x - 1.3$ if $C = 2.4$. (2 points procedural)

2. Find the equation of the line that goes through the points (-2,-3) and (0,5). (2 points procedural)

3. The value of a computer when it was put on the market was $2,200. Three months later, the value of the computer had decreased to $1,975. If we assume the decrease in value is linear, find the linear equation that describes the value of the computer in terms of the number of months the computer has been on the market. (2 points conceptual)

4. The cost of the TI-85 the year it was first put on the market was $150. Every year since then, the cost of the TI-85 calculator has dropped $5. Write the linear equation that gives the cost of the TI-85 calculator in terms of the number of years it has been on the market. (2 points conceptual)

5. Graph the equation $4x - 3y = 12$ and label the coordinates of the x and y intercept. (2 points procedural)



6. What is the slope of the line whose equation is $4x - 3y = 12$? (2 points procedural)

7. The value of a Pentium computer is dependent upon the number of months it has been on the market. The value of the computer can be determined by the linear equation $v = -50t + 2400$ where v is the value of the computer in units of dollars, and t is the number of months the computer has been on the market. (1 point each conceptual)

A. According to the equation $v = -50t + 2400$ what is the value of the computer after it has been on the market for 6 months?

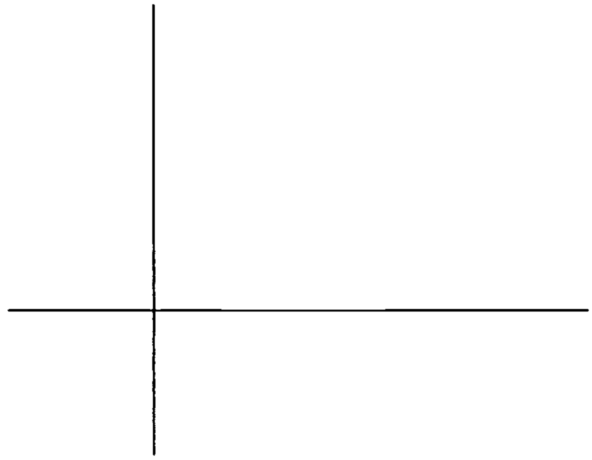
B. According to the equation $v = -50t + 2400$ when will the computer have a value of $2,000?

C. According to the equation $v = -50t + 2400$ when will the computer have no value?

D. For the equation $v = -50t + 2400$ what does the -50 tell you about the value of the computer in relation to the number of months the computer has been on the market?

E. For the equation $v = -50t + 2400$ what was the value of the computer when it was initially put on the market?

F. Graph the equation $v = -50t + 2400$ and on your graph indicate the points that answer questions 7C and 7E.

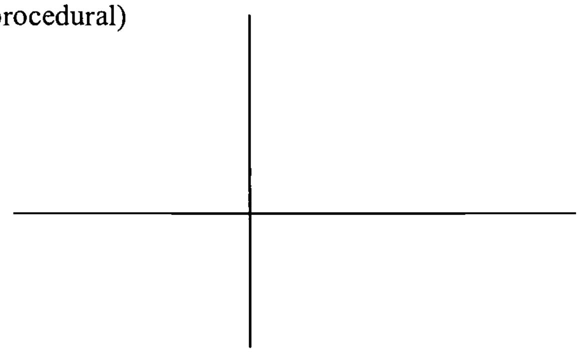


8. Complete the table for the equation $y = -4x + 7$ (2 points procedural)

x	y
-1	
0	
1	

9. Solve the system of equations (2 points procedural)
$4x - 3y = 18$
$y = 4x - 14$

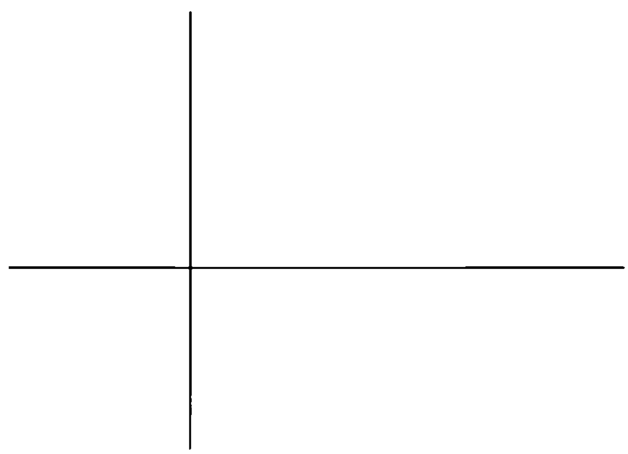
10. Graph the equations $y = 2x + 6$ and $y = -4x$ and give the coordinate of their intersection point. (2 points procedural)



11. The value of a Pentium computer can be determined by the equation $v = -50t + 2400$ and the value of a AMD computer can be determined by the equation $v = -40t + 2000$. For both equations v is the value of the computer in units of dollars, and t is the number of months the computer has been on the market.

A. How many months must the two computers be on the market until they have the same value? What is that value? Explain how you determined your answer.
(2 points conceptual)

B. Graph both of the equations and show the point that answers problem 4A.
(2 points conceptual)



12. Solve the system of equations (2 points procedural)

$3x - 4y + z = -1$
$4x - 5y = 3$
$-x + 5y - 3z = 12$

13. Suppose that you have \$150,000 to invest. You decide to invest the money in three investments options: AA bonds yielding an annual interest rate of 12%, Utility stock yielding an annual interest rate of 8%, and a Bank Certificate yielding an annual interest rate of 4%. How should the investment of the \$150,000 be allocated to obtain \$16,000 per year in interest, if the total amount allocated to the stock and Bank Certificate option is \$30,000?
(2 points conceptual)

14. Complete the table and graph the equation $Y = 2x^2 - x$ (2 points procedural)

x	y
-1	
0	
1	

15. Solve the equation $2x^2 + 5x = 12$ (2 points procedural)

16. The value of a computer is dependent upon the number of months it has been on the market. The value of the computer can be determined by the quadratic equation $v = -1.25t^2 + 2000$ where v is the value of the computer and t is the number of months the computer has been on the market.

A. Complete the table for the equation $v = -1.25t^2 + 2000$. (1 point conceptual)

t	v
0	
20	
50	

B. According to the equation $v = -1.25t^2 + 2000$ when will the computer have no value? (1 point conceptual)

17. Find the minimum point on the graph of the equation $y = x^2 + x - 6$ (2 points procedural)

18. The model $C = 20t^2 - 200t + 640$, describes the concentration "C" of bacteria per cubic centimeter in a body of water t days after treatment to control bacterial growth.

a. Find the day(s), to the nearest hundredth, when the concentration of bacteria will be 150 per cubic centimeter. (2 points conceptual)

b. Find to the nearest hundredth of a day when the concentration of bacteria will be at its minimum level. Explain how you got your answer. (2 points conceptual)

19. Did you use a graphing calculator on this test?

If so, circle the graphing calculator functions you used on this test.

1. Equation Solver
2. Polynomial Solver
3. Simultaneous Solver
4. Statistics menu
5. Table menu
6. Graphing features (specify the graphing features)______________________
7. Any other graphing calculator function (specify)______________________

Student graphing calculator knowledge and usage questionnaire

1. Do you know how to use a graphing calculator?

If so, explain how to use the following graphing calculator functions on your graphing calculator. For example, write the form the equation needs to be in and give the buttons you would need to press to perform the following functions.

A. How do you solve linear equations on your graphing calculator?

B. How do you solve quadratic equations on your graphing calculator?

C. How do you solve systems of equations on your graphing calculator?

D. How do you graph equations on your graphing calculator?

E. How do you find x intercepts, y intercepts, or points of intersection on your graphing calculator?

F. How do you find equations of lines on your graphing calculator?

G. How do you make tables of values on your graphing calculator?

2. What type of graphing calculator did you use for your math class?

3. How many weeks have you used your graphing calculator?

4. Approximately what percent of class time did you use your graphing calculator to do algebra?

5. Were you shown in class how to use your graphing calculator to do algebra?

6. Do you feel you are better doing algebra with a graphing calculator or without a graphing calculator? Why do you feel this way?

7. Do you believe that students should use graphing calculators in math 90 and math 93? Why?

8. Approximately what percent of class time did you spend working in groups or working with others in your class on problems during class?

9. Approximately what percent of class time did you spend working on applied problems or word problems?

10. Approximately what percent of class time did you spend working on non-word problems or non-applied problems?

Scoring Rubric

For the Problems worth 2 points, students needed to have the correct answer to receive both points. For the Problems worth 1 point, students needed to have the correct answer to receive a point for that question. There was no partial credit given on problems worth 1 point.

A student would receive 1 point on a 2 point problem if they either had the correct steps in solving the problem but made a arithmetic mistake, or if they answered half of the question correct. For example a student would receive 1 point on problem 1 if they went through the paper and pencil steps to solve the linear equation, but made an arithmetic mistake. A student would receive 1 point on problem 2, 3, or 4 if they only found the correct slope. A student would receive 1 point on question 5 if they only found either the x or y intercept. There was no partial credit given on problems 6, 7, 8, 10, 11, 16, or 18b. A student would receive 1 point on question 9 or 12 if they found either the x, y, or z part of the solution. A student would receive 1 point on problem 13 if they set up the equations correctly. A student would receive 1 point on problem 14 if they either completed the table or graphed the equation. A student would receive 1 point for problem 15 or problem 18a if they only gave 1 of the correct solutions. A student would receive 1 point for problem 17 if they either gave the x or y part of the coordinate for the minimum point.

Appendix B

Graphing Calculator Supplement

1. ON and OFF-To turn on the calculator press the **AC/^{on}** key. To turn the calculator off, press the **SHIFT** key then the **AC/^{on}** key. The **SHIFT** key causes the calculator to do the function in yellow above the key.

2. **SHIFT**-To get any of the functions in yellow, you must press the **SHIFT** key first.

3. **ALPHA**-To get any of the letters in red, you must press the **ALPHA** key first.

4. **MENU**-Use the **MENU** key to go to the different menus. Once you are at the main menu, you can press the number associated with the menu you want to go into (for example **RUN** is 1) or use the arrow keys to highlight the menu you want to enter.

5. **EXE**-Use the **EXE** key in blue to get the solution to a problem after you type it in.

6. **Negative numbers and Subtraction** -To type in negative numbers use the **(-)** negative key located below the + key. This key is for entering negative numbers. The – key, located to the right of the + key, is for subtraction.

   Example- Turn on the calculator and then press 1 for the **RUN** menu. To type in -5 – (-2) type in the negative key,**(-)**, then 5, then the subtraction key,-, then the left parenthesis key,(, then the negative key **(-)**, then 2, then the right parenthesis key, **)**, then **EXE**. You should get -3.

7. **F1-F6**- These are your function keys. They activate a function on the screen directly above the function key. If **F6** has an arrow above it, this means that there are more choices. You can then press **F6** to see the rest of the choices.

8. **EXIT**- To exit out of a function menu press the **EXIT** key. The **EXIT** key will take you back one level of functions.

   Example – Turn on the calculator and then press 1 for the **RUN** menu. Now press the options button **(OPTN)** next to the yellow **SHIFT** button. Now you should see words on the screen above the function keys. For example, on the screen above **F3** is **CPLX** (complex numbers). Press the **F3** button, and now you have a submenu of choices under complex. For example above **F2** you should see **Abs** (absolute value). Press the **F2** button and you will see Abs on the screen. Now finish typing in the rest of this expression: **Abs(-5)** Remember to use the negative key **(-)** under the + sign and not the subtraction key. Now press **EXE** and you should have an answer of 5. Now press the **EXIT** key once to go back to the previous menu and press **EXIT** again to get out of all the menus. If you would like leave the Run menu and go to the Main Menu, press the Menu key.

9. **Totally Stuck**- If you ever get stuck and turning off the calculator doesn't help, use the reset button on the back of the calculator. Just put a writing pen in to reset the memory.

The RUN menu

To get to the RUN menu, turn the calculator on then press the number 1.

1. **Negative numbers and Subtraction** -To type in negative numbers use the (-) negative key located below the + key. This key is for putting in negative numbers. The – key is for subtraction it is located to the right of the + key.

Example To type in -5 – (-2) type in the negative key,(-), then 5, then the subtraction key,-, then the left parenthesis key,(, then the negative key (-), then 2, then the right parenthesis key,), then EXE. You should get -3.

2. **Fractions**- Use your fraction key, $a\frac{b}{c}$, to put in fractions.

Example to add the fractions, $\frac{-5}{4}+5\frac{3}{10}$, just type in (-)5 $a\frac{b}{c}$ 4 + 5 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 10 then EXE. You should get $4\frac{1}{20}$.

3. **Parenthesis**- You will need to use parenthesis any time there is parenthesis in the problem or if you have more than one term in the numerator, denominator, or exponent.

Example If you want to find the answer to $\frac{5+3}{8+3\div 2}$, you will need to put parenthesis around the numerator and parenthesis around the denominator. So you should type in $(5+3)\div(8+3\div 2)$. You should get .8421.

4. X^2-To square a number, type in the number then press the X^2 key, then EXE.

Example to get 5^2, type in 5 then press the X^2 key, then EXE. You should get 25.

5. $\sqrt{\ }$ To get the square root of a number or a variable, press the square root key (above the X^2 key) and type the number or variable you want to take the square root of.

Example if you want to get the square root of 30, press the square root key (SHIFT X^2) then type in 30, then EXE. You should get 5.477.

6. ^To raise a number or a variable to any power, put in the letter or the number then press the ^ (power key), then type in the power that you want to raise the number or variable to.

Example to get 5^{10}, type in 5^10, then EXE. You should get 9765625. So you could have got 5^2 by either using the squared key or the power key.

Example- If you want to find the answer to $\left(\frac{3}{4}\right)^{\frac{-2}{3}}$ type in $\left(\frac{3}{4}\right)$^$\left(\frac{-2}{3}\right)$ You should get 1.211. Notice you need parenthesis around the base and the exponent.

7. $^{x}\sqrt{\ }$ To get any root of a number or a variable, type in the root, then press the X root key (above the ^ key) and type the number or variable you want to take that root of.

Example if you want to get the 4th root of 30, $\sqrt[4]{30}$, type in 4, then press the X root key (SHIFT ^) then type in 30, then EXE. You should get 2.340. You can check this by raising your answer, 2.340347319 to the 4th power. Just type in ^4 then EXE. You should see Ans^4. (Ans means the answer you had on the previous line). You should get 30 when you press EXE.

8. **ANS-** Use the **ANS** key (SHIFT then the negative key) to get the answer you had on the previous line.

9. **Editing your expressions and equations-**If you type in something wrong you can use your arrow keys to edit your expressions. If you have already hit EXE and want to edit an expression, then use your left arrow to get you back to the end of the expression, you can then arrow to the part of the expression you want to edit. If you have already hit EXE and want to edit your expression, you can use the right arrow to get to the start of your expression. You can overwrite a symbol, or you may need to delete or insert something.

10. **DEL-**Use the DEL (delete) key to delete any part of your expression.

11. **INS-**Use the INS (insert) key (SHIFT then DEL) to insert something into your expressions.

12. **Editing expressions farther back-** To edit expressions used earlier, Press the ON button, to clear the screen, then use the up arrow to get to the expression you want to edit. Then use the left or right arrow to make changes.

13. **Substituting values into expressions and equations-**To have your calculator substitute values into an equation, you need to use the : COLON function.

Example- To put -5 in for f and 15 in for v in the equation $w = 91.4 - \dfrac{(10.45 + 6.69\sqrt{v} - .45v)(91.4 - f)}{22}$ type in -5 $\rightarrow$ f:15 $\rightarrow$ v: $91.4 - (10.45 + 6.69\sqrt{v} - .45v)(91.4 - f) \div 22$ The $\rightarrow$ key is the key above the ON key. To get the letters use the alpha key. To get the COLON function: press SHIFT then VARS. Then F6 (for more choices). Then F5 (for the **:**. You should get – 38.346.

14. **EXIT-** To exit out of a function menu press the EXIT key. The EXIT key will take you back one level of functions.

15. **Scientific Notation-**There are two ways to calculate scientific notation on your calculator.

Example-To find the answer to $\frac{6.024 \text{ X } 10^{-24}}{8.23 \text{ X } 10^{-8}}$ type in 6.024 EXP-24 ÷ 8.23 EXP-8. You should get 7.319E –17. This means 7.319 X 10^{-17} in scientific notation, or expanded out .00000000000000007319. You could have also type it in using either the ^ key or the $\mathbf{10^x}$ key, but you will need to put parenthesis around the numerator and the denominator. To find the answer to $\frac{6.024 \text{ X } 10^{-24}}{8.23 \text{ X } 10^{-8}}$ without using the EXP key, type in the following (6.024 X 10^-24) ÷ (8.23 X 10^-8) then EXE. You should get the same answer as before, 7.319E –17.

16. **The X key-** You **must** use the X key (X,θ,T) when you are solving or graphing equations.

17. **Solving linear equations- NOTE:** The solver function will only work with equations that have one solution (linear, logarithmic, and exponential). It will not work for quadratic or cubic equations. To solve these see EQATIONS MENU #1 on the next page.

To solve linear equation on the **Casio 9850gA plus** go to the EQUATION menu. Choose SOLVER (F3) then type in the equation. The equals sign is SHIFT then decimal point. Then press EXE, then press F1 for solve.

To solve linear equations on the **Casio 9850g**, you need to set the equation equal to 0. Then on the calculator make sure you are in the RUN menu. Then press the OPTN (options) key, then press the F4 key (calculate), then press the F1 key (Solve). The word Solve will show up. Then type in your equation (you must use the (X,θ,T) key for the independent variable no matter what the letter is in the equation you must use the the (X,θ,T) key. Do not type in = 0. After you type in your equation press the**,** key. Then type in the number 1, then close the parenthesis, then EXE.

Example- To solve the equation 2x – 5 = 10 on the Casio 9850 ga Plus, just go to the Equation menu, Press F3 for Solver, then just type in the equation, press EXE, then F1 for SOLVE. To solve the equation 2x – 5 = 10 on the Casio 9850g, first set the equation equal to 0, and you get 2x – 15 = 0. Now press (OPTN) then F4, then F1. You should now have the word Solve up on the screen with left parenthesis. Then type in the left side of your equation $solve(2x-15,1)$ then EXE. You should get 7.5. You wouldn't even had to combine the like terms,. For example you could have typed in $solve(2x-5-10,1)$ then EXE

18. **Imaginary and Complex numbers-** To work with imaginary and complex numbers, press Options (OPTN), then F3 for complex, then above F1 is the number **i** when you need it. However you can **not** use the ^ key with imaginary numbers. However the x^2 key will work.

Example- To find $(2i+3)^3$ type in (2i +3)(2i +3)(2i +3) then EXE. You should get -9 + 46i. Or you could get the answer to $(2i+3)^2$ then on the next line type in X (2i +3).

19. **Absolute value-** To get the absolute value. press (OPTN), then F6 for more choices, then F4 for numbers, then F1 for abs (absolute value).

20. **Log-** Log base 10 key. To find the $\log_{10}(1000)$ type in log(1000). You should get 3

21. **ln-** Natural log key. To find the $\log_e(1000)$ type in ln(1000). You should get 6.9077

22. **e^x-** To get the e^x key, press SHIFT then the natural log key (ln).

Example- To solve $50 = 100e^{(-2t)}$ go to the solver, then type in solve(100e(-2x)-50,1) then EXE. You should get .34657. Notice that with the e^x key you **do not use the ^ key.** Note: with the Casio 9850ga Plus you could go to the equation menu and use the solver and you would not have to set the equation equal to 0.

The TABLE menu on the Casio 9850
To get to the TABLE menu, turn the calculator on then press the number 7.

The table menu is for calculating tables of values. To make a table, you must use the **X,θ,T** key for the independent variable (x). For example, if wanted to type in the equation y = 2x + 5, you would only type in 2x + 5. After you type in the equation, press F5 for **RANG**. You will then need to enter the **START** of the values you want to use for x, the **END** of the table, and the **PITCH**. The pitch determines if your table is increasing in units of ones, tens, tenths, or whatever you choose. You must press **EXE** after each change. Then press **EXE** again to get out of the Range menu, then press F6 for **TABLE**. You can use the **arrow keys** to scroll through your table. You can also type in a value for *x* right into the table. For example, if you wanted to get a table of values for the equation y = 2x + 5 for the following x values;

x values	y values determined by the equation y = 2x + 5
0	
2	
4	
6	

you would go to the **TABLE** menu. Type in 2x + 5 (don't for get to use the **X,θ,T** key for x). Press the **EXE** key. Press F5 for **RANGE**. Type in the **START** of your x values, which is 0, then **EXE**. Type in the **END** of your x values, which is 6, then **EXE**. Then type in the **PITCH**, which is 2, then **EXE**. The press the **EXE** key again to leave the **RANGE** area. Then press the F6 key for **TABLE**.

The EQUATION menu

1. **Solving Quadratic and Cubic equations-**To solve quadratic or cubic equations, go to the equation menu choose polynomial **F2**, make sure the equation is set equal to 0 and is in standard form. Type in your coefficients hitting **EXE** after each one. Then **F1** for solve.

Example- To solve the quadratic equation $(2x-3)^2 + 5 = x + 3$ you must first put the equation in standard form. Expanding $(2x-3)^2$ we get $4x^2 -12x + 9$. So we now have $4x^2 -12x + 9 + 5 = x + 3$. Moving all the terms to the left side of the equation we have, $4x^2 -12x + 9 + 5 - x - 3 = 0$. Combining like terms we have, $4x^2 -13x + 11 = 0$. Now we are ready to put this equation into our calculator. Go to the EQUATION menu, chose Polynomial, F2, then F1 for 2nd degree. Now type in the coefficients of your equation. Type in 4 **EXE** then -13 **EXE** then 11 **EXE** then F1 for Solve. You should get 1.625 + .3307i and 1.625 - .3307i. This means both solutions to this equation are complex numbers. In other words, this equation has no real solution.

2. **Solving Systems of Equations-**To solve systems of equations, go to the equation menu choose Simultaneous F1, make sure the equations are in standard form (ax + by = c). Chose the number of unknowns. Type in your coefficients hitting **EXE** after each one. Then **F1** for solve.

Example- to solve the system of equations:

$x + 2y - 4 = -3Z$
$2y - 3z = 7$
$5y + 4x = -2$ Go to the equation menu. Choose Simultaneous (**F1**). Then choose the number of unknowns (**F2** for 3 unknowns) then type in your coefficients pressing **EXE** after each one. You need to enter a coefficient for every variable in every equation. So, it is worth it to rewrite the system of equations above as, $x + 2y +3z = 4$, $0x + 2y - 3z = 7$, $4x + 5y + 0z = -2$Now type in 1 EXE 2 EXE 3 EXE 4 EXE 0 EXE 2 EXE -3 EXE 7 EXE 4 EXE 5 EXE 0 EXE -2 EXE. The press **F1** for solve. You should get x = -5.727 y = 4.1818 and z = .4545.

3. **Solver-** The Casio 9850GA Plus and Casio 9970 have a solver under the equations menu. Go to the equations menu, then press (**F3**) for solver. Then simply type in your equation (the equals sign is **SHIFT** then decimal point). Then **F1** for solve. The Solver will only give you one solution to an equation, so do not use the Solver for quadratic or cubic equations. You can use the Solver for linear, exponential and logarithmic equations.

Note: If you are using the Casio 9850G see #17 under the run menu directions.

The GRAPH menu

To get to the GRAPH menu, press the menu key, then press the number 5.

1. **Entering equations you want to graph-** The equations need to be solved for Y, and you must use X, the (X,θ,T) key, as the independent variable. You do not type in the Y part of the equation, since it is already there. Just type in the right side of your equation, then **EXE**. You can now press **F6** to Draw the graph, or **F1** to select or deselect the graph (when the graph is selected the equals sign will be highlighted), or **F2** to Delete the equation.

2. **Zoom-** Use the zoom key to zoom **OUT**, or zoom **IN** (this will zoom in on the location of the cursor) or zoom **AUTO** (this will get some part of your graph showing).

3. **V-Window-** Use v-window to change your viewing window and scale, you must hit **EXE** after each change you make. It's generally a good idea to have your Ymax and Xmax be positive and your Ymin and Xmin to be negative by about the same amount so you can see the x and y axis. Scale just lets the calculator know how much each hash mark is worth on the x and y axis. You can also set the viewing window to standard. Once you press the V Window button, you can press (**F3**), STD, for standard viewing window. The standard viewing window sets the x max and y max to 10, and the x min and y min to –10.

   Example- If you want to graph $Y = x^2 + 20$, you will need to type in the equation, then press **F6** for draw, You won't see the graph because it is out of the standard viewing window. You can then Zoom **AUTO**, Zoom **OUT**, or adjust your viewing window as needed.

4. **TRACE-** Trace is for tracing your graph. When your graph is on the screen, press **TRACE** then hold down the right arrow until the cursor shows up. If you use **ZOOM IN** now, you will zoom in on the area that the cursor is on.

5. **G-SOLVE- G SOLVE** is for finding x intercepts (roots), Maximums, Minimums, Y intercepts (Y-ICPT), points of intersection (ISCT), calculating y values (Y-CAL) and calculating x values (X-CAL). However, to find any of these, they need to show on your viewing window.

The CONICS menu

Use the conics menu to find vertex points of parabolas.

To get to the CONICS menu, press the menu key, then press the number 9.

Choose the fourth type of conics equation ($y = ax^2 + bx + c$). Enter your A, B, and C, then **F6** for draw. In the conics menu it does not matter if the graph shows on the screen. Then press **F5** for **G-solve**. Then **F4** for VTX (vertex).

The STAT menu on the CASIO 9850

The STAT menu is for plotting points and finding lines and curves of best fit.

To get to the STAT menu, turn the calculator on and press the number 2

1. **F6-** If there is an arrow on the screen above the **F6** key, that means that there are more choices for the function keys. Use **F6** to see the other choices. If you press **F6** again you will get back to your original choices for the function keys.
2. **DEL-** If you press the **F6** key, then you will get to more choices. One choice is **DEL** above the **F3** key. Use **DEL** to delete an entry from a list. Note: you can not use the **DEL** key

next to the **AC^{on}** button to delete an entry from a list.

3. **DELA-** Use **DELA** above the F4 key to delete all the entries in a list.

4. **Lists-** List 1 is for your x values (independent variable) and list 2 is for your y values (dependent variable). You will need to press **EXE** after each entry. You can use **F6** to get more choices, of which one is **DEL** for deleting an entry or **DELA** for deleting all the entries of a list.

5. **Graph-** If you do not see **GRAPH** on the screen above **F1**, then use **F6** for more choices. After you input your datapoints, press **GRAPH (F1)**.

6. **Gph1-** Once you press **GRAPH**, you have the choices of Gph 1, Gph 2, etc. Gph 1 (**F1**) is set up to graph the datapoints in list 1 and list 2. List 1 is for your x values (horizontal variable) and List 2 is for your y values (vertical variable).

7. **X-** Once you press **Gph1** you will see your datapoints plotted. You can now find the linear equation of best fit of the form $y = ax + b$ (where a is the slope and b is the y intercept) by pressing (**F1**). The reason X is above **F1**, is because for a linear equation the independent variable, x, is to the first power. The graphs of linear equations are lines.

8. **DRAW-** Once you get the equation of the line of best fit, you can press **F6** for **DRAW**, to draw the graph of the equation. You will be able to see how well your model fits the datapoints. You can then press **EXIT** to get back to your lists.

9. **COPY-** Once you get the equation of the line of best fit, you can press **F5** for **COPY**, then press **EXE**. This copies your equation to the graph and table menu. You can then press the menu key and go to the graph menu or table menu to do further analysis of the graph. For example, once you get the equation in the **STATS** menu, you may want to copy the equation and go to the graph menu to find the roots, y-intercept, or some other feature of the graph.

Examples Using STAT menu on the CASIO 9850

Example 1- Let's say we know that 32 degrees Fahrenheit is equal to 0 degrees Celsius, and 212 degrees Fahrenheit is equal to 100 degrees Celsius. However, we do not know the conversion formula from Fahrenheit to Celsius. To get the linear conversion formula we need at least two points. We could set up our points (F,C). So, from the fact that 32 degrees Fahrenheit is equal to 0 degrees Celsius, we get the point (32,0) and from the fact that 212 degrees Fahrenheit is equal to 100 degrees Celsius, we get the other point (212,100). To find the equation of the line that fits the points (32,0) and (212,100) go to the **STAT menu** and enter the Fahrenheit values on list1 and their associated Celsius values for list2.

LIST 1	LIST 2
32	0
212	100

Now press **F1** for Graph, then **F1** for Gph1 (your data points should now be plotted), then **F1** again for X (for the line of best fit) and you will get $a = .555555$, $b = -17.7777$ and r = 1 (we will not need to use the r) and the general equation of a line $y = ax + b$. So the equation of the line of best fit is $y = .5555x - 17.7777$. You can now either Draw the line (**F6**) or copy the line to the Graph menu or Table menu (**F5**).

Example 2- Let's say you want to find the equation of the line (linear equation) that best fits the points (5,3) (4,2) and (1,-3). Go to the **STAT** menu and enter the x values on list1 and the y values for list2. Then press F1 for Graph, then **F1** for Gph1 (your data points should now be plotted), then F1 again for X (for the line of best fit) and you will get $a = 1.538$ $b = -4.46$ and r = .9962 and the general equation of a line $y = ax + b$. So the equation of the line of best fit is $y = 1.538x - 4.46$. You can now either Draw the line (**F6**) or copy the line into the Graph menu (**F5**).

The Matrix menu

1. **Entering a matrix-**To enter a matrix, go to the matrix menu, then use the right arrow to get to the word (NONE). When you arrow over, the word NONE is replaced with 0 X 0. Replace the first 0 with the number of rows in your matrix and the second 0 with the number of columns in your matrix, then **EXE**.

2. **Operations with matrices-**To add subtract or multiply the matrices you made, go to the RUN menu, then press (OPTN) and F2 to access the matrix operations functions.

Example- If you want to multiply the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ go to the matrix menu. Go to matrix A. Then use the right arrow to go to the word none. Replace the first 0 with a 3 and the second 0 with a 3 then EXE. Then put in your matrix elements. Then press EXIT. Then go to matrix B. Then use the right arrow to go to the word none. Replace the first 0 with a 3 and the second 0 with a 1 then EXE. Then put in your matrix elements. Then press EXIT. Now go to the RUN menu, then press (OPTN) and F2 to access the matrix operations functions. Then press the F1 key for Mat, then ALPHA A then the multiplication key. Then F1 for Mat, then ALPHA B, then EXE. You should get the matrix

How to use the TI-85

1. ON and OFF-To turn on the calculator press the on key in the lower left corner. To turn the calculator off, press the yellow 2nd key then the on key. The yellow 2nd key causes the calculator to do the function in yellow above the key.

2. 2ND-The yellow 2nd button on the upper left hand side causes the calculator to perform the function in yellow above each key. You must press and release this button every time you want to use a function written in yellow

3. ALPHA-To get any of the letters in blue, you must press the **ALPHA** key first.

4. ENTER-Use the **ENTER** key in the lower right hand corner to get the solution to a problem after you type it in.

5. Negative numbers and Subtraction -To type in negative numbers use the (-) negative key located below the 3 key. This key is for putting in negative numbers. The – key is for subtraction .

Example To type in -5 – (-2) type in the negative key,(-), then 5, then the subtraction key,-, then the left parenthesis key,(, then the negative key (-), then 2, then the right parenthesis key,), then **ENTER**. You should get -3.

6. Fractions- If the fraction is a mixed number you must first write it as an improper fraction. For example if the fraction was $-2\frac{3}{5}$ you would have to write this as $\frac{-13}{5}$. To enter fractions you must manually enter the division sign. You then need to press the 2nd key, then the multiplication sign for the math menu, then F5 for Misc, then MORE for more choices, then F1 for FRAC. To get rid of the menus at the bottom of the screen hit EXIT until they disappear.

Example to add the fractions, $\frac{-5}{4}+5\frac{3}{10}$, just type in $(-5 \div 4) + (53 \div 10)$ Before you press ENTER, press the 2nd, then MATH, then F5 for MISC, then MORE, then F1 for FRAC, then ENTER. You should get 81/20 which is equal to $4\frac{1}{20}$.

7. Parenthesis- You will need to use parenthesis any time there is parenthesis in the problem or if you have more than one term in the numerator, denominator, or exponent.

Example If you want to find the answer to $\frac{5+3}{8+3\div 2}$, you will need to put parenthesis around the numerator and parenthesis around the denominator. So you should type in $(5+3)\div(8+3\div 2)$ You should get .8421.

8. X^2-To square a number, type in the number then press the X^2 key, then ENTER.
Example to get 5^2, type in 5 then press the X^2 key, then ENTER. You should get 25.

9. $\sqrt{\ }$ To get the square root of a number or a variable, press the square root key (above the X^2 key) and type the number or variable you want to take the square root of. **Example** if you want

to get the square root of 30, press the square root key (2nd X^2) then type in 30, then ENTER. You should get 5.477..

10. Power Key ^ To raise a number or a variable to any power, put in the letter or the number then press the ^ (power key), then type in the power that you want to raise the number or variable to.

Example to get 5^{10}, type in 5^10, then ENTER. You should get 9765625. So you could have got 5^2 by either using the squared key or the power key.

Example- If you want to find the answer to $\left(\frac{3}{4}\right)^{\frac{-2}{3}}$ type in $(3 \div 4)\wedge(-2 \div 3)$ You should get 1.211. Notice you need parenthesis around the base and the exponent.

11. $\sqrt[x]{\ }$ To get any root of a number or a variable, type in the root, then press the multiplication sign to enter the math menu, then press F5 for MISC, then press MORE, then F4 for $\sqrt[x]{\ }$, then enter the number you want to take the root of.

Example if you want to get the 4^{th} root of 30, $\sqrt[4]{30}$, type in 4, then 2nd, then MATH, then F5, then MORE, then F4, then 30, then ENTER. You should get 2.340. To get rid of the menus at the bottom of the screen hit EXIT until they disappear. You can check this by raising your answer, 2.340347319 to the 4^{th} power.

12. Editing your expressions and equations-If you type in something wrong you can use your arrow keys to edit your expressions. If you have already hit ENTER and want to edit an expression, press 2nd then ENTRY (ENTER).

13. DEL-Use the DEL (delete) key to delete any part of your expression.

14. INS-Use the INS (insert) key (2ND then DEL) to insert into your expressions.

15. Substituting values into expressions and equations-To have your calculator substitute values into an equation, you need to use the ALPHA key, the STO key and the **:** colon key (2nd then the decimal point).

Example- To put -5 in for f and 15 in for v in the equation

$w = 91.4 - \frac{(10.45 + 6.69\sqrt{v} - .45v)(91.4 - f)}{22}$ type in -

$5 \to f{:}15 \to v{:}\,91.4 - (10.45 + 6.69\sqrt{v} - .45v)(91.4 - f) \div 22$ Press -5 then STO then ALPHA f, then 2nd : then 15 then STO then ALPHA v, then 2nd : then type in the equation, then ENTER.

You should get –38.346.

16. EXIT- To exit out of a function menu press the EXIT key. The EXIT key will take you back one level of functions.

17. Scientific Notation-There are two ways to calculate scientific notation on your calculator. **Example**-To find the answer to $\frac{6.024 \text{ X } 10^{-24}}{8.23 \text{ X } 10^{-8}}$ type in 6.024 EE-24 ÷ 8.23 EE -8 then ENTER. You should get 7.319E –17. This means 7.319 X 10^{-17} in scientific notation, or expanded out .00000000000000007319. You could have also typed it in using either the ^ key or the **10^x** key, but you will need to put parenthesis around the numerator and the denominator. To find the answer to $\frac{6.024 \text{ X } 10^{-24}}{8.23 \text{ X } 10^{-8}}$ without using the EE key, type in the following (6.024 X 10^-24) ÷ (8.23 X 10^-8) then ENTER. You should get the same answer as before, 7.319E –17.

18. The X-VAR key- You **must** use the X key (**X-VAR**) when you are solving or graphing equations.

19. Solving linear equations- To solve linear equations on the TI85, or any equation that only has one solution, you need to go to the solver function (2nd then GRAPH). After you do that, the letters **eqn:** will show up. Then type in your equation. You must use the **X-VAR** key for the independent variable no matter what the letter is in the equation. Then ENTER. Some numbers will come up that say right and left, but that is not your solution. You then need to press F5 for solve, and you will get the answer. **NOTE:** The solver function will only work with equations that have one solution (linear, logarithmic, and exponential). It will not work for quadratic or cubic equations. To solve these see EQATIONS #1 on the next page. **Example-** Solve the linear equation $2x + 4 = 10$. Use the solver function (2nd GRAPH). After you enter the equation hit ENTER, then F5 for solve. You should get $x = 3$

20. Imaginary and Complex numbers- Complex numbers are entered (real , imag) using the comma key located below the x^2 key.

21. Absolute value- To get the absolute value, press 2nd, MATH, F1 (for NUM), then F5 (for abs), then whatever you want to take the absolute value of, then ENTER.

22. Log- Log base 10 key. To find the $\log_{10}$ (1000) type in log(1000). You should get 3

23. ln- Natural log key. To find the $\log_e$ (1000) type in ln(1000). You should get 6.9077

24. e^x- To get the e^x key, press 2ND then the natural log key (ln).

TABLE and VERTEX PROGRAMS on the TI-85

You can link your calculator to your instructors calculator and copy the programs off of your instructor's calculator. Two of the programs you should copy are called TABLE and VERTEX. These programs allow you to calculate tables of values and calculate the vertex point of a parabola.

1. TABLE- After you have copied these programs from your instructor, press the program button **PRGM**, then **F1** for **NAMES**, then the **MORE** button for more choices until you get to **TABLE**. Once you see the word **TABLE**, press its function key, then press **ENTER**. To make a table, press F1 for equation, then type in the right hand side of the equation. You must use the **X-VAR** key for the independent variable (x). After you type in the equation, press **ENTER**. You will now need to set the **range (F2)**. You will then need to enter the **START** of the values you want to use for x, the **END** of the table, and the **PITCH**. The pitch determines if your table is increasing in units of ones, tens, tenths, or whatever you choose. Then press **ENTER** for the first value of your table. Keep pressing enter to see more values of your table. For example, if you wanted to get a table of values for the equation y = 2x + 5 for the following x values;

x values	y values determined by the equation y = 2x + 5
0	
2	
4	
6	

you would press the **PRGM** key, then **F1** for **NAMES**, then the **MORE** key until you see the word **TABLE**. Press the function key that has the word TABLE on the screen above it, then press **ENTER**, then press F1. Type in 2x + 5 (don't for get to use the **X-VAR** key for x). Press the **ENTER** key. Press F2 for **RANGE**. Type in the **START** of your x values, which is 0, then **ENTER**. Type in the **END** of your x values, which is 6, then **ENTER**. Then type in the **PITCH**, which is 2, then **ENTER**. Press **ENTER** to get back to the table program. You could use **Eval**, to evaluate the equation at a particular point.

2. **VERTEX -** Press **PRGM** button, then **F1** for names, then the **MORE** button for more choices until you get to **VERTEX**. Once you see the word **VERTEX**, press its function key, then press enter. Then enter the A, B, and C from your quadratic equation that is in standard form. Then press enter to find your vertex point.

3. **Other Programs-PolyM** is a program for multiplying polynomials. **Polyfact** is a program for factoring polynomials. **Graph** is a program that sets the *y*min and *y*max automatically on your range in the graph menu when you set the *x*min and *x*max.

EQUATIONS

1. **Solving Quadratic, Cubic equations and higher-** To solve quadratic or cubic equations or higher powered press 2nd, **PRGM** (to get into **POLY**), then enter the order of the equation (Quadratic = 2, cubic = 3, etc.). Hit **ENTER**, then enter your coefficients hitting **ENTER** after each one. Then **F5** for solve. Your equation must be in standard form and set equal to 0.

 Example- To solve the quadratic equation $(2x-3)^2 + 5 = x + 3$ you must first put the equation in standard form. Expanding $(2x-3)^2$ we get $4x^2 -12x + 9$. So we now have $4x^2 -12x + 9 + 5 = x + 3$. Moving all the terms to the left side of the equation we have, $4x^2 -12x + 9 + 5 - x - 3 = 0$. Combining like terms we have, $4x^2 -13x + 11 = 0$. Now we are ready to put this equation into our calculator. Hit 2nd, **POLY**, 2 (for the order), **ENTER**. Type in 4 **ENTER** then -13 **ENTER** then 11 **ENTER** then **F5** for Solve. You should get 1.625 + .3307i and 1.625 - .3307i

2. **Solving Systems of Equations-**To solve systems of equations, press 2nd then the **STAT** button (this enters you into the **SIMULT MENU**). Enter the number of equations. Make sure the equations are in standard form (ax + by = c). Type in your coefficients hitting **ENTER** after each one. Then **F5** for solve.

 Example- to solve the system of equations:

 $x + 2y +3z = 4$
 $2y - 3z = 7$
 $4x + 5y = -2$

 Press 2nd **STAT**,3, then type in your coefficients. You need to realize that the system of equations above need to be written as

 $x + 2y + 3z = 4$
 $0x + 2y - 3z = 7$
 $4x + 5y + 0z = -2$

 So type in 1 ENTER 2 ENTER 3 ENTER 4 ENTER 0 ENTER 2 ENTER -3 ENTER 7 ENTER 4 ENTER 5 ENTER 0 ENTER -2 ENTER. Then press F5 for solve. You should get x = -5.727 y = 4.1818 and z = .4545

3. Solving linear equations- To solve linear equations on the **TI85**, or any equation that only has one solution, you need to go to the SOLVER function (2nd then **GRAPH**). After you do that, the letters **eqn:** will show up. Then type in your equation. You must use the **X-VAR** key for the independent variable no matter what the letter is in the equation. Then **ENTER**. Some numbers will come up that say right and left, but that is not your solution. You then need to press **F5** for solve, and you will get the answer. **NOTE:** The solver function will only work with equations that have one solution (linear, logarithmic, and exponential). It will not work for quadratic or cubic equations. To solve these, see **EQATIONS** #1 on the next page. **Note:** The solver may not solve equations that have the variable in the denominator.

 Example- Solve the linear equation $2x + 4 = 10$. Use the solver function (2nd **GRAPH**). After you enter the equation hit **ENTER**, then **F5** for solve. You should get x = 3

GRAPHING

1. Entering equations you want to graph- To enter the graph menu, simply press the **GRAPH** key, then press **F1** for **y(x)**. The equations need to be solved for Y, and you must use **X-VAR** key as the independent variable ,then **ENTER**, then the 2nd key and **F5** key to **GRAPH** the equation.

2. Zoom- To zoom press **F3** for zoom. You will now have several zoom functions such as **BOX**, **ZIN**, **ZOUT**, **ZSTD**, **ZPREV**, and an arrow. The arrow to the right of **ZPREV** means that there are more choices under the zoom function. To see more of the zoom functions use the **MORE** key. You will see **ZFIT** above **F1**. **ZFIT** stands for zoom fit. **ZFIT** will at least get some part of your graph to show on your calculator. If you press the **MORE** key twice you will get back to your original zoom functions. Now you could use **F3** then **ENTER** to zoom out, or **F2** then **ENTER** to zoom in. If you use zoom in, you will zoom in on the location of the cursor. Move the cursor with the arrow keys to the area you want to zoom in on, then press **ENTER**. To exit out of the zoom functions press the **EXIT** key.

3. Adjusting the Range - Use **F2** (Range) to change your viewing window and scale. It's generally a good idea to have your *y*-max and *x*-max be positive and your *y*-min and *x*-min to be negative by about the same amount so you can see the x and y axis.

Example- If you graph $Y = x^2 + 20$, you will need to type in the equation, then press 2nd then **F5** for graph, You won't see the graph because it is out of the standard viewing window. You can then zoom out until you see the graph then adjust your range as needed.

4. TRACE- Trace is for tracing your graph. When your graph is on the screen press **F4** for **TRACE** and a cursor will appear. Then use your arrow keys to move the cursor to where you want it. If you use **ZOOM IN** now, you will zoom in on the area that the cursor is on.

5. MATH-You can press the **MORE** key, to see more choices. One choice is **MATH (F1).** After you press F1 for **MATH**, you will have choices for **LOWER, UPPER, ROOT, dy/dx, f(x), and more choices.** Use these to find the roots, max, mins, y intercepts, intersection points, and inflection points on your graphs.

The STAT menu on the TI-85

The STAT menu is used for plotting points and finding lines and curves of best fit.
To get to the STAT menu simply press the STAT key.

Function Keys- There are sometimes two rows of function keys on the **TI-85.** To use a function on the bottom row, simply press the function key you need (**F1 -F5**). To use a function on the top row, press the 2^{nd} key then the function key you need (**F1-F5**). If you see an arrow on the screen after **F5**, then that means there are more functions. To see the other functions, press the **MORE** key.

Before you type in your datapoints in the **STAT** menu, you need to make sure your **RANGE** is set in the **GRAPH** menu. Press the **GRAPH** key under the blue **ALPHA** key. You will now see words at the bottom of the screen like;

y(x)=	RANGE	ZOOM	TRACE	GRAPH

Note that the little arrow to the right of the word **GRAPH**, means that there are **MORE** choices. If you would want to see the other choices you would press the **MORE** key. What we need to do though is set our **RANGE**. The word **RANGE** is above the function key **F2**. So press **F2** to set your **RANGE** of your graph. Once you press **F2** for range you will see;

xMin=
xMax=
xScl=
yMin=
yMax=
yScl=

Use the calculator's four gray arrow keys to change the values of your range to match what you need for your datapoints. For example, if your datapoints were

x values	y values
2	10
6	36
10	50

you would need to set your xMax to at least 10, and your yMax to at least 50, and your xMin no higher than 2 and yMin no higher than 10. It's a good idea to go a little higher and lower than what you need. The xScl and yScl stands for the scale on the x axis and the scale on the y axis. You really do not need to change these.

Once you set the range press the **STAT** key.

After you press the **STAT** key you will see these words at the bottom of the screen.

CALC	EDIT	DRAW	FCST	VARS

1. **EDIT- EDIT (F2)** is used to enter your *x* and *y* values. After you press the **STAT** key, press **F2** for **EDIT**, then **ENTER** twice. Now press **F5** (CLRxy) to clear any datapoints. Enter your first *x* value, then press **ENTER**, then the y value, then press **ENTER** and continue through all of your points. After you type in all your datapoints you will see a cursor blinking at x = __ and y = 1. Do not worry about the y = 1.

2. **DRAW-** Press the 2nd key then press **F3** for **DRAW**, then press **F2** for **SCAT** to draw a scatter plot of your datapoints.

3. **CALC- CALC** (2nd key then **F1**) is used to calculate lines or curves of best fit.

   Press the 2nd key, then F1 for **CALC**. Press **ENTER** twice, then F2 for **LINR.** You will get $a = 35.30785$ $b = 1.3992$ and corr = .8744 . The general equation of a line is y = slope(x) + y-intercept. On the TI85 the a is the y-intercept and b is the slope, so the equation of the line of best fit is $y = 1.3992x + 35.307$. To see the graph of this line, press the 2nd key then **F3** for **DRAW**, then press **F4** for **DRREG**. You can press the **CLEAR** key to clear the menus at the bottom of the screen.

Matrix Menu

1. **Entering a matrix-**To enter a matrix, hit 2nd **MATRX**, then name your matrix with a letter, and put in the size of your matrix. Then enter the elements of your matrix (they go in as columns but they are the rows of your matrix.

2. **Operations with matrices-**To add subtract or multiply the matrices you made, go to **NAMES** and type in the name of your first matrix (maybe you named it A) then times or whatever operation you want, then the name of your second matrix (maybe B) , then **ENTER**.

Appendix C

Course Syllabi for Elementary and Intermediate Algebra

Elementary Algebra course syllabus
Spring 1999

GENERAL INFORMATION

Course title and Number: Math 90 Beginning Algebra

Text: Jerome Kaufmann, Elementary Algebra 5th edition. PWS Publishing Company 1996.

Graphing Calculator: Required (This statement was on the course syllabus for the classes that required a graphing calculator)

I. PURPOSE

A. Description:
A course designed to enable the student to develop sufficient skill in working with fractions, integers, signed numbers, variables, algebraic expressions, roots, linear equations, linear inequalities, ratios and proportions, word problems involving; percents, area, mixtures, distance and interest, formulas, graphs of linear equations, systems of equations, rules for exponents, scientific notation, negative exponents, polynomials, and solving quadratic equations. The course will have applied problems. Recommended for all students who have never taken algebra or whose placement test scores indicate the need. This course does not meet AA graduation requirements. 3 semester hours.

B. Course Objectives:
1. To encourage the student to think in a logical fashion and gain mathematical maturity by developing a work ethic.
2. To learn how to use a graphing calculator. (This statement was on the course syllabus for the classes that required a graphing calculator)
3. To supplement or enrich a student's background by developing in greater detail the concepts and applications of Elementary Algebra.
4. To examine the concepts and applications of graphing.
5. To enable the student to solve various types of statement problems using the techniques of algebra.
6. To provide the student with the mathematical background needed for Math 93 (Intermediate Algebra) and Math 105 (Elements of Math).

II. COURSE POLICIES

A. Attendance: Attendance is required. The instructor may withdraw students that have more than two unexcused absences from class. No notification will be given to the student when he or she has reached the limited amount of absences. If a student has documentation of a college sanctioned activity, death in the family, or illness, that absence will not count if the instructor is notified in **advance** either by phone, e-mail, or in person.
B. Participation: Class Participation has been shown to help a students understanding of mathematics, although no points are awarded for class participation. Working with your group is required.

C. Grading:

Tests- 4 tests at 70 points each	280
Homework, Quizzes and Attendance	50-100
Total number of points possible	330-380

Grading Scale: 90 to 100% = A
80 to 89% = B
70 to 79% = C
60 to 69% = D
0 to 59% = F

Note: The grade of X will be given to students that do not pass, but have done all homework and attended all classes.
Note: If student passes placement test at the end of the semester, student will receive at least a grade of "C" in the course.
Note: If you receive a "D" in the course, you still must repeat the course. You need a "C" or better to go on to math 93.

III. COURSE REQUIREMENTS

A. Outline of Recommended Topics:
1.1 - 1.5 Review of Real Numbers (fractions, signed numbers, exponents, roots, order of operations)
3.1 - 3.4 Solving Linear Equations
TEST I
2.5 - 4.4 Formulas, Problem Solving, ratio and proportions, Distance problems and Inequalities
TEST II
8.1 - 8.7 Graphs, slope, equations of lines, and systems of equations (2X2)
TEST III
5.1 - 5.6 Exponents and Polynomials
6.1 Factoring Polynomials
10.3 and 10.5 Quadratic formula and applications
TEST IV

Calculators - Calculators are a **must**. Get a graphing calculator Casio 9850GA Plus or Casio 9850g or the TI-85. (This statement was on the course syllabus for the classes that required a graphing calculator)

Intermediate Algebra course syllabus
Spring 1999

GENERAL INFORMATION

Course title and Number: Math 93 Intermediate Algebra

Text: Allen R. Angel, Intermediate Algebra for College Students 4th edition. Prentice Hall Publishing Company, 1996.

Graphing Calculator: Required (This statement was on the course syllabus for the classes that required a graphing calculator).

I. PURPOSE

A. Description:

A course designed to enable the student to develop sufficient skill in working with linear equations and inequalities, absolute value equations, word problems involving; percents, area, mixtures, distance and interest, literal equations, graphs of linear equations and inequalities, systems of equations 2x2 and 3x3, rules for exponents, scientific notation, radicals, polynomials, quadratic equations, and applications of all topics. Exponential and logarithmic equations may also be included.. This course does not meet AA graduation requirements. 3 semester hours.

B. Course Objectives:

1. To encourage the student to think in a logical fashion and gain mathematical maturity.
2. To learn more advance features of the graphing calculator. (This statement was on the course syllabus for the classes that required a graphing calculator).
3. To supplement or enrich a student's background by developing in greater detail the intermediate concepts and applications of Intermediate Algebra.
4. To examine the concepts and applications of graphing and systems of equations.
5. To enable the student to solve various types of statement problems using the techniques of algebra.
6. To provide the student with the mathematical background needed for Math 102 (College Algebra), Math 119 (Pre-calculus), or Math 221 (Statistics).

II. COURSE POLICIES

A. Attendance: Attendance is required. The instructor may withdraw students that have more than two unexcused absences from class. No notification will be given to the student when he or she has reached the limited amount of absences. If a student has documentation of a college sanctioned activity, death in the family, or illness, that absence will not count if the instructor is notified in **advance** either by phone, e-mail, or in person.

B. Participation: Class Participation has been shown to help a students understanding of mathematics, although no points are awarded for class participation. Working with your group is required.

C. Grading:

Tests- 5 tests at 70 points each	350
Homework, Quizzes and Attendance	50-100
Total number of points possible	400-450

Grading Scale: 90 to 100% = A
80 to 89% = B
70 to 79% = C
60 to 69% = D
0 to 59% = F

Note: The grade of X will be given to students that do not pass, but have done all homework and attended all classes.
Note: If student passes placement test at the end of the semester, student will receive at least a grade of "C" in the course.
Note: If you receive a "D" in the course, you still must repeat the course. You need a "C" or better to go on to math 93.

III. COURSE REQUIREMENTS

A. Outline of Recommended Topics:
1.1 - 1.4 Review of Real Numbers (fractions, signed numbers, exponents, roots, order of operations)
2.1 - 2.6 Solving linear equations, inequalities, literal equations, and absolute value equations
TEST I
3.1 - 3.6 Graphing linear and non-linear equations
TEST II
4.1 - 4.3 Systems of equations and applications with two and three unknowns
TEST III
5.1 - 5.6 Exponents and Polynomials
6.1 Factoring Polynomials
TEST IV
8.1, 8.2, 8.6, 8.7 Roots, radicals, radical equations, and Complex Numbers
9.2, 10.3 Quadratic formula and applications
11.1, 11.2 Exponential and logarithmic functions
TEST V

Calculators - Calculators are a **must**. Get a graphing calculator Casio 9850GA Plus or Casio 9850g or the TI-85. (This statement was on the course syllabus for the classes that required a graphing calculator)

Appendix D

Script for Instructors to read to students

Script to read to students for Mark Shore's dissertation study

A mathematics teacher at Allegany College is conducting a statistical study to determine the effect of graphing calculators on college students' ability to solve procedural and conceptual problems in developmental algebra (math 90 and math 93). We would like to thank you for agreeing to participate in this study. Your participation is entirely voluntary. Neither your class standing, athletic status, or grades will be affected by refusing to participate or by withdrawing from the study. Your responses will remain anonymous and confidentiality will be maintained.

This study consists of a pre-test and a post-test. Also complete the calculator questionnaire that is attached to each test. You will need to go to the Instructional Assistance Center (room H-58) and identify yourself as either a math 90 or 93 student (depending on the class you are currently enrolled in). Please bring the calculator that you use in your math 90 or math 93 class and a photo ID with you when you take the pre-test and post-test.

When you take the pre-test do not get discouraged if you are unable to answer very many of the questions. Most of the questions on the test you have not been exposed to at this point in the course. The test is untimed, you are allowed to use either a scientific or graphing calculator, and you are not allowed to use your notes or your book. Please try your best on the test.

Thank you again for agreeing to participate in this study.

Appendix E

Vita

Mark A. Shore
Phone 301-777-0437
E-mail marks@acc7.ac.cc.md.us

Education

Present West Virginia University Morgantown, WV

Ed.D. Mathematics/Education

- Presently accepted into candidacy

1986 Frostburg State College Frostburg, MD

M.Ed. Mathematics/Education

- Twenty-one hours of graduate level mathematics courses

1984 Frostburg State College Frostburg, MD

B.S. Mathematics and Education

- Dual major

Professional experience

1994 - Present Allegany College of Maryland

Assistant Professor of Mathematics

- Primary responsibilities are teaching mathematics and statistics courses ranging from Developmental Mathematics to Multivariable Calculus. Other duties include development of hypermedia computer programs, instruction over distance learning, Pro Active learning Steering Committee, Learning Communities Project, and academic advising. I have also held workshops on graphing calculators, computer algebra systems, internet programs, and spreadsheets for high school mathematics teachers and college mathematics faculty, and mathematics teaching strategies, workshops, and summer institutes for elementary school teachers.

1988 - 1994 Potomac State College

Assistant Professor of Mathematics

- Primary responsibilities were teaching mathematics and statistics courses ranging from Developmental Mathematics to Differential Equations. Other duties included coordinator of the Mathematics and Science computer lab, coordinator of the Developmental Mathematics Program, Acting department head (1989), and academic advising. Member of the Biological Science Curriculum Study Grant from the National Science Foundation.

1986 - 1988 Frostburg State University

Visiting Lecturer of Mathematics

- Primary responsibilities were teaching mathematics and statistics courses. Other duties included assisting in the Annual Mathematics

Symposium, and coordinator of biannual Mathematics Seminar.

Publications

The Effect of Type of Courseware on the Achievement of College Students Enrolled in a College Algebra Course toward Problem Solving, Educational Multimedia/Hypermedia and Telecommunications, 1997

College Algebra: Applications and Models - Numerically, Algebraically, and Graphically, Allegany College of Maryland, College Algebra Textbook
Elementary and Intermediate Algebra: Applications and Models - Numerically, Algebraically, and Graphically, Allegany College of Maryland, Elementary and Intermediate Algebra Textbook

Professional activities

Allegany County Board of Education:

- Series of workshops for Elementary School Teachers
- Series of workshops for Secondary School Teachers of Mathematics
- Summer institute for Elementary School Teachers for Mathematics and Science (1996, 1997)

Presentations

- **The Effects of Type of Courseware on the Achievement of College Students Enrolled in a College Algebra Course toward Problem Solving, (p. 56), Paper presented at the Educational Multimedia/Hypermedia and Telecommunications, 1997, Calgary, Canada.**
- Writing Hypermedia Programs, presentation at the Frostburg State University Mathematics Symposium, 1997, Frostburg, Maryland.
- Recurrence Relations, Paper presented at the West Virginia University High School Mathematics Symposium, 1997, Morgantown, West Virginia.

Papers Written

- Curves of Best Fit. One of two papers written for comprehensive finals in Ed.D program at WVU.
- The Effect of Graphing Calculators on College Students' Ability to Solve Procedural and Conceptual Problems in Developmental Algebra, 1998, Prospectus for Dissertation, West Virginia University.

Member of Core Learning Goal Committee
Member of the Pro-active Learning Steering Committee
Coordinator of the Learning Communities Project for Mathematics, Social Science and Nursing
Developed Testing materials for Regent's Test

Professional memberships

West Virginia Developmental Educators Association
National Council of Teachers of Mathematics
Mathematics Honor Society
Member of Maryland State Standards and Assessment Committee
Who's Who in American Teachers (1996-98)

Community activities

American Heart Association: Heart Ball, Jump-rope for Heart, and Walk for Heart (1992-96)

Heart (1992-96)

References Jim Winner, Mathematics Department Chairman, Allegany College of Maryland
Dr. James Zamagias, Division Head of Humanities, Allegany College of Maryland
Dr. Gerald Wilcox, Division Head of Mathematics and Science, Potomac State College
Karen Bundy, Coordinator of Mathematics and Science, Allegany County Board of Education

Accreditations First Test of Actuarial Exam

Courses taken relevant to position

Education Courses	**Mathematics Courses**
Instructional System Design	Differential Equations
Curriculum Development	Real Analysis
Curriculum Evaluation	Complex Analysis
Survey of issues in Mathematics	Applied Regression Analysis
Advanced issues in Mathematics Education	Math Models and Applications
Hypermedia in Education	Mathematical Programming
Teaching to the adult learner	Probability and Statistics
Advanced Teaching Strategies	Research Statistics
Teaching in Higher Education	Linear Algebra

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