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## ABSTRACT

This paper reports on a case study that was conducted at five high schools from a large, urban school district located in South Texas. The purpose of the study was to gain an understanding of Algebra 1 teaching strategies. The research questions were: (1) What is the predominant mode of instruction for Algebra 1? and (2) What is the level of achievement of Algebra 1 students? The schools consist primarily of students of Mexican descent. The teachers were also of Mexican descent. Both qualitative and quantitative data were collected. Field observations revealed a teaching methodology that focused on skills and procedures facilitated through lecture and supervised practice. The instruction was heavily influence by the state's high stakes test. Student achievement reflected the instructional methodology, which focused on particular skills and processes. Students failed to complete open-ended and higher order thinking questions on a standardized measure of algebraic skills and concepts developed by a publisher. Recommendations are included for improving algebra instruction. (Contains 19 references/ASK.) (Author)

ED 441 675

Running head: SCHOOL ALGEBRA REFORM: MEETING THE GRADE?

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## School Algebra Reform: Meeting the Grade?

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## Abstract

This paper reports on a case study was conducted at five high schools from a large, urban school district located in South Texas. The purpose of the study was to gain an understanding of Algebra I teaching strategies. The research questions were:

1) What is the predominant mode of instruction for Algebra I? 2) What is the level of achievement of Algebra I students? The student consists primarily of students from Mexican descent. The teachers were also of Mexican descent. Both qualitative and quantitative data were collected. Field observations revealed a teaching methodology that focused on skills and procedures facilitated through lecture and supervised practice. The instruction was heavily influence by the state's high stakes test. Student achievement reflected the instructional methodology, which focused on particular skills and processes. Students failed to complete open-ended, and higher order thinking questions on a standardized measure of algebraic skills and concepts developed by a publisher. Recommendations are included for improving algebra instruction.

## Introduction

Algebraic knowledge is considered a requirement for a new literacy. The point is made by Schoenfeld (1995) who stated:

With too few exceptions, students who do not study algebra are therefore relegated to menial jobs and are unable often to undertake training programs for jobs in which they might be interested. They are sorted out of the opportunities to be productive citizens in our society (p. 11-12).

More over, algebra is a mathematics class that has been described as the “new civil right” (Moses, 1995). The view of algebraic knowledge as a “civil right” implies that this knowledge should be made accessible to all students. Algebra I is the first non-arithmetic class encountered by both high school and some middle school students. If students are not successful at this stage, then they may not be able to take advantage of this “new civil right.” Since the mathematics education community considers algebraic knowledge a contributing factor toward success in life, it becomes necessary to provide learning opportunities that have the best chance to engage students’ interests while improving their understanding of algebraic concepts. The purpose of this paper is to report findings from a study that examined Algebra I instructional methodology in high schools with a predominantly Mexican American student population.

## Theoretical Background

Algebra has historically been considered a gate keeper course in high school (Kaput, 1999; Schoenfeld, 1995). Successful completion of the course leads to higher-level mathematics course taking. Students who experience more courses in mathematics tend to have higher scores on standardized achievement tests (Catsambis, 1994). Secada and Williams Berman (1999) pointed out that students from diverse backgrounds are not well served by the mathematics instruction since much of it is focused on lower order skills. According to Croom (1997), an equitable mathematics classroom recognizes the richness of cultural diversity and creates an

opportunity to engage all students in an interactive learning process from which higher-level concepts must emanate. Reform efforts are under way in various sections of the United States to make mathematics accessible to students who have a wide range of ability, such as the Quasar Project (Stein, Schwan-Smith, Henningsen, & Silver, 2000). The researchers focused their efforts on inner city middle schools; to provide teachers with a knowledge base for increasing the cognitive demands of mathematical tasks engaged by students. The implementation of mathematics instructional practices that develops understanding represent efforts to bring about a mathematics curriculum that is high quality and fosters equity.

The traditional algebraic pedagogy, described by Kaput (1999), focused on the simplification of algebraic expressions, solving equations, learning the rules of symbol manipulation, and it is taught without regard to making connections to other mathematical knowledge and students' world views. A conception of how the procedures fit into the overall structure of algebra is necessary for developing understanding in students. Kieran (1992) has suggested a framework to encase the teaching of Algebra. It is the procedural-structural cycle where both conceptions are stressed during instruction; procedural refers to "arithmetic operations carried out on numbers to yield numbers . . . the objects operated on are not the algebraic expressions but their numeric instantiations" (p. 392). The operations are characterized as computational, yielding a number. Structural is considered a different set of operations that are carried out on algebraic expressions, not numeric instantiations. There must be a two-way transition between the two perspectives. This implies that students should receive instruction in algebraic procedures as well as how the procedures are related to algebraic concepts and the foundational structure of the concepts.

According to Schwartz (1992), the content of secondary school mathematics should be "made coherent and pedagogically workable" (p. 303). The coherence of the algebra curriculum can be achieved through a central concept such as functions because the concept of functions is considered a unifying topic in algebra and other secondary mathematics courses (Bednarz, Kieran, & Lee, 1996; Romberg, Fennema, & Carpenter, 1993). Algebra can be made

“pedagogically workable” through appropriate use of problem-solving activities designed to enhance students’ understanding of concepts. The algebra curriculum centered around problem solving and real-world applications offer students an opportunity to take advantage of their “new civil right.”

In today’s atmosphere of accountability, teachers are often under pressure to teach toward particular goals and outcomes, especially in a curriculum driven by high stakes testing, which produces an intended curriculum (Telese, 1998). The intended curriculum may not emphasize meaningful mathematics teaching. A “survial trait” of teachers in this setting is to continue teaching with the textbook and worksheets in preparations for the testing. The intended curriculum is one aspect that influences mathematics teachers’ actions and decisions. Three other factors are: 1) teacher cognition (Putman, Heaton, Prawat, & Remillard, 1992), 2) previous experiences of the teacher which include pedagogical training (Pearce & Loyd, 1987) and 3) characteristics of the teacher’s environment (Haimes, 1996). These factors describe influences on how algebra is taught. For example, if a teacher has both an in-depth understanding of algebra and is pedagogically skillful, then the teacher is likely to teach the structural and procedural aspects of algebra. Moreover, teachers need support in order to teach in a manner that shifts from the traditional pedagogy to one that emphasizes meaning and understanding.

The shift away from the traditional teaching of algebra toward a more meaningful approach that emphasizes understanding is possible through the use of various pedagogical strategies. Generally, mathematics teachers may employ a combination of modes or utilize one particular mode. Farrel and Farmer (1988) have identified eight modes of mathematics instruction, which can be adapted for algebraic instruction. The Lecture mode involves a preponderance of teacher talk with some use of the chalkboard or overhead, while students are expected to listen. This may occur for an entire period or for smaller segments of the class period. The Question/Answer mode occurs when the teacher asks a question, a student response to the question is given, the teacher reacts and poses another question, and another student

responds. A Discussion is a planned student-to-student talk with occasional verbal intervention by the teacher. A Demonstration occurs when a teacher "shows" something, for example, using a cone and cylinder, having equal heights and radii, filled with rice to illustrate the relationship between their volumes. The Laboratory mode allows students to manipulate concrete objects or equipment under the direction of the teacher, for example, the extensive use of algebra tiles to gain a thorough understanding of polynomials, not just to cover an objective on the end of course exam. The category of Individual Student Projects is defined as students working individually on different manipulative activities or varied library research, or different problem-solving tasks. Audiovisual and technological activities involves the use of videos, filmstrips, laser discs, or audio tapes, and the use of graphing calculators to teach conceptual understanding of mathematical topics, not just for calculations. Supervised Practice involves students performing some practice tasks, either at their seats or at the chalkboard, while the teacher observes their progress and gives help as needed. The use of technology may be accompanied by any of the above modes of instruction and permits understanding of concepts to develop (Heid, 1996).

With the call from the National Council of Teachers of Mathematics (1989; 1998) that every student has an equal access to substantive mathematics education, and the view that algebraic knowledge is a new civil right, it becomes necessary to evaluate the extent to which reform efforts have reached various parts of the country. This study is an attempt to describe the Algebra I in a large public school district of south Texas with a predominantly Mexican American student and teacher population. The study sought to answer two research questions:

- 1) What is the predominant mode of instruction for Algebra I?
- 2) What is the level of achievement of Algebra I students?

## Methodology

Both qualitative and quantitative data were collected. The qualitative data included transcripts from classroom observations, student interviews, and artifacts. The quantitative data consisted of student achievement scores from published instruments that measured algebraic

readiness and knowledge levels, the *Orleans-Hanna* Algebra Prognosis and the *Key-Link* exam, respectively. The KeyLinks result is accompanied by qualitative data from the classroom teachers who administered the test. The purpose for this data collection phase was to gain an understanding of the students' algebraic strengths and weaknesses during the same time period as the observations.

*Setting.* The study was conducted in an urban school district. The district consists of five high schools. It is located in southern Texas along the Rio Grande River. The student population is predominantly of Hispanic descent and, primarily, has English as a second language. All of the students in the district receive free or reduced lunch. A majority of the teachers in the district are also of Hispanic descent. The study involved 13 teachers, two of them were White. At the time of the study, the district was experimenting with different formats for block scheduling. Three schools maintained the 90 minute block, one school split the block into 45 minutes for math and 45 minutes for reading, and the fifth school taught Algebra I for the whole year rather than in one semester.

*Field-observations.* In order to gather data concerning teaching methodology, a total of 13 Algebra teachers were randomly selected for observation. Observations were 90 minutes in length. The total number of observations was 84. The observations were performed in 1998 during the months of February, March, and April. The observers were university researchers and undergraduate and graduate assistants trained in the process of scripting events in the classroom, and they were expected to select at least two students to interview. The collected artifacts consisted of items such as worksheets, lesson plans, and tests. The qualitative data were analyzed using grounded theory. Codes were attached to field notes drawn from observation scripts and sorted and sifted to identify patterns of instruction.

A preliminary series of observations were conducted in the Fall of 1997, and they were used for training in the scripting and coding process. During the observation phase, the observers



met three times to discuss the process of scripting. The observers were directed to focus their scripting on the modes of instruction based on Farrel and Farmer's (1988) view of teaching methods in the mathematics classrooms. The eight modes of instruction included a) lecture (teacher talk), b) question/answer, c) discussion, d) demonstration, e) laboratory, f) individual student projects, g) audiovisual and technological activities, and h) supervised practice.

*Instrumentation.* Quantitative data were collected using published instruments, the Key-link and the Orleans-Hanna Algebra Prognosis exam. Both instruments were administered to students in the observed classes. The Orleans-Hanna Algebra Prognosis (Harcourt Brace, 199?) exam was employed to gauge student readiness for algebra, which was administered early in the spring semester. The administration took place over a period of two weeks, depending on the teacher's schedule. The *Orleans-Hanna* scores were collected in three of the five high schools.

The *Key-Links* Exam was used to obtain information related to students' algebraic knowledge. There were 10 enhanced multiple choice questions with sections for students to explain their thinking, and two items for problem solving, communication and reasoning categories, which were scored using a rubric, which ranged from zero to three. Teachers were asked to take an active role in this phase of the data collection. The teachers scored the exams. The university researcher suggested that the teachers consider going over the results with their students as a review for the End-of-Course (EOC) test, a statewide Algebra I achievement test. Also, in order to increase motivation in the students to do well on the test, the teachers offered a grade or extra credit for their students' participation. The teachers were asked to write a brief, 1-2 page, analysis of what information was learned about their students from the process. For example, they were to address whether or not their previous notions of their abilities were confirmed by their performance on *Key Links*.

*Achievement Data Analysis.* The data were analyzed using univariate one-way ANOVA with Schools as a main factor. There were three schools, A, B, and E that were using the traditional model of block scheduling, 90 minute class periods. At one other high school, D, the school established a modified version of block scheduling where the students attended algebra class for 45 minutes, followed by a reading class for 45 minutes. At the fifth school, C, all students enrolled in a year long Algebra I course. The different programs were considered the treatment effects. The dependent variables were achievement test scores.

## Results

### Qualitative Data

Three high schools had three of their Algebra I teachers observed while the other two high schools had two of their Algebra I teachers observed. Table 1 provides the date and the number of observations.

Table 1

#### *The Date and Number of Observations*

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#### **Date and Number of Observations**

February 24	x x x x
February 25	x x x x x x
February 26	x x x x x
February 27	x
March 3	x x x x
March 4	x x x x x x x
March 5	x x x x x
March 6	x x x x x

March 7	x x
March 17	x x x
March 18	x x
March 19	x x x x x
March 20	x x
March 21	x x x x x
March 24	x x x
March 25	x x x x x x
March 26	x x x x x
March 27	x x x x
April 1	x x
April 2	x x
April 3	x
April 7	x x
April 8	x
April 10	x

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The artifacts consisting of worksheets revealed that 95 percent of them were made available as supplements to the textbook, while the teachers created the remaining five percent. The topics of the worksheets varied, including finding slope, determining equations of lines, area, factoring and percents, and ratios. The topics were closely aligned with the time sequence established for the school district. The major concepts discussed were linear equations, functions, and inequalities. The tests that were collected were primarily tests that accompanied the textbook. The tests' topics coincided with the presented worksheets and classroom discussions. This data indicated that the teachers used the supplements to the textbooks as instructional aides.

*Classroom Observations.* Brief vignettes are presented, which are considered a representative sample of the observations. Teachers often began class by going over the previous assignment whether it was homework or assigned during class, lectured, had students work problems from a worksheet or text, or they were tested. For example, a teacher at School A said, "We will do last night's homework problems one through 14." The content, during this first observation, was fractions. The instruction was procedural in nature.

Teacher: What is the common denominator for  $(7m/8) - (5m/5) = 11/2$ ?

Student: 40.

Teacher: Reduce them by 8, 5, 2, multiply by common denominator. The fraction will then go away; we got rid of the fraction.

Student: Do we have to follow the pattern?

Teacher: Yes.

When the teacher moved along during the lecture, the topic switched to solving equations. He appeared to make mathematics look like a mysterious invention when solving equations. For example:

Teacher: What should we invent so that we can solve this equation,  $-2x + 11 = 10$ .

Student: -11.

Teacher: What do we invent again?

Student: The reciprocal of -2

Teacher: They look different [referring to the first equation], but they are the same after 2 extra steps.

This was followed by an assessment in the form of a quiz. The students were allowed to use "opened-books, notebooks, or whatever you need." The second observation on the second day involved the topic of percents. The objective was written on the board followed by "7-5", referring to the chapter and section in the text. Again the instruction focused on procedures and shortcuts rather than conceptual understanding of the relationship between percentages and their representation as decimal numbers. For example:

Teacher: What is  $33 \frac{1}{3}\% \times 900$ ? There was no response from the students.

Teacher: We will try to change percent into decimals.  $29\% =$  what? Take the percent sign off, move the decimal two spaces to the left. What is one-fourth of a dollar?

Student: 25,  $.25\% = .0025$ .

The student displayed confusion about both the percent sign and the rule to move the decimal two places to the left, which he did, but is incorrect. The student appeared to have a poor understanding of the percent concept and associated proportions. The teacher failed to comment on this response and continued with his lecture.

Teacher: Later we will try to change decimals into percents.

Student: We move decimals 2 places to the right.

The teacher did not respond to the comment, although it was the correct rule.

Teacher: This worksheet will be easier if we follow simple rules.

In the below passage, the teacher's comments reflect a poor perception of the students' willingness to accomplish tasks, as well as his use of the word "try" in the above dialogue.

Although the teacher stated the necessity to eliminate the percent sign, he chastised the students for wanting to get rid of the symbol.

Teacher: Please, lazy people don't be just getting rid of the percent sign. Change  $7/8$  into percents.

Student: The bell is going to ring.

Teacher: Who cares? Beto, Celia, Liza, Frank, what do you get?

Student: Just do it for us.

Student: No, No, wait a minute.

Teacher: Here is Supermario,  $7/8 = x/100$  .

The lesson seemed to come to an abrupt end. The students, with the exception of Supermario, left the room apparently confused about percents. The teacher emphasized rules and procedures for finding percent, and the cognitive demand was low. Conceptual development was not apparent in this episode. Although one student understood that one-fourth of a dollar is a quarter, the instructional focus was teacher demonstration of the problems while the students watched. Moreover, the student's comment, "just do it for us" is reflective of an apparent dependency on the teacher to do the work for the students.

In a second teacher's class, the topic of slope was observed as the main focus of instruction for two weeks. The teaching modes were Lecture Demonstrated Practice (LDP) and Questions/Answer. The lesson presentation focused on procedures. The class began by going over homework. The questions she asked were low-level, recall questions. A representative question is "How many points determine a line?" Referring to this equation,  $\frac{x}{2} + \frac{y}{2} = 12$  , the teacher proceeded:

Teacher: What type of numbers must be for  $x$  and  $y$ ? If we put 24 in for  $x$  what must  $y$  be? What other numbers can we put in for  $x$ ? If  $x = 5$ , what is  $y$  equal to?

After a pause of a few seconds,

Teacher: For the last three days, we have been finding the slope of a line.

She used, during her lecture, a mental image of a skier for slope. This was an incongruent metaphor to use since a vast majority of students could not relate to the example because the local, tropical climate does not foster snow skiing. She then proceeded to tell the students about her ski trip to Colorado. This was followed by the procedure of finding slope. A typical example is the following:

Teacher: From here we have to go over then up, what do we get? What direction did I go? Yesterday, we went in depth and there is another formula for slope. Remember we wrote it  $(-10, 1) = (x_1, y_1)$  and  $(-5, 5) = (x_2, y_2)$ , and I sang the song, plug it in, plug it in. We used the formula  $(y_2 - y_1) / (x_2 - x_1)$ .

The teacher proceeded in a similar manner until the students found coordinate pairs for the equation. This was followed by supervised practice. She assigned problems from a worksheet and walked up and down the aisles and repeated the lesson, regarding plotting points and lines, almost to every student. For example, she was heard to say, "Pick a point, go up or down to the next point . . . put them in your formula" [for slope], at another student's desk she said, "pick a point, find the change in  $y$  over the change in  $x$ ." As the class was coming to an end, she then assigned homework from the text, a few minutes following the assignment, the bell rang ending the class. While she was supervising the students' practice time, the teacher attributed their low performance on a previous test to not paying attention in class. The teacher was making an attempt to motivate the students to pay attention to the lecture so they would be able to pass the next test. Most of the students disregarded the comment as evidence from statements such as, "ah miss, we are always having a test," or "we don't care about no test." The above teaching event illustrates the procedural nature of the instruction. The concept of slope

was presented in a decontextualized fashion as a plug and chug process rather than a rate of change.

A teacher from another high school was observed teaching ordered pairs and slope. An example of the teacher's supervised practice routine dealt with ordered pairs. Following a brief explanation of ordered pairs, she asked, "What is an ordered pair?" The students responded in unison, "(x, y)." The teacher then assigned problems two to 10 from the text. "You must work hard because you only have 4 weeks to get ready for the end-of-course exam." The teacher's statement suggests an urgency on the teacher's part in preparing students for the test. She then moved around the room helping students.

The teacher's knowledge of mathematics appeared shallow as a result of the following encounter. A student asked, "Why do you have to divide by the negative one?" The teacher responded, "You always want what ever you are solving to be positive. Which, in this case, the y was negative but it became positive after being divided by a negative." She proceeded to solve,  $x + 2y = 8$ . After the students worked on solving equations, the teacher began to plot various points. She said, "Tomorrow you will have 50 ordered pairs to plot. During the lesson the teacher mentioned that the slope is the change in y over the change in x. Another method is to use this formula,  $m = (y_2 - y_1)/(x_2 - x_1)$ , when you have two points. A student asked the question, "Why can't we put (x1, y1) first? The teacher said, "No, you have to do it the way the formula says to do it." The students experienced a rule orientation to slope rather than encountering the slope concept as a rate of change. She used the LDP mode of instruction followed by supervised practice during a lesson where students had to identify the slope and y-intercept from equations in the form of  $y = mx + b$ . Using "m" and "b," students were asked to graph the line. She asked students to identify them, graph the line, and there was no response. So, she said, "To graph it, first plot the y-intercept, then follow the slope."

The above examples illustrate a heavy reliance on procedural perspective rather than a structural perspective, and gimmicks for teaching algebra. The instruction focused on procedural processes, which appears to contribute to the students' difficulty in understanding or applying the

slope concept. The students could not graph the lines when given the slope-intercept form of the equation, which is an indication of their lack of understanding of what a line represents and perhaps failure to recall the procedure. The emphasis was on memorizing the formula rather than on an understanding that slope is a rate of change.

In a third high school, a similar instructional mode, which was procedural in nature, continued during a lesson on operations with polynomials. A typical lesson, in this case, dealt with factoring and multiplication of polynomials using FOIL. The methodology was coded as LDP, followed by a worksheet. The teacher emphasized the short cut method for the difference of squares, squaring the first term and the second term when the polynomial is in the form of  $(a + b)(a - b)$ . The teacher presented 16 examples. Without prior warning, the teacher used algebra tiles and associated handouts. The examples dealt with finding the perimeter and area of a rectangle with dimensions of  $(2x - 3)(x - 2)$ . The teacher connected the use of FOIL to finding the area of the rectangle. The teacher reminded students about the benchmark test on Friday. For the remaining 40 minutes of class, the students were required to complete a worksheet.

The teacher used over half of the class presenting examples that were representative of the state's algebra achievement test. On this day the topics included, fractions, equations, perimeter, and proportions. The teacher asked "How many of you find these hard?" A student responded, "They are not hard, but the thing is if we will remember them on the test." This exchange illustrates the lack of deep understanding and confidence students have in themselves. The idea that they are not hard to do, but will not remember how to do them is reflective of the many procedures the students are expected to recall.

The following indicates that yet another teacher was teaching to the state's algebra achievement test, which lasted for the entire class period.

Teacher: The test from last year had 34 problems and we worked on six. Could you do these on the test? Last year, about two students per class were passing. We're going to work on yesterday's work now, but put these papers in you notebook. OK, let's go over yesterday's work. Let's go over number nine,  $7x + 2y = 10$ , translate this into slope intercept form. When they say to translate, it means put it in  $y = mx + b$  form. The



answer should look like this:  $7x + 2y = 10$ ,  $2y = 10 - 7x$ ,  $y = 10 - (7x)/2$ . After you find this, they want you to find the slope. What is it? How do we find the intercept class?

There was no response. So he explains once more how to find the intercepts. He does another problem and asks for students to pay attention and the bell rings. This example further illustrates the LDP methodology. The teacher, in referring to the assignment's directions, used the word "they", implying that it is not him who wants them to finish the worksheet, and it's us against them.

The LDP and procedural teaching pattern, which was followed by Supervised Practice, is evident in this brief excerpt.

Teacher: Yes, you will be using yesterday's techniques. There should not be any fractions.

He walked about the room and talked to a student in Spanish, then addressed the whole class,

We have to change the signs and explains further that the system  $9x - 10y = 2$  and  $9x + 2y = -22$  may be solved by multiplying the second equation by negative one and then adding to eliminate the  $x$  variable and finding the value for  $y$ , which is then substituted into the first equation. Any questions on this method?

He waited, everyone said no.

Teacher: Now copy the problems on the front board.

The class then worked the problems while he walked around the room. He assigned 12 problems from the text, 2 -24 even. The observer summarized his observation by mentioning that the teacher wrote problems on the board and worked them out asking questions; most students seemed to understand and appeared focused on the lesson.

During an observation of another teacher, it was noted that the procedures to graph a line were written on the board:

How to graph a line (equation) by using the slope-intercept form ( $y = mx + b$ ) of a linear equation.

- I. Be sure the equation is written in the slope-intercept form ( $y = mx + b$ ). If not, solve for  $y$ .
- II. Identify the slope and  $y$ -intercept of the point.

## III. To graph the line

- A) Plot the y-intercept point which is always located on the y-axis.
- B) From the y-intercept point, use the slope to find the steepness of the line, connect the points and draw a line.
- C) Name the line.

The following shows the degree of emphasis placed on teaching procedures. During a lesson on graphing lines, the teacher used the LDP methodology. "I am going to do some problems for you. You need to find the equation of the line passing through points A and B. First, you need to draw a line through the two points. Then you need to find the y-intercept. The next step is to find the slope. Finally, you use the slope and y-intercept to write the equation. You will now do the rest on your own. Remember the steps you need to follow. First, draw the line, second, find the y-intercept. Third find the slope. Fourth, write the equation. If you follow these steps it will be easy for your to write the equation."

**Student Interviews**

The students were asked five questions, a) What is algebra? b) What do you do in your algebra class? c) What is easy for you to do in algebra? d) What is difficult for you to do in algebra? e) Are you a good algebra student? Why or why not? Their responses were categorized and are presented in Table 2.

Table 2

*Student Interview Responses*

Question	Code	Selected Responses
What is Algebra?	Arithmetic	(pp. Algebra is math problems. (qq. Solving problems. Arithmetic. Boring. (rr. Numbers put into questions.
	Generalize	♦ It's just math sometimes we work with numbers.

	d Arithmetic	<ul style="list-style-type: none"> <li>◆ In algebra, we work more with letters than in simple math.</li> <li>◆ Numbers and variables.</li> <li>◆ Algebra is anything that has numbers and variables.</li> <li>◆ It's just math with adding, subtracting, etc. but a little more complicated.</li> </ul>
	Effort	<ul style="list-style-type: none"> <li>◆ After working a lot with it, you get used to it and get better at it.</li> <li>◆ A class that prepares you for the big test.</li> </ul>
	Mental process	<ul style="list-style-type: none"> <li>◆ Algebra is something that has to do with logical thinking and problem solving.</li> </ul>
	Avoidance	<ul style="list-style-type: none"> <li>◆ It is a very hard subject...it should not be required in any type of school.</li> <li>◆ I don't know. I don't find its point! I don't know how we will use it in the future.</li> <li>◆ I don't really know what it means.</li> <li>◆ Algebra is a subject where one wastes his time.</li> </ul>
What do you do in algebra class?	Various Topics	<ul style="list-style-type: none"> <li>◆ Fractions, variables, system of equations to find intersections, plot points, draw lines, write equations.</li> <li>◆ Solve problems, find slope, y-intercept, write equations.</li> <li>◆ I do math.</li> <li>◆ Graphing, finding slope, lines, worksheets, assignments, quizzes, and tests.</li> </ul>
	Work	<ul style="list-style-type: none"> <li>◆ We do what the teacher tells us to do.</li> <li>◆ Too much work!</li> <li>◆ Work!</li> </ul>
	Solve Problems	<ul style="list-style-type: none"> <li>◆ Everything we've done in math since elementary, but you know it's different because in algebra you use variables.</li> </ul>
	Nothing	<ul style="list-style-type: none"> <li>◆ Sleep I took it in the 8<sup>th</sup> grade and made a C.</li> <li>◆ I just sit there...Talk to my friends.</li> </ul>

(Table 2 continues.)

Question	Code	Selected Responses
What is easy for you to do in algebra?	Procedures	<ul style="list-style-type: none"> <li>◆ Addition</li> <li>◆ The basic stuff</li> <li>◆ Graphing</li> <li>◆ Simplifying, it is fun looking for numbers that will be equal to the answer you have at the end of your equations.</li> <li>◆ Everything is easy if you do it one step at a time.</li> </ul>
	Nothing	<ul style="list-style-type: none"> <li>◆ This is my third time to take this class.</li> <li>◆ It's all so complicated and hard to understand.</li> </ul>
	Everything	<ul style="list-style-type: none"> <li>◆ Most of it actually.</li> <li>◆ Algebra is very easy as long as you listen to the teacher when he explains.</li> </ul>
What is difficult?	Procedures	<ul style="list-style-type: none"> <li>◆ Division and fractions.</li> <li>◆ The signs are hard.</li> <li>◆ Percents</li> <li>◆ I have difficulty when you multiply and cross multiply, I mix them up. I cancel the wrong number.</li> </ul>
	Formulas	<ul style="list-style-type: none"> <li>◆ I have trouble finding areas of circles and perimeters and all that stuff because of so many formulas to remember.</li> </ul>
	Ratios	<ul style="list-style-type: none"> <li>◆ I'm not able to understand it. I've never understood them. Many teachers have explained it and I've never been able to understand them.</li> </ul>
	Everything	<ul style="list-style-type: none"> <li>◆ When I go to class, I don't understand anything.</li> <li>◆ I get frustrated with all these letters.</li> </ul>
Are you a good algebra student?	Confident	<ul style="list-style-type: none"> <li>◆ Yes, great.</li> <li>◆ It doesn't take too much time for me to learn what the teacher is telling us.</li> </ul>
	Grade Reliance	<ul style="list-style-type: none"> <li>◆ Average, my grades are in the 80's with a few 70's and 90's.</li> <li>◆ Very good, I have 80's and 90's.</li> <li>◆ Sort of I make 70's.</li> </ul>
	Definite	<p>(ss. No, I seriously doubt that I am a good student. If I were I think that I would have already advanced to another high school course, don't you?</p> <ul style="list-style-type: none"> <li>◆ No, I don't know why, I've never been a good math student.</li> <li>◆ Yes, since I got here, I have been passing, so I think that I am a good student.</li> </ul>
		<ul style="list-style-type: none"> <li>◆</li> </ul>

In regard to question one, most of those interviewed saw algebra as generalized arithmetic.

The students seem to recognize that algebra involves variables. However, it appeared that the students do not really view the 'letters' as mathematical objects with an understanding that variables have an important role in Algebra. There were students who viewed algebra as arithmetic, uncertain about its nature, or avoided the question. For these students, difficulties may arise because they may be unable to view algebraic expressions and equations as objects so that higher level processes may be carried out. Consequently, these students are unable to recognize the structural features of algebra, the different set of operations that are carried out, not on numbers but on algebraic expressions (Kieran, 1990).

A view of how Algebra I was taught can be seen through the eyes of the students. They frequently reported topics that were under current study or specific processes when asked what they do in algebra. The idea of what is done in algebra equates to the procedural aspect of algebra, the memorization of rules and processes that are applied to algebraic expressions, failing to achieve structural understanding. One student described the class routine, the teacher going over examples and then assigns problems with a test on Friday. Other students reported that they solve problems in algebra from a textbook or worksheets, and that there was an emphasis placed on practice. The phrasing of the question may have had an effect on the mode of response relating to specific class activities.

A majority of the responses to what is easy in algebra was related to specific processes. The topic mentioned most often was graphing and plotting points. Apparently, a strong emphasis was placed on this topic. Many students appeared confident in their ability to plot points. The process approach to learning algebra is indicated by statements like, "algebra is easy when you do it one step at a time and you listen to the teacher when they explain." The responses also hint at the methodology that students are experiencing, lecture and doing problems. An implication is that graphing could become a vehicle for teaching function concepts rather than as a process to memorize for the End-of-Course exam.

There were a variety of areas that students reported having difficulties. For some students it was fractions and its related concepts such as ratios and proportions. This lack of understanding may contribute to their failure in other areas of algebra, such as solving fractional equations, or inverse and direct variation. For other students, the areas of difficulty include the use of formulas, factoring, and integers. The difficulties cited here may be indicative of a lack of understanding related to the structural features of algebra.

Typically, students responded to question five by referring to their grades in order to determine whether or not they were good algebra students. If students are getting 80's and 90's, then they see themselves as good algebra students. If they are making 70's than they are average students. The students could not access any other information concerning what they are capable of doing in class other than seatwork and tests. This indicated that there is a need for other forms of assessment that communicates to the students their strengths and weaknesses. The students appeared lacking in this knowledge, rather they are relying on another's view, the teacher, in order to determine whether or not they are good algebra students. One student, however, did possess a metacognitive view of her ability when she said, "I don't understand anything, so I'm not a good student." She did not refer to her grade average.

In summary, the students seem to have had a narrow view of algebra, in particular, they viewed graphing and plotting points as merely procedures and regarded algebra as a subject that requires practice and hard work. The idea of practice reflects the procedural nature of the instructional practices. Success occurs when they pay attention or "listen to the teacher". They are having difficulty seeing the structural features, how the various topics relate to one another, and algebraic expressions are seen as abstract entities. This was evident when students responded to having problems with integers, fractions and formulas. Different students reported various areas of difficulties and successes, perhaps this is due to when they pay attention or when they do not pay attention as they indicated. However, the ratio concept does appear to be a common thread in the difficulty category, and graphing the common success story.

## Quantitative Data Results

*Student Achievement.* Table 3 presents the mean scores and standard deviations for the Orleans Hanna and the Key-Link exam results. In general, according to the *Orleans-Hanna Prognosis Test Manual*, it can be expected that nearly two-thirds of students with a raw score near 70 or above will pass algebra. There were only three sites that administered the *Orleans-Hanna*, A, B, and D, School C only had five students, and School E did not administer the exam. There was a wide variation in student predicted ability, as indicated by the large standard deviations.

Table 3

*Mean Scores for the Orleans-Hanna and Key Link Exams*

School	Orleans-Hanna Raw Score	Key Link Percent Correct
A	60.4 (16.64) n = 47	42.7 (30.5.) n = 26
B	68.7 (14.36) n = 52	42.6 (22.8 ) n = 54
C	Not available	31.2 (19.2) n= 33
D	62.8 (19.87) n = 61	38.8 (18.2) n = 18.2
E	not available	42.7 (18.7) n = 30

The one-way ANOVA, conducted on the Orleans-Hanna, indicated no statistically significant differences among the three schools; the students had similar capabilities in algebra  $F(2, 156) = 2.28, p > 0.05$ . The mean scores indicated that a majority of the students were under-prepared to enter algebra class.

The percentage of students with correct responses on the KeyLinks exam are presented in Table 6. The large standard deviations reflect the students' wide variety of algebraic skills and knowledge. The one-way ANOVA, with KeyLink scores as the dependent variable, showed no statistically significant differences,  $F(4, 206) = 1.83, p > 0.05$ , among the school's programs. Regardless of which type of structure of the block scheduling, the students performed similarly on the KeyLinks exam.

Table 4 presents the topics of questions one to 10 and the percentage of responses to each item. For most of the items, approximately one-third of the students chose the correct response. and, nearly one-third of the students did not choose an answer for the items. The remaining one-third chose an incorrect answer. In particular, item four, related to solving inequalities, had the highest percentage of correct responses. Item nine, determining a quadratic equation from a picture of algebra tiles, had the next best percentage of correct responses. This indicated that 37% of the students tested are able to read a diagram representing algebra tiles, and to translate the information into an equation that represents the area of the figure. Again, one-third of the students on items six, seven, and eight responded correctly. This indicated that students can match information from a table to its equation (33%), identify a slope and its line (32%), and match an equation of a function to its graph (34%). Consequently, there is a one-third, two-thirds breakdown in the students' ability in Algebra. Hence, one-third of the students, apparently, seemed to understand the basic concepts of algebra while two-thirds of them do not have a good understanding.

The general student performance on the open-ended questions was dismal. The students rarely attempted the problems. There were six items that were scored using a four-point scale rubric, ranging from zero to three, paired with each of the three categories: problem-solving, communication, and reasoning, each of which had two tasks. Two schools had means of zero for the communication problems, while the other three schools had means of 0.10, 0.26, and 0.26. The problem solving means were zero for three schools and 0.07 and 0.15 for the remaining two



schools. The student performance on the reasoning problems was slightly better with means for the schools of 0.0, 0.01, 0.05, 0.46 and 0.79.

Table 4

*Percentage and Frequency of Responses to each KeyLinks Item*

Objective	Percentage of Responses	Frequency
<b>1. Evaluate Polynomials</b>		
a	16.9	43
b*	33.6	89
c	12.5	33
d	8.3	22
no response	29.4	78
<b>2. Identify an equation or inequality that represents a problem situation</b>		
a	13.6	36
b	14.3	38
c	10.6	28
d*	32.8	87
No Response	28.7	76

Table 17 (cont.)

Objective	Percentage of Responses	Frequency
<b>3. Solve Equations with Radicals</b>		
a	21.5	57
b	11.3	30
c*	29.8	79
d	9.1	24
No Response	28.3	75
<b>4. Solve Inequalities</b>		
a	16.6	44
b*	41.9	111
c	9.4	25
d	5.3	14
No Response	26.8	71

5. Use factoring to Solve Problems		
a	25.3	67
b*	23.0	61
c	12.8	34
d	6.8	18
No Response	32.1	85

(Table 4 continues.)

Objective	Percentage of Responses	Frequency
6. Functions-Identify the equation of a function given a table of values		
a	16.6	44
b	4.5	12
c*	32.8	87
d	14.7	39
No Response	31.3	83
7. Functions-Identify the graph of a function given its slope.		
a	12.1	32
b	23.0	61
c*	31.7	84
d	6.8	18
No Response	26.4	70
a	7.5	20
b	15.5	41
c	11.3	30
d*	34.3	91
No Response	31.3	83

9. Problem Solving strategies-Solve problems using non-routine strategies		
a*	37.0	98
b	10.6	28
c	4.9	13
d	16.2	43
No Response	31.3	83
10. Problem Solving strategies-Solve problems using non-routine strategies		
a*	23.0	61
b	14.3	38
c	18.5	49
d	10.2	27
No Response	34.0	90

\*Correct Answer to Item

*Teacher Comments*

From the results of the KeyLinks test, the teachers were asked to describe their students' algebraic abilities. One teacher reported that some of the students did not attempt the problems, which according to the teacher, "...is consistent with both their efforts in the classroom and their attendance." She summarized her class results by concluding that her students "had more success with the graphing type problems, including the inequalities because this was the most recently covered material in this class."

Another teacher, from the high school that splits the 90 minute block into 45 math/45 reading, concluded that the low scores from his class were due to four factors, "i) test was given to an inclusion class, 16 out of 25 students who took the test (65%) are classified as special education, ii) most of these students have a weak math background, iii) most of these students have a weak reading and writing background, and iv) retention is very low." He described the rest of his class as "ESL students, some of which have some problems with the English language. The students were further described as having "a very weak math background . . . especially the special education students whose math level is third or fourth grade." A related issue to the low scores was attributed to the "extensive reading and writing" required on the test. The teacher also mentioned that his students had poor memories, "Most of the students I am teaching have a poor memory retention. In his view, one factor that contributes to poor retention was related to the length of his class, 45 minutes, "Since student spend less time in my class we do not do much of the guided practice; therefore, more independent practice is required from them, ... less than 50% of this class do their assignments."

Students from another class were confounded by the format of the exam. Their teacher stated, "When the students were given the test, most of them were flabbergasted." His rationale was similar as the previous teacher's comments, "They saw too many words, what was worse was that the words were in English. . . for some reason, reading and writing seemed to be very scary

for the students. . . exposing them to more word problems will alienate the weaker student even more because of their low self-esteem." The teacher seems to be equating the lack of language skills to mathematical ability and self-esteem.

Turning to another teacher at School B, she reported that 21 out of 25 students attempted the test. Teachers were informed that they may use more than one day to administer it. However, she choose to administer it in one day, "Given only one 90-minute period to work the test individually, the students may not have had enough time to try all the problems." The teacher reported that students used various methods or tools such as the TI-81, guessing, and the process of elimination on the test. As a result of her experience, she plans to "revise the curriculum to include more writing about the learning they are accomplishing and to introduce more illustrated/elaborated written problem situations."

Although the objectives on the KeyLinks correlate with the Algebra EOC exam, one teacher suggested that they did not. He stated that the content was far above the capabilities of his students, "The instrument tests information at a level far above the level to which Algebra is learned by most students." He then suggested that this was caused by the textbook used in the district, "The textbook that is currently used in the district's algebra courses does not facilitate the teaching or learning of material at this level." This teacher seemed to express frustration and failed to really analyze what were his students' strengths. The curriculum is to blame for their lack of performance.

A sixth teacher provided a report from School D. His students followed the same pattern upon encountering the test. He reported that "many of his students were complaining that the test was too hard." His students' attitude took a turn for the worse when he told them that he "could not help them on the test, so the students automatically gave up before starting." This teacher felt that this test was within their sphere of ability. "I think that most of these problems were within the grasp of their abilities, but the students used their usual excuses not to think. It boils down to the students not wanting to think." The teaching methodology of which the students were

accustomed is reflected in the following quote, "These students have been accustomed to mimicking examples and if they are not given examples their whole world stops."

A teacher from School A provided some student quotes in her report. "What if we knew how to solve it, but didn't know how to show it?" "I knew what to do, but couldn't say how." "How will the 'clouds' be graded?" "If you didn't know how to put it in the calculator, you could get the square root problem wrong." "Numbers 11 to 16 [the open-ended questions] were hard!" The teacher also revealed that in her opinion, "This assessment was a fair indication of the problems that will be on the End-of-Course exam." The students appeared to say that they knew the concept, but they could not explain it. This leads to the question: Do the students truly know the concept if they can't explain it?

## Discussion and Conclusion

This study supports research (e.g., Kaput, 1999) that describes algebra instruction which focuses on procedures disconnected from both other mathematical knowledge and from experiences that students can relate to. The cognitive demands of the tasks that were presented to the students in this study were low or nonexistent. This is more problematic for minority students who suffer ill-effects from the gate keeping nature of Algebra I. Chamblis (1993) reported that Hispanic students are prevented from entering the mathematics and science career pipeline.

The result of this study indicated that the predominant teaching methodology was Lecture Demonstrated Problems mixed with Supervised Practice. The observations revealed that the teachers maintained a procedural focus, in a small number of cases, the teachers attempted to teach at a conceptual level. It appears that the teachers' expectations include a belief that the students cannot grasp the material unless it is presented in a cookbook fashion. Classroom discourse was held to a minimum as most teachers answered their own questions and seldom required students to explain their thinking or share their ideas. The communication that did occur was related to students answering the teacher's low level factual questions. In other words,

discussion was generally absent in the algebra classroom. The laboratory mode was non-existent. One teacher used algebra tiles, but not in the fashion that would assist in the development of the conceptual understanding of polynomials. Tools are rarely used such as graphing calculators. When they are used, they are used in a haphazard manner; basically, they were used for calculating or as a crutch. The teachers and students seem uncomfortable with their use to enhance conceptual development.

In light of the current emphasis being placed on accountability and high stakes testing, i.e., the Algebra End-of-Course Exam, the teachers addressed and in some cases, taught directly the questions from both the district's Benchmark test and the End-of-Course exam. There appears to be a sense of urgency regarding these exams. The teachers demonstrated that if they cover the questions in class that their students will do well, however, this is not the result. It seems that the students are overly tested and have a lack of interest in algebra.

Classroom testing, with the exception of one teacher, includes going over the material prior to the test and offering any type of assistance. This may develop in the students the tendency to rely on teacher help, notes, books, or other assistance. As a result, when the End-of-Course exam time arrives, the students do not have the confidence in themselves or are permitted to use any resources. There appears to be a great deal of dependency on the classroom teacher. Student autonomy in learning also is lacking. There was very little evidence of student persistence while problem solving. In fact, there was very little problem solving or application oriented teaching, with the exception of one teacher who presented five quadratic function application problems.

Very few students were able to respond to the open-ended questions, and fewer even made attempts to solve them. This indicated that the students were not confident in their ability and that their self-perception was based on grades rather than from their meta-cognitive reflection on their ability. The majority of students in this study have English as a second language. Their lack of confidence to attempt the open-ended problems may be due to the language issue, as well as, a lack of opportunity to develop the higher order thinking skills. The

algebra content area in which the students have strength, revealed by KeyLinks, is in graphing or linear functions. With this ability, the teachers could capitalize on the students' strength and use real world examples of functions and how they pertain to the students' daily lives in some manner.

The teaching of algebra for understanding should be the goal of school algebra instruction. A classroom environment that allows all students to learn with understanding should be a priority (Kaput, 1999). As a result of this study, it is recommended that instruction in Algebra I for all students, and in particular, minority students, include aspects that include the use of informal knowledge, application to real-world settings, applying mathematical thinking, and discourse. Technology, for example, in the form of graphing calculators should be utilized to aide in the teaching of linear functions. The development of contexts built around student experiences would assist students in maintaining interest and provide a vehicle for increasing motivation. In conjunction with the procedural aspects, algebra instruction should begin to focus on strengthening the structural perspective. If language is an issue, than a sheltered English approach to teaching algebraic vocabulary may be appropriate. Algebra instruction should involve active learning of concepts and procedures through discourse, and making connections to students' experiences and applying the concepts and skills to other subjects. A strong background in arithmetic is necessary for success in algebra instruction (Pillay, Wilss,& Boulton-Lewis, 1998). Another aspect for establishing an algebra classroom environment where students can be successful is building on arithmetic and pre-algebraic skills. Those who have a weak background should have the opportunity to strengthen their arithmetical skills. Unless changes are made in how algebra is taught, a road-block will continue to exist on the paths to higher mathematics and careers in mathematics or science for minority students.



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