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ABSTRACT

This paper presents the results from a paper-and-pencil instrument designed to elicit students' thinking about the rate of change in particular situations. A scoring rubric was designed to classify students' solution strategies to algebra problems. The purpose of the study was to validate the results in different ways. First, the results from this scoring system were compared with students' performance on the initial instrument and students' performance on a different test. Later, the same instrument was given to another group of students and their responses were scored using the same system. The results from both groups were compared in an attempt to understand the relevance of the scores and the value of the implications of the scores' interpretation. (Contains 14 references.) (ASK)

Examining Relationships between Students'Solution Strategies, Algebraic Reasonsing, and Achievement

by

Cristina Gomez

Examining relationships between students' solution strategies, algebraic reasoning, and achievement

Cristina Gómez

Understanding the development of students' mathematical thinking has shown to be a successful strategy to help teachers in making decisions in the classroom (Carpenter, Fennema, Peterson & Carey, 1988) and, as a result, to improve student achievement (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Much of the research on this aspect has focused on early arithmetic, especially in addition and subtraction problem types and children's solution strategies. Our goal is to extend this analysis to other content areas of school mathematics. We have directed our work toward algebra and algebraic reasoning in high school. The final goal is to have descriptions of students' thinking on this particular content area that teachers can use in the classroom and that will have an effect in student achievement.

This paper presents the results from a paper-and-pencil instrument designed to elicit students' thinking of rate of change in particular situations. A scoring rubric was designed to classify students' solution strategies. The purpose was to validate the results in different ways. First, the results from this scoring system were compared with students' performance on the instrument and with students' performance on a different test. Later, the same instrument was given to another group of students and their responses were scored using the same system. The results from both groups were compared to understand the relevance of the scores and the value of the implications of the scores interpretation.

Understanding algebra

The definition of understanding we used corresponds with Hiebert et al.'s (1997). "We understand something if we see how it is related or connected to other things we know" (p.4). Understanding, then, involves constructing relationships, extending and applying mathematical knowledge, reflecting, communicating, and being an active participant in building new knowledge (Carpenter & Lehrer, 1999). The development of conceptual understanding does not have to sacrifice the learning computational skills. Both have to be developed in a dual relationship. Learning skills gives opportunities to reflect about processes and to make generalizations. Learning with understanding helps students to construct skills they can use in different situations.

Algebra is one of the cornerstone topics of high school mathematics. Until recently, this course was reserved only for those who were successful and had an interest in mathematics. Recent studies have shown the influence of taking more advance mathematics courses on bachelor's degree completion (Aldeman, 1999) including algebra, geometry and calculus. At the same time, documents such as the Principles and Standards for School Mathematics (NCTM, 2000) are promoting the inclusion of algebraic ideas since kindergarten to help students be successful in taking algebra courses.

Students who learn algebra with understanding are able to apply algebraic reasoning to new topics and are able to adapt this reasoning to new and unfamiliar problems. Studying patterns and functional relationships, representing these relationships in different ways, using symbolic forms to represent and analyze mathematical situations, and using mathematical models to analyze change in real and abstract contexts form the core of algebra (Kaput, 1999). Being able to understand how some algebraic processes work, to go from the answer to back to the starting point, to recognize patterns, to organize data, and to think about computation independently of the particular numbers used are all characteristics of algebraic reasoning (Driscoll, 1999).

The tasks

We are interested in developing accounts of students' thinking when they work with linear models. The ideas of slope, covariation, and rate of change are all related with the notion of linear function and they underlie other more advanced concepts such as function, limit, and derivative. The tasks we prepared address those ideas in different ways. We present situations using different representations-tabular, graphic, and verbal-and different contexts familiar to the students.

The first task presents a table showing a schedule of payments over a period of time. Students are asked to describe the pattern in the table, find some missing values, find the rate of change, and figure out the total time to pay off the loan.

The second task presents the linear graphs of a 100-meter race run by a mother and her daughter. The mother started 20 meter ahead. Students must answer the first two questions based on a graph without numbers on the axis. These questions ask who won the race and who was the faster runner. The next four questions require the analysis of the rate of change of the distance in relation to time for each of the runners and of the equations that relate distance and time for each runner.

The third task presents a verbal description of two phone company charges. The call rate and the monthly charge are given. Students are asked to compare the charges of both companies for a given number of calls, the number of calls for a given bill amount, and to choose the more economical company.

In order to get these accounts of students thinking we have been working with students in the classrooms, giving tasks that elicit algebraic thinking and analyzing the solution strategies students use to approach each task. In a previous study (Gomez, Steinthorsdotir, Uselmann, 1999) we gathered initial information about how students understand rate of change and linear models. We developed broad levels of understanding for each task based on students' responses and explanations. The next step was to refine these levels and pose some hypothesis related with student thinking, use of strategies, and performance.

Scoring system

In order to reach our goal we have designed a rubric to score students solution strategies. A rubric is a rating system by which it is possible to determine at what level of understanding a student is able to perform a task or display knowledge of a concept (Moskal, 2000). The process of developing a scoring rubric has two steps. During the first step the qualities of the top level are described and then, during the second step, the evidence suggesting the different levels is identified.

In the present study, the rubric for scoring students' work was developed using the results from our previous study and results from research. During the first step we looked at the underlying concepts involved in these tasks. There are certain concepts that are necessary for students' successful use of advanced strategies. In this case, the concept of covariation was identified as necessary to understand the tasks. Covariation involves the relationship between changes in one quantity and changes in another. It involves two quantities varying simultaneously in a linked manner (Thompson & Thompson, 1996; Coulombe & Berenson, 1997). Being able to think about relationships between varying quantities is the heart of functional thinking, required for more advanced mathematical topics. For each task, the underlying concepts can change but overall the ideas of recognizing patterns, covariation, and linear change are behind.

In Task 1, the ability to recognize and extend patterns is an integral component of mathematical reasoning. By analyzing the table of data students develop an understanding of the dynamics of change and recognize how change in one quantity effects change in another. Students who really understand rate of change are able to describe the patterns in each of the columns of the table, they can coordinate the two patterns in a way that they are able to interpolate data for unknown values, and they are able to describe the situation in terms of the rate of change (Confrey & Smith, 1995).

In Task 3, the understanding of covariation requires the recognition of the dependency relationships, that is, the effect that the number of calls has on the monthly charge. It requires also the identification of multiple patterns of covariation, one of each company and that those patterns were linear. Finally, the concept of generalizability expressed in the ability to generalize rules from the given patterns of variation, and make predictions are required to fully understand the situation (Coulombe & Berenson, 1997).

During the second step of defining the rubric, three levels of understanding were identified for each question. These levels correspond with the dimensions involved in the situation and the coordination of those dimensions. *Dimension* refers to the algebraic concepts involved in the question and *coordination* refers to the relationships between those concepts. For example, in the first task, a table that shows the change in time of two variables—number of months and amount owed—is presented. Students are asked to describe the pattern in the table. The dimensions involved in this question are the variables and the coordination is the relationship between these variables.

The rubric was designed to identify three different levels: single dimension, multiple dimensions non-coordinated, and multiple dimensions fully coordinated. For the task presented above, a single dimension answer focuses only on describing one variable; a multiple dimensions non-coordinated answer would refer to both variables but not in relation to each other; a multiple dimensions fully coordinated answer would describe the variation of both variables in relation to each other.

In the first level, a student is able to recognize some isolated characteristics of the situation. In the second level, all relevant features are identified but not connected. In the last level, a true understanding of the situation, features, and connections are clearly described. This last level implies also an ability to reflect and communicate thinking.

Not all problems we posed to students give us this range of knowledge about student thinking. Two of the three tasks given to students were real windows to student understanding.

It means that students' solution strategies could be classified using the levels mentioned before. The second task did not offer this type of analysis. Thus, the results presented later include only students' responses to the first and third task.

Validity

The concept of validity we used for this study corresponds with Messick's (1989). Validity is "an integrated evaluative judgment of the degree to which empirical evidence and theoretical rationales support the *adequacy* and *appropriateness* of *inferences* and *actions* based on test scores or other forms of assessment.(…) What is to be validated is not the test or observation device as such but the inferences derived from test scores—inferences about score meaning or interpretation and about the implication for action that the interpretation entails" (p. 5). In order to validate the inferences it is necessary to be certain that multiple lines of evidence are consistent with the inferences. The sources of validity come then in different forms. We can look at how the content of the test relates to the content of the domain, we can examine relationships among different parts of the test, we can survey relationships with other test scores, we can investigate uniformities and differences across groups, we can probe the way individuals cope with specific tasks to illuminate the processes underlying item response, or we can appraise the value implications of interpreting the scores in particular ways.

In the present study we will focus on four of these validation forms. First of all, we look at how the content of the test related with the content of the domain. This is done based on the scoring system. The levels for each question were decided based on specific aspects of the content. Second, we examine relationships among different parts of the test, comparing the use of strategies with the overall performance on the test. Third, we examine the relationship with another test, comparing the use of strategies with the results in a new instrument created based on NAEP items and school district performance tasks. Finally, we look at the consistency of the results when the test was given to a new group of students. Other forms of validations are presented elsewhere. Rousseau (2000) interprets the results from Task 1 in a slightly different way and Brendefur (2000) describes how individual students deal with one specific task, focusing specially on translations among different representations.

We are interested in looking at how the use of more sophisticated strategies relates with student achievement measured in two different ways. The first way is the performance of student in the whole test. The second way is the performance of students in a test constructed using NAEP items and school district performance tasks. Finally, we are interested in validating the results with a new group of students. The same test was given to a new group of students from a different city. Students' solutions were scored using the same rubric and the results from these two samples were compared.

The study, then, has two main parts. The first presents the results of the test given to a group of bilingual students. This group of students is called Group B. The second part presents the results when the test was given to the new group of students. This group is called Group N.

Group B

Participants

This section reports the results of the test given to 46 high school bilingual students, Group B, from a mid-sized city in the Midwest. Of these students, 38 were enrolled in Algebra, and 8 in a remedial algebra course. These classes included students who were in the 9^{th} through 12^{th} grades. Fifty-two percent were female and forty-eight percent were male.

Procedure

The paper-and-pencil test was administrated by the teachers in their classrooms during the 1998-1999 school year. Students were given a one-hour class, 55 minutes, to complete the test. Each question in the test was presented in English and Spanish. The test consisted of three questions, each one constructed to bring into focus one of the representations of rate of change — table, graph, or natural language (see Appendix A for the actual test).

The test was scored using a rubric designed to take into account different levels of understanding evidenced from student's responses. Students' responses were coded in two different ways. First a strategy-code was used coding as 1 if they used a single dimension strategy, 2 for a multiple dimensions non-coordinated strategy, 3 for a multiple dimensions fully coordinated strategy and 0 for no attempt to answer the questions and answers that did not fit any of the other categories. This scoring will be called Strategy use. Secondly, a performance-code was used. Responses were also scored as right or wrong. This score corresponds to Performance on the Rate of Change test variable and it is on 9-point scale.

The second test was administrated by the teachers in their classrooms at the end of the 1998-1999 school year. Students were given a one-hour class, 55 minutes, to complete the test. Each question in the test was presented in English and Spanish. The test was constructed using NAEP items and school district performance tasks and consisted of seven questions including multiple choice and open-ended problems. It was scored using a 20-point scale, considering only right answers. All questions involve covariation, rate of change, or linear patterns. Tables, graphs, and verbal descriptions were used to present the problems (see Appendix B for the actual test). For simplification this test is called NAEP in the results and analysis sections.

Results

Table 1 shows the results for Strategy use. The percentages of student responses coded at each level of strategy and for each item of Task 1 and Task 3 are presented. The results show that for Task 1 students preferred strategies at levels one and two. Less than one fourth of the students used strategies level 3. The high percentage that did not answer the question or did not give an explanation to the answer, that is solutions coded at level 0, was surprising. For Task 3, the percentage of students using strategies of level 3 is lower. In the first question only 28% used more advanced strategies. For the other questions students used mostly level 1 strategies and again the percentage of answers coded at level 0 was very high.

Table 1

Percentage student who used each strategy (n=46)

Strategy	1a	1b	1c	1d	1e	1f	3a	3b	3c
0	26	14	26	26	55	72	41	70	76
1	11	50	48	46	4	22	22	22	15
2	41	15	9	2	20	0	9	2	9
3	22	22	17	26	22	7	28	7	0

Table 2 presents the percentage of students who gave a correct answer for each strategy. It is clear students using more advanced strategies (Level 3) are more successful in solving the problems. The high percentage for question 1a at level 2 means that 63% of the students who were able to describe the pattern from the table recognized both variables but they had some misrepresentation of the data. The percentages at level 0 indicate students who answer the question correctly but they did not offer any explanation.

Table 2

Percentage student who solve it right (n=46)

Strategy	1a	1b	1c	1d	1e	1f	3a	3b	3c
0	0	0	17	0	7	30	5	0	3
1	0	0	5	0	0	0	0	0	0
2	63	14	0	0	0	0	0	0	0
3	100	100	88	100	80	100	100	67	0

As mentioned before, we were interested in looking at how the use of more sophisticated strategies relates with student achievement measured in two different ways. The first way is the performance of student in the whole test. The second way is the performance of students in a test constructed using NAEP items and school district performance tasks.

Table 3 presents the mean in Performance for the whole test. Each question was scored with 1 for correct answer and 0 for incorrect answer. Nine questions were scored, six from Task 1 and three from Task 3. The results show that students who used more advanced strategies performed better in the whole test.

Table 3

Mean in performance for the Rate of Change test (n=46)

Strategy	1a	1b	1c	1d	1e	1f	3a	3b	3c
0	0.8	0.3	1.7	0.7	0.7	2.0	1.1	2.0	2.1
1	0.6	0.8	0.7	0.7	1.5	1.0	0.7	1.3	1.6
2	1.6	2.3	1.5	2.0	1.1	0.0	3.5	1.0	2.5
3	5.2	5.9	6.8	5.9	6.4	6.3	4.1	5.7	0.0

To study the relationship between the strategy used and the over all performance we considered only the answers scored at level 1, 2, or 3. Tables 4 and 5 and Figure 1 present a linear regression analysis for these two variables. The independent variable was Strategy and the dependent variable was Performance on the Rate of Change. The adjusted R^2 says that Strategy Use accounts for about 50% of the variance in Performance of the Rate of Change test.

Table 4

ANOVA

Performance on Rate of Change vs. Strategy

	DF	Sum of Squares	Mean Square	F-Value	P-Value
Regression	1	922.303	922.303	221.115	<.0001
Residual	226	942.679	4.171		
Total	227	1864.982			

R^2 =.495

Table 5

Regression Coefficients

Performance on Rate of Change vs. Strategy

	Coefficient	Std. Error	Std. Coeff.	t-Value	P-Value
Intercept	-1.711	.314	-1.711	-5.442	<.0001
Strategy	2.318	.156	.703	14.870	<.0001

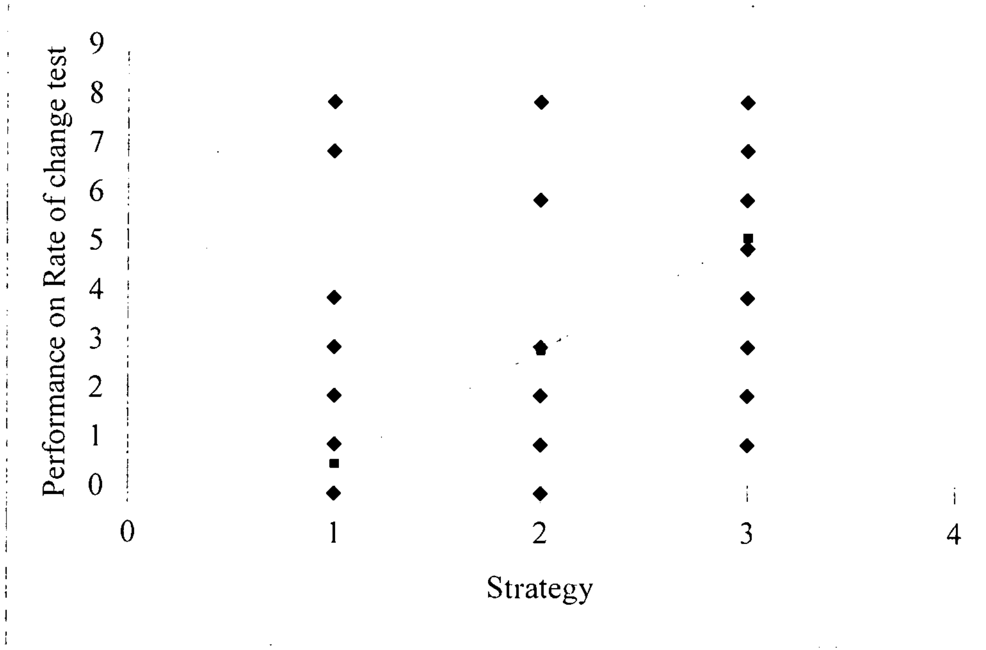

Figure 1. Regression Line fit plot for Strategy vs Performance on the Rate of Change test.

Table 6 presents the mean by question in the test constructed with NAEP items and school district performance tasks. Even though some scores for strategies level 1 and 2 are high, it is clear that students who used strategies level 3 scored better.

Table 6

Mean in performance for the NAEP test (n=46)

Strategy	1a	1b	1c	1d	1e	1f	3a	3b	3c
0	5.6	4.5	7.5	5.0	5.9	8.6	6.9	8.8	8.9
1	10.2	8.7	8.4	9.0	13.0	9.5	7.8	8.5	8.7
2	9.1	9.8	9.0	6.0	10.4	0.0	10.7	14.0	13.5
3	13.5	12.7	14.3	14.0	15.7	16.0	13.3	15.0	0.0

The following tables present the results of the linear regression analysis for Strategy Use as independent variable and performance on the NAEP test as the dependent variable. The adjusted R^2 says that Strategy Use accounts for about 21% of the variance in Performance on the NAEP test (see Tables 7 and 8, and Figure 2).

Table 7

ANOVA

NAEP vs. Strategy

	DF	Sum of Squares	Mean Square	F-Value	P-Value
Regression	1	1076.302	1076.302	58.286	<.0001
Residual	226	4173.325	18.466		
Total	227	5249.627			

R^2 =.205

Table 8

Regression Coefficients

NAEP vs. Strategy

	Coefficient	Std. Error	Std. Coeff.	t-Value	P-Value
Intercept	6.095	.661	6.095	9.216	<.0001
Strategy	2.504	.328	.453	7.634	<.0001

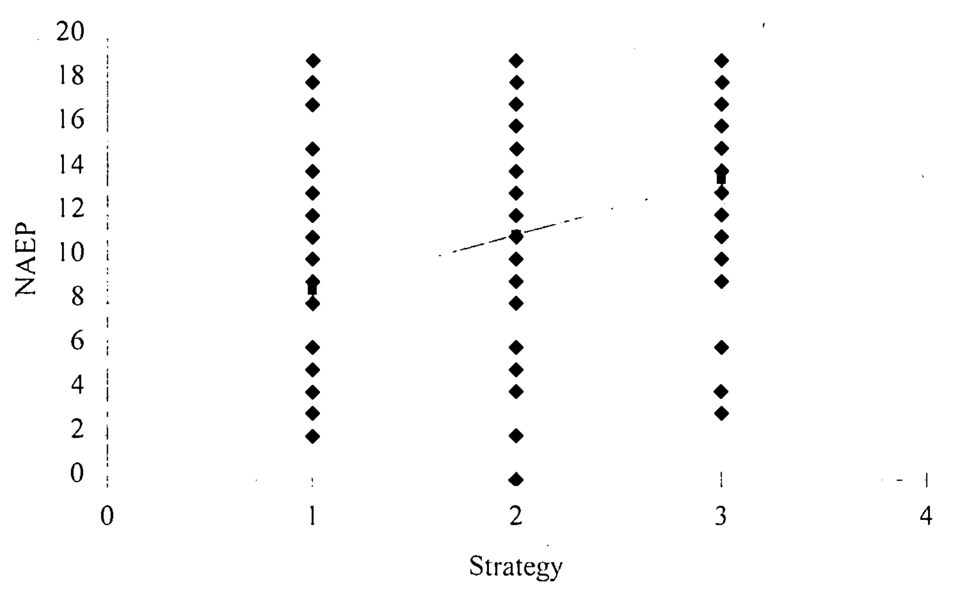

Figure 2. Regression Line fit plot for Strategy vs Performance on NAEP test.

Analysis

The results presented in this section show interesting relationships between student strategies, performance, and algebraic thinking. The test was given after the unit on linear models was taught and yet the percentage of students using advanced strategies is clearly low.

In Task 1 almost half of the students used strategies level 1 or 2, which correspond with lack of coordination of all dimensions involved in the problem. For these students the relationship between the variables involved is not clear. They do not see changes in one variable depending on changes in the other. Besides, these students do not see a pattern in the data. They look at the values of both variables as independent from one another and they are not able to generalize a relationship among the variables involved.

Solutions coded at level 3 show evidence of algebraic understanding of the situation. A student needs to identify the dimensions involved in the question and the way these dimensions are coordinated in order to be able to use a strategy level 3. These dimensions change from one question to another, giving students opportunities to show evidence of reasoning at different levels and in different situations. From the results showed in Table 3 we see students using more advanced strategies make less systematic errors and then they are more successful in solving the problems.

Furthermore, this relationship to performance could be extended. The results of the regression analysis shows that Strategy Use is a good predictor for performance in both, the whole test and in a different test designed using NAEP items. In other words, students who use advanced strategies to solve specific items have understanding of the underlying concepts involved in linear models. More specifically, the concepts of covariation and rate of change showed to be essential to solve these and other tasks involving linear models.

Extending the analysis, Group N

Participants

This section reports the results of the test given to 105 high school students, Group N, from a mid-size city in the Midwest. These students were enrolled in five different Algebra classes. These classes included students who were in the 9th grade. Forty percent were female and sixty percent were male.

Procedure

The paper-and-pencil test was administrated by the teachers in their classrooms during the 1998-1999 school year. Students were given two class periods (roughly an hour and forty minutes) to complete the test. The test was scored again in two different ways, one for Strategy use and the other for Performance. The rubric used was the same.

Results

Table 9 presents the results for this new group of students. The preference for strategies at levels 0 and 1 is evident, though more students used strategies at level 3, with exceptions in questions 1a and 3a. Again, the percentage of students who did not answer the questions or did not give any explanations is very high, especially for Task 3.

Table 9

Percentage of student in pre-algebra who used each strategy (n=105)

Strategy	1a	1b	1c	1d	1e	1f	3a	3b	3c
0	16	18	47	31	56	50	33	59	88
1	10	46	27	29	3	14	19	18	3
2	20	7	10	3	17	8	2	11	9
3	54	30	17	38	24	29	46	12	1

The percentage of correct answers by strategy and by question shows that students who used more advanced strategies were more successful in solving the problems (see Table 10). Compared with Group B (Table 1), Group N has higher percentages of students using advanced strategies and this might explain why the percentages of success are lower.

Table 10

Percentage of student who solve it right (n=105)

Strategy	1a	1b	1c	1d	1e	1f	3a	3b	3c
0	0	21	20	15	6	0	11	11	0
1	0	0	0	0	0	6	0	0	0
2	57	0	20	0	0	0	0	18	0
3	98	100	94	95	92	73	89	84	0

Looking at the overall performance in the test, Table 11, we see again high scores for students who used advanced strategies. These scores are not much different from the scores for Group B as shown in Table 3.

Table 11

Mean in performance for the Rate of Change test (n=105)

Strategy	1a	1b	1c	1d	1e	1f	3a	3b	3c
0	0.8	1.7	2.1	1.4	1.7	1.6	1.7	2.6	2.6
1	1.7	1.4	1.7	1.4	0.3	1.5	1.4	1.6	3.0
2	2.3	3.0	3.4	2.3	2.6	2.3	4.0	3.6	4.4
3	3.7	5.6	5.9	5.0	5.8	5.6	4.1	4.5	7.0

Analysis

Extending the analysis to the new group, Group N, showed the scoring system is consistent. We did not find new strategies non-scorable with the rubric. Group N has more students using strategies level 3 in most of the problems and also more students using strategies at level 0. One way of interpreting these results is to say these students have overall better understanding of linear models. Looking at tables 2 and 10 we see students from Group B executing the strategies more successfully, although students from Group N using strategies at level 3 were still very successful.

The performance in the test was similar for both groups. The mean from Group B was 2.51 with standard deviation of 2.86 and for Group N the mean was 2.78 with standard deviation of 2.38. Given that both groups correspond to two very different groups of Algebra 1 students in 9^{th} grade, these results tell us the tasks and the scoring system are good indicators of student understanding of covariation and rate of change in linear models.

Conclusion

Looking in detail at students' solutions to the tasks presented here, we see how the use of strategy is constrained by the representation of the problem. In Task 1, where the information is presented in a table, students prefer to keep looking at the data as set of discrete points with no connection between one another, making it difficult to see the pattern or rate of change of one variable in relation to the other. In Task 2, presented in a graph, students preferred to use the graph as the main source for explanations, making difficult for us to see patterns of thinking in students' responses. In this case students did not give explanations because they simply looked at the graphs to answers the questions and because of that this task was not considered in the analysis. In Task 3, presented in verbal form, students preferred to use words to explain the way they solved the questions. We did not find any students using formal equations or other algebraic representations to solve this task.

Because the test was given after the unit on linear models was taught in both groups, we can argue that many students try to apply formally taught procedures with no understanding or they do not use at all the procedures taught in class. This explains the high percentage of students using strategies level 1 and 2. These students have had experiences working with tables where the pattern is given by the difference between two consecutive values. In Task 1 it is necessary to fully understand the relationship between the variables involve in order to solve the problems. For Task 3, students have worked with similar situations and they know how to solve some of the missing value problems but they have a difficult time generalizing the results, which explains the low percentage of students using strategies level 3 for both groups in question 3c. Instruction, then, should focus on connecting procedures to problem solving, developing algebraic sense, and giving student opportunities to take a more active role in constructing strategies for solving problems using different representations and reflecting and communicating their thinking.

The results of performance in both groups show the use of more sophisticated strategies lead to less systematic mistakes. Because the tasks presented here are non-routine tasks we can assert student who really understand these algebraic ideas not only use more sophisticated strategies but also they are more successful executing the strategies. In other words, for these students the development of conceptual understanding does not have sacrificed the learning of computational skills.

The use of the scoring rubric has shown to be a useful tool to look at the relationships proposed in this paper. We believe that with more advance students we will need to change the rubric to include the use of formal representations and translations to different representations. These two aspects are fundamental in developing algebraic thinking and have to be included to have a complete trajectory of students' development of algebraic ideas. We need also to identify more tasks that are real windows for understanding. We have seen that Tasks 1 and 3 give us good information about student thinking while Task 2 does not require students to show evidence of their thinking.

Finally, we need to develop ways of giving this information back to the teachers and look for ways to use this information in the decision making process of the classroom in real time. Teachers need access to these windows for understanding in order to gather actual information about students' thinking and design leaning environments that promote understanding.

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