This paper discusses uses of TI-73 graphing calculators for middle school mathematics students. It indicates that with the appropriate use of the TI-73, students can develop understanding about variables and basic concepts of algebra, and explore mathematical topics. Background on middle school students' difficulties with those subjects is provided along with examples of teaching some middle school mathematics concepts with the TI-73 such as order of operations, solving equations, and variables. (ASK)
Using Technology to Prepare All Students for Success in Algebra

By

Franklin Demana
The Ohio State University
Columbus, Ohio USA
Email: Demanaf@math.ohio-state.edu

Introduction

The TI-73 is a middle school (and upper elementary in my opinion) graphing calculator. It has many of the important features of the Math Explorer Plus and the TI-80 graphing calculator. It also has many new data analysis features. It is a flash ROM based product which means the chip can be changed electronically, and applications can be installed electronic.

More importantly, with appropriate use of the TI-73, students can develop understanding about variables, basic concepts of algebra, and explore mathematical topics. The key is to introduce these ideas through numerical computation and problem solving involving a balance of paper-and-pencil and calculator techniques.

Background

Students have difficulty with algebra because of their lack of understanding of variables. Researchers generally agree that there are four levels of understanding about variable. For example, Level 1 means that a variable or letter is understood to stand for a specific object, or number. Lesley R. Booth gives a report of the strategies and errors in secondary mathematics in England in a monograph titled, Algebra: Children's Strategies and Errors (1984). The students in this project were 13 to 15 year olds. The list of references in this monograph provide a rich source of information about children misunderstanding about algebra.

Here is an example of a student at Level 1 in understanding about variables reported by Booth. The Interviewer is denoted by I, and the 15-year-old student, Trevor, by T. The question posed was, "Can x + y + z ever equal x + p + z?"

T: 'It won't be true, never.'
I: 'Never?'
T: 'Never, because, it'll have different values... because p has to have a different value from y and the other values, so that'll never be true.'
I: 'So p has to have a different value... why do you say that?'
T: 'Well, if it didn't have a different value, then you wouldn't put p, you'd put y. You see, you put a different letter for every different value.'

Of the 35 students interviewed, 23 (about 66%) had the same understanding about variable. That is, a variable stands for a specific number.
Another interesting question asked was, "What is the perimeter of the shape with \( n \) sides all of length 2?" This time the student is 14-year-old Michelle (M).

I: 'Can you write down anything for the perimeter?'
M: 'No, because \( n \) probably stands for a number and if that's \( n \), I can't get how many sides that is. And I've got to add up all the sides, all the 2's, so I need to know how many.
Unless \( n \) stands for ... like say \( n \)'s in the alphabet, somewhere along the numbers ... if \( n \) stands for one of those, then you can say what \( n \) is.'
I: 'How do you mean?'
M: 'Well, say \( n \)'s 14?'
I: 'How did you get that?'
M: 'Same as I said before, to get \( n \) for a number, I got 14. It's 14 along.'
I: 'Oh, did you count along the alphabet?'
M: 'Yes, so that's 14, so I took that for \( n \), so it should be ... 28, to make up the quantity, the perimeter.'
I: '28?'
M: 'Yes, you add up all the 2's.'
I: Could \( n \) be another number, or does it have to be 14?'
M: 'Well it could be any number, but if you write the question out like that, without any indication of the number, then there's nothing. The only thing you can do is go along the alphabet and take whatever number \( n \) comes under to tell you how many two's you need.'

Only about 11% of the students got this type of item correct.

**Order of Operations**

Using a calculator with algebraic logic with hierarchy requires students to understand basic arithmetic properties that are important in algebra. We have all had students who report the incorrect answer 36 to the following problem:

Evaluate \( 2x^2 \) for \( x = 3 \).

The student performs the operations in order from left to right to get the incorrect answer. That is, multiple 2 by 3 to get 6, and then square 6 to get 36. Appropriate use of technology can help students gain a deeper understand about order of operations.

**Example 1** Show that addition and multiplication cannot be interchanged. In other words, multiplication has higher hierarchy than addition.

**Solution** Set the constants \( C_1 \) and \( C_2 \) as shown in Figure 1a. The first line of Figure 1b shows the result of entering 3 and applying \( C_1 \) followed by \( C_2 \). The second line shows the reverse order. By experimenting, students can discover that the order in which operations are applied matters.

Copyright by Franklin Demena
Students can do numerous activities like Example 1 to truly understand the meaning of the oft given teacher reminder, remember "My Dear Aunt Sally," when students are doing paper-and-pencil computation. Students are actually able to discover the rules and hierarchy themselves.

Solving Equations

We found in earlier projects that using guess-and-check or guess-and-test was an effective technology based approach to help students understand the meaning of solving equations. The Constant feature of the TI-73 is very useful for this method.

Example 2 Solve $2x + 5 = 14.5$ using the Constant feature of the TI-73.

Solution Enter $x \cdot 2 + 5$ for the single constant C1. Figure 2 shows the result of using guess-and-check to solve the equation.

Figure 2 The result of pressing 3, CONST, then 4, CONST, and so forth.

It is important to start with problems that have answers that can be obtained quickly. Then, students can be given more difficult problems. This method helps students build number sense.

Using Lists to Introduce Variables

Copyright by Franklin Demana
Using LIST and STAT is a great way to introduce graphing with students. I will not go into this in this paper.

List based arithmetic can help students deepen their levels of understanding about variables. Consider a rectangle whose length is 5 more than its width. In our earlier work in the Approaching Algebra Numerically (AAN) Project (Demana, Leitzel, & Osborne, 1988) with seventh and eighth graders, we had students build tables like the following one that shows some of the possible values for perimeter and area. We found that the project students had little understanding about area and perimeter, and frequently interchanged them. That is why we usually made tables with columns for both area and perimeter.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5 + 5  = 10</td>
<td>5 + 10 + 5 + 10 = 30</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>10 + 5 = 15</td>
<td>2(10) + 2(15) = 50</td>
<td>150</td>
</tr>
<tr>
<td>15</td>
<td>15 + 5 = 20</td>
<td>2(15 + 20) = 70</td>
<td>300</td>
</tr>
<tr>
<td>20</td>
<td>20 + 5 = 25</td>
<td>4(20) + 10 = 90</td>
<td>500</td>
</tr>
<tr>
<td>w</td>
<td>w + 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We had a good deal of trouble getting students to write expressions such as 5 + 5, 10 + 5, and so forth in the second column. This is important if students are going to be able to write w + 5 in the last row. The study in England reported on earlier found that students would simplify w + 5 to 5w. Objects like 5 + 10 and w + 5 were not legitimate expressions—rather they were directions to add. Notice the nice patterns in the second and third columns. This can be taken advantage of when solving problems.

In the third column, I showed some of the ways our project students would calculate the perimeter. Only two of my students used 4(20) + 10, and many used the "add them up" procedure in the second row. Several equivalent expressions result for perimeter in the last row.

\[
\begin{align*}
P &= w + (w + 5) + w + (w + 5) \\
P &= 2w + 2(w + 5) \\
P &= 2(w + (w + 5)) \\
P &= 4w + 10
\end{align*}
\]

When we solved problems about the perimeter, classroom discussion led to student understanding that these expressions were equivalent. They also found that the fourth one was "best" for solving problems about perimeter using guess-and-check. (The two students who initially used this method would always get the answers first.)

Unfortunately, graphing calculators were not yet invented when we were in this project. I would like to illustrate how we could have tackled this problem with the list feature of graphing calculators. Figure 3 shows the first three columns. The maximum number of letters possible for a column heading is 5, so I used L length and P for perimeter.

Copyright by Franklin Demana
List based arithmetic allows us to use the fact that list $L$ can be obtained from list $W$ as shown in Figure 4a. Pressing ENTER results in all computations being done with the calculator.

Now students can complete the third column ($P$) using their own expression for perimeter (Figure 5). Comparing their $P$ list with other students using different expressions helps convince them that the expressions for perimeter are equivalent.

Let's look at the problem of finding one of these rectangles with perimeter 73. Students learn that solving such equations is just finding the right $w$ so that 73 shows up in the $P$ column. They can observe that the required $w$ is between 15 and 20. This concept of "bracketing" is fundamental in mathematics of all levels. This approach allows students to experience this technique at a younger age.

If students insert their guess, say 17, in the first column, they will find out that nothing happens to the second and third columns. They would be forced to recalculate the second and third columns. Fortunately, there is an easy way to make the second column compute
automatically when the first column is changed. We just need to enter the expression we used before with quotes around it as suggested in Figure 6a. Now any changes made in the first column will automatically be calculated in column 2. We do the same for the P column (Figure 6b).

![Table](a)

![Table](b)

Figure 6

Now students can see that a width of 17 results in a perimeter of 78, a little too big. They can try 16, then 15.5, and their next guess might be 15.75 (Figure 7). Of course, students will proceed at their own rate and with their own guess. Eventually, the problem will be solved. That is, a rectangle with width 15.75, length 20.75 has a perimeter of 73.

![Table](c)

Figure 7

Besides solving this problem numerically, students will begin to appreciate that variables can be replaced by sets of values. Notice that this procedure illustrates that variables can be introduced using words like width, length, and perimeter.

Our results from the ANN project suggested that basic concepts of algebra are accessible to students through numerical computation and problem solving. In particular, the ideas of variable and function can be established to lay the foundation for success in algebra and advanced mathematics courses. Now with the power of graphing calculators, the successful approach we used can be enriched with activities similar to the ones illustrated in this paper.

Using Real Data

The use of real data can help students learn to value and appreciate mathematics. Use a variety of regression models to fit the data in the table below. Use the models to predict the population of the U.S. in 1990 and compare with the actual value of approximately 250 million.

Copyright by Franklin Demana
<table>
<thead>
<tr>
<th>Year</th>
<th>Population (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880</td>
<td>50.2</td>
</tr>
<tr>
<td>1890</td>
<td>63.0</td>
</tr>
<tr>
<td>1900</td>
<td>76.0</td>
</tr>
<tr>
<td>1910</td>
<td>92.0</td>
</tr>
<tr>
<td>1920</td>
<td>105.7</td>
</tr>
<tr>
<td>1930</td>
<td>122.8</td>
</tr>
<tr>
<td>1940</td>
<td>131.7</td>
</tr>
<tr>
<td>1950</td>
<td>151.3</td>
</tr>
<tr>
<td>1960</td>
<td>179.3</td>
</tr>
<tr>
<td>1970</td>
<td>203.3</td>
</tr>
</tbody>
</table>


Bibliography


Copyright by Franklin Demana


I. DOCUMENT IDENTIFICATION:

Title: USING TECHNOLOGY TO PREPARE ALL STUDENTS FOR SUCCESS IN ALGEBRA

Author(s): FRANKLIN DEMANA

Corporate Source: 

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, Resources in Education (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign in the indicated space following.

The sample sticker shown below will be affixed to below will be affixed to below will be affixed to all Level 1 documents all Level 2A documents all Level 2B documents

Level 1 Check here for Level 1 Level 2A Check here for Level 2A Level 2B Check here for Level 2B

Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g. electronic) and paper copy.

Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media.

Check here for Level 2B release, permitting reproduction and dissemination in ERIC archival collection.

Documents will be processed as indicated provided reproduction quality permits.

If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche, or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Signature: 

Printed Name/Position/Title: 

Organization/Address: 

Telephone: 

Fax: 614-846-9836 

E-mail Address: DEMANA@MATH.OHIO-STATE.EDU 

Date: 11-29-00 

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to
cите the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:
If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:
Name:
Address:

V. WHERE TO SEND THIS FORM:
Send this form to the following ERIC clearinghouse:

ERIC/CSMEE
1929 Kenny Road
Columbus, OH 43210-1080

Franklin Demana