Over the past five years, Pesek (Simoneaux), Gray and Golding have been actively involved in the Louisiana Systemic Initiative Program (LaSIP) and the Louisiana Collaborative for Excellence in the Preparation of Teachers (LACEPT) grants through Southeastern Louisiana University. Through these grants teachers from the region are inserviced on implementing the directives and philosophy of the Mathematics Association of America's (MAA) "A Call for Change" and the National Council of Teachers of Mathematics' (NCTM) "Curriculum and Evaluation Standards for School Mathematics", and preparation courses for elementary majors are being redesigned. This paper reports some tasks being implemented across the mathematics content and methods courses in one curriculum strand, namely rational numbers. Elementary education majors at Southeastern are required to take 12 hours in mathematics (algebra, probability and statistics, number sense, and geometry), three hours in mathematics education, and are certified for grades 1-8. (Contains 24 references.) (Author/ASK)
Rational Numbers in Content and Methods Courses for Teacher Preparation

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Over the past five years Simoneaux, Gray and Golding have been actively involved in the Louisiana Systemic Initiative Program (LaSIP) and the Louisiana Collaborative for Excellence in the Preparation of Teachers (LACEPT) grants through Southeastern Louisiana University. Through these grants teachers from the region are inserviced on implementing the directives and philosophy of MAA's *A Call for Change* (1991) and NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989), and preparation courses for elementary majors are being redesigned. This paper reports some tasks being implemented across the mathematics content and methods courses in one curriculum strand, rational numbers. Elementary education majors at Southeastern are required to take 12 hours in mathematics (algebra, probability and statistics, number sense and geometry) and 3 hours in mathematics education and are certified for grades one through eight.

The rational number domain is one that causes great difficulties for students and their teachers. This struggle with rational numbers has been well documented (Behr, Lesh, Post, & Silver, 1983; Kieren, 1976; Post, Harel, Behr, & Lesh, 1988). Students fail to "internalize a workable concept of rational number" (Behr, Wachsmuth, Post, & Lesh, 1984, p. 323) and therefore their overall performance with rational numbers has been poor. This poor performance may be a direct result of inadequate conceptual understanding on the part of the teacher (Lester, 1984).
While the cause of the lack of conceptual understanding in the domain of rational numbers cannot be attributed to a single source, the curriculum has played a major role. The lack of conceptual knowledge of our teachers has resulted in their delivery of a curriculum which emphasizes procedures rather than understanding (Behr et al., 1983). Students have memorized the algorithms, often incorrectly, but have no knowledge of the concepts underlying the procedures (Mack, 1990). Difficulties with rational numbers are heightened by misconceptions that arise as students try to give meaning to the teacher-taught algorithms (Fishchbein, Deri, Nello, & Marino, 1985; Mack, 1990). As students become exposed to the notion of rational numbers, they attempt to find a connection with something already familiar, like whole numbers. They try to fit the new idea of rational numbers into their existing schemes and frames of whole numbers. When a natural connection is not identified, misconceptions occur as the new knowledge is forced to conform to preexisting schemes (Skemp, 1987). One of the more common misconceptions that surfaces in operations of rational numbers is the student-generated strategy called whole number dominance (Behr et al., 1984). An example of this strategy is students attempting to add rational numbers by adding the numerators and adding the denominators. Studies by Graeber, Tirosh, and Glover (1989) indicate that the misconceptions established by children are not outgrown. Many of our preservice teachers possess the same misconceptions.

Another major factor in poor understanding of rational numbers is the pedagogy used by teachers in the mathematics classroom as well as by the instructors of these teachers. “An individual who has only rote-level mastery of a topic cannot be expected to guide others to more than rote-level mastery of the topic” (Lester, 1984, p. 55). Also, rote-level mastery prior to an understanding of a concept creates an interference to meaningful learning of that same topic (Simoneaux, 1992). Many students and teachers have developed a very weak understanding of rational numbers and have been exposed to
extensive rote instruction on the topic; thus professors of mathematics content and methods courses have a major task to accomplish during the teacher preparation program. Sowder, Bezuk, and Sowder (1993) recommend, "Prospective teachers need to be provided with opportunities to examine their personal understanding of rational number concepts. This should be done in the context of problem situations that force them to confront any misconceptions they carry from earlier experiences, to come to new understandings of connections and relationships that underlie the mathematics of rational numbers, and to reflect on these new understandings and how they were reached" (p. 243). This recommendation is the basis of tasks selected and shared through this paper.

**Number Sense Course: An Alternative Algorithm**

It has been suggested that the formation of units is informal knowledge that can aid rational number understanding (Lamon, 1992). A rational number can be defined as any number of the form m/n where m and n are integers and n is not equal to zero. Children's initial understanding of rational numbers is not derived from m and n, but rather from physical embodiments (Post, Wachsmuth, Lesh, & Behr, 1985). These embodiments might be pictures of a pie cut into n equal pieces with m of them shaded or a set of n circles with m of them shaded. In any case, the embodiment involves partitioning or "fractioning" (Freudenthal, 1983) of some physical or mental object. This object is a unit.

Work by Behr, Harel, Post, and Lesh (1993) has focused on the unit. As children deal with whole numbers, most traditional problems focus on units of one. In the domain of rational numbers, the focus turns to the unit fraction. In an addition problem such as one-third plus one-half the process of finding a common denominator involves the reconceptualization of one-third as two-sixths and one-half as three-sixths. The next step in the traditional algorithm is merely to add the numerators and "bring over" the denominator. Research by Lamon (1992) suggests a more conceptual approach through the notion of unitizing. Unitizing refers to the formation of composite units. That is, the ability to
recognize two-sixths plus three-sixths as 2(1/6-unit)s plus 3(1/6-unit)s. By focusing on the unit, addition and subtraction of rational numbers is merely an extension of addition and subtraction of whole numbers. This provides a natural connection between the whole number domain and the rational number domain.

To aid in conceptualizing unit fractions, students in a mathematics content course for elementary teachers are given fraction pieces. These pieces consist of colored rectangular pieces that represent different fractions. Students spend a lot of time “sizing” each piece based on different units. For example, if the black piece is the “ruler” then the orange piece represents one-half, the purple piece represents one-fourth, etc. If the orange piece becomes the ruler, the black piece represents two and the purple piece represents one-half. By using the fraction pieces students begin to conceptualize a unit fraction as a composite unit. For example, one orange piece is equivalent to two purple pieces (i.e., 1(1-unit) = 2(1/4-unit)s).

After working with the fraction pieces, alternatives to and explanations of the traditional algorithms are formed. The addition of 2/3 + 1/6 becomes 4/6 + 1/6 or 4(1/6-unit)s + 1(1/6-unit). Much like, 3 balls + 5 balls. Students are no longer tempted to add or subtract denominators. A natural alternative to the traditional algorithm for division also surfaces. When given a problem like 9/12 ÷ 3/12, students write

\[ \frac{9}{12} \div \frac{3}{12} = 3. \]

\[ \frac{3}{12} \]

In a problem with different denominators like 3/5 ÷ 2/3 students write

\[ \frac{9}{15} \div \frac{2}{3} = \frac{9}{10} \]

\[ \frac{10}{15} \]

This approach reduces the confusion generally caused by the invert-and-multiply algorithm.
Using the unit approach with operations of fractions provides a nice alternative to the traditional algorithms and promotes conceptual understanding of these algorithms. It also provides a much-needed connection to the domain of whole numbers.

**Number Sense Course: Multiple Embodiments and Use of Writing**

The dialectic that has been blazing during the last ten years since the publication of the Standards (1989) has actually been smoldering in mathematics education for more than thirty years. Mathematics educators accuse the content teachers of teaching algorithms that get answers quickly without using reasoning. Mathematicians, on the other hand, accuse educators of “dumbing down” the curriculum by carrying blocks to class and not teaching the “meat” of the course. This is a very important debate. It is probably better to not take sides, as emphases on teaching styles have been cyclic throughout the history of mathematics education. And after all, there is only one way to teach mathematics, and that is “to teach it well.”

If we focus the debate on the content teaching of rational numbers, then clearly the question is “how best can we assure that future elementary teachers understand and use rational numbers correctly?” The preservice content course in number sense is typically populated with students who have not been taught in the light of any kind of “standards” nor have they been exposed to any type of manipulatives as elementary students. They often do not distinguish well the difference in the fraction 2/3 and the ratio 2:3, among other mangled concepts. Even though they have memorized standard algorithmic processes, many of their misconceptions are revealed in faulty application of these processes.

There is a natural tension in the mathematics content classroom. The instructor would prefer to do problem solving, but the students cannot perform the operations necessary to find answers. When students want quick fixes (“Just show me how to do it, I don’t want to understand why it works!”), it sometimes seems a more efficient use of class
time to lecture on the system of rational numbers with its structure, formal operations and rules. If the content teacher can ignore this impulse, the students who allow themselves to discover meaning will be most grateful.

Ten years ago, a search for some good ideas for teaching rational numbers to middle school students for use at a workshop revealed some interesting discoveries. Old copies of The Arithmetic Teacher from the 1950s revealed that even before the "new math" of the 1960s it was suggested that students need manipulatives for learning fractions. Several articles (Glenn, 1957; Hoffman, 1958) described unit strips, circle pieces for the felt board and colored counters among other models for teaching fractions. The authors seemed to be well aware of the four principles of Zoltan Dienes' Theory of Mathematics - Learning (Dienes, 1960). The most fascinating of these principles was what Dienes called "multiple embodiment" (p. 44). He stated that "multiple embodiment means that every concept should be presented in as many different ways as possible" (Dienes & Golding, 1971, p. 55).

If students only use the manipulatives which involve strips in different colors to represent a unit and parts of a unit, they may begin to think of \( \frac{1}{3} \) as "the orange piece." And according to Dienes' theories "it is important for us to understand that the learning situation which is ideal for one child may not be right for another" (Dienes & Golding, 1971, p. 56). Students in preservice content courses must be given the opportunity to see multiple embodiments of concepts.

Hence, to help develop good rational number concepts, one may want to use fraction bars and fraction strips, but also lay out 4 foot long unit strips on a board or table top that are marked as shown here:
To find the sum of $\frac{1}{2}$ and $\frac{1}{3}$, for example, students use a long piece of packaging string to measure the distance from 0 to $\frac{1}{2}$, and then, holding a finger at the $\frac{1}{2}$ mark on the string, they tack on the length from 0 to $\frac{1}{3}$ so that the string length is now the sum of the two distances. They can move the length of string along the various rulers to find a match for $\frac{1}{2} + \frac{1}{3}$. A few “ah-ha’s” are audible signs that convince the instructor that some students did not get the concept that others saw as obvious with the unit strips.

Content instructors should also use commercially prepared pizza pieces and two-colored chips. Students ought to construct circle wheels, and use transparent fraction grids to show multiplication with an area model. In spite of all of these excellent models, it is still possible that some students will not have a good concept of fractions or of operations with them.

After the students have had a chance to use the area model for fractions, a good problem for a quiz might be:

Antonio owns a very popular pizza business. He decides to give $\frac{1}{3}$ of the business to his son and $\frac{1}{4}$ to his daughter. Use an area model to show how much
of the business Antonio keeps for himself. Tell whether he keeps more or less than
one-half of the business for himself. How do you know that?
A typical error in this problem shows that Antonio still has _ of the business:

\[
\begin{array}{|c|c|c|}
\hline
\text{Son} & \text{Daughter} & \text{Daughter} \\
\hline
\text{Son} & \text{Daughter} & \\
\hline
\text{Son} & & \\
\hline
\text{Son} & & \\
\hline
\end{array}
\]

Antonio’s Business

This student has confused a model for multiplication with a model for addition.

Sometimes students will add the two fractions using the standard algorithm and get 5/12 for Antonio’s part, but fail to see the inconsistency between their answer and the area model.

Another way to use the multiple embodiment principle is in the use of writing for the purpose of discovering what the students understand. This type of writing is not free, but prompted by a statement to which they can react. The directions encourage the students to write at least three sentences in response to the given statement. (In addition, data on
this prompt is currently being gathered from students at the beginning and the end of the mathematics education course which follows this number sense course.)

A typical prompt given before the study of rational numbers is:

Write two or three sentences commenting on this statement written by a student:

"I think 0.8 is about an eighth."

About half of the class did give satisfactory comments. Some of them were incensed that anyone would have such a silly misunderstanding. Some even drew pictures of circles and shaded in $\frac{8}{10}$ and $\frac{1}{8}$ to show that the two are not the same. Here is a selection of a few typical faulty responses to show how many misconceptions can be formed.

1) I think that 0.8 is about an eight because of its position behind the decimal. The 8 is in the tenths place.

2) I think 0.8 is about an eighth because 10 is ten, therefore 0.8 would be an eighth because it is not 10.

3) 0.8 is about an eighth of an inch because we are told it is an eighth of an inch.

4) Yes, because to get to the number 1 there is 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8. 0.9, 1. 0.8 is one eighth of 1.

5) I am no good with decimals or fractions but I do not believe this statement is correct. In my mind (which does not think in math terms), I would think that $\frac{1}{8}$ would be better defined as .08. I remember learning how to convert fractions to decimals in elementary school but I haven't done it in years and cannot remember the process.

6) 0.8 is not a whole #; therefore, it is not 8. It is in the tenth's place, and is read as an eighth.

7) I think this sentence is true because 0.8 means eighth. But an eighth of what? (inch, foot, sandwich). Also because of the point it means gth.

8) 0.8 is an eighth of a tenth. This statement is partially true.
Another example of a prompt with some responses is:

**Write a decimal between 3.2 and 3.21. Comment on this.**

1) **Impossible** I don't think it is possible because there is nothing that comes between 3.20 and 3.21. Without anything behind the 3.2 you would assume it is 0.

2) **3.2** I could not think of anything else. I started to put a third number behind 3.2, but you've already done that.

3) _____ I didn't realize there was one. Is this a trick question?

4) **3.202** This caused me much anxiety.

5) _____ This sentence is asking to put a decimal between 3.2 and 3.21, which could be interpreted in two ways. One way is 3.2 . 3.21. The second way is to think about the exact measurement between the two numbers.

At the beginning of the unit on rational numbers, these responses will be shown to the class so that they can see what students learn or remember from current instructional practices.

**Mathematics Education Course: Teaching for Understanding**

To help overcome math anxiety about and develop further understanding of rational numbers, five components are a part of the syllabus of the elementary mathematics education course. The components include a math-autobiography, a diagnosis of a child (In this paper “child” refers to an elementary student; “student” refers to a preservice student,) on his or her understanding of rational numbers, a two- to three-hour model lesson on teaching rational numbers and a performance-based oral exam on multiplication and division of rational numbers.

The first assignment of the semester is the math-autobiography. The students are asked to reflect on their personal mathematics experiences and write a two-page essay describing the earliest mathematics experience they recall, their best experience, their worst experience, pedagogy they themselves experienced that they know is effective, a pedagogy
they know should be eliminated at all costs, and finally they identify their place on a continuum of one to ten indicating their phobia about mathematics. Many preservice elementary education majors have very severe math-phobias; many of the phobias center around fractions. Despite the very creative approaches to rational numbers that some of these students presently experience in their mathematics content courses, very few students feel confident in their understanding or their ability to teach rational numbers. Recognizing and acknowledging one's fears is a first step to overcoming that fear.

Another early semester assignment in the mathematics methodology course is the diagnosis of a middle school child on his or her understanding of rational numbers. To prepare for this assignment, the instructor diagnoses a child while the preservice students observe.

The instructor's plan for diagnosis is shared with the students. The students' task is to diagnose a middle school child on rational number concepts and skills and to write a one-page report discussing what the child knows, what he or she does not know, and the preservice student's reaction to the experience. The students are allowed to use the instructor's plan for diagnosis. The purpose of this task is to motivate the students to understand rational numbers, to experience some of the many facets of understanding rational numbers, and to have the students experience and be amazed by the thinking of a child. (The instructor believes that if a teacher becomes fascinated with the thinking of one child, he or she will perhaps seek to have that same experience with each child in the classroom.)

Very soon after the demonstration of the diagnosis of a child, the instructor conducts a two to three-hour lesson on rational numbers. The purpose of this lesson is to deepen the students' understanding of rational numbers through experiencing tasks that can be duplicated at least in part in any elementary classroom. This lesson includes the development of rational number concepts, comparing of fractions and finding equivalent
fractions through the construction of fraction tiles (This lesson will be submitted to one of the NCTM school mathematics publications within the year) using different colored construction paper strips, fractional parts of sets and pattern blocks. Modeling operations on rational numbers with fraction tiles is also a part of this lesson. In the evaluation of the course, students usually give this lesson a perfect score. The lesson includes questions like the following: Which is larger, _ or 1/4?; How much larger is it?; Which is larger, 7/8 or 6/7? Can you defend your answer?; Which is larger, 9/8 or 8/7?, Can you prove it?.

Reflections, reasoning, communication by the students and connections to other fractional concepts are also incorporated throughout this lesson. The instructor applies much effort to accepting each student’s level of understanding and encouraging each student to have the opportunity to share thinking.

Finally, one day is set aside for the preservice students to take an individual oral exam. The exam takes approximately five minutes per student. Several simple multiplication and division of fraction problems are placed face down on the table. The student randomly selects one problem for each operation and, using the provided fraction tiles, demonstrates and explains the meaning of the problem and its solution. Sample problems include the following: _ x 1/4, 1/4 x _, _ + 1/4, and 1/4 + _. The exam is worth one-tenth of the course grade. The rubric for the exam follows.

Oral Exam Rubric

_____ (A) Did an outstanding job. Knew the mathematics and the manipulatives. Fluid. Used good vocabulary and clear explanations.

_____ (B) Did a good job. Knew the mathematics and the manipulatives. Used minimum explanations.

_____ (C) Adequate. Knew the mathematics and the manipulatives. Had some trouble with appropriate vocabulary and/or connections.
Knew the mathematics. Had some trouble with manipulatives. Had problems with explanations.

Did not know the mathematics. Got confused. Faltered.

Conclusion

We believe that understanding rational numbers is one of the most important objectives in the elementary school curriculum and yet, presently, is one of the least understood. It is therefore imperative that content and methods instructors on the same campus work together to develop programs that empower future teachers to break the present cycle that results in inadequate development of the rational number system. Much research is available on rational number teaching (Behr et al., 1992; Bezuk & Bieck, 1993; Carpenter, Fennema & Romberg, 1993; Langford & Sarullo, 1993). These methods need to be applied to college mathematics content courses so that future teachers themselves understand rational numbers, and good pedagogy must be experienced, planned and practiced in mathematics education courses if change is to occur.

References


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