DOCUMENT RESUME

ED 445 909 SE 063 974

AUTHOR	Campbell, Stephen R.
TITLE	Bringing Insights from Research into the Classroom: The Case of Introductory Number Theory. Discussion: Treating Symptoms?
PUB DATE	2000-00-00
NOTE	14p.; Paper presented at the Annual Meeting of the Association of Mathematics Teacher Educators (Chicago, IL, January 1999).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Elementary Education; Higher Education; *Knowledge Base for Teaching; *Mathematics Education; *Number Concepts; Preservice Teacher Education; Preservice Teachers

ABSTRACT

This discussion focuses upon potential implications of research conducted on preservice teachers' understanding of introductory topics from elementary number theory. Comments interlace three levels of consideration and recapitulate what is striking as some of the most interesting and important findings raised in the presentations, and flesh out some of the theoretical presuppositions underlying these findings. The potential of these findings with respect to practical classroom applications is also discussed. Finally, consideration is given as to why these kinds of phenomena are observed, what may be responsible for their emergence, and what might be done differently in elementary mathematics teacher education. (Contains 14 references.) (Author/ASK)

Bringing insights from research into the classroom:
The case of introductory number theory
— Discussion: Treating symptoms? —

Stephen R. Campbell
University of California at Irvine

Summary:
In this discussion I focus upon potential implications of the research my colleagues and I have conducted in the area of preservice teachers' understanding of introductory topics from elementary number theory. My colleagues have presented some of our findings, and provided examples of how our research has influenced our practice as teacher educators. My comments interlace three levels of consideration. I recapitulate what strikes me as some of the most interesting and important findings raised by my colleagues in their presentations and flesh out some of the theoretical presuppositions underlying these findings. I will also discuss the potential of these findings with respect to practical classroom applications. Finally, I give some consideration as to why we are observing these kinds of phenomena, what may be responsible for their emergence, and what we might do differently in elementary mathematics teacher education.

Introduction:
Much research in mathematics education of late has focused upon, or at least acknowledged, differences between informal, "day-to-day," concrete meanings of words with the more formal abstract mathematical meanings of the same or similar terms. From a linguistic perspective, e.g., Durkin and Shire (1991), such ambiguities can be taken to signal a difference in the "natural" or "mathematical" registers in which such terms are evoked: that is to say, the realms of discourse in which these terms are used. From a cognitive perspective the difference between informal and formal meanings signal a difference between concrete lived experiences and abstract procedures and concepts. Reconciling differences between informal and formal meanings of mathematical symbols and terms can present mathematics educators with a wide variety of pedagogical options.

The simplest approach, of course, and traditionally perhaps the most prevalent, is to ignore differences between informal and formal meanings altogether. An approach such as this is typically evident when mathematical terms are introduced through formal, or even quasi-formal, definitions with the hope that learners will develop a meaningful understanding of those terms through their contextual use in calculation. Indeed, as our research resoundingly indicates, preservice teachers' understandings of symbols and terms from number theory appears to be procedurally bound in exactly this way.

Another approach that also tends to ignore differences between informal and formal meanings of mathematical terms is to focus on applications of mathematics to more concrete real world, or quasi world, situations. Such an approach, which usually exists in tandem with a procedural bound approach of manipulating symbols and buttons, yields results that situate the more formal meanings of mathematical terms exclusively within the natural register of the problem context in which they have been applied. However, little or no insight is gained by the students as to what mathematics is and why it works.

Procedural and problem-solving approaches, in and of themselves, may do little to resolve whatever conflicts that natural and concrete meanings of the symbols and terms used in mathematics have with their associated formal meanings. More progressive approaches to resolving the differences between informal and formal meanings of mathematical terms along with their associated concepts typically involve building upon and/or transforming the former into the latter. I will defer further discussion of this issue for now, as it has not been a central focus of our research to this point. Rather, to date, most of our research in this area has focused on subject content. That is to say, we have focused mainly on what preservice teachers know and do not know about introductory number theory and elementary arithmetic.

The purpose of this symposium is, on the basis of our research into preservice teachers subject content knowledge, to explore what can be done about improving these students' understanding of this material and their ability to learn it. It is evident to my colleagues and I, and perhaps now to some of you as well, that our research readily translates into practical methods for teacher educators in teaching subject content courses in mathematics to elementary school teachers. As you have seen, some insights from that research have been presented that have informed our own practice as teacher educators. I will now to discuss some of those insights in more detail. I will then attempt to carry the implications of our work a step further.

On Divisors and Quotients:

In the opening presentation of the session, Rina Zazkis pointed out that some of the most common terms of mathematics involve ambiguities, not just between the natural register and the mathematical register but rather, within the general register of mathematics *itself* and, more specifically, within the "sub-register" of elementary classroom mathematics as well. In particular, she considered the conflict in the meaning of the term "divisor" as it is commonly used in arithmetic and its more specialized meaning in number theory. For example, in the former 5 *is* the divisor in the expression 12÷5, however, 5 *is not* a divisor of 12 in the latter. From a linguistic perspective, this would seem to implicate yet other kinds of mathematical "sub-registers" (i.e., sub-registers of arithmetic, number theory, geometry, algebra, etc.). Zazkis went on to consider two interpretations of divisor, one that "attends to the number's role in a binary operation," and another that "attends to a relation among numbers." She suggests that the intended meaning of the term "divisor" can be identified by attending to the grammatical context within which it is used: The phrase "the divisor *in*" (a particular equation or expression) referring to its role as part of an operation, and the phrase "a divisor *of*" referring its role in relation to another number. See Zazkis (1998) for a more detailed treatment of these issues.

Zazkis noted a more troublesome ambiguity in the meaning of quotient within (what I will refer to here as the "sub-register" of) arithmetic itself. For instance, the quotient of 12 divided by 5 can be expressed in different ways depending upon whether or not one is referring to whole number or rational number division. Perhaps this would provide grounds for considering further numerical domain specific "sub-sub-registers" within the "sub-register" of arithmetic as well? Perhaps and perhaps not. The criteria required for a more detailed stratification of linguistic registers is not readily evident to me. Aside from the case where a "remainder" is specifically mentioned or requested, there appears to be no definitive set of grammatical queues as to which kind of division is being referred to apart from simply being more explicit about what kind.of division is intended. However, even when a whole number quotient is implicitly requested, as our research has shown, many students seem to remain caught up with the notion that the term refers to the result of rational number division. Moreover, even when whole number division is explicitly referred to, Zazkis has shown that there is a propensity for students to continue to interpret division as rational number division. These phenomena may correlate with the development and entrenchment of an implicit belief that the outcome of an arithmetic operation must yield a singular result. After all, how often do teachers speak of the quotient and remainder as the "results" of a whole number division?

On LCM's, Prime Decomposition, Divisibility, Factors, Divisors, etc.

Anne Brown, Georgia Tolias, and Karen Thomas presented results stemming from their collaborative study of their students' understanding of least common multiples (LCM). Their research was motivated by an interest in students' use of prime factorization to determine LCM's "without having any idea why it produced the correct answer." This work led to a further exploration of their respective students' understanding of subsidary concepts involved in deriving LCM's such as multiples, factors, divisors, and divisibility.

The standard approach to learning to calculate LCM of two numbers is first to decompose those numbers into their prime factor representation. Once that is done, either by using "factor trees," or by some other means, the student is instructed to take the primes with the largest exponents from each decomposition and construct a new number, the desired LCM. An obvious advantage of this approach is that one can readily adapt it to obtain the greatest common factor (GCF) simply by taking primes with the smallest exponents instead. In either case, why the procedure actually works is either not questioned or, to the extent it is questioned at all, remains a mystery to students.

It should not be surprising that many are mystified as to why mathematics works. Why mathematical procedures work quite often remain a mystery to both students and teachers alike. This is hardly a trivial problem. Some professional mathematicians even claim to rely solely on formal definitions and procedures as the basis of their understanding of mathematics. Indeed, formalism, the belief that mathematics is nothing more than a consistent set of formal definitions and procedures, in and of itself, has traditionally constituted a major position in the history of the philosophy of mathematics.

Be this as it may, mathematics educators are generally challenged with the task of making mathematics meaningful to students. If mathematics can not be made meaningful in and of itself, then meaning must come, well, by some other means. This usually translates, as noted above, into problem-solving involving various admixtures of procedural calculations and contextual applications. If one is to make mathematical concepts meaningful in and of themselves, one must first ask what is a mathematical concept. Many argue that mathematical concepts, such as the elementary arithmetic concept of whole number, are fictional and not real. Terms like "one," "two," and "three," are names given to abstract representations of collections of real objects.

To have numerical symbols and terms "stand" for concrete but arbitrary collections and magnitudes is one thing. That much, at least, seems reasonably meaningful. However, once one begins to define new operations, derivative concepts and representations, such as negative collections and magnitudes, their meaning can become progressively more difficult for teachers to articulate and for learners to grasp. When students have difficulty understanding higher order mathematical concepts, it would seem that the only viable recourse is to turn to the mathematical concepts and procedures upon which they are based. This is the approach that we have all been taking in our research. In our research on divisibility and prime decomposition, Zazkis and I quickly found ourselves having to focus on students' understandings of divisors, factors, multiples, etc. Likewise with

Brown, Tolias, and Thomas in their study of students' understanding of LCM. Clearly these introductory concepts and procedures of elementary number theory stand in some relation to each other, but those relations can often be interdependent, subtle and complex. If there are natural hierarchical progressions by which an understanding of one concept or procedure can be built upon others, they can, at times, be far from obvious.

As a case in point, we were posed with an interesting problem designed by Anne Brown. We were asked to determine subsequent elements in a sequence of numbers in their prime factored representation:

$2^2 \times 3^4, 2^3 \times 3^4, 2^2 \times 3^5, 2^4 \times 3^4, 2^2 \times 3^4 \times 5, 2^3 \times 3^5, \ldots$

The answer to this wonderful problem proved very challenging for most of us to find. In observing myself and those around me, it was evident that the natural thing to do was to search for a pattern amongst the numbers given in the sequence. Upon inspection, though, there are no obvious patterns between the primes and exponents in this sequence. This difficulty gave rise to a temptation to transform these numbers into their decimal representation with the hope that a pattern would emerge. However, even without the assistance of a calculator, it is readily evident that we were dealing here with some pretty big numbers. As it turns out, of course, this kind of problem is much easier to construct than it is to solve. Moreover, this problem is constructed in a way that attempts to defeat our previous experience in solving sequence problems. Speaking for myself, the key to my solving this one was based on many enjoyable, yet fruitless, hours trying to find a pattern in the sequence of prime numbers. If a predictable pattern for the primes exists, it has yet to be found. This immediately led me to think that this problem was somehow exploiting *that* problem—and then, suddenly, all was revealed. Eureka!

Fortunately, the prerequisites for understanding some things are more evident than others. Brown, et al., noted that for students to understand the procedure for determining LCM's and GCF's requires more experience with, and a better understanding of, the prime decomposition of whole numbers. Thus they were led to develop exercises—that ask if one number is divisible by another—that require transforming one or more of those numbers from one representation to another. Prime decomposition and divisibility, however, open up whole new levels of problems in students' understanding (See Zazkis and Campbell, 1996a, 1996b). These concepts and procedures, in turn, open into more basic levels of understanding factors, multiples, divisors. These introductory concepts of number theory eventually collide with the more familiar meanings of these terms with respect to the operations of arithmetic multiplication and division (Campbell, 1998).

Lessons for the Classroom

Collectively, our research in this area has helped to bring out the deeply layered and textured nature of understanding even the most basic of concepts and procedures of elementary number theory. The sedimented nature of understanding concepts such as LCM's, divisibility, and prime decomposition, has been brought to light mostly via a regressive analysis of these concepts. That is, an analysis of the concepts, procedures, symbols, and terms, that they seem to be associated with, constructed from or dependent upon. Our motivation for proposing this symposium, however, was to provide some indication as to how this research can serve to inform our practice as teacher educators, and my colleagues have presented some important insights in this regard from their own classroom experiences with preservice teachers.

Zazkis suggested that, on the basis of what we now know, we must confront and resolve ambiguity in the classroom wherever we find it: even within the mathematical register itself. As we have seen, this is particularly important with respect to terms and concepts associated with arithmetic division. Zazkis demonstrated for us her "democratic" approach to allowing learners themselves bring these ambiguities to light. The ambiguities are partially resolved through individual student "research" and collective "debate," then ultimately, if any disagreement remains, she resolves the issue by "laying down the law" regarding the intended meanings to be adopted by the class.

Tolias presented an approach to determining LCM's (and by extension, GCF's) that first provides students with more experience with and a better understanding of prime factorization and the concept(s) of multiple (factor and divisor). Then, starting with one of the two numbers, that number is progressively multiplied by factors from the other number that are "missing" in the first number. The idea is that the student should be in a better position to see 1) that the result of each step is a multiple of the first number, and 2) that once the first number has been multiplied by all the missing factors of the second number, that the result finally becomes a multiple of that second number as well. Laying such groundwork will hopefully give students a better chance of developing a more meaningful understanding why the LCM is not only a common multiple of two numbers, but is also the *least* common multiple at that. This approach for LCM is readily adaptable for helping students to develop a more meaningful understanding of GCF's as well.

Brown, et al., also brought out an issue close to my heart: the potential utility of the quotitive model of division for understanding divisibility and whole number division. It has been commonly assumed in research in mathematics education that the partitive model is more germane to understanding division with a whole number divisor and that the quotitive model is more germane to understanding division with rational number divisors — particularly those which fall between zero and one. This is an understandable assumption to make, especially if one fails to distinguish between whole number division and rational number division. Be that as it may, even if one is careful to make such a distinction, it seems tempting to assume that this propensity to relate the partitive model to whole number divisors and quotitive model to rational number divisors applies more generally to whole number and rational number division as well. Elsewhere (Campbell, 1998), I have argued that exactly the opposite should be the case. Indeed, the fact that the quotitive model is directly applicable to teaching whole number division, is immediately evident from the form of the division algorithm ($A=QD+R$, where $0 \leq R < D$), which serves to define that operation (and hence, when $R=0$, divisibility as well). Moreover, it can be seen that the partitive model assumes rational number division insofar as model assumes a rational quotient (the partitive quotient is not constrained to be a whole number). As amazing as it might sound, many, if not most, preservice elementary school teachers have no formal or intuitive understanding of the concept of a multiple. The quotitive model of whole number division, as exemplified by the division algorithm, offers teacher educators an important pedagogical tool for addressing this deficiency.

As the designated discussant for this symposium, I'll now take some license in stepping out upon a more speculative limb, although I am not sure to what extent my colleagues would wish to join me. Let's say that we are all in agreement that insights into students' understanding such as these are important and should be implemented in teacher education. If it is true that teachers have a tendency to teach as they were taught—and even if it isn't—I think it is of the utmost importance to reflectively and critically consider how germane these very same insights may be for teaching children. I am particularly interested in discussing what the practical implications *of that* would be for both the elementary school curriculum and how we approach teaching mathematical *methods* courses to elementary preservice teachers.

Treating Symptoms or Causes?

It appears that many difficulties young students confront in understanding arithmetic in their earlier years may be left unresolved into adulthood. The immense number of potential permutations of unresolved obstacles to understanding elementary arithmetic may account for the wide diversity of difficulties encountered our research into preservice teachers' understanding of number theory. For instance, in the more abstract contexts of symbolic computation it is well known that middle school learners' understandings of connections between whole, fractional and decimal numbers can be quite problematic (e.g., Mack, 1995; Markovits & Sowder, 1991; Resnick et al., 1989). Greer (1987), quoting Hiebert and Wearne (1983, April), notes that learners "... see little connection between the meaning of a decimal and a whole number or a fraction." For preservice teachers, my own research (Campbell, 1998) indicates that this seems especially true regarding what Silver (1992) has referred to as "different forms of expression for remainders" (e.g., 10R1, 10 1/2, or 10.5).

Much of our research has been conducted into preservice teachers' understanding of procedures and concepts that they have already been "taught." Simply put, most of these students simply aren't getting it. Why? it is evident that the difficulties preservice teachers have in understanding introductory concepts of number theory can run deeply right into the heart of their understanding of basic arithmetic. Surely, the insights our research brings to the classroom will help us at least to "patch up" some of the difficulties preservice teachers are having with understanding the introductory concepts of elementary number theory. And there are certainly better connections that we, as teacher educators, can help students make as well. We can also be much more careful in explaining what is intended by the symbols and terms that we use. But that, in itself, may not be enough. What I believe our research is suggesting is that, as teacher educators, we may have to go back and start over, right from the beginning, with the basic concepts and operations of whole number arithmetic. It is, after all, this material that preservice teachers will soon be teaching to kids. If so, the question is how are we to do that?

We can certainly focus, as we have been here, on improving preservice teachers' formal, abstract understanding of arithmetic and number theory. I think we must ask ourselves, if it is the case that teachers tend to teach in the way that they were taught, if this will help. That is, are we really addressing the cause of preservice teachers' poor comprehension and poor retention of introductory concepts of elementary number theory, or are we, as I believe our research suggests, just addressing symptoms of a deeper problem? If we are just addressing symptoms, if we are just putting better patches on preservice teachers' understanding of formal abstract mathematics, then we are not doing what we need to do as teacher educators. If we are treating sympoms rather than causes then we are not getting to the bottom of the problem and we may not, in the long term, be able to say that we have made a significant difference in mathematics education.

If we are just treating symptoms of a deeper problem, what is that problem? If it is a pedagogical problem, then teaching preservice teachers formal abstract mathematics may not be too helpful to the children they will be teaching. Besides, matters of pedagogy are usually treated in methods courses, not content courses. To what extent will a better formal understanding of subject content inform elementary teachers in their teaching of mathematics? It simply does not follow that a good formal understanding of mathematics is sufficient to effectively and successfully teach mathematics to a child. Indeed, it may not even be necessary. In the elementary grades, what is important, in my opinion, is that the mathematics that is taught, is taught in a manner that is meaningful and comprehensible to children. Presumably this is what methods courses are all about.

What impact then are treating these symptoms likely to have on pedagogy? I don't know the answer(s) to that question, but I suspect that the impact could just as readily be negative as positive. Negative for reasons I alluded to above, if teachers tend to teach like they have been taught. Do we really want elementary preservice teachers teaching mathematics to children subject content in the same ways in which we are teaching them? Having said that, it would be absolutely foolish to claim that an improved understanding of formal abstract mathematics wouldn't be helpful to a prospective teacher! It is always helpful to have had some previous experience with your destination. The point that I am trying to make is that that experience may not be too helpful if you don't know where you started from or how you got there. And even if you do, it may not be too helpful if you can't show others how to get there in a way in which they are able to follow.

Identifying the Problem:

The low levels of elementary preservice teachers' understanding of elementary mathematics is evidence of the overall ineffectiveness of their own K-12 mathematics education. If treating the symptoms of this problem is not likely to have much impact on how these teachers teach children more effectively, then it is not too likely that the problem, or problems, whatever they are, are going to be fixed for the next generation. The cycle will remain unbroken. But I think it would be better to say that the *broken cycle will remain unfixed*. I say this because I think the problem is the one I raised in my introductory comments: mathematics education in general has failed to help learners successfully make the transition, bridge the gap, (or however one wishes to put it) between concrete, informal, lived experience, and abstract, formal, conceptual understanding of mathematics.

There seems to me, as I have stated elsewhere (Campbell, 1998b), and I think it bears repeating, to be a common implication, too readily drawn by traditionalists and progressivists alike: given an appropriate amount of experience with problem-solving and calculation, a conceptual understanding of mathematics will, somehow, ensue. Many traditionalists seem to think that if a learner is well grounded with basic computational skills, eventually the learner may experience some form of conceptual enlightenment. A learner will either "get it" or not. Many progressivists seem to think that making the appropriate "connections" between symbols and real world experiences, gained through problem-solving, is what eventuates conceptual understanding. Here again, whatever those connections happen to be depends, for the most part, on the learner.

There seems to be little doubt that most children can be quite proficient at memorizing how do a particular calculation. They can also be very good at emulating how to apply the results to a particular problem. Such things are relatively easy to teach, and place relatively little responsibility upon teachers' own conceptual understandings of mathematics—especially in the elementary grades. However, without an understanding of the underlying mathematical concepts involved, it is difficult, if not impossible, to properly understand those procedures, or to recognize new ways of applying them in problem-solving contexts. This, I think, has implications for both elementary mathematics education and elementary teacher education in mathematics.

I take it to be a common educational goal motivating traditionalists and progressivists alike to promote a better conceptual understanding of mathematics in learners. I think that there are important elements of truth in both the traditionalist and progressivist views regarding mathematics education. However, I also think that the underlying problem may be that proponents at both ends of this spectrum share impoverished visions regarding the conceptual nature of mathematical understanding and how to properly nurture it. Unfortunately, it seems to me that all points between these extremes suffer from an impoverished view of what kind of curriculum and pedagogy will more effectively enable learners to develop a meaningful, experientially grounded, conceptual understanding of mathematics; an understanding that would help learners to make better sense out of both algorithms and applications, rather than relying primarily upon algorithms and applications to make sense out of concepts.

Why do we have this problem and how might we go about fixing it?

I think we have this problem of bridging informal and formal mathematical understanding (and here, of course, I am most concerned with the basic concepts of arithmetic and number theory) because we have forgotten how we came into this understanding in the first place. Mathematical understanding wasn't developed overnight. It wasn't handed down to us mortals by some Promethean happenstance. Unfortunately, however, even when considering the matter historically, it would seem most mathematicians and mathematics educators think of mathematics, essentially, as we know it today, began with Euclid and his Elements. The reality of the matter is that the development of mathematical understanding has occurred over thousands of years. Moreover, Euclid's Elements represents a compilation of work that took centuries to develop. Yet we tend to teach mathematics today as if it has existed forever in the state that we currently teach it.

When we puzzle over how to bridge the gap from informal lived experience and formal conceptual understanding of mathematics, I think it would be helpful look into our cultural history, as it may offer an exemplary case of how that can be done. At the very least, we can take solace in the fact that we at least have an "existence proof" that the task can be done. I think it is important to understand, to the extent that is possible, how it was that humanity in general, and Western culture in particular, came into a conceptual understanding of mathematics. I believe that if we approach this question in the right ways, that the answers we get may serve to inform both our views on the mathematics curriculum and mathematical pedagogy, especially in the early grades.

I will close this discussion by briefly providing an example of what I mean. In my study of ancient Greek thought, I have found that the conceptual evolution of the most fundamental concept of mathematics, the concept of a unit, has undergone at least three major shifts (Campbell; 1996, in press). Perhaps the most important shift was from thinking of units as concrete objects, such as pebbles and sheep, to a Pythagorean way of thinking that considered units as the fundamental bits of matter composing all things. These little units had no attributes to speak of except spatial extension. This theory formed the foundation of what we refer to today as the atomic theory of matter. The second shift was Plato's removal of this last sensible attribute of spatial extention, consequently elevating the unit to the status of a pure conceptual entity. This unit was also given a purely conceptual attribute: indivisibility. The third shift was Aristotle's conceptualization of the arithmetic unit as a divisible unit of measure.

Understanding what motivated these conceptual shifts, and how they came about can inform mathematical pedagogy in the early grades. Understanding that we are dealing here with fundamentally different concepts of the unit should alert us to be more careful in distinguishing between them in our teaching. Yet, today, it is a common didactic practice to consider whole numbers, for all practical intents and purposes, as a subset of rational numbers (e.g., Freudenthal, 1983, p. 103). This practice carries with it the implication that any particular whole number is also a rational number and therefore can, both in principle and in practice, be divided. To conflate these aspects of number is to lose sight of the historically and conceptually crucial distinction between divisible and indivisible units in pre-Euclidean Greek mathematical philosophy. More importantly, such conflation appears to increase the potential for many of the conceptual confusions and semantic ambiguities that we have been trying to clarify and resolve here today.

Campbell, S. R. (1996). On preservice teachers' understandings of division with remainder. In L. Puig & A. Gutiérrez (Eds.), *Proceedings of the 20th International Conference for Psychology of Mathematics Education* (vol. 2; pp. 177-184). Valencia, Spain: Universitat de València.

Campbell, S. R. (1998). *Preservice teachers' understanding of elementary number theory: Qualitative constructivist research situated in within a Kantian framework for understanding educational inquiry.* Unpublished Ph. D. Thesis, Simon Fraser University.

Campbell, S. R. (in press). *The problem of unity and the emergence of physics, mathematics, and logic in ancient Greek thought.* Paper presented at the 1997 International History and Philosophy of Science and Science Teaching, Calgary, AB, Canada.

Durkin, K., & Shire, B. (1991). Lexical ambiguity in mathematical contexts. In K. Durkin & B. Shire (Eds.), *Language in Mathematical Education: Research and Practice* (pp. 71-84). Milton Keynes: Open University Press.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Boston, MA: Dordrecht, D. Reidel Publishing Company.

Greer, B. (1987). Nonconservation of multiplication and division involving decimals. *Journal for Research in Mathematics Education, 18*(1), 37-45.

Hiebert, J., & Wearne, D. C. (1983, April). *Student's conceptions of decimal numbers.* Paper presented at the American Educational Research Association, Montreal.

Mack, N. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research in Mathematics Education, 26*, 422-441.

Markovits, Z., & Sowder, J. T. (1991). Students' understanding of the relationship between fractions and decimals. *Focus on Learning Problems in Mathematics, 13*(1), 3-11.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education, 20*(1), 8-27.

Silver, E. A. (1992). Referential mappings and the solution of division story problems involving remainders. *Focus on Learning Problems in Mathematics, 14*(3), 29-39.

Zazkis, R., & Campbell, S. R. (1996a). Divisibility and multiplicative structure of natural numbers: Preservice teachers' understanding. *Journal for Research in Mathematics Education, 27*(5), 540-563.

Zazkis, R., & Campbell, S. R. (1996b). Prime decomposition: understanding uniqueness. *Journal of Mathematical Behavior, 15*, 207-218.
Zazkis, R. (1998). Divisors and quotients: Acknowledging polysemy. *For the Learning of Mathematics*, 18(3), 27-30.

SE063974

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
[Image] [Image]
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

Reproduction Release
(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: Bringing Insights from Research into the Classroom: The Case of Introductory Number Theory

Author(s): Stephen R. Campbell
Corporate Source: University of California, Irvine
Publication Date: 2000

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, Resources in Education (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign in the indicated space following.

The sample sticker shown below will be affixed to all Level 1 documents	The sample sticker shown below will be affixed to all Level 2A documents	The sample sticker shown below will be affixed to all Level 2B documents
[Image]	[Image]	[Image]
Level 1	Level 2A	Level 2B
[Image]	[Image]	[Image]
Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g. electronic) and paper copy.	Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only	Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits.

If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche, or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Signature: Printed Name/Position/Title:

Stephen R. Campbell
Professor

Organization/Address: 2001 Berkeley Place, Irvine, CA 92697-5500 Telephone: (949) 824-7736 Fax: (949) 824-2965

E-mail Address: sencael@uci.edu Date: October 26, 2000

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:

Address:

Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:

Address: