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ABSTRACT

This document contains the following papers on issues related to mathematics in technology and teacher education: "A Case for Strong Conceptualization in Technology Enhanced Mathematics Instruction" (Michael L. Connell and Delwyn L. Harnisch); "Faculty/Student Collaboration in Education and Math--Using the Web To Improve Student Learning and Teaching" (P. Gavin LaRose and Michael L. McDonald); "Examining the Feasibility of Using Computer Supported Collaborative Work-Space in Pre-Service Mathematics Methods: A Vygotskian Perspective" (Christina Wotell Charnitski and Fred Croop); "Implementing Technology in Secondary Science and Mathematics Classrooms: A Perspective on Change" (Scott W. Slough and Gregory E. Chamblee); "Tools for the 21st Century Classroom: How Digital Video and the Internet Can Engage Learners in Math and Science" (Evangeline S. Pianfetti and Brian M. Pianfetti); "Actions on Objects: A Theoretical Framework for Mathematics Education" (Michael L. Connell); "Some Psychological Aspects of Using Information Technologies in Teaching Linear Algebra" (Mikhail M. Bouniaev); "Factors Related to Teacher Use of Technology in Secondary Geometry Instruction" (David A. Coffland and Albert W. Strickland); "Using WebCT To Deliver a Finite Mathematics Course to Preservice Teachers" (J. Harley Weston); "Online Conversion of a Technology Based 'College Algebra' Course" (Lisa R. Galminas and Kathy Autrey); "Issues in Developing an On-Line Mathematics Learning Series for Middle School Teachers" (Jennifer Knudsen); "Symbolic Computers and Mathematical Objects" (Michael L. Connell); "Exploring Whole Number and Rational Number Division within a Computer-Generated Conceptual Domain" (Stephen Campbell and Gil Fonthal); "Serendipity in Interactive Mathematics: Virtual (Electronic) Manipulatives for Learning Elementary Mathematics" (Lawrence O. Cannon, E. Robert Heal, and Richard Wellman); "Using Web Pages To Teach Mathematical Modeling: Some Ideas and Suggestions" (Elliott Ostler and Neal Grandgenett); "Technological Tools To Develop Mathematical Problem Solving" (John E. Bernard and Olga M. Ramirez); "The 'Graph' System for Function Graph Construction" (Vladislav Katkov); and "San Antonio Technology in Education Coalition: A Mathematics and Technology Curriculum Integration and Staff Development Project" (Paul Tisdel and Lorraine Bratcher). Individual papers contain references. (MES)

MATHEMATICS

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In examining this year's set of papers for the mathematics section I cannot help but be impressed by the growth which this area has seen over the past 10 years. Initially, we were lucky to have papers which showed competence in the use of technology and which tended to be written from a "Golly-Gee-Whiz! It actually worked" perspective. The mathematics in many cases was at a rudimentary level, and often came in second to the computer itself.

We have indeed progressed far since those days. As you are aware, to even get consideration in this year's section you had to be able to submit your papers in electronic form via the Internet. This marks a major transition period for SITE. To this writer it marks the end of the early exploratory era where "Golly-Gee, it works" was good enough. Today you have to be a relatively savvy user of technology to even participate in the discussion. In many ways this is unfortunate, as it means that there will be some voices left out of the dialog. I raise this as an issue because to even enter the SITE discourse from this point on you must already be a fairly high end user.

Although this will certainly raise the bar on the technological sophistication that one might expect within the papers, I will miss the diversity. Part of the excitement of SITE has always been the mix of talents and abilities that it drew. It was common to find novice classroom teachers rubbing shoulders with professional instructional designers and I feel that as an organization we were richer for it. Let's make sure we continue the long-standing SITE tradition of making the "newbies" feel welcome.

As you shall see, we have indeed progressed to the point where we are now getting an abundance of extremely strong papers. In this new wave not only is "Golly-Gee" not good enough, but the papers are written within well-developed and articulated theoretical frameworks for what is being done, a high-end use of technology in some very innovative and creative fashions, and the creation of some new and powerful mathematical tools with which to teach and think. All of these serve as strong indications that this is truly a fruitful area and one in which should see much growth and research over the next few years.

The Papers

This years mathematics papers tended to cluster around three major themes. The first of these themes might be thought of as *Why do we do things we do?* Papers that were classified in this area tended to feature the psychological and pedagogical implications of technology. They often were concerned with how we might take advantage of what we know concerning how best one can teach, how best students can learn, and how the technology can be used to enhance existing construction.

The second major theme concerned more content specific issues and might be thought of as *What should we teach?* These papers were often very specific regarding their mathematical content, occasionally to the extent of limiting their generalizability to other instructional settings. The central focus of these papers is what is it that we are going to teach. However, they serve as lovely reminders of the extent to which mathematics instruction can be enhanced via technology and a reminder of the growing maturity of the field. In reading this set it is fascinating to see the various pedagogical methods that were adopted.

The third theme was that of tool construction for instruction. This area might be thought of as *What do we have to teach with?* These papers describe some truly cutting edge and innovative efforts of interest to all mathematics educators. As I reflect upon the new methods and opportunities for instruction represented in this section I am convinced that a careful reading would make a technology using teacher educator out of the most die-hard critic.

Why do we do things we do?

These papers contained many important philosophical and logistical concerns. Common to nearly all of them, however, is the question of why should we teach in a technologically enhanced fashion and what does this change mean to our teacher candidates and to their eventual students. In Connell and Harnisch we read of the need for strong conceptualization within technology enhanced mathematics instruction - one could easily extend the argument to mathematics instruction in general. This paper describes why it is important that we do not abandon personalized individual understanding of concepts. Upon first exposure to educational computing many teachers

candidates comment that they no longer need to understand the concepts as the computer will bail them out. This is a dangerous perception and is addressed head-on in this paper

The model for interdisciplinary collaboration put forth by LaRose and McDonald reflects awareness of this concern. This paper describes a well-developed effort in using the World Wide Web as a method of instruction. Their rich descriptions of both the day-to-day usage and the manner in which they evaluated and assessed their project reflect a nicely developed set of ideas. In particular, there was an attempt to match the technological method to appropriate methodological approaches. As published here, this particular paper is a work in progress. I would encourage interested readers to attend their session at the conference. It will be exciting to see how their Fall and Spring semester turn out! It should be noted that this paper attempts to meet both the more rigid instructional design requirements and capture the fluidity and dynamic aspects of constructivism and social interaction.

Our next look at why we teach the way we teach is by Charnitski and Croop and offers beautifully developed discussion Vygotskian notions relative to computer enhanced mathematics instruction. Given the tendency of many technologically intensive courses to become depersonalized and machine-centered I feel that this is a very important paper. It describes in easy to understand terms some of the key issues of Vygotsky and his psychological theories. The notion of creating a collaborative learning space within a computer-enhanced environment should be must reading for all teacher educators. This important paper raises issues that we need to be looking at very carefully lest we find ourselves trapped with content delivery systems in lieu of interactive learning environments.

In Slough and Chamblee's paper we read their approach toward implementation of technology in math and science at the secondary level. For those readers who are not familiar with CBAM, or the Concerns-Based Adoption Model, it would be well worth the time necessary for reading their theoretical framework section. The variation of CBAM used in this paper involves the five major stages and reflects the dynamic nature of the change process. A major finding from this paper is that CBAM model does address the perceived concerns of the teachers. We also find further examples of the use of technology to enhance and transdisciplinary work across content areas.

The last paper in this section comes from Pianfetti and Pianfetti and illustrates some absolutely beautiful work using the World Wide Web to teach in a new fashion. The study itself can serve as a template for other researchers and I was particularly impressed by their use of actual students in actual classrooms schools throughout the creation, design, and evaluation of their project. This paper could serve as a model for researchers wishing to field based their work.

What should we teach?

In the paper concerning Actions on Objects, Connell describes a methodological framework for the technological enhancement of mathematics education. In this paper we see examples showing the essential parallelism of this approach, together with the expressed notion that good thinking is good thinking regardless of what tools or developmental level might be used in its' generation.

Bounieav deals with some of the psychological aspects of using information technologies in the second paper within this subsection. In this case to teach a very specific set of concepts from linear algebra. For past readers of the SITE mathematics section you are probably aware of Mikhail's work - Step-by-Step Development of Mental activities (SSDMA). A major strength of this paper lies in its' carefully delineated set of actions which derived from the mathematics itself. The instructions his examples are based upon are quite well worked out, in each case showing the operations underlying the mental activities that will later be built. This paper builds a strong case for the importance of doing some type of structural analysis concerning each new action that is to be developed. I find is to be a healthy suggestion. In some constructivist classrooms there occasionally exists the notion that every construction of meaning is of equal worth.

The next view of why we do the things we do in teaching comes to us from Coffland and Strickland. They report the results of a survey of variables related teacher technology and geometry. In reading this report a noteworthy point is that one's attitude towards use of computers appeared to be independent of other types of characteristics such as a technology awareness, attitude, training or usage. Perhaps it is the case that one becomes an adopter of technology in the mathematics classroom because of other factors not related to formal instruction in educational technology. We also find evidence supporting a reluctance toward use. In particular, almost every respondent stated that technology required more time to learn and implement. Although the field has come a long ways, as we will later see in the tools to teach with section, there is still a need for streamlining the technology preparation process.

A further examination of how technology can be used to deliver course material is offered in Weston's paper. The content in this case is a finite mathematics course and the technological tool was WebCT – an extensive set of tools created by faculty at the University of British Columbia. As we take a look at the types of tools contained within WebCT we see a rich set of communication tools, study tools, evaluation tools and management tools. I would encourage people planning on designing their own online course to take a careful look at this selection. The paper provides important details regarding the nature and organization of these tools well worth examining.

In Galminas and Autrey's paper we read of issues that emerge in the conversion of existing courses into a technology-based or technology enhanced course. The content in this case is that of College Algebra. Given College Algebra's place in nearly every curriculum it is clear that this paper should be read with great care. They correctly point out that technology comes in many different forms and one can be skilled in one form and not another. In planning such conversions it is likewise important to remember the differences in classroom interactions and the impact this plays on the mathematical cultures which is created.

The final paper in this section serves as a true bridge between the content and tools subsections. Knudsen's discussion of the issues dealt with in developing online learning for middle school teachers serves as both a carefully written justification piece for reform based instruction and as a rich description of a potentially powerful resource for mathematics education. In this paper we see a notable example of interactive Web pages that serve dual roles as both manipulative and as resource textbook. It is also possible to observe a keen sensitivity between the mathematics content and the manner in which the technology is used to enhance its instruction.

Together, these papers each have a well-developed reason for teaching in the way in which they do. Technology clearly not just slapped on as an afterthought but is an integral part of the instruction. They serve as evidence to a growing maturation in terms of creating interactive learning environments for both student, teacher candidate, and practicing teacher / teacher educator.

What tools do we have to teach with?

The third and final group of papers deals with the tools which are now available and which technology enables for us to use in both teaching and learning. It is at this level that we often see the true power of the computer in terms of both representation and instruction most fully utilized. Connell begins with two parallel examples showing the student creation of a personally meaningful, computer-enabled, referent for their mathematics. These examples show that the objects we can create to think with significantly impact thinking. This influence is felt not only in what the content is about but also in terms of what might be done in teaching and learning. The concept that these objects can themselves have built-in intelligence is something that should not be lost upon the mathematics educator.

In Campbell and Fonthal's paper we can read how a specific tool was created for use to explore whole and rational number concepts. As examine how this was done it seems to have resulted in a good match between the need of the content and the pedagogical and teacher requirements. Indeed, what sets this paper apart is that the program that was created as a thinking tool was very carefully aligned with what is technologically possible and pedagogically important. I find the recommendations for designing future software packages to be quite informative and helpful.

In the Cannon, Heal and Wellman's paper we see the fruition of a longtime dream which I sure many mathematics educators have shared in the creation of the library of virtual electronic manipulatives on the World Wide Web. These interactive web-based manipulatives are well worth the time it takes to visit. Needless to say www.matti.usu.edu has been bookmarked on my browser and I imagine that it soon will be on yours as well. It's amazing to think that this is just a preliminary paper. I look forward to following this work as it grows over the next few years.

In Ostler and Grandgennett's paper we see how Web pages can be used to teach mathematical modeling. In particular there some nice observations and suggestions in terms of instructional considerations. I find it very helpful to see that care was taken in terms of not only how we can use the computer to teach mathematical modeling, but also how do we teach the students. I heartily recommend this paper for anyone who is considering expanding their own mathematical models and representations as well as those considering the use of Web pages in the mathematics classroom.

In Bernard and Ramirez paper we see a beautifully expanded problem, *The biker and the nearby town*, as it is worked out with three different technological tools – Cabris Geometry, a spreadsheet, and a TI 83 calculator. It's fascinating to observe how each of the tools to think with subtly and some cases quite blatantly influenced the problem solving approach which was taken. This paper offers a rare insight into the manner in which technology influences student thought and problem solving.

The final paper in this section by Katkov concerns a new tool currently under development for the graphing of functions. Although not immediate related to teacher education, per se, it appears to be a powerful tool that looks to be quite extensible. The screen shots and a list of the permitted actions serve to remind us that even at a highly abstract level in the technologically enhanced mathematics classroom it is possible to see a series of well-defined actions taken upon objects of well defined properties. This tool appears to be currently under development and not having had the opportunity of using the tool myself, I would do like to examine its' robustness in an actual classroom setting.

Concluding Remarks

This year's crop of paper serves to illustrate how far we have come in this interesting intersection of mathematics, educational technology, learning theory, psychology, and pedagogy. We are far beyond the early days of "let's plug it in and see what is can do" and are well into integrating the

computer in the mathematics classroom. Furthermore, applications and methods are emerging which are not only pedagogically meaningful but also powerful from a mathematics perspective.

As we look to the future it is easy to envision classrooms where Internet connectivity is taken for granted, where every student has the expertise which was only held by experts in the previous year, and hardware and raw computational ability rivaling those of many research institutions of only five years ago. From this vision it is clear that we are in for some very exciting times ahead. However, despite whatever educational hardware and software we might have at our disposal we must bear in mind that in order for teachers to use these tools effectively they must understand the underlying mathematics.

A Case for Strong Conceptualization in Technology Enhanced Mathematics Instruction

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Abstract. It is extremely important for students to develop strong concepts regarding the nature of the objects they are manipulating and how these objects are to be used mathematically. This is increasingly important when the objects with which one is thinking are themselves active – as is the case within a technologically enhanced mathematics classroom. This serves to further underscore the need for increased mathematical conceptual awareness on the part of our teacher candidates.

Introduction

Mathematics educators have long espoused the use of hands-on manipulative objects as being essential for children's creation of meaningful mathematical concepts. As a result of this long-standing commitment to their use many of these manipulative objects have become widespread and nearly universal in mathematics classrooms around the world. Through the years educators everywhere have come to understand many of the properties of these powerful objects to think with together with many strategies for their effective use and cautions for their abuse.

Today we are seeing the growing awareness and use of technology enhanced mathematical objects, many of which possess considerable "awareness" of their function and capable of offering suggestions to the learner concerning their own use. Little is known, however, of the emerging technology enhanced environment, where the very tools with which one thinks are active as well as interactive. This paper attempts to describe some of the potential difficulties I have observed over the years. I will further argue for the creation of a strong set of mathematically relevant understandings on the part of both teacher candidates and their eventual students in order to be effective learners within this new technologically enhanced mathematical culture.

Two types of mathematical objects[1]

One of the first things which many teachers notice when using manipulatives is that they appeal to the children across a broad sensory array. A set of Dienes' $Base_{10}$ blocks, for example, will simultaneously present to the student experiences with mass, density, smell, color, texture and so forth. And, although this is a part of the manipulatives appeal to the student, not all of these sensory experiences are beneficial to the construction of the eventual concept for which they were selected. Furthermore, traditional manipulatives - such as the $Base_{10}$ blocks – will also serve to illustrate many other concepts than that for which they were originally created and in a similar fashion not all of these will be of benefit to the mathematics classroom.

[1] For a further discussion of this viewpoint see *Symbolic Computers and Mathematical Objects* by *Michael L. Connell* within these proceedings.

This is typically viewed as strength of the material and as an asset to the teacher. In this scenario, we have the potential to generate more than just the mathematical topic of the day. This is a generally a very good situation to have as it provides much flexibility and great linking power in a given manipulative.

Of course this multifaceted presentation of a broad array of experience to the senses is not always a good thing. It is possible that these additional added features are actually seductive details that distract from the core mathematical concepts that we would hope are being developed. This leads to the observation that the narrowing of focus brought about via a sketch is the first step toward the later highly abstracted representations such as those utilized in the algebra. Remember Whitehead in "Process and Reality" when he states that "...every advance in human understanding is brought about via an advance in the symbol systems used to think with" (Whitehead, 1929).

The technology enabled manipulatives and sketches lack many of the multi-faceted features of the traditional manipulative and serve as an important step in such a concentration of focus. For example, I have long suggested that an excellent use of a computer based manipulative is as a Visual Representation of a previously encountered physical manipulative which had been used to teach an earlier concept (see Connell, 1988). In particular, one of the key differences between a computer manipulative and the real world manipulative upon which is based lies in the degree of abstraction that occurs due to the use of the computer to generate the object of thought.

This is not such a strange idea for most teachers and teacher educators. In the case of mathematics quite often we are trying to teach a specific representation as opposed to a broad multiple uses of the manipulative. Thus as a domain we utilize standard representations such as Dienes blocks which are specifically constructed to carry a single meaning at the expensive other potential meanings which the material might be used for.

A Case for Strong Conceptualization

Due in part to this increased abstraction inherent in many computer enabled objects it is extremely important for children to develop strong concepts. There should be well developed understandings regarding what the objects they are manipulating are to be used for mathematically and how these objects are appropriately and inappropriately use. One analogy comes immediately to my mind is the smart wizards we see more and more of in Microsoft applications. I can envision a case where the technologically enabled objects that we provide for the children think with become more and more self aware.

To see how this might play out, let's imagine that we have identified two quantifiable sets that we wish to begin working on. Let us further imagine that we have identified various operations which it is possible to use on these two sets of objects and that they are of the same class such that the operations identified are appropriate. So the student selects a set addition object using the operating software and passes to it the instruction to it to combine these two sets.

The set addition object – which has more then a bit of intelli-sense and wizardry (to use Microsoft terminology for a moment) - does the cybernetic equivalent of looking at the task ahead of it and then responds back to the student, "Are you sure that you really want to do this?" In order to be successful in this new environment the child has to know whether or not this really is an appropriate operation to perform upon these objects, whether or not he's asked the right object to do the job, and how to interpret the results that he or she will eventually receive.

This is not a fanciful example. Such scenarios are becoming all to common and indeed are more than a bit of a nuisance showing up all the time in intelli-sense technologies. For example, you can be writing a letter and before you can even finish the first paragraph the Wizard de jour will pop-up. "Hi there! It looks like your trying to write a letter. How about I help you?" Typically I really don't want or need the help because it doesn't fit with either my writing style or the way I want to put this on paper. After all let's face it as an academician things are written differently all the time. For whatever reason, however, it's important to note that in this scenario I am the expert. I can override the suggestions of any object or Wizard when it's not inline with the tasks that I need to have done.

In this New World I'm envisioning, however, of computer enhanced mathematics and mathematics instruction via active objects this may not always be the case. Let's imagine that we are doing some integration and a wizard makes a suggestion on the boundary conditions over which to integrate which nine

times out of ten would be right. If your problem happens to fit the tenth condition and you succumb to the wizards' advice in the face of your own lack of concept you are in major troubles.

This is problem occurs on a daily basis in computer programs commonly used in the statistical analyses of data. A very real problem has occurred as more and more researchers are getting access to higher and higher levels of statistical programs. In many cases programs such as SPSS and SAS will enable processing beyond the interpretive levels of the users. It is very common to find data sets that are spherical and never checked for, and post-op comparisons that are performed correctly but selected inappropriately. All of this because the tools given for the individual to think with were, in many ways, smarter than the people thinking with them. This trend is one that shows every tendency of continuing and accelerating.

This plays out with a vengeance in the educational arena. We are already able to design and implement intelligent objects with more "number-sense" than the beginning students who will be utilizing them will have. We are very close to being able to come up with objects to think with which are more intelligent then the people who working with them. This is not intended as a callused or a mean comment. We don't expect a tremendously high level of mathematical metacognitive knowledge at the first grade level. After all, the learner is just putting all this stuff together – in many cases for the very first time. So to return to our earlier example, it would be very easy to imagine a set addition object that basically says, "I will take any two numbers you give me and combine them using the operation of addition". In terms of sheer processing power this could easily be at a higher skill level then the child using the object.

If we are to be effective teachers in this new technology enhanced environment we need to make sure that our students truly understand the concepts. If this is not done, all of our lovely correct answers are meaningless. This was a major concern in the calculator based reform effort of 15 years ago, it is even more crucial in the computer environment. Let us see why this should be the case. Using a traditional calculator you still had to plug everything in yourself – much like the old command line DOS interfaces or line-by-line BASIC compilers. The new computer environments and many of the newer calculators are becoming increasingly object-oriented. Therefore, it is entirely possible that we may end of having the terminal smarter than the user. I have always thought we were better off having dumb terminals and smart users in computing – without extreme care we will soon be facing the reverse.

Examining the Tools to Think With

The creation of new objects of thought or tools to think with can become very powerful pedagogical tools assuming we understand the concepts underlying them. The hidden danger surfaces when we cannot understand the underlying mathematical concepts upon which the active objects are operating on and we simply take them for granted, follow their recommendations blindly, and accept their results at face value.

This would be analogous to letting your writing be totally edited by wizards in your word processor. The following poem, which has enjoyed wide popularity among information technology faculty through the years, will serve to illustrate the dangers of such an approach.

SPELL BOUND

I have a spelling checker,
It came with my PC.
It plainly marks four my revue
Mistakes I cannot sea.
I've run this poem threw it
I'm sure your pleased too no,
Its letter perfect in it's weigh,
My checker tolled me sew.

Because of our in-depth knowledge of words and word usage it is easy to see the humor in this piece of writing. The errors are obvious and mostly harmless. If, however, we are looking at a

computational object which was created with a flaw in the underlying logic the errors are not nearly so obvious, harmless, or humorous.

Consider the following:

Depth	The Total Relation	Diff. From 1	Radical Portion	Diff. From .5
1	2.00000000000	-1.00000000000	1.00000000000	-0.50000000000
2	0.00000000000	1.00000000000	0.00000000000	0.50000000000
3	1.08239220029	-0.08239220029	0.54119610015	-0.04119610015
4	0.99401088611	0.00598911389	0.49700544305	0.00299455695
5	1.00019859327	-0.00019859327	0.50009929664	-0.00009929664
6	0.99999679901	0.00000320099	0.49999839950	0.00000160050
7	1.00000002540	-0.00000002540	0.50000001270	-0.00000001270
8	0.99999999990	0.00000000010	0.49999999995	0.00000000005

In this example it may not even be possible to identify the purpose for which the "object" was created, the assumptions that underlay it's creation, or the use to which it's results might be applied. The calculations are done correctly, of that we have no doubt, but to what use are they to be put? I am reminded of Douglas Adams Science Fiction classic "Life, the Universe, and Everything". In this classic text we learn that the answer to all of the truly deep problems of philosophy, metaphysics, etc., is actually 42. The difficulty is we do not know precisely what these questions are and in what form this answer fits.

Conclusion

In conclusion, the mathematics classroom of today is a far cry from that of 15 years ago. The technological objects upon which we act now routinely have intelligence built into them. If you made an error in using a slide rule, as was quite common when I was in school, all that would happen is that your result would be inaccurate. Today's intelligent objects have the potential notifying you of the error, suggest new options for you consider, and quite possibly lead you astray through giving information at a level which does not match with your understanding.

Ironically, the importance for understanding the underlying mathematical concepts in this scenario is significantly stronger than during the previous introduction of technology – the calculator. The calculator, despite the vast hue and cry of the time, proved to be a relatively benign intervention. Helping, as it were, with the numeric processing of skills and procedures that the child would have to construct, apply, and evaluate. The computer with its increasingly powerful objects of thought is a much more insidious problem.

On one hand it allows us to leverage our thinking tremendously forward through interaction with tools which themselves have rudimentary problem solving abilities. Furthermore, the natures of the data organization made possible through using technology lend themselves to types of approaches with known mathematical pay-off. On the other hand, however, we are in a very real danger of having our tools become more intelligent than the people using them. It is the responsibility of mathematics educators everywhere to ensure that our teacher candidates, teachers, and students are able to use the tools and not be used by them.

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Faculty/Student Collaboration in Education and Math—Using the Web to Improve Student Learning and Teaching

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Abstract: In this paper we describe a model for interdisciplinary collaboration between students interested in teaching and faculty teaching courses in the students' area of study—math and education. This collaboration involves the education students in the development of Web materials, video recording of classroom activities, and development of rubric for pedagogy effectiveness. It has as its objectives: the enhancement of the education students understanding and development of effective teaching strategies, including knowledge and application of instructional technology skills; the improvement of student learning in an existing course being taught by one of the collaborating faculty; and the enhancement of the teaching expertise for the faculty member teaching the course. These goals are accomplished through the use of the Web to provide immediate feedback to students in the class on topics that they find difficult, and through the faculty and education students' reflection on classroom dynamics through the use of digital video.

Introduction

The World Wide Web (or, more simply, Web) is rapidly becoming a ubiquitous feature of all aspects of society, including higher education. However, in the collegiate setting it is all-too-frequently used only to reproduce information that is available in other formats or media. Thus while it provides a new and effective vehicle for the dissemination of course information, it is difficult to determine whether its use is improving student learning. This is of increasing concern given the continual expansion of responsibilities felt by many university faculty, for whom adding Web page development to their repertoire may be part of a zero-sum game the loser in which is sleep. We are interested in the synergy afforded by the collaboration of faculty and students, especially those students interested in education, and the manners in which this cooperation may be used to enhance teaching and learning at the interrelated levels of the faculty member's course, the students learning in the course, and the education students' development as effective teachers. The involvement of the education students allows the distribution of the workload associated with the use of the Web and results in the students applying instructional technology skills acquired from a required education course, "Instructional Technology," which introduces them to a variety of educational software applications.

The use of the Web in this model has two interrelated parts, which are carried out with differing frequencies throughout the semester. First, it is used by the faculty member(s) teaching the course as a vehicle to respond to student confusion as expressed in specific classes through the use of a classroom assessment technique similar to the "Minute Paper" and "Muddiest Point" methods of Angelo & Cross (Angelo & Cross 1993). Second, it is used in conjunction with videotaped classroom sessions, taped by the education students, to similarly address students' misperceptions and confusion. The videotaped sessions are discussed by the faculty and education students using a rubric on pedagogy (developed by both the math education students and both faculty members) to isolate areas of confusion, and the education students then develop Web materials to address these. In either part, the Web provides a unique and valuable vehicle for the dissemination of the information to be conveyed: it allows for a dynamic information base that may change on a more-or-less daily basis, and provides a universally

accessible resource for the information. Its use in this manner also allows these confusions to be addressed simultaneously with the usual progression of the course, thus avoiding the loss of the time in the classroom which is so often already in short supply. Finally, there are multiple levels of learning occurring as a result of the second component, as the students in the class are learning through the Web materials being developed and the education students are also learning through the discussions of pedagogy and development of these materials.

In the remainder of this paper we explain how we implemented this model, describe in greater detail the different components indicated above, describe the manners in which we assessed the success of the model, and draw conclusions as based on the results of the assessment in the Fall Semester.

Implementation

All of the activities described in this paper were implemented in the environment of a small (about 1500 full-time students), somewhat selective, liberal arts based university. Upper-level courses in both mathematics and education, the two subjects studied, are small (10-15 students large), as are even lower-level courses offered out-of-sequence. This has clear advantages in teaching, as well as disadvantages when it comes to assessing the impact that the methods we discuss here actually have.

We are implementing these uses of the Web in the Fall semester 1999 in a second semester calculus course in the mathematics department. In the Spring semester 2000, we apply the same methods to the "Secondary Methods" education course.

Day-to-Day Web Use

The "day-to-day" use of the Web to respond to students' confusion regarding the material covered in class was accomplished through a classroom exercise resembling the "Minute Paper" or "Muddiest Point" techniques in "Classroom Assessment Techniques" (Angelo & Cross, 1993). At the end of each class period, approximately five minutes were reserved for student consideration of the material covered during the preceding hour. In this time, students answered the questions "what was the central theme of this course period?" and "what about this was least clear?" in a couple of sentences each. These were collected by the instructor, who then reviewed the comments and, if appropriate, articulated a response. Appropriateness in this case was defined by whether a significant number of the students in the class expressed confusion on the same topic or posed questions that were of sufficiently general application as to merit a response.

The response articulated was implemented as a Web page designed to elucidate the material and resolve questions and confusions. The materials developed were limited to a single Web page per day.

Videotaping and Web Materials

Three classroom sessions were selected in advance of the commencement of the semester as being on topics that would prove especially difficult for the students taking the course. These class periods were videotaped by the education students involved in the project using a digital video camera. The faculty and education students then watched the video later in the day and assessed the effectiveness of the class presentation, degree to which the instructor accurately adjusted to the classroom dynamics, and areas that were particularly confusing to the students in the class. As a vehicle for this assessment, a rubric was cooperatively developed by the students and faculty to evaluate the instructor's teaching method, effectiveness of questioning, assessment of students' understanding, and class organization. This rubric appears in an appendix.

Following this discussion and assessment, the faculty and education students determined what type of Web-based materials would be appropriate to resolve those issues left 'hanging' by the classroom presentation. These include demonstrations, excerpted sections of the videotape with further explanation and references, text and graphical instructional pages, and links to other on-line resources providing background information for the topic being covered. The outline of the mathematics and substance for the materials was determined in advance by the faculty and education students, collaboratively, and the actual pages were then developed independently by the

education students. Excerpts from the video were obtained through video editing Final Cut, MotoDV and included in the resulting Web pages, and the resulting materials reviewed and posted by the faculty member on a class website.

The technology skills of math education students were gained in Education 187 "Instructional Technology" and the development of this project allowed these students to apply these skills. In addition, these students were able to analyze the role and impact of interactive video and website development. This approach supports the Davis' (1999) principles for technology in teacher education including "...students should experience innovative technology-supported learning environments in their teacher education programmes" (p. 9).

Assessment

This project has three primary goals, namely, 1. the promotion of the education students' awareness of effective teaching strategies and the use of reflection and assessment in the development of the same, 2. the improvement of student learning in the courses involved with the project as a result of the use of the Web to provide "immediate" feedback on difficulties encountered with specific course topics, and 3. the enhancement of faculty teaching expertise. In order to determine the degree to which these were accomplished in the course of our work, several assessment instruments were employed. However, the assessment of any of these objectives is constrained by their inherently qualitative nature and by the small size of the classes and small numbers of education students involved, and we are therefore constrained to use generally indirect measures to determine our success in accomplishing them.

To assess the first objective, the education students each developed in the course of the semester a portfolio describing their thought processes at the beginning and end of the program, as well as their reflections on the nature and success of each of the videotaping sessions. They were provided with a number of prompts to which to respond, as described in the appendix. Samples of the students' entries are also included in the appendix. The reflections in these by the students and the assessment of the cooperating faculty members of the students' portfolios and their overall development were used to obtain a picture of the effectiveness of the project in accomplishing this goal.

The second objective is assessed through three different measures. First, we surveyed the students in the course at the beginning, middle, and end of the semester to determine their perception of the effectiveness of the course and usefulness of the Web materials produced. Second, the quality and relevance of the materials were evaluated by the faculty in the program at the end of the semester, taking advantage of knowledge of the difficulties students experienced on homework and exams. And third, we monitored the number of "hits" on the course Web throughout the semester, assuming that a continued or increased number of hits as the semester wears on is indicative of perceived value by the students. It is worth observing that while we would prefer to have more direct measures of the success of this objective it is difficult to obtain them in the face of the size and number of courses taught at a small university. We have therefore instead used these measures of effectiveness of the materials rather than try to directly assess the improvement of student learning.

The third objective suffers from the same difficulty in assessment as the second, for the "faculty teaching expertise" that we seek to enhance should be measured in improved student learning. As noted above, this is notoriously difficult to determine, and we therefore used subjective assessment measures to determine its success, having the education students and faculty member who was not teaching the course evaluate the instructor's effectiveness using the rubric developed for the examination of the videotapes and through general reflection

While it is not within the scope of this paper, the most significant assessment of the impact the program had on the education students involved in it would be an examination of those students' teaching (ideally, as compared with the teaching of students who did not participate in the project). This is again complicated by the small numbers of students in the program, but we look forward to being able to evaluate this as the education students do begin their student teaching, at the end of their university experience.

Results

Because we are still in the process of the first semester in which this program has been implemented, final results of the assessment program described above are not available here and will be presented in the conference presentation. In particular, the portfolios and faculty teaching assessments require the comparison of results from

the beginning and end of the semester, the latter of which are not available at this time. Similarly, complete results from the surveys of the students in the class are unavailable. We are, however, able to provide the preliminary results from the midterm surveys, which are positive.

In the midterm survey given in the Fall, students in the calculus class indicated that they were using the Web materials (with a majority indicating periodic or regular use, and 100% using it when an assignment referring to the materials was given). When asked to rate the usefulness of the materials on a four point scale (0=not useful, 4=immensely useful), they gave an average score of 3.13 (with some students simply responding "yes," it was useful; no students gave the materials less than a 2 in their rating). In addition, the number of hits on the Web pages did increase in successive months of the semester, from an approximate rate of 3 hits per student in the class in the first two months to approximately twice that in the last two. This provides some material evidence that the Web pages were perceived as useful by the students in the class. Further results will be presented at the SITE Conference on 1999 Fall Semester work as well as initial research in 2000 Spring Semester.

Conclusions

We have discussed an innovative, collaborative method of using the Web to respond directly to students' misperceptions and confusions as they take a class. The course primarily discussed here was second-semester calculus, but the method is applicable to courses in any area, and will be implemented for a course in secondary education methods in the Spring semester. In this method, we have involved faculty from our education and mathematics departments, as well as students who are studying mathematics education, to promote interedisicplinary learning at multiple levels—in the classroom, by the education students involved in the program, and by the faculty teaching and observing the class. Results from assessment of the method provide evidence that these goals are accomplished. The use of the Web is in integral to the project, as it provides a universally accessible medium that admits frequent addition of material on a regular basis and is an instructional technology to which the education students will both have access and be expected to use as they begin their careers.

Appendix

Assessment rubrics and materials are included in this appendix. The assessment rubric used to evaluate the videotaped class sessions is shown in figure 1, below. Note that the columns 3-5 headed in the second row should follow to the right of the first row in the table.

	1	2
Teaching Method Appropriate to Material	No method, or no formal connection between teaching material being covered	One evident tie of teaching method with content
Demonstrated Effective Use of Basic Questioning	No questions asked of students	Few questions asked of 25% of students or topics
Effective Assessment of Students' Understanding of Class Material	No method of assessment	Limited assessment of limited number of students
Effective Class Organization (Intro/Overview, Sequence of Instruction, Closure)	No effective organization	A minority of the lesson structure is present: one element present and good, or more present but all abysmal
3	4	5
Two evident ties of teaching method with content	Three evident ties of teaching method with content	Several (four) evident ties of teaching method with content
Some questions asked of	Similar, 75%	Questions asked of all

50% of students or topics		students and topics
Some assessment of a select few students	Many assessments of a majority of students	Many assessments of all students
Mixed or muddled lesson organization: all three elements present but ineffective	Majority of lesson structure is present and effective: at least two elements present and good, or three present but not fully effective	Effective organization including clear intro/overview, good sequence of instruction and effective closure.

Figure 1: Class assessment rubric.

The prompts given to the education students at the beginning and end of the semester, and following each of the videotape discussion sessions, are shown in figure 2.

At the beginning and end of the semester:

1. Outline a classroom lesson on a topic that you would expect to teach following your graduation or when you student teach.
2. Comment on the terms "assessment" and "feedback." In particular, what do you think about when you hear them? And how do you think they are related to teaching and the classroom.

After each videotape session:

3. Comment on what happened in the classroom and in the development of the supporting materials.
4. What key ideas or issues relating to teaching did you encounter in the course of working on this part of the project?

Figure 2: Portfolio prompt questions.

Samples of student responses to these prompts (prompt #2) are shown in figure 3.

Student response one--"When somebody uses the terms assessment and feedback, the first thing that comes to mind is tests! However, there are many other things that are involved with assessment and feedback besides just testing. First of all, assessment is...".

Student response two—"Feedback: The information students give to their teachers that convey confusion, also not always verbal. Things as simple as seating arrangements or eye contact can be considered a type of feedback for a teacher...Assessment and feedback work together in a well-organized classroom".

Everyone was really well engaged in the discussion and development of the rubric. It looked like everyone had fun with it, too".

Figure 3: Sample portfolio responses.

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Examining the Feasibility of Using Computer Supported Collaborative Work-Space in Pre-Service Mathematics Methods: A Vygotskian Perspective

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Abstract. Many elementary teachers lack the conceptual knowledge needed to teach even basic mathematics and thus create a weak link in the chain of mathematical learning for students. The socio-cultural theory of L. S. Vygotsky may offer direction for the design of a concept-building mathematics experience for elementary education majors. Computer supported collaborative workspaces have the potential to support and enhance a Vygotskian approach to mathematics learning for the preservice teacher.

Introduction

According to Rech, Hartzell and Stephens (1993), the quality of teaching at the elementary level is a critical factor in the future mathematical success of the student. High quality teaching is heavily dependent upon the teacher's conceptual knowledge of the subject matter. Yet, when questioned, many elementary teachers communicated a sense of personal inadequacy in mathematics and a lack of confidence in teaching mathematics (National Science Board, 1998; Stepans et al., 1986). Simon (1993) demonstrated that teachers who do not possess solid understanding of mathematics principles are unable to teach mathematics in a conceptual manner. Simply stated, teachers cannot teach what they do not know.

The competency of those who are teaching elementary level mathematics has long been questioned (e.g., Ball, 1990; Glennon, 1949; Zaslavsky & Peled 1996). Research conducted by Zazkis and Campbell (1994) supported the general claim that elementary teachers often possess weak content knowledge and insufficient conceptual understanding to teach mathematics. Rech, et al. (1993) found that "...the elementary education majors possessed deficits in almost all knowledge and content areas in mathematics when compared with the established norms of a general college population" (p.144)

Assuring that prospective teachers possess adequate mathematical content knowledge should be a primary concern in the preparation of elementary teachers; however, it is an area that continues to be neglected. Monroe (1984) stated that the mathematics education of elementary level teachers was less than adequate "by whatever standards are used" (p.23). Leitzel (1991) contended that the weakest link in the nation's system of mathematical education is the mathematical preparation of elementary teachers. According to Hungerford (1994) colleges and universities have done little to remedy the weak backgrounds of pre-service elementary teachers; "...the lack of attention being paid to mathematics courses for prospective elementary teachers is astounding" (p.16).

Teachers' mathematics backgrounds are particularly relevant in light of the research that suggests that a teacher's academic background (i.e., both level of courses and grades earned) may be related to student outcomes (Chaney, 1994). In 1985 Galambos et al. documented a pattern that reflected the absence of college-level mathematics requirements in teacher education. There is no indication that this pattern has changed.

Courses in pedagogy influence how preservice teachers think about teaching and learning mathematics. Yet, merely training teachers in mathematics pedagogy appears to be insufficient. Chaney (1994) demonstrated that mathematics pedagogy provided added benefit for teachers only if they possessed adequate mathematical concepts. Attending to mathematics teachers' beliefs about teaching and learning is unproductive when content knowledge is absent (Brown, Cooney, & Jones, 1990).

Advocacy for a combined content-methods mathematics course in teacher education is not new (Berg et al., 1993; Glennon, 1949; Heddens & Speer, 1997; Reisman, 1981). The combined content-methods approach recognizes that pre-service teachers may not possess the content knowledge and understanding needed to competently teach elementary mathematics. This curriculum emphasizes: (a) building a strong background understanding of mathematics concepts that are taught in elementary programs; (b) addressing and replacing misconceptions with conceptually sound knowledge; (c) reducing teacher anxiety and increasing teachers' self efficacy relative to doing mathematics; (d) demonstrating appropriate mathematics strategies and methodologies.

Conceptual learning differs from the rote memorization of facts and procedures (Bruner, 1963, 1971; Piaget, 1971; Vygotsky, 1986, 1997) that have historically characterized most mathematics classrooms (Cuban, 1984). Assuring conceptual knowledge of mathematics in preservice teachers will require change in both the learning environment and the emphasis of instruction. The socio-cultural-historical theory of learning proposed by Vygotsky (1986) may offer guidance in creating such an environment.

Vygtosky's Theory of Socio-Cultural Learning

Vygotsky's sociocultural theory of mind serves as a middle ground between formalism and constructivism (Kozulin, 1990; Minick, 1987; Moll, 1990; Vygotsky, 1986; Wertsch, 1985). According to his theory, concept formation is a dynamic interaction between the concrete and the abstract; neither requiring the learner to reinvent information, nor expecting the learner to conceptualize abstractions without first engaging in concrete activities that support the formation of mental models. Vygotsky proposed several key ideas relative to the levels and types of concept formation, the role of language and collaborative interchange in concept development, and the characteristics of an instructional environment that guide and promote mature concept development. Vygotsky viewed the ultimate goal of the learning process as the development of mature concepts which are characterized by the learner's ability to: (a) synthesize abstracted traits; (b) use the resulting abstract synthesis as his or her main instrument of thought without any reference to the related concrete situation or impression; and (c) use the concept in the formation of judgments and new concepts.

Language.

Language, and its relationship to thinking, is at the foundation of Vygotsky's theory (Vygotsky, 1986). Vygotsky regarded language as an indispensable requisite for all intellectual growth. He asserted that the merging of practical intelligence with a system of symbolic representation (i.e., speech) is the essence of complex behavior (Vygotsky,1978). According to Vygotsky (1978), speech has a particular organizing function that when combined with tool use produces fundamentally new forms of behavior. "...the most significant moment in the course of intellectual development... occurs when speech and practical activity, two previously completely independent lines of development, converge" (p. 24).

Two critical observations Vygotsky (1986) made were that: (1) the role of speech is equally as important as the role of action in attaining a solution to a problem; and (2) as situations demand more complex and indirect actions in finding a solution to a problem, speech plays a more important role in the solution process as a whole. Speech acts to organize, unify, and integrate the many aspects of the learner's behavior including perception, memory, and problem solving (Vygotsky, 1978).

According to Vygotsky (1986), individuals negotiate meaning and form concepts through verbal interactions with more knowledgeable others and by sensory interaction with their culture. Vygotsky noted that the mere association of words with objects does not imply concept formation and suggested that concept formation begins with a problem that cannot be solved other than through the formation of new concepts. The attainment of a true concept results in a qualitatively new type of thinking, therefore, merely quantitatively increasing associations of words with physical objects would never culminate in the higher intellectual activity that is characteristic of mature concepts (Markova, 1979; Moll, 1990; Ratner, 1991, 1997; Vygotsky, 1986; Wersch, 1985). Vygotsky considered the defining moment in concept formation to be the point at which the learner is able to use words as functional tools to facilitate communication, understanding, and problem-solving.

Spontaneous and Scientific Concepts

According to Vygotsky (1978, 1986), conceptual development does not occur in a vacuum, nor does it develop in a one dimensional, linear fashion. Vygotsky viewed concept formation as a multidimensional interaction of the child's social, historical, and cultural development. This view of concept development

considers the concept itself as being qualitatively more than the sum of its individual parts (Davydov, 1990; Forman, Minick, & Stone, 1993; Ratner, 1991; Vygotsky, 1986; Wersch, 1985).

Vygotsky identified two different types of concepts; *spontaneous concepts and scientific concepts.* Spontaneous concepts are those concepts that result from an individual's everyday exposure to his or her social and cultural environment. Typically, spontaneous concepts are unsystematic and highly contextualized. Scientific concepts are grounded in mediated instruction and are characterized by hierarchical, logical organization. Vygotsky (1986) proposed that spontaneous and scientific concepts differ in their development as well as their functioning, yet these two variants of the concept formation process influence each other's evolution.

Zone of Proximal Development

Vygotsky (1978) identified a construct that he called the Zone of Proximal Development (ZPD) which he defined as: "...the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p.86). An individual's movement through his or her ZPD requires a mediated environment that is rich in verbal communication and collaborative exchange (Forman, Minick, & Stone, 1993; Vygotsky, 1997; Wersch, 1985). Vygotsky (1986) stated that an individual's progression through his or her ZPD involves intelligent, conscious imitation, which requires an understanding of the field structure and the relationship between objects. Vygotsky and Luria (1993) cautioned that imitative behavior that is characteristic of movement through the ZPD should not be confused with automatic imitative behavior that shows no signs of conscious understanding.

Implications for Instruction

Vygotsky's theory clearly supports a mediated, collaborative classroom environment. He asserted that every function in cultural development appears twice "...first on a social, and later on the psychological level; first between people as a interpsychological category, and then inside [the learner] as an intrapsychological category" (Vygotsky, 1978, p. 128). According to John-Steiner and Souberman (afterward in Vygotsky, 1978) teaching represents "...the socially elaborated contents of human knowledge and the cognitive strategies necessary for their internalization" (p. 131). In the socio-cultural context, instruction should be configured not so "...that the student is educated, but that the student educates himself" (Vygotsky, 1997, p. 48) through the social negotiation of meaning.

Socio-cultural learning environments stand in contrast to traditional classrooms that historically have made little use of socially shared tasks (e.g., Cuban, 1984; Ferraro, Rogers, & Geisler, 1995; Wilson, Teslow, & Taylor, 1993). Learning environments consonant with Vygotsky's theories facilitate collaborative student engagement in: (a) the active process of sense-making through meaning negotiation; (b) the use of shared signs (i.e., speech) and symbols (i.e., objects of meaning representation) as cognitive tools for concept development; and (c) multidisciplinary approaches to learning that reflect the learners' social, cultural, and historical context.

As an instructional tool, computer-mediated-communications (CMC) have the potential to support the learning outcomes, orientation of instruction, and underlying pedagogical beliefs espoused by Vygotsky. Jonassen (1996) classified CMC as a knowledge representation tool that may engage learners in critical thinking activities. He contended that CMC has the potential to involve students in active learning that is built on cumulative (i.e., prior) knowledge, and supports integrative, reflective, goal-directed and intentional learning. Current technologies, particularly functions of computer networking, provide a mechanism for developing collaborative environments that transcend the constraints of time and place, thus increasing students' opportunity for socially supported engagement (Laffey, Tupper, Musser, & Wedman, 1998; Reeves & Reeves, 1997; Riel, 1996; Romiszowski, 1997).

Computer Supported Collaborative Learning Space Environments

Computer -mediated communications (CMC) is a general term used to define any form of organized interaction between individuals or groups of individuals that facilitates and/or mediates communication utilizing computers or computer networks as the medium of communication (Hawisher, 1995; Kahn, 1997; Jonassen, 1996). CMC includes both *synchronous* and *asynchronous* communications. Synchronous communications are those that take place when two or more individuals communicate simultaneously over a network (real time), while asynchronous communications are those that do not require concurrent communication (delayed).

CMC is a medium that can be used to more closely link collaborative classroom experiences with formal instruction to create a highly student-centered learning environment (Jonassen ,1997; Riel 1996). Results from studies suggest that teachers in a traditional classroom contribute up to 80% of the total in-class verbal interaction, whereas the total verbal contribution of teachers using CMC conferencing techniques is between 10-15% (Reil & Harisim, 1994). Curtis and Reynolds (1997) found that CMC-based interchanges resulted in more frequent exchanges between students and Scott (1993) demonstrated that students working within groups via CMC participated more evenly, and accomplished more task objectives than students not using CMC. CMC mediated environments often decrease the students' inhibitions to participate, while increasing student opportunity for reflection and knowledge accommodation (Romiszowski, 1997). This medium also offers a high degree of learner control, a wide range of environmental flexibility, and global connective capablities (Kahn , 1997; Relan, & Gillani, 1997) that support both formal and informal learning environments at various interactivity levels (Hiltz, 1994; McLellan, 1997).

Groupware

Groupware is any type of software designed for group work or group communication. Organizational and educational trends toward increased teamwork along with the availability of networked computing has stimulated the development of computer supported collaborative workspaces (CSCW) as a means of supporting tasks carried out by participants who are physically or temporally removed from one another (Brinck, 1998; ter Hofte, 1998). Ter Hofte (1998) used the term "working apart together" to describe the essential nature and function of groupware systems. The types of groupware that may prove beneficial to preservice mathematics learning include: (a) computer conferencing systems; (b) chat systems; (c) workflow management systems; (d) shared whiteboards; and (e) coauthoring systems.

Computer conferencing systems (also known as bulletin boards) can be regarded as a variation of electronic mail (e-mail) systems. E-mail systems support interpersonal communication by sending computer mediated messages to one or more persons, while computer conferencing systems facilitate the transference of messages to a uniquely identified address or location on the Internet (URL) that is devoted to discussion about a particular topic. Conference areas allow individuals to post and to retrieve information just like they might do on a physical bulletin board. Early conferencing systems supported only textual messages, but more current systems support the posting of other types of documents, such as word processor documents, spreadsheets, graphics, etc.

Chat systems provide synchronous text-based computer-mediated discussions between and among users. Discussions take place through rapid turn-taking entries by the participants. These systems provide "live" interpersonal communication for an arbitrary number of users who are connected via personal computers.

Workflow management systems have embedded workflow cooperative tasks models that coordinate actions by users by prompting the appropriate contributions at the right time by the right users. These systems can also track the progress of the workflow, provide relevant information required for particular actions, and block information entries that contradict the model.

Shared whiteboard systems are designed to support text, drawings, and sketches that are often shared at formal and informal meetings to point out particular items, clarify relations, or illustrate complex materials. Objects drawn in the shared workspace are immediately visible for all other users. These systems allow all participants to simultaneously refer to such illustrations, propose modifications by altering the drawing, or add textual comments. Entries can be saved and distributed.

Co-authoring systems are designed to support multiple users in creating a collaborative document. Classes of co-authoring systems support diverse phases of the writing process such as brainstorming, researching, planning, writing, reviewing, editing, and revising. These systems facilitate communication between authors and assist the coordination of the authoring process.

Conclusions

There is a documented need for mathematics reform in preservice teacher education. A long history of research suggests that preservice mathematics methods courses must play a dual role of delivering content in the form of reeducating students in basic mathematical concepts, and instructing students in pedagogy that supports concept development. Vygotsky's theory appears to offer guidance for structuring a learning environment that facilitates conceptual learning and promotes sound methodological strategies.

Among the roadblocks that may hinder change in preservice mathematics reform are internal institutional issues such as curriculum reform and time limitations imposed by course instructional hours, scheduling conflicts, required field work, and the length of school terms. Applying Vygotsky's theories to the application of computer-mediated collaborative workspace may help ameliorate problems associated with these issues. Computer supported collaborative work-spaces have the potential to add dimension to an existing course by extending facilitated collaboration beyond the limited classroom hours. The combination of synchronous and asynchronous communication options opens the possibility for student engagement in collaborative enterprises that otherwise may have been impossible because of distance and scheduling conflicts.

Networked technology and various categories of groupware systems offer a new spectrum of tools that, with proper implementation, may prove useful in preservice mathematics experiences. The inherent communicative nature of these tools is compatible with the underlying tenets of both Vygotsky's theories and the goals of the content/methods curriculum. As a tool, computer supported collaborative workspace has the potential to incorporate the extended boundaries of the student's social and educational milieu, thus providing an environment conducive to the application and generalization of knowledge.

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Implementing Technology in Secondary Science and Mathematics Classrooms: A Perspective on Change

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Abstract: The purpose of this paper is to examine and describe the change process as technology is implemented in secondary science and mathematics classrooms. This paper synthesizes the results from two studies, a qualitative study on implementation concerns of secondary science teachers resulting from the use telecommunications (Slough, 1998) and a quantitative study on technology implementation concerns of middle and secondary mathematics teachers and potential teachers of first-year algebra in North Carolina (Chamblee, 1996). The Concerns Based Adoption Model (CBAM) provided a theoretical framework for both studies.

This paper synthesizes the findings from two studies, a qualitative study on implementation concerns of secondary science teachers resulting from the use telecommunications (Slough, 1998) and a quantitative study on technology implementation concerns of middle and secondary mathematics teachers and potential teachers of first-year algebra in North Carolina (Chamblee, 1996). Both studies used a Concerns-Based Adoption Model approach to frame data collection and analysis. Commonalties and differences from separate content areas—math and science—and separate methodologies—qualitative and quantitative, were analyzed in an effort to triangulate data and findings. From this approach, commonalties that exist across content and methodologies are strengthened and differences that exist point to possible content specific idocentricities or need for additional corroboration. Thus, by comparing and contrasting the findings from the two studies, a more complete picture on implementation concerns for math and science teachers with respect to technology emerges, including a more holistic picture of the technology-based change process.

Theoretical Framework

The implementation of technology will require change in the classroom. One model that has been utilized to inform the decision-making process when innovations are introduced is the Concerns-Based Adoption Model (CBAM). CBAM states that successful implementation of an innovation is a process not an event (Hall & Hord, 1987; Fullan, 1991; Friel & Gann, 1993), developmental in nature (Hall & Hord, 1987), and a highly personal experience for each teacher (Hall & Hord, 1987). Thus, for any change to be successful the concerns of each individual teacher must be considered as important and their individual needs must be met.

Hall, George & Rutherford (1986) define concerns as the feelings, thoughts, and reactions that individuals have about an innovation or a new program that touches their lives. To measure these concerns, Hall, Wallace & Dossett (1973) developed the Stages of Concern Questionnaire (SoCQ). Initial research on the instrument construction verified the existence of seven stages in the concerns process: awareness, informational, personal, management, collaboration, and refocusing, with internal reliability for individual scales ranging from $r=0.64$ to $r=0.83$ (Hall, George & Rutherford, 1986).

Bailey and Palsha (1992) proposed a modification of the CBAM model to include only five stages: awareness, personal, management, impact, and collaboration. The awareness stage is characterized by teachers having little knowledge about the innovation but interested in learning more about it. The personal stage is characterized by teachers who are primarily concerned with how the innovation will affect them, with a specific focus on required changes in roles and tasks. The management stage is characterized by teachers who are primarily concerned with time management, organization, and prioritization of responsibilities. The impact stage is characterized by

teachers who focus on the effects of the innovation on learners and how this innovation can be used to change or improve learning. The collaboration stage is characterized by teachers who focus on working with others to implement the innovation as well as sharing information about the innovation with other teachers.

Data Sources

Data for the qualitative study (Slough, 1998) of secondary science teachers' concerns was collected through open-ended ethnographic interviews of twenty-four high school science teachers who had been in an emerging telecommunications-rich environment for at least two and one-half years as of the Fall Semester, 1997. The Bailey and Palsha (1992) five stage model was utilized to frame the analysis. The emerging telecommunications-rich environment was defined as including a district-wide infrastructure that had been in place for two and one-half years that included 24 network connections in each classroom, full Internet access from the network, four computers per classroom (teachers were required to attend training before receiving), and a variety of mandated and optional professional development opportunities within and outside the district. The teachers were from a single, large, suburban school district with five high schools. Teachers in the study were well distributed across each of the five high schools, across typical high school science courses, across all levels of educational attainment, and included fairly new to veteran teachers. Research questions focused on teacher and students' use of telecommunications, barriers and supporting conditions to telecommunications implementation, and the effect of telecommunications on the teaching and learning of science.

Data for the quantitative study (Chamblee, 1996) on first-year algebra teachers' concerns was collected using the Stages of Concern Questionnaire (SoCQ) by Hall, Wallace & Dossett (1973). A total of 132 middle and secondary mathematics faculties from 72 North Carolina school districts were mailed the Stages of Concern Questionnaire (SoCQ) and a teacher demographic data questionnaire during March, 1995. The SoCQ is a 35 item Likert-scale instrument that contains seven levels of responses. The teacher demographic data questionnaire consisted of 19 questions designed to obtain information in three areas: (1) standard demographic data such as age, gender, degree, years of experience, (2) teachers' school experiences such as current teaching load, and (3) teachers' technological experiences such as computer workshop experience, graphing calculator workshop experience, and in-class technology teaching experience. A total of six hundred and sixty surveys were mailed. Two hundred and sixty-six surveys were returned, with one hundred and fifty-one surveys from current teachers of first-year algebra.

Initially, SoCQ mean stage scores and total concerns score were calculated for each respective subgroup. To determine overall concerns, two analyses were performed. First, mean stage scores were converted to percentile ranks based on the norms presented by Hall, George & Rutherford (1986). Second, a peak stage score analysis was calculated. Peak stage scores are defined as the stage at which an individual has his or her highest percentile rank score on the seven concern stages (Hall, George & Rutherford, 1986). Analysis of variance (ANOVA) was utilized to determine differences for each mean stage score and overall total concern scores. Step-wise regression analysis was utilized to determine possible predictors for each of the seven stage scores and total concerns score.

Results

Results of the qualitative study (Slough, 1998) of secondary science teachers' concerns are summarized in Table 1. The data were analyzed vertically and horizontally; more succinctly, each interview was read in its entirety and then across each individual question. The data were grouped into CBAM stages by specifically looking for particularly enlightening themes, recurring ideas or language, and tacit descriptions of the social culture of the emerging telecommunications-rich high school science classroom. These themes evolved into the unique descriptors found in the summative descriptors.

Summative Five Stages of Concerns Descriptors	
Stage	**Descriptor**
Awareness (n=4)	Little knowledge of telecommunications beyond its existence Unaware of how to use telecommunications Unaware of barriers and supporting conditions for implementation Unaware of the potential for telecommunications to change science teaching and

	learning
Personal (n=6)	Some knowledge of telecommunications, mostly of others' use Some knowledge of the mechanics of telecommunications use Primary concerns dealt with how telecommunications would affect them--their time, their energy, their curriculum Vaguely aware that telecommunications could change science teaching and learning, just not sure of the mechanism
Management (n=6)	Some, if not frequent, personal use of telecommunications Little student use, often restricted Used designated local resources to learn telecommunications Concerns dealt with how telecommunications would affect their system of management over time, students, and curriculum Primary concerns dealt with students access to inappropriate material and support from administration Unaware of the potential of telecommunications to change science teaching and learning, sometimes in open opposition to it
Impact (n=4)	Frequent personal/professional use of telecommunications Required/allowed some student use in and out of class Primary telecommunications learning resource was local teachers and occasionally students Primary concerns involved how telecommunications would impact student learning Aware of potential for telecommunications to change science teaching and leaning and had openly embraced it
Collaboration (n=4)	True inventors with telecommunications Group use of telecommunications Required and independent student use of telecommunications Used students, local resources, and "off-site" professionals as resources Served as resource for local and "off-site" teachers Primary concerns were not with mechanics of telecommunications, but with how to sustain the momentum Aware of the potential of telecommunications to change science teaching and learning and spreading the message to all who will listen Enjoyed the challenge that change brings

Table 1: Summative Five Stages of Concerns Descriptors

Results of the quantitative study (Chamblee, 1996) of secondary mathematics teachers also generally supported the CBAM model. Teachers had their highest concerns at the lowest developmental levels (awareness and information) and their lowest concerns at the higher developmental levels (consequence and collaboration). Several reported teacher characteristics portrayed teachers of first-year algebra as very involved with integrating graphing calculator technology into first-year algebra. First, over 50% of the teachers rated themselves as better than novices at using the graphing calculator. Second, most teachers stated that they have been using graphing calculators in their classrooms for several years. Third, approximately 81% said that they had been exposed to some in-service training. But when teachers' SoCQ concerns profile was compared with the self-reported teacher characteristics, there was a contradiction between self-perceptions and actuality. Teachers considered themselves to be competent users of the graphing calculator. The overall concerns profile analysis generated a non-user profile. Only 52% had progressed beyond the awareness and information concerns stage (considered awareness stage in the Bailey and Palsha Model), which are the lowest developmental implementation stages.

ANOVA results demonstrated that female teachers of first-year algebra had lower overall concerns than male teachers of first-year algebra, which means female teachers were more likely to implement graphing calculators than male teachers. Personal concerns differed according to age. Specifically, teachers in their twenties and forties had lower concerns related to graphing calculator proficiency for the lowest stages. Gender, age, graphing calculator expertise rating and graphing calculator training had a significant relationship to teacher concerns. All significant relationships were at the lower stages, awareness, information, and personal (awareness and personal stages for the Bailey and Palsha Model). No significant characteristics for higher level concerns were found.

Stepwise multiple regression analysis found few predictions of the seven stage scores and total concerns score. At the awareness stage, graphing calculator training, graphing calculator expertise rating, gender and years of teaching experience were significant predictors. At the information stage, graphing calculator expertise rating, number of years using microcomputers in mathematics teaching and years of teaching experience, were significant predictors. For the personal stage, graphing calculator expertise rating was the only significant predictor. For the refocusing stage, graphing calculator access was the only significant predictor. Finally, for total concerns score, only graphing calculator expertise rating was a significant predictor.

Finally, the results of the study indicate that teachers of first-year algebra were beginning to focus more on the universal consequences of graphing calculators in the classroom. Many teachers reported that they are becoming proficient at using graphing calculators on their own and not through in-service. But, in actuality, all of the teachers in this study were still focused on the lowest levels of concern (awareness, information, and personal).

Conclusions

These studies show that: (a) math and science teachers can implement technology if they are given adequate resources, including copious access to the technology and a variety of formal and informal professional development; (b) even with access to the new technology and a variety of formal and informal professional development, most math and science teachers had not progressed to adoption; (c) the Concerns-Based Adoption Model (CBAM) addresses many of the perceived concerns of the teachers; (d) gender, age, and technology experience/expertise were not generally found to be adoption determinants; (e) the non-static nature of technology adds to teacher concerns; (f) teachers can and do decide *not* to implement technology in their classroom; and (g) a new change model that incorporates changing innovations and non-progress to adoption needs to be explored (Slough, 1999).

Science teachers were able to implement telecommunications at the two highest stages. They were able to accomplish this in a relatively short period of time when the combination of available technology, formal, and informal professional development were *all* in place. Of these three, the formal professional development was considered least important to teachers who had begun to successfully implement telecommunications. To a lesser degree, math teachers were able to implement graphing calculators at the highest levels (a smaller percentage of math teachers reached these stages). This is possibly due to the fact that less informal professional development opportunities were present. Regrettably, the majority of math and science teachers had not begun to implement technology after several years. According to the concerns-based approach, this is due to the fact that their individual concerns had not been met. But, both studies identified individuals who had made the decision *not* to adopt. This is significant because, CBAM does not factor in an individual's decision to not adopt.

Although age, gender, and technological proficiency were found to be statistically significant for mathematics teachers when using graphing calculators, the significance was only found at lower stages and were not found to be predictors at higher stages (Chamblee, 1996). In other words, males and females, young and old, and novice and expert alike were able to implement technology in both studies. In fact, Slough (1998) reported that female teachers comprised half of the teachers at the highest stages of adoption and that all of the teachers at the highest stages had taught at least six years. Technological proficiency was over reported by mathematics teachers based upon the CBAM model (Chamblee, 1996) and at least two of the four science teachers at the impact stage were relative newcomers to technology who had found telecommunications to be worth the effort to learn.

In general, the basic assumptions of CBAM were supported by both studies (Chamblee, 1996; Slough, 1998). Teachers at different stages of the implementation of technology did have different concerns; and further, teachers at similar stages of implementations did have similar concerns. Two observed shortcoming of CBAM as a theoretical framework for each study were CBAM assumes a static innovation and all individuals progress to adoption. Technology is not a static innovation. Teachers at all levels reported that one of the difficulties with implementing technology was that it was always changing, in effect constantly creating a new innovation. Also, CBAM assumes a general linear model where teachers go though each successive stage. There are no accommodations for individuals who reject an innovation. Both studies identified individuals who had, or appeared to have, rejected technology and were not progressing to adoption. The need for a change model that addresses the challenge of implementing changing innovations and non-progression to adoption must be explored.

Significant findings found in the study of science teachers (Slough, 1998) not found in the study of math teachers include: (a) administrative support, particularly from the building principal, is perceived by teachers to be a critical factor; (b) safety concerns, particularly related to wires and cables in and around sinks in science labs presents an additional safety concern in science classrooms that may not be present in other classrooms; and (c) loss of precious laboratory space, particularly related to loss of bench top space presents an additional concern in science classrooms that may not be present in other classrooms. Teachers perceived the support by the principal to be a primary concern. Where teachers perceived the principal to be supportive of telecommunications, teachers were more likely to adopt. Where perceived support was lacking, teachers were less likely to adopt. Lack of corroboration in the math teachers and graphing calculators (Chamblee, 1996) is primarily an artifact of methodology. The interview protocol (Slough, 1998) allowed teachers to express concerns about the principal that were not available on the quantitative SoCQ Instrument. But, when fourth and fifth grade mathematics teachers were interviewed about technology implementation concerns using a interview protocol modified from Slough (1998), they did perceive the principal as the primary change agent (Slough & Chamblee, in press)

Bringing telecommunications into the science classroom creates special problems dealing with loss of laboratory space and safety concerns. The science classrooms in this study, and in many other schools, typically had a lecture area and a lab area. Computers were typically placed in the laboratory area. Teachers in this study who were implementing telecommunications had done so at the expense of laboratory space and/or laboratory efficiency. Computers were either taking up permanent bench space or, where computer carts were used, they were constantly being moved for labs to occur. Neither situation is satisfactory in the long-term. Also related are the safety concerns associated with running electric wires in and around sinks and gas jets. If telecommunications are to become integral parts of the science classroom, long-term planning needs to address the potential need for lecture space, laboratory space, and computer space in a safe environment.

Significant findings found in the study of math teachers (Chamblee, 1996) not found in the study of science teachers include: (a) technology training needs to be matched to immediate teacher concerns; (b) some teacher characteristics were effective predictors of teacher concerns level, but only at the lowest stages; and (c) teachers self-rated themselves higher than the SoCQ rated them for graphing calculator expertise. While concerns models differ in the number and description of individual stages, researchers (Hall & Hord, 1987; Bailey & Palsha, 1992; Chamblee, 1996) conclude that technology training needs to be matched to the needs and concerns of individual teachers at appropriate times. Currently, many staff development models lack this feature. Integration of this feature will require more time pre-assessing teacher needs and personalizing instruction. It will also require having more content specific technology experts available in the schools and creating more specialized staff development opportunities with follow-ups throughout the year.

At the present time, much in-service is based on introducing teachers to new classroom innovations only at the awareness level. If studies continue to support the predictive validity of these teacher characteristics and more characteristics can be found that predict awareness concerns then selective screening can be used to place teachers with higher developmental level concerns (consequence and collaboration) in more appropriate in-service programs. For example, teachers who rated their expertise as intermediate vs. those who rated themselves novices differed on three concern stages: awareness, information and personal and overall concerns intensity. This implies that teachers who rated their expertise as intermediate have already began the process of gathering, synthesizing and personalizing the uses of graphing calculators in their everyday classroom instruction. Yet, the finding that these two groups of teachers did not differ at the other levels of concerns needs to be further explored. According to the model, to reach higher developmental levels of concern these teachers should begin to go through a refocusing phase in the near future. These data do not support this premise for these teachers. However, since neither non-users nor expert teachers were included in this analysis, further conclusions regarding any substantial differences in the groups is not possible at this time. The inability of teacher characteristics to predict higher developmental levels of concern (consequence and collaboration) was both discouraging and intriguing. One explanation of this lack of prediction is that as a teacher moves from a non-user (high awareness and information concerns) to an experienced user (high collaboration and consequence concerns) the more internal the innovational adaptation process becomes. Awareness and information about an innovation are externally controlled.

Overall, these studies demonstrate that commonalties and differences do exist between mathematics and science teachers going through the process of adopting technological change in their classrooms. These commonalties

provide opportunities for non-content specific professional development. Differences provide opportunities for content specific professional development. To become an experienced user of an innovation means a teacher has made a conscious and subconscious effort to focus on integrating the innovation into their everyday classroom activities and feels very comfortable with using the innovation as a tool for learning. If this is true then more emphasis has to be placed on being able to define the process teachers go through as they progress along the developmental continuum. Until we are able to do this, professional development opportunities which attempt to focus only on helping teachers meet low level concerns (awareness, personal, and management) are less likely to be successful.

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Tools for the 21st Century Classroom: How Digital Video and the Internet can Engage Learners in Math and Science

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Abstract: In a recent report commissioned by the Milken Exchange through Education Week, Zehr (1999) suggests that although schools have invested billions of dollars on computer hardware, the challenge now facing educators is in selecting appropriate software for use in their curriculum. The concern is that despite an emphasis on technology integration in the classroom, the use of computers fail to promote creativity, problem-solving and life-long learning unless the applications encourage these skills. The purpose of this paper is twofold: 1) to introduce educators to the Motion Media Grapher (MMG), a Web-based software utility designed by the authors of this paper in response to this growing need for effective classroom applications; and, 2) to share with educators the lessons learned when 9-12th grade mathematics students interacted with this utility.

Introduction

In a recent report commissioned by the Milken Exchange through Education Week, Zehr (1999) suggests that although schools have invested billions of dollars on computer hardware, the challenge now facing educators is in selecting appropriate software for use in their curriculum. The concern is that despite an emphasis on technology integration in the classroom, the use of computers fail to promote creativity, problem-solving and life-long learning unless the applications encourage these skills. The purpose of this paper is twofold: 1) to introduce educators to the Motion Media Grapher (MMG), a Web-based software utility designed by the authors of this paper in response to this growing need for effective classroom applications; and, 2) to share with educators the lessons learned when 9-12th grade mathematics students interacted with this utility. Additionally, this paper will show how the MMG can help teachers meet standards currently employed by the National Council for Teachers of Mathematics (NCTM, 1989) and the National Science Education Standards (NSES, 1996).

Based on a year-long research study (Pianfetti, 1998), this paper begins with a detailed description of the MMG and its convergence with mathematics and science standards. The next section presents a discussion of the research conducted on the efficacy of the MMG as a classroom resource. The last section examines three lessons learned emerging from the data. This paper concludes with insight from one mathematics teacher and her perspective on how this software utility could easily be adopted into her existing curriculum.

The Motion Media Grapher (MMG)

The impetus for developing the Motion Media Grapher was a result of observations made by one of the authors in a high school mathematics classroom of students using technology as an integral part of the curriculum. The students were effortlessly using such technologies as TI-82 graphing calculators to solve

algebraic expressions. In most instances, the technology was giving the students correct answers, but the students were failing to ascertain whether or not the answer was reasonable given the context of the problem they were trying to solve. The idea behind the MMG is that three networked-based interconnected components would give students multiple and situated representations of a single concept while they engage in authentic learning activities. Hence, the MMG would encourage the students to think critically about the answers that they were receiving because the answers would be presented to them through three different representations. These three interconnected components are a digital video component, a graph component and a table component. Students have the option of plotting points either by clicking on the graph or the digital video or setting points in the table. The other two perspectives are automatically updated to match the point initially plotted by the student. The digital video component allows the concept to be situated in an actual event and not just an abstract representation. Furthermore, this component contextualized the concept taught by the teacher. For example, in learning about nonlinear functions, such as acceleration, the digital video could be illustrating a car stopping and proceeding through a stop sign. The students could then plot a points directly on the video clip. This way, students see the concept in terms of real world events. The graph component permits the visual display of data and through its connection with the digital video component may help students better interpret what the points on the graph represent in more concrete terms. The numeric representation in the table component with its connection to the other two components may help the students understand what numbers mean and how they are portrayed in actual events. The MMG's emphasis on multiple perspectives reaches students with varying learning abilities and learning strengths while adhering to mathematics and science standards.

The NCTM Standards include among other things: 1) the use of problems representing applied settings to motivate and apply theory; 2) the use of computer utilities to develop conceptual understanding; and, 3) the use of computer-based methods for learning. Decreased attention will be given to word problems, simplification of radical expressions and pencil and paper graphing equations (NCTM, 1989, p. 125-9). Furthermore, the NSE Standards have stated that beyond basic skills and understanding, students in the middle schools should have heightened sense of inquiry that would help them understand the relationship between a concept and its explanation. By so doing, students become better problem solvers and are better able to communicate their reasoning. Students easily interact with the MMG. They may capture their own video, develop problem sets to accompany the video and by making two changes in the "html" source code of the MMG, they become creative and critical thinkers while engaged in authentic learning activities.

Moreover, there is growing concern that in introductory math and science classrooms, teachers have a tendency to oversimplify abstract and complex concepts (Kaput, 1994). This oversimplification may prevent mastery and create difficulty for the transfer of knowledge in advanced level courses (Feltovich, Spiro, & Coulson,1989; Chi, Glaser, & Farr (Eds.), 1988). The challenge becomes finding a way for students to learn knowledge flexibly so that they may situate it into a variety of unique contexts and not simply those in which they were learned (DeGroot, 1978; Lave,1988; Lave & Wenger, 1991). The MMG attempts to bridge all these issues by providing multiple and situated representations of an abstract concept and by permitting hands-on engagement with this network-based learning tool.

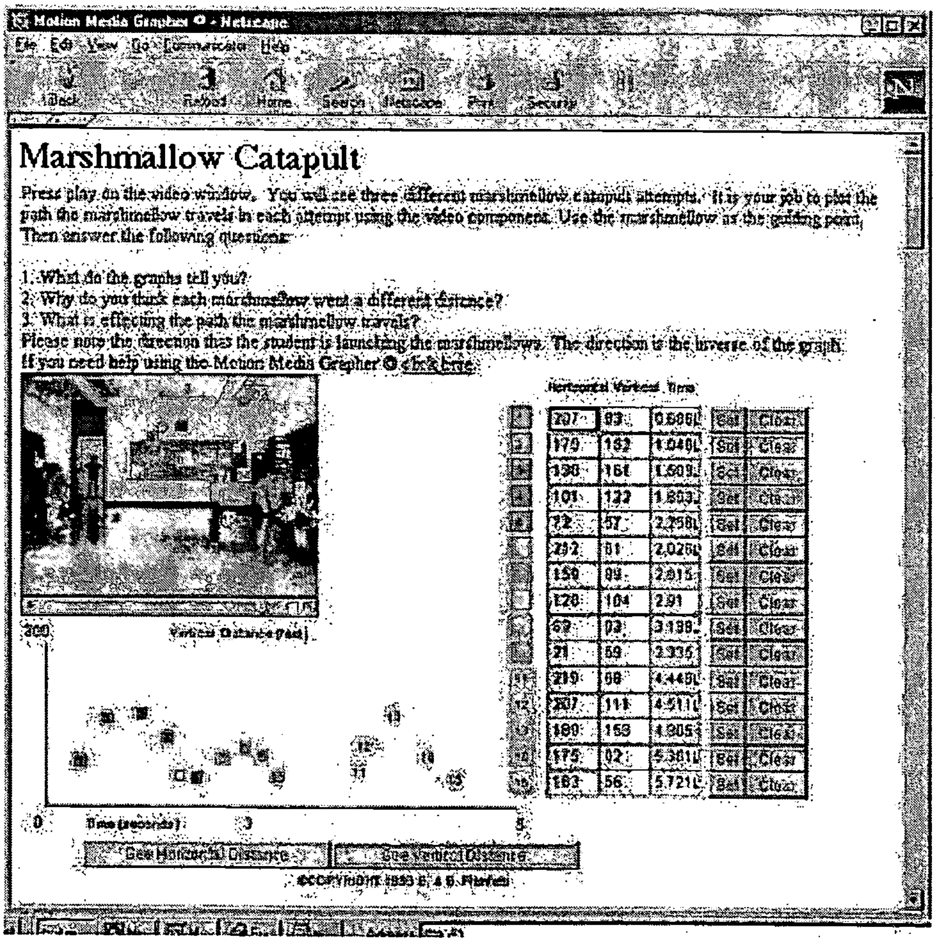

Figure 1: Screen capture of the Motion Media Grapher (MMG)[1]

The Study

This year-long study consisted of three phases. Phase 1: The Developers' Phase involved four high school upperclassmen who were primarily responsible for collecting and digitizing video, creating problem sets and building Web pages. These upperclassmen eventually developed the MathNet Web site that consists of several pages that incorporate the MMG and ask students to solve mathematical problem sets regarding linear and non-linear functions. Phase 2: The Evaluators' Phase involved four teams of high school sophomores who conducted a formative evaluation of the MathNet Web site three times throughout its development. The sophomores' evaluations were subsequently reported back to the upperclassmen and systematic changes were made to the MathNet Web site in response to the evaluators' comments. Phase 3: The End-users' phase involved twelve high school freshmen in an introductory-level algebra class. The freshmen used the MathNet Web site as part of their mathematics class while they were learning about functions. A situated evaluation was employed as the main methodology (Bruce & Rubin, 1994). Additional measures used in the data collection included: 1) audio and videotaped interviews, 2) field notes, 3) an analysis of artifacts such as the Web pages created by the students, 4) an analysis of the

[1] http://www.cyber-joe.com/education/Motion_Media_Grapher.html.

answers given by the freshmen to determine if they were using the multiple representations to answer the problem sets, and 5) pre and post surveys.

There were three core research issues that guided this research study. The questions addressed were built upon these core issues: 1) Technical: How can technology, specifically digital video and the Internet, be used to help students understand and interpret functions in applied settings?; 2) Cognitive: What impact does technology have on the way students learn to understand and to interpret functions in applied settings?; and, 3) Instructional: How can technology, specifically the MathNet Web site that incorporates the MMG, be integrated within a high school mathematics classroom?

Lesson learned from the data collected include: 1) The selection of visuals is important because misleading visuals may obscure the learning. The technology should not transcend the learning; 2) Collaboration is a key factor is the transfer of knowledge. Students can exchange their ideas and discuss concepts in terms that they know and they understand; 3) Digital video is a good tool that permits an abstract concept to appear more concrete in the learner's mind. The use of digital video because it can show a variety of contexts helps students better articulate their understanding. What was learned from this study will be briefly examined in the remainder of this paper[2].

Lessons Learned #1: Learning transcends the technology

The high school senior described in this mini-case study showed significant growth in her understanding of functions as well as in her understanding of how visuals can empower students by authenticating what they are learning. Conversely, she came to understand that the inappropriate selection of visuals could in fact misrepresent the content. She learned that technology should not transcend learning. In her own words,

> I think like we almost have to pump them up with math. You know take it a little away from the focus of 'wow, look at the graph, look at the movie' and say, 'okay so why does this work' ... we [shouldn't] take away from the purpose of the page - learning math.

Katina was working on a problem set that centered on the descent of a fire escape. She wanted the students to determine if the fire escape descended in a linear or non-linear path. In shooting the video, the upperclassmen were limited in where they could position the camera. Consequently, when the video was digitized, Katina realized that in considering the Cartesian plane, the fire escape moved solely within the second quadrant. Hence the corresponding plots on the graph would have negative numbers for the 'x' and the 'y' axes, but a positive slope. Katina believed that this perspective would confuse the freshmen who had only completed one semester of algebra; therefore, she attempted to horizontally flip the video.

> The freshmen students might think that the slope was negative because of the negative 'x' and 'y' values and because visually the students would see the fires escape moving downwards. [The freshmen] may associate the downward movement with negative numbers and forget that a negative divided by a negative number is positive. They might think it had a negative slope.

Katina was considering the larger impact the video might have on the students. Katina did not want to confound the understanding of math by misrepresenting the mathematical concept because of a limitation of the video. Katina knew that the video she collected was misleading. She did not want the visuals to "obscure the math" and subsequently obscure the learning.

Lesson Learned #2: Collaboration is key

[2] This paper is based three in-depth case studies. For more detail on these case studies, contact esecaras@uiuc.edu.

In reviewing the three evaluation phases and tracing the trends of comments that were made, it appears that there was significant growth made over the duration of the evaluation phase. For the first evaluation, these sophomores primarily submitted individual evaluation forms with aesthetic recommendations. By the second and third evaluations, however, the sophomores were more critical in their comments as they considered the overall operation of the Web site and worked together to give comprehensive feedback. As one evaluator stated:

> We were never asked to evaluate something before ... and now when we do [the upperclassmen] listen to us. It's a great feeling. It's forcing me to really think about the math and think about these pages. We're working more as a team because a lot of the uhm ideas we have we can talk about and see if we are on the right track. It's good to work with [our teammates] because I'm learning a lot from them about the math and about Web pages.

Comments such as the one above were echoed in many of the evaluators' comments. Other tangible evidence included the extensive recommendations that they made on their final evaluation forms. For example, one team of evaluators gave illustrated suggestions for how better to layout the three components so that the end users would be able to utilize more efficiently the multiple representations. They wanted the digital video clip to appear in the upper left-hand corner while the graph would cover a larger portion on the bottom of the screen. They included with this illustration a justification for why they wanted the layout changed.

> If you put the video at the top, then you can look at the video and work your way over to the table and the graph. The video is what draws your attention first, but the graph and the table are important, too. And for the graph you need more space to really spread out the [plots].

Lessons learned #3: Digital video as a situating medium

From the data collected, it was apparent that the freshmen end-users had a more concrete understanding of function. They were able to articulate their ideas about functions more directly than before their interaction with the MMG.

> A linear function is where you put something in and get the same thing out. Like when you talk on the phone and someone talks back. ... and like gas, the more miles you go, the more gas you need.
>
> Or like when you are reading and fall asleep. You're taking your time, then you slow down and then you fall asleep ..
>
> ... and the line on the graph continues going, but the number of pages your read stay the same.

Although the pre and post survey results indicated that the freshmen still had difficulty understanding functions, dialog exchanges such as the one above suggest that the students are able to transfer the knowledge to familiar events when they are asked to make the association.

Conclusion:

One of the major findings of this study suggests that proper integration of the MMG is instrumental for its functionality as an effective learning tool. Its placement in the curriculum should reflect what the teacher is currently teaching. The teacher in this study considered how it could be used within a mathematics class.

> Oh, now this is interesting. I could definitely see myself using this problem in my class. ... do you remember that worksheet I gave [the class] - the one the students had to draw graphs on different functions based on what they thought the graph would look like?

> Well, they had trouble drawing the graphs ... so now see here with this problem, I could have the students draw the graph and then have a student make a graph using the Web page. Yes, I think that the video would really add to the instruction ... you know, it might not even be a bad idea to have video that represents all the graphs [the students] are asked to draw.

In this teacher's opinion, the strength of the MMG was that the Internet could now be used for more than just browsing or research. Her students could be 'constructors of knowledge'. In terms of engaged learning, they were learning by doing. In addition, this software utility could, with minor adaptations, be used in a variety of classrooms and in a variety of disciplines, including science. Since conducting this research, the MMG has been modified for use in middle school science classrooms.

In essence, the MMG is geared towards the shifts in learning and instruction that are stated in the NCTM and NSE standards. The use of the Internet to support multiple media as a means to present interconnected multiple representations of a single concept is a key feature. The network-based interconnected components encourage the students to see how the different representations of a single event each reflect a perspective of the same underlying concept. Ideally, these different perspectives would foster problem solving and inquiry because the students would have to learn to negotiate the different representations and the meanings of the variables. Moreover, the digital video component offers a concrete representation of a natural event and as such can encourage the use of contextual problems to motivate and apply theory.

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Actions on Objects:
A theoretical framework for mathematics education

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Abstract: When one thinks of reasoning, problem solving, communication and connecting related ideas the tool of choice in nearly every discipline is the microcomputer. Furthermore, unlike the traditional calculator, the modern classroom computer has an unparalleled ability to implement both graphical and procedural components of mathematics understanding in a single unified object.

Through creation and utilization of mathematically relevant computer-based objects this dual encapsulation enables the students a unique opportunity to see both the form of representation and their actions utilizing this representation simultaneously. For this reason alone it would be a natural tool for both classroom use and theoretical musings. This paper explores the potential for classroom uses that blend learning theory and practical realities of student actions. It addresses the extent to which the object orientation metaphor, found in the modern windowed operating systems and programs, transfers to the "tools to think with" notions of current action upon objects models of mathematics teaching and learning.

In particular, we will suggest that the object-oriented environments which modern technology has created are ideally suited to parallel and facilitate the ability of students to take a broader variety of action upon objects of a nature and kind hitherto unknown.

Technologically Enhanced Mathematics Learning.

Over the past ten years I have noticed that a major theme that has been emerging in my research and writings has been that of *action upon objects*. This has in turn lead to some foundational questions surrounding the nature of the objects and the types of actions that one might be expected to perform upon them. I have found this framework of actions upon objects to be very powerful in both the laboratory as well as in the predictive power they enable in the minds of the students. I feel that they also capture quite a bit of current interest in the field as evidenced by recent thinking on object reification (Sfard, 1994).

What I am trying to add to the mix is the notion of a firmly developed and articulated way of looking at what these objects might be and in particular how we might utilize them to develop mathematical thinking. In particular, I have developed an approach that results in students developing mathematical thinking regardless of the developmental level and nature of the object. When this method is followed we repeatedly observe markedly similar patterns of thought on the part of the students. This is all the more significant when we consider that this parallelism shows itself in the same type of thinking taking place at each developmental level.

I am taking this opportunity to share some of my notions concerning the action upon objects models I have been using in my university classes and in my own personal thinking. As we will see, my approach toward addressing these questions has been very heavily influenced by readings of and work with Mikhail Bounieav and Sergei Abramovich (Abramovich, 1998; Abramovich, 1995). Together with Mikhail Bounieav, we have been developing a way of thinking about step-by-step development of mental activities as enhanced by technology (Bounieav & Connell, 1999; Connell & Bounieav, 1997). With Sergei Abramovich we have been looking at the nature of the new tools to think with which technology provides (Connell & Abramovich, 1999). The development of this theme includes developing concepts through mental picturing, and the notions I've been developing over the last 20 years or so regarding actions upon objects of various types.

Background examples.

At the elementary level, the objects the children are capable of thinking with (or acting upon) is influenced by both their developmental level and their prior experience. In particular, we find that young children are not able to think with formalized abstracted mathematical objects. This should not be a major surprise, as it has been part of our understandings of human growth and development for some time. This limitation, however, at first glance would appear to limit the degree of mathematics which might be made.

As Whitehead (1978) correctly noted, it is only when one reaches the abstracted levels of mathematical formalism that we can really leverage forward our thinking. It is at that point that the tremendous growth in the intellectual potential of the individual might happen. This has led many to speculate and even to promote the notion that young child is incapable of rigorous mathematical thinking, and in fact that much of what we do that young childhood level is basically preparatory for the real mathematics which they'll be developing later.

Based upon my experiences in the classroom and my own theoretical musings I have taken a very contrary position to this. As I have stated in many different venues the young child is capable of very well developed mathematical thinking if the objects with which they think and questions upon which they think are of an appropriate level and type for their developmental abilities. I will be the first to acknowledge that this is a different type of mathematics content than we often see in more formal mathematics, but the thinking strategies is in direct parallel to that exhibited in the higher levels.

Let us work through a few examples to see how this might play out. For a preschooler working with pattern blocks we can ask questions concerning these blocks that require acting upon these blocks and the use of some quite elaborated thinking. This thinking is to a large degree rooted in the physical actions the child is taking upon the physical object presented to them for their use.

Suppose, for example, the child creates a base pattern composed of a square followed by a triangle and then a parallelogram.

Figure 1: Pattern block base.

The child can easily and correctly predict what would come next from extensions to this pattern based upon this starting sequence as it is continued and repeated. Indeed, for any given starting sequence the children will quickly learn how to extend that pattern and to create their own patterns from bases of their ownchoice.

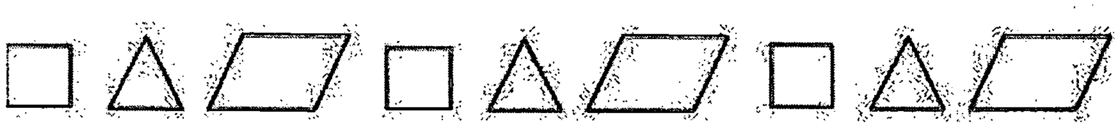

Figure 2: Extended pattern derived from the earlier base.

Now some might suggest that this type of thinking is more replication them prediction. However, we commonly see the same type of thinking occur when we observe algebra students using a guess and check strategy to fill in values in a function like the following:

x	F(x)
1	3
2	6
	9
4	
5	15
6	
7	21
8	
9	27
	30
11	33
⋮	⋮

Table 1: Function illustrating a guess and check solution strategy.

I really believe that we are not just laying a foundation for later mathematical thinking but that we are actually seeing mathematical thinking which is appropriate for the objects which the student is able to think with.

A model for use.

It is clear, however, that if this approach is to be effective that the objects to think with must be developmentally appropriate for the student. For the past 15 years I have been researching and writing about one such model which has proven to be very helpful in identifying the level of objects to think with and some of their properties (Connell, 1998; Connell & Ravlin, 1988; Connell, 1986). Figure 3 serves to illustrate these developmental levels.

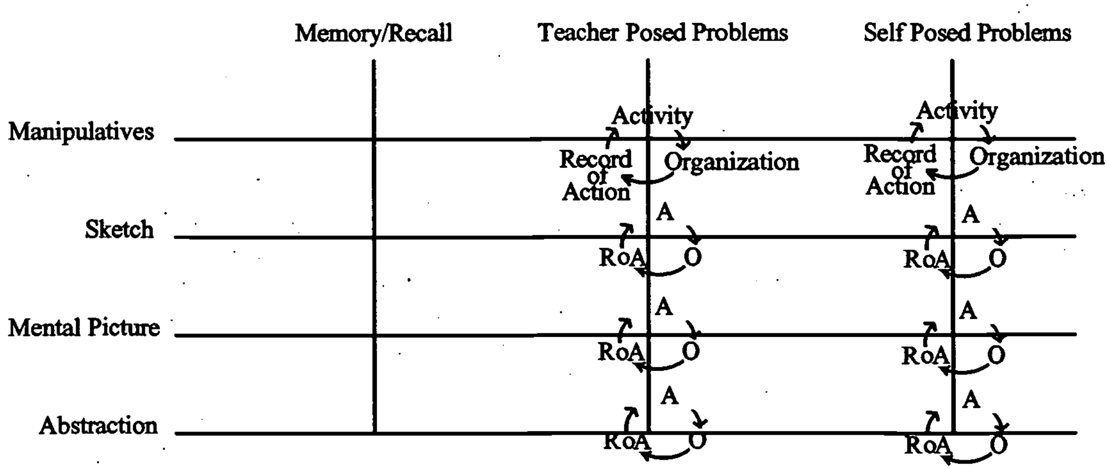

Figure3. Model illustrating the type of *actions* as performed upon the *objects of thought*.

Let me provide a quick general overview of this approach. The instructional goal is to enable gradual student construction of meaning through the use of manipulatives through abstraction via four transitional object types. Within each problem type three types of activities will typically be encountered by the student: 1) memory/recall - often of terminology; 2) teacher posed problems - related toward student

construction of concepts; and 3) student posed problems - based upon developing understanding of the problem space presented and it's relations to other problem types. These activities will be experienced by the student in the form of related problems requiring the use of the developmentally appropriate object to think with: manipulatives, sketches, mental pictures, and abstractions.

Furthermore, at each location in the model where a student will encounter a problem – either teacher or self-posed – the student will solve this problem via activities which are then organized and recorded for later reference.

An extremely elementary example of this model showing actions upon all of the object types might include initial actions upon a manipulative - such as a pile of counters used to develop elementary addition. A sketch might then be drawn recording the actual counters, which in turn would serve as an object of thought for further construction of meaning. Mental pictures, in addition to serving as a further representation of the problem space, provide natural entry points for technology – which will be utilized in technology-aligned classrooms. Abstraction would occur when it is no longer necessary for the student to use countable counters but is capable of reflecting upon the constructed representations in the construction of new knowledge, a process that Piaget referred to as "reflective abstraction". As we thus expand our earlier notions of action upon object we can see that we are working with a carefully selected set of developmentally appropriate primitive objects[1] and experiences with these objects to build up a working vocabulary and subsequent conceptualization.

What should occur next, regardless of the students' developmental level, would be for a skilled teacher or instructor to pose follow-up problems or questions relating to the newly instantiated and defined object of thought. This would again hold true whether we're talking about a physical ***manipulative** object*, a ***sketch** object* of predictive power such as an interactive fractions object, or a ***mental picture** object* such as a comparison of mass based upon remembrances of experience, or a formal and logically ***abstracted** object* such as function or some other mathematical construct. In each case we observe a skilled teacher using newly developed objects as a venue within which questions are to be asked and problem situations explored via student actions upon these very same objects of thought.

Good Thinking is Good Thinking, At Every Level at Which it Occurs

It should also be noted that the nature and form of the thinking and reasoning strategies runs parallel across each of these developmental levels. We can easily observe the young child reasoning with the manipulative objects, communicating their findings with others using the manipulative objects, connecting their most recent experiences with previous experiences with the manipulative objects and using these same manipulative objects in making quality judgments regarding their work. We can likewise easily observe the same strategies being used at each of the other developmental levels of objects within classrooms utilizing this approach.

If the question that has been posed relies upon action upon objects that the child is capable of manipulating and has developed personally meaningful understandings for, then the child is typically successful in their problem solving efforts. This holds true whether the form of the action upon the object is via a physical manipulation, a symbolic manipulation, or a more abstracted application of logical formalisms. It is important to note that I am not suggesting that we are necessarily observing extreme mathematical sophistication. However, what I am arguing for is that it is possible to observe a parallel form of mathematical thinking as students perform their respective actions upon their understood objects at each of these developmental levels. Thus I am quite comfortable making the claim that we can observe quality mathematical thinking at the preschool level as well as at the graduate level.

Furthermore, as we extend this model the children are given the opportunity to develop problems of their own based upon the objects that they recently defined, worked with, and developed problem solving skills and schemata for. This is an important part of the instructional strategy, for without this piece of the puzzle the children will always look to someone else to serve as the source of their problems and as final judge as the answers to the problems they face. This ability, to pose one's own problems and to

[1] I am using the term *primitive* to refer to a foundational material from which later concepts will be built – not necessarily as referring to simple or unsophisticated. This is in keeping with the use of *primitive* in computer systems. Thus, Dienes $Base_{10}$ blocks might well be considered as an experiential primitive upon which later conceptualizations would be built.

then successfully solve these problems, provides further opportunity for growth in mathematical thinking and problem solving. In this, we also see once again the direct parallelism between each of these developmental stages.

Conclusion

Since their inception in 1989 the NCTM Standards for Curriculum and Evaluation have been changing the face of what constitutes mathematics and how we think about its teaching and learning. Consider the following: Mathematics is communication; Mathematics is reasoning; Mathematics is problem solving; and Mathematics is connections. These statements are drawn directly from the process strands of the 1989 standards document and constituted a major revision as to what counts in mathematical thinking.

These process strands have become the de facto class of acceptable actions that are to be performed in mathematics education. These actions are much different in both form and substance than earlier procedural and content driven actions of the past. And it is in this very difference that we see that it is possible to perform these actions - to reason, problem solve, connect, and communicate mathematically - at every developmental age provided that an appropriate object for this action to act upon is present.

With the addition of the technology enhanced object there is tremendous evening of the playing field in terms of what counts as mathematical thinking. With this addition we can observe that the both the thinking processes and sophistication as shown by students of various ages begins to parallel each other. The technology serves an important role in this process. In particular, the computer can serve as a tool to record the information that has been generated by the students' activities. It can then capture the essence of the activity by allowing the students to organize their work in powerful structures. For example, a table might be built using developed data in a row and column structure, or an organizing graphic might be constructed reflecting the series of activities the children have performed. The technology can then be used to create formal records of action that may be shared or used in later problem solving endeavors. These records of action can even be shared with others outside the immediate sociocultural milieu within which the student is working via the World Wide Web.

The computer, with its object oriented interfaces and tools geared specifically to enable the user to perform specific actions upon specific objects, lends itself perfectly to an action upon object model of learning and instruction. Some of these include Step-by-Step Development of Mental Activities (Galperin & Talizyna, 1979; Leontiev, 1972), Action Reification (Sfard, 1994), and Emergent Structural Theory (Connell, 1996) to name a few. This linkage between suggestions from learning theory and the object classes engendered by the technology is far too priceless to allow going to waste.

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Some Psychological Aspects of Using Information Technologies in Teaching Linear Algebra

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Abstract: Despite the indisputable advantages of using information technologies in teaching mathematics in general and linear algebra in particular certain problems related to technology-oriented education have become quite evident now. The presentation reviews and analyses efficiency and expediency of using information technologies at different stages of teaching linear algebra. Our research is based on the results of the experiment conducted during the last five years in teaching linear algebra with MATLAB and modern calculators. This paper is a follow up of an earlier article "Linear Algebra With MATLAB Package In Preservice Teacher Education" (Bouniaev, 1997).

Introduction. Basics of DMA Theory

Our analysis of using information technologies in teaching linear algebra is based on the theory of stage-by-stage development of mental actions (DMA theory) developed by the Russian school of psychology (Galperin 1960, Leontyev 1975, Talizina 1975) as applied by the author in teaching mathematics (Bouniaev 1991, Bouniaev 1996, Bouniaev &Connell 1996). Without going into a detailed description of this theory we will outline some of its basic concepts that are essential for this paper. We also assume that in a linear algebra course an instructor can use various technologies at different stages of instruction, such as modern calculators and software packages like MATLAB or Derive. So we are not going to specify what software or a calculator should be used in the course of instruction . In most cases this problem of choice may be solved by different means and depends on their availability. According to the DMA theory the major goal of instruction is developing mental actions with objects of the studied field. Finding a solution of a system of linear equations, proving that systems of vectors is linear independent (dependent), reducing matrices to the reduced row echelon form, finding eigenvalues or eigenvectors are examples of the actions to be developed in a college linear algebra course.

Instruction is viewed as organizing and controlling students' activities and hence organizing and controlling the process of development. Thus, instruction efficiency is determined to a great extent by a well developed system of control and management.

Analysis of component operations of any action shows that they perform different functions. A part of performing an action is taken up by some preliminary work, a preparation for an action in a certain sense. This preparatory part is called *orientation part of an action*. The performance itself is called *executive part* of an action. Analysis of an action and experiments carried out in this respect show that there also exists a *control part* which takes place after the executive part, when an individual compares the results achieved

in the executive part of an action with the goals of an action and the draft plan of the execution planned in the orientation part.

As a rule, the performed actions consist of other, more primitive actions and in their turn can be part of other actions. Actions that are part of a given whole, are called operations. That is, operations are also actions; hence the term emphasizes only a hierarchical subordination among actions. The DMA theory specifies four independent characteristics of any action used to determine the level of development of an action. The first characteristic is a *form of action*. An action can be in a materialized (material), speech or mental form. The materialized form of action is connected with manual activities (manipulation, hand-on activities, etc.); objects of action (or their models) are presented in a material form; results of action should be real transformations of these objects or of their models. For example, entering an equation into a graphic calculator could be considered as an action in materialized form.

There is no need to discuss the speech form of an action. It should be noted that according to the DMA, writing belongs to the same form, i.e. the speech form. If we consider an example of solving a system of linear equation then the speech form will mean *articulation* of the performed actions. There is no need for these objects to be present in the material form. For example, in the process of instruction a student may be asked to comment on all the operations of entering an equation into the calculator. The action can be an answer in the form of oral speech or a note in a workbook.

The mental form of action is the highest form of action development. An action in this form is imperceptible for one's associates and its results are recorded in an imperceptible for others form also. This form of action means that its objects are representations, notions and concepts. All operations are performed to oneself. The ability to perform a whole action in the mental form indicates that it has gone through all the stages of development and interiorization.

Structure of the actions to be developed in a linear algebra course

The primary goal of the first linear algebra course is introduction of basic concepts of the subject such as systems of linear equations and their solutions, the concept of a matrix, linear space and linear operator. Activities-oriented learning theories claim that development of concepts takes place through development of actions aimed at the objects that fall under these concepts as well as at the objects that fall under the concepts immediately connected with the developed one.

All actions can be referred to two categories: general logic actions and specific actions. The characteristic general logic action for a linear algebra course is that of recognition. For example recognize if a matrix in the reduced row echelon form is an augmented matrix of the inconsistent system of linear equations. Another example of a general logic action is an action of classification, for of matrices into subclasses of singular and nonsingular matrices. We will demonstrate that developing general actions is crucial in the course of linear algebra study.

Specific actions are basically inherent to a given subject field. For example, in linear algebra they are: reducing a matrix to the reduced row echelon form, matrix multiplication, finding determinant of a given matrix, etc. We will show that in developing basic concepts of linear algebra most specific actions can be performed with the help of technologies.

Let us consider the structural composition of some actions aimed at developing the basic concepts of a linear algebra course. Naturally, if we consider the development of the systems of linear equations concept, then the basic action to be developed is that of finding a solution of systems of linear equations. This action is not homogeneous, it consists of a sequence of certain operations.

Example1 Action of solving a system of linear equations.

- Operation 1. Recognizing the system as a linear system (general logic action).
- Operation 2. Recognizing variables, coefficients, constant terms (general logic action).
- Operation 3. Rewriting the system in the standard form (specific action).
- Operation 4. Finding a method of solution (general logic action).
- Operation 5. Creating a matrix **A** – augmented matrix of the system (specific action).
- Operation 6. Reducing matrix **A** to the reduced row echelon form (specific action).
- Operation 7. Making a conclusion based on the form of rrefA (general logic action).
- Operation 8. Determining the method of solution that depends on the results of performing operation.
-Operation 9. Solving the system given in the reduced row echelon form.

In developing the concept of vector space an important role belongs to developing such actions as recognizing linear independent (dependent) system of vectors; determining whether this particular vector is a linear combination of vectors of the given system and recognizing the basis of a vector space. Let us consider the operational composition of these actions.

Example2 Action of recognition of linear independent (dependent) system of vectors.

Assume we have a system of vectors $\mathbf{a}_1=(a_{11}, \ldots, a_{1m}),\ldots, \mathbf{a}_n=(a_{n1}, \ldots, a_{nm})$ in m-dimensial eucledian vector space $\mathbf{R}^m$. The problem is to determine whether this system is linear independent or dependent.

- Operation 1. Designing a master plan for solution (general logic action).
- Operation 2. Creating the system of linear equations $x_1\mathbf{a}_1^T+\ldots+x_n\mathbf{a}_n^T = 0$ (specific action)
- Operation 3. Creating the matrix $\mathbf{A} = [\mathbf{a}_1^T \ldots \mathbf{a}_n^T\ \mathbf{0}]$ (specific action).
- Operation 4. Reducing the matrix **A** to the reduced row echelon form (specific action).
- Operation 5. Making a conclusion based on the form of rrefA (general logic action).

Example3 Action of representing a vector as a linear combination of the system of vectors.

Assume we have a system of vectors $\mathbf{a}_1=(a_{11}, \ldots, a_{1m}),\ldots, \mathbf{a}_n=(a_{n1}, \ldots, a_{nm})$ and the vector $\mathbf{b}=(b_1, \ldots, b_m)$ in m-dimensial eucledian vector space $\mathbf{R}^m$. The problem is to determine whether the vector **b** is a linear combination of vectors $\mathbf{a}_i$, i=1…n.

- Operation 1. Designing a master plan for solution (general logic action).
- Operation 2. Creating the system of linear equations $x_1\mathbf{a}_1^T+\ldots+x_n\mathbf{a}_n^T = \mathbf{b}$ (specific action)
- Operation 3. Creating the matrix $\mathbf{A} = [\mathbf{a}_1^T \ldots \mathbf{a}_n^T\ \mathbf{b}^T]$ (specific action).
- Operation 4. Reducing the matrix **A** to the reduced row echelon form (specific action).
- Operation 5. Making a conclusion based on the form of rrefA (general logic action).

Example 4 Action of basis recognition.

Assume we have a system of vectors $\mathbf{a}_1=(a_{11}, \ldots, a_{1m}),\ldots, \mathbf{a}_n=(a_{n1}, \ldots, a_{nm})$ in m-dimensional eucledian vector space $\mathbf{R}^m$. The problem is to determine whether this system is a basis for $\mathbf{R}^m$ or not.

- Operation 1. Designing a master plan for solution (general logic action).
- Operation 2-5 as in example 2.

Then we have to check whether any vector in $\mathbf{R}^m$ is a linear combination of vectors $\mathbf{a}_1,\ldots, \mathbf{a}_n$. It may be done in different ways. Usually we choose the method that assumes the use of technologies, but not symbolic computation systems. So the idea is to check whether the vectors of standard basis $\mathbf{e}_1,\ldots, \mathbf{e}_n$ are linear combinations of vectors $\mathbf{a}_1,\ldots, \mathbf{a}_n$. This problem can be substituted by the problem of determining whether n systems of linear equations are consistent or not. Fortunately all these systems have the same matrices of coefficients, so it makes sense to determine their consistency simultaneously.

- Operation 6. Creating the matrix $\mathbf{A} = [\mathbf{a}_1^T \ldots \mathbf{a}_n^T \mathbf{e}_1^T \ldots \mathbf{e}_n^T]$ (specific action).
- Operation 7. Reducing the matrix **A** to the reduced row echelon form (specific action).
- Operation 8. Making a conclusion based on the form of rref**A** (general logic action).

From the point of view of our analysis it is instructive to compare the operational composition of the action of the previous example with the action of finding a matrix that is inverse to the given one.

Example 5 Action of finding matrix inverse .

Assume we have matrix $\mathbf{A}= [\mathbf{a}_1^T \ldots \mathbf{a}_n^T]$. The problem is to find its inverse.

- Operation 1. Designing a master plan for solution (general logic action).
- Operation 2. Creating the matrix $\mathbf{A} = [\mathbf{a}_1^T \ldots \mathbf{a}_n^T \mathbf{e}_1^T \ldots \mathbf{e}_n^T]$ (specific action).
- Operation 3. Reducing the matrix **A** to the reduced row echelon form (specific action).
- Operation 8. Making a conclusion based on the form of rref**A** (general logic action).

Developing Actions In A Linear Algebra Course

Comparison of the operational composition of actions of the above examples shows that actually the only "transformational" operation in all of these actions is that of reduction of the matrix to the reduced row echelon form. Analyzing the actions to be developed in a linear algebra course one may come to the conclusion that the majority of actions to be mastered by students in this course can be presented as a sequence of three absolutely identical operations. The first operation is construction of a certain matrix, second – reducing this matrix to the reduced row echelon form, the third is interpretation of results, i.e. comparison of the original matrix with the one in the reduced row echelon form.

It is easy to see that the above problems require practically identical treatment not due to the limited number of concepts we operate with in linear algebra but because these operations form only the executive part of action. The executive part of action in many linear algebra problems comes down to construction of a matrix and its reduction to the reduced row echelon form.

In real life situation (not in class) that require performing any of the above described actions it would be natural to perform explicitly only the executive part of the action. An engineer or a mathematician would enter the matrix into the computer or the calculator and then push the button controlling "rref" command irrespective of the fact whether one is looking for the solution of the system of linear equations or proving that system of vectors is linearly independent. The situation is completely different in class since all students actions are not aimed necessarily at getting the right answer for the given problem but more at acquiring skills to solve problems of this category.

In the course of the experiment while studying the linear independence and the concept of basis in the experimental group we allowed the students to start the solution of problems right from the executive part of the action. Thus, for example, in developing the action of recognizing the linear independent system (example2) students were not required to start with the system of linear equations but with operations 2 –3 (entering the matrix in a computer or a calculator and finding its reduced row echelon form). In the control group the students were required to start with the system of linear equations and write down an explanation how this system is related to the problem in question. Thus the experimental group in the course of study was able to solve almost twice as many problems as the control group but 60% of the students experienced considerable difficulities substantiating their actions, and they also provided substantiation that could be related to a different class of problems. They could not extend the same idea to a similar class of problems.

While determining the fact that given vectors generate the entire space (example 4) the students of the experimental group were allowed to perform all operations (except 6 and 7) mentally. The students in the control group were required to write down all operations with detailed substitution. The day before we discussed these topics we found an excuse to remind the students how to find an inverse matrix, i.e. actually reviewed the operational composition of the action of example 5.

Proceeding from the assumption that the executive operations of example 4 (operations 6 and 7) and example 5 (operations 2 and 3) are absolutely identical the students from the experimental group came to the independent conclusion that actions of examples 4 and 5 are based on the same idea (which is a misconception). All the attempts to provoke students in the control group to come to the same conclusion failed. The control group students were fully aware of the fact that despite the superficial similarity of the execution parts of these actions their orientation parts are absolutely different and thus these actions cannot be similar.

According to the DMA theory, besides the form any action has another three independent characteristics:

- *degree of generalization;*
- *degree of completeness;*
- *degree of assimilation.*

Generalization of an action means the ability to determine and discriminate essential for performing an action properties as well as the ability to apply them to objects of different nature . For example, if the action of solving systems of equations is developed at a high enough level of generalization the student does not find it difficult to progress from solving the system of three equations with three unknowns to solving any system. It also does not matter how the unknowns are designated. The degree of completeness indicates whether all the operations that were to be performed in the process of performing an action have been actually completed. If the action is already developed, then the subject of the action (the student who performs it) practically does not discriminate operations from each other, i.e.

the action takes place in the compressed form. If we assume that the previous learning was successful, then failure to perform an action (without any time limits) often indicates that a student can not present the action in operation-by-operation form when all the operations are present and are clearly identifiable. This indicates that performance of the action is not completed. The ability of the student to perform an action in the operation-by-operation mode giving justification for their performance shows that the action was developed at the sufficient degree of completeness.

Going back to discussing different approaches to organizing the learning process in the experimental and control groups we can come to the conclusion that in the experimental group the development of actions did not reach the required level of completeness which in its turn affected negatively the degree of generalization of the developed action. On the other hand, the degree of assimilation was higher in the experimental group.

In the experimental group we had to spend additional time to develop the actions at the required level of generalization. For this purpose we created a class of exercises that we called inverse problems. The students were given only the executive part of the action of the problem to be solved. The task was to restore the full operational composition of the action.

This experiment as well as other experiments conducted in the course of teaching linear algebra with information technologies demonstrated that each part of the action should be singled out and developed separately. In the process of instruction the executive part of an action should be performed by a computer or a calculator.

The executive part usually is a specific action for the subject. The orientation part as a rule is a general logic action aimed at the objects of the studied field. The orientation part of an action includes intermediate goals, reducing a problem to the already familiar ones, selection of definitions and theorems related to the performed actions.Thus in the above examples the problem is reduced to determining the consistency of the sysem of linear equations. The expediency of this problem substitution is determined by the corresponding definitions and theorems.

In developing any new action in the course of linear algebra its orientation part should go through all the forms of development starting at least with written speech. For the above discussed examples 2-4 it means that at the initial stage of development a plan of action should be written down as well as all necessary theorems and definitions on which this plan is based. The system of linear equations should be presented in a written form. Only after completing all these actions a student can use a computer or a calculator to perform the corresponding operations. Thus major problems in teaching linear algebra arise not in developing specific actions of the course but in developing general logic actions aimed at the objects of the studied field.

The DMA theory presupposes *five stages in organization of instruction*. At the first stage the instructor presents new material. Taking into account a relatively abstract nature of the linear algebra course it is expedient in presenting a new material to illustrate it with practical problems and to create computer models of these problems. Thus in teaching the theme Least Squares Solutions it is worth starting with the discussion of linear regression and demonstration of vivid programs modeling linear regression. In teaching the projections it is hard to overestimate the value of visually enhanced programs illustrating the geometry of the performed actions. At the second stage the actions are developed in the materialized form. At this stage of instruction it is expedient to organize the work with blocks of texts and illustrations and move these blocks to different parts of the screen.

The third stage is development of actions in the external speech form. At this stage it is important for students to articulate their thoughts out loud and write down necessary comments. As experience showed at this stage traditional pencil and paper are hard to substitute. Group work is also very useful. Computers can be used for conducting different

experiments requiring discussions with other students. And only the fourth and the fifth stages are developing actions in the form of internal speech and mental form.

In developing every new action the executive part can be delegated to a computer. Thus for example in developing an action of reducing matrix to the reduced row echelon form it is expedient to use programs like MATLAB m-file "RREF", providing executions of elementary row operations at students instructions. At further stages of instruction it is expedient to delegate this action fully to a computer or calculator. In finding eigenvectors and eigenvalues computers can solve the systems of linear equations and characteristic equations. At the same time development of the orientation part of this action should go through all the stages and forms.

Conclusion

In the course of study of linear algebra it is expedient to carry out the structural analysis of every new action to be developed. It is preferable to present the orientation part with all the necessary operations to be performed and to develop it going through all the stages starting with the material form of an action. The executive part of an operation can be given over to computers or calculators. At further stages the orientation part can become an operation of the executive part of an action and be performed by a computer. It is worth pointing out that in developing the orientation part of an action the use of computers can be highly efficient in creating visual models and a bank of basic concepts, definitions and theorems.

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Factors Related to Teacher Use of Technology in Secondary Geometry Instruction

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Abstract: This survey sought to identify variables related to teacher use of technology in secondary level geometry classrooms in southeastern Idaho. The primary variables examined in the study were teacher technology awareness, teacher attitude toward technology, teacher technology training, and teacher computer use for instruction. This study also tested for associations between these primary variables and principal attitude toward technology and a selected group of demographic variables: geometry teaching experience, number of sections of geometry taught, college mathematics major, and computer lab access. Four significant relationships were found. An inverse relationship was found between teacher computer use and the number of geometry sections taught. Direct relationships were found between teacher attitude and both teacher technology awareness and principal attitude. Finally, a direct relationship between type of teacher training and teacher instructional computer use was reported.

Introduction

The use of technology in education, especially mathematics instruction, has been recommended on multiple occasions by the National Council of Teachers of Mathematics (NCTM, 1996, 1998). In the state of Idaho, technology use has even been mandated (Watson, 1996). At the secondary level, many articles have recommended the use of computer technologies in geometry classes; however, few studies have been published on the factors which relate to teacher use of technology within geometry classrooms.

The purpose of this study was to discover relationships of teacher technology awareness, teacher technology training, teacher and principal attitude toward technology, and teacher computer use among secondary, geometry teachers in the southeastern region of the state of Idaho. This study was broken into three groups of research questions. The first set of research questions sought to discover associations between selected demographics variables (experience, number of sections taught, mathematics major, and hardware availability) and the primary variables of teacher technology awareness, teacher technology training, teacher attitude toward technology, and teacher technology use. The second research question examined relationships between the primary variables and principal attitude toward technology use in geometry. Finally, the last three research questions sought to determine associations between the four primary variables of the study.

Methods

Description of the Population

Idaho teachers and principals were considered in this study for several reasons. Since 1994, Idaho has invested heavily in technology for schools (Idaho Council on Technology in Learning, 1998). The State of Idaho has provided, and is still providing, monies earmarked for technology directly to school districts, requiring a plan for integrating technology into instruction from school districts as a condition of receiving that money. In addition, a private foundation has provided large dollar amounts for Idaho schools which incorporate technology into instruction. Finally, the state of Idaho has mandated the appropriate use of technology in instructional settings (Watson, 1996). Therefore, Idaho teachers were considered because Idaho schools have had both the money and the incentive to incorporate technology into the classroom for the last five years. Geometry teachers were examined in this study because dynamic geometry environment software offers

opportunities for students to interact with the constructs of a field which has been described as a hurdle for high school mathematics students (National Research Council, 1989).

Data Collection Techniques

The data were collected via a mail survey instrument. The surveys consisted of both closed and open response items. The surveys were sent in packets to high schools and junior highs in southeast Idaho school districts during the latter part of the school year. Each packet contained a principal survey and teacher surveys for each geometry teacher in the school building.

After allowing several weeks for the return of the instruments, it was noted that some districts had not returned the superintendent permission form for using their districts data as required by the Human Subjects Committee. The superintendents of these districts were contacted again and asked to return a faxed copy of the permission form. The data from those districts were then added to the total data list.

Participants

The study participants were restricted to secondary school geometry teachers and their principals in southeastern Idaho. All junior and senior high schools which offered at least one section of geometry were invited to participate in the study. The principals of each of the selected schools made up the principal section of the sample. If more than one teacher from a given school was in charge of at least one geometry class, then all such teachers from that school were asked to participate in the study.

There were 75 secondary schools from 52 school districts in the three southeastern Idaho regions which fit the criteria for this study. The number of geometry teachers in each school was determined by calling all secondary schools in southeastern Idaho. A representative of each school was asked if geometry was taught in the building and, if so, how many teachers were assigned at least one section. From this initial phone survey, it was found that 75 secondary schools in 52 school districts located in southeastern Idaho offered at least one section of geometry. Each of the school districts was asked to participate in the study. One district refused to participate and two districts closed prior to being contacted. This meant that the potential sample consisted of 136 teachers in 72 schools from 49 districts across southeastern Idaho.

Completed surveys were received from 52 teachers and 33 principals from 26 school districts. This resulted in a return rate of 38% for teachers and 46% for principals. The demographic data obtained from the teachers and the principals are reported in the tables below.

Table 1: Teacher Demographic Data

Description	Responding Teachers	%
Gender		
Male	34	65
Female	18	35
Geometry Teaching Experience		
1 to 5 years	17	33
6 to 15 years	19	37
16 or more years	16	31
Number of Geometry Sections Taught		
1	22	42
2 to 3	26	50
4 or more	3	6
Mathematics Major		
Yes	33	63
No	19	37

Table 2: Principal Demographic Data

Description	Number of Principals	%
Gender		
Male	29	88
Female	4	12

Experience as a Principal		
1 to 5 years	14	42
6 to 15 years	12	36
16 or more years	7	21
Mathematics Teacher Background		
Yes	1	3
No	32	97

Summary of Findings

The research questions for this study were divided into three sections. The first section, of four research questions, examined the relationships between the primary variables listed above and the selected demographic variables. The next research question sought associations between the primary variables and principal attitude toward technology. The final section, comprised of the last three research questions, tested for relationships between each of the different pairs of primary variables. For each of the research questions, the critical alpha value was set equal to .05.

Relationships Between the Primary Variables and Demographic Variables

The first set of research questions sought to discover associations between selected demographic variables (geometry teaching experience, number of geometry sections taught, mathematics major, and hardware availability as measured by computer lab access) and the primary variables of teacher technology awareness, teacher technology training, teacher attitude toward technology, and teacher technology use. Chi-square procedures were used to test for associations. The findings for each of the first four research questions are summarized below.

The first research question examined relationships between teacher technology awareness and the selected demographics variables. For each of the demographic variables, no significant relationship was found.

Research question two tested for relationships between teacher technology training and the selected demographic variables. Once again, no significant relationships were found for the demographic variables.

The third research question looked for associations between teacher attitude toward technology and the selected demographics variables. The results indicated that no significant relationships were found in these tests.

Research question four sought to discover associations between teacher computer use and the selected demographic variables. No relationships were found between computer use and geometry teaching experience, mathematics major, and computer lab access. However, teacher computer use was found to be significantly related to the number of geometry sections taught. The chi-square value for the comparison between number of geometry sections taught and teacher technology use was χ^2=9.776, df=4, p=.044. The strength of the association, as measure by Cramer's V, was V=.31. Examination of the data revealed that the more sections of geometry which a teacher was assigned, the less likely that teacher was to make use of technology in teaching geometry.

Relationships Between the Primary Variables and Principal Attitude

The fifth research question examined relationships between the primary variables and principal attitude toward technology use in geometry. In three cases: teacher technology awareness, teacher technology training, and teacher computer use, no significant associations were found. However, principal attitude toward technology was found to be significantly related to teacher attitude toward technology. The chi-square test for the comparison between principal attitude towards technology use and teacher attitude towards technology use was statistically significant, χ^2=6.297, df=1, p=.012. The strength of the association was measured using Cramer's Φ as Φ=.351. It was found that those teachers with high attitudes toward technology use tended to work for principals with high attitudes toward technology use.

Relationships Among the Primary Variables

The last three research questions sought associations among the four primary variables of the study: teacher technology awareness, teacher technology attitude, teacher technology training, and teacher computer use. For these comparisons, correlation statistics were used.

The sixth research question tested for relationships between teacher attitude toward technology and the other three primary variables. No significant relationships were found between teacher attitude and teacher technology training or teacher computer use. However, a significant association was found between teacher attitude and teacher technology awareness. The Pearson product-moment correlation value was $r=.30$. Those teachers with higher awareness of the capabilities of computers tended to have higher attitudes toward technology.

The seventh research question examined relationships between teacher technology awareness and teacher technology training or teacher computer use. The results of this study indicated that teacher technology awareness is significantly associated with neither teacher technology training nor teacher computer use.

The eighth and final research question looked for a relationship between teacher technology training and teacher computer use. The results of this study indicated that there was a significant relationship between these two variables. The Kendall tau value was $\tau=.34$. Those teachers who were trained in the integration of subject specific software into their geometry classes were more likely to make use of technology when teaching geometry.

Conclusions

Relationships Between the Primary Variables and Demographic Variables

Three of the four demographic variables tested in this study showed no significant relationship to the primary variables. From these findings, it may be concluded that years of geometry teaching experience, college mathematics major, and access to a computer lab were not related to teacher technology awareness, technology attitude, technology training, or teacher computer use. This is in keeping with Dupagne & Krendel's (1992) finding that attitude towards computers was independent of personal characteristics.

The fourth demographic variable, number of sections of geometry, was not significantly related to technology awareness, attitude, or training. Therefore, the results of this study lend evidence to the conclusion that no such relationships exist in the general population of secondary geometry teachers.

The only significant finding involving a demographic variable developed in this study was the relationship between the number of sections of geometry which a teacher was assigned and the use of computer technology in the classroom. It is interesting to note, however, that none of the high users taught more than three sections of geometry per day. Further, the majority of those teachers in the medium use group only teach one section of geometry. This may be a result of the large number of small schools in the sample which can only offer one geometry section per year. However, if these results are indicative of a more general population, it may represent a trend of those teachers which have a larger number of geometry classes proving to be less willing to experiment on a new teaching technique. Additionally, it may be that unequal access to computer technology may lead teachers to adopt a least common denominator strategy: if it isn't available for all students, it won't be used by any students. Several teachers specifically mentioned the lack of time as a factor in their decision not to use technology. As one teacher put it, "We don't have time to teach the current curriculum; much less add time with technology." Many of the respondents stated that technology required more time to learn and implement than they had available or were willing to give. This is in agreement with findings from previous studies on both principals (MacNeil & Delafield, 1998) and teachers (Cooper, 1998).

Relationships Between the Primary Variables and Principal Attitude

This study found no significant relationships between principal attitude toward technology and the variables of teacher technology awareness, technology training, and teacher computer use. The lack of a significant relationship between principal attitude and teacher computer use is in contrast to Stegall's (1998) finding that enthusiastic principal leadership was related to high technology use. This discrepancy may be explained by Stegall's definition of enthusiastic leadership. That term encompassed actions as well as attitudes. This study examined only the principals' attitude. In order to effect classroom practice, it may be necessary for the principals to act upon their beliefs about the usefulness of technology. In other words,

principal attitude may be a necessary but not sufficient condition for changing teachers' technology practices.

The interpretation of the significant relationship between teacher and principal attitudes towards the use of technology is straightforward. As the principals' attitudes goes up, so do the teachers' attitudes. These findings are consistent with Drake & Roe's (1994) assertion that the principal should be able to foster change in teacher attitudes. It should be noted, however, the results of this study could also be explained as teachers effecting a change on their principals' attitudes.

An examination of the open response items indicated a potential problem. In both the teacher and the principal samples, approximately one third of the respondents indicated that the amount of use was their primary gauge of appropriate technology use in the classroom. Since Roberts & Stephens (1999) found that merely increasing the amount of time students spend at the computer does not increase achievement in geometry, those who advocate simply more technology access or higher usage levels in secondary geometry classrooms have no current research to support their position.

Relationships Among the Primary Variables

This study found that teacher attitude was not significantly related to technology training or teacher computer use. These findings are in contrast to Okinaka's (1992) results. One possible reason for the difference is that Okinaka surveyed pre-service teachers' interest in taking more computer courses and their intent to use computers after being hired. It may be that inservice teachers have enough demands on their time that their attitude toward technology does not always lead to training on technology and use of technology in the classroom.

The variable of teacher technology awareness was not significantly related to teacher technology training or teacher computer use. This is somewhat in opposition to the conclusions of Okinaka (1992) and Sheingold & Hadley (1990), that awareness is necessary for technology implementation. It may be that mere awareness of the capabilities of technology is insufficient to guarantee technology training or use.

In spite of their non-significant associations with technology training and teacher computer use, teacher attitude toward technology and teacher technology awareness were significantly related to each other. This finding does not contradict Okinaka's (1992) conclusion that teacher attitude toward technology can be positively affected by making teachers aware of the capabilities of technology.

Several papers have recommended additional training for teachers in order to increase their level of technology use (Cooper, 1998; NCTM, 1998; Mathews et al., 1996). Yet none of these studies have shown that technology training and teacher computer use are related. Therefore, the significant association between teacher technology training and classroom technology use found in this study is a step toward justifying the recommendations for teacher technology training.

Recommendations

Recommendations for Future Research

This study found a relationship between teacher and principal attitudes. A portion of the relationship was based on the attitude that appropriate computer use could be described as an amount of time or level of access. Since merely increasing the amount of time spent in geometry class on a computer has been shown to be unrelated to achievement (Roberts & Stephens, 1999), the shared attitude has no supporting evidence. Therefore, it is recommended that one topic for future research should be an investigation of how teacher and principal attitudes towards the use of technology can be changed.

The relationship between type of technology training and teacher computer use also has more room for exploration. This study did not determine any causal relationship between these two variables. If a specific type of training is found to cause a higher level of computer use, that type of training should become standard. Therefore, it is recommended that the relationship between type of technology training and teacher computer use be a topic of future study.

Since the use of technology is both recommended (NCTM, 1998) and mandated (Watson, 1996), the inverse relationship between computer use and the number of sections of geometry taught becomes important. If it is a goal to use technology, the teachers who teach the most students should be using technology. Discovering the reasons behind this lack of technology use by these teachers should also be a topic for future research.

Recommendations for Practice

The results of this study can provide school districts with several recommendations for practice. The inverse relationship between technology use and the number of sections of geometry taught provides one such recommendation. School administrators need to be aware of this relationship and take steps to discover if it holds true in their district. If those administrators should find the inverse relationship among the geometry teachers in their district, an attempt should be made to determine the reasons behind the lack of technology use. At that point, the administration could seek to alter the factors which lead to low technology use among teachers with the most sections of geometry.

The teachers in this study were asked to report on the type of training they have received in the use of geometry specific software. Of the 52 teachers who responded, 25 (48%) indicated that they have received no training in the use of geometry specific software. Another 9 (17%) reported that they had undergone training only in how to use geometry software. Since the use of technology has been both recommended and mandated, another recommendation for school districts is to even out the levels of training received by providing integration training to all geometry teachers.

The results of this study indicate that 26 of 51 (51%) of the respondents indicated that they had low or medium levels of awareness of the capabilities of technology in geometry classrooms. It is recommended that school districts make an effort to assure that their teachers are kept up to date with the latest products available in their field. Since it is not possible to use technology that the teacher is unaware of, this will remove a potential impediment to technology use.

Finally, this study offers recommendations for current practice in teacher education. Schools of education should provide opportunities for pre-service teachers to become aware of the capabilities of technology in the teaching of geometry. Pre-service teachers should also be trained in the integration of technology into their specific subject areas. In this way, colleges of education will assist school districts in accomplishing the previous two recommendations.

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Using WebCT to Deliver a Finite Mathematics Course to Preservice Teachers.

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Abstract: In the spring of 1998 five faculty members at the University of Regina in Saskatchewan, Canada received funding to implement an online version of an introductory mathematics course. This course, Math 101, was designed to meet the needs of students in the elementary education program in the Faculty of Education and to satisfy the "critical thinking" requirement in the Faculty of Arts. This paper chronicles the development of Math 101 Online from the initial conception through the first full offering of the course in the Fall of 1999. It includes an overview of the tools that are available in WebCT. Strengths and weaknesses of WebCT are addressed from the course developer, instructor and student points of view. The paper also addresses the difficulties in online communication with students when such communication involves diagrams and mathematical notation.

Introduction

Math 101 is an introduction to finite mathematics which is delivered through a standard lecture format to 500-600 students per year on campus, and through regional and community colleges at various locations in Saskatchewan. The content contains arithmetic and numeration systems; problem solving; number theory; rationals, irrationals, ratios and percent; sets; and logic. In the spring of 1998 funding to implement an online version of Math 101 became available from the provincial government which has a desire to expand the number of university level courses that are accessible to learners in remote communities. Our motivation was to use this opportunity to tailor the course more closely to the needs of Saskatchewan learners and to improve accessibility for students who, for whatever reason, find it difficult to attend lectures. The proposal was made by five faculty members, four in the Department of Mathematics and Statistics and one from Mathematics Education.

Once the approval for funding was received a variety of possible software platforms were considered and a decision was made to use WebCT, a collection of course tools created by academics at the University of British Columbia. A number of factors influenced this decision. Software features that were required for delivery of the course were used to narrow the range of possible platforms and then versatility of the tools available and cost were the final determining factors. An added bonus was that WebCT was being used at the University of Regina to deliver a non-credit course in genealogy and thus there were people on campus with experience using the software. In the intervening eighteen months WebCT has been used to deliver and supplement other courses at the University of Regina (Maeers, SITE 2000).

Content Creation and Organization

Two undergraduate students were hired in the spring of 1998 to begin collecting, organizing and inputting the content. Both students were familiar with the content of the course and one of them, a geography major,

had experience the previous summer creating web pages. The second student, an elementary education major, had only minimal experience with HTML.

During the summer of 1998 the students took course notes and problem sets from three of the faculty members involved and began to design the course around this material. Each of the six units in the course was assigned to a faculty member who wrote the first draft which was then formatted for web delivery by the students and uploaded to the WebCT server. Each unit went through several iterations as all the participants read, edited and reread the material.

The content was prepared for WebCT delivery using a text editor and most of the HTML coded from scratch. Adobe Photoshop was used to draw the diagrams and MathType for construction of the mathematical symbols. Creating the HTML from scratch is certainly not necessary as any page creation software that produces HTML could be used. The File Manager in WebCT was used to easily move the content pages from the local computer to the server. Collections of files were zipped together, moved to the server as one file and then unzipped. The course material was organized on the server into units and sections using the Path Editor and problem sets were collected into the quiz database.

Pilot, Revisions and Delivery

In the fall of 1998 a pilot section of Math 101 Online was offered to five volunteer students. These students met with the instructor in a laboratory setting, three hours per week for the semester and worked online. The students, for the most part, communicated with the instructor through the Bulletin Board and the Private Mail Tools in WebCT. The scheduling of the class at a particular time and place was to observe the students and identify any problems they might have with the technology as well as allow for the class to be continued in a lecture format should some major problem develop. The class was completed without incident and the students performed well on the common final examination. The students had some constructive criticisms of the course layout and the number of worked examples but there were no serious criticisms of the technology or format of the class.

In the summer of 1999 revisions were made to the course material based on the comments from the students in the pilot section. The page layout was changed to improve the readability. The material was expanded in areas where the students found to be thin, more worked examples and practice questions were added and the quiz database was expanded. Work also began on translation the course into French.

Math 101 was scheduled in an online format for the first time in the fall of 1999. Twelve student registered and subsequently three dropped. Of the nine remaining students four live in Regina where the university is located, four live in other locations in Saskatchewan and one lives out of the province. At the time of writing this paper the students are completing the sixth quiz in the course and the final examination is scheduled in three weeks. The quizzes are written online but the final examination, which is common for all students in all sections, will be written with pen and paper.

WebCT Tools

The tools in WebCT can be roughly categorized into four sets, communication tools, study tools, evaluation tools and management tools. The specific tools used in Math 101 Online are outlined below.

Communication Tools

Bulletin Board

This is the main tool for communication among the students and the instructor. Messages posted to the Bulletin Board are viewable by everyone in the class. Messages are threaded and can be organized into fora by the instructor. The Bulletin Board is hypertext based which allows graphics, mathematical symbols and hypertext links to be included in posted messages.

Private Mail

The Private Mail Tool is used by the students and the instructor for private communications. Private mail between students or between students and the instructor is visible only to the individuals involved. In many instances students asked the instructor questions about the course material using Private Mail Tool. In these situations response were posted to the Bulletin Board without identifying the student. This helped to open up the discussion and show individual students that they are not alone with their difficulties.

Calendar

Important dates were posted on the Calendar, dates of quizzes, the final exam and university holidays when it might take the instructor longer to respond to questions. The instructor can post to the Calendar for everyone to see and the students can post notes to the Calendar which they alone can see.

Chat Room

The Chat Room was made available to the students in the course although the instructor had no plans to use it.

Study Tools

Compile notes for printing

Using this tool a student can collect pages into one file for printing. Since the course is organized into units and sections, with each section being an individual file, this tool make it very convenient for students to print individual sections, collections of sections or whole chapters.

Resume Reading Notes Where You Left Off

This tool, which uses the WebCT Path facility, allows students when they log on to return to the last page they were previously reading.

Glossary

A glossary of important terms used in the course.

Search Content
Students can search the course material for a particular word or phrase.

Evaluation tool

Quiz tool
Quiz problems and solutions are organized in a database. The questions can be multiple choice, matching, short answer, paragraph answer or calculated answer. All except paragraph answer can be graded automatically. In Math 101 the Quiz Tool was used for six quizzes given throughout the semester, one at the end of each unit. Students had a forty-eight hour window in which to write each quiz, but once a student began a quiz there were sixty minutes to complete it. One additional practice quiz was given in the first week to allow students to become familiar with the Quiz Tool. All questions were graded by the instructor. After each quiz was graded the student was able to see the questions, their responses, the correct solutions from the quiz database, their grade and any comments from the instructor.

Student management

The instructor is able to create and delete student accounts and manage the student password file. Students are able to change their own passwords. Grades on quizzes are automatically stored in a spreadsheet-like format which can be used to manipulate grades and compute averages. The instructor is able to view tracking information on each student which shows the distribution of hits on each page and tool in the course. The only time this facility was used in Math 101 was to ensure, in the first few days of the class, that each student was able to successfully log on.

WebCT: Strengths and Weaknesses

WebCT has strengths and weaknesses that need to be considered by anyone considering it as a delivery platform. From the designer's perspective it has a rich array of tools which can be used in a variety of ways to present course material. The course material is created using standard HTML with no special of custom code creation software required. The ability to structure the material using Path Editor was extremely valuable in making Math 101 function as desired. This reliance on standard HTML does however have its drawbacks. A designer can use a WYSIWYG editor to produce the HTML code but in the author's experience there are times when the HTML code has to be edited directly to get the page to perform exactly as desired. Also the fact that the WebCT File Manager is used over the web to manipulate the files means that it is slower than if you could manipulate the files on your own computer. The main means of support for a designer is the WebCT mailing lists, and this support is excellent. Replies to questions posted to the mailing list is very quick and helpful, both from the employees of WebCT and other subscribers to the list.

As the instructor in Math 101 one of the most useful features was the Bulletin Board. Since the Bulletin Board is a hypertext environment it allows the instructor to respond to students' questions using diagrams and mathematical notation when required. This however requires that the instructor have a knowledge of HTML or create the reply using a WYSIWYG editor and upload it to the server. The Private Mail Tool has also proved to be valuable. The instructor found it very helpful that the private correspondence with students in the class stays within the WebCT environment. The course record facility which keeps the grades on quizzes in a spreadsheet-like format was found to be limited and slow since it has to be accessed over the web. It is, however, easy to download the grades from this facility to a spreadsheet on the instructor's local computer.

Each student in the class is being asked to complete a course evaluation form but at the time this paper is being written these forms are unavailable. The students have used the Bulletin Board, the Private Mail Tool, the Quiz Tool and the ability to compile and print the notes but their reaction to these and other tools, and to the course in general is not available at this time.

Concluding Remarks

There are no easy solutions to many of the problems that arise in designing, implementing and teaching an online class. One of the first problems to face is the choice of software. The author has been satisfied with the selection of WebCT despite the weaknesses mentioned. No other course delivery software has been found which contains its versatility and array of tools. The requirement of mathematical notation and diagrams adds an additional level of complexity to a mathematics course. For Math 101 the requirements of mathematical notation are minimal and the use of MathType by the designers and instructor met these requirements. The students however have difficulty with mathematical notation when posting to the Bulletin Board or answering quiz questions. There are also times when a student asks a question that would be easier to articulate if a diagram were possible. The White Board, which is available in WebCT and in other web delivery software, may help to fill this need but its synchronous nature made it unsuitable for Math 101.

On reflection the author has misgivings about the choice of Math 101 as a first attempt in creating an online course. Many of the students who register in Math 101 have a fear of mathematics and would not be expected to enroll in a web based class. The participants, however, have learned from this experience and have produced a version of this existing course that complements the lecture format and expands the options for students wishing to enroll in Math 101.

A version of Math 101 can be seen at http://online.math.uregina.ca/public/math101sampler/. This sampler version of the course contains all the structure of Math 101 and a sample quiz but most of the content has been removed.

Online Conversion of a Technology Based "College Algebra" Course

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Abstract: The aim of this paper is to report on the authors' experience in designing and implementing an online version of a technology (graphing calculator) based "college algebra" course. College algebra is one of several courses mandated by the State of Louisiana as a component of the core curriculum requirements for undergraduate students. This is a course, which many K-12 educators will go on to teach in their classrooms. We identify some of the problems we encountered and the solutions we employed in order to resolve them. In particular, we discuss the unique challenges we faced in communicating mathematics online and utilizing the graphing calculator technology. Finally, we examine the compatibility of the standards advocated by the National Council for Teachers of Mathematics (NCTM) with the online paradigm, and we describe what we believe to be strengths and weaknesses of online learning models for college algebra.

Introduction

"College Algebra", a course offered by every university in the State of Louisiana and by many universities around the country, contains material which should have been covered in the K-12 curriculum completed by many of our incoming students. Nonetheless, few Louisiana students (less than 2%) place out of this course. Thus, it appears that many students are not learning this material before entering college. Eight years ago, the Department of Mathematics at NSU was motivated to "reform" the course. A major component of our reform effort involved making the course technology-based. Namely, graphing calculator technology was heavily integrated into the course.

Several considerations led to the development of an online version of the course. NSU has a large proportion of nontraditional students (i.e. students who work full time and/or commute). For many of these students, the online alternative is not only more convenient, but may also be the only alternative. A second reason is the university's current mission of making it possible for a student to complete an associate degree program completely online by the year 2005; it will be necessary to have this course in place if the university is to achieve its mission. A third consideration for the authors was the compatibility of the course goals and objectives of our reform based college algebra course with the online paradigm. At first, we were cautiously optimistic that we would be able to provide a "reasonable" alternative to the traditional on-ground class for nontraditional students who were sufficiently motivated. As the development of the course progressed, we knew that we could do much better than simply provide a "reasonable" alternative. After only a minimal amount of training, it seemed apparent to us that the principles which guide current reform efforts in undergraduate mathematics education went hand and hand with those which tend to guide the design of online learning models. We began to think that perhaps the online learning environment was more conducive to achieving some of these "reform" goals than the "on-ground" one was.

Section 1: Challenges/Problems Encountered -- Some Resolutions and Advice.

In this section we discuss some of the challenges we faced and some methods we used to resolve (or at least attempt to resolve) the problems we encountered. Some informal advice and recommendations are offered.

Training & Non-technical Support

We knew that the project could not even get off the ground without certain preliminary resources, as well as general support from our institution – both departmental and administrative. We needed time to develop the course, new personal computers, and professional training. We obtained these through a grant from our state Board of Regents (with matching support from the university). The grant provided us with course release time, money to take training courses in the design and delivery of online courses from the UCLA Online Extension Program, and new computers. We strongly encourage anyone developing an online course or program to get some training. Only a very short time into our own training process, we were amazed at our initial naiveté and at some of the misconceptions we had held. The online experience is inherently different than the on-ground one, especially when the online mode is asynchronous. Bedore (1997) maintains that online experience and training, as well as content expertise, are necessary prerequisites to a successful online curriculum conversion. This is particularly important for those who are facilitating more interactive learning models. Bedore also recommends having a "champion" in your corner, someone dedicated to the success of your project who will do what it takes to keep development and support for the project on track. In our case, we had this from the dean of our college and our departmental coordinator. Finally, for the purposes of doing justice to your course and preserving your sanity, we recommend against developing an online course for the first time without release time and/or help from a graduate assistant (both would be preferable).

Technical Support.

As concerns technical support, we did run into some problems. Use of the course management system TopClass was mandated by the university; a lack of training in its use, and a lack of support when things went wrong were major stumbling blocks. We strongly recommend that, before jumping in, you find out about the existing technological infrastructure at your institution and the level of technical support you can expect it to deliver. Unreliable or difficult to use technology and poor support services could doom a program to failure and put a permanent dent in your market. Fortunately, NSU has been a Louisiana leader in distance learning. There were many people around for us to talk to during the process of developing our course. We strongly recommend actively seeking the advice and expertise of people who have experience with whatever technology you are using.

Course Management Systems

We found Top Class, the course management software, to be somewhat inflexible (especially in terms of our need for graphical representations). As it turns out, that problem may be resolved as the university is switching to a system called Blackboard (which appears to be more user friendly). For anyone in the position of being able to choose a course management system, we would recommend giving this careful consideration. Learn something about the different alternatives and make an informed choice rather than just choosing one randomly. They really do differ, and it doesn't seem to be the case that the more expensive options are necessarily the better ones.

Accomplishing Learning Objectives & Choosing Technologies

We wanted to accommodate different learning styles, yet at the same time keep it simple. We did not want to burden students with complicated technological demands or overwhelm them with distracting state-of-the-art "bells and whistles." We were especially concerned about the availability and quality of technical support services. We felt there was no reason to use a given technology unless it would clearly support/enhance the learning objectives for the course. We did know that something beyond plain text and "lecture" was needed to convey problem solving techniques and develop problem solving skills. One develops problem solving skills by solving lots of problems. Some mechanism needed to be in place to provide students with an opportunity to attempt problems and get immediate feedback. While we would be available to answer students' questions via email, we did not feel this would be sufficient to provide

students with an appropriate level of guidance and feedback. Initially, we considered requiring students to purchase an interactive CD ROM device (such as Math Systems' *Mathpert Assistant*, Quant Systems' *Adventures in Algebra*, or Academic Systems' *Mediated Learning* software). But we decided against this for two reasons. We wanted to keep student costs at a reasonable level and we discovered that there were already wonderful free resources available on the Internet. In addition to the text and TI-83 graphing calculator which we also require for our on-ground class, we required students to purchase one additional piece of software called Graph Link that allows communication between calculator and computer.

The course was structured as follows: students received weekly assignments that included readings from the text, problems to solve, and discussion question(s) requiring responses in our class "discussion" area. Each assignment was accompanied by a "lesson enhancement." The lesson enhancements were Web pages (written by us) containing summaries of key concepts, examples, suggested interactive activities, and links to interactive tutorials and self-quizzes. Many of the tutorials offered multi-layered dynamic presentations of concepts with visual/graphic explanations, opportunity for experimentation, and self-quizzes with instant grading and feedback. Weekly quizzes were given for credit. (We provided solutions to assigned problems before posting quizzes-for-credit.) Another integral component of the course was the use of both group and individual projects. These provided students with an opportunity to apply the concepts learned in the assignment to "real world" situations (i.e., choosing the best cell phone plan from options offered or predicting market value of property for a given geographic location). Finally, we gave midterm and final examinations (administered online via the TopClass testing feature).

Graphing Calculator Technology

Research has suggested that use of graphing calculators enhances students conceptual understanding of mathematics and tends to foster a more positive attitude toward mathematics in general among both students and faculty (Hubbard, 1998). One of the major issues involves the need to effectively use the graphing calculator within the academic setting and its incorporation in the solution of real-world problems (Testone, 1998). Based upon our own experiences and that of others (Hollar, 1999), students using graphing calculators tend to have a better understanding of functions than those who do not use this technology and, therefore, have a better grasp of the implications in problem solving. The calculator allows students to easily view problems and principles from multiple perspectives, including visual or graphic, numerical, and symbolic, and it allows them to see the relationships between them. It also makes it feasible for students to attack "real world" problems without losing the forest for the trees.

As we mentioned, the graphing calculator is an essential component of our current on-ground course; also our text is intended to be used in conjunction with the graphing calculator. Knowledge of the graphing calculator is assumed in all courses for which "college algebra" is a prerequisite; it was essential that we find a way to incorporate this technology into our online course. That is where the Graph Link software came into play; this software allows the user to send images generated by the calculator to the computer for inclusion in documents. We used this technology to send graphical representations, tables, instructions, etc. to our students and the students were able to send graphs to us. The biggest problem was the way that must be done; check your course management software. Graph Link saves images as .tif files but the software would only accept .gif files; the conversion was time-consuming but not impossible (you need good technical support here to find out exactly what is required). In addition, we created a "Calculator Corner" where students could go for help in using both the Graph Link and their calculator. In some cases they were referred to various sites for assistance; in others, we wrote simple instructions and provided images from the graphing calculator using Graph Link. Other than that, the students used an ordinary keyboard for most of their work, using the same symbol conventions as they do with the graphing calculator. There are nice websites out there that explain "how to type mathematics" on an ordinary keyboard.

Students' Computer Literacy

We did encounter some problems with computer literacy on the part of students. The university does not officially require a computer literacy prerequisite. We strongly encouraged literacy in a course

"disclaimer;" nonetheless, we did end up with some students who had very little or no computer experience. In the future, we would like to see a computer literacy requirement for incoming freshmen.

Other Considerations

We had high hopes about the discussion component of our course. Based on our experience with online learning, we imagined an active and stimulating "virtual classroom" where we as facilitators would ask leading questions, steer the conversation in the appropriate direction and bring about understanding of the key concepts. We have since reconsidered our assessment of the potential value of the class discussion. Many of the learning objectives for our algebra course involve specific "skills" which must be mastered. These are probably best demonstrated through an interactive component that can provide meaningful feedback. We still have hopes that discussion of the class projects can be fruitful. We intend to make the projects our main vehicle for assessing the achievement of learning objectives centering upon "real world" application of mathematical knowledge, critical thinking and communication skills, and building cross-disciplinary knowledge.

Section 2: Compatibility of Online Paradigm with NCTM Standards.

In 1989, the National Council of Teachers of Mathematics (NCTM) published standards for mathematics education. The document provides general guiding principles for mathematics instructional programs and specific standards about mathematical content and processes students should know and use as they progress through school. The chief premise is that the underpinnings of everyday life are increasingly mathematical and technological; students will live in a world where intelligent decisions often require quantitative understandings. Just as the level of mathematics needed for intelligent citizenship has increased dramatically, so too has the level of mathematical thinking and problem solving needed in the workplace. The guiding principles for instructional programs include promotion of the learning of mathematics by *all* students and the use of technology to help students understand mathematics and to prepare them to use mathematics in an increasingly technological world. We believe that the online paradigm is particularly compatible with these basic principles. The abundance of Internet resources and emergence of new technologies are creating opportunities to accommodate a wider variety of learning styles. Concepts can easily be presented from many points of view. Mathematical software, computer algebra systems such as MAPLE, and graphing technology are being integrated ever more seamlessly online.

The Standards are divided into two categories – those which focus on content and those which focus on process. Embedded in the standards we found many of the learning objectives for our college algebra course, as well as other lower level math courses offered here at NSU. One of the content standards includes a recommendation that attention be given to patterns and models so that students acquire the ability to "use mathematical models and analyze change in both real and abstract contexts." Another is that attention be given to data analysis and statistics so that students learn to "pose questions and collect, organize, and represent data to answer those questions," "interpret data using methods of exploratory data analysis," and "develop and evaluate inferences, predictions, and arguments that are based on data." We believe that the combination of graphing calculator technology with Internet resources offers outstanding potential in these content areas. Numerous Internet resources are available for data gathering and experimentation with patterns and models. The TI-83 graphing calculator is particularly well suited for statistics and data analysis. Students can readily test predictions of models by collecting information from the Internet or by running simulations on their calculator.

The process-oriented standards focus on problem solving capabilities, reasoning abilities, communication skills, making connections, and mathematical representation. It is not only recommended that students learn to express mathematical ideas coherently and clearly to others, but also that they extend their mathematical knowledge by considering the thinking and strategies of others. It is also recommended that students develop a repertoire of mathematical representations and learn how to use them to "model and interpret physical, social, and mathematical phenomena." We believe that the process-oriented standards can be met most effectively through interactive discovery-oriented learning models in which the student is

expected to actively assume responsibility for his/her learning process. The capability of the online learning model to accommodate asynchronous interactions allows much greater opportunity for students to learn by drawing on their own experiences and the experiences of others to apply knowledge in a real world context, and to make connections between math and other disciplines. It is worth noting that asynchrony, as well as being a convenience is what makes the high level of interaction possible. Internet resources further facilitate the goal of having students draw on a number of resources to answer questions and solve problems. Internet resources also facilitate multiple representations of phenomenon and the illustration of ideas from different points of view (e.g. graphic, numerical, and symbolic).

It struck us from the beginning that the guiding principles advocated by proponents of online learning models tend to be similar, if not identical, to those which continually pop up in the undergraduate mathematics reform literature. For example, Berge (1996) recommends avoiding lecturing, encouraging group interaction , and intentionally giving students little direction. De-emphasis on lectures, cooperative learning, and student-centered learning are all cornerstones of the current reform efforts and appear to have positive effects on student learning (Reynolds, 1995, Rogers, 1988). Other "reform" goals include emphasis on concepts and the "big picture and de-emphasis of rote memorization (particularly of isolated facts and techniques for which no context is provided.). Research supports the contention that students learning objectives are better achieved when "teachers regularly utilize the computational, graphic, or symbolic capabilities of technological tools to develop mathematical ideas. (Carpenter 1998, Heid 1988, Hiebert & Wearne 1996).

Section 3: Strengths and Weaknesses.

Strengths of an Online Program

Offering college algebra online accommodates both "distance-free" and "time-free learning." The asynchronous element of the program opens educational opportunities for students with varying backgrounds and experiences. Each brings a unique mathematical perspective to the "class" which promotes interest and may provide each with a better understanding of why mathematics is so important. Furthermore, the online environment is especially suited for fostering the development of critical thinking skills as a vehicle for students to apply course content in a "real world" context. Not only does the online discussion encourage students to draw upon their own experiences to tie theory to practice, it also provides an environment conducive to cooperative learning activities. In addition, the lack of time constraints gave students an opportunity to digest the material and relate it to their own experiences.

Conversion of traditional algebra courses to online courses will vary but utilizing grant resources, as we did, can provide some budgetary relief. Naturally, as faculty experience grows and technology changes, these costs can be expected to decline. According to Bedore (1997), curriculum conversion costs should be about $500 per course with setup and technology costs ranging from $0 to $1,000. Complete course development costs would be substantially more (about $2,000).

Weaknesses of an online program

With the need for major modifications to the existing infrastructure, the cost of implementing an online program could be prohibitive. Without the kind of support we had for this project in terms of equipment, release time, and faculty education, this project would have been doomed from the start. In addition, poor promotion of the course could lead to frustration on the part of both faculty and students. We had problems with students who were registered for the course but really had no desire to participate in an online course (their advisors put them in the section because it was the only one open at the time).

Research suggests that the dropout rate for online courses may be higher than that of traditional courses (Sherry, 1996; Starr, 1995). Students taking a college algebra course online (or any other online course) must be sufficiently motivated, self-directed and dedicated to the task. They must understand that they share a somewhat greater responsibility for their learning than might be expected in a traditional class. One

of our main concerns was student interaction; the students were reluctant to "discuss" mathematical concepts with each other for a variety of reasons. The lack of face-to-face contact was a real problem for some; this may have been due, in part, to the novelty of the situation since these students had limited online experiences.

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Issues in developing an on-line mathematics learning series for middle school teachers

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Abstract: Online learning could be an important resource in meeting the need for helping teachers learn more mathematics, which is critical to the success of long-term change in mathematics instruction. We report on a pilot of an online learning series for middle-school mathematics teachers. An existing Web-based professional development center was utilized together with specially designed interactive web pages to structure and support six weekly synchronous meetings. The mathematical content of the series was proportions. The pilot yielded initial success in fostering mathematical discourse among participants and raised issues, technological and other, for the future.

As a decade of reform work in mathematics education—based on the National Council of Teachers of Mathematics' Curriculum Standards (NCTM, 1989)—comes to a close, reformers are looking to technological solutions for the problem of delivering high-quality professional development to teachers nationwide. Subject-matter expertise is of particular importance as recent research (Ball, 1989; Schifter, 1997; Darling-Hammond, 1998; Ma, 1999; Sowder et al, 1996) highlights the mismatch between teachers' knowledge of mathematics and what and how they are expected to teach. Neither simply exploring student activities nor taking traditional college-level mathematics courses will help teachers develop the mathematical practices they need. Reformers are designing new programs focusing on developing content knowledge in support of change. Online learning communities hold promise for providing professional development that is convenient to schedule; can be offered to urban, suburban and rural teachers; can fit in with ongoing reform programs rather than be simply "another inservice." The question of how well subject matter learning can be supported online is an open one.

This fall, we at IRL developed and offered a pilot version of an online seminar for teachers, designed to contribute to this reform work. The pilot, *Brush up on Proportions*, is part of our WebMath online math learning community for middle school teachers, which will expand into a larger program next year. The intended audience includes teachers who are already engaged in implementing a Standards-based math program or are at least using some materials or processes recommended by the Standards. The seminar consisted of six hour-long sessions, offered as a series. Each session was devoted to exploring and deepening teachers' proportional reasoning—a central topic in Standards-based middle school programs—through activities and discussion. Conceptual and procedural knowledge were both emphasized, as well as the relations between them. We also aimed to establish norms of discourse that will encourage teachers to develop mathematical practices such as making conjectures and constructing mathematical explanations.

This paper describes features of our online seminar environment and issues that arose during the pilot.

Features

A suite in TAPPED IN. Rather than build and support our own interactive web infrastructure, we contracted with TAPPED IN to house the virtual classroom in which *Brush up on Proportions* took place. TAPPED IN is a virtual professional development center designed to house activities for teachers throughout their career. The classroom is part of a suite where various middle-school math-related events sponsored by IRL take place. Organized as a campus, with building, suites and offices occupied by tenants who provide professional development services for teachers, TAPPED IN provides an infrastructure with a graphical interface in addition to a text window through which participants can communicate with each other and use objects in the environment. Participants' main venue for participation in class is a text window in which their comments, "emotings" and interactions with objects are seen by others, and they can see what others say and do. Objects and occupants of the classroom appear in a graphics window too. The graphics window gives participants a sense of location through maps, floor plans, and sometimes, a 3/4

view into a room. One room in our suite is portrayed visually as a corner of a classroom with objects on the walls, through which teachers can view the course syllabus, get more information about IRL and access the web pages that form the text for our seminar.

Web pages and other tools. One of our initial questions was how to build a discussion of mathematics in the relatively thin online medium of text exchange. Current math teaching and professional mathematicians' practices include the use of many different materials and representations. In teaching, handouts, texts, manipulatives or other curriculum materials are resources for mathematical thinking and discourse. Also, there is some medium for whole class display of ideas: a white board or overhead projector. The overhead can display pre-made representations or spontaneous ones. The practices of working mathematicians often include discussion centered around a just-in-time scribbled diagram, sketch or equation.

The current TAPPED IN tools that provide similar capabilities are text-only whiteboards, which we determined to be of limited use in displaying mathematical ideas. The whiteboard did begin to serve an agenda structuring purpose, though. In future iterations, we plan to design, with TAPPED IN staff, and use a graphics-capable whiteboard and a special mathematics notation tool. In addition we are looking at inexpensive commercial devices for bringing scribbles to the digital environment. In this first pilot, however, we worked with software and capabilities we had already to create web pages.

We created interactive web pages that play the roles of both manipulatives and textbook in our virtual classroom. These pages are textbook-like in that they structure the course content; they serve as manipulatives in that their interactivity allows users to have experiences on which to base mathematical conjectures. The facilitator, or anyone in the room, may project the pages for all to see and work with. Four of these web pages were used in *Brush up on Proportions*:

- Page one shows the iconic proportional man by Leonardo DaVinci, where a circle is inscribed around his armspan and height and a square is constructed around him in another way. There is no interactivity to this page. Text asks users to consider what is commonly meant by the term "in proportion", using the picture as a springboard for discussion.
- Page two shows a floorplan of a room with some furniture in it. When users type in a scale for the room, the resulting width and length of a rectangular table appear. Text asks users to set the scale so that the floorplan rectangle represents a table that is 130 cm wide.
- Page three presents a more complicated floorplan problem: selecting a table from one scaled diagram to insert in a floorplan with another scale. The ratio between the sides of the table is the first determiner of which table is a good choice.
- Page four shows a proportion form, $\frac{a}{b}=\frac{c}{d}$. In a set of challenges, numbers replace a, b, c and d to make a false proportion and users must change one number to make a true proportion. The proportion form is tied to a picture of two rectangles—a by b and c by d—superimposed on a grid, so users can note how the rectangles compare visually when the proportions are true or not. They are asked to use this experience to help them think about why the cross-multiplication algorithm works. They are encouraged to use other representations or arguments based on arithmetic, too. In other words, they are to explain why if $a/b=c/d$, then $ad=bc$, without resorting to algebra.

These experiences are designed to help teachers develop their proportional reasoning and connect it to formal representations. The problems on pages two and three are not difficult for many mathematics teachers to solve; however, we have found that their connection to proportion and the relationship among all the multiplicative relationships in the pictures are not always explicit. For example, one participant said, "we don't need proportions" to solve the problem on page two, indicating that dividing 130 by the width of the rectangle in floorplan units would suffice. This is certainly a correct procedure for solving the problem, based on an intuitive understanding of its underlying multiplicative structure. However, this solution to the problem does not help teachers help students develop their own multiplicative sense. In discussion of the problem together in seminar, we asked ourselves to justify each stage of the answer and to connect the work to the proportion $a:b=c:d$.

The web pages worked well as a resource for mathematical discussion. Participants actively engaged in trying out solutions in the interactive structure. The discussion was anchored in the particulars of the representation but also led to more general mathematical insights. We will improve the representations based on insights we gained through using them. For example, setting the scale may not be the most helpful interaction to offer in solving the problem presented on page three. We will also build new sequences of web pages that follow other learning trajectories through proportional reasoning: for example, supporting the connection between the proportion form and linear variation.

Objects. The TAPPED IN environment, in true "moo" fashion, includes a number of objects with which users can interact. A few of these have been used as instructional devices. As a simple example of proportionality, we created a virtual pet, Shrinky, who responds to the text command, "shirnk by 50%" by shrinking from 100 to 50 centimeters. This prompted others to display two cows, drawn in the text window in ASCII symbols. The cows then became the topic of discussion: what percent of the first's length was the second? Conjectures were made and the facilitator was able to model mathematical questioning and justifying in a non-threatening way around objects that were displayed with humor.

In the future, we hope to have objects that can accept more flexible commands than "upon text x, print text y." For example, Shrinky Jr. might be able to know her current length and calculate her new length based on the percent she was commanded to shrink. Participants could try to get Shrinky to shirnk from 160 to 25 feet in exactly three shrinks and learn about the concatenation of percents, part of multiplicative and proportional reasoning.

Using research findings. We are building on existing insights into cultivating and growing Web-based communities (Reingold, 1993; Rochelle and Pea, 1999). From this research and reflection, we know, for one thing, that attention must be paid to norm setting. Our facilitators set the tone of discussion in the community and establish mathematical norms. Essential to this tone setting are both providing and requesting mathematical explanations. We have also encouraged a sense of play with the TAPPED IN objects and with light banter in between and throughout the mathematical insights.

The literature also informed us that personalities can be an important factor, and that social events and personal talk help bind people together to do the more content oriented work. In the future, we plan to focus on these aspects of community building: We will encourage participants to create profiles and use pictorial "avatars" when they are online. We will host online social events and encourage "idle chatter" as a way of establishing commonalties. We will utilize TAPPED IN's resources and existing culture to help "enculturate" course participants.

Issues

Mathematical discourse. We aim to help teachers connect their mathematical intuitions with standard forms through discourse. This is a two-way street: teachers can use standard forms such as proportions to construct mathematical explanations of numerical insights they have; they can also use the situation presented in the interactive web pages to help support explanations of their use of the standard forms. In *Brush up*, this meant that we helped teachers connect the proportion form, a:b=c:d, to their own multiplicative reasoning. Figure one shows one of the interactive web pages that sets up a simple scale problem: to set the scale of the floorplan so that the rectangle represents a 130 cm long table. For adults fluent in intuitive proportional reasoning, it is not too difficult to divide the 130 cm by 5 squares of the rectangle's length to get 26 as number to put into the scale. But our participants did not immediately construct explanations for why division was the operation to use and why those two numbers were involved. Through discussion together, we came up with two explanations: that the 130 cm of the real-world length needed to be distributed across the 5 units of length of the rectangle, and that this distribution called for division. We also saw that we could set up the proportion 1 unit:? cm = 5 units:130 cm. The proportion related the scale to the paper and real tables. Building these kinds of multiplicative connections is facilitated by use of the proportion form, a:b=c:d. The result of all this should be more conceptual resources for teachers to draw on as they help students reason about and solve similar problems.

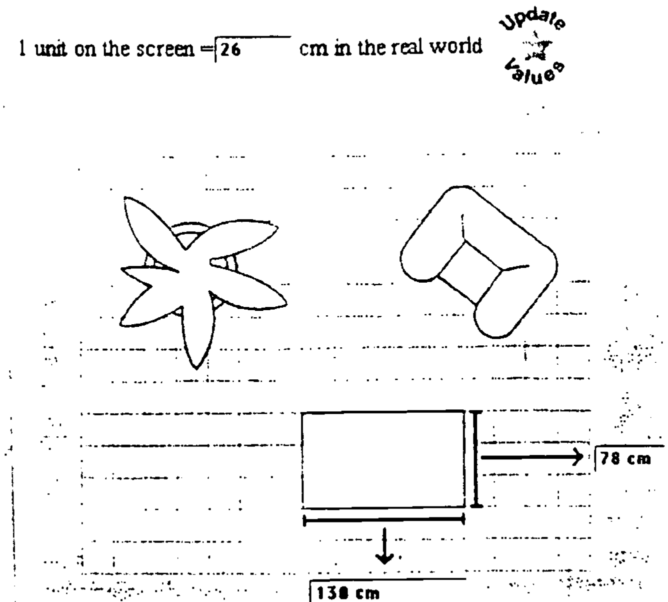

Figure one: an interactive web page to support mathematics learning

<u>Pace of interaction</u>. As compared with a face-to-face discussion in a classroom, the pace of conversation is slow in our online course. One hour was not sufficient time to do an activity, discuss it, and summarize the important mathematics in the activity and discussion. The initial course outline covered the material in three weeks; our just-prior-to launch outline was stretched to six sessions, and we ended up covering about two thirds of the outlined material. This phenomenon is, of course, not restricted to syllabi of online courses, but it is our sense that the problem was exacerbated by the slow rate of conversation online. The next seminar will have ninety-minute sessions. We will also institute two leaders: a facilitator, keeping the discussion flowing and helping everyone participate, and a recorder, keeping track of insights on the white board.

There are some benefits to the slowed pace of idea exchange in the online classroom, however. The pace of the overall discussion leaves room for side conversations to emerge. And obviously, these are not disruptive the way two people whispering to each other in class can be. Two staff members participated in *Brush Up* from computers that were housed side by side. They were able to carry on a face-to-face exchange about a mathematical topic related to that week's seminar: they were trying to informally prove or disprove that "three points determines a circle." Scratch paper and white boards were involved. They made a fair amount of progress while still participating seamlessly in the discussion of Leonardo DaVinci's iconic proportional man.

This interaction and, additionally, phone calls to solve technical problems during the class, has led to the plan to encourage participants to pair off into "buddies", between whom some kind of communication outside but during the seminar is encouraged. Online, phone and face-to face communications are three means for this. The online environment allows participants to whisper to each other: in other words, to type comments that only the two in the exchange can see on their screens. Phone conversations are possible if long-distance phone bills are not a problem and if the Internet connection does not take up the phone line. Alternatively, participants from the same school could participate together from their school's computer lab.

<u>Abandoning linearity.</u> Giving up conventions of linearity that apply to oral conversation seems to be key to becoming adept at online, text-based discussion. In oral conversation, we allow parenthetic remarks, but they have some labeling, and they are limited in scope and frequency (depending on the speakers, of course). Online, two or more conversational topics can become quite interleaved. Online participants develop conventions for marking this, too, if the comments in the same conversation become widely spaced

in the text. For example, a participant might preface her comment with "re table size" if table size had been the topic several lines earlier.

Our preliminary experiences suggest that the non-linearity of online, text-based conversation can allow for greater access to participation. Though new users sometimes say that the moment for their comment was lost so they kept quiet, in fact, they probably could insert their comment way down the line, with a marker, and make a meaningful contribution to the conversation. We will track data on this hypothesis in the next pilot, and we will seed, by example, appropriate markers.

<u>Attracting participants and supporting change.</u> We had no idea how many participants to expect at our first series. We announced the series on our Middle-school Math through Applications Web site and related Web sites visited by teachers and curriculum decision makers. We sent email to teachers with whom we had worked in the past and also announced the series on TAPPED IN's calendar and newsletter. Seven was our highest attendance. Additionally, no one teacher attended all six sessions. These numbers were adequate for our pilot purposes. We didn't want to have a crowd in our initial pilot, where our intention was to try out the new environment and course materials.

Our initial pilot confirmed one of our starting premises: that this online subject-matter learning series needs to be linked to ongoing professional development programs for teachers. We believe that this is important not only for attracting participants, but also for making the experience effective in supporting long-term change. We already have in place connections for the future. Connections can be made through curriculum adoption or through state and district programs. For example, our next seminar series will be offered to 20 teachers who are piloting a unit from a new Standards-based comprehensive math curriculum. They will be teaching a unit focused on scale factors and proportions, and the *Brush up* seminar will directly support their teaching. We have developed interactive web pages that present problems similar to those found in current Standards-based texts. We have also connected with state and district leaders to see how *Brush up* can be of service to their programs.

<u>Divergence vs. convergence.</u> We experienced the tension true of all problem-centered, conceptual-understanding-oriented teaching: constraining the conversation to keep the participants' discussion and insights in the range of the mathematics topics defined in the lesson plan, as opposed to encouraging divergence to capture the range of mathematical insights possible based on a given situation.

For example, one week, the entire session's discussion focused on page one of the "web text", displaying a man with arms stretched and figures circumscribed about it. The depth and breadth of the conversation around this image surpassed our initial planning expectations. The page was designed simply to ground proportion in a familiar image, helping us tease out the meaning in everyday usage. But the discussion went beyond that: Questions were asked and conjectures made about the relationship between the circle and the square. The connection to ratio and proportion was made. Someone raised the question, "What was he trying to tell us with this image?" We agreed upon a relation among the geometric figures and the body parts in the image and, finally, shared insights about the meaning of "proportional" in common usage.

As we plan for balancing convergence and divergence in the next iteration, we are structuring the session so that time is left at the end for the facilitator, and others, to mount a virtual soapbox in order to summarize the mathematical territory covered and make connections that might have otherwise remained implicit.

<u>Technological capacity.</u> We did not want to limit teachers' access to the series with high-tech demands. Yet we wanted to provide a rich online learning experience. This led to tensions. Web pages with text and graphics, like the DaVinci activity, work for any teacher with an Internet connection and a browser. The TAPPED IN environment offered both Java-based and Web-based versions of the classroom. In order to create a more interactive experience, we decided to use JavaScript. JavaScript does not take significantly longer to download than a traditional web page, but requires a Java-enabled browser. This did not seem too limiting since Java-enabled browsers are quite common. In order to support the majority of these browsers, we limited ourselves to JavaScript 1.0, the earliest incarnation of the scripting language.

In this pilot we also required participants to get the Shockwave plug-in and provided a link to acquire it. Java applets will run without a plug-in, but would take longer to download and were more difficult for us to develop. When we combined these different technologies (particularly the Java TAPPED IN window and the Shockwave activities), we found that the browser could run into memory problems. The teachers needed to be able to allocate more memory to their browser if they had enough RAM to do so. This was too difficult for some participants to do while maintaining their role in the discussion.

Conclusion

Online learning environments hold promise for providing professional development opportunities for teachers. Our initial pilot showed that a focus on subject matter, as called for in recent research on teaching, can be accomplished through discourse centered on interactive web displays. Issues of problem-centered instruction are mirrored on line and new issues arise due to text-based pacing. Additionally, online learning programs can be "just another workshop" unless integrated into ongoing professional development programs. Balancing the need for rich representational tools for math learning with low-tech user limitations is a constant challenge.

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Symbolic Computers and Mathematical Objects

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Abstract: Technology is best utilized as a tool for exploring the foundational objects and exploring and varying the underlying nature of the object upon which the abstracted mathematical symbol is constructed. When this is done appropriately it can serve to elicit the formation of selected concepts. This paper will serve to illustrate how the symbolic computer can serve to create psychologically foundational mathematical objects of thought for the learning.

Introduction

The potential inherent in the modern microcomputer to serve as a bridge between and among instructional theories has not been lost upon many researchers in mathematics education and curriculum theory (Brownell & Brownell, 1998; Garofalo, Shockey & Drier, 1998). The computer easily enables a dynamic and active learning environment within which each of the process strands of the National Council of Teachers of Mathematics (NCTM) standards might find a home (NCTM, 1989). After all, when one thinks of reasoning, problem solving, communication and connecting related ideas the tool of choice in nearly every discipline is the microcomputer.

Furthermore, unlike the traditional calculator, the modern classroom computer has an unparalleled ability to implement both graphical and procedural components of mathematics understanding in a single unified object. By their creation and utilization of mathematically relevant computer-based objects this dual encapsulation enables the students a unique opportunity to see both the form of representation and their actions utilizing this representation simultaneously. For this reason alone it would be a natural tool for both classroom use and theoretical musings1.

This has not been lost upon teachers and despite those who would use the computer solely for the presentation of pre-packaged instructional units, there is a growing consensus that the computer can be an ideal tool for knowledge construction at an individual or group level (Harvey & Charnitski, 1998). As we explore the potential for classroom uses of computer technologies we have a once-in-a-lifetime opportunity to blend the best of learning theory and the practical realities of student actions.

Creating a New Class of Mathematical Object

Let me address for a moment the extent to which the object orientation metaphor, found in the modern windowed operating systems and programs, transfers to the "tools to think with" notions of current action upon objects models of mathematics teaching and learning. In particular, the object-oriented environments which modern technology has created are ideally suited to parallel and facilitate the ability of students to take a broader variety of action upon objects of a nature and kind hitherto unknown. These student controlled actions upon these mathematically powerful and computer enabled objects have the potential for creating classroom environments which both surpass the pale hopes of the integrated learning system and would surprise those wedded to a conservative view of Piagetian developmental levels.

Given the long history of cognitive science of borrowing the most current metaphors from computer science it should come as small surprise to see the applicability of many of the object oriented programming strategies to reflect aspects of human cognition (DuPlessis, 1995). And, indeed this new metaphor plays out very well for the case of mathematics education. The tools supported by the modern computer enable a new class of objects with which to think.

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[1] For further background on the importance of what I am calling dual encapsulation Sfard offers a highly informative account of the intertwined roles of process and objects in mathematics (Sfard, 1991).

In order for this to happen, however, we must attempt to leave behind any preconceived notions about the role of the technology as being most useful for information presentation and delivery. A much healthier perspective, at least in terms of understanding this new approach, would be to envision the technology as being used to providing a milieu within which knowledge construction can occur (Connell & Abramovich, 1999). To see what this might entail, let me begin by telling a brief story from a very early research study using the Windows based authoring program ToolBook.

I had created a ToolBook program that I called a "Cognitive Playspace" for children to explore various foundational notions of mathematics. My goal at the time was to examine the extent to which traditional physical manipulatives might be augmented by technology. One activity, in particular, involved the use of various geometric shapes. I had intended these to be analogous to the traditional pattern blocks with which the children were already familiar, but to not be identical to them. In this activity, the children were presented with a variety of the puzzles to solve.

In the puzzle set for this particular day one of the tasks was for the child to reconstruct a pattern made using the provided shapes through manipulation of the geometric forms on the computer's screen. I had developed some fairly simple tools that would enable the geometric shapes to be rotated, translated, and generally moved about the screen.

Of particular interest for the notion of object creation is an event took place when I brought a group of Kindergarten and First-grade students to the University to work with this exploration package. By this time they had had other experiences in both using ToolBook and the relatively primitive Windows machines. By the time of this incident the children typically had very little difficulty relating to the computer itself. In looking into their level of expertise I would say there were easily the equal of a computer savvy kindergarten student of today. If this is somewhat surprising given the fairly primitive state of technology at the time I should mention that these were 386 and 486 computers running Windows for workgroups. At the time of this study they represented an extremely high end product. Furthermore, for the type of software that we were running, these were more than adequate to give reasonable performances even by today's standards.

As an aside, the notion of using the most powerful technology and authoring system then available to work with kindergarten students struck many as being amazing. I made the point then, which I still believe, that in order for the computer (of that day) to be meaningful it required the highest end product. One of the major pleasant surprises of the last nine years is how well these earlier findings hold up. Indeed, the off-the-shelf machines of today are quite comparable (in terms of the visualization and manipulability of objects they engender) with these early efforts. Not only was this work ahead of the curve, it was outside the box totally.

But what I remember most, relative to the notion of object reification, came from the experiences of one young man who in his manipulation of the graphic objects managed to drag all of them off of the computer screen. This was a bit more of a problematic than it might at first appear for in the nature of "it's not a bug it's a feature" I had not created a simple button to return objects to the screen once they had been dragged off the working area.

I must confess that this was a complete oversight on my part. The program was under development and I had not yet thought as to what would happen if objects were to be dragged off the screen. In the course of developing the activities I had always just clicked to the next screen and then back to reset the screen – this had become so much a habit that I had taken it for granted at the time work with the children began. Of course, this child was blissfully unaware of that strategy. When he dragged them off of the screen for all he knew they were gone forever.

I'll never forget his eyes looking up to me as they swelled with tears as he said, "Help me, Dr. Connell! I have lost all of my toys."

His painfully sincere statement serves as a powerful reminder that in the mind of this child these computer images represented real objects upon which he was carrying out real operations. Furthermore, these operations carried with them a heavy kinesthetic component that was totally unexpected at the time. Subsequent observations and interviews illustrated that the physical manipulation of his eyes and hands with the motions of the mouse as translated to the computers screen translated to his mind as a series of actual actions upon real objects.

To put it in more poignant terms, this was no virtual reality to this child. These were his real toys and they were really lost. I'll never forget his joy as I showed him how to get his toys back.

The experiences of this child that the technology had engendered were extremely powerful for him. Furthermore, they proved to be similar to the real world pattern blocks from which the geometric shapes had been designed. The combination of kinesthetic motion required manipulating the objects with the mouse and the geometric shapes on the computer screen appeared to provide a direct analog to the kinesthetic motion

required to move the pattern blocks on the desktop. To these children differences between moving the virtual object on the computer screen and moving the physical object of the pattern walks on the desktop were so small as to be indistinguishable.

For this child this represented a valid action upon a real object. To that mathematics does represent a series of action upon reified objects of growing complexity and abstraction, then this provides an excellent clue in terms of starting place for effective technology use.

Two Parallel Examples.

Let us see how this model plays out for the case of a young child who is just beginning the process of acquiring basic number concepts. The initial task for the young child is to perform sufficient actions upon a foundational set of manipulative objects to develop a working vocabulary for later use. This vocabulary must include the terminology used for the manipulative itself, relevant properties of the manipulative, and the canonic problems to which the manipulative is used to explore.

Consider the Dienes $Base_{10}$ blocks as an example. Typically a child begins by using the blocks to build with – just as with any other set of building blocks. Through carefully guided activities the young child will come to explore more of the mathematically relevant properties of the blocks and begin to assign the appropriate terminology to them.

Thus, the smallest block is soon recognized as a unit. Building from this basic foundation they learn that it is this unit that we count when using the $Base_{10}$ blocks. This is the set of primitive objects serving as the source for later representations within this system of modeling. From this beginning other vocabulary relating to the blocks is carefully developed, such as Hundreds Flat, Tens Rod, and Thousands Cube.

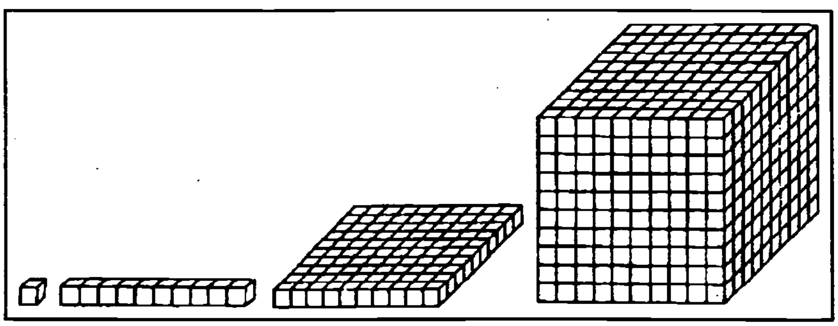


Figure 1: Traditional manipulative - Units, tens, hundreds and thousands Dienes $Base_{10}$ blocks.

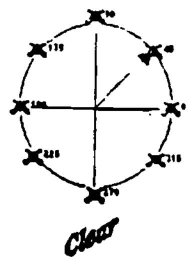
With this vocabulary in place, experiences are designed to explore the relationships between the numbers represented in these objects to think with and problems are posed which require the child to consciously and strategically act upon these objects in order to solve. The child will then pose problems of their own, which will end up involving further actions that the child will perform upon the primitive objects that they have been working with.

To see the parallelism of approach this method incorporates, let us contrast this example from early elementary mathematics with one from introductory trigonometry[2]. First of all, the objects these students are thinking with are quite a bit more abstract. Let us look at a technology-enhanced Sketch object designed to allow exploration of the sine and cosine function via the unit circle. Just as was done with the young child we begin by developing a working vocabulary relevant to a developmentally appropriate object – in this case derived from examining selected points along a unit circle.

This Sketch object is indeed more sophisticated and is best thought of as a dynamic sketch, due to the changing nature of it's constituent parts. Despite this increased sophistication, however, we can observe the same pattern of thinking as the student uses it in exploration of their developing notions on trigonometric functions. The student must first acquire a working vocabulary regarding the object and some of its properties prior to any meaningful questioning or problem solving to take place. Careful examination of the unit circle

[2] Thanks to Mr. Robert Crotteau for permission to include these figures.

shows that there are two colored lines that are formed each time a ray crosses the unit circle at any particular angle, their lengths corresponding to the sine (shown in green) and cosine (shown in red) of the angle which is used. As the student explores various angle combinations the associated sine and cosine are displayed in both numeric and graphic form providing multiple representations for the same concept.



Theta	Theta	Cos Theta
45	0.7071	0.7071

Figure 2: Technology Enhanced Sketch – An active unit circle object from trigonometry.

Just as was done with the Base10 blocks, once this vocabulary is in place experiences are designed to explore the relationships between the functions represented in this technology-enabled object to think with and problems are posed which require the student to consciously and strategically select actions using this object in order to solve. The child will then pose problems of their own, which will end up involving further actions that the child will perform upon the objects in order to solve. This same sequence may be likewise followed for the sketch and mental picture levels of the model.

What should occur next, regardless of the students' developmental level, would be for a skilled teacher or instructor to pose follow-up problems or questions relating to the newly instantiated and defined object of thought. This would again hold true whether we're talking about a physical manipulative object, a sketch object of predictive power such as an interactive fractions object, or a mental picture object such as a comparison of mass based upon remembrances of experience, or a formal and logically abstracted object such as function or some other mathematical construct. In each case we observe a skilled teacher using newly developed objects as a venue within which questions are to be asked and problem situations explored via student actions upon these very same objects of thought. As these examples serve to illustrate, these objects of thought become the basis upon which later mathematical thinking occurs.

Summary

The type of technology enhancement illustrated within this paper has the potential to significantly alter the nature of the mathematics classroom. This goes far beyond simple changes in content scope and sequence. The very nature of what counts as a mathematical thought must be examined when technologically enhanced active objects are a part of the learning environment. If we are to successfully prepare teachers to be comfortable within this new environment it strikes me as essential that we go beyond typical preparation programs, which for the most part use technology to present information. We must model the creation of active objects in our own methods classrooms if we expect to see them in those of our teacher candidates.

I have found that this is nowhere near the ordeal that many would suggest. What it requires, however, is a willingness to experiment with foundational representations to identify those which best benefit from technological enhancement. Although much more research needs to be done concerning such *objects of thought* might play it is increasingly clear that if we are to be successful in the mathematics reform effort they must find a home in our methods courses – and in the classrooms of our teacher candidates.

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Exploring Whole Number and Rational Number Division Within a Computer-generated Conceptual Domain

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Abstract: In this preliminary report of work in progress we discuss a prototype computer-generated conceptual domain designed to enable learners to explore various visual and symbolic representations of whole number, fractions, and decimals, and other fundamental differences pertaining to division with regard to whole number and rational number arithmetic. Feedback from prospective teachers were included as an integral part of the design process. Other instructional design considerations regarding developmental methods, platform limitations and constraints, and future directions for improvement are discussed.

Introduction

The difficulties learners have in understanding procedural and conceptual connections and differences between whole numbers, fractions, and rational numbers is well known (e.g., Mack, 1995; Markovits & Sowder, 1991; Resnick et al., 1989; Silver, 1988). Potential causes and solutions to these difficulties usually focus on the semantic or informal experiential, concrete, contexts in which arithmetical equations can be more meaningfully formulated as opposed to focusing on the purely formal conceptual relationships expressed in arithmetical equations with respect to the particular numerical domains involved. Previous work researching preservice teachers understanding of divisibility and multiplicative structure, however, found that many students do not have a meaningful conceptual grasp of the formal differences between either whole and rational numbers, or between whole and rational number arithmetic (Zazkis & Campbell, 1996). These difficulties where particularly evident with respect to understanding whole number division with remainder (Campbell, 1996).

In this paper, we draw on Greeno's notion of number sense as situated knowing in a conceptual domain (Greeno, 1991). The basic idea is to construct a spatially-oriented environment using computer graphics that presents the learner with a variety of inter-related visual and symbolic representations of whole numbers, fractions, and decimals, along with some interactive tools for working with those representations that collectively serve to accentuate connections and differences between them. The central thematic of the environment was to render visible the division algorithm -- a fundamental theorem of elementary number theory that provides the formal basis for whole number division (as most mathematics educators tend to think of the "division algorithm" as referring to an algorithm for long division, we will henceforth refer to this theorem as the "division theorem" in order to avoid any confusion). As will be illustrated below, the environment we are designing is intended to take advantage of the division theorem's potential for illustrating procedural and conceptual differences between whole number and rational number arithmetic (Campbell, 1997).

This is a work in progress. Here we offer a preliminary report on design criteria, with particular emphasis given to the guiding design thematic of a transition model between whole number and rational number arithmetic that is based on the division theorem defining whole number division. We will then briefly discuss constraining features of the development platform, some aspects of the development process and the role of learners in that process and some ideas on how that role can be optimized. We will conclude with a critical assessment of the prototype, and our ideas for future directions with this environment.

The prototype software, developed on a Macintosh platform using Real Basic, can be downloaded at:

http://www.gse.uci.edu/doehome/Deptinfo/Faculty/Campbell/divfact.html

Guiding Thematic: A "Transition" Model for Whole and Rational Number Arithmetic

It often comes as a surprise to learners and teachers alike to realize that whole number arithmetic is fundamentally different from rational number arithmetic. This is principally due to the fact that division is not closed over the whole numbers. That is to say, there are cases, such as 10÷3, where there is no whole number solution. The surprising part is that this entails completely different definitions of division for whole number and rational number arithmetic. The part that is not so surprising is that if one is not clear about these differences, many problems relating to the conflation of whole and rational numbers, between whole number and rational number arithmetic can result (Campbell, 1996, July). This prototype has been developed to help learners explore, discover, and make procedural and conceptual connections regarding these fundamental differences. To avoid ambiguity in what follows, capital letters are used to designate whole, non-negative integer, number variables and small letters will be used to designate non-negative rational number variables

The formal basis for whole number division is a fundamental theorem of number theory is *the division theorem*. According to this theorem, for any whole numbers, A and (non-zero) D, referred to as the dividend and divisor respectively, there exists ***unique*** whole number quotient, Q, and remainder, R, where $0 \le R < D$, such that $A=QD+R$. Most learners are implicitly familiar with the division theorem, in so far as it is used iteratively in long division and as a means of "checking" the results thereof.

Rational number division, when thought of solely in fractional terms, may be more intuitive than whole number division, but conceptually it is much more involved. Formally, rational number division depends on the fact that for any (non-zero) rational number, d, there exists a unique multiplicative inverse, or reciprocal, d^{-1}. In dividing a dividend, a, by a divisor, d, the quotient, q, is defined as the product, ad^{-1}. The quotient can be expressed in a number of equivalent ways: $ad^{-1} = a/d = a \div d = q$; or implicitly, in an equation more closely related to the division theorem and divisibility, $a = qd$. *There is no such thing as a remainder in rational number division.* Instead, as is characteristic of rational numbers, the quotient has a decimal, or *fractional component* (possibly zero) in addition to a whole number, or *integral component*. The relationship between remainders and fractional components is not always evident to learners (Campbell, 1997).

The division theorem offers an effective starting point from which to illustrate the transition from whole numbers to fractions to decimals. Consider the following derivation:

$A =$	$QD+R$	whole number division:	where $0 \le R<D$	(1)
$=$	$QD+R(D/D)$		where $D/D = 1$	(2)
$=$	$D(Q+R/D)$		distributivity	(3)
$=$	Dq		where $Q+R/D = q$	(4)
$A(1/D) =$	$Dq(1/D)$		where $X=Y \Leftrightarrow X(1/D) = Y(1/D)$	(5)
$A/D =$	q	rational number division	where $D^{-1} = 1/D$	(6).

This derivation highlights the central role of arithmetic division in making some of the fundamental procedural and conceptual shifts involved in numerical transitions from whole numbers to fractions to rational numbers. More specifically, step (1) is essentially the division theorem defining whole number division. By definition it serves to exemplify the fact that whole number units cannot be divided. Step (2) remains implicit in the prototype as it currently stands, and yet it is both subtle and important to consider in understanding the transition from whole numbers to decimals. Here is an example of where learners' familiarity with whole numbers may be brought to bear on understanding fractions and rational numbers more generally. Conceptually, it seems to be much easier to go from whole numbers to fractions than it is to go from whole numbers to decimals. This is because the representational differences between the former are more intuitive than the representational differences between the latter. Consider, for instance, the difference between "1/8" and ".125". Moreover, the transition from fractions to decimals is much easier to make than it would be to jump straight from whole numbers to decimals. Consider, for instance, that ".125" is readily interpretable as "125/1000". Placing initial pedagogical emphasis on D/D, say in contrast so something like 1/D, helps to place emphasis on

the division of a whole number unit, rather than on a previously divided part of that unit. That is to say, there is an important way in which D/D is both logically and intuitively prior to 1/D. Step (2) is also important in that it is a necessary precondition to the distributive step (3). It has been shown that distributivity can be problematic in a wide variety of ways for prospective teachers (Campbell & Zazkis, 1994, November). Nevertheless, the importance of this step in the transition model is crucial, in that it results in a form from which the integral and fractional components of the rational quotient of step (4) is readily identifiable. The transition from step (3) to step (4) marks a clear transition from the concept of fractions to rational numbers. Of course, one could resist, or postpone, this step and consider the rational quotient as the improper fraction (QD+R)/D. One way of so doing would be to apply steps (2) and (3) once again, only this time applying the identity (D/D) to the whole number quotient Q and then distribute out (1/D). Either way, the next step would be to isolate the rational quotient, and this would require multiplying both sides of the equation by the proper fraction, 1/D, in step (5). Finally, the crucial property of every whole number except zero having a multiplicative inverse in the domain of rational numbers, and the product of a whole number and its multiplicative inverse being unity, is exemplified in step (6). For the sake of expedience, other important relations such as commutativity and the more general concept of a multiplicative inverses of arbitrary non-zero rational numbers have either been left implicit in this analysis or simply fall outside the scope of the transition model. Also note that one can move logically either forwards from whole number division to rational number division or backwards from rational number division to whole number division. Pedagogically however, moving forwards through the transition model, as illustrated here, may be the preferable approach. Be this as it may, the transition model offers a rich venue for unpacking a wide variety of procedural and conceptual differences between whole numbers, fractions, decimals, and between whole number and rational number division. It is this characteristic that makes it such a rich thematic guide for designing a computer-generated conceptual domain for learners to explore.

Guiding Thematic: Visual and Symbolic Aspects and Related Factors

Using the transition model as a guiding thematic for the development of a computer-generated conceptual environment provided part of the original motivation of this project. Another guiding thematic of this project has been to provide an environment whereby learners are provided with a number of tools for exploring relations between the visual and symbolic representations. What exactly is involved in this and how to do it effectively is a deeply problematic. A number of factors pertaining to the relationship between visual and symbolic aspects of mathematics and mathematical cognition have been implicated from a wide variety of disciplines ranging from philosophy (e.g., particular and general), mathematics (e.g., geometry and. arithmetic), logic (e.g., sense and reference), computational theory (e.g., procedural and declarative), psychology (e.g., concrete and abstract), and linguistics (e.g., object and attribute) (Campbell, forthcoming). These factors pertain directly to the learners' understandings of the relations between visual and symbolic aspects of mathematics and mathematical cognition. There are, however, many other design factors involved regarding the environment in and of itself. Particularly regarding the very idea of a computer-generated conceptual environment. To what extent should one attempt to anticipate the kinds of things that learners might or might not do when they are actually engaged within such an environment? To what extent is it even possible? Such design issues are intimately bound up with assessment issues which will discuss in more detail below. We will now say a words regarding design with respect to the development platform and the developmental process.

Development and Delivery Platforms: Design Constraints

Instructional design is always going to be expedited or limited by what is possible with the hardware and software constituting the development and delivery platforms. Fortunately, as computer technology is rapidly evolving, these factors are placing fewer and fewer constraints on instructional design while opening up more and more possibilities. It is unlikely that one can ever be completely free of such constraints. We will discuss some factors that have limited our design. Predominantly due to the existing Apple cultural environment that we are in, the development and testing of the prototype software developed on and geared toward that delivery platform. Unfortunately, however, we have yet to find a robust programming language for Rapid Application Development (RAD) with the possibility to develop a Graphic User Interface (GUI) in the Macintosh environment. The only high level object-oriented programming language with the characteristics mentioned

above that we have found is Real Basic (RB). This is a relatively new implementation still in development with many similarities to Visual Basic from Microsoft in the Widows environment -- which constitutes [the latest] a more robust programming language. However, the alternative to use an object-oriented language was very attractive because we included in our design objectives the possibility to construct a virtual scenario in the computer where the students have an interactive and visual environment to represent whole numbers. Numbers can be manipulated using scroll bars and can be represented in spatio-geometric form as unitary squares (area) and as non-unitary rectangular products. Using the visual characteristics of the environment the learners can make changes in the dividend or the divisor and then see instantly their graphical representation (of these two values) and also the values of quotient and remainder not only as numerical representation but also graphically.

There are, however, ways in which RB is limited with respect to VB. These limitations are particularly evident with the inflexibility of RB in the handling of the screen. We will discuss one particularly bothersome example: refreshing of the screen after it has been hidden by other windows and in the handling of the z-order of graphics -- i.e., the order in which several graphics appear one on top of the other. In other words, if the screen is in the x-y plane, several graphics can be represented as occupying layers in the z-axis.

We developed a "trace" function that requires overlapping windows, so that the effect of different divisors with respect to a given dividend can be explored. The dividends and divisors are constrained, by design, as whole numbers, and the divisor is constrained to be less than or equal to the dividend. When a given divisor divides the dividend exactly we have used an areal graph -- one that displays the product of the divisor and the resulting (whole number) quotient -- that is colored blue. When the divisor does not divide the dividend exactly, then the areal graph that displays the product of the divisor and the resulting (rational number) quotient is colored red. When using the Trace function over several such configurations, the areal graphs, to some extent, overlap upon one another. We can easily trace either by increasing the value of the divisor or by decreasing it. The order (z-order) in which they appear, unfortunately in this particular case, affects the way the resulting information is displayed on the screen to the learner. For the case in point, if one uses the trace function when decreasing the divisors, then the overlapping areal graphs indicate which values of the divisors that divide evenly into the dividend very well. Unfortunately, they are not so clearly indicated when tracing proceeds using an increasing divisor. If we could manipulate this order (which in RB is not possible) we could provide much more consistent picture of the configurations, irrespective of the particular order in which they are explored by the learner.

Development Process: Learner Interaction, Assessment, and Feedback

All these design factors, whether they are originate in the minds of the designer, the learner, or the computer system, will always be, to one extent or another, bound up with what learners do within the environment. Prima facie, there are a number of different ways of assessing this. For instance, designers can rely on occasional verbal or written feedback from learners regarding their experiences or from teachers reporting on their observations regarding their students experiences. Other approaches involve various ways of having the computer keep track of how students use the environment, or to use audio-video equipment to record learners' interactions, possibly in tandem with various kinds of "talking aloud" protocols. We are currently developing more rigorous approaches along these lines (see Campbell, forthcoming). If all goes well, we will be able to report in more detail on our data and methods when we present this paper. Here, we will just say a few words about how we have developed the prototype as it currently stands.

The authors have brought complementary backgrounds, interests, and skills into this project which has proved beneficial to this collaborative effort. The first author conceptualized the guiding thematics of the transition model with an eye toward developing an environment suitable for both teaching and learning arithmetic and for further research into the relation between visual and symbolic aspects of mathematics and mathematical cognition. The second author has a strong background in educational software design and development. These complementary orientations regarding design and development gave rise to a important synergy that led to rapid implementation of an initial prototype suitable for classroom use.

This prototype was then introduced in undergraduate mathematics education course called "Thinking Mathematically: Learning and Teaching Mathematics" -- an undergraduate course designed by the first author for prospective teachers in mathematics. The guiding thematic of the course was essentially the same as for the

prototype: to explore ways of visualizing elementary arithmetic in ways that were conceptually meaningful. Around the middle of the quarter, the students worked with the software for about an hour and half, while remaining focused on how the environment was contributing to their conceptual understanding of whole number and rational number division. The students reported on their insights and difficulties in using the environment, and the software was adapted and refined wherever possible in accord with those reports for the next class. The students were then able to use the environment again and explore first-hand the impact of their feedback. In this way, they were able to contribute to the evaluation and design process. In what follows we will discuss a few of those contributions.

In the original prototype, there were two symbolic equation schemas that allowed for numerical instantiations immediately beneath them depending on the values selected for the dividend and the divisor:

$$A = Q * D + R$$

$$A/D = Q + R/D$$

Many students experienced significant difficulties in relating these values to the visual graphic representations. Particularly the one based on the product of the quotient and divisor. The revised prototype included the following equation schemas, again with numerical instantiations included immediately beneath each one:

$A = Q * D + R$ (with the explicitly noted constraint that: $0\ R < D$)

$$A = D(Q + R/D)$$

$$A/D = Q + R/D$$

With the revised prototype students were more successful in linking visual with symbolic representations. Other factors likely implicated with this success may have related to greater familiarity and overall involvement with the environment. In the upper right of the revised (and as of this writing, the current) prototype is a bar graph of each of the values of the division theorem from left to right. In the original prototype the bar graph was "top-down" in scale rather than the more usual "bottom-up" bar graph representation. Some students found the "top-down" graph distracting or disconcerting. They felt much more comfortable with the standard "bottom-up" representation.

In the original prototype, only changes in the divisor were possible for a given dividend. Some students felt unduly constrained by this, so the prototype was revised so that the dividend could be changed for a given fixed divisor, thus adding greater user-flexibility to the system.

In the revised version the remainder was also represented as small blue dots in the Quotient-Divisor graph. These dots provided a good focus task for the students. Many of them worked individually or in pairs, trying to see some connection. Gradually, the connections were made. Two students finally conceded that they could not figure it out and asked their neighbors. Once they were informed that they related to the remainders, they instantly made the connection. It seems that they had not expected yet another representation of that value, and thus were blocking themselves, a priori, from considering it as a possibility.

Overall, the students' first exposure to the original prototype allowed them to feel as though they were actively participating in beta-testing and de-bugging. Surprisingly, the students seemed to enjoy this role, rather than be dismayed by it, especially when they were able to see that many of the problems that they pointed out had been fixed in the revised version. Some of these bugs and suggestions for improvement included things like:

a) No windows refresh after hidden by another window.
b) No Restart function after closing the main window of the program
c) Suggestion to include up to 100 numbers in the dividend
d) Introduce cross hairs or some form of indicator as to the current operation in the graph

Critique and Future Directions

We think that the prototype it is still unclear the transition from whole numbers to real numbers (with decimals). There is much more that can be done to help bring out visual meanings that remain implicit within the symbolic representations, as evidenced by the analysis above. Perhaps a graphical representation of those decimals could be more appropriate adding a new window (floating) to illustrate that. In this window a magnified unitary square could be depicted to show fractions of it.

Other improvements can be made regarding more familiar representations of division with remainder. In particular, whole number division is often expressed as A÷D=QremR. Some caution must be exercised here. Note that this "equation" is imprecisely formulated and *does not* conform with standard arithmetic meaning and use. Some learners may be prone to operating on this common expression of whole number division as if it were a well formed arithmetic formula.

The software could be refined (despite current limitations of Real Basic) in the following ways:

a) Including the floating window mentioned above to enhance the "vision" of decimals
b) Shows in a notorious way the presence of prime numbers
c) Increase numbers up to 100
d) Redesign of the color bar graph for more meaningful information
e) Define three independent levels of difficulty for the division theorem: (1) including only whole numbers; (2) including real numbers; (3) including prime numbers.
f) Include a "help" service in the main menu for some directions
g) Include a "print" service for hard copies
h) Possibility to include a "freeze" function to insolate desired configurations
i) Work on the suggestions and difficulties (above) manifested by the students.

We will be running additional tests with the current prototype with preservice teachers and making further refinements prior to presenting this paper at the conference and hope to have a fuller report available then. One of the areas that we hope to gain much greater insight into between now and then is in how preservice teachers navigate their attention within the environment. Toward this end, we will be videotaping the screen during some of their sessions using talk-aloud and cursor-pointing protocols and analyzing the results using video-processing software. We will be presenting some of the preliminary results of these experiments at the meeting.

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Serendipity in Interactive Mathematics: Virtual (Electronic) Manipulatives for Learning Elementary Mathematics

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Abstract: The authors are co-principal investigators for a National Science Foundation project to create a web-based National Library of Virtual Manipulatives for learning mathematics in the elementary grades (K-8 emphasis). Many of our virtual manipulatives are based on physical manipulatives commonly in use in the schools (i.e. geoboards, tangrams, pattern blocks, fraction bars, etc.); others are concept manipulatives especially designed to teach or reinforce basic mathematical concepts. Our emphasis is on interactivity for the user, so the learner controls the variable aspects of the manipulative and is not only free, but encouraged, to explore and discover important mathematical principles and relationships. Teachers or parents can provide direction, but control of the activity remains with the user. This paper is preliminary and descriptive only, but it describes aspects of our experience in the design of manipulatives and some of the delightfully unexpected advantages of interactive technology for discovery learning.

Introduction and History

This paper must partake of some aspects of self-contradiction, intended as it is to describe in words some of the advantages of not using words. Our goal is to convey some of the unexpected, or at least unanticipated, consequences of interactive software in the educational experiences of both elementary school children and their teachers, both pre- and in-service. Since we have no choice but to rely on words in proceedings such as these, we will do what we can using verbal descriptions, supplemented by static figures. At the same time, however, we invite readers to visit our web site (www.matti.usu.edu), where it is our hope that they may experience for themselves some of the things we describe.

The authors of this paper have a history of working with interactive computer-aided mathematics instruction that predates their current National Science Foundation funded National Library project. The first two authors earlier received funding from the state of Utah to develop an interactive pre-calculus project for use in the state system. We learned early on, however, that despite rather extravagant claims to the contrary, there were no commercially available tools adequate to support the kind of interactive mathematical experience of the sort we envisioned. It was our extreme good fortune to add to our design team a colleague (Dr. Wellman, also in mathematics), whose interest in computer programming led him into the creation of a mathematics editor and related tools sufficient for our purposes.

With the completion of our pre-calculus project, we recognized the potential of our approach for both elementary students and for pre-service and in-service elementary teachers. That potential was magnified by the growing possibilities of a new programming language, Java. It was the Java language that has really made web-based instruction possible.

Java has many advantages over other object-oriented programming languages, especially C++. One of the most obvious—and critical—advantages is multi-platform and web accessibility. But while Java improves on a number of features of C++, it does not have the maturity of some of the older programming languages or the extensive libraries of built-in functions readily at hand. It has been necessary to build many of our own fundamental classes and our own mathematics editor to accomplish some of the things we considered essential to our long-term goals.

Our Utah State University team, in collaboration with a colleague from Fayetteville State University (North Carolina), proposed to the National Science Foundation that we build a National Library of Virtual Manipulatives, interactive applets for learning and reinforcing concepts in elementary mathematics. We intended our applets for the use of elementary (primarily K-8) students, but recognized that the same instructional tools could also serve to strengthen the mathematical preparation of pre-service and in-service teachers.

In the process of our negotiations with the National Science Foundation, it was suggested that we communicate with people working on the new Principles and Standards for School Mathematics ("Standards 2000") for the National Council of Teachers of Mathematics (NCTM). The NCTM had released their original Standards document in 1989 and recognized the need to update their national guidelines for the teaching of K-12 mathematics. As part of the planned publication of the new standards (which also address changes in technology) an Electronic Format Group was established to create a web-version of Standards 2000, including a new approach, "E-examples," to suggest and illustrate ways to incorporate technological resources into the mathematics classroom.

At the 1998 International Conference on Technology in Collegiate Mathematics, we demonstrated to some members of the NCTM Electronic Format Group some prototypes of the kind of virtual manipulatives we had developed for our proposal to the National Science Foundation. The upshot of that meeting has been a very productive collaboration. We are now working very closely with the Electronic Format Group and the writers of the new Principles and Standards to provide appropriate E-examples for all four grade bands. The majority of E-examples that will appear with the web version of the Standards, to be released at the annual national meeting of NCTM in April of 2000, will be our interactive applets.

In addition to productive exchanges with writers and teachers associated with NCTM, we have had the opportunity to work with several groups of in-service elementary teachers. In both web-based distance instruction (in Utah and in North Carolina) and in on-site classes (in North Carolina and Ohio), we have used our virtual manipulatives as part of our technology-supported mathematics courses. Participating elementary teachers in our current North Carolina project are designing ways to incorporate our materials into their curriculum and will conduct in-service efforts in their home school districts to make these resources available to elementary students throughout their districts.

Design Philosophy

Initially, as we contemplated constructing electronic manipulatives that could be used by children, we were guided by existing physical manipulatives. We were confident that we could make electronic versions that, because of their residence on the Internet, could be made available to students, teachers, and parents, at any time and in any location having a web-connection.

One of our first creations was a virtual geoboard, mimicking the common "nails in a board" version using rubber bands. As we shared an early version with teachers and asked for suggestions, several teachers said how nice it would be to be able to color the regions inside the rubber bands. So we did it. Later reviewers wondered if it would be possible to translate an elaborate construction around the board without having to shift the band from every vertex. We are now working on incorporating both translations and rotations, illustrating the mathematical concepts of "slides" and "turns."

Building such features into a virtual manipulative allows users to do things that aren't conveniently possible with corresponding physical manipulatives. We constructed a circular geoboard and another one using "nails" spaced to form an equilateral triangular grid instead of squares. The circular board has immediate applications as diverse as creating pie-chart fractions and illustrating trigonometric functions. But in the design of all of our virtual manipulatives, we try strenuously to avoid doing too much. We would rather have five manipulatives, each doing a well-defined task, than a single applet that requires more complicated operations to accomplish the same five tasks. Each applet has a tightly designed focus and the simplest interface we can create. The mathematics underlying the functionality is often very sophisticated; what the user sees and does is very simple.

Another principle that guides all of our design is that the student must interact with the applet to accomplish something. There is never simply a "watch this clever animation" or "see what happens when we do this" attitude. Mathematics, perhaps more than any other discipline, cannot be learned by watching someone else do it, no matter how elegantly. Any student, to be successful, must be involved, engaged in

the activity. And thoughtful engagement requires participation. Interactivity is thus essential to the design of every one of our applets, the one feature that we absolutely require.

The kind of interactivity we have in mind requires the user to think about a specific task, to formulate strategy to achieve a specific goal (almost always informally, and seldom articulated), to engage in some physical action (clicking to select something, dragging an object, or moving a slider), and to observe (and perhaps, describe) consequences of the action. The goal is to allow students to control events and to discover relationships. Differences between coincidence and causal relationships become clearer when we allow the user to repeat an activity as many times as desired. We can ask questions to direct explorations and, we hope, guide meaningful discovery, but control remains in the hands of the user. Nothing happens until the user takes action, and an activity can be repeated until there is satisfaction; the computer never tires of repetition.

Unexpected Consequences

With NSF support for our three-year project to build a National Library of Virtual Manipulatives, we could begin to focus our effort to accomplish some of our objectives. But even with early prototypical examples, before we had any formal structure to our project, we were learning something of both the limitations and advantages of working in an electronic environment and more specifically within the virtual machine of Java programming.

It is impossible to convey in words an accurate picture of what can be done on-screen with our interactive applets, so we compromise by giving a few pictures of some of our applets, simple snapshots of the kinds of screens produced by students actually working with these manipulatives. We will accompany each figure by a description of some of the functionality of the applet and some of the dynamic features available to the user. We hope thereby to communicate at least a little of what we have discovered in both the design and implementation of some of the virtual manipulatives.

Again, we invite interested readers to visit our web site (www.matti.usu.edu) and to explore freely. Everything is, of course, in a state of continual flux as we get feedback and suggestions from teachers, students, and evaluators. Applets already on the site are also changed as we build tools to support one particular concept applet and learn that such a tool would add value to another.

The serendipity to which we refer in the title of this paper is of two varieties. One is unexpected responses or capabilities that an electronic manipulative possesses in contrast to the physical model that inspired the computer implementation; the second occurs when we are led to unanticipated design changes that enhance the educational value of a virtual manipulative. We will try to illustrate both.

Figures 1 and 2 show two screen shots of a virtual manipulative on the Platonic solids. To help develop spatial visualization, we wanted students to be able to see all sides of a solid object. What neither figure can show is that by moving the mouse the student can rotate the given object freely in space. In future variations of this applet, we will paste different images onto the sides of a cube and ask questions about, say opposite faces. But as we worked with this applet, we realized that there was an opportunity to let students discover Euler's relationship among vertices, faces, edges ($V - F + E = 2$). As the user rotates the solid in space, a Shift-Click changes the color of a face, an edge, or a vertex. In Figure 1, we have changed the color of five faces with their surrounding edges and corners. The color changes make it easy to determine when you have counted everything. In the implementation shown, there is a running tally of the numbers, which the teacher can either choose to show or not.

One of our most exciting experiences with this applet took place during a visit to several inner-city schools in Cleveland. The settings and circumstances were less than ideal, but we took CDs containing some of our materials to leave with the teachers, some of whom we had worked with in an earlier NASA project. We took a projector and a lap top computer into the classroom and let the children (first, second, fourth and fifth graders) take turns with the mouse, rotating objects, selecting and changing colors while the entire class kept a (collective verbal) running count. The sense of ownership—and excitement—felt by the children as they controlled the magic of rotating the image and changing colors was palpable.

Figure 2 illustrates both kinds of serendipity we mention above. In the original discussions we had not thought of illustrating the idea of a dual of a solid (taking as vertices the mid-points of each face), but after seeing the solids in space, we serendipitously realized that we had another teaching opportunity. And, as may be seen from Figure 2, the electronic setting (using some fairly sophisticated mathematics) allowed us to show something that would probably never be constructed physically.

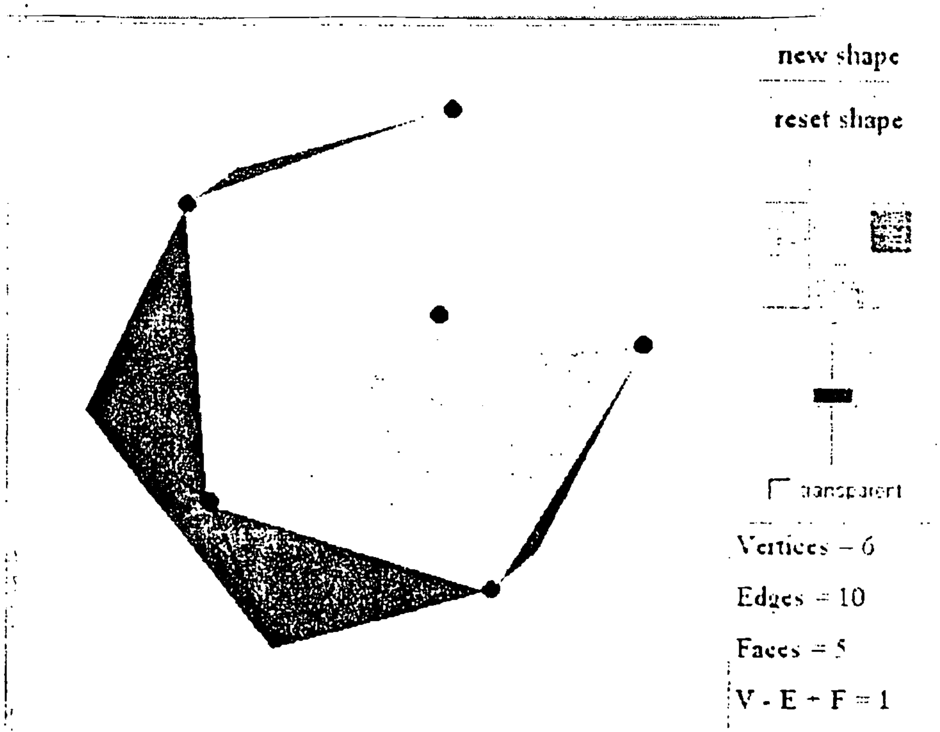

Figure 1. An icosahedron, showing the selection of some faces, vertices, and edges.

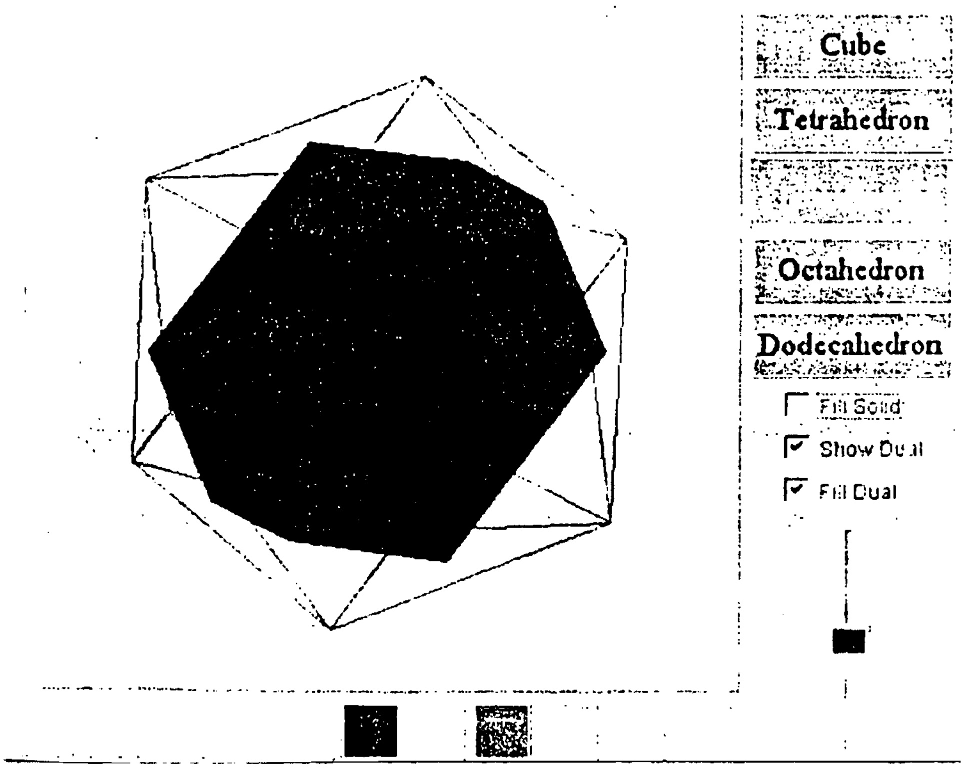

Figure 2. A transparent (wire-frame) icosahedron with its dual, a dodecahedron.

Figures 3 and 4 together also demonstrate both kinds of serendipity. The virtual geoboard in Figure 3 is "standard" in the sense that the nails or pegs form a square grid, as do most physical geoboards, although few boards have as many nails as ours. With the added size, the electronic version makes it much easier to explore such concepts as similarity, transformations, or symmetry. The smaller physical geoboards have that built in limitation. As mentioned above, it was teachers with whom we were working who suggested the addition of color to geoboard regions. We thought it a nice idea, but we did not anticipate some of the advantages we have since observed. With color, the comparison of sizes of regions or geometric figures is much more natural. When we ask students to use their rubber bands to divide a square region into halves, challenging them to find ways that are different from the obvious one in the upper left of Figure 3, the color stimulates choices. In particular, it seems less likely that the solution in the lower right of Figure 3 would be as apparent without the use of color to put together the two outside bands to form a region matching the central band.

Having the standard spacing of nails with our first implementation of geoboard, we realized that a very minor change would allow us to keep all the functionality while illustrating a number of different mathematical concepts. Thus we constructed a geoboard with a triangular grid, leading to a natural tiling of the plane with equilateral triangles or regular hexagons, and we have two different geoboards making use of circular symmetry. Now, taken together, our set of geoboards permit a completely unanticipated flexibility and versatility. Figure 4 shows how one student constructed a pie chart to answer the question, "What is 1/3 of 1/4?" Again, the ability to add color greatly aids the visualization of the answer.

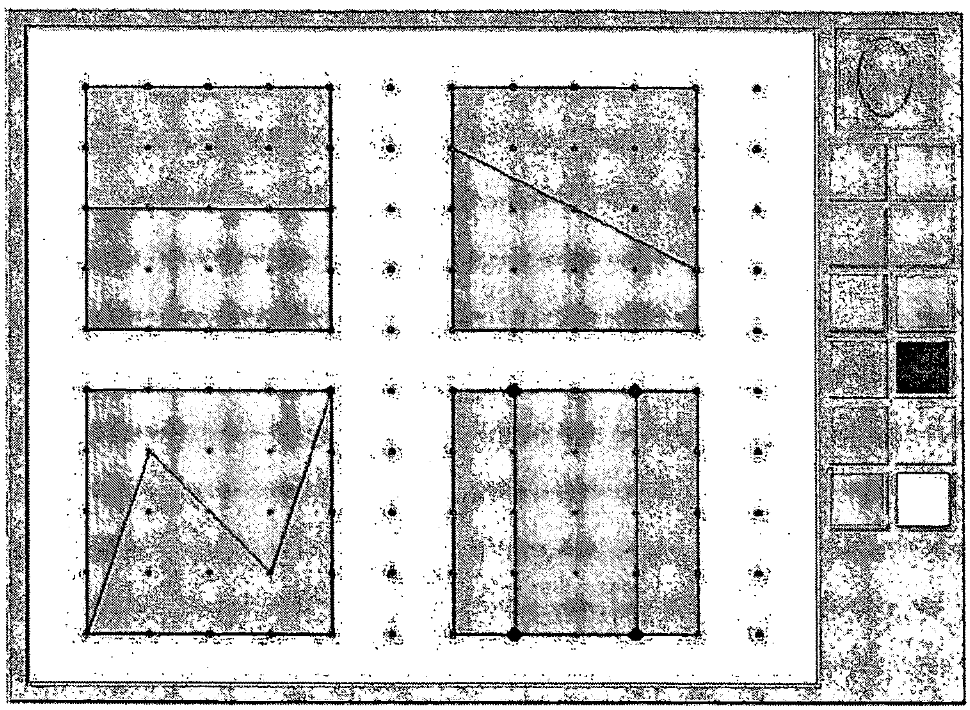

Figure 3. A "standard" geoboard illustrating different student pictures of "one-half."

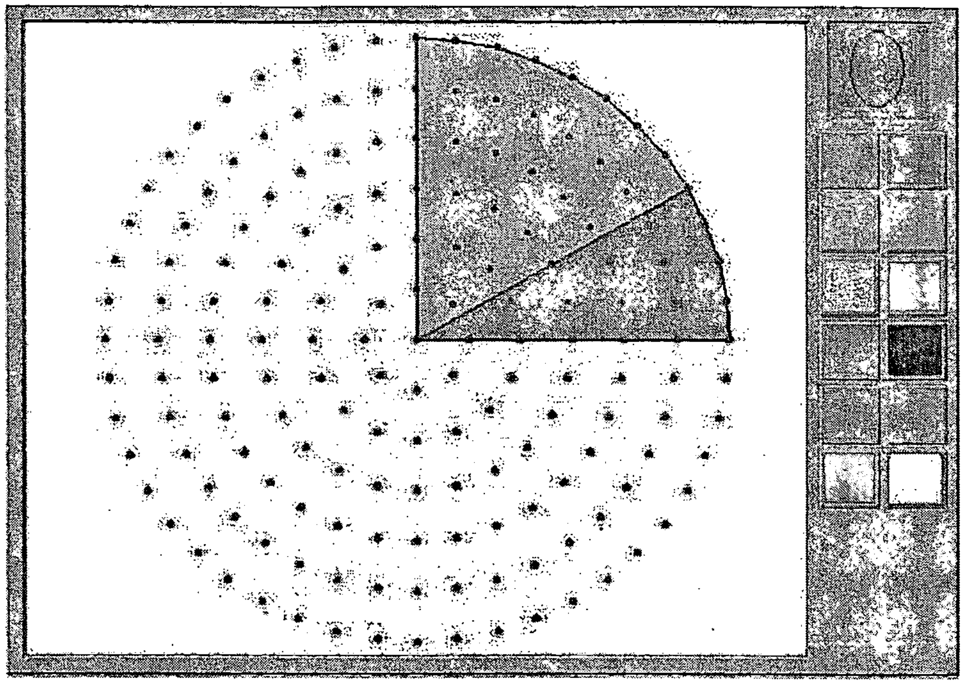

Figure 4. A circular geoboard, with a student's rendering of "1/3 of 1/4."

Conclusions

As is obvious at this point, we are submitting here only a preliminary report. This paper is intended to be neither technical nor definitive. In the future there will be a number of evaluation studies of the virtual manipulatives in our National Library. The summative evaluation for our project will be handled by Douglas Clements and Julie Sarama of the University of Buffalo. Given the direction of their previous educational research, Professors Clements and Sarama are uniquely qualified to assess the effectiveness of computer technology in early childhood education. Other teacher educators, including another of our co-principal investigators, James Dorward, will be examining the use of our materials in public school settings and plan comparative studies.

What we describe in this paper is purely descriptive, based on our own observations and information passed along to us from in-service teachers who are learning with and using our materials. However limited our observations, we have been delighted by the response of students actually using our manipulatives. The measurement of learning of specific mathematical content at different grade levels over an extended period of time is an important task that will extend into the future. But in our own development work, as we try to better understand, and address, the needs of various groups of students, the reactions have been wonderfully heartening. As typified by our visit to the Cleveland inner-city schools, the level of engagement, the enthusiasm, excitement, and participation of children will not soon be forgotten.

Using Web Pages to Teach Mathematical Modeling: Some Ideas and Suggestions

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Abstract: Mathematical modeling can perhaps be best defined as "the process of scientific inquiry" for mathematics. This is obviously a comfortable mode for teachers of science, but is rarely seen in the mathematics classrooms of today. This paper explores the possibilities of using interactive web pages to help facilitate an understanding of practical applications based mathematics. Because the scientific process is emphasized as the general operating framework, situations where students can hypothesize and experiment, and create data tables are most valuable. Special emphasis is placed on the fact that students and teachers both need to re-conceptualize effective mathematics instruction in order to really embrace a modeling approach.

Introduction

An important aspect of the continually changing reform movement in secondary level mathematics is that teachers are able to absorb and integrate what they have learned from both the classroom dynamics and from new research. It is perhaps most important that teachers of mathematics continue to grow with respect to the pedagogical techniques that have the greatest classroom potential. Although finding these techniques requires a great deal of effort, good teachers would certainly agree that the resources they bring to bear on behalf of their students set a foundation of success or failure for those students from both a competence standpoint and from a motivational standpoint as well.

The reform efforts of the past decade have resulted in a mass of professional documents, curriculum standards, and reports, all of which are intended to strengthen a teacher's profile of techniques. Yet with all of the various forms of assistance, the mathematics classrooms of the 21st century will probably look very similar to those that have been so common for the past 50 years. The fact that we know so much more now than we did 50 years ago, at least from a scientific standpoint, has appeared to have very little impact on what is taught or how things are taught in the secondary math classroom. Agreed, technology has brought flavor to the mathematics classroom, but the textbooks along with their very familiar format still seem to be the preferred method of instruction. Although there are instructional perks to this classroom format, the fact that students aren't internalizing the information would suggest that other formats merit exploration.

Instructional activities using a mathematical modeling approach have proven to be both effective and engaging for students. Additionally, some of the most valuable curriculum-application considerations in today's mathematics classrooms can be revisited in the context of an interactive web based format that preempts the "what do we need this for" question. The mathematical modeling approach to instruction is indeed a "front heavy" technique for teachers, but allows for the kind of valuable exploration in mathematics that has been absent to date.

Classroom Considerations for Mathematical Modeling

Mathematical modeling can perhaps be best defined as "the process of scientific inquiry" for mathematics. This is obviously a comfortable mode for teachers of science, but is rarely seen in mathematics classrooms. Students engaging in mathematical modeling activities would spend the majority of time experimenting in applied physical situations in an attempt to find patterns and consistencies in sets of data. Data sets could already exist in a number of different forms, or they could be collected as part of a classroom activity. Part of the impetus for mathematical modeling activities in the classroom is to help students understand that mathematics is not a discipline where complex solutions to problems are innately obvious or solvable in a matter of just a few minutes. In fact, any good mathematical modeling activity should be appropriately vague so that the students don't get the impression that the activity is just another textbook assignment. The teacher designing the activity has the difficult task of articulating the problem in such a way as to provide clarity without being too prescriptive. This is done to emphasize that mathematical modeling is a process of continual refinement and modification. In most cases, this process of refinement serves two distinct tasks. First, the refinements are intended to create a working model that is more efficient, faster, or more accurate in some way than any previous model. Secondly, refinement and modification are natural processes of building any axiomatic system of notation. Students in essence build their own mathematical system of notation and in turn, greater mathematical understanding. Some instructional considerations related to the use of mathematical modeling activities in the classroom are as follows:

1. Students have some control over how they approach a problem. This is not typically the case with most textbook problems.
2. Good modeling activities are adaptable to many different ability levels.
3. Good modeling activities are easily scalable to different grade levels.
4. Problem solving and mathematical modeling are different processes. Problem solving typically acts as a process oriented approach whereby students find a specific solution to a specific problem. Mathematical modeling is an experimental approach where a problem is solved and continually refined over time in order to be more efficient, faster, or more accurate. Problem solving in many cases has a solution that is either correct or incorrect. Mathematical modeling is a process where few answers are incorrect, they just require continual revision.
5. Mathematical modeling focuses primarily on the "general case." Students must at least generally understand the concept of a variable, which is why modeling activities below the fifth grade are difficult for teachers to construct.
6. Mathematical modeling activities are difficult to assess. An elegant solution may be an approach that works in a way that appears to be coincidental, but a student can justify why. Another solution to the same problem may utilize some specific procedure from the textbook, yet the student has no understanding of why they chose that method nor why it works.

The premise of the mathematical modeling concept is not that the traditional courses in the curriculum need to be replaced, but rather accented in the appropriate spots to better emphasize the practical use of the concepts we do teach. Because mathematical models can take on many forms, the processes by which problems are approached are numerous and varied. Some of the more basic modeling structures lend themselves very well to established secondary level curriculum (i.e. numerical tables and patterns, graphs, systems of equations, etc.). Others may be more algorithm-based problems that require a computer or graphing calculator as an extension. Although no one set of rules is inherent to all mathematical modeling activities, the following set of steps can act as general guidelines for students engaged in mathematical modeling activities:

1. Identify what the problem and resulting model should look like
2. Establish the factors that affect the outcome
3. Define which of the factors are parameters and which are outcome variables
4. Establish a relationship between the parameters and the variables to derive a formula or alternately defined model or algorithm
5. Test the model with known values from previously collected data
6. Refine the model for accuracy and efficiency

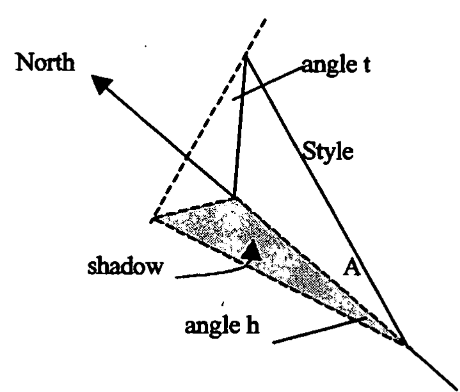
Using Web Pages for Modeling Activities

Although modeling activities in the mathematics classroom don't have to be technology driven, the interactive nature of Java applications on many web pages can provide a physical context which students can use to test conjectures and build generalizations. Because the scientific process is emphasized as the general operating framework, situations where students can hypothesize, experiment, and create data tables are most valuable. Well designed web pages using Java allow for the kind of interactive experiments needed for success without the hassle of setting up a physical lab situation. The following example illustrates a possible modeling problem that could be practically used on the web:

Problem

Suppose we wanted to find the time of day without using a clock. In ancient times, sundials were used for this purpose, and were fairly accurate. The first step in finding the time without using a clock is to use the relative movement of the sun and earth to predict how shadows might fall at different times of the day. Assuming that the meridian line (or noontime mark) has been established and the gnomon has been angled, we must find a way to mark the hour lines on the dial plate. Create a mathematical model that uses the angle of the sun on the style (top of the gnomon that creates the shadow) to mark hour lines on the dial plate of the sundial. Using angle A as the base angle of the gnomon, and angle t as the angle of the arc the sun passes through in a given time frame, we should be able to calculate angle h by using the length of the resulting shadow. This is illustrated in figure 1.

Figure 1: Shadow used to mark the dial plate



One Possible Solution

Angle t is perhaps the first angle that needs to be defined. Because the earth rotates through a central angle of 360° in 24 hours, we can assume that each hour is defined by a 15° arc of the sun's apparent movement over the surface of the earth. This is true at any longitude. Angle t then measures 15° for each hour away from the noon hour.

1. If the length of the style on the gnomon is known, the vertical side of the gnomon can be calculated as follows: Height = sin A (S) ; where S is the length of the style.
2. At 11:00, angle t = 15°. So, if we want to mark the 11:00 hour on the dial plate, angle h can be calculated by measuring the length of the shadow from the noon mark and subsequently estimating the length of the adjacent side as equal to the length of the style, so that the tangent ratio can be used. The side of the shadow opposite angle h = (tan t)(sin A)S.

3. Also, tan h = [(tan t)(sin A)S]/S since the tangent ratio is opp/adj and S is being substituted for the adjacent side in this ratio.
4. Therefore, our model could be as follows: $\tan h = (\tan t)(\sin A)$.

Because we know the longitude of our specific location, we also know the measure of angle A. Let us assume for the sake of easy calculations that we are at a 30° longitude, and that our style length is 8 inches. Because we are marking the 11:00 hour line, angle $t = 15°$. We need to find angle h for several different hours in order to mark the dial plate appropriately. The following is a test calculation:

1. $\tan h = \tan 15°(\sin 30°)$
2. $\tan h = (.268)(.5)$
3. $\tan h = (.268)(.5)$
4. $h = \tan^{-1}(.134) = 7.63°$

Therefore the 11:00 hour line would lie at an angle of 7.63° to the left of the meridian mark. Also, since the hour marks are symmetrical with respect to the noon mark, it is an angle of 7.63° to the right of noon for 1:00. We could continue in this fashion to mark the rest of the dial plate for each hour of daylight.

Other modeling activities would be used as primers to get the students to the point where they could successfully manipulate the web experiment in such a way as to define an answer. Because students and teachers both like self contained educational packages, much of what is provided on the web page can really help smooth out problems that might arise during the course of the activities. Other hints that create successful web experiments are as follows:

1. Help students define and modeling heuristic similar to that found on page 2 of this paper, and have this listed on the page as they progress through the activity.
2. Create an on-line hint button that directs students when they are off track. This may even be a step by step derivation of a sample approach to the problem.
3. Use data tables on the web page where students can enter the data from their experiments.
4. Provide a virtual notepad where their models can be entered.
5. Have specific objectives that the activity will highlight, but don't be afraid to deviate. Modeling activities sometimes lead us in directions we may not think of, but turn out to be valid solutions.

As in any educational activity, good planning and lesson design are key to successful implementation. It is important to remember that for every minute of planning on the front end, we save ourselves headaches during the activities themselves.

Conclusion

Of course the models being presented here take some time to develop and even more time for the students to research. Workload is perhaps the biggest obstacle when it comes to developing on-line math modeling activities. In addition, students and teachers both need to reconceptualize effective mathematics instruction in order to really embrace a modeling approach. Those students who are comfortable in an environment where a math problem can be done in a matter of a few seconds will need to embrace a more realistic view of "scientific" mathematics. In some cases, the students may not come up with the models that we have intended even after hours of work; however, through careful investigation, and with some guidance, students can learn many things that we haven't even thought about. Yet, it is important to remember that many of the greatest inventions of our times have been accidents. Mathematical modeling though provoking problems appears to be a great way to pave the road to accidental learning, and history has taught us that accidents can sometimes be good!

Technological Tools to Develop Mathematical Problem Solving

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Abstract: Problem-solving processes should extend beyond mere working problems by type where students are provided algorithmic approaches to fit situations (e.g., rate, mixture, coin, investment, work) since redncing these typical problem situations to "algorithmic processing" is counter-productive relative to higher-level problem-solving goals (NCTM, 1989,1991, 1996). By incorporating technological tools (CABRI Geometry II, spreadsheets, and graphing calculators) coupled with the problem solving principles espoused by George Polya (famous mathematician and teacher of mathematics and mathematics teachers), secondary school algebra problems can be taught as recommended by the NCTM curriculum standards to appropriately meet recommended problem-solving goals. Even typical problems can therefore be used to expose students to multiple problem-solving approaches that extend understanding and meta-cognitive abilities.

In the quest to teach Atype@ problems as genuine problems versus exercises as defined by Baroody (1993), technological tools, when appropriately used, bolster opportunities for students to think mathematically. For example, using computer and graphing calculator simulations and spreadsheets, even with typical problems one can expose students to multiple approaches that extend cognitive abilities to learn mathematics concepts, discover patterns, detect relationships between variables, and to apply processes and strategies to explore systems and structures of mathematics. By specifically utilizing George Polya=s (1957) four-step problem solving model and appropriate technological tools, typical Arate@ problems can be explored and solved so that students further their intuitions and knowledge about the interrelationships between mathematical ideas, thereby fostering the learning of problem-solving skills in a responsible and enriching manner.

Paraphrased, in the problem to be solved, entitled **AThe Biker and the Nearby Town Problem@** shown on the next page the goal is to find how many kilometers it is to the town from where Bill started. By utilizing three technological settings: (1) a simulation model programmed in **Cabri Geometry II**; (2) a trial and error simulation model programmed in **Microsoft ExCEL**; and (3) an algebraic and related geometry representation shown in a **TI-83 Graphing Calculator** setting, the traditional Arate@ word problem selected which normally provides a context for using particular formulas or algorithms will be re-visited in such a way that critical thinking will be encouraged In each setting, the aim besides solving the problem, as per Polya=s Problem Solving Model, is to promote better understanding of what problem solving is while concurrently enhancing the learner=s problem solving ability. In the **AUnderstanding the ProblemA** phase of Polya=s model, the problem solver is encouraged to spend time and effort developing an understanding of the problem. Prominent in this, is to identify that which is the goal, what it is that is to be found, proved, accomplished, or whatever. Also among the essentials is to obtain a clear understanding of the given(s) and requirements or conditions of the problem. In the **ADevising the Plan@** phase, it is common that a plan or at least a part of a plan evolves from an understanding of the problem. This may involve knowing relevant definitions (concepts), non-definitional generalizations (axioms, postulates, formulas), principles (generalizations, theorems), skills (operations, processes, algorithms, prescriptions), logical reasoning, and problem solving strategies (heuristic or otherwise) that might be applied, and in what order. Often plans change as steps are carried out (see the next phase) and/or new understanding arises. Thus, one should not restrict this phase to a lock-step position that keeps one from moving on if a complete plan is not known, nor should one erroneously keep from returning to a search for understanding after embarking on this phase. In the **ACarrying Out the Plan@** phase, a systematic, step-by-step guided "straight-line" sequence of operations, deductions, and processes is ideal, but often unrealistic when one is working on a REAL problem. Expect some blockage, some dead-ends, some back-tracking, and so on. Persistence (related to motivation) may be one of the most essential qualities of a problem solver. There is a need to be tolerant of confusion, uncertainty, frustration, errors, and the like. Expect also that new insights and understanding may lead to changes in plans. Assuredly, REAL problem solving is often a dynamic situation. In the last phase, the **ALooking Back@** phase, is the idea of CHECKING a possible solution to see if it really satisfies the requirements of the problem, has the problem really been solved? But, there is more beyond mere checking. Here, one should be encouraged to check if there are other solutions, other strategies for solving the problem, or if applicable, whether there is an extension to the problem. Looking back should not be a mere review of what has occurred to arrive at a solution,

but rather, looking back should allow for additional thought-provoking stimulation about the problem-solving process undertaken. Polya, in his model, views the looking back phase as an opportunity to have the student rise to a higher level of metacognition relative to the problem-solving process in general, as well as to the local subject-specific matter.

The Problem

Bill bikes to a nearby town at 12 kilometers per hour and returns at 10 kilometers per hour. How far away is the town if the entire trip took 3 and 2/3 hours?

A. The Cabri Geometry II Setting

Figure 1

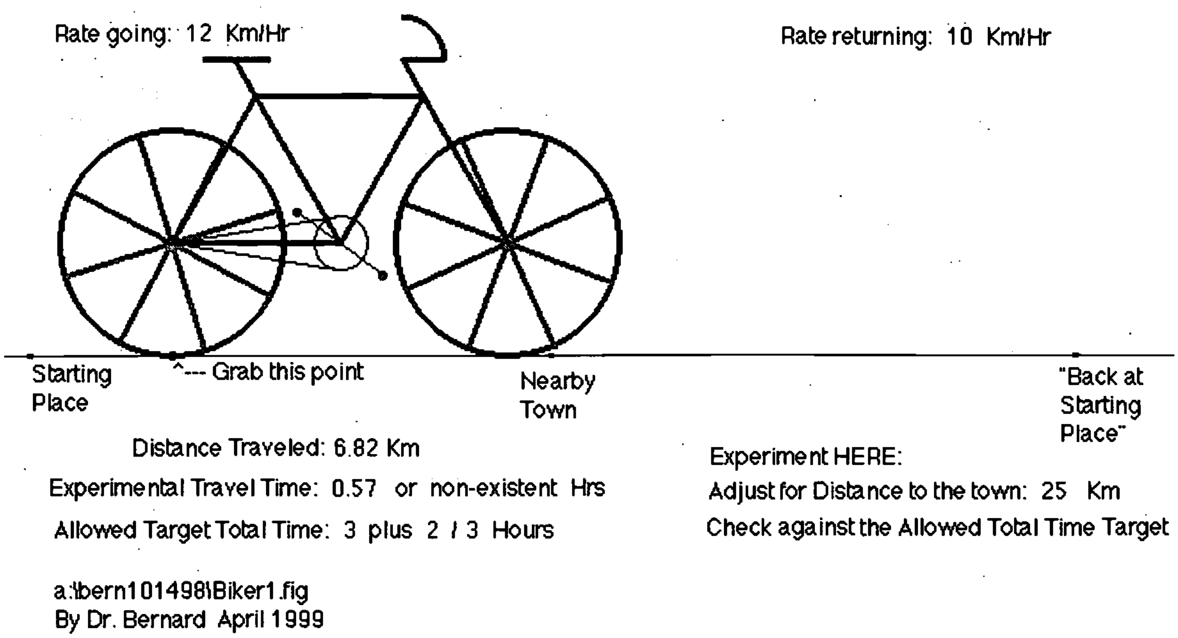

Figure 1, programmed using **Cabri Geometry II** (see Keyton, 1996, Laborder et al., 1994-1997, Vonder Embse, et al. 1994-1997, Vonder Embse et al., 1996) suggests the use of a simulation to depict how Bill, in the problem, might be traveling to the nearby town. The figure makes a major adjustment from reality in laying the trip out linearly and consecutively without overlap. It explicitly shows the two parts, first the part going to the town, then the part returning from the town to the starting point. Other things "fall into place" once viewers have made this adjustment. The model provides for experimenting (guessing) with the distance to the town. Following a given guess, one can then act the trip out using the **Grab** point, a built-in feature in **Cabri Geometry II.** Readouts of the distance traveled and the lapsed travel time (Experimental Travel Time) enable one to evaluate what is happening as the bicycle moves from the starting point to the town and from the town back to the starting point; i.e., over the course of the entire trip. If the distance guess is correct, the dependent total trip time (when the **Grab** point is at the right "starting Point") will match the target total time. Experimenters can distinguish when the dependent time does not match the target total time and separate such into two cases; those when the dependent time is too large, and those when it is too small. One compensates for the former by reducing the distance setting and for the latter by increasing the experimental distance setting (guess).

B. The Spreadsheet Setting

In the first spreadsheet setting (see Spreadsheet #1), **AA = the distance to town traveled at 12 km/hr and B = the distance to town traveled at 10 km/hr@.** Once students understand that the distance to and from town Amust@ be equal they can be encouraged to Aguess and check@ which time to town and from town, at the rates given, yield the

closest distance between **A** and **B**. This can occur by having students guess, as shown below, in the column denoted by the variable x for the **"Time in hours it takes Bill to bicycle into Town"** and the other information calculated on the basis of this guess. Students and teacher can develop the spreadsheet together, in parts, until Adiscovery@ of the need to approximate the difference between **A** and **B** to zero is developed.

Spreadsheet #1

Rate	Time	Rate	Time	Distance	Time	Distance	Distance
12	Fixed Total Hours	10	x	A = x/12	(3+2/3)-x	B = x/10	A - B
Bicycle Speed To Town in Km/hr	Total Time Held Constant	Bicycle Speed From Town in Km/hr	Time in hours it takes Bill to bicycle into Town	Distance in miles to Town at 12 km/hr	Time in hours it takes Bill to bicycle from town at 10 km/hr	Distance in miles to Town at 10 km/hr	Difference Closest To Zero
12	3 2/3	10	1.0000000	12.0000000	2.6666667	26.6666667	-14.6666667
12	3 2/3	10	1.3333333	16.0000000	2.3333333	23.3333333	-7.3333333
12	3 2/3	10	1.6666667	20.0000000	2.0000000	20.0000000	0.0000000
12	3 2/3	10	2.0000000	24.0000000	1.6666667	16.6666667	7.3333333
12	3 2/3	10	2.3333333	28.0000000	1.3333333	13.3333333	14.6666667

In the second spreadsheet (see **Spreadsheet #2**) that follows, a different approach to Aguessing and checking@ occurs. The guess is for distance to the town being varied until the total time given, 3 and 2/3, is reached after multiple guesses at the rates provided. This approach coincides with the **Cabri Geometry II** simulation shown previously.

Spreadsheet #2

Distance	Rate	Rate	Time	Time	Total Time
X	12	10	x/12	x/10	x/12 + x/10
Guess (distance in Km to town)	Bicycle Speed To Town in Km/hr	Bicycle Speed From Town in Km/hr	Time in hours it takes Bill to bicycle into Town	Time in hours it takes Bill to bicycle back from Town	Time in hours it takes Bill to bicycle to and from Town
1	12	10	1/12	1/10	11/60
2	12	10	1/6	1/5	11/30
3	12	10	1/4	3/10	11/20
4	12	10	1/3	2/5	11/15
5	12	10	5/12	2	11/12
10	12	10	5/6	1	1 5/6
12	12	10	1	1 1/5	2 1/5
15	12	10	1 1/4	1 2	2 3/4
20	12	10	1 2/3	2	3 2/3

C. The TI-83 Graphing Calculator Setting

Using a **TI-83 Graphing Calculator** (see Texas Instruments Guidebook, 1996), techniques that are similat to the spreadsheet methods shown above can be used. In this case, a method referred to as the **AIntersection Method@** for solving a system of equations has been utilized. The equations **$Y_1 = 3 + (2/3)$ and $Y_2 = X/12 + X/10$** (where X is the varying guesses for distance) shown in **Figure 2** represent the target total time to and from the town and the partial times necessary to travel to town and back, respectively. After graphing the equations (**see Figure 3**), the intersection point **(20, 3.6666667)** indicates that when the distance is 20 miles to and from the town, the exact total time traveling to and from town is 3.6667 hours (an approximation to 3 and 2/3). Using the **TRACE** feature of the **TI-83 Graphing Calculator** to Atravel@ along the linear graph of **Y_2** toward the horizontal line **Y_1** helps to visualize the relationship between the total time and the partial times, **as X or distance varies**, to and from town. **Figure 4** obtained using the **TABLE** feature of the **TI-83 Graphing Calculator** captures this same relationship by denoting the incremental changes between total time and partial times as distance varies. This is a powerful way to bridge algebra and geometry to solve problems!

Figure 2.

```
Plot1 Plot2 Plot3
\Y1■3+(2/3)
\Y2■X/12+X/10
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Figure 3.

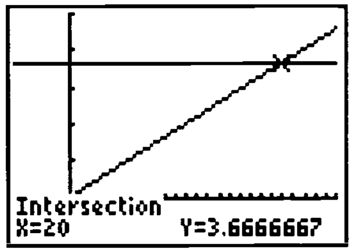

Figure 4.

X	Y_1	Y_2
15	3.6667	2.75
16	3.6667	2.9333
17	3.6667	3.1167
18	3.6667	3.3
19	3.6667	3.4833
20	3.6667	3.6667
21	3.6667	3.85

X=20

Regarding The Problem-Solving Phase

In either setting, or cumulatively, **understanding** began as a result of a diagram, table of data, graph, and or probes, and continues throughout. What can we say about the nature of the answer? We can say that it is a number. Moreover, it cannot be negative and is unlikely to be zero; so it must be some non-negative real number (imaginary numbers would not make sense in this situation). Distance is a kind of measure and non-negative real numbers are the kind of numbers used for measures. **More understanding;** but first a **devising a plan** idea - That algebra can be used to solve this problem; specifically, this is a place where a letter may be used to represent the unknown. The letter, in this case, serves as a "pronumeral," that is, a symbol representing a number. Here the interpretation may be that of a fixed unknown number rather than a variable, per se. Let d correspond to the number of kilometers measured from Bill's starting point to the town.Thus, d 0 {x 0 Y | x > 0 }, where Y represents the set of real numbers. **More understanding** (real-world knowledge): The problem mentions the entire trip, this includes Bill's traveling to the town and the return trip from the town to where Bill started. From the fact is that the distance for the return trip is the same as that for initial trip, we can conclude that the return trip was d kilometers as well, and that the distance measure for the entire trip was d + d or 2d kilometers. **More understanding** (more real-world knowledge): We should not only think about kilometers of distance since the problems, in addition, also mentions time/hours of travel - "the entire trip took $3\frac{2}{3}$ hours." Our real-world knowledge tells us that this is made up of the time it took to travel from the starting place to the town, plus the time for the return trip; the $3\frac{2}{3}$ is the sum of two other numbers/times. **More understanding**: Reflecting on the information generated thus far, we seem to have two disconnected pieces/ideas. The first part talks about three related distances. The second part talks about three related times. What we seem to need is some connection between these two things (some way to "bridge the gap", this is a problem-solving meta-level notion or heuristic. The solver "sees" that this problem has a "missing link" or "gap to be filled" quality to it, and brings into play problem-solving organizational and planning strategies to make the connection/fill the gap.). **More Understanding** (prior knowledge): Prompting recall, we need to "hit" on a relationship that involves distance and time. This relationship need not be to the exclusion of other entities/concepts; we just need one or more relationships whose ingredients include distance and time. This need (sub-goal) identified may be enough to activate the solver's memory, bring to mind some such relationship as "distance is equal to rate times time." Should this not naturally occur, perhaps that potential solver never learned the principle or temporarily experiences recall difficulties at a particular point in time, more consciously controlled or directed mental processing activities may be generated. This is a carrying out the plan item for the particular sub-goal, within the activities for solving the problem as a whole. However **memory is activated** or even if the solver gets help or looks up/finds the relationship in a book (on the internet, or whatever) is not of concern for this discussion, only that it (the relationship "rate times time equals distance") does surface. **Variations are acceptable,** from "rt = d" to $\text{Time} = \frac{\text{Distance}}{\text{Rate}}$; whatever equivalent forms one might generate (restricted to avoid division by zero and negative numbers for measures). The formula/relationship $\text{Time} = \frac{\text{Distance}}{\text{Rate}}$ provides a mental frame or "template" that can and should be applied in at least two ways in this problem, for the time traveling from the starting place to the town and for the time of the return trip; measured in hours. Thus, we have the following execution (**carrying out the plan**) activity.

More understanding coupled with carrying out part of a plan:

The time going $= \frac{\text{The distance to the town}}{\text{The rate traveled going}} = \frac{d}{12}$ and The time returning $= \frac{\text{The distance from the town}}{\text{The rate traveled returning}} = \frac{d}{10}$.

More understanding, "seeing" mental connections and acting to integrate the "pieces of the puzzle" that fit together (**carrying out the plan**):

The latter two specific instances of the time and distance relationship connect to the real-world knowledge mentioned earlier citing that the total time is equal to the time going plus the time returning (which itself is semantically equivalent to the equation: The time going + the time returning = the total time). Instantiating more concretely, we have

$$\frac{d}{12} + \frac{d}{10} = 3\frac{2}{3}$$

At this point in time, a major problem sub-goal has been attained. Whether stated explicitly or left implicit, **producing a model** equation involving the unknown and givens in a valid solvable relationship is part of the algebraic problems solving method/strategy. In essence, this "translates" the problem from one expressed in words (in this case) to one expressed in the "realm" of algebra. Solving the problem is thereby reduced to solving an equation (here, a linear equation in one unknown) which is a skill that is an object of learning in the algebra curriculum.

Carrying out the plan has been going on already. At this point in time, however, an appropriately educated/trained solver might exhibit relatively routine activity in a relative short time. Say:

$\frac{d}{12} + \frac{d}{10} = 3\frac{2}{3}$ >—[Multiplying through by the LCM 60]→ $5d + 6d = 220$ >—[Adding like terms]→ $11d = 220$ *and* $11d = 220$ >—[dividing by 11]→ $d = 20$. Here again, **a sub-goal has been reached within this problem-solving episode**. Interpretation mental activity takes over, noting that unknown value d stood (stands for) 20 has been revealed. Thus, 20 is the number of kilometers distance from the starting place to the town.

Looking Back: Looking back includes, but is **not limited to checking**. Checking will be followed by additional **Looking Back** ideas.

Checking: Here, checking might take the form of critically re-reading the problem in "light" of now knowing that the town is 20 kilometers from the starting point. In the critical re-reading process, one might do associated computations that reveal the specific values of other conceptually defined and formerly not known values. To illustrate: Bill bikes to a nearby town >—[now becomes]→ Bill biked 20 kilometers. He did this at 12 km/hr. Hence, it took him $\frac{d}{12} = \frac{20}{12} = \frac{5}{3} = 1\frac{2}{3}$ hours to bike to the town.

Bill returns by bike from the nearby town >—[now becomes]→ Bill biked 20 kilometers, this time at 10 km/hr. Hence, it took him $\frac{d}{10} = \frac{20}{10} = 2$ hours to return from the town.

Our real-world knowledge directs us to add these two amounts, giving us $2 + 1\frac{2}{3} = 3\frac{2}{3}$ hours; which is consistent with the given total time. Observe that revealing 20 as the value of d is consistent with the understanding the d be an positive real number. **Checking may take or include other forms** such as checking the steps in the algebraic solution of the equation, but consider checking for this presentation to now be completed. **Looking Back** includes seeing the solving of this problem in the bigger context of solving problems. One can sub-categorize this problem as one which can make efficacious use of elementary algebraic structures (a model equation) and processes. It demonstrates the value of the "try using algebra" problem-solving heuristic. **Looking Back** includes transfer, both near and far. The following item might seem routine (near transfer) for someone who was able to develop the above solution.

An airline flight to a certain airport travels 500 kilometers per hour and returns at 550 kilometers per hour. How far is the airport if the entire trip took 11 hours?

To add some transfer distance, consider the item and comments that follow.

Bill bikes to a nearby town at 12 kilometers per hour and returns at 10 kilometers per hour. If the town is 20

kilometers away, how long did the entire trip take? [Like the variation before this, roles are switched. Here, the formerly unknown distance is now given, and the formerly known time is now unknown.

Overall, what=s important in promoting NCTM=s problem-solving goal is not that technology has been used, but that it has been used appropriately. As in the activities suggested by this paper, sufficient and appropriate utilization of technology ought to be the norm rather than the exception if one is to meet the goal proposed.

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The *Graph* System for Function Graph Construction

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Abstract: The system *Graph* intended for drawing of the function graphs is described. The general algorithm of the graph drawing is described. The function can be set in an explicit, implicit, parametric form and as a sequence of the more simple formulas. With drawing the asymptotes of graph and the limits of function in points specified by the user are calculated. For the analysis of the graph (allocation of maximums, minimums and inflections) derivatives of function can be calculated in an analytical form. The allowable elementary transformations above the constructed graph are listed. The resulting figure can be made out as the appropriate file for the most popular textual and graphic editors. The function and its derivative can automatically be presented as the descriptions in languages C++, Fortran and Pascal. In system there will be a library of frequently used functions, set of the helps and recommendations, and also mode of examples demonstration. The system Graph is realized in language C++ under Windows 95/97/98.

Introduction

The function graph is powerful and convenient tool for research of functional dependencies in mathematics and applications. The graph drawing is widely used in various activities at schools, colleges, universities, R&D institutes, in business, in a science, industry etc. This activity is realized in many applied programs, for example, in (Baiakovski et al., 1985), and also in a number of universal systems of numerical and analytical manipulations (Mathematica, 1989, MATLAB, 1979). The universality and the multipurpose of such systems results that the user is compelled in the beginning honesty them to study, and only after that he/she be able to modest solve the simple task - to draw the graph of interesting his/her function. (For example, the *Mathematica* user's guide occupies 1400 pages).

The purpose of our work - to create the simple and effective system for graph drawing not requiring any appreciable efforts to its mastering by the novice. At the same time for the experienced user a lot of additional possibilities allowing to build and to analyze the complex graphs of functions and to represent result in a convenient form is offered. It is permitted to make some transformations over graph for reception of the required image or analysis of function. The system will be realized in two variants: *school*, destined for the beginners, and *basic*, in which all opportunities of system will be accessible. The *Graph* system will work in a semi-automatic mode and addresses to the user in inconvenient situations at the help. Basically it occurs with updating of the singular points and work to infinity.

The system will have the built-in library of the most widespread functions, in which for each function will be given its formula, graph with some typical values of parameters and brief explanation. The library will consist of two parts: libraries of school functions (algebraic functions, trigonometrical functions and inverse to them, exponent, logarithm, power, hyperbolic and other functions) and basic library (special functions).

The *Graph* system will have the interconnected set of the helps and demonstration examples. There will be rich means of polygraph representation of the graphs for the most popular textual and graphic editors.

Construction of Function Graph

The process of graph drawing consists from the following steps:

1) To determine domain of function definition.

2) To calculate the singular points, points of maximum and minimum, points of inflection and points of function discontinuity.
3) To calculate asymptotes vertical, horizontal and sloping. To find limits of function in the chosen points, including indefinitesses of a kind 0/0, ∞/∞, $0\cdot\infty$, $\infty-\infty$, 1^{∞}, 0^0, ∞^0.
4) To set the sizes of the graphs, to draw axes to put inscriptions.
5) To draw the graph of function.
6) To edit a figure before printing or insert in the document.

The domain of definition. The *Graph* system calculates the domain of definition for what writes out restrictions on argument of function: the denominator is not equal to zero, the argument of the "log" function is positive, the subradical expression for a root of an even degree is non-negative etc. For example, for function $y = [(2x-x^4)^{1/2} - x^{1/3}] / \log(1-x^{3/4})$ the *Graph* system will give out inequalities $2x - x^4 \geq 0$, $\log(1-x^{3/4}) \neq 0$, $1 - x^{3/4} > 0$ and also will ask the user to refine domain of function definition. By solving inequalities, the user can receive restrictions on allowable values of argument, i.e. the segment $[a, b]$, on which the function is given, can appear broken on smaller segments $[a_1, b_1]$, $[a_2, b_2]$, ..., $[a_n, b_n]$, on which the function is meaningful and can be calculated.

The singular points and function breaks points, points of maximum and minimum, points of inflection and points of function discontinuity are defined. The points of a maximum and minimum, and also point of inflection are calculated with use of the first and second derivatives.

The calculation of asymptotes vertical, horizontal and sloping. The parameters k and b of sloping asymptote $y = k\cdot x + b$ calculated on formulas

$$k = \lim_{x\to\infty} f(x)/x, \quad b = \lim_{x\to\infty} [f(x) - k\cdot x].$$

The uncovering of indefinitesses 0/0, ∞/∞ and others with use of l'Hospitale's rule is made up in a semi-automatic mode, when the user can see intermediate result and to operate the analytical calculations.

The setting of the graph sizes, drawing of coordinate axes and inscriptions. Before drawing of the graph it is necessary to specify its sizes, to draw axes of coordinates, to choose types of scales for axes of coordinates (usual, logarithmic, half-logarithmic), to superscribe numbers on them, to set drawing a coordinate grid, if it is necessary. The graph can be supplied with inscriptions and explanatory text.

Drawing of the graphs. The *Graph* system works in a dialogue mode and can simultaneously build the graphs of two functions $f(x)$ and $g(x)$ in two different windows or in one window by superposition. The graphs are drawn in Cartesian coordinates.

The function can contain no more than one parameter. The *Graph* system can build family of the graphs with various values of parameters. Each graph is represented by a curve, at which the color, thickness and type of a line (from an offered set) are easy selected.

The system updates simultaneously two functions, which graphs can be imposed one on other. It is convenient for the various purposes, for example, for the approached definition of roots of the equation $f(x) = g(x)$.

The function is set analytically in one of the following forms:
1) Explicitly, as $y = f(x)$, $a \leq x \leq b$.
2) Parametric, as $x = X(t)$, $y = Y(t)$, $a \leq t \leq b$.
3) Implicitly, as $f(x, y) = 0$, $a \leq x \leq b$ or $c \leq y \leq d$.
4) It is possible to set function as consecutive of formulas. For example, the function $f(x)$ can be set as $u(x) = (x-1)/(x+1)$, $v(x) = \exp(2*u-x*\delta)$, $f(x) = u$^2-3*sin(v)+δ, where p is a parameter.

Edition of a figure for printing or insert in the document. The user can transform a figure to a textual or graphic file of the appropriate textual or graphic editor. It is supposed to use such formats, as .doc, .rtf, .pdf, .bmp, .html, .pcx, .gif, .jpg. It is possible also to convert symbolic expression for function and its derivatives into a textual file of WinWord, Word Perfect, Latex, Adobe Acrobat etc. The *Graph* system can translate the function definition and its derivatives in languages C, Fortran, Pascal.

We will demonstrate an example of work of *Graph* system. Let user needs to select parameters a and b by such, that the equation $a/(1+x^2) = b\cdot \sin(x)$ had two positive roots on [-3, 3]. With the help of an input field "f = " in the bottom part of a window (see Fig. 1) user sets the first function "f = a/(1+x^2)" and in a field "[f] = " specifies its limits of change " 0, 4". The trial value of parameter a, equal "4", is set in an input field "prm = ". The second function "g(x) = b*sin(x)" and corresponding parameters is similarly given. In a field "[x] = " the segment, interesting for the user, [-3, 3] is set, in which the limits of functions $f(x)$ and $g(x)$ are analyzed. Then

the graphs of these functions are drawn: $f(x)$ in left window, $g(x)$ in the right one. Finally after pressing a key "g→f" the graph of function $g(x)$ is lain on the graph $f(x)$. Resulting picture is shown on Fig. 1. It is obviously, that with values of parameters $a = 4$ and $b = 2$ there are two positive roots of the analyzed equation on a segment [-3, 3]. (Note: In an input field "df/dx = " the symbolic record of derivative f(x), received by the user on one of stages of processing f(x), was stored).

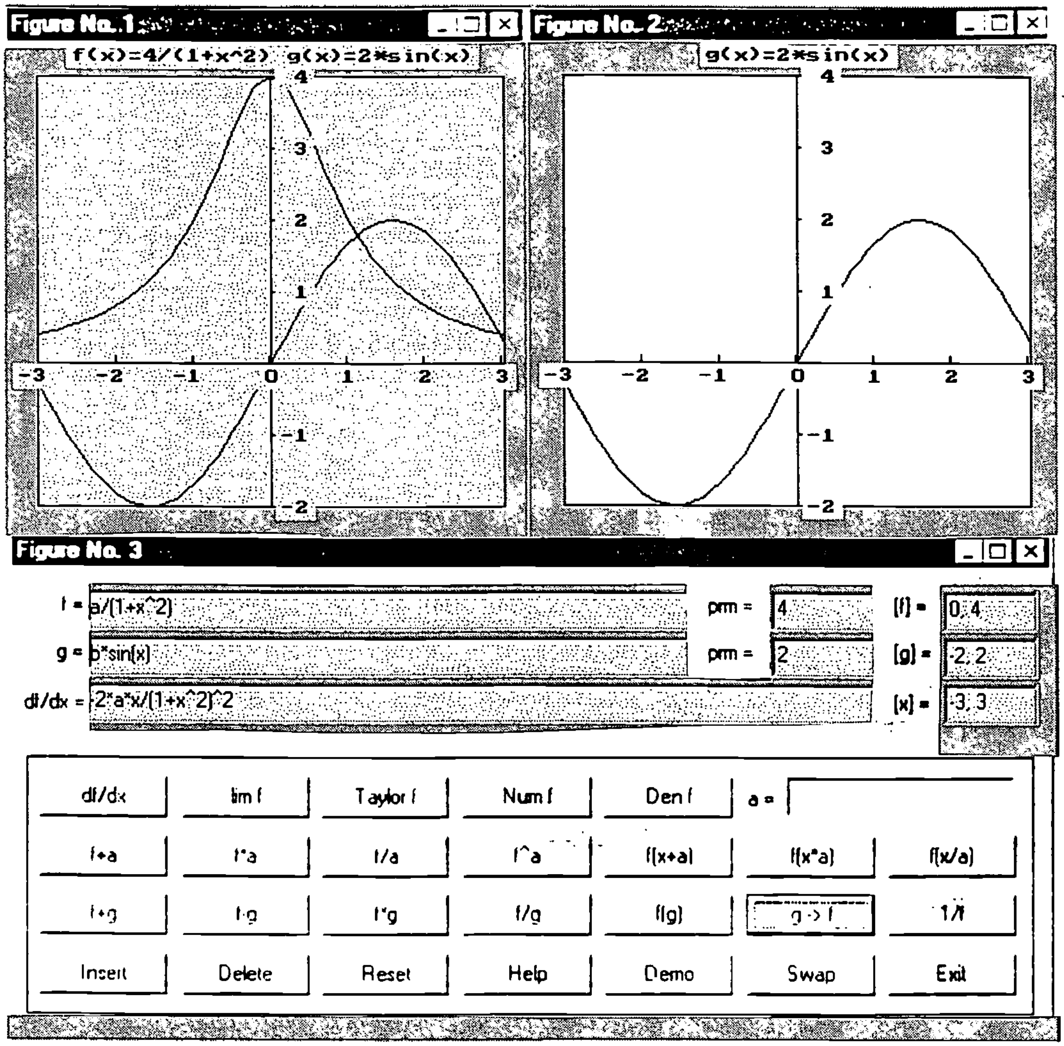

Fig. 1: The example of work of *Graph* system

Transformations of Functions and Graphs

It is possible to execute simple transformations above function and its graph with help of the ñîntrol menu (see Table 1). Such transformations are necessary in themselves, and also for the purposes of training of pupils and students at schools, colleges and universities. All transformations are carried out above function f(x). However, by changing f(x) and g(x) by button "Swap", it is possible to execute necessary transformations above function g (x).

Button	Action (a is given value)	Button	Action (a is given value)
df/dx	To differentiate function $f(x)$	f(x / a)	To replace on $f(x / a)$
lim f	To calculate a limit of $f(x)$ at õ → à	1 / f	- " - $f(x)$ on $1/f(x)$
Taylor f	To expand out $f(x)$ in Taylor's series	f + g	- " - $f + g$
Num Den	To extract from $f(x)$ numerator and denominator of rational expression	f − g	- " - $f - g$
f + à	To replace $f(x)$ on $f(x) + a$	f * g	- " - $f * g$
f * a	- " - $f(x) * a$	f / g	- " - f / g
f / a	- " - $f(x) / a$	f(g)	- " - $f(g)$
f ^ a	- " - $f(x) \wedge a$	g → f	To lay $g(x)$ on $f(x)$
f(x + a)	- " - $f(x + a)$	Swap	To swap the functions $g(x)$ and $f(x)$
f(x * a)	- " - $f(x * a)$		

Table 1: The list of transformations over functions and its graphs

The *Graph* system stores in inner list (with name f_list) the restricted set of functions and their graphs, with which the user worked earlier and has stored for future use. The set of actions above this list is given in table 2.

Button	Action	Button	Action
Insert	To insert current function in f_list	Show	To show selected function from f_list
Delete	To delete selected function from f_list	Reset	To set f, g and f_list in an initial state
Help	To give out the help or explanatory	Demo	To run a demonstration example
Exit	To finish work and to exit from system		

Table 2: The set of actions over f_list

Debugging of the *Graph* system nowadays is completed. The system is designed on language Visual C++ and will work on IBM PCs under Windows 95/97/98.

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San Antonio Technology in Education Coalition: A Mathematics and Technology Curriculum Integration And Staff Development Project

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Abstract: SATEC seeks to connect student learning to concrete experiences through the seamless and appropriate integration of technology into instruction. A training and application model is being piloted in the coalition's critical need area of mathematics through a hands-on, data-driven approach to the learning of mathematical concepts using such tools as computer-interfaced probes, image analysis software, and spreadsheet based simulation activities. While radically changing the environment of the teaching/learning process for mathematics, these tools are allowing teachers to introduce students to concepts by permitting them to discover patterns in their own collected data. The previous focus on skills has shifted to a focus on concepts and connections to the real world.

Introduction

Many adults who took high school algebra remember calculating the slope of a line from a memorized equation: m=¯y/¯x. Unfortunately, few learned the applications of m or understood the concept of slope. Though this abstract approach was deemed adequate for some children, most of today's students are not responding to this approach, and they are not learning. For example, only 17% of students taking Algebra I in the San Antonio Technology in Education Coalition (SATEC) target schools passed the state mandated End-of-Course Algebra I Exam in 1996. We believe that sound teaching begins with questions about real events that are interesting and familiar, not with abstractions out of context. Students cannot learn to think critically, analyze information, make logical arguments, explain natural phenomena, or work as part of a team unless they are often permitted and encouraged to do so. When students connect their learning to concrete experiences, they develop a foundation for understanding more complex ideas. Because computers help to rapidly collect, organize, and analyze data, technology enables students to quickly and easily replicate time consuming and laborious experiments. Inductive reasoning is used more frequently when students collect their own data and make generalizations about properties and events in the natural world. This approach allows students to grow in their ability to make observations and generalizations, reason logically, manipulate symbols, and derive formulas. The ability to communicate abstractions and to connect that kind of reasoning with the world around them makes students more likely to succeed in a society that demands problem solving rather than repetitious tasks.

Over the past twenty years educational institutions have embraced technology as a means to improve school and student performance with a major emphasis on hardware acquisition and on building a sophisticated network infrastructure. Only recently has there been a realization that staff development and quality curriculum modules are important components of successful technology curriculum projects. We cannot expect technology alone to impact student performance. Knowledgeable, well-trained teachers must participate in the development of rich curriculum that incorporates technology where it is appropriate to do so. Putting computers in the classroom of an unmotivated and untrained teacher may do more harm than good. It is certainly not cost effective. While our project is in the preliminary stages of data collection, we are able to suggest several outcomes from our study.

Suggested Outcomes

First, technology training for the teachers out of the context of curriculum content does not transfer in to appropriate integration of technology tools into teaching. Many teachers do not see the relevance of the tools to their instruction unless they are given sample lessons to work through as the venue in which the technology tool is taught. Second, since most teachers have been trained in presenting pencil and paper algorithms, the transition into conceptual learning in very difficult for them. We are also seeing that when faced with a concept to explore and no exploration lesson readily available, the teacher will abandon conceptual learning aided by technology and return to the comfort zone of teaching a pencil and paper process. Third, many teachers are deficient in mathematical content. Asking teachers to let students explore the bigger picture of how all the concepts fit together is not part of their comfort level. Usually this is due to the fact that mathematics teachers have traditionally asked students to imitate and memorize procedures to accomplish disconnected tasks. Our staff development always includes the "bigger picture." Fourth, teachers are limited in their understanding of mathematical application to the real world. We feel this is primarily due to the fact that teachers have been trained to use procedures to produce solutions to abstract mathematical problems. Consequently, all of our training is done with respect to a real world application. Fifth, professional development must model the methods the teacher is expected to reproduce in the classroom. Teachers must experience the training in the same way with the same materials that we expect them to use with the students. Finally, teachers must be held accountable to implement what they have learned in the classroom. We are attempting to accomplish this with actions plans and with commitments to use the things learned in professional development in an actual lesson with in the month of attending training.

Sample Activities

The following activities are two of the units we use to work with teachers. They are expected to then use these materials with the students in their classroom. The teacher is actively engaged in using the tools the student will be using to complete the lesson. The activity called "Letter of the Law" provides teachers with training and experience in the use of a motion detector. During the training, the trainer uses questioning techniques that mirror those the teacher will be expected to use in the classroom. The central concept covered in the lesson is mathematical functions. The second activity, "Life Expectancy," incorporates the use of a software piece called Graphical Analysis. With this software the teacher is expected to create a scatter plot with a given set of data. Next the teacher will work with creating an equation that best models the data. Central to the lesson is the concept of correlation.

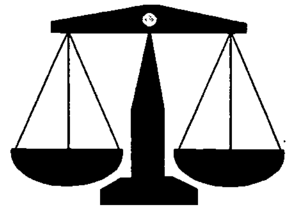
As you examine the activities, you will notice that our training models the points we have mentioned above. We mention them again for reinforcement:

- Teachers learn to use the technology tools in the context of a lesson they will use.
- Mathematical concepts are stressed throughout the lessons.
- The rich discussion by the trainer during the training session improves teacher content knowledge.
- The lessons are drawn from real world experiences.
- The training experienced by the teacher models the way the lesson should be presented to children.
- The teacher will be expected to present the lesson in a class.

Conclusion

We believe that training accomplished in this way will improve teacher self-confidence and content knowledge. Teacher growth in these areas will improve their ability to provide instruction in the classroom that engages the students. Improved instruction in which the students are working instead of watching will result in improved student achievement.

Letter of the Law



Objective

In this activity, you will

- Use the motion detector to create graphs that resemble letters of the alphabet.
- Predict which letters of the alphabet you can make using the motion detector.
- Analyze why you can do some but not other letters.

Setting up your experiment
- *Logger Pro* Program
- ULI with Motion Detector connected.

Instructions:
1. Launch *Logger Pro* on the computer (Make sure the ULI is properly connected)
2. Open the **Motion Detector** file.
3. From the **Window** menu, select **New Graph.**
4. Stand about five feet from the motion detector.
5. Start collecting data.
6. Move in the appropriate directions to make the letter V on the data collection screen (This may take several attempts).
7. What direction(s) did you move in order to create the letter V? ____________________

   ____

8. Repeat Steps 1- 3 to create an M.
9. What direction(s) did you move in order to create the letter M? ____________________

   ____

10. Predict what direction(s) you should move in order to create the letter W? ________

    ____

11. Try out your suggestion to see if the letter W is drawn.
12. Make a sketch in the empty space at the right of what graph appeared when you followed your instructions.
13. Predict what actions would you have to take to make a P. ____________________

    ____

14. Try out these actions to determine if you were correct. Were you?____________________
15. What are some other letters you could make?____________________
16. What are some other letters you could not make? ____________________
17. Describe why you could not make the P but you could make the V, M and W.

Correlation Study: Life Expectancy

LIFE EXPECTANCY		
Year	**Male**	**Female**
1900	49.6	49.1
1910	50.2	53.7
1920	54.6	56.3
1930	58	61.4
1940	60.9	65.3
1950	65.3	70.9
1960	66.6	73.2
1970	67.1	74.8
1980	69.9	77.5
1990	71.8	78.6

Data from
Parade Magazine,
"Must We Age?"
Hugh Downs, 1994
Used by Permission

1.Double click on the icon for **Graphical Analysis** on your desktop.

2. Label the axes appropriately. To do this, click once on the blue X in the Data Table window. A **Column Options** window will appear. Type the word **YEAR** and hit the **Return** key on your keyboard. Repeat the procedure for the Y-axis Label but use the words **LIFE EXPECTANCY**.

Which is the independent variable? Explain.______________________________

____

Which is the dependent variable? Explain.______________________________

____

3. Input data for *either* gender on the x-y table. To do this, click once in the first cell of the left-hand column of data. This is cell X1. Type in the number for the first year reported in your data set, 1900. You may use left and right arrow keys to move from cell to cell or you may use the **Return** or **Enter** keys to move from cell to cell. Experiment with these keys to find a method that works well for you. Enter the rest of the data - year in the X column and **life expectancy** (*either* male *or* female) in the Y column. The points will automatically be plotted on the graph.

Looking at the data table, what do you notice about the life expectancy as the years increase from 1900 to 1990?______________________________

4. When examining how variables relate to each other in a scatter plot, you are studying **CORRELATIONS**. There are three types of CORRELATIONS (relationships between variables): positive correlation, negative correlation, or no correlation.

Positive Correlation As the values of one variable increase, the values of the other variable also increase.
Negative Correlation As the values of one variable increase, the values of the other variable decrease.
No Correlation As the values of one variable increase, you cannot tell if the values of the other variable are increasing or decreasing.

Identify the type of correlation illustrated in your scatter plot.________________

5. Give two other examples from your experiences of this type of correlation.________

____

____

6. Identify the domain and range for the data displayed in the scatter plot.

DOMAIN____

RANGE____

- Change the scale for your "x-values" by clicking on the last value at the right. Replace this value with 2050. Repeat this procedure for the "y-values" and replace the value with 110.
- Title the graph. To do this, select **Double click on the title. After the current title,** type the gender you selected (male or female) and then type a dash followed by your initials.
- From the **File** menu, Click on **Print** and choose **Selected Display.**

Using a straight edge, draw a **Line of Best Fit** on your graph and use it to answer the following questions.

7. According to this information, what is the best prediction of the Life Expectancy for your gender in the year 2000?____________________

8. According to this information, what is the best prediction of the Life Expectancy for your gender in the year 1958?____________________

9. What is the best prediction of the Life Expectancy for your gender in the year 4 B.C.?____________________

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Double click on your Data table window. Click **Edit** and choose **Select All.** Now hit the icon containing R² at the top of the screen. Notice how the computer creates a **Line of Best fit.** Now select **Analyze.** and click **Interpolate.** ***Using your cursor, scroll up and down the line of best fit.*** With the aide of this feature, answer the following questions:

10. What is the best prediction of the Life Expectancy for your gender in the year 2000? How does this compare to your previous answer?____________________

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11. What is the best prediction of the Life Expectancy for your gender in the year 1958? How does this compare to your previous answer?____________________

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Click **File** menu. Select Exit. **DO NOT SAVE ANY PREVIOUS DATA.**

NOTICE

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