Researchers are often faced with the dilemma of having to use intact groups. Analysis of covariance (ANCOVA) is sometimes used as a means of leveling unequal groups. However, only after five assumptions have been met is it safe to use ANCOVA. Part and partial correlations also invoke statistical correlation procedures. This paper reviews these covariance correction procedures, explains the relevant assumptions and potential pitfalls related to the analysis, and discusses research situations when their use is warranted or ill-advised. (Contains 1 figure, 5 tables, and 13 references.) (Author/SLD)
Covariance Corrections: What They Are and What They Are Not

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Abstract

Researchers are often faced with the dilemma of having to use intact groups. ANCOVA is sometimes employed as a means of leveling unequal groups. However, only after five assumptions are met is it safe to use ANCOVA. Part and partial correlations also invoke statistical correction procedures. The purpose of the present paper is to (a) review these covariance correction procedures, (b) explain the relevant assumptions and potential pitfalls related to the analyses, and (c) discuss research situations when their use is both warranted and ill-advised.
Covariance Corrections: What They Are and What They Are Not

Researchers often find themselves at the mercy of intact groups (e.g., classrooms, schools, etc.) that limit, if not eliminate, the possibility of conducting true, randomized experimental research (Henson, 1998). For example, given a particular intervention for special education instruction, it is arguably unethical to randomly assign students into treatment and control group participants. Accordingly, there remains a great need for experimental validation of educational programs in general (cf. Welch & Walberg, 1974).

One result of this dynamic is the tendency (perhaps necessity) to either (a) use existing groups (e.g., two special education classrooms) and attempt to statistically control for preexisting differences between groups or (b) use one treatment group and attempt to statistically control for preexisting variables that may impact the final outcome(s). These types of analyses are all related and go by various names such as analysis of covariance (ANCOVA) and part and partial correlation.

Covariance corrections vary in their frequency of use. ANCOVA, for example, appeared in about 4% of the published literature in American Educational Research Journal articles (Goodwin & Goodwin, 1985). More recently, Baumberg and Bangert (1996) reported that the use of ANCOVA accounted for 7% of the statistics used in the research found in the Journal of Learning Disabilities for 1989 through 1993. Similarly, in a review of 17 well-respected education and psychology journals, Keselman and colleagues (1998) found that univariate ANCOVA was used in 7% of the articles examined (45 out of 651). Furthermore, this hit rate does not include applications of regression analyses with covariates or multivariate ANCOVA. Thompson (1988, 1994) found frequent use among doctoral students. Furthermore, it is arguable that other related corrections, such as part and partial correlation, are more frequently utilized in
the literature, particularly when researchers attempt to control for variables that may influence the outcome of a single treatment group, such as using a pretest correction on posttest scores.

Like most analytic methods, it is important to realize that these corrections have certain benefits and limitations. ANCOVA, for example, is generally inappropriate when used with intact groups such as established classrooms of students. And yet, ANCOVA does continue to be utilized in such situations. Of the 45 uses of ANCOVA reported, by Keselman and colleagues (1998), full two-thirds of the studies "involved non randomization of the experimental units" (p. 373). As suggest by Henson (1998),

Perhaps the lingering use of ANCOVA is due to the mystical promise that ANCOVA is a statistical correction of all pre-treatment problems and that it will provide increased power against Type II error. Such an argument is particularly compelling to doctoral students who find themselves aggressively seeking and even praying for statistically significant results! Unfortunately, ANCOVA has multiple assumptions that must be met before it can be accurately utilized. (p. 4)

Similarly, part and partial correlations should be used only when it is meaningful to do so. While these correlational procedures may not have the same assumption as ANCOVA, there are important ramifications of their misuse.

The purpose of the present paper is to (a) review these covariance correction procedures, (b) explain the relevant assumptions and potential pitfalls related to the analyses, and (c) discuss research situations when their use is both warranted and ill-advised. To make the discussion concrete, heuristic data sets are presented and analyzed to illustrate appropriate and inappropriate use of covariance corrections.
A Review of Procedures

ANCOVA is a form of statistical control used to determine whether some variable, other than the intervention, is responsible for the difference on the dependent variable between groups (Borg & Gall, 1989; Shavelson, 1996). To do so, a continuously scaled variable that is highly correlated with the outcome measure is identified that contains information on individual subjects and is collected before the intervention begins. This variable is called the covariate. Covariates may take many forms and are best determined by the theoretical model of the researcher. Examples of covariates in the social and behavioral sciences may include a pretest, aptitude, age, experience, previous training, motivation, and grade point average (Ruiz-Primo, Mitchell, & Shavelson, 1996). Importantly, ANCOVA is designed to be used when individuals have been randomly assigned to control and experimental groups. That is, the ANCOVA statistical analysis is appropriate when the study has an experimental design. It cannot make a study’s design experimental when, in fact, it is not. As Henson (1998) noted,

In a researcher’s zealous attempts to have an experimental or quasi-experimental design, the use of ANCOVA comes to be seen as a way of making such a design happen. In fact, a design is either experimental or it is not. Statistical analyses employed on the data obtained from the design do not magically transform a study into a true experiment . . .

This thinking (or lack thereof) reflects confusion regarding the cooperative but separate roles of methodological design and statistical analysis. (p. 6-7)

In an ANCOVA, covariate regression first removes the covariate variance from the dependent variable ignoring treatment group membership, thus reducing the error variance from within groups or the error caused by systematic differences among subjects. The idea is that this gives a more accurate estimate of error when asking if two or more groups differ on a dependent
variable. If the covariate and the dependent variable are linearly related, the use of the covariate in the analysis will serve to reduce the unexplained variation between groups. The greater the magnitude of the correlation, the greater the reduction in dependent variable variance due to within groups variation. Next, the error (or residualized) scores that remain from this regression of the covariate and dependent variable become the new dependent variable, and an analysis of variance (ANOVA) is performed using this residualized dependent variable (Henson, 1998).

Since removal of the covariate ideally reduces the variance of the error scores in this ANOVA analysis, the size of the ANOVA F statistic purportedly increases, as does the likelihood of obtaining statistical significance. If error variance is reduced in this manner (which assumes that certain assumptions have been met), the ANCOVA will be more powerful than the ANOVA (Shavelson, 1996).

In sum, ANCOVA partitions the total variability among the scores on the dependent variable into three sources of variability (Ruiz-Primo et al., 1996) but does so in a two step sequence. First, variance attributable to the covariate is removed from the dependent variable. This is done for all subjects, regardless of treatment group membership. Second, a classical ANOVA partitions the residualized dependent variable into variance attributable to the treatment group (or explained) variance and error.

As previously stated, one reason ANCOVA is used concerns its “promise” of power against Type II error. However, there are several conditions that must be met for this to occur. Statistically, ANCOVA requires one degree of freedom be lost from the error term for each covariate used (Shavelson, 1996). This actually results in a larger critical value for the F statistic and makes it more difficult to find a significant F statistic. In order for power to increase, the variance attributable to the covariate must be sufficient enough to compensate for the loss of
Covariance Corrections

power due to degree of freedom manipulations. This compensation occurs by lowering the sum of squares error (because some of the variance was due to the covariate) and thereby increasing power in the ANOVA. Thus, power is fostered only when the covariate has a sufficiently large correlation with the dependent variable. If the covariate is not highly correlated with the dependent variable, the use of ANCOVA may actually lower statistical power.

A similar procedure of covariance correction is employed in part and partial correlations, which are used to identify unique relationships between two variables. Part correlation removes the influence of a third variable from one of two variables being considered (Borg & Gall, 1989). Here, a regression is performed between the third variable and one of the two variables being considered. The remaining error scores (residuals) are then correlated with the second variable in consideration, yielding the part correlation. Theoretically, the analysis is intended to explain the unique relationship between the two variables of interest after removing the spurious influence of some identified third variable. Partial correlation involves a similar dynamic but removes the influence of a third variable from both of the variables being considered. Here, two regression analyses are conducted between a third variable and both variables in consideration. Then, the remaining error scores from both of these analyses are correlated, yielding the partial correlation. Theoretically, a partial correlation explains the relationship between the variables after removing the spurious effect of the third variable from both variables of interest.

For example, in considering factors that influence end-of-the-year test scores, one would have to consider what the student knew coming into the class (beginning-of-the-year test scores) as well as the teaching method that was used. Naturally, the beginning-of-the-year test scores would influence the teacher's decision of which method of teaching to use. Teachers with high achievers will probably instruct them differently than teachers of low achievers. By using partial
correlation analysis, the influence of the beginning-of-the-year achievement level can be removed from the end-of-the-year scores and from teacher instructional factors. This same scenario could employ part correlation analysis to remove the beginning-of-the-year achievement from just one variable, such as the end-of-the-year scores, to purportedly account for differences between students prior to instruction. In this case, if a large correlation were then observed between teaching method and the residualized end-of-the-year scores, the researchers may be more inclined to believe that the instructional method had some real effect on student achievement. (Note that this assertion could only ultimately be supported through an experimental study.) However, in such pre/posttest cases, there are serious conceptual problems with using part and partial correlations that will be discussed later.

Relevant Assumptions and Potential Pitfalls

If ANCOVA is to be used appropriately, then multiple conditions must be satisfied. Misinterpretation and misapplication of ANCOVA data may result from ignoring required assumptions. Researchers, both novice and experienced, should heed warnings against using this analysis as a “correction” of the dependent variable scores to adjust for data set problems (Buser, 1995), particularly when treatment groups were not randomly assigned. Loftin and Madison (1991) listed several assumptions underlying the successful use of ANCOVA. They are as follows:

1. The covariate (or covariates) should be an independent variable highly correlated with the dependent variable.
2. The covariate should be uncorrelated with the independent variable or variables.
3. With respect to the dependent variable, (a) the residualized dependent variable ($Y^*$) is assumed to be normally distributed for each level of the independent variable, and (b)
the variances of the residualized dependent variable (Y*) for each level of the independent variable are assumed to be equal.

4. The covariate and the dependent variable must have a linear relationship, at least in conventional ANCOVA analyses.

5. The regression slopes between the covariate and the dependent variable must be parallel for each independent variable group. (p. 134)

Each assumption has important ramifications. For example, Assumption 1 states the need for high correlation between the dependent variable (Y) and the covariate (X). The covariate will not do much to reduce the error sum of squares if the correlation is not high, and power may actually be reduced due to the loss of error degree(s) of freedom as the price to use the covariate(s). To demonstrate this point, Table 1 represents three sets of summary tables illustrating a regular ANOVA without a covariate, an ANCOVA in which the covariate (X) has a small correlation with the dependent variable (Y), and an ANCOVA involving a covariate with a large correlation with the dependent variable. The covariate is perfectly uncorrelated with the treatment conditions.

Notice that when the covariate has a large correlation with the dependent variable, the statistical adjustments include the loss of one degree of freedom (df) for each covariate, a larger mean square (MS) error, and larger calculated F values. While the lowering of degrees of freedom error actually increases the critical F, the decrease in error sum of squares counterbalances the loss of degrees of freedom in error due to the high correlation between the
covariate and the dependent variable. If the covariate is perfectly uncorrelated with the
dependent variable, the sum of squares error would remain totally unchanged after the covariate
is removed. In addition, degrees of freedom error would decrease by one for each of the
covariates, mean square error would increase, and $F$ calculated would become smaller. All of
these results make it less likely the researcher will get statistical significance.

Assumption 2 obligates the covariates and the independent variables to be uncorrelated.
Table 2 presents three summary tables to illustrate the importance of this assumption. The best
situation is illustrated in the (a) portion of Table 2, where the covariate ($X$) and the independent
variable are perfectly uncorrelated. This allows unique explanation of different portions of the
total variance of the dependent variable by the covariate and independent variable. However,
reality is such that most social science variables are correlated to some degree. When the
covariate and independent variable are indeed related, it is very possible that the covariate will
“rob” the treatment effect of explanatory power.

In part (b) of Table 2, the covariate and the treatment variable are moderately correlated.
Since a covariance regression is conducted in ANCOVA prior to the ANOVA analysis, some of
the sum of squares that could have been attributed to the treatment variable were instead
accounted for by the covariate. In fact, all of the shared variance between the covariate and the
treatment will be attributed to the covariate, potentially “robbing” the treatment variable of
dependent variable variance.

Finally, part (c) of Table 2 illustrates the extreme ramifications of this problem. Here, all
of the sums of squares of the treatment are accounted for by the covariate, and no credit is given
to the treatment variable. The unsuspecting researcher may conclude that the treatment had no
effect whatsoever.
This problem is not unlikely. As noted, most social science variables are correlated to some degree. Furthermore, when treatment groups are not randomly assigned, then there is an increased likelihood that the covariate will be correlated with the treatment groups. For example, if a researcher wished to study achievement of students in two special education classrooms, he or she may choose to use some pretest measure of achievement as a covariate to “level” any differences between the classes since they were not randomly assigned. It is very possible that the pretest achievement scores (covariate) will be related to what classroom the students attend. In this case, it may be ill advised to use that covariate since it would remove some of the variance from the treatment variable, thereby lowering observed effect of the treatment in the ANCOVA analysis. Thompson (1994) explained this quandary:

When the covariate is related to the treatment variable, use of the covariance correction will alter the effects attributed to the treatment itself. For example, one might have a very effective intervention that looks completely ineffectual, because the covariate is given credit for the variance that would correctly otherwise be attributed to the treatment variable. Here the ANCOVA correction actually destroys power against Type II error. (p. 27)

Paradoxically, when groups are not randomly assigned, ANCOVA is needed the most to “level” for group differences; but in these cases, ANCOVA is less appropriate as an analysis when groups are significantly different. This is a paradox that severely limits the usefulness of
ANCOVA outside of studies that use random assignment, which, of course, are rare in educational research.

In the third assumption, the residualized dependent variable must meet the requirements of any ANOVA analysis. Since the residualized dependent variable becomes the dependent variable in an ANOVA, it must be normally distributed and meet the homogeneity of variance assumption. Homogeneity of variance means that the variance of the residualized dependent variable is the same for all levels of the independent variable. It is only reasonable to compare the means of each group when the groups’ distributions are equally variable.

Assumption 4 addresses the regression analysis assumption that a linear relationship exists between the covariate and the dependent variable. If the covariate is not linearly related to the dependent variable, use of conventional ANCOVA may bias the data. To meet this assumption, change in the covariate should correspond to a proportional change in the dependent variable for each level of the independent variable. It is important that the proportional change exist for all levels of the independent variable to avoid biasing the responses from one or more of the groups (Loftin & Madison, 1991).

Finally, assumption 5 demands that an ANCOVA analysis possess what is known as homogeneity of regression. This assumption states that "The regression slopes between the covariate and the dependent variable must be parallel [or homogeneous] for each independent variable group" (Loftin & Madison, 1991, p.134). If the slopes are not parallel, then one line cannot represent both the covariate and dependent variable accurately (Henson, 1998). In this case, the data will be biased rather than corrected since the relationship of the independent variable to the dependent variable will be changed (Loftin & Madison, 1991). It is important to remember here that the initial regression between the covariate and the dependent variable is
conducted ignoring group membership (as defined by the independent variable). Therefore, a “pooled” regression line is used to represent the regression slopes between the covariate and the dependent variable for each individual group. This is only appropriate when the “pooled” regression line accurately represents (or has the same slope of) the individual group regression lines.

To illustrate this assumption, an ANCOVA analysis will be conducted on a small heuristic data set taken from Henson (1998). The SPSS syntax used to run the analysis is included in the Appendix. In this example, a researcher is investigating whether a note-taking intervention will positively impact achievement of special education students. Unfortunately, the researcher is compelled by the school’s administration to provide the intervention to all special education classrooms, thereby leaving no possibility for a true control (or even a comparison) group. Rather than having no comparison group, the researcher decides to use a regular education class as a comparison group and statistically “equate” the groups with ANCOVA using a reading and writing achievement measure as a covariate on the achievement dependent variable. Table 3 presents the data for the treatment (special education) and comparison (regular education) groups. Scores are given as T-scores. The results of the ANCOVA analysis are given in Table 4.

In this example, when the regression between the covariate (READ) and dependent variable (ACHIEVE) was conducted, a “pooled” regression equation was derived that ignored group membership \( Y^*=-9.158192 + 1.405085 \times \text{READ} \). In other words, the relationship
between READ and ACHIEVE was examined simultaneously for all 12 students. However, it is intuitive to note in this case that these 12 students are not the same. As would be expected, the group means on the covariate variable are notably different (Treatment group: M = 38.33, SD = 6.83; Comparison group: M = 56.67, SD = 6.83). In this case, the covariate variable is strongly related to the treatment variable (see Assumption 2 above).

Furthermore, the use of the "pooled" regression line to represent all groups is appropriate only when it accurately represents the regression lines between the covariate and the dependent variable for each individual group (i.e., when the lines are parallel). In this case, the "pooled" regression line does not represent either of the individual group regression slopes. Figure 1 presents the "pooled" slope and the individual group slopes. Thompson (1988) presents the researcher with the "nasty dilemma" of ANCOVA:

If the controls are not needed, then they should not be used. But if statistical control is needed because the groups in a study are not equivalent, then often the homogeneity of regression assumption cannot be met and the use results in seriously distorted inferences. (p. 23)

As previously noted, part and partial correlations are close cousins to the ANCOVA procedure, and all are part of a statistical correction family that attempts to control for influences on variables. While part and partial correlation do not have the same assumptions of ANCOVA, they do require that the general assumptions of linear regression and correlation analysis be met
since both are hybrids of a regression analysis followed by a correlation analysis with error scores.

Importantly, in all statistical correction procedures, one must be able to interpret the residualized variance after the covariance correction has occurred. In the above ANCOVA example, one is compelled to question what is left of the dependent variable after the covariate had a 96.01% effect size (i.e., the covariate accounted for nearly all of the dependent variable variance). Of the variance that remained, was it still achievement as originally expected, or was the dependent variable some conglomeration of error and other constructs? These conceptual difficulties of using covariance corrections should not be overlooked. As Thompson (1992) noted, "Statistical corrections remove parts of the dependent variable and then analyze whatever's left, even if whatever's left no longer makes any sense. At some point we may no longer know what it is we're analyzing" (p. xiii-xiv).

For example, it is not uncommon to find researchers employing some form of part or partial correlation when using pre and posttest measures. Most commonly, a pretest is given as a baseline measure and used as a covariate on the dependent variable posttest. Then, a third variable may be correlated with the residualized posttest variance (part correlation). The intent of the procedure is to control for differences between subjects prior to the intervention and then analyze differences that occurred during the time between pre and posttest.

While the logic of the analysis is appealing, it is conceptually troublesome to use a pretest as a covariate for a posttest measure. Interpretation of the dependent variable becomes a tricky matter when a researcher has removed variance due to the construct at pretest. To illustrate this dynamic, consider the following heuristic example. Suppose a researcher wishes to study the effects of collaboration during an academic year-long teacher inservice on teachers' sense of self-
efficacy. The same pre and posttest measure of efficacy is given prior to and after the inservice. The researcher then removes pretest efficacy variance from posttest efficacy (the covariance correction) and then correlates the residualized dependent variable (posttest efficacy error scores) with a collaboration variable.

As an example, Table 5 presents two sets of hypothetical data for five teachers. In the first set of data, all of the teachers gained exactly the same during the inservice with all teachers increasing by one point on a Likert scale from pre to posttest. This may be considered a best case scenario in that all teachers benefited equally from the inservice. Unfortunately, when the pretest scores are covaried with the posttest scores, there is exactly zero residualized variance that remains. This is because the pre and posttest efficacy scores were perfectly correlated. As such, there can be no relationship between the collaboration variable and efficacy since there is no remaining efficacy variance to predict! In the second set of data, a more realistic and less than perfect gain is illustrated (i.e., not all teachers gained the same). Here a part correlation can be conducted, but the question remains of what is left of the posttest teacher efficacy after removing the influence of pretest teacher efficacy? Is it still efficacy or is it something else? The common interpretation is that the remaining variance is uniquely due to the intervention. However, it is doubtful that this unique variance is conceptually the same as the originally defined efficacy construct. Essentially, covariance corrections of this nature call into question the construct validity of the residualized dependent variable. While researchers may begin with a posttest measure of teacher efficacy, they may be left with a residualized dependent measure of something else.

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INSERT TABLE 5 ABOUT HERE.
While there are no rules governing the interpretation of the dependent variable in such cases, informed researcher judgment should be invoked to evaluate the constructs that are being analyzed. At some point, the dependent variable may not be what we think it is, potentially leading to misinterpretation of results.

Conclusion

Researchers, especially those doing educational research, are often faced with the dilemma of having to use intact groups. Since it is impossible to employ true experimental control in these situations, ANCOVA is often employed. By taking out variance attributable to a covariate from the dependent variable, ANCOVA "promises" to increase the size of $F$ calculated and the possibility of reaching statistical significance. However, problems arise when using ANCOVA since the use of intact groups almost guarantees that assumptions necessary for the successful use of ANCOVA will not be met. Only after these assumptions are met is it safe to use ANCOVA. Part and partial correlation also invoke statistical correction procedures. While the assumptions required for part and partial correlation are not identical to those of ANCOVA, there are conceptual difficulties in interpreting the residualized dependent variable in most cases. After removing variance attributable to the covariate, it is important that the remaining variance is comprehensible and maintains appropriate construct validity.
Covariance Corrections

References


Table 1

Three Summary Tables for Three-level One-Way Design (n=30)

(a) ANOVA--treatment Ho not rejected

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(b) ANCOVA with smaller correlation between X and Y and zero correlation between X and treatment--treatment Ho not rejected

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(c) ANCOVA with higher correlation between X and Y and zero correlation between X and treatment--treatment Ho rejected

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Table 2

Summary Tables Illustrating the Necessity for the Covariate and the Treatment to be Uncorrelated

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(a) Treatment and covariate (X) uncorrelated

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(b) Treatment and covariate (X) partially correlated

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<td>3.33</td>
<td>0.0%</td>
</tr>
<tr>
<td>Error</td>
<td>155</td>
<td>29</td>
<td>5.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total (Y)</td>
<td>200</td>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Covariate subsumes Treatment SOS
Table 3

Heuristic Data for Treatment and Control Students on Reading (READ) and Achievement (ACHIEVE) Measures (n = 12)

<table>
<thead>
<tr>
<th>Student</th>
<th>READ</th>
<th>ACHIEVE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Special Education Students Receiving Intervention</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jennifer</td>
<td>30</td>
<td>34</td>
</tr>
<tr>
<td>Suzanne</td>
<td>30</td>
<td>36</td>
</tr>
<tr>
<td>Greg</td>
<td>40</td>
<td>46</td>
</tr>
<tr>
<td>Kyle</td>
<td>40</td>
<td>46</td>
</tr>
<tr>
<td>Natascha</td>
<td>45</td>
<td>49</td>
</tr>
<tr>
<td>Alfred</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>M</td>
<td>38.33</td>
<td>43.50</td>
</tr>
<tr>
<td>SD</td>
<td>6.83</td>
<td>6.80</td>
</tr>
<tr>
<td><strong>Students in Regular Education Without Intervention</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stephanie</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>Bob</td>
<td>50</td>
<td>68</td>
</tr>
<tr>
<td>Timothy</td>
<td>55</td>
<td>67</td>
</tr>
<tr>
<td>Micah</td>
<td>55</td>
<td>70</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>65</td>
<td>80</td>
</tr>
<tr>
<td>Leah</td>
<td>65</td>
<td>85</td>
</tr>
<tr>
<td>M</td>
<td>56.67</td>
<td>72.67</td>
</tr>
<tr>
<td>SD</td>
<td>6.83</td>
<td>9.18</td>
</tr>
</tbody>
</table>

Table 4

ANCOVA Summary Table (n = 12)

<table>
<thead>
<tr>
<th>Source</th>
<th>SOS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate</td>
<td>2912.038</td>
<td>1</td>
<td>2912.038</td>
<td>397.383</td>
<td>96.01%</td>
</tr>
<tr>
<td>Group</td>
<td>54.926</td>
<td>1</td>
<td>54.926</td>
<td>7.495</td>
<td>45.44%</td>
</tr>
<tr>
<td>Residual</td>
<td>65.952</td>
<td>9</td>
<td>7.328</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3032.917</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The Covariate effect size calculated was $r^2 = .9601$ or 96.01%. The Group effect size calculated was eta2 = .4544 or 45.44%. The eta2 result was found by dividing the Group SOS by the total SOS after the covariate effect was removed ($54.926/(54.926 + 65.952) = .4544$). In ANCOVA, the treatment effects are found in relation to the error scores that are created after regressing the covariate (READ) on the dependent variable (ACHIEVE). As such, the Group SOS is not divided by the Total SOS but rather by the Residual SOS after removing the covariate.

Table 5

Heuristic Data for Pre and Post Teacher Efficacy as Predicted by Teacher Collaboration.

<table>
<thead>
<tr>
<th>Subject/Statistic</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Teacher Collaboration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerri</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Joy</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>John</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Carolyn</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Shawna</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ r_{\text{pretest} \times \text{posttest}} = 1.00 \]
\[ \text{Residualized sum of squares} = 0.00 \]
\[ \text{Part } r_{\text{collaboration} \times \text{residualized posttest}} = \text{Cannot be computed since the residualized Posttest scores are all 0 (constant).} \]

Example 2

<table>
<thead>
<tr>
<th>Subject/Statistic</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Teacher Collaboration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerri</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Joy</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>John</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Carolyn</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Shawna</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

\[ r_{\text{pretest} \times \text{posttest}} = .981 \]
\[ \text{Residualized sum of squares} = 1.071 \]
\[ \text{Part } r_{\text{collaboration} \times \text{residualized posttest}} = .926 \]

Note. All scores were on a 5-point Likert-type scale.
Figure 1. Slopes of Subgroups and Total Sample

Appendix

TITLE 'ANCOVA heuristic analysis'.
COMMENT 'n=12 see Table 1'.
GET FILE= 'a:ancova.sav'.
SET BLANKS=SYSMIS UNDEFINED=WARN PRINTBACK LISTING>
TEMPORARY.
SELECT IF (group=1).
DESCRIPTIVES
    VARIABLES=achieve read
    /FORMAT=LABELS NOINDEX
    /STATISTICS=MEAN STDDEV MIN MAX
    /SORT=MEAN (A).
TEMPORARY.
SELECT IF (group=2).
DESCRIPTIVES
    VARIABLES=achieve read
    /FORMAT=LABELS NOINDEX
    /STATISTICS=MEAN STDDEV MIN MAX
    /SORT=MEAN (A).
COMMENT 'determine pooled regression equation'.
REGRESSION
    /MISSING LISTWISE
    /STATISTICS COEFF OUTS R ANOVA
    /CRITERIA=PIN(.05) POUT(.10)
    /NOORIGIN
    /DEPENDENT achieve
    /METHOD=ENTER read.
COMMENT 'compute yhat using weights from all students'.
COMPUTE yhat = -9.158192 + (1.405085 * read).
EXECUTE.

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