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ABSTRACT

This paper describes an intensive 3-year classroom- and district-based study of the process of integrating the development of algebraic reasoning into elementary school mathematics in an educationally underachieving school district based on a year-long pilot effort in 1997-98 with two 5th grade teachers and one 6th grade teacher. In 1998-99, the program involved working on a bimonthly basis with approximately 20 grade 2-5 teachers from 8 different elementary schools. Future research will link extended elementary school classroom mathematics teaching in a third grade class to pre-service education and district-wide in-service teaching. This will enable continued documentation of the following: (1) both pre-service and in-service teacher development of algebraic reasoning; (2) the process of "algebrafying" elementary mathematics instructional materials; (3) the evolution of classroom practice and culture towards forms that support the development of students' algebraic reasoning; (4) student learning, as measured by progressive statewide 4th and 8th grade tests and supplementary assessments; (5) the design of appropriate experiences for pre-service teacher development; (6) the dual shaping of teacher-support materials for use with pre-service and in-service teachers; (7) the process of implementation of innovations as they spread across a district with 30 elementary schools (Fall River, Massachusetts); and (8) a comparison with publicly available student assessments from a very similar district in the region (New Bedford, Massachusetts). This paper provides the larger view of algebra reform on which current and planned work is based and then reports on pilot work and current work relating to (2), (3), and (7). An Appendix provides the "Details of Pilot Work with Grade 5-6 Teachers." (Contains 47 references.) (CCM)

ALGEBRAIC REASONING IN THE CONTEXT OF ELEMENTARY MATHEMATICS: MAKING IT IMPLEMENTABLE ON A MASSIVE SCALE[1]

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The Context for the Research Reported in this Paper

We are engaged in an intensive 3-year classroom- and district-based study of the process of integrating the development of algebraic reasoning into elementary school mathematics in an educationally underachieving school district based on a year-long pilot effort in 1997–98 with two 5th grade teachers and one 6th grade teacher. Currently, in 1998–99, we are working on a bimonthly basis with approximately twenty grade 2–5 teachers from eight different elementary schools. Next year we will link extended elementary school classroom mathematics teaching in a third grade by the second author to pre-service education and district-wide in-service teaching by both authors and Project-developed teacher leaders. This will enable us to continue to study and document the following:

(a) both pre-service and in-service teacher development of algebraic reasoning;

(b) the process of "algebrafying" elementary mathematics instructional materials;

(c) the evolution of classroom practice and culture towards forms that support the development of students' algebraic reasoning;

(d) student learning, as measured by progressive statewide 4th and 8th grade tests and supplementary assessments;

(e) the design of appropriate experiences for pre-service teacher development;

(f) the dual shaping of teacher-support materials for use with pre-service and in-service teachers;

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(g) the process of implementation of our innovations as they spread across a district with 30 elementary schools (Fall River, MA);

(h) a comparison with publicly available student assessments from a very similar district in the region (New Bedford, MA).

This paper will provide the larger view of algebra reform on which our current and planned work is based and then report on pilot work and current work relating to (b), (c), and (g).

Confronting America's Algebra Problem

Our larger research program confronts a major national problem, what we term "America's Algebra Problem," which is the intolerably poor outcomes of our highly dysfunctional, late, abrupt, isolated, and superficial approach to school algebra, and our collective inability to move beyond it. Our intent is to make progress on the Problem by contributing to a badly needed research-based Early Algebra Story (described below). Solving this Problem ensures simultaneous progress on four major national goals:

- opening curricular space for 21st century mathematics desperately needed at the secondary level, space locked up by the 19th century national high school curriculum now in place;
- adding a new level of coherence, depth, and power to K-8 mathematics;
- eliminating the most pernicious and alienating curricular element of today's school mathematics: late, abrupt, isolated, and superficial high school algebra courses;
- democratizing access to powerful ideas by transforming algebra from an engine of inequity to an engine of mathematical power, thereby making opportunities for achievement more equitable.

Changes of this magnitude require deep rethinking of the core algebra enterprise and will not be achieved by minor adjustments such as attempts to "fix" Algebra I and Algebra II, starting Algebra I a year earlier, or legislating two years of algebra for all. Solving America's Algebra Problem involves deep curriculum restructuring, changes in classroom practice and assessment, and changes in teacher-preparation—each a major task. Further, each must be achieved within the capacity constraints of the teaching population, within the limited time and resources available for in-service and pre-service teacher development, and within the constraints of widely used instructional materials. This is why we have adopted a "critical nexus" approach that deals *simultaneously* with (1) the evolution of ("algebrafying") existing curricular materials, (2) the enhancement of classroom practice, and (3) the coordination of pre- and in-service education reform.

Aspects of the Algebra Problem

Recent TIMSS results have shown (a) US students' declining performance in mathematics from 4^{th} to 8^{th} to 12^{th} grades (U.S. Department of Education, 1997a; 1997b; 1997c); (b) close connections with a disorganized and superficial curriculum, especially from the middle grades onward (McNeely, 1997); and (c) classroom teaching practices focused on procedural skills instead of conceptual understanding (U.S. Department of Education, Office of Educational Research and Improvement (OERI), 1997a; 1997b). In significant measure, these disappointing findings can be traced to our nation's approach to algebra because of the central role of algebra in mathematics and the grossly dysfunctional approach to algebra that has been accepted as the norm in the US for over a century. (We note that this approach occurs virtually nowhere else, least of all in the mathematically successful countries of the world.) Beyond its role in alienating the general citizenry from mathematics for generations, the traditional approach to algebra has also served as an engine of inequity through its role in academic tracking (Moses, 1995; Kaput, 1998).

Across the country, well-meaning repair attempts at the middle school level have taken the form of pre-algebra courses or opportunities for students in 7^{th} or 8^{th} grades to take versions of the traditional Algebra I courses. Moreover, except in recent reform documents (e.g., National Council of Teachers of Mathematics (NCTM) 1998b) and new standards-based curricula (e.g., see the recent American Association for the Advancement of Science (AAAS) review of middle school curriculum materials), these attempts and standards have not substantially involved changes at the earlier grade levels, especially before Grade 5. Indeed, the appropriateness of this approach to algebra reform has been called into question in emerging standards. In particular, the NCTM Principles and Standards 2000 draft document (NCTM, 1998a) suggests that moving

Algebra I into middle school is *not* a good idea, in part because "students are likely to have less opportunity to learn the full range of mathematics content, especially topics in geometry and data analysis, that are expected in the middle grades" (p. 214). In addition, it points out that "there is considerable value in connecting the learning of algebraic concepts and skills to the study of other mathematical content topics" (p. 214). Finally, the strategy of pushing the traditional first algebra course back into 8th grade and leaving virtually everything else untouched leaves little room for desperately needed restructuring and modernization of the high school mathematics curriculum to include the critically important new mathematics for the next century (Kaput & Roschelle, 1998; Romberg & Kaput, in press; NCTM, in press).

Our work, following a line of work begun by the PI a decade ago (Kaput, in press; 1995a) and supported by OERI through the National Center for Research in Mathematical Sciences Education (NCRMSE) and more recently by its successor, the National Center for Improving Student Learning & Achievement in Math & Science (NCISLA), directly attacks this national problem by changing teaching and learning *prior to* the middle grades.

A Research-Based (Early) Algebra Story: Our Central Research Objective

Just as algebra has acted as a constricted gateway to significant mathematics and all that follows from mastery of that mathematics, algebra reform is proving to be the gateway to K-12 mathematics education reform for the next century (Kaput, 1998). Among the reasons that algebra reform has foundered in the past decade is the simple fact that no research-based consensus has developed around a coherent "Algebra Story" on which sustained reform could be based. Such a story would include:

(a) a sufficiently explicit account of the nature of mature algebraic reasoning – a content analysis of algebraic reasoning;

(b) descriptions of what its immature forms may take (i.e., what does it look like, especially in the context of elementary school mathematics?);

(c) descriptions of what its teaching looks like and what classroom practices and cultures promote its development;

(d) descriptions of plausible developmental trajectories for the different forms of algebraic reasoning;

(e) examples of the kinds of instructional materials that render it learnable by mainstream students in ordinary classrooms, especially as a means by which it can be integrated with, and hence deepen, elementary mathematics;

(f) how it might be made intelligible to and implementable on a massive scale by typical K-8 pre-service and in-service teachers.

We have made substantial progress on (a) and some progress on (b) and (c)—see (Kaput, in press) and the papers downloadable from www.simcalc.umassd.edu/EABook.html, which provide illustrations of (b) and (c) based on (a)—but relatively little on (d)–(f). *Furthering progress on (b)–(f) constitutes our Central Research Objective.*

Critical Nexus Strategy: Support Teacher Development & Curricular Evolution by Enabling Teachers to Algebrafy Existing Instructional Materials & Classroom Practice

A Content Analysis: Broader and Deeper Conceptions of Algebra Is a Critical First Step

In order to describe our "algebrafying" implementation strategy, we first need to outline the content analysis [see (a) directly above] that we use as a starting-point. Our first step is to move beyond the traditional narrow view of algebra as primarily syntactically-guided, symbolic manipulations. This narrow view not only grossly understates the multiple sides of algebra historically as mathematics, it is also an inadequate foundation for reform of algebra in school. We need a broader and deeper view of algebra that can provide *school* mathematics with the same depth and power that the many facets of algebra have historically provided mathematics. Unlike the narrow syntactical algebra, this broader and deeper conception of algebra can support the integration of algebraic reasoning across all grades (K-12) and all topics (Kaput, 1998; NCTM, 1998b). We argue (Kaput 1995b; 1998; in press) that mature algebraic reasoning is a complex composite organized around five interrelated forms, or strands, of reasoning:

1. Algebra as Generalizing and Formalizing Patterns & Constraints, especially, but not exclusively, Algebra as Generalized Arithmetic and Quantitative Reasoning;

2. Algebra as Syntactically-Guided Manipulation of Formalisms;
3. Algebra as the Study of Structures and Systems Abstracted from Computations and Relations;
4. Algebra as the Study of Functions, Relations, and Joint Variation;
5. Algebra as a Cluster of Modeling and Phenomena-Controlling Languages.

Forms (1) and (2) underlie all the others, with (1) based both within and outside of mathematics, and (2) done in conjunction with (1). It is difficult to point to mathematical activity that does not involve generalizing and formalizing in a central way. It is one of the features of thinking that makes it mathematical. Also, the actions one performs with formalisms identified as (2), the kinds of manipulations that dominate current algebra courses, should typically occur as the result of prior formalizing of situations and phenomena, so that they can be related to those situations and phenomena. The formalisms may be of many different types, not merely variables over sets of familiar numbers, or transcendentals over some field, (for example, matrices and strings of characters of any kind (as occur in codes, for example)). Furthermore, it is possible that the manipulation can yield general patterns and structures at another level of generalizing and formalizing - which is the essence of the third, structural form of algebraic reasoning. In order to use or communicate generalizations, one needs languages in which to express them, which leads to (5), which in turn permeates all the others. While (3) is a school mathematics topic strand that typically occurs at advanced levels (although see Strom & Lehrer in the downloadable papers for an illustration of 2^{nd} graders coming to terms with the key issues associated with transformations of the square), it is also an important "live" domain of mathematics in its own right - abstract algebra. On the other hand, topic strand (4), functions, is more a school mathematics domain, and lives in the world of mathematics more as a general purpose conceptual tool rather than a branch of mathematics. The study of variation and the properties of functions leads, of course, to calculus. One hint of the breadth of this school algebra is the fact that (3) and (4) lie on opposite sides of a deep boundary in mathematics separating algebra and analysis, respectively. Both appear in school algebra.

Traditional school algebra focuses on (a part of) (2) at the expense of all the others. And while calls for a functions approach to algebra went ignored for almost a century, some of our contemporaries tend to see (4) as all of school algebra (Yerushalmy & Schwartz, 1993). But the list, and history, suggest that algebra is more than functions, although the idea of function is an extremely powerful organizer of mathematical activity across topics and grade levels. But so are all the other forms of algebra listed, which is exactly why algebra can play the key role across K-12 mathematics that we and others suggest. This wider view of algebra emphasizes its deep, but varied, connections with all of mathematics. Lastly, we should note that this content analysis is consistent with that provided by the NCTM Algebra Working Group and appearing in (NCTM, 1997, 1998b). Moreover, it is consistent with that used in the current draft NCTM Principles & Standards (1998a). [Much more can be added here regarding the concrete specifics of this analysis.]

The Role of Elementary Teachers (and Their Basal Texts) in Algebra Reform

A central part of an integrated strands approach (*vs*. a traditional approach) to algebra reform is recognizing that the genesis of algebraic thinking lies in younger children's mathematical activities, in opportunities for them to generalize and formalize their thinking. Because of this, it has been argued that the degree to which elementary teachers are capable of developing children's algebraic thinking may determine the depth of algebra reform and hence, mathematics reform in general (Kaput, in press; Schifter, in press). An integrated strands approach to algebra reform absolutely requires that the various forms of algebraic reasoning be intelligible to typical teachers, especially in elementary mathematics classrooms. Yet, most elementary teachers are products of the algebra instruction we need to replace and have little experience with the rich and connected kinds of algebra described below (see also www.simcalc.umassd.edu/EABook.html) and that need to become the norm in schools (Swafford, Otto, & Lubinski, (at the above address)). Furthermore, the default instructional materials in K-5 schools today are the basal texts, which are increasingly supplemented by, but far less frequently replaced by, standards-based instructional materials. Moreover, even the best of commercially available standards-based materials (e.g., *Investigations*, 1991; *Everyday Mathematics*, 1998) do not yet embody systematic approaches to the development of algebraic reasoning. *This suggests that elementary teachers and their basal texts are in the critical path to solving America's Algebra Problem.* Any large-scale change requires that we work with the innovation capacity and existing instructional resources of the million-plus elementary teachers already in our schools as well as with the pre-service teachers educated in university programs over the next decade.

Defining and Extending Our "Algebrafication" Implementation Strategy

Our approach is to work with teachers to "algebrafy" their existing instructional materials, and, when these materials are inadequate, to supplement them. We use the term "algebrafy" to indicate three dimensions of teacher-based classroom change:

- the process of building algebraic reasoning opportunities, especially generalization and progressive formalization opportunities, from available instructional materials;
- the building of teachers' "algebra eyes and ears" so that they can spot opportunities for generalization and systematic expression of that generality and then act upon these as they occur;
- the process of creating classroom practice and culture to support active student generalization and formalization within the context of purposeful conjecture and argument, so that algebra opportunities occur frequently and are viable when they occur.

Generalizing and expressing generalizations in gradually more formal and conventional ways underlie all aspects of algebraic reasoning as discussed above. As such, these processes should occur in arithmetic, in modeling situations, in geometry, in data analysis, and in virtually all of the mathematics that can or should appear in the elementary grades (see Kaput, in press, as well as the downloadable papers for a varied set of examples). By integrating existing content into the broader and deeper conception of algebra outlined above, teachers enhance their understanding of both that basic content as well as its place in the bigger algebra (for example, how the notion of "=" changes as one moves from arithmetic to algebraic statements (Kieran, 1992)). At the same time, they are collecting and generating instructional resources to support their own practice, resources that we will assemble as described below. Our ongoing work in Fall River elementary schools uses the most recent edition of the Scott-Foresman basal text series (Charles, et al., 1998), supplemented with appropriate materials eclectically drawn from available resources. (Prior work has generated and culled from many sources a substantial body of instructional tasks and activities. Our consultants and advisory Early Algebra Research Group likewise contribute materials. We will provide specific examples in the Preliminary Results section below.)

Note that the act of determining that certain instructional materials are inadequate as stepping off points for generalizing and formalizing is itself a powerful professional development strategy. We are currently using this strategy with our Fall River in-service teachers as well as pre-service teachers being taught by the second author.

Algebrafying Instructional Materials and Classroom Practice with Pre-Service Teachers

The purpose of our work with pre-service teachers is (a) to understand the evolution of pre-service elementary teachers' algebraic reasoning and ability to build classroom cultures that promote children's algebraic reasoning; and (b) to describe types of instructional contexts in *teacher education* which support this development. Indeed, a major obstacle in solving America's Algebra Problem is understanding how teacher educators can make the various forms of algebraic reasoning, and the associated pedagogies, intelligible to pre-service teachers who, ultimately, carry the responsibility for long-term algebra reform. Our pilot and current work indicates promise for a strategy that both parallels and interacts with our algebrafying strategy for in-service teachers. The second author is currently teaching a course for pre-service teachers during which algebrafying work takes place. This work will be extended in the coming year to include a field-based approach to support pre-service teachers in actively algebrafying carefully chosen tasks from the elementary curriculum, reflecting on the process and the different results, examining videos of exemplar algebrafied classroom practices, and interning in classrooms where algebrafication is underway. Our research with pre-service teachers is guided by the following questions:

- What capacity do elementary pre-service teachers have for making and expressing generalizations, formalizations, and purposeful arguments, and how do these constructs emerge in their mathematical discourse? [We expect this to contribute to other relevant research (see e.g., Simon (in press), Simon & Blume, 1996)];

- How does elementary pre-service teachers' algebraic reasoning emerge throughout the field experience? (In particular, we will study this from the perspective of the first form of algebraic reasoning: Algebra as Generalizing and Formalizing Patterns & Constraints, especially, but not exclusively, Algebra as Generalized Arithmetic and Quantitative Reasoning)

- What essential knowledge and experiences do elementary pre-service teachers need in order to reason algebraically and to create classrooms that foster students' algebraic thinking?
- To what extent does a field-based approach transform elementary pre-service teachers into future practitioners who can think algebraically as well as cultivate this type of thinking in their classrooms?
- What kind of instructional strategies, or pedagogies, (including elements of the CGI approach described below) can the *teacher educator* use to promote elementary pre-service teachers' algebraic reasoning and ability to build classrooms that support student's algebraic reasoning?

Applying the CGI Teacher-Empowerment Approach to Algebraic Reasoning District-Wide

Among the more successful approaches to empowering teachers in the past 15 years is that used by the Cognitively Guided Instruction (CGI) Project (Carpenter & Fennema, 1992), which empowers teachers by enabling them to understand student thinking and learning and to apply this understanding within the contexts of their own classrooms and available instructional materials. Under joint NCISLA support, we have been working closely with Carpenter and Lehrer to extend the CGI approach beyond K-3 elementary arithmetic to include broader subject matter (Lehrer & Chazan, 1998; Strom & Lehrer, in press). More recently, Carpenter and colleagues are concentrating on the early development of certain forms of algebraic reasoning, especially form (1) listed above (Carpenter, 1999), and even form (3) with 2nd grade students (Kaput, in press; Strom & Lehrer, in press; Lehrer & Chazan, 1998). Our plan is to continue to extend this CGI-style approach, *in the context of our algebrafication strategy*, to the broader conception of algebraic reasoning outlined above for both elementary school mathematics (grades 1-5) and pre-service teacher education. By focusing on student thinking as it relates to activities of generalizing and formalizing and how to support those activities on a daily basis beginning with existing instructional materials, we are able to work in all three algebrafication dimensions simultaneously. That is, we can upgrade instructional materials and build teacher awareness and capacity while changing everyday practice and classroom culture to support the use of the enhanced (algebrafied) materials.

Preliminary Results from Pilot & Current Work

Pilot Work in 1998-99

We met bimonthly for 2 hours with 3 teachers from grades 5 & 6, respectively, and base this report on field notes from those meetings as well as notes taken from several class visits to the 5th and 6th grade classrooms by the first author and a research assistant during the Spring of 1998. The K-6 school in which this work took place is in a low-income neighborhood of an urban center. Seventy percent of the students receive reduced or free lunch. The teachers, each of whom had at least 20 years of experience, mainly at that site, were "off the shelf" in the sense that they had not been involved in any sustained in-service professional development work in mathematics and had well-established, traditional forms of instruction from very traditional materials in very traditional settings.

Two of the three continuing teachers had parallel 5th grade classes, one of which was designated as an "enrichment" class and the other as "standard." These two categories are the only distinctions beyond Special Ed groupings. The third teacher had a standard 6th grade class. All mathematics classes used the Heath mathematics series, and the 5th and 6th grades were using the latest (1994) edition. As can be expected, the teachers felt very obligated to "cover" their textbook material, despite the fact that, according to their new principal, no specific requirements were in place for the 1997-98 school year. The reason is because the city, as part of the statewide reform process, is committed to adoption of curriculum guides for each grade level that are in turn based on the statewide mathematics frameworks accepted in 1997 by the State Board of Education.

We will now describe in summary form the pilot work and our conclusions from it. The Appendix contains more detail describing specific tasks and teachers' reactions to and uses of them.

Reading Teachers' Classroom Stories: Setting the Problem, Building Trust. The first 4 seminars were centered on readings from teacher-based reports of classroom episodes:

"Classroom stories: Examples of elementary students engaged in early algebra" (by D. Schifter and V. Bastable, available at www.simcalc.umassd.edu/EABook.html) and "Operation sense as a foundation for algebraic reasoning" (D. Schifter, in press). These readings emphasize natural connections between typical arithmetic topics (treated in reform-oriented classrooms, however) and issues of generalization and student expression. While the teachers seemed to understand the readings in an abstract sense, they did not seem to be able to relate the readings to their own practice. Their notion of mathematics was very much procedural - linked to arithmetic operations - and their notions of student thinking and understanding were tied to procedural competence in these operations. In other words, a student understood "it" (i.e., some procedure) if he or she could do "it" across varied numerical contexts (see Thompson & Thompson JRME procedurality article). Also, not surprisingly, the teachers regarded the classroom stories as reflecting a practice beyond the reach of *their* students. The teachers repeatedly emphasized their students' weak academic backgrounds, low SES, associated family problems, etc., despite the fact that some of the classrooms in the readings reflected the same kinds of students. The overall effect of the first sessions was to instill an ill-formed worry among the teachers that improvements were needed in their classrooms, but the form of these changes were unclear, and that changes would be very difficult. Up to this point, no classroom interventions or visitations took place—we regarded two months primarily as a period of trust-building and orientation to issues of student thinking and classroom culture.

Summary of Results from the Next 4 Months. After four additional months of almost weekly sessions, either after-school with the researchers or through classroom visits and interventions by the research assistant, two of the teachers began in the Spring to team-teach a weekly special session. The sessions were with a combined class of their 5th grade students, who remained in their respective classes while their classmates were in a Title I pull-out program for reading instruction. Importantly, this was *not* regarded as regular class time. Their teaching took the form of assigning specially designed activities based on activities completed by the teachers in the after-school sessions. As they were piloted and refined with the teachers, the activities involved much discussion with and among the teachers and, thanks to the comfort built over several months, genuine learning on their part. However, the teachers "workbooked" the activities with their students, handing out the activities with procedural instructions to work in groups of three, to raise hands if they had questions, etc. And, of course, the students likewise assimilated the activities to their own expectations and ways of working—including "working alone in groups." All this led to superficial and routinized approaches to generalization and formalization that lacked generativity and viability.

While these findings that both students and teachers assimilated our approaches and interventions to a classroom culture and forms of practice that had little place for substantive mathematical discussion or inquiry, we were struck that *under these conditions, our goal for the development of algebraic reasoning in the context of elementary mathematics was impossible! It was not merely difficult, or challenging, but impossible.* Students' mathematical thinking was quite clearly unaffected by the dozens of interventions (either ours or those of the teachers), which were held at arms length from daily practice and the real objectives. Indeed, our interventions seemed much like traditional field trips—interesting and engaging when they are happening, but separate from real school.

The distance between where these teachers are and where they need to be in order to implement a successful approach to the early and rich development of algebraic reasoning is not likely to be covered through a mode of professional development that depends on external seminars and imported activities. They need either the full immersion of an entirely new curriculum, with full support, including in-class assistance and out-of-class work, or some other form of development that systematically connects with their everyday practice and instructional resources. They also need a larger, district-based reform context to support change in their practice. None of these was in place when the pilot work occurred. Our findings guided our planning for the current work, described below.

Current Work with In-Service Teachers

Our current work involves a much larger group of 20 elementary teachers (grades 2–5) whose participation reflects a coordinated, district-wide effort to introduce algebraic reasoning into elementary school mathematics. It takes place in a neighboring school district that is economically and culturally similar to that in which our pilot work occurred. However, it differs from our previous work by providing a systematic connection to daily practice and currently available instructional materials (the "algebrafying" strategy), as well as a strong, district-based reform context.

Implementing the "Algebrafication" Strategy. For the 1998-1999 academic year, we have worked bimonthly (2 hours per session) with the current group of teachers to "algebrafy" their instructional materials, their instructional practices, and ultimately, their classroom culture. In this initial stage of implementation, we are attempting to determine how and whether the algebrafication strategy might make sense in practice. Teachers are grouped by the grade level they teach to identify problems and activities from the district-adopted curriculum (the most recent edition of the Scott-Foresman basal text series [Charles, et al., 1998]) that suggest opportunities to promote students' algebraic thinking. In coordination with this, we have gathered activities from assigned readings (e.g., Carpenter, et al. personal communication; Shifter, in press), illustrating real classroom examples of the rich kinds of activities and discussions that can support the development of students' algebraic thinking, for our teachers to adapt to their particular grade level. When implementing these activities (see selected examples below) in daily classroom instruction, teachers are encouraged to focus on exploring students' thinking, (e.g., Carpenter & Fennema, 1992). Unlike our pilot work, this has been an integral part of our strategy in the current work and it seems to have forged a connection between otherwise esoteric readings and teachers' own practices. The bimonthly sessions then provide a context for collegial sharing of teachers' classroom experiences, respective to explorations of students' thinking and their own instructional strategies, as these experiences relate in the context of the teachers' daily experience to examples of students' thinking and implicit or explicit instructional strategies represented in the readings. We are now gradually introducing the various strands of algebraic reasoning as an organizer of teacher's thinking (although the heaviest emphasis has been on #1; in discussions we referred to the "five faces of algebraic thinking").

Selected Examples of Algebrafiable Tasks Used in Teachers' Classrooms. Based on teachers' collective work of algebrafying instructional tasks and the reflective writings of one of our teachers (Jan (pseudonym), a 3rd grade teacher and one of our more energetic teachers), we include here a selected set of algebra-rich problems and the classroom scenarios in which they occurred. The set includes problems from the basal texts, as well as adaptations of activities culled from our readings and other resources. Besides illustrating algebrafiable opportunities in the elementary classroom, the excerpts below suggest the empowerment of a teacher as she "discovers" students' thinking, emerging instructional strategies that support her students' algebraic thinking, and a developing classroom (and school) culture that values doing mathematics.

Jan's Reflective Writings

Multiplication Tables. I was teaching the 5 table. The children of course find this table very easy, so I figured this would be a good place to start to find missing factors. I gave the class the problem '$5 \times a = 20$'. John immediately raised his hand and gave the answer "4". I asked him how he knew the answer so quickly. He answered that he knew that $5 \times 4 = 20$, so the answer had to be 4. I then asked if you did not know this, how could you find the answer? Jeff responded with 'count by 5's until you get to 20'. I asked for any other ways. Dan said that he could find the answer by subtracting. I asked him to explain his thinking. He said take 20 – 5, 15 – 5, 10 – 5, and 5 – 5. Then he said to count the number of times you had to subtract 5. There were 4 times that you subtract, so the missing number had to be 4. I was impressed!

Missing Numbers. After doing the problem 5 + 8 = __ + 9 (adapted from Carpenter, personal communication), I decided to make up more of these kinds of problems for them to do:

$\Delta + \Delta = 6$

$9 + \Delta = 12$

Δ = ___

This activity started out with just about every student asking how to do this. I wanted to see what strategies they would use to figure out the problems, so I only told them that the triangles had to have the same number. Some of the children did guess and try. They did not understand that the numbers had to be the same. Then all of a sudden a few of them noticed that if you did the doubles first, it was much easier to get the remaining number. I asked if their answers made sense. They didn't like the idea at first to check it on their own, but after a while they felt good about knowing their answer was correct.

Introducing "Letters". I wanted to get the class started using letters to represent a number or numbers. So we talked about what we could use to replace a number or an empty box. After some discussion, someone came up with the idea to use a letter. I put 1 x *b* on the board. John finally figured out that the answer was *b*. I then asked what other number sentence could I use and another student answered $b \times 1 = b$. So then we started to replace the *b* with all different numbers and multiplied the number by 1. They used a calculator to check these problems. I then asked them to make up a rule that they thought they could always follow. They had to write the rule. Every one of the students was able to say in their own way that any number times 1 always equals the other number.

I tried the same thing using 0. I wrote $d \times 0$ on the board. Because they had done the problem with the 1, they knew that $d \times 0 = 0$. Again, they were able to see that $0 \times d = 0$. We once again replaced the letter with any numbers. The numbers got bigger. (This brought in them reading large numbers for a review.) Again, they used the calculator to check. The first time they put in their large number, Dan announced loudly, "Hey, that '0' took away my number!" So then we tried adding and subtracting numbers on the calculator before multiplying by 0 (e.g., 569 + 222 – 123 + 256 x 0 = 0). They were quite surprised to see that everything they did became a '0' when they multiplied by 0. So we took it a step further and replaced the (nonzero) numbers with letters: A + B = C, C – D = E, E + F = G, G x 0 = ?. They all knew the answer would be '0'. They are having a lot of fun with numbers!

The 100th Day of School Challenge. On the 100th day of school, I challenged my 3rd grade and Julie's 5th grade to come up with as many number facts as possible that would equal 100. This had a few of my students very excited. Many of the students started to randomly come up with the facts that equaled 100 in no particular fashion. However, 2 boys who do not appreciate homework went home and started making up problems (without being assigned to do so) to put on the chart. This was rather interesting. One of the boys started making problems this way:

1+99 =100; 2 + 98 = 100; 3 + 97 = 100.

He continued this pattern all the way to 50 + 50 = 100. Another boy, without knowing what the other student had done, had a similar response:

200 – 100 = 100; 199 – 99 = 100; 198 – 98 = 100.

I thought it was interesting that the boys discovered a pattern to follow, but did so in different directions.

In order for the children to put their facts on the chart, they had to check to see if the problem was on the chart already. They found the approach used by the 2 boys easy to check their facts against. Adding competition with the 5th grade class really motivated my 3rd grade class.

More than half the teachers reported activities of this type in our meetings, although only about a half dozen actually wrote about them "for the record" in the sense of submitting writings for sharing.

Shifts in Teachers' Instructional Practices. As teachers iterate on algebrafying their instructional materials and exploring students' thinking through the implementation of algebrafied tasks in their own classrooms, their awareness of pedagogical choices that support students' algebraic thinking seem to be emerging in a natural way. For example, in a recent discussion, teachers reported that, in contrast to their previous practices, they were now asking students to demonstrate their strategies, to look to their peers for ideas about solving problems, to explain (verbally and in writing) how they arrived at a particular solution, to determine if their solutions were different from that of their peers, to articulate patterns in data, to demonstrate real world connections, to test the validity of their solutions, to make connections to previous mathematical activities, and to model problems through graphs, charts and tables. While this span of change is not reflected in an individual, it has come to be reflected in the group, which is developing its own norms for "interesting" and report-worthy activity. Additionally, teachers reported a decreased focus on students getting the correct answer and an increased attention to posing questions for which students could explore various solution paths. In connection with this, we have also observed that (in some cases, such as those in the above selected examples) teachers' reflective writings seem to mirror these changes in instructional practices, as they shift from an evaluative focus on students' problem performance to rich details that capture students' thinking.

One other revealing development was the teachers' sensitivity to the levels of participation of their students in the generalizing and formalizing activities. Several teachers, among those who were followers rather than leaders, worried publicly that only a minority of their students seem to be engaged in these activities, that others merely watch and listen, and some even seem to withdraw from participation entirely. These teachers wondered how they might "motivate" more students to become involved. Our guess is that their classroom practice was perhaps the most routinized and computationally oriented, and that their students' expectations are especially out of line with what their teachers are now attempting to do.

Clearly, we are in the early stages of a longer-term process for this group of teachers. However, the fact that they are becoming reflective, and in the case of about a third of the group, generative in the sense of creating their own generalization and conjecturing activities, suggests that we are moving in a promising direction. We are confident that, for about a third of the teachers, their practice will continue to evolve in positive directions without any further intervention on our part and without the support of their peers. For another third or more, we see a dependence upon their colleagues and us for support in attempting new ideas and activities. The remaining teachers, perhaps 4 or 5, seem uncertain about what the whole enterprise is about, and appear to be in a position similar to those students whose expectations are out of line with what is happening around them.

Future Work: Linking Pre-Service and In-Service Teacher Education

Combining an Extended Teaching Experiment with Pre-Service And In-Service Work to Produce an Early Algebra Resource As a Community-Wide Initiative

Our work will proceed simultaneously in university-based pre-service education and district-based in-service education. Both will draw upon the year-long 3rd grade classroom teaching experiment led by the second author during 1999–2000 and which will be extended in the next two years by cooperating teachers. We expect this to lead to the development of two preliminary versions of an Early Algebra Resource, one each for in-service and pre-service teachers. This twin resource will be compiled as both a loose-leaf, bound hardcopy "book" organized by grade level and algebra-aspect, and as a web-resource. The in-service version will also be utilized by Scott-Foresman as a set of resources specifically tailored to its elementary level basal text series. It will embody, in teacher-resource form, our best version of the Early Algebra Story.

Importantly, the entire effort will be treated as a major community initiative. Since it will require substantial and extended cooperation across the district, it will initiated by a workshop for the mayor and school committee and then followed by a half-day workshop for all elementary school principals. The first author is currently co-leading a year-long Leadership Academy for all middle and high school administrators as well as the superintendent and assistant superintendents, and the mayor is participating in the first author's university seminar on education reform for faculty. (As a former legislator, the mayor was a key architect of the education reform law in Massachusetts.) Indeed, our effort is part of ongoing, city-wide educational renewal begun at an Education Summit in June, 1998, led by the Mayor.

The Pre-Service Teaching Experiment

We will use a field-based approach beginning with a systematic teaching experiment in which the 2nd author will teach in a representative 3rd grade classroom with the assistance of the regular classroom teacher (fall semester) and then supervise a pre-service student intern in that same classroom (spring semester). The pre-service intern will have participated in the fall semester portion of the teaching experiment as an observer. Both the teacher and the intern will be chosen from our current pilot group of 20 teachers and pre-service course on the basis of their commitment to the algebrafying process and their articulateness in reporting on their efforts. During the fall semester, the elementary classroom will serve as a laboratory in which pre-service teachers will participate in:

- identifying opportunities within the classroom of and for generalizing, formalizing, and making purposeful arguments (that is, opportunities to "algebrafy" the classroom);
- establishing a classroom culture that supports children's algebraic reasoning.

In addition, the investigators and the pre-service teachers will meet weekly to:

- dialogue about these early algebra opportunities in the classroom laboratory;
- algebrafy curricular materials used in the classroom;
- deepen understanding of the various 5 aspects of algebraic reasoning.

During the spring semester, the 2nd author, along with the regular classroom instructor, will supervise the chosen pre-service teacher-intern in the same classroom. Our dual purpose is to study how that pre-service teacher builds the classroom culture in the more intensive activity of student teaching, and in what ways, pedagogically, the supervisor can support this (Blanton, Berenson, & Norwood, 1998b). We, along with the cooperating teacher, will observe and meet with the pre-service teacher on a weekly basis during the student teaching semester to (a) explore how the pre-service teacher builds a classroom culture that supports students' algebraic thinking, (b) identify obstacles to this process, and (c) construct ways to overcome these obstacles and scaffold the pre-service teacher's development. In addition, the 2nd author will lead a pre-service seminar that builds on the previous semester's work with pre-service teachers, the goal of which is to refine a suitable program that integrates algebra reform into pre-service elementary teacher education.

In-Service Work and Interaction with the Pre-Service Teaching Experiment

Simultaneous with our pre-service work, we will hold bimonthly, in-service sessions for Fall River elementary teachers. These sessions, co-led by both authors and 3 teacher-leaders from our current study, will begin in the fall (1999) with 24 teacher leaders, 3 from each of 8 district schools. The 3 teacher leaders will also lead parallel bimonthly (2 hour) seminars for approximately 10 teachers each in their own schools (not included in the 8 schools). Our purpose will be to collectively algebrafy instructional materials and classroom practice, using anecdotal insights, selected videos, and field-tested resources from the pre-service teaching experiment to support this activity. The in-service group will contain approximately 6 additional teacher leaders from our ongoing 1998–99 pilot work representing as many of the 8 schools as possible. The teacher leaders will be designated as in-school assistants for their participating school colleagues. The total group of 24 plus 30 will build a Fall River Early Algebra Resource (FREAR) in 1999–2000 for use, elaboration, and refinement by their colleagues in the next two years. The FREAR will also include instructional resources gleaned from the pre-service teaching experiment and will exist both in hardcopy draft form and as a continually updated web-based resource. Teacher leaders will also provide advice and, through the FREAR, potential instructional materials for the pre-service teaching experiment. In 2000–2001, six of the most successful teacher leaders, representing a cross-section of Grades 1–5, will each lead a bimonthly seminar (based on the FREAR) for 24 colleagues representing 4 elementary schools. This group of six will also meet on alternative weeks with the authors in the pre-service seminar. By the 3rd year there will be at least two (and in most cases several) teachers *in each of the 31 elementary schools* whose practice is "algebrafied", as well as an elaborated FREAR. In each school, teacher leaders will be asked to run an in-service seminar and provide algebrafication support for their previously "untouched" school colleagues so that by the end of the project, all Grade 1–5 Fall River teachers will have had an extended algebrafication experience. The more successful of the teacher-leaders will be supervisors of pre-service interns who are simultaneously taking the pre-service course being taught by the 2nd author in the two years following the teaching experiment year.

We will omit details regarding methodology and measures of student and teacher learning except to note that excellent, state-mandated, high-stakes student measures (MCAS) are available at the 4th grade level. These measures, particularly when extended with additional tasks, will provide efficient opportunity to gather large scale data to complement clinical and classroom-based detailed data.

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Appendix: Details of Pilot Work With Grade 5-6 Teachers

Moving Towards the Classroom: Unifix Patterns. After 4 sessions based on readings and discussions of classroom episodes, we turned to more concrete, classroom implementable activities involving the notion of algebra as an ability to identify, construct and represent patterns in various ways. We intended to pull the teachers away from the underlying belief that serious mathematics is equivalent to numeric computation, and so we began with non-arithmetic patterns at the outset, using Unifix Cubes as the medium. This proved to be much more difficult for the teachers than we expected and constituted the first real surprise of our pilot work. All three teachers showed difficulty in describing and then creating patterns in strings of Unifix Cubes. The idea of a minimal generating set seemed new to them, and they did not seem to see the significance of non-numeric patterns in mathematics, despite our best attempts which included some classic Unifix Cube activities as well as extensions involving clapping and finger-snapping rhythms (represent a given rhythm using Unifix Cubes, and the reverse, "clap & snap" a pattern given in Unifix Cubes). We had intended that these would make easy low-risk entry points to our first classroom activities, but the teachers chose not to use them. The plan was to use patterns as a route into algebraic description - that is, rule-based descriptions of the patterns that not only can be used to reconstruct the pattern, but can be used to answer questions about the pattern (e.g., what color is the 233rd block? How many blocks in the 50th cycle? How is this pattern different from the last one? If you do __ to the description, what happens to the pattern?) These are nicely organized in the Maths in Context 5th grade unit on Patterns & Algebra, and we will borrow from these for the planned work.

First "Breakthroughs" - Based on Published Curriculum Materials. After 7 sessions, we were finally able (a) to get enthusiastic buy-in on the idea that new, more interesting math was not only possible in their classes, but necessary, and (b) to get real focus on students' thinking and strategies. The two breakthrough events were based on the use of well-designed and proven curriculum materials—Lesh's "Million Dollar Robbery" Packet (we did the first part of it) and the Maths in Context 6th grade unit on Quantitative Reasoning. Of course, we introduced the materials by having the teachers work through them with us. The activities were sufficiently interesting to the teachers to convince them that students would be engaged, and students were indeed engaged. The activities also provided experiences sufficiently rich to reflect upon in many ways. Our Research Assistant, who had used the materials in Providence in Lesh's NextStep Project, led the classroom activity, beginning with the Million Dollar Robbery packet, in one week in all three classes, and then in the following week, the first two Bartering and Rope-Pull activities in the MiC Quantitative Reasoning unit (involving the thinking patterns underlying proportional reasoning). The MiC unit proved to be a real breakthrough in two ways. First, it convinced the teachers that their students could be excited about real mathematics (where some legitimacy was conferred on the content by its connection with a topic known to cause difficulty in later grades). Second, teachers saw that they too could understand it and help facilitate its occurrence in their own classrooms. After acting as active observers in the Million Dollar Robbery unit, all three teachers took a much more active role in the second unit. This included enabling some of the students to create cut-outs of the objects discussed in the Bartering activity and paste them in sequences on large sheets of paper that reflected their strategies for solving the problems. The teachers also encouraged the students to take the problems home to challenge their parents and siblings over the weekend, which most students did. Reports in the next week showed much excitement on the part of the students, and discussions, which we encouraged (and which would not have happened otherwise), involved very explicit comparisons of strategies among the students. It is a mark of the professional development yet needed that the teachers were not able to follow up on this activity when we were not present, despite some opportunities offered involving number patterns and further activities in the MiC unit. This led to our next approach.

Order of Operations Activity—Linking into Existing Curriculum Topics. The last activity before Christmas involved order of operations. This choice reflected a decision to try to tie our activity more explicitly into what the teachers regard as their daily curricular responsibilities based on their textbooks, which happened to involve an idea that is important to the transition from arithmetic to algebra, order of operations. We saw this issue as having two sides, reading and writing. Beyond the competencies associated with parsing or producing sequences of operations are the important pre-algebraic experiences of dealing with operations without necessarily executing them, determining whether or not expressions are equivalent, stringing together sequences of expressions connected by equal signs, and so on. In our meeting with the teachers, (and with the help of Kaye Stacey, who had been visiting), we generated a series of activities that we feel involves both creating and parsing arithmetic statements involving grouping and different operations. Our first, tried with the teachers, was to answer the challenge: "Who can make the most different numbers using four 2s?" The next was the same involving four 3's. The first author then suggested having a contest across the classes: Who can make the most numbers between 1 and 100 using four 4s? (This is a classic problem with a high ceiling but easy entry.) The teachers agreed to produce a Hundreds Chart that would be posted on each classroom door that would include students' solutions. The activity was postponed 3 weeks to mid-January due to the interruption of the Holiday. Note that, in addition to the link with a topic that appears in the teachers' texts, we chose this topic because it admits of easy teacher-extension - simply change the numerical parameters of the task, for example. As with in-class work done later, this activity was experienced as interesting, even exciting by some students, but not part of their real classroom work.

Discussion of Classroom Videos and Classroom Culture. We obtained access to a series of classroom videos being produced via NSF funding to BBN Labs that provide discussion material involving interesting teacher-led inquiry and student argumentation in 6th grade classrooms. Viewing and discussing this material, focusing on forms of classroom discourse and activity, was intended to help us address the need to change classroom culture to support inquiry, conjecture and purposeful generalization.

As described in our summary, we had hard evidence of the centrality of appropriate classroom culture in the development of algebraic reasoning, evidence that appeared early in the interventions. Without a classroom culture that invites and sustains rich mathematical discussion, generalizing and purposeful argument, the root activities that underlie our educational objectives, are simply impossible. And the forms of generalization that occur are isolated and thin – the standard numerical and geometric pattern-finding, that are easily "work-bookable" into silent seatwork. Hence we spent two full sessions systematically reviewing two 25-minute videos of "interesting" classroom discussions, one on fractions and ratio reasoning in the context of tangrams, and the other on least common factors. Just as with the imported curriculum activities, these seminar episodes and the videos themselves were regarded as fascinating, generated animated discussion, but did not impact observable performance in the first year nor change the way the teachers talked about themselves—at least in ways that we could document.

In retrospect, this is not surprising. After all, the teachers were chosen deliberately to represent the kind of teacher who has not been part of the reform effort, who has not been a regular participant in professional development activities in mathematics, and whose district has not been a leader in reform (the kind of teacher and district that represent the great majority of teachers and districts across the US). They had decades of investment in a different view of education and well-honed competence within the context of that view. The only viable learning that was possible in their classrooms was provided by the research team, and then only after preparatory work, which was then marginalized by students and teachers alike as an "unusual event." Not surprisingly, the teachers' attempts to use ideas and materials intended (by us) to promote discussions, investigations, and reflection, were assimilated into the tightly woven web of teacher expectations, habitual forms of practice, and classroom culture that rendered them at most marginally effective.

The Constraints of Teachers' Mathematical Competence. We mentioned above the difficulties that the teachers had with patterning activities, which was evidence to us of the computational narrowness of the teachers' preparation. They found all of our mathematical activities challenging, and, in one revealing case, impossible. This activity involved a series of variations of the "comparing differences" problems used by Thompson (1993). The two 5th grade teachers, after some difficulty, were able to form and compare differences in the context of the "Marble Problem." This problem involved various comparisons between the difference in the number of marbles a student Mary and her mother, respectively, could hold in a closed fist with the difference in the number of marbles a second student, Jane, and *her* mother could hold in a closed fist. The 6th grade teacher was unable to deal with this problem, despite help from her colleagues, pictures, diagrams, etc. She was visibly upset with her inability to make sense of the problem situation—so much so that we moved on to another activity to avoid further embarrassment. This and other episodes convinced us that yet another reason for the teachers' rigidity of practice and curricular allegiances was their mathematical fragility, narrowness, and diffidence. They would require much more professional development in order to move significantly beyond their present practice.

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