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ABSTRACT

This study explored the possibility of positioning the sociomathematical norms construct to more centrally reflecting the quality of students' mathematical engagement in classroom processes. Given the challenges of implementing reform ideals, the sociomathematical norms construct can be critical in understanding whether reform-oriented teachers use classroom social structure effectively to develop students' mathematically significant beliefs and values, and to enhance their conceptual understanding of mathematics. The classroom practices of 17 reform-oriented second grade U.S. teachers were observed and analyzed. This study supports the growing realization of the reform community that reforming mathematics teaching involves reconceptualizing how students' engagement in the social fabric of the classroom may enable them to develop increasingly sophisticated ways of mathematical knowing and valuing. (Contains 26 references.) (ASK)

Challenges of Reform: Utility of Sociomathematical Norms[1]

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Background to the Study

Educational leaders have sought to change the prevailing teacher-centered pedagogy of mathematics to a student-centered pedagogy (NCTM, 1989, 1991, 1995, 2000). The term *teacher-centered* refers to a teacher's explanations and ideas constituting the focus of classroom mathematical practice, whereas the term *student-centered* refers to students' contributions and responses constituting the focus of classroom practice.

Cobb and his colleagues developed an "emergent" theoretical framework that fits well with the reform agenda (Cobb & Bauersfeld, 1995). In this perspective, mathematical meanings are seen as emerging in a continuous process of negotiation through social interaction. In investigating students' mathematical learning within the emergent perspective, Cobb and his colleagues addressed *sociomathematical norms* as "the normative aspects of whole-class discussions that are specific to students' mathematical activity" (Cobb & Yackel, 1996, p. 178). They differentiate general social norms as applicable to any subject matter area from sociomathematical norms which are unique to mathematics. The examples of sociomathematical norms have included the norms of what count as an acceptable, a justifiable, an easy, a clear, a different, an efficient, an elegant, and a sophisticated explanation (Bowers, Cobb, & McClain, 1999; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Yackel & Cobb, 1996).

The reform movement has been successful in marshaling large-scale support for instructional innovation, and in enlisting the participation and allegiance of large numbers of mathematics teachers (Knapp, 1997). However, despite the widespread endorsement of reform, there is concern that many teachers have not grasped the full implications of the reform ideals (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Oliver, & Wearne, 1996; Research Advisory Committee, 1997). Teachers too easily adopt a new teaching techniques, but without reconceptualizing how such a change in teaching strategies relates to fostering students' conceptual understanding or mathematical dispositions (Burrill, 1997; Stigler & Hiebert, 1998).

Numerous studies have examined the challenges faced by dedicated and committed teachers as they struggle to understand and come to terms with reform (Ball, 1993; Fennema & Nelson, 1997; Lampert, 1990; Schifter, 1996; Schifter & Fosnot, 1993; Wood, Cobb, & Yackel, 1995). However, past research has been limited mainly to studying general social norms of typical classes or exploring social and sociomathematical norms of specific reform classes wherein researchers are supporting teachers in their attempt to change their instructional methods. Naturalistic studies of social and sociomathematical norms in unsupported reform-oriented classrooms have not yet been undertaken. Moreover, the previous research trend was to provide an extensive analysis of one single reform-oriented classroom. Close contrasts and comparisons of unequally successful reform classes have rarely been conducted in previous research on reform.

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Research Project

Objectives

This study explored the possibility of positioning the sociomathematical norms construct as more centrally reflecting the quality of students' mathematical engagement in classroom processes. Given the challenges of implementing reform ideals, the sociomathematical norms construct can be critical in understanding whether or not reform-oriented teachers use classroom social structure effectively to develop students' mathematically significant beliefs and values and to enhance their conceptual understanding of mathematics. Within this project, I pursue the possibility that the breakdown between teachers' adoption of reform objectives, and their successful incorporation of reform ideals implicates the sociomathematical norms that become established in their classrooms.

Methods

This study was an exploratory, qualitative, comparative case study (Yin , 1994) using *constant comparative analysis* (Glaser & Strauss, 1967; Strauss & Corbin, 1998) for which the primary data sources were classroom video recordings and transcripts (Cobb & Whitenack, 1996). The purpose of exploratory study is to articulate new issues and problems, rather than to definitively answer questions (Yin, 1993, 1994). The small number of classes, and small number of observations of each class, do not provide a basis for firm generalization. However, qualitative case study is well established as a methodology for generating theoretical and empirical insights to be pursued in subsequent broader based studies (Yin, 1994).

This study provided detailed descriptions of the processes that constituted unequally successful student-centered pedagogy in the elementary mathematics classrooms. Moreover, the comparison and contrast between more successful and less successful reform classes provided a unique opportunity to reflect on possibly subtle but crucial issues with regard to reform implementation. Since the principal concern of reform is to connect changes in teaching practices with changes in learning opportunities that students will encounter in the class, students' learning opportunity was analyzed in the target classrooms.

Data Collection and Analyses

As a kind of purposeful sampling (Patton, 1990), the classroom teaching practices of 17 second grade U.S. teachers recommended as reform-oriented were preliminary observed and analyzed. Two classes, class UE and class UM, were selected that clearly aspired to student-centered classroom social norms, but that appeared to differ in the extent to which students' ideas became the center of mathematical discourse. Seven mathematics lessons in each of these classes were videotaped, audio-taped, and transcribed. A total of 12 hours of interviews was taken with the two teachers, Ms. E and Ms. M, to trace their construction of their teaching approaches. These interviews were audio-taped and transcribed. Additional data included students' papers and projects.

Data analyses for this study had two stages: Individual analysis of each classroom setting, and comparative analysis of the two classes. The analysis of each teaching practices were very carefully scrutinized in a bottom-up fashion and constituted the empirical portion of the study. The individual analysis covered classroom flow, teacher's approaches, students' approaches, and students' learning opportunities. The comparative analysis was done by comparing and contrasting social norms and sociomathematical norms established in each class. The interview data were employed in order to explore how more successful and less successful teaching practices have been constructed.

Results

Individual Analysis of Class UE

Ms. E was successful in establishing classroom social norms compatible with a student-centered approach. Among other things, Ms. E actively facilitated students' participation in the classroom mathematics activities and discussions by employing enjoyable formats, emphasizing visualization of the given problem situations, giving students many opportunities to solve problems individually or collectively and to present their methods to the class, expressing excitement about students' novel ideas, and asking students to author story problems within a group.

Despite this exemplary *form* of student-centered instruction, the content and qualities of Ms. E's teaching focussed primarily on procedural knowledge. To be clear, in some cases, Ms. E expressed her interest in conceptuality but those cases were somewhat infrequent. Ms. E listened to students' various contributions but usually turned out to control the classroom discourse toward one direction -- using standard algorithm or a specific equation for a given mathematics problem. This concern occurred across different classroom activities.The following two episodes illustrate the characteristics of Ms. E's teaching practices.

Episode UE-1 reveals Ms. E's consistent curricular intention of using formal algorithm in the estimation activity. The class was estimating the number Brazil nuts in a jar. A student counted the nuts Ms. E had taken out. Ms. E wrote an equation 100-12=(blank) in vertical format and asked students to explain how to subtract 12 from 100.

<Episode UE-1: Alex's idea against the teacher's expectation of using algorithm>

Ms. E: Who can tell me how to do it? Someone who has not had a turn yet, a new person. Alex, ... what do I have to do first?

Alex: If I took away ... 100 take away 10, it will equal 90. And so if I take away 2 more, it will equal 88.

Ms. E: Good gentleman! Look at here. He looked at this 12 and he said, that 12 is close to 10. He said, 100 minus 10 is 90, I just know that (writing 100-10=90 in a vertical format). And I take 2 more away. Good thinking! Super! How can we do it this way now without mental math? (Points to the equation on the board.) How can we do it if it is a little hard for us? Who can explain what we can do? Arterrion?

Arterrion: 0 minus 2 is 2 and 10 minus 1 is 9.

When Ms. E initiated the discussion regarding how to solve 100-12, she expected students to attempt to take away 2 from the 0 in the ones column, as evidenced by the statement, "What do I have to do *first?*" Unaware of her expectation, Alex provided a different idea that 100 minus 10 is equal to 90 and 90 minus 2 is equal to 88. Ms. E explained what Alex said to the whole class and praised him for his novel idea. But she immediately returned to her initial interest of using the standard algorithm by asking, "How can we do it this way now without mental math? How can we do it if it is a little hard for us?" Responding to Ms. E's expectation, the class was then involved in doing subtraction with algorithm. Note that the standard algorithm departed from the holistic meaning of subtracting 12 from 100 for estimation (i.e., there were 100 items and 12 of them were taken out) by focusing on each individual digit (i.e., how to subtract 2 from 0). In this way, estimation was shifted from an intuitive conceptual activity into an opportunity to practice standard algorithms.

Episode UE-2 illustrates Ms.E's approach of reinforcing specific mathematical equations or standard algorithms over students' various ideas. At one level, Ms. E accepted students' different ideas.

At another level, she revealed her own ultimate interest in students' various contributions. Students were collectively solving the Shell Problem[3] in their small groups. Ms. E encouraged students to discuss how they would solve the problem with their group members. She also asked them to write their solution methods after they reached consensus. Soon she called for whole class discussion and asked for each group's method.

<Episode UE-2: Students' various ideas and teacher's interest in using equations>

T: On the Plan [written in the worksheet as a sequence for problem solving], how can you solve the problem? I would like to have one volunteer from every group to read what you decided by consensus in your group. One volunteer, Um, Mary. Listen.
Mary: Solve the problem by adding.
T: She said, they are gonna solve the problem by adding. Is that all you wrote?
Mary: Yes.

T: Okay, is there anyone who would like to read over here? Okay, Alex? Are you listening? Stop one moment. Someone over here has to listen because he may have something different. You need to learn from your friends.
Alex: How can you solve the problem? [He reads the question in the worksheet.] Add 12 plus 18, and you can find your answer.

T: (To Kelsey's group) What would you say?
Kelsey: We say, how can you solve the problem? [She reads the question.] They can go to the store and buy more shells or they can go to the beach.

T: Okay, this group, what would you say? Billie Jo.
Billie Jo: I say, add 12 to 18, and see how many they need to make 60.

T: Okay, who will say at this table?
Kayla: They need to buy 30 more shells.

T: You are giving us an answer now. We wanna know how we're gonna get to that answer. All right, okay, boys and girls. There are many different ways some of you tried to solve the problem. What you wanna do is... what are they going to do? They're just going to buy more shells. That's true. But we want mathematical problem solving. Some of you said, they have to buy, they have to add 12 and the 18. Um.. But we know they've done that, and we found out what that was. So you added that. But then what does that tell you? Yes, Terrance.

Terrance: They buy 30 more shells to make it.
T: Where did you get the..., adding 30 more?
Terrance: How can you solve the problem? [Reads the question in the worksheet.]
T: Where does the number 30 come from?
Terrance: Because, I know that, 15 plus 15 is 30, and then I say, 18 plus 12 is 30, and so, then 30 more is 60.

T: Pretty good thinking. 15 and 15 is 30, you know. 18 and 12 is 30. Um. Now, though, Sam. We need 60 all together. We have 30. There is an equation we must make right there to have that number. Raise your hand if you think what kind of equation we must make? Boys and girls. You have told me that 12 plus 18 equals 30 (writing the equation, 12+18=30 on the board in a vertical format). Terrance said to me, 15 plus 15

[3]Roy and Reba are buying shells to make jewelry. They need sixty shells in all for their projects. Roy buys 12 shells. Reba buys 18 shells. How many more shells do they need?

equals 30 (writing the equation, 15+15=30 on the board), which is really a nice thought and very true. But now we wanna have 60 all together. And there is an equation we must make. Tell us what we need. So, in writing you plan, what you're gonna have to do is, you still have one more equation you didn't talk about. One more thing you must do in your planning, talk with your neighbor to figure out.

(Students discuss and the teacher walks around for a while and calls for whole-group discussion.)

T: Now, we need to find out how many do they have? It says, Roy buys 12 shells. Reba buys 18 shells. Who can tell me what you already did, what you already told me? You said, 12 plus 18, you get 30. This is how many they have now. (She writes "12+18=30 have") But they need, how many, class? They have this (pointing to the number 30). We wanna find out how many more they need to buy. Then, many of you keep telling me the answer. You haven't talked about the equation to get it. You are using intuitive thinking, which is good. What equation do I write to find how many more I need to buy? What is it, Lara?
Lara: You need to write 30 plus, 30 plus...(looking at her group members).
T: We have, how many do we have? Lara, look, how many do we have?
Lara: 60 shells.
T: 60. We have... No, we want to have 60. I made a mistake. We have 30. How many more do we need?
Lara: We need, 30 more.

T: Correct. But how did you come up with the number?
Lara: Because I know that 3+3=6. That's easy. I know that 3+3=6. So, I know that 30+30=60.
T: So, your thinking is, what can I add, 30, to get 60? Is that what you are saying?
Lara: Yes.

T: There are several ways to doing that. You can say that 60 minus 30 will be
S: 30.
T: 30 more, you must buy.

In the episode, when Ms. E initiated the discussion, most students presented their ideas with some ambiguity. Ms. E accepted their contributions but was interested in the two specific equations: 12+18=30 and 60-30=30. Some of the students came up with the answer 30, but did not use the subtraction to figure it out. When Ms. E asked how they got the answer, Terrance explained, "I know that, 15 plus 15 is 30, and then I say, 18 plus 12 is 30, and so, then 30 more is 60." He seemed to use the equation 30+X=60, after adding 12 to 18. Note that Terrance said, "Then *30 more* is 60." Given the problem was "*how many more* shells do they need?", it seemed natural or intuitive for students to come up with the equation 30+X=60. As Ms. E kept asking where the answer 30 came from, Terrance provided a rather irrelevant equation (15+15=30) as well as a reasonable explanation, "30 more is 60." Ms. E acknowledged his contribution but she recognized the equation, 15+15=30, was irrelevant to getting the second addend. Ms. E seemed not satisfied because the students did not use the second equation (60-30=30) she expected. She kept telling, "You still have one more equation you didn't talk about." At this point, it was not clear whether Ms. E thought of the possibility that students might have made another equation, 30+30=60.

Ms. E then gave students time to discuss more and then led whole class discussion again. Ms. E evaluated students' contributions to get the answer as "intuitive" and praised for their thinking. When she said, "You haven't talked about the equation to get it", Ms. E implied that students had not used an appropriate equation. In other words, she seemed not to regard the equation 30+30=60 as mathematically valid. Responding to the teacher's consistent question of writing an equation, Lara provided a rationale to use the equation 30+30=60 to get the answer: "I know that 3+3=6. So, I know that 30+30=60." Ms. E checked Lara's explanation whether she thought of what to add to get 60, and accepted her idea. Acknowledging that there were several ways to get the answer, Ms. E finally revealed her interest in using the equation 60-30=30.

There are two possible interpretations of Ms. E's insistence on 60-30=30. She might have intended to provide semantic (conceptual) grounding for the missing addend interpretation of subtraction. However, Ms. E did not connect the conceptual relationship between 30+X=60 and X=60-30, when she had the opportunities to do so. For instance, when she asked to Terrance, "Where did you get the ..., *adding 30 more*?", Ms. E might know that he was using the equation, 30+X=60. Lara later explained the rationale of using the equation 30+X=60. Instead of connecting the two equations (30+X=60 and X=60-30), Ms. E directly introduced the subtraction equation 60-30=30 as *another* (or alternative) way to get the answer 30. This leads to a second and more plausible interpretation of Ms. E's insistence on the subtraction equation. She was interested in using the prescribed form (i.e., the two equations, 12+18=30 and 60-30=30). She had been waiting for the answers based on a desire to follow the form. Because students' did not come up with the subtraction equation, Ms. E introduced it even after students' reasonable thinking. This interpretation was consistent with Ms. E's further instruction. When students were supposed to review their solution process following the problem solving sequence specified on the worksheets (understand, plan, solve, and review), Ms. E asked them to check whether they wrote the two specific equations explicitly. There was little room for students to reflect on their "intuitive" thinking of using 30+X=60 and to develop conceptual grounding for a connection of 30+X=60 and X=60-30.

Reflecting Ms. E's practices, students often expressed keen excitement when they got right answers. But they sometimes waited for their teacher's confirmation rather than develop their own rationales or arguments while engaged in group activities. As a result of Ms. E's and of their own approaches, students' learning opportunities were somewhat limited to acquire procedural skills to solve routine problems with accuracy and confidence. Whereas the students were actively involved in classroom mathematical activities, they had little chance to develop the mathematical understandings that could inform their activities. In these respects, the important sociomathematical norms of this class included mathematical accuracy and automaticity.

<u>Individual Analysis of Class UM</u>

Ms. M also established classroom social norms by which students' contributions and ideas were focused. Like Ms. E, she was concerned about students' participation in classroom mathematical activities and discussions. Unlike Ms. E, Ms. M focused on students' own sense-making processes while they were participated in the classroom community. Her primary interest was to create an effective classroom community in which students invent, explain, and justify their own solution methods or ideas. Ms. M encouraged her students to argue or debate for extended periods of time — especially when there were competing solution methods or ideas — rather than providing her judgement. Only after students' full contributions to the discussion did she summarize the main argument in each position. While focusing on students' mathematical thinking, Ms. M urged them to specifically mathematical ways of valuing and communicating. Producing only a correct answer without a mathematically justifiable process was rejected.The following two episodes illustrate the characteristics of Ms. M's teaching practices.

Episode UM-1 is an example demonstrating Ms. M's concern of students' thinking, in particular when students provided the right answer without understanding. She focused not on whether they got the answer but on how they got the answer.

<Episode UM-1: <u>Mathematical explanation over memorization</u> >

(Ms. M drew 6 circles and 5 stars in each on the board.)

Ms. M: All right. Now I am gonna practice writing down our multiplication facts. ... Who can tell me what I've got here? (A few students raise their hands.) How many groups do I have? ... Or how many sets do I have? I wish everybody's hand this time. How many groups do I have? (Points to each circle one by one.) These are sets or groups (pointing to each circle again). How many do I have? Trenea?

Trenea: 6 times 5.

Ms. M: Okay.

Trenea: Equals 30.

Ms. M: Trenea says, I've got 6 sets with 5 in each one. So, I've got 6 groups of 5 (writing 6X5), 6 groups of (pointing to "X"), that's what times means, 6 groups of 5, she said that's 30 (finishing the equation, 6X5=30). How did you figure it out, Trenea?

Trenea: Because, I said like, I just knew because I have a times table ... (Interrupted.)

Ms. M: We are just in second grade. We don't just know that. We have to have ways of saying or finding out.

Trenea: I have a times table chart in my house and ... (Interrupted.)

Ms. M: But I need to know a better way to find out.

Trenea: 5, 10, 15, 20, 25, 30 (pointing to the circles with her finger).

Ms. M: Aha! 5, 10, 15, 20, 25, 30 (pointing to the circles one by one). So, 6 groups of 5 is 30.

Ms. M initially did not accept Trenea's contribution because Trenea knew 6X5=30 from a multiplication table at home. Ms. M encouraged her to figure out the answer in a way that might make sense at a second grade level. When Trenea came up with the idea of counting by 5s, Ms. M excitedly accepted her contribution. Episode UM-2 is a representative episode illustrating how Ms. M facilitated students' mathematical thinking through classroom discussion. After demonstrating how to play the game How Close To 20[4], Ms. M asked which case would be better, when they had the highest score or when they had the lowest score. She encouraged students to present rationales for their answer. As students had two different choices, highest or lowest score, Ms. M used the explanations of students perceived as strong in math to support that the lowest score would be the correct choice and why.

<Episode UM-2: <u>Teacher's challenging question and students' responses</u>>

T: I have a very difficult question to ask. This is the thinking man's question. Do you want to have the highest score or the lowest score? Jonathan, this is important. Brandon, you don't know? So ... do you know? You know? What do you wanna have, Brandon?

Brandon: The highest.

T: Why do you wanna have the highest?

[4] A player sorts out cards to have only number cards. Each player has 5 cards per turn and chooses three out of them attempting to get as close to 20 as possible. In each turn, a player figures out a score by the absolute difference between 20 and the sum of the three cards. The winner is whoever has the lower sum after five turns.

Brandon: Because the higher is the best.

T: In a lot of games, the higher is better. And in a lot of games, Brandon is right. Is he right in this game?

Ss: No, yes, no.

T: Some people say yes; some people say no. Is he right in this game? If you think yes, raise your hands. If you think no, raise your hands. (More students raise hands for no.) Why are you thinking no, Derrick?

Derrick: Because in this game you have to see who can come close to 20. If you have the highest score, then you are not close to 20.

T: Tell me more. Keep talking, keep talking. Tommy?

Tommy: Like, if we get to 20 closer then we have a lower score.

T: Tell me more. Keep going. Who else can tell me more, David?

David: It was like, 23, you have 3 more extra than 20.

T: Okay, 3 is better score than 1, David? What's the better score?

David: 1.

T: So, what do we wanna have?

T: The what, Lainey?

Lainey: The lowest.

T: The lowest score. All right. What, Chase?

Chase: It's just like in golf. You try to get the lower score, because if you get the higher score, it's bad. You are trying to get the 20. But if somebody gets 78, it's not gonna be good, because that's not 20.

T: I see. That's right. That will be too far away from 20. You got 78. You would be really far away from 20, wouldn't you, Chase. Everybody, look at 20 and look at 78 (pointing to 20 and 78, respectively, on the number line).

Ss: God!!!!

T: You would be really far away. All right.

In conjunction with Ms. M's approach, her students tended to use their own ways of approaching a given task both in whole group and in small group settings. Moreover, they were often engaged in mathematical debates without the teacher's initiation or mediation. In these approaches, the students had the learning opportunities to construct conceptual underpinnings of the mathematics they were studying, even as they were continually exposed to mathematically significant ways of knowing, valuing, and arguing.

Comparative Analysis between Class UE and Class UM

The two classrooms in this study established very similar social norms including an open and permissive learning environment, stressing group cooperation, connecting concrete representation by manipulative materials to numerical computation process, employing enjoyable activity formats for students, orchestrating individual or small group session followed by whole group discussion, emphasizing multiple solution methods, expecting students' active participation, and providing the teacher's amplication of students' contributions (see Table 1). These are general social norms that are compatible with current reform recommendations (NCTM 1989, 1991, 2000).

Table 1. Comparison: Social Norms

<table>
<tr><th>Degree of Similarity*</th><th>Ms. E's Class</th><th>Ms. M's Class</th></tr>
<tr><td>●</td><td colspan="2">Classroom Flow
- Open, permissive, dynamic atmosphere
- Small group activity & whole class discussion
- Enjoyable activity format

Teacher's Expectations & Roles
- Stressing independent solving & presentation
- Positive expectations: praise & encouragement
- Repeating/amplifying students' contributions
- Excitement about students' ideas

- Soliciting different solution methods
- Using manipulative materials
- Emphasizing the process of learning
- Asking for authoring story problems

Students' Participation
- Excitement
- Listening to peers' explanations
- Compliance with teacher's instruction
- Collaboration in group activities
- Pointing out others' mistakes</td></tr>
</table>

*Note: Degree of similarity: ● = a lot; ◑ = somewhat; ○ = very little

Despite these similar social participation structures, the two classes were remarkably different in terms of sociomathematical norms (see Table 2). In Ms. E's class, students experienced mathematics on the basis of rather fixed procedure the teacher consistently emphasized. The students' mathematical ways of thinking and valuing were limited to find out the pre-determinded rules. Similarly, their mathematical ways of arguing and justifying were concerned mainly with following the rules, rather than with their own sense-making. In these respects, being accurate or automatic was evaluated as a more important contribution to the classroom community than being insightful or creative.

In contrast, the students in Ms. M's class learned mathematics on the basis of their own sense-making processes. A specific solution method or idea was little emphasized over students' various ways of approaching to a given mathematics problem. The students were continually engaged in significant mathematical processes by which they could develop an appreciation of characteristically mathematical ways of thinking, communicating, arguing, proving, and valuing.

Table 2. Comparison: Sociomathematical Norms

Degree of Similarity*	Ms. E's Class	Ms. M's Class
	Mathematically Significant	
	- Standard algorithm or specific equations - Being accurate - Being automatic	- Making sense by individual students
	Mathematically Acceptable	
○	- Confirmation by Ms. E	- Reasonable argument - Logical explanations - Convincing others - Classroom mathematical community
	Mathematical Engagement	
	- Checking Ms. E's response - Practicing skills - Being confident in basic skills & procedure - Expressing excitement to right answer	- Using their own solution methods - Students' own debate - Asking for more challenging problems - Being autonomous

*Note: Degree of similarity: ● = a lot; ◑ = somewhat; ○ = very little

Discussion

Cautionary Notes on Reform

This study supports the growing realization of the reform community that reforming mathematics teaching involves reconceptualizing how students' engagement in the social fabric of the classroom may enable them to develop increasingly sophisticated ways of mathematical knowing and valuing. This reconceptualization never comes easily, even for teachers who are dedicated and committed to aligning their teaching practices to reform. Ms. E's case warns of the possibility that simply changing classroom social norms promotes neither students' conceptual learning opportunities nor their social engagement toward characteristically mathematical ways of thinking and communicating. Although the students in both of these classrooms had positive and enjoyable experiences in their mathematics classes, opportunities for enhancing their specifically mathematical development were somewhat limited. Ms. M's case shows the possibility that students may acquire conceptual underpinnings of mathematics they are studying as they actively participate in the social processes which include explanation, justification, argumentation that are specific to mathematical activity and discourse. This is a case where reform efforts turn out to be successful.

The similarities and differences between the two teaching practices clearly show that students' learning opportunities do not arise from general social norms of a classroom community. Instead, they are closely related to its sociomathematical norms. Given the challenges of implementing reform ideals, the sociomathematical norms construct is critical in understanding whether or not reform-oriented teachers use classroom social structure effectively to develop students' mathematically significant beliefs and values and to enhance their conceptual understanding of mathematics. Thus, this study suggests that the construct of sociomathematical norms, not general social norms, should be focused for initiating and evaluating mathematics education reform efforts as they occur at the classroom level.

The two classroom teaching practices examined in this study also reveal that the simple dichotomy between student-centered and teacher-centered pedagogy obscures the variety of mathematics education reform possibilities. Class UE displayed student-centered instruction at one level. The general social norms established in the class, which were compatible with reform recommendations, were very different from those norms in typical teacher-centered mathematics classes. However, the detailed analyses of Class UE illustrated that they displayed teacher-centered instruction at another level, because the ultimate focus of mathematical activity and discourse was on the teachers' methods, rather than on the students' contributions.

Utility of Sociomathematical Norms

The construct of sociomathematical norms evolved out of a classroom teaching experiment in which Cobb and his colleagues attempted to account for students' conceptual learning as it occurred in the social context of an inquiry mathematics classroom wherein the teacher and the students together constituted a mathematical community and negotiated mathematical meanings (Cobb & Bauersfeld, 1995). The researchers designed instructional devices and sequences of specific mathematical content and extensively supported the classroom teacher to foster students' mathematical learning using those sequences. They analyzed how sociomathematical norms became constituted and stabilized in those project classrooms (e.g., Bowers, Cobb, & McClain, 1999; Cobb et al., 1997; McClain & Cobb, 1997; Stephan, 1998). The frequent examples of sociomathematical norms included what counts as a different, clear, efficient, easy , or acceptable mathematical explanation. However, previous studies tend to briefly document sociomathematical norms (and also social norms) mainly as a precursor to the detailed analysis of students' conceptual learning established in the classroom community.

The construct of sociomathematical norms is intended to capture the essence of the mathematical microculture established in a classroom community rather than its general social structure (Yackel & Cobb, 1996). This study explored the possibility of positioning the sociomathematical norms construct as more centrally reflecting the quality of students' mathematical engagement in collective classroom processes and predicting their conceptual learning opportunities. This notion allows us to see a teacher as promoting sociomathematical norms to the extent that she or he attends to concordance between the social processes of the classroom, and the characteristically mathematical ways of engaging. In this way, the construct of sociomathematical norms include, but in no ways needs to be limited to, teacher's mediation of mathematics discussions.

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