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ABSTRACT

This paper considers some decisions that must be made by the researcher conducting an exploratory factor analysis. The primary purpose is to aid the researcher in making informed decisions during the factor analysis instead of relying on defaults in statistical programs or traditions of previous researchers. Three decision areas are addressed. First, the importance of determining the number of participants and variables is discussed. Second, the similarities and differences in the two most common factor extraction methods, principal components analysis (PCA) and principle factor analysis (PFA), are addressed. Finally, the paper compares different methods for retaining factors (e.g., K1 rule, scree plot, parallel analysis, and others). Overall, although PCA and PFA results converge with an increasing number of variables and communality estimates, PFA may be the better choice for attempting to determine underlying factors in a data set. Furthermore, parallel analysis shows great promise for facilitating precision in factor retention, but a researcher should take into consideration a combination of methods in order to be as accurate as possible in developing the factor model. Empirical studies indicate that defaults in statistical packages are frequently used and that superior methods, such as parallel analysis, are grossly underused. (Contains 18 references.) (Author/SLD)

Running Head: FACTOR ANALYSIS

Toward Reflective Judgment in Exploratory Factor Analysis Decisions: Determining the Extraction Method and Number of Factors to Retain

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Paper presented at the annual meeting of the Mid-South Educational Research Association, November 15-17, 2000, Bowling Green, KY.

Abstract

The present paper considers some decisions that must be made by the researcher conducting an exploratory factor analysis. The primary purpose is to aid the researcher in making informed decisions during the factor analysis instead of relying on defaults in statistical programs or traditions of previous researchers. Three decision areas are addressed. First, the importance of determining the number of participants and variables is discussed. Second, the similarities and differences in the two most common factor extraction methods, principal components analysis (PCA) and principle factor analysis (PFA) are addressed. Finally, the paper compares different methods for retaining factors (e.g., K1 rule, scree plot, parallel analysis, etc.). Overall, although PCA and PFA results converge with increasing number of variables and communality estimates, PFA may be the better choice for attempting to determine underlying factors in a data set. Furthermore, parallel analysis shows great promise to facilitate precision in factor retention, but a researcher should take into consideration a combination of methods in order to be as accurate as possible in developing the factor model. Empirical studies indicated that defaults in statistical packages are frequently used and that superior methods (e.g., parallel analysis) are grossly underutilized.

The initial development of factor analysis occurred almost a century ago with the development of exploratory factor analysis. In 1901, Pearson visualized a method that would allow latent, unobserved variables to be detected from data sets, and in 1904, Spearman developed a theoretical explanation for factor analysis (Kieffer, 1999). There are two broad types of factor analysis. First, exploratory factor analysis (EFA) is utilized when the researcher maybe has some minor theoretical and/or empirical support but is basically trying to reduce his/her variables and uncover latent factors through a more parsimonious model. Second, confirmatory factor analysis (CFA) is used to confirm a model that has an adequate theoretical and empirical basis. The former method is the focus of this paper. According to Fabrigar, Wegener, MacCallum, and Strahan (1999), there are five procedural decisions that must be considered when conducting an EFA. First is considering variables and sample size. Second is judging the appropriateness of using EFA. Next, one must determine what method will be used to extract the factors. Then, the researcher must figure out how many factors to retain. Finally, s/he must decide on rotation of the factors. This paper will briefly discuss the issue of variables and sample size, but the focus will be on extracting methods and how to retain factors. In statistical procedures, there are many guidelines developed through tradition and research. Factor analysis is no exception. It is important to consider the options along the factor analysis decisional pathway and not depend on the default in SPSS or other statistical packages to present the best options.

<u>Sample Size and Number of Variables Used</u>

In general, a large sample size is required for whatever type of factor analysis one's research is utilizing. Simplified rules of thumb have suggested 5-10 subjects per variable and others have worked from a continuum with 1000 participants as an excellent sample size, 300 as good and 100 as poor (Tinsley and Tinsley, 1987). Tinsley and Tinsley, also, indicated that a sample over 300 individuals is pretty robust to the ratio of subjects to variables. In 1985, Arrindell and van der Ende demonstrated some flexibility in determining the sample size by illustrating that a ratio and/or a definite number was not as critical as once thought for determining factor stability (Tinsley & Tinsley, 1987). Tinsley and Tinsley recommended that one can increase the precision of the analysis by determining theoretically the potential number of factors and ensure that a sufficient amount of variables for each factor are included in the study. Of course, the use of theoretical basis in factor analysis makes the differentiation between CFA & EFA less clear. However, in the case of true EFA, there may be very little theoretical basis for making these decisions, which makes the researcher default back to using a rule of thumb (i.e., 5-10, some say 20, subjects/variable up to about 300). Other articles have slightly different guidelines. For example, Kieffer (1999) suggested a minimum of 300 individuals and at least 5 subjects per variable. Also, Fabrigar, et al., (1999) stressed the importance of overdetermination (i.e., 3-4 observed variables per factor) and high communalities (i.e., .70 or higher) which have the ability to obtain the factor model with a sample size of just 100 people. They stated as these two conditions decrease the need for a large sample increases (e.g. moderate conditions need a sample size of 200). Although though there are some strong suggestions, the variance in the suggested numbers illustrates how difficult a decision this can be and why research literature tends to rely simply on guidelines. Of course, in an EFA context, often little information may be available to allow the researcher to make definitive decisions.

Related to determining sample size, the number of variables are important in being able to extract the factors from the data. Kieffer (1999) states that since the object of EFA is to reduce the number of measured variables to a more simplified structure consisting of latent factors, theoretically the number of variables that the experimenter desires to explore is infinite. However, since factors are more stable with multiple variables appropriated for each factor, the study needs to have an adequate number of variables to adequately represent each factor. Fabrigar, et al., (1999) determined from other research that the total amount of variables needs to be 3-5 times greater than the expected number of factors (i.e., 3-5 variables/factor). Furthermore, if there is little information on how many factors to expect, then the researcher should try comprehensively to determine what are the possible observed variables and attempt to include as many as possible into the study.

Principle Components Analysis vs. Principle Factor Analysis

The two most common EFA extraction methods are the principal components analysis (PCA) and Principle Factor Analysis (PFA), sometimes referred to a principle axis factoring or common factor analysis. In factor analysis, the matrix of association that is most commonly used in the analysis are correlational matrix (Kieffer, 1999). The idea of EFA is to account for as much variance represented in the matrix as possible with the smallest number of latent factors. In correlational matrices, unities (1.0) are usually found on the diagonal which indicates that the item is fully correlated with itself. PCA utilizes this method for extracting factors.

Statistically, variables contain three types of variance: common, specific, and measurement error (Tinsley & Tinsley, 1987). The latter two are sometimes considered two subcomponents of unique variance. Common variance refers to the variable relating to another variable in the data set, presumably their correlation is accounted for by an underlying factor (Fabrigar, et al., 1999). Specific variance represents reliably measured variance that is not shared and indicates that the variable is unique within the data set (i.e., no relationship with other variables) (Tinsley & Tinsley, 1987). Finally, measurement error is also unique to a variable but it is random variance that is not replicable and is unpredictable. As such, it is important to use measures that have historically yielded reliable scores. If the variable was unreliably measured, then there is little chance of reproducing the variable sufficiently within the factor because random error can not be correlated. Because it uses unities on the diagonal, PCA does not differentiate among these variances and has all three represented in the analysis. Therefore, PCA technically transforms the original data in its entirety into new linear combinations of variables rather than an actual factor analysis (Tinsley & Tinsley, 1987; Fabrigar, et al., 1999). Therefore, some of the factors could represent simply correlated error, which is uninterpretable. Fabrigar, et al. (1999) argued that this conceptual difference distinguished PCA from a type of EFA.

Before continuing the discussion of extraction methods, two terms must be defined. First, an eigenvalue reflects how much of the original variance is being accounted for by each factor. Second, a communality coefficient indicates the portion of a variable's variance that is captured by all factors. PFA is interested in the common variance because the goal of this method is to identify latent variables/concepts as opposed to simply a data reduction method like PCA. When using a PFA extraction method, the diagonal in the matrix of association is not represented by unities because PFA takes into consideration measurement error variance. The unique variance is controlled for by placing communality estimates on the diagonal. The estimated communality represents a value attenuated by measurement error. According to Gorsuch (1983), there is no mathematical solution available to determine communalities before running the factor analysis. Thus, researchers must attempt to estimate the communality coefficient. While there are several methods to estimate the communality estimate for a variable, the most commonly recommended is the squared multiple correlation (SMC) (Gorsuch, 1983; Fabrigar, et al., 1999). It is considered a conservative method because SMC is a lower-bound estimate, which ensures that it is capitalizing on the reliable variance rather than error. The next step in PFA is to take the initial communality estimates and place them on the diagonal of the matrix of association and run the analysis. The communalities that result from the factor analysis are compared to the initial ones and if the difference is more than marginal, then the matrix is analyzed with the new communalities on the diagonal (Gorsuch, 1983; Tinsley & Tinsley, 1987). These steps are repeated until the final communality estimates are as close to the original estimate as possible. The repeating of the process is called an iteration and is suppose to produce results closer to the "true" communalities because the values converge. There has been some controversy as to whether to do iterations and how many. Gorsuch (1983) recommended limiting the number of iterations because one may reach a point where some estimates wobble back and forth without converging; moreover, at that reference time such configurations consumed an inordinate amount of time. Today, however, several iterations can be done within seconds on SPSS. Tinsley and Tinsley (1987) also suggested limiting the iterations to no more than 25-50 to prevent the same wobbling or looping effect when communalities will not converge. Furthermore, as the number of iterations grows, the analysis will increasingly capitalize on sampling error while trying to control for measurement error. This dilemma is problematic but often overlooked by researchers. Thus, in general, iterations should be kept to a minimum.

Other differences exist between the two extraction methods. With PFA, one can conceptually test out the fit of the model because hypotheses were developed. However, since it is an EFA, future research must be conducted to confirm the model (i.e., increase generalizability). Fabrigar, et al. (1999) state that since PCA lacks hypotheses one can not decide whether the model was a good fit. Thus, although in certain conditions they yield similar results, they differ conceptually. PCA assumes that the variables contain an insignificant amount of unique variance, while PFA presumes that not focusing on the common variance can alter your conclusions. The reason that PCA is the default on SPSS is because more researchers are familiar with its concepts and before the ease of such programs, PFA was much more time intensive in an already bulky statistical procedure. A potential problem in PFA, although it carries some diagnostic information, is what has been referred to as "Heywood (also seen as Haywood) cases". In these uncommon situations, the procedure used to estimate the communalities produces an estimation equivalent to one or greater (Gorsuch, 1983; Fabrigar, et al., 1999). Since it is impossible to account for more than the full variance of a variable in the factors, this could be problematic. Advocates of PFA and other EFA procedures state that this indicates that a factorial model has violated some assumptions of PFA and/or the model is misspecified and such valuable information would not be discovered with PCA (Fabrigar, et al., 1999). On the other hand, Gorsuch (1983) stated that using the SMC will decrease the probability of such cases occurring since it is a lower bound estimate and when all factors are extracted, the correlated variances of variables will be captured by the factors. Finally, advocates of PCA state that individual composite scores can be computed by PCA, which is not possible with common factor extraction methods (Fabrigar, et al., 1999). A counter argument is that individual composite scores are not relevant for most research conducted by EFA, which generally uses it to determine the underlying constructs.

As stated earlier, PCA and PFA can produce similar results under certain conditions. Gorsuch (1983) states that one situation is when communalities are reasonable high ($\geq .7$), which indicates that the variables are demonstrating reasonable score reliabilities because communality estimates are a lower bound estimate of reliability. Gorsuch further stated that once variables have increased to around 30-40 in number, then effects of placing communality estimates as opposed to unities in the diagonal are minimal because of the ratio of off-diagonal elements in relation to diagonal estimates. In 1989, Snook and Gorsuch conducted a Monte Carlo study to determine whether PCA or PFA was more accurate and at what point did results merge to equivalent findings. Generally, loadings of PCA were higher than PFA indicating inflated loadings which may first appear to indicate that the variable clearly is being captured within that factor. This effect was noted primarily for nonzero, as opposed to zero, loadings and thus may be missed in previous comparison studies that use a statistic that didn't differentiate between these two loadings. Due to this bias, the authors suggested that methods should not be considered to converge to similar findings until around 40 variables are present in the matrix. Also, PFA was found to be less variable than PCA in all conditions particularly when variables were low (i.e., 9) and when loadings were low (i.e., .40) with moderate variables (i.e., 18) (Snook & Gorsuch, 1989). Overall, PFA was more likely to replicate the matrix than PCA. As number of variables and the loadings become large, PCA and PFA are equal in accuracy. However, each research situation is often unique. Decisions regarding extraction method should be a function of reflective researcher judgment, not defaults in statistical packages.

Number of Factor to Retain

When considering how many factors to include in one's study, the researcher generally attempts to explain as much variance as possible while maintaining a parsimonious model. Choosing the correct number of factors in a matrix is critically important. Extracting different numbers of factors can dramatically affect the results of the study. Underfactoring/extraction is the process of extracting not enough factors for the model. Overfactoring/extraction is the term used to indicate that too many factors were specified in the model. A common consensus is that underfactoring can lead to more distorted results than overfactoring (Fabrigar, et al., 1999; Tinsley & Tinsley, 1987). When underextracting factors, the observed variables that are better accounted for by a not yet defined factor could be mistakenly represented by another factor. In addition, variables that should be captured by a particular factor may erroneously appear to poorly defined, if at all, by that factor (Fabrigar, et al., 1999). Such substantial error is likely to significantly misconstrue the model that is defined from the data. On the other hand, overfactoring should also be avoided because it can make minor latent factors seem to be major as well as tempt researchers to define constructs that are unique to that data set (i.e., not generalizable) without really having any theoretical basis (Fabrigar, et al., 1999). Since all general linear model analyses capitalize on sampling error, minor factor may not be replicable. However, overextraction may be less erroneous that underextraction because additional factors often poorly represent the variance in the variables and probably have minimal (or poor) loadings.

As mentioned before, eigenvalues are variance-accounted-for statistics for a specific factor. Eigenvalues are the basis for several methods of retaining factors. Probably the most commonly method used is the K1 rule which was developed by Kaiser (1960) and based on a concept originally proposed by Guttman (1954). Basically, the procedure is to retain any eigenvalue greater than unity (one) because it is thought that any higher value would have more summarizing power than a single variable (Zwick & Velicer, 1986). The number of eigenvalues provided equal up to the number of original variables with the first one representing the factor accounting for the most variance and so on ordered (Tanguma, 1999). This method has been criticized for being simplistic and rather arbitrary. While some have reported it to underestimate factors (Tinsley & Tinsley, 1987), the majority of researchers report that it tends to overestimate

(Zwick & Velicer, 1986). In SPSS, this is the default method. Given K1's potential inaccuracy and strong tendency to overestimate the number of factors, it is unfortunate that many researchers depend heavily on K1 as the decision rule. In fact, in a review of 60 published EFA studies, Henson and Roberts (in press) found that K1 was the sole or primary means of determining the number of factors about half of the time.

Zwick and Velicer (1986) conducted a Monte Carlo study evaluating different procedures for retaining factors. Their study found that the accuracy of using eigenvalues was positively affected by large sample sizes, while negatively affected (i.e., overestimation) by larger sets of variables and by low common variance (and, in essence, poor loadings onto factors). This finding is consistent with Stevens (1996) description of study characteristics that potentially affect the eigenvalues and lead to inaccurate estimation (usually overestimation) of how many factors to retain. In Zwick and Velicer's (1986) Monte Carlo study, the K1 method always overestimated, and with a moderate saturation of .50, it was consistently at least three standard deviations away from the actual number of factors. Accordingly, the K1 is not a recommended approach to determining factor number. However, it can provide a higher end estimation of factors, from which one can use other methods to narrow down the specific number of factors.

Developed by Cattell (1966), the scree plot is another method to use to determine factors. It also is based on the eigenvalues, but it graphs the magnitude of the eigenvalue on the y-axis and the factor number along the x-axis. Cattell conceptualized that extraction is best determined by figuring out the "non-trivial common variance". The non-trivial factors are considered to be on the curve of the scree plot, while the "rubble" or trivial factors will be platykurtic and located along the horizontal line at the end of the "mountain". The general rule is to cut at the uppermost point of the flat line and retain factors before the break.

Several complications occur with the scree plot resulting in criticisms related to its ambiguity. Sometimes there exists a "double-scree" which involves two possible slopes. Cattell (1966) recommended taking the higher scree's breaking point and ignore the lower one. As Tanguma (1999) reported, there is uncertainty on how to effectively handle making a determination if there is a gradual slop with no obvious breaks, more than one break in the line, and more than one potential line that can be drawn through the trivial items. Taken together, interrater reliability also becomes an issue. In the Monte Carlo study, there was overall moderate reliability (.60 and higher) between the two naïve raters trained by the authors and the expert's judgement with a mean of .80 agreement (Zwick and Velicer, 1986). However, the level of inconsistency between researchers remains a threat to the factor model. Zwick and Velicer (1986) found that the estimation of factors to retain became more accurate with increasing saturation of variables onto the factor and a larger sample size and was unaffected by number of variables. The scree plot in their study was consistent with the actual number of factor approximately 42% of the time with a saturation of .50 and about 71% of the time with a saturation of .80 (Zwick & Velicer, 1986). When error was made, it was generally an overestimation and usually only one standard deviation from the correct model. Macrosson (1999) investigated the effects of random variance on the scree plot through visual portrayal. The author demonstrated that the spacing between the scree and the steep curve diminishes to the point that they merge into one as noise (error) was introduced into non-orthogonal and orthogonal structured data. Thus, the more unique (specifically error) variance is present in the matrix, the more the scree plot will be ambiguous. One can concluded that, although the scree plot may be useful, it should not be used solely by itself to make the final determination.

Yet another method for determining factors is the Bartlett's chi-square test which is a statistical test of null hypothesis. It was developed in the early 1950's and appears not to be heavily used in the literature (Henson & Roberts, in press). Each eigenvalue is extracted in order until the chi-square typed null hypothesis test is not rejected (Zwick and Velicer, 1986). Like all statistical significance tests, this method is sensitive to increasing sample size, which leads to more factors retained. Also, an increasing number of variables have overestimation effects on this test. Zwick and Velicer (1986) found the Bartlett's test was quite inaccurate and variable with sensitivity to many influential study characteristics such as sample size, variables, and saturation. With its high variability and 21-38% estimation accuracy, Bartlett's test is not recommended.

The Minimum Average Partial (MAP) method was developed by Velicer (1976) and is based on a matrix of partial correlations. The method is said to be exact and can be applied with a covariance matrix (Zwick & Velicer, 1986; Tanguma, 1999). Zwick and Velicer (1986) found that it tended to underfactor but improved with increasing variables and saturation. Also, previous research by Zwick and Velicer (1982) included improvement with sample size, which was not found in their 1986 Monte Carlo study. MAP appears to be the most accurate procedure discussed so far but tends to be conservative. This method neglects to recognize less well-identified major factors and would be useful to the researcher trying to ignore smaller factors (Zwick & Velicer, 1986; Tanguma, 1999).

The final method discussed in this paper is Parallel Analysis (PA) which was developed by Horn in 1960. PA is a sample based adaptation to the K1 rule, which is population based (Zwick & Velicer, 1986). PA involves factoring an additional matrix, which has the same number of subjects and variables as the original (Tanguma, 1999). However, this second matrix is comprised of random data (i.e., as if people responded randomly to the variables). After the random matrix is factor analyzed, its own eigenvalues emerge. However, theoretically a random matrix will not have any underlying factors. Therefore, if the eigenvalue of the real data matrix is greater than the associated random eigenvalue, then the factor is retained as a latent construct (Tanguma, 1999). Two criticisms have been made: (a) it is still somewhat arbitrary in narrowing down the number of factors (Fabrigar, Wegener, MacCallum, and Strahan, 1999) and (b) it requires a rather large set of generated random correlation matrices (Zwick & Velicer, 1986). Developing random matrices are not available in a "click and point" method in SPSS; it requires the researcher to compute it through syntax. Thompson and Daniel (1996) provided the syntax necessary to implement this procedure in SPSS. In the Monte Carlo study, PA was generally found to be the more accurate procedure at varying complexities (Zwick & Velicer, 1986). Overall, it accurately retained factors approximately 84% of the time at .50 saturation and nearly 100% of the time at .80 saturation. In conclusion, of the most common approaches, this method appears to be the most precise way to determine the number of factors to keep. By using the PA method, the scree plot, and the K1 rule (as a high estimation), the researcher can easily ensure close precision when making this decision. All three of these are available and relatively easy to access in SPSS. Although MAP appears to be fairly accurate in retaining factors, its use may be further hindered by not currently having a simple way to access this method. Reflective researchers would do well to use these methods rather than relying solely on the default K1 rule.

Conclusions

Being an informed researcher will promote the accuracy of EFAs in the literature base. It is somewhat alarming that about half of published EFAs rely heavily on the K1 rule (Henson & Roberts, in press). As such, many studies have included poorly defined factors, which my not be replicable or possess adequate construct validity. In counseling psychology, for example, tests are constantly being developed to measure underlying constructs and it is crucial to know the literature on factor analysis before arbitrarily using guidelines, traditions, and defaults to determine the structure of the test and its subscales. Of course, with any EFA, a follow-up using CFA should be used to confirm the factors which would hopefully detect errors and lack of generalizability. This is assuming the person is familiar with interpreting CFA methods. Unfortunately, Henson and Roberts (in press) found that CFA was seldom used even when authors had theoretical rational for the expected factor structure. Even if CFA is warranted, conducting two factorial analysis studies requires a lot of time, subjects, and hard work. Therefore, the researcher most likely would not intentionally desire to be "spinning his/her wheels and getting nowhere" (i.e., conduct a CFA only to discover that EFA had many faults that could had been prevented). Fabrigar, et al. (1999) conducted an analysis of two journals in the literature to determine what factor analysis methods were being used in psychology. The results were from the *Journal of Personality and Social Psychology* and the *Journal of Applied Psychology* with the publishing dates from 1991-1995. The percentages were quite similar across journals, which indicates a trend in the literature. Overall, they found that approximately half of the time researchers used PCA, with only about 20% using PFA and slightly over 20% not reported. The procedures for retaining factors were not reported in about 40% of the cases. Multiple methods were used approximately 20% of the time, K1 was the sole method used in 15-20% of the articles, and the Scree Test was utilized 15% of the time. Parallel analysis was reported less than 1% of the time. In their study of four journals in psychology and education, Henson and Roberts (in press) found similar trends. The K1 rule and scree test were heavily utilized, while the more accurate MAP and parallel analysis were almost nonexistent.

Furthermore, in almost all occasions, the authors of the studies failed to report sufficient information to inform the reader of the decisions made and to allow an external evaluation of the factor analysis. Given the subjective nature of many decisions in EFA, external evaluation is critical to developing a sound literature base and correct usage of tests.

These results are disturbing and speak to the necessity of educating psychological experimenters of the importance of decision making in factor analysis. Also recommended is a standard means of reporting those decisions in the literature, because without such indications in the article, the factor analysis findings may hold little credibility.

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