Best practice is now understood by many to include reporting effect sizes when submitting manuscripts for publication since some (and potentially more) journals in the education field require, or strongly encourage, authors to report magnitude of effect measures with their statistical interpretation discussion. Therefore, it is important that researchers familiarize themselves with various effect size measures and how to interpret them. This paper reviews many of the effect size statistics available, including both corrected and uncorrected measures such as eta squared, omega squared, epsilon squared, the Wherry formula, the Herzbung formulas (both fixed and random models), and the Lord formula. These statistics are explained and demonstrated with heuristic data. Several recommendations are presented to facilitate appropriate interpretation of effect sizes, including the consideration of estimate bias, fixed or random effects design use, and whether analyses were conducted using univariate or multivariate statistical approaches. One table shows actual calculations using general linear model analyses. (Contains 42 references.) (SLD)
Understanding and Interpreting Effect Size Measures in General Linear Model Analyses

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Abstract

Best practice is now understood by many to include reporting effect sizes when submitting manuscripts for publication since some (and potentially more) journals in the education field require (or strongly encourage) authors to report magnitude of effect measures with their statistical interpretation discussion. Therefore it is important that researchers familiarize themselves with various effect size measures and how to interpret them. This paper reviews many of the effect size statistics available, including both corrected and uncorrected measures such as eta squared, omega squared, epsilon squared, the Wherry formula, the Herzburg formulas (both fixed and random models), and the Lord formula. These statistics are explained and demonstrated with heuristic data. Furthermore, several recommendations are presented to facilitate appropriate interpretation of effect sizes including the consideration of estimate bias, fixed or random effects design use, and whether analyses were conducted using univariate or multivariate statistical approaches.
Understanding and Interpreting Effect Size measures in General Linear Model Analyses

Introduction

When reporting the statistical significance of research findings, investigators should include in their discussion the "meaningfulness" of the outcomes of their research. Thomas and Nelson (1996) defined meaningfulness as the "importance or practical significance of an effect or relationship" (p.109). Many authors and the APA Publication Manual (American Psychological Association, 1994; Cohen, 1990; Rosnow & Rosenthal, 1989; Thomas, Salazar, & Landers, 1991; Thompson, 1994) indicated the need to provide an acceptable estimate of "meaningfulness" with all tests of statistical significance. These statements are predicated upon one of the most important issues in statistics which is the use of reasonable judgement in interpreting statistical findings.

"Meaningfulness" judgements made by researchers must be based upon the context of theory, previous work, and a sound understanding of relevant basic concepts which undergird the information needed to determine whether a study's outcomes have merit. Statistically speaking, several important issues which should be considered in this merit interpretation involve the relationship between alpha, statistical power, sample size, and effect size (Kraemer & Thiemann, 1987; Murray & Dosser, 1987; Thompson, 1989; Thompson, 1994; Thompson, 1997).

Magnitude of Effect Measures

Over the past three decades, researchers (Bakan, 1966; Cohen, 1969; Cohen, 1990; Cooper & Hedges, 1994; Glass, McGraw & Smith, 1981; Hedges & Olkin, 1985; Kupfersmid, 1988; Thomas & French, 1986; Thompson, 1989) have argued that one of the most effective methods for interpreting the true meaningfulness of a reported "statistically significant"
difference between means (traditional, comparative study) or a relationship (correlation/regression-based study) is the calculation and reporting a research study’s effect size (i.e. determining the magnitude of the effect.). The previously noted researchers have supported the inclusion magnitude of effect measures in all research presented and/or published for which these measures are relevant. Thomas and Nelson (1996) and Hedges and Olkin (1985) illustrated examples of how small differences/relationships can be interpreted as statistically significant based upon the presence of large sample sizes, or conversely, large differences/relationships can be declared statistically non-significant due to small sample sizes. Indeed, Cohen (1990) pointed out that the focused purpose of research should be to measure the magnitude of effect rather than traditionally reported “statistical significance” which relies on p values. Carver (1978), Chow (1988), Franks and Huck (1986), Huberty (1987), Thomas, Salazar, and Landers (1991) all have indicated that a single study resulting in the dichotomous decision of “failing to reject” or “rejecting” a null hypothesis at a predetermined alpha value level, demonstrates minimal impact on theory development, or practice. On the other hand, reporting of estimates of the magnitude of effect observed may offer comparative standards with past and present research, and assist the researcher in identifying important characteristics for subsequent follow-up research.

Carver (1978) and Rosnow and Rosenthal (1989) pointed out that one of the primary contributions of a magnitude of effect statistic is providing the reader with a sense of how much of the dependent variable can be controlled, predicted, or explained by the independent variable(s). In behavioral research, investigators are often interested in determining the amount of variance accounted for or explained by the presence of another variable (Chow, 1988; O’Grady, 1982). The use of estimates regarding the magnitude of an effect can assist the
researcher in determining whether statistically significant findings are of any meaningful significance in the world of professional practice. Snyder and Lawson (1993) illustrated this point with an excellent example:

For example, use of an instructional method that increases the performance of an experimental group on a dependent measure by 5 points over a control group will result in statistically significant findings, if the sample size is large enough. Whether or not such a 5-point difference (i.e. magnitude of effect) between groups is meaningful from an instructional standpoint depends on many factors besides the statistically significant $p$ value. (p.335)

Snyder and Lawson also pointed out that it is important for research to avoid misinterpretation of a small $p$ value. A relatively small $p$ value does not necessarily mean that there is a strong relationship between the independent and dependent variables of interest in a study.

A concern which has been expressed by many in the behavioral sciences, particularly in the fields of health, physical education, and recreation (Franks & Huck, 1986; Looney et al, 1994; Thomas & French, 1986; Thomas & Nelson, 1996; and Thomas, Salazar, & Landers, 1991), is the apparent lack of attention researchers in these fields demonstrate by omitting discussion of effect size in their papers, or even including in their reporting components necessary for accurate estimation of effects size such as sample sizes, Ms and SDs, and full disclosure of all factors/variables within the design. Thomas and Nelson (1996) pointed out that such failures to report these data not only make accurate estimation of effect size impossible, but also create great difficulties for other researchers in making comparisons from past to future research and in conducting meta-analyses. Thomas, Salazar, and Landers (1990) aptly
summarized this perspective, "Fundamental to the reporting of good science is information for comparison to other work; at a minimum that includes the Ms and SDs of the variables of interest (main effects and interactions)" (P.346). Looney, Feltz and VanVleet (1994) concurred with their health, physical education and recreation colleagues.

Post-hoc estimates of effect size (ES) and omega squared ($\omega^2$) provide methods which can assist in the evaluation of meaningfulness. A standardized effect size such as Cohen's $d$ (ES = $[M_1-M_2]/s$) may be used to represent an estimate of meaningfulness concerning the difference between two levels of comparison (groups, for example) very much like a follow-up comparison ($t$ test), while omega squared ($\omega^2$) may be used to estimate the proportion of total variance accounted for by the independent variable(s). Kraemer and Thiemann, (1987) discussed the use of a priori, post hoc, or both methods of analysis in using ES, $\omega^2$, and the statistical indices of power and alpha in the interpretation of a study's "meaningfulness".

Importantly, Cohen (1988) suggested that a useful a priori procedure is the calculation of power for each of the statistical procedures planned for application within a study. The computation of power requires information on three of four indices; alpha, beta, sample size, and effect size. Researchers can also estimate the sample size needed to detect a certain effect (ES) given a specific alpha level, beta level, and anticipated effect size (Kraemer & Thiemann, 1987). It is possible, of course, that a valid a priori estimate of ES is not available, and power cannot be calculated. Whether this is the case or not, Cohen (1988) argued it is indeed useful to calculate a post hoc estimate of ES and $\omega^2$ for comparison of greatest interest. Thomas, Salazar, and Landers (1991) argued that not only should magnitude of effects be calculated, but that they should be interpreted for the reader (e.g. $.2 =$ small, $.5 =$ moderate, $.8 =$ large for standardized
Effect Size Measures

...differences [Cohen, 1988]) and compared to interpreted findings in relevant studies.

Magnitude-of Association Measures

Several authors have noted that researchers use many different terms when discussing magnitude of effect estimates (Hedges, 1982; Hedges, 1984; Franks & Huck, 1986; Murray & Dosser, 1987; Murphy, 1997; O’Grady, 1982; Thomas & French, 1986). These labels may include; (a) magnitude of effect, (b) magnitude of experimental effect, (c) explained variance, (d) effect size, (e) strength of relation, and (f) strength of association. Maxwell and Delaney (1990) and Thomas, Salazar, and Landers (1991) discussed the organization of magnitudes-of-effect indices into two broad categories; measures of effect size and measures of association strength. The first category includes indices which directly involve investigation of differences between means. Many of these indices include estimates typically used in meta-analysis techniques such as; (a) mean differences, (b) effect parameter measures, and (c) standardized differences between means, such as Cohen’s d (Cohen, 1988). There exists numerous works which discuss effect sizes for mean differences, including the necessary calculations (Camp & Maxwell, 1983; Cooper & Hedges, 1994; Hedges & Olkin, 1985; Maxwell & Delaney, 1990; O’Grady, 1982; Thomas & Nelson, 1996). Interested readers are referred to the previously mentioned citations for a thorough review of calculations regarding these effect sizes.

Since all analytical procedures are inherently correlational in nature (Cohen, 1968; Cohen & Cohen, 1983; Hedges & Olkin, 1983; Knapp, 1978; Thompson, 1991) the focus of the present paper will be on the discussion of issues regarding magnitude-of-association. Indices within this second broad category can be used on data from either experimental or correlational design frameworks.
Magnitude-of-association indices attempt to elucidate association strength between variables (e.g. proportionality of variance associated between dependent and independent variables). Snyder and Lawson (1993) pointed out several magnitude-of-association measures which have been developed over the past 40 years in attempts to address the interpretation of "meaningfulness" issue, including eta squared, partial eta squared (Cohen, 1973), omega squared, epsilon squared, $R^2$ (Stevens, 1992), partial $R^2$ (Cohen & Cohen, 1983), the Wherry formula, the Herzberg formula, and the Lord formula. Snyder and Lawson noted that the literature covering magnitude-of-association indices generally includes discussions of the following categories; (a) biased or unbiased computations, (b) whether the indices are based upon population or sample calculation estimates, (c) fixed or random effect model utilization, and (d) univariate or multivariate statistical analyses. As Snyder and Lawson observed, these various indices of effect are all part of the same general linear model (Henson, in press) and therefore are similar conceptually and computationally. Researchers are encouraged to use such indices as appropriate within a given research design context.

Computing Magnitude-of Association Measures: Some Considerations

Biased and Unbiased Estimates:

Thompson (1990) noted that eta squared (ANOVA) and $R^2$ (regression), both of which express ratios of explained variance to total variance, tend to overestimate the proportion of variability explained in the population or future samples. Discussion offered by Stevens (1992) concurred with this perspective indicating that simple magnitude-of-association indices may cause overestimates due to the "mathematical maximization principle" operating in general linear model analyses. Variation due to sampling error can potentially cause simple magnitude-of-
association indices to be positively biased. As such, these estimates are often called biased or uncorrected effect size measures. Stevens (1992) and Thompson (1997) both noted that even if no relationship existed between X and Y variables within a population (R= 0), the sample $R^2$ or eta squared would probably not be exactly zero. Thompson (1994) indicated that three characteristics of a research study may impact the amount of positive bias in an uncorrected magnitude of association indices, including (a) sample size (larger sample/less bias), (b) number of variables (fewer variables/less bias), and (c) true population effect size (effect size larger/less bias).

In an attempt to statistically correct for potential positive bias in the calculation of magnitude-of-association indices, statisticians have developed “unbiased” or “corrected” formulas. Synder and Lawson (1993) provided an excellent summary of both corrected and uncorrected formulas including $R^2$ and eta squared, omega squared (fixed & random effects), epsilon squared, Wherry, Herzberg (fixed and random effects) and Lord formulas. The authors indicated that the Herzberg and Lord are designed to correct for estimates potentially realized in future samples, and these indices will always be smaller than the other formulas discussed. These two estimates will result in the most shrinkage in the size of the magnitude-of-association estimate because they adjust for sampling error in both a given research study and some future research study. Omega squared, epsilon squared, and the Wherry formula are recommended for use in research designed to develop population expectations, since these are designed to estimate association strength most likely to be realized in the population.

**Fixed or Random-effect Design Models:**

Factors in a fixed-effect model ANOVA design and values within a fixed-effects
regression model design are assumed as just that, fixed, i.e. they do not change. When factors are randomly selected in ANOVA designs, or when predictors are randomly selected in a regression model design, they can change from one replication of the study to another. Therefore, the potential increase in sampling error (with people or variable levels) should be corrected for in the magnitude-of-association estimate calculations. Stevens (1992) presented several different formulas for the Herzberg correction for fixed and random design models. Snyder and Lawson (1993) and Thomas and Nelson, (1996) included an omega squared formula in their discussions of effect size computational formulas. Murray and Dosser (1987), Thomas and Nelson (1996) and Tolson (1980) have emphasized that investigators must pay attention to the underlying assumptions of various models during the selection process of a bias-correction formula for magnitude-of-association estimates.

Univariate or Multivariate Estimates:

Most formulas discussed by Snyder and Lawson (1993), Stevens (1992) and others discussed thus far were developed from a univariate perspective. However, Huberty (1972), Thomas and Nelson (1996) and Tolson (1980)) described formulas which may be used in multivariate cases. Stevens (1992) noted that the multivariate $D^2$ (mean vectors and sample covariance matrix $S$) can replace their univariate counterparts $d$ and $s$. Pedhazur (1982) discussed $1$-$\lambda$ as analogous to the univariate $R^2$ or eta squared.

Perspectives of the General Linear Model

Maxwell, Camp, and Avery (1981) noted that magnitude-of-association strength estimates developed from a particular GLM perspective may be expected to produce similar statistical values to other techniques that essentially come from the same source. For example,
epsilon squared (fixed effects ANOVA) will be numerically equal to the Wherry formula (regression). Omega squared (random effects ANOVA) will equal the squared intra-class correlation (correlation model) (Snyder & Lawson, 1993). Maxwell, et al. (1981) espouse viewing the use of all measures of magnitude-of-association from the general linear model. As previously noted, all of the effect size measures are conceptually similar. A full understanding and interpretation of these measures are facilitated by considering their common origin.

Selection of Effect Estimates

Numerous researchers have addressed issues regarding what difference it makes in the selection and subsequent use of various magnitude-of-association estimates (Knapp, 1978; Snyder & Lawson, 1993; Thomas & Nelson, 1996; Thompson, 1991; Thompson, 1994; Uhl & Eisenburg, 1970). While no clear cut answer is patently available which would cover all types of situations, research has noted that when sample sizes and effects sizes are both large (50-100+), biased and unbiased correction formulas appear to produce similar estimate values. When sample sizes and effects sizes become smaller, statistical corrections tend to be larger (Thompson, 1990). The amount of statistical correction varies dependent upon the formula used in the estimate. Snyder and Lawson (1993) concurred with Uhl and Eisenberg (1970) by noting that the Lord formula produces the most conservative estimates across all sample sizes. The omega squared formula develops more conservative population estimates, and the Lord formula develops more conservative sample estimates. Of course, when sample sizes are smaller (< 30), most bias corrected formulas produce more conservative estimates than unbiased formulas.

Conclusion

Magnitude-of-effect measures are important tools in assisting investigators to develop
further insights into the true meaning of their research outcomes. Maxwell, et al (1981), Murray and Dosser, (1987), O'Grady, (1982, and Thompson, (1997) discussed the need for researchers to understand the appropriate application and interpretation of these indices, so that the most productive use of these measures can be utilized. Some general considerations regarding use of magnitude of effect indices are offered by Cooper and Hedges (1994), Hedges and Olkin (1985), Snyder and Lawson (1993), and Thomas and Nelson (1996).

1. Generalizations about effect estimates should be limited to studies that involve the same levels of variables, and similar numbers/types of subjects. Estimates tend to be very context dependent.

2. Confidence intervals should be constructed for magnitude-of-association measures, since these measure may be viewed as point estimates of populations which may not take into account sampling error (Fowler, 1985).

3. While Cohen (1988) proposed general guidelines for assessing the relative size of an effect estimate, practical significance must rest with the individual researcher’s judgment, the importance of the research questions posed, the design of the study, and ultimately with the true societal impact of the findings. Since there are many contextual issues which may effect the estimates interpretation, no strict or arbitrary guideline should be employed in making the final judgement on practical meaningfulness of a study’s outcome.
Effect Size Measures

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ACTUAL CALCULATIONS USING
GLM ANALYSES

\[ \text{eta squared (} R^2 \text{)} = \frac{SS_{\text{explained}}}{SS_{\text{total}}} = \frac{143.846}{176.000} = .817 \]

\[ \text{omega}^2 (\omega^2) = \frac{SS_{\text{explained}} - [(v-1) MS_{\text{error}}]}{SS_{\text{total}} + MS_{\text{error}}} = \frac{143.846 - [(8-1) 1.891]}{176.000 + 1.891} = \frac{130.609}{177.891} = .734 \]

\[ * \text{omega}^2 (\omega^2) = \frac{F (k-1)}{[F (k-1)] + (n-k) + 1} = \frac{10.865(7-1)}{10.865(7-1) + (25-7) + 1} = \frac{59.190}{83.190} = .712 \]

\[ \text{Epsilon}^2 = \frac{SS_{\text{explained}} - [(v-1) MS_{\text{error}}]}{SS_{\text{total}}} = \frac{143.846 - [(8-1) 1.891]}{176.000} = \frac{130.609}{176.000} = .742 \]

\[ \text{Wherry} = 1 - \frac{(n-1)}{(n-k-1)} (1 - R^2) = 1 - \frac{(25-1)}{(25-7-1)} (1 - .817) = 1 - (1.412) (.183) = .742 \]

\[ \text{Herzberg} = 1 - \frac{(n-1)/(n-k-1)] [(n+k+1)/n]}{(1 - R^2)} = 1 - \frac{(24/25-7-1)] [(25+7+1)/25]}{(1 - .817)} = 1 - (1.412) (1.32) (.183) = .659 \]

\[ * \text{Herzberg} = 1 - \frac{(n-1)/(n-k-1)] [(n-2)/(n-k-2)] [(n+1)/n]}{(1 - R^2)} = 1 - \frac{(24/25-7-1)] [(25-2)/(25-7-2)] [(25+1)/25]}{(1 - .817)} = 1 - (1.412) (1.438) (1.04) (.183) = 1 - .386 = .614 \]

\[ \text{Lord} = 1 - (1 - R^2) \frac{(n+k+1)/(n-k-1)]}{[(25+7+1)/(25-7-1)]} = 1 - (.183) [(25+7+1)/(25-7-1)] = 1 - (.183) (1.941) = 1 - .355 = .645 \]

* Random effects model
COMPARISON OF EFFECT SIZE COMPUTED ON HEURISTIC DATA

SUMMARY OF RESULTS

\[
\begin{align*}
\text{eta squared } (R^2) & = .817 \\
\text{omega squared } (\omega^2) & = .734 \\
*\text{omega squared } (\omega^2) & = .712 \\
*\text{epsilon squared} & = .742 \\
\text{Wherry Formula} & = .742 \\
\text{Herzberg Formula} & = .659 \\
*\text{Herzberg Formula} & = .614 \\
\text{Lord Formula} & = .645
\end{align*}
\]
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