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ABSTRACT

This paper reviews graphical and nongraphical procedures for evaluating multivariate normality by guiding the reader through univariate and bivariate procedures that are necessary, but insufficient, indications of a multivariate normal distribution. A data set using three dependent variables for two groups provided by D. George and P. Mallery (1999) is used to analyze histograms, stem-and-leaf plots, box-and-whisker plots, kurtosis and skewness coefficients, Q-Q plots, the Shapiro-Wilk or Kolmogorov-Smirnov statistic, and bivariate scatterplots. A procedure programmed by B. Thompson (1990, 1997) is used to explore multivariate normality by plotting Mahalanobis distances against derived chi-square values in a scatterplot. Four appendixes contain Statistical Package for the Social Sciences (SPSS) commands for two study groups, the SPSS syntax for evaluating univariate and bivariate normality, and SPSS commands for a new dependent variable. (Contains 2 tables, 17 figures, and 39 references.) (Author/SLD)

Running head: EVALUATING MULTIVARIATE NORMALITY

Evaluating Univariate, Bivariate and Multivariate Normality Using Graphical Procedures

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Abstract

This paper reviews graphical and non-graphical procedures for evaluating multivariate normality by guiding the reader through univariate and bivariate procedures that are necessary, but insufficient, indications of a multivariate normal distribution. A data set utilizing three dependent variables for two groups provided by George and Mallery (1999) is used to analyze histograms, stem-and-leaf plots, box-and-whisker plots, kurtosis and skewness coefficients, Q-Q plots, the Shapiro-Wilk or Kolmogorov-Smirnov statistic, and bivariate scatterplots. A procedure programmed by Thompson (1990, 1997) is used to explore multivariate normality by plotting Mahalanobis distances against derived chi-square values in a scatterplot.

Evaluating Univariate, Bivariate, and Multivariate Normality Using Graphical Procedures

Reality is complex. Over time, researchers in the social sciences have become increasingly aware that simple univariate methods comparing an experimental group with a control group on a single dependent variable are inadequate to meet the needs of the complex phenomena that dominate educational and psychological research. In the majority of social science research, two or more dependent variables are necessary, because nearly every effect has multiple causes and nearly every cause has multiple effects. Even when studying a single construct, such as self-concept, it is often helpful to use multiple tools to measure elusive constructs (called "multi-operationalizing").

In a methodological shift that increasingly emphasizes honoring the complexity of reality, Grimm and Yarnold (1995) reported that the use of multivariate statistics in research has accelerated in the last 20 years and that it is difficult to find empirically based research articles that do not employ one or more multivariate analyses. In a comparison of the 1976 and 1992 volumes of the Journal of Consulting and Clinical Psychology (JCCP) and the Journal of Personality and Social Psychology (JPSP), Grimm and Yarnold found that the use of multivariate statistics in JCCP increased from 9% to 67% in that 16 year period. For JPSP, the use of multivariate statistics increased from 16% to 57% in the same time frame.

In an exhaustive study of statistical methods used in studies published in three major social science journals during a 16-year period, Emmons, Stallings, and Lane (1990) found increases for MANOVA, MANCOVA, multiple regression, and multiple correlation methods. For example, the use of MANOVA increased from 44% of the studies published from 1977 to 1981 to 54% of the studies published from 1982 to 1987. Emmons, Stallings, and Lane (1990, p. 14) concluded that "the

results of this research project indicate that empirical research studies across the three social science disciplines of education, psychology, and sociology have begun to realize that the world is multiplicative, multivariate, and curvilinear." Undoubtedly, another major factor contributing to the widespread use of multivariate statistics is the proliferation of computer software packages that reduce complex processes that took years to complete in the pre-microcomputer age (like factor rotation in factor analysis) to a simple "point and click."

Daniel (1990) noted that multivariate methods usually best honor the reality about which the researcher wishes to generalize. McMillan and Schumacher (1984) compellingly argued against the limitations of viewing the world through an overly- simplified univariate lens:

> Social scientists have realized for many years that human behavior can be understood only be examining many variables at the same time, not by dealing with one variable in one study, another variable in a second study, and so forth ... These [univariate] procedures have failed to reflect our current emphasis on the multiplicity of factors in human behavior ... In the reality of complex social situations the researcher needs to examine many variables simultaneously. (pp. 269-270)

Thompson (1994) elaborated on the need for epistemological and analytic consistency by pointing out that all researchers have a presumptive model for the nature of reality and that it is critical for presumptive and analytic models to closely align because analytic models are used as tools to test assumptions and draw conclusions about the preconceived (albeit not always conscious) model. Earlier Thompson (1986, p. 9), stated that the reality about which most researchers strive to generalize is usually one "in which the researcher cares about multiple outcomes, in which most

outcomes have multiple causes, and in which most causes have multiple effects." Given this conception of reality, only multivariate methods honor the full constellation of inter-relating variables *simultaneously*.

Experimentwise Error Rates

Whereas "testwise" error rates refer to the probability of making a Type I error for a given hypothesis test, "experimentwise" error rates refer to the probability of having made a Type I error anywhere within the study. Inflation of "experimentwise" error rates can be attributed to two factors: (a) the number of dependent variables in the study; and (b) the amount of correlation between the factors--if two factors are perfectly correlated there is no inflation. On the other extreme, very low correlations produce highly inflated "experimentwise" error rates. The Bonferroni inequality can be used to calculate the "experimentwise" error rate when the hypotheses or variables tested using a single sample are perfectly uncorrelated:

$$_{EW} = 1 - (1 - _{TW})^{K}$$

As noted by Thompson (1994):

> ... if three perfectly uncorrelated hypotheses (or dependent variables) are tested using a single sample, each at the $_{TW}$ = .05 level of statistical significance, the "experimentwise" Type I error rate will be:

$$
\begin{aligned}
_{EW} &= 1 - (1 - _{TW})^{K} \\
&= 1 - (1 - .05)^{3} \\
&= 1 - (.95)^{3} \\
&= 1 - (.95)(.95)(.95) \\
&= 1 - (.9025\ (.95))
\end{aligned}
$$

$$= 1 - .857375$$

$$_{EW} = .142625$$

> Thus, for a study testing three perfectly uncorrelated dependent variables, each at the $_{TW}$ = .05 level of statistical significance, the probability is .142625 (or 14.265%) that one or more null hypotheses will be incorrectly rejected within the study. Mos unfortunately, knowing this will not inform the researcher as to which one or more of the statistically significant hypotheses is, in fact, a Type I error. (p. 10)

As illustrated by Fish (1988) and Maxwell (1992) using heuristic examples, invoking multiple univariate tests instead of multivariate tests can also lead unwary researchers to fail to identify statistically significant results. The wrong-headed use of the so-called "Bonferroni correction" coupled with use of univariate tests is also inappropriate, because the application (a) severely attenuates power and (b) still does not honor a multivariate reality. Multivariate analyses can detect interaction effects between independent variables that would go undetected if multiple univariate measures were used in place of multivariate measures. Independent variables may have small, but noteworthy effects on multiple dependent variables that add up to an important pattern when examined as a composite, but otherwise appear meaningless in a univariate test (or series of tests) of a single dependent variable.

<u>Assumptions of Multivariate Statistics</u>

Because use of multivariate statistics has become commonplace, it is imperative that researchers understand the assumptions that underlie their use. There are several central assumptions of multivariate statistics. The first assumption of most multivariate statistics is that the

variance/covariance matrices across the k groups must be homogeneous (equal); and the second assumption, which is the focus of this paper, is that the interval response variables across the k groups must be multivariate normally distributed. The test for homogeneity of variance in multivariate statistics is Box's M (Box, 1949; 1954), which is a statistically powerful test of bivariate correlations (unstandardized r) that is analogous to the Levene test in univariate analyses. If Box's M is favorable, you do not reject the homogeneity of variance assumption, which means that you have met the first assumption of multivariate analyses. Box's M tests the first assumption, but it is also sensitive to the second assumption of multivariate normality. In other words, if you don't reject the homogeneity of variance assumption, you may have a problem with multivariate normality (see Tabachnick & Fidell, 1983; 1989; 1996 for a detailed elaboration of the homogeneity of covariance assumption).

Univariate Normality

Determining univariate normality is helpful when assessing multivariate normality, because one can do so even with a small sample size ($n < 25$) and because univariate normality is a necessary precondition for multivariate normality (Gnanadesikan, 1977; Johnson & Wichern, 1992). The advantage of proceeding from a univariate to bivariate to multivariate examination of the data is that such a procedure provides useful information on which dependent variables to use before conducting a multivariate analysis. In order to build a foundation for a complete understanding of multivariate normality, a review of univariate normality is in order.

Parametric tests require that the sample data be drawn from a population with a known form, most typically the normal distribution, so that at least one population parameter can be estimated from the sample (Munro & Page, 1993). As noted by Bump (1991), the normal curve is determined

by a mathematical equation that uses the mean and standard deviation values to determine two additional statistics--skewness and kurtosis. Both statistics are used to assess the normality of a univariate distribution. Skewness refers to the degree of symmetry of the distribution, which is determined by calculating the third-order moment of the score deviations from the mean, formulated by Glass and Stanley (1970, p. 89) as:

$$S_x = (\text{Sum of } Z_i^3) / n .$$

A perfectly symmetrical distribution has a skewness coefficient of zero. Negative coefficients indicate a negative skew (tail to the left) and positive coefficients indicate a positive skew (tail to the right). However, a distribution can be completely symmetrical (as in the case of a bimodal distribution), but not normal. Therefore, it is essential to also examine the kurtosis, which refers to the shape of the distribution against the normal distribution, by comparing relative height to width. The mean and standard deviation are used to convert the measured scores to z-scores, which are then used to compute the kurotosis, as explained by Glass and Stanley (1970, p. 91): "$K_x = ((\text{Sum of } Z_i^4) / n)$, most researchers and statistical packages, however, apply an additive constant of (-3) so that the kurtosis will be equal to 0 in a univariate normal distribution."

Contrary to popular misunderstanding, there are *infinitely* many normal distributions (Henson, 1999) and it is impossible to know for certain whether a distribution is normal or not by "eyeballing" the shape of the distribution since kurtosis is mathematically derived (Bump, 1991). The standard normal distribution is the classic bell-shaped curve featured in countless statistics books which is really a special case of the more general case of all normal distributions (including both z- and non-z score forms) in which the skewness, kurtosis and mean equal zero and the standard deviation is one. While it is true that all normal univariate distributions are bell-shaped, the bells can

take an infinite variety of shapes--many textbook authors erroneously imply that the classic bell-shaped curve (standard normal distribution using z-scores) is the *only* normal distribution.

The normal curve has the following properties:

1. The curve is symmetrical. The mean, median, and mode coincide.
2. The maximum ordinate of the curve occurs at the mean, that is, where $z = 0$, and in the unit normal curve is equal to .3989.
3. The curve is asymptotic. It approaches but does not meet the horizontal axis and extends from minus infinity to plus infinity.
4. The points of inflection of the curve occur at points +/- 1 standard deviation unit above and below the mean. Thus, the curve changes from convex to concave in relation to the horizontal axis at these points.
5. Roughly 68 per cent of the area of the curve falls within the limits +/- 1 standard deviation unit from the mean.
6. In the unit normal curve the limits $z =$ +/- 1.96 include 95 per cent and the limits $Z =$ +/- 2.58 include 99 per cent of the total area of the curve, 5 per cent and 1 per cent of the area, respectively, falling beyond these limits (Ferguson, 1971, pp. 93-94).

However, Glass and Stanley (1970) noted that in a univariate distribution, skewness has a very minor effect on alpha or power in ANOVA if the design is balanced (i.e. there are an equal number of observations in each cell) and kurtosis also has a very slight effect on alpha levels and only effects the power of a test when the distribution is platykurtic (flattened as compared to the normal distribution). The severity of the effect of kurtosis on power increases proportionately with

the presence of kurtosis in more than one variable.

Q-Q Plots

According to Stevens (1996), one of the most popular graphical methods for testing univariate normality is the normal probability plot or Q-Q Plot (quantile-versus-quantile) in which observations are ordered in increasing degree of magnitude and then plotted against expected normal distribution values. The closer the line is to a straight line, the more correlated the observed score is with the expected score and the more normal the distribution. In these plots, the scores are ranked and sorted from lowest to highest. Then an expected normal value is computed and compared with the actual normal value for each case. The expected normal value is the z-score that a case with that rank holds in a normal distribution. The normal value is the z-score it has in the actual distribution. If the actual distribution is normal, then the points for the cases fall along the diagonal running from lower left to upper right, with some minor deviations due to random distribution of scores. Deviations from normality shift the points away from the diagonal processes (see Tabachnick & Fidell, 1996, for examples of interpreting Q-Q plots).

Additional Univariate Graphics

Three additional graphical tests are the box-and-whisker plot, stem-and-leaf plot, and a histogram of the dependent variables. These tests allow a quick and simple means of evaluating the shape of the univariate distribution for each dependent variable. You can think about the box-and-whisker plot as though you are looking down at the distribution from above with the median as the line in the middle of the box. The box itself defines the width of the interquartile range, the thin line represents the range of scores that fall within two standard deviations (95%) and outliers appear beyond the extreme ends of the range on both the positive and negative sides of the distribution. The

stem-and-leaf plot has the advantage of identifying the specific values of each observation and how the shape of the distribution compares roughly to a normal distribution. The histogram is very easy to read and interpret and can compare the shape of a distribution to a normal distribution superimposed on it. It is very helpful with quickly identifying outliers and skewness in a distribution.

However, Stevens (1996) cautions that "eyeballing" the shape of the distribution can be misleading with small to moderate sample sizes due to sampling error that increases as sample size gets smaller. With samples of less than 50 cases, he recommends that prudent researchers use non-graphical tests such as the chi-square goodness of fit, Kolmogorov-Smirnov, the Shapiro-Wilk test, *and* an evaluation of the skewness and kurtosis of the distribution to make an evaluation about univariate normality. The Shapiro-Wilk test (Wilk, Shapiro, & Chen, 1968) was developed to detect a wide variety of variations from a normal univariate distribution. The smaller the W value, the greater the departure from normality. As a guideline, Gnandesikan (1977) stated that for $p_{calculated}$ values of .1 or higher, normality is a reasonable assumption.

Statistical Tests of Univariate Normality

Wilk, Shapiro, and Chen (1968) conducted an extensive Monte Carlo study of statistical tests for normality and determined that the Kolmogorov-Smirnov test was not as powerful as either the Shapiro-Wilk or the combination of using the skewness and kurtosis coefficients and that the chi-square is limited because it depends on the number of intervals used for each grouping. They concluded that for sample sizes under 20, the combination of the skewness and kurtosis coefficients or the Shapiro-Wilk method were most sensitive to detecting extreme non-normality. Stevens (1996) recommended that researchers evaluate unvariate normality by examining the Shapiro-Wilk statistic and examining the kurtosis and skewness coefficients (along with their standard errors) because

Shapiro-Wilk has the most power and a review of the skewness and kurtosis can help determine the cause of non-normality whenever it is present. The Shapiro-Wilk test is recommended for samples of less than 25 and the Kolmogorov-Smirnov test is recommended for samples greater than 25. As a default, SPSS 8.0 performs a Lilliefors correction with the Kolmogorov-Smirnov test. Both the Shapiro-Wilk and the Kolmogorov-Smirnov tests perform an aggregate test of skewness and kurtosis in the univariate case. You do not want to find statistical significance because the null says the distribution is normal and you do not want to reject the assumption of normality.

Univariate Normality Compromised by Outliers

According to Tabachnick and Fidell (1983, 1989, 1996) one of the most serious limitations of MANOVA is its sensitivity to outliers which can make the statistical test uninterpretable because outliers in MANOVA can lead to both Type I and Type II errors in the analysis. Therefore, Frane (1977) recommended that researchers test for outliers whenever conducting a MANOVA or MANCOVA analysis. Tabachnick and Fidell (1983) recommended that researchers remove or transform univariate and multivariate outliers before conducting a MANOVA or MANCOVA and report this in any subsequent written analysis of the data:

> Although univariate normality does not guarantee multivariate normality, the probability of multivariate normality for most real variables in social science is increased if all the variables have normal distributions. Univariate normality is desirable, anyway, because of the known and deleterious consequences of skewness and outliers on robustness of many significance tests. Therefore it is probably a good idea to transform variables to achieve normality ... unless there are good reasons not to (e.g., the variables were measured in meaningful units).

(p. 79)

Statistical procedures in general, and multivariate procedures in particular, can be very sensitive to outliers. It is very important to identify them and decide what to do with them because we want the results of our analyses to reflect the data as a whole and not the disproportionate influence of one or two deviant scores. Outliers are not inherently "bad." In fact, persons with extremely high or low scores on a variable can provide an exceptional opportunity for further study of a particular construct, skill, or ability. Generally, it is not wise to eliminate a person's scores from the data set just because their scores on one or more variables are anomalous.

There are several steps that can be taken to help support the decision to eliminate an outlier. If you talk to the individual about their unusually low score, you may find out that he or she did not want to take the test or complete the instrument, but chose to stay in the testing situation and half-heartedly complete the instrument rather than face the embarrassment of getting up and leaving. In addition, the researcher can review the raw data and look for clues as to why the person's performance was anomalous--perhaps the bubble sheet was filled out in the pattern of a Christmas tree by a disinterested participant or the errant score was due to instrumentation error or a difference in how an individual's score was derived. In cases such as these where justification can be given, removal of an outlier is acceptable.

Stevens (1996) noted that if an investigation suggests that the extreme scores are due to recording or entry error, the analysis can be redone. If no reasonable explanation can be found, Stevens (1996) suggested that researchers consider reporting two analyses of the data--one with the outlier and one without it. In lieu of any form of justification for the aberrant score, transforming the score is more prudent than deleting the score. If there are just one or two outliers in an otherwise

normal distribution, Tabachnick and Fide!l (1996) suggested that researchers change the scores on the outlying variables so that they remain deviant, but not as deviant as they were originally:

> For instance, assign the outlying case(s) a raw score on the offending variable that is one unit larger (or smaller) than the next most extreme score in the distribution. Because measurement of variables is somewhat arbitrary anyway, this is often an attractive alternative to reduce the impact of a univariate outlier. (p. 69)

Tabachnick and Fidell (1996) offered several common procedures for transforming variables due to a non-normal distribution or numerous outliers, but cautioned that even if one or more variables are transformed to become univariate normal, there is no guaranteee that bivariate relationships between variables will become normal or that multivariate normality will be obtained. Also, transforming a variable in a set of variables that are all moderately skewed in the same direction is unlikely to help. A square root transformation is tried first if the distribution differs moderately from normal. If the distribution differs considerably, a log transformation (logarithim) is tried and if the distribution differs severely, the inverse is tried.

According to Bradley (1982) the inverse is the best course with J-shaped or L-shaped distributions. Often, a researcher needs to try one transformation and then another until finding a transformed set of scores that produces a near zero skewness and kurtosis and the fewest outliers. If departure from normality is severe and no transformation seems to help, Tabachnick and Fidell (1996) recommended considering dichotomizing the variable. The major disadvantage to transforming a variable is that it may become difficult for the researcher to relate to what he or she is analyzing--the distribution may become more normal, but it is hard to assess exactly what you are working with. The interested reader is referred to Box and Cox (1964) or Mosteller and Tukey

(1977) for a more detailed elaboration on transforming variables.

Bivariate Normality

As noted by Stevens (1996), in addition to establishing univariate normality, two additional characteristics of a normal multivariate distribution are that the linear relationship of any combination of variables is distributed normally, and that all possible subsets of the sets of variables are normally distributed. The relationship between bivariate and multivariate normality is complex. Statistical significance tests like those used in MANOVA require that the distribution of each dependent variable are normally distributed about each of the other dependent variables in any "X_1 and X_2" comparison.

Two distributions that are univariate normal might also be bivariate normal, but just because two distributions are univariate normal does not mean that they will be bivariate normal. For bivariate normality, univariate normality is a necessary but insufficient condition. In a bivariate comparison, we compare each person's score on two measures, so we are thinking in three dimensions--the X-axis, Y-axis and a third axis to demonstrate frequency of scores. This requirement means that a circular or elliptical pattern will emerge in a scatterplot when examining the correlation of any two dependent variables in a bivariate normal distribution. The narrower the ellipse in the bivariate scatterplot, the greater the correlation between the dependent variables, and subsequently, the greater the likelihood that the assumption of multivariate normality will hold.

Figure 1 is a graphical representation of a bivariate frequency distribution in two-dimensional form. In this drawing, the viewer is looking down at the distribution from above. The largest concentric circle is the footprint or floor of the bell or mound. The footprint of the bell is not a circle in this example, because the standard deviation for each person on the X-axis is roughly twice as

large as the standard deviation on the Y- axis. A series of "contour" lines is used to demonstrate a series of ellipses with varying amounts of distance from the common center, called the "centroid." The advantage of drawing the centroid with contour lines is that you can graphically demonstrate the probability that a random bivariate observation (plotted on the X_1X_2 plane) will lie within the elliptical region, which is equivalent to the area under a portion of the normal curve in a univariate distribution of scores (Neter, Kutner, Nachtsheim, & Wasserman, 1996).

INSERT FIGURE 1 ABOUT HERE

Statistical significance testing applies to the bivariate case in terms of the distance from the centroid or Cartesian coordinate for each person on the X and Y axes. The closer the scores aggregate toward the centroid, the greater the chance of being included in the sample because of nearness to the Cartesian coordinate. The first contoured line shows a value of .8 meaning there is an 80% chance of being included in the sample. The last contoured line has a value of .2 meaning that there is only a 20% chance of being included in the sample.

If a group of 400 people is measured in two ways--for example, each person's composite (Verbal + Quantitative) GRE score (X) and self-esteem (Y)--the data can be represented in a bivariate frequency relationship as shown in Figure 2. If we had bivariate normality, the circles would be concentric in a sense. We are comparing two variables, but have three axes. The third axis is height which graphically shows the frequency of the bivariate scores. In this example, height is a measure of frequency and not a third variable. For each person, there is a pair of scores, a score on X and a score on Y. A bivariate frequency distribution is a picture of the frequency with which different pairs of X and Y scores occur in a group of persons. In Figure 2, a bivariate frequency

distribution is displayed for about 400 people on GRE Composite (Verbal + Quantitative) Score (X-axis) and self-esteem (Y-axis). In this example, the highest frequency of scores is a GRE Composite Score of 1000 and a self-esteem score of 30. This point is the Cartesian coordinate for the two sets of scores and also forms the highest point of the distribution of scores. When the height of the line is compared to the vertical scale of frequency, we can determine that approximately 20 persons had a composite GRE score of 1000 and a self-esteem score of 30.

INSERT FIGURE 2 ABOUT HERE

A surface or "roof" drawn on the top of a large number of scores in a bivariate frequency distribution takes the shape of a three-dimensional bell or hat as demonstrated in Figure 3. The shape is formed by conceptualizing the one-dimensional bell-shaped normal distribution and stretching it in the X and Y directions and rotating it around its center (i.e. the cartesian coordinate) in the XY plane. All bivariate normal distributions have the following characteristics:

(a) For each value of X, the distribution of its associated Y value is a normal distribution and vice-versa.

(b) The Y means for each value of X are linear (i.e., they fall on a straight line) and the same is true for the X means for each value of Y.

(c) The scatterplots demonstrate homoscedasticity--the variance in the Y values is uniform across all values of X and the variance in X values is constant for all values of Y.

INSERT FIGURE 3 ABOUT HERE

If you were to multiply all of the z-scores on the X axis by 2 in Figure 3 and place those scores on the Y axis, the base of the three-dimensional bell will be an ellipse instead of a circle because the Y scores will be twice as spread out as the X scores. However, a non-circular base can still be normal because a multiplicative constant of two will not change the skewness, kurtosis, or mean of zero.

If we sliced through the mean intercept along the axis of Figure 3 and then took that slice and laid it down on the table, it would be wide, but symmetrical and univariate normal. If you took a slice through the mean of the Y-axis, and laid it down on the table, the distribution would be narrower than the first slice, but it would still be univariate normal. If the scores were z-scores, the first slice would have a skewness, kurtosis, and mean of zero (i.e., a normal distribution) and a standard deviation of one. The second slice would also be normal, but it would be wider and appear twice as long if the variable's z-scores were multiplied by 2, but the mean, skewness, and kurtosis would remain zero.

We could cut a slice through every other location along the centroid (intersection of means for X and Y) because there are infinitely many choices and every univariate slice would be normal even though some of the slices would be narrower or wider than others. In a normal bivariate distribution, the footprint for the distribution is only circular when the standard deviation for the variable on the X-axis and the variable on the Y-axis are exactly the same.

The shape of the mound or hat is determined by the amount of correlation between the two variables. If both dependent variables are expressed in standard deviation units, the more correlated the variables, the narrower the mound or hat because correlation causes the probability to concentrate along a line (see Figure 4; $r = .8$). In the extreme case that dependent variable X_1 is completely

correlated with dependent variable X_2, all points would be exactly on the regression equation, the standard deviation for X_1 and X_2 would be equal to zero and the "contour" would all be straight lines with no areas.

INSERT FIGURE 4 ABOUT HERE

Furthermore, if the distribution is bivariate normal, any plane perpendicular to the X_1X_2 plane will cut the surface into a normal curve and a plane parallel to the X_1X_2 plane will cut in an ellipse. The bivariate normal distribution has the property that the regression of X_1 on X_2 is linear. Therefore, if we have a bivariate normal distribution, we know that if we trace the means of X_2 for each X_1, the result will be a straight line. It does not necessarily follow, however, that if the regression is linear, the joint distribution is bivariate normal.

Multivariate Normality

For a data set of two or more dependent variables, all of the variables must be univariate normal and all possible pairs of the variables must also be normal as necessary but insufficient conditions for multivariate normality. The mathematical model that serves as the basis for MANOVA and other multivariate techniques is based on the multivariate normal distribution. This means that both the sampling distributions of the means of dependent variables in each cell are normally distributed as are the linear combinations of dependent variables. The central limit theorem states that for large samples, the sampling distribution of means in the univariate case will approach normality. Mardia (1971) demonstrated that MANOVA is robust to modest violation of normality if the violation is caused by skewness rather than outliers. According to Tabachnick and Fidell (1983),

> A sample size that would produce 20 degrees of freedom for error in the univariate case should ensure robustness of the test, as long as sample sizes are equal and two-tailed tests are used. Even with unequal n, a sample size of about 20 in the smallest group should ensure robustness with a few DVs. (p. 232)

In some instances, researchers can examine multivariate outliers by simply examining z-scores and looking for extreme scores on each dependent variable. However, this technique does not identify a set of scores for a person that are slightly deviant on several variables. Fortunately, a statistic called Mahalanobis distance (D^2) can be used to detect scores that deviate from the mean (above or below) for a group of dependent variables as a set. Detecting multivariate outliers from a set of dependent variables is a much subtler process than detecting univariate or bivariate outliers.

The Mahalanobis distance is the distance of a case from the centroid where the centroid is the point defined by the means of all the variables taken as a whole. The Mahalanobis distance demonstrates how far an individual case is from the centroid of all the cases for the predictor variables. When the distance is great, the observation is an outlier. According to Krzanowski (1988) and Stevens (1996), the Mahalabonis distance is accepted by researchers as the measure of distance between two multivariate populations and it is independent of sample size. The Mahalanobis distance can be written in terms of the covariance matrix S as:

$$D_i^2 = (x_i - x)' S -1 (x_i - x),$$

where D_i^2 is the Mahalanobis distance for a given individual, S is the covariance matrix with variances on the diagonal and covariances off the diagonal. The rank for S is the number of rows and columns for the covariance matrix, which is 3 x 3, if there are three dependent variables.

The assumption of MANOVA, for example, is that in each group, multivariate normality

holds regarding the dependent variables, so if there are a total of 105 cases (as in the heuristic example below) with 64 cases in the female group and 41 cases in the male, both have to have multivariate normality. In group 1, there are three interval variables and the rank of the correlation matrix is 3 x 3. X_i is the composite of three scores of a given individual with a rank of 3 x 1. Person #1 has three scores with one column. The matrix of means also has a rank of 3 x 1 (three means with one column) which yields a product of 3 x 1 and is not conformable to 3 x 3. The transpose (') notation means you flip the 3 x 1 and it becomes 1 x 3. The right most part of the matrix is also a 3 x 1 but it does not have a transpose symbol, so it is not flipped on its side.

From the formula, the Mahalanobis distance is descriptive of how far each case's set of scores is from the group means adjusting for correlation of the variables (in the example, a measure of the distance of the each person from the group means adjusted by how correlated the three variables are). In Figure 5, the smallest Mahalanobis distance is for participant #32 because each of the three scores (3.0, 6.1, and 9.8, respectively) is closest to the mean for each variable (2.89; 6.23; and 10.3, respectively).

INSERT FIGURE 5 ABOUT HERE

Having correlated dependent variables is commonplace in social science research. The correlation of dependent variables must be taken into account when calculating the Mahalanobis distance because deviations from the means of two highly correlated dependent variables are partially redundant whereas the deviations from the mean for two highly uncorrelated dependent variables are not redundant. More concretely, say in a set of three dependent variables all with a standard deviation of 5, that the mean of X_1 is 10, the mean of X_2 is 11 and the mean of X_3 is 2, X_1 is highly correlated with X_2 but X_1 is highly uncorrelated with X_3 and X_2 is highly uncorrelated with

X_3. If person #1 has a score one standard deviation above the mean on X_1 (X_1=15) and X_2 (X_2=16) and scores at the mean of X_3 (X_3=7), that person will have a smaller D^2 than person #2 who scores at the mean on X_1 (X_1=7) and one standard deviation above the mean on X_2 (X_2=16) and X_3 (X_3=7). The D^2 for person #1 includes redundant distance from the means because the scores on X_1 of 15 and X_2 of 16 are very similar. In a sense, X_1 and X_2 are measuring the same thing, so the deviation from the means is due in part to similarity in the variables. Person #1 will have a lower D^2 because the deviation from the means is redundant whereas the D^2 for person #2 will be much greater because the Mahalanobis distance is not due to distance from similar means of the variables but rather to substantial distance from *dissimilar* means (X_1=10; X_2=16; X_3=7).

There are two evaluations to be done when examining the Mahalanobis distance by chi-square scatterplot--the first is whether or not the points form a straight line or not. If the points on the scatterplot form a straight line, you have multivariate normality. The second consideration is whether or not there are anomalous persons with scores on the scatterplot that are a noteworthy distance from the centroids. You can have a perfectly straight line and still have outliers in the data set, but it is rare to have a person whose scores are outlying on all of the dependent variables in a data set. Before eliminating outliers, a prudent researcher will examine whether or not the extreme score on the multivariate scatterplot is due to an anomalous score on one dependent variable by examining each univariate distribution before eliminating the person from the data set. If only one score is anomalous, it is more prudent to transform the score on that variable rather than eliminate valuable information from the analysis, or to eliminate that variable from the data set.

Evaluating Univariate Normality: a Heuristic Example

To make the discussion about testing univariate, bivariate, and multivariate normality more concrete, a data set developed by George and Mallery (1999) will be analyzed using SPSS version

8.0 to test the assumption of univariate, bivariate, and multivariate normality for 64 female and 41 male students taught by the same professor in three sections of a course. The three dependent variables in this analysis are each student's GPA previous to taking the course (PREVGPA), final exam grade (FINAL) and total points for the course (TOTAL). In such a data set, it might be interesting to examine the differences between males and females (an independent variable with two levels) on all three dependent variables--previous GPA, final exam grade, and total points in the course. The SPSS syntax for the female group (n = 64) appears in Appendix A and the syntax for the male group (n = 41) appears in Appendix B. For the sake of brevity and clarity, only the output from the female group will be analyzed in detail in this paper.

Before conducting a MANOVA on this data, it is necessary to examine the univariate, bivariate, and multivariate distribution of the dependent variables for each group. As noted by Marascuilo and Levin (1983), multivariate normality is a requirement for utilizing the statistical inference procedure that is the basis of all "OVA" designs. As noted earlier, a preliminary analysis of two or dependent variables for normality begins with an evaluation of univariate normality for each of the variables. The test for univariate normality for the grades data for the female group was done by using the MULTINOR program developed by Thompson (1990, 1997) on SPSS 8.0 (Appendix A). The MULTINOR program generates graphical and non-graphical analyses of the distribution of each dependent variable separately.

Univariate Normality

When examining histograms for the female group in Figure 5 (n = 64) PREVGPA appears relatively normal, but has high number of scores on the positive end, so a cursory examination does not determine normality. FINAL appears more normal with better prospects for univariate normality. TOTAL does not appear to be normal because the distribution is negatively skewed. In Figure 7, we

can review the box-and-whisker plots for these three dependent variables. For univariate normality, we would expect the box (interquartile range) to be centered between the ends of the whiskers and the line representing the median to be in the middle of the box. The boxes in the box-and-whisker plots for PREVGPA and FINAL are towards the positive end indicating that both are positively skewed. The box for TOTAL sits right in the middle of the distribution, but two persons (case 62 and case 64) are outside the 95th percentile and appear to be outliers. In Figure 8, the stem-and-leaf plot provides both a visual distribution of scores and the precise values of the score taken to one decimal point.

INSERT FIGURES 6, 7, AND 8 ABOUT HERE

In Figure 9 we see the Q-Q plots which plot the observed scores with the expected scores ranked from low to high so you expect a line from lower left to the upper right--the straighter the line, the more correlated the observed scores are with the expected scores. For PREVGPA, FINAL, and TOTAL we have a fairly straight line but there are outliers off the line at both ends of the scatterplot for all three distributions, particularly with lower observed values for TOTAL, so univariate normality is inconclusive.

INSERT FIGURE 9 ABOUT HERE

Moving on to a descriptive evaluation of univariate normality, the skewness and kurtosis statistics along with their standard errors are shown in Table 1. According to Stevens (1996), we can divide the skewness and kurtosis for each DV by its standard error and if that value is greater or less than roughly 2.0 we have statistical significance for skewness or kurtosis for that variable. Since the null hypothesis is the assumption of normality, we cannot reject the null for PREVGPA or FINAL for skewness or kurtosis because the computed statistics fall within the desired normal range. For

TOTAL, however, both skewness (-3.09) and skewness (3.32) are outside the normal range and we have statistical significance for non-normality.

INSERT TABLE 1 ABOUT HERE

An aggregate test for skewness and kurtosis for samples of 50 cases or more is the Kolmogorov-Smirnov test performed by SPSS 8.0 using the Lilliefors Significance Correction. While p values of .5 or greater are preferred, Table 2 shows that the p values for all three DVs are .2 and all three distributions are not statistically significant for non-normality at the .05 alpha level. In this example, for the TOTAL dependent variable, the skewness and kurtosis values are fairly extreme, but the univariate normality is not bad when consulting the Kolmogorov-Smirnov statistic. At the close of the univariate analysis of variables, we cannot rule out the tenability of multivariate normality.

INSERT TABLE 2 ABOUT HERE

Bivariate Normality

If the three dependent variables displayed univariate normality (bearing in mind that univariate normality is a necessary, but insufficient foundation for multivariate normality), the next step would be to examine the bivariate correlations between each of the dependent variables. You can attain univariate normality, but fail to demonstrate bivariate normality, which examines each pair of variables--PREVGPA with FINAL, PREVGPA with TOTAL and FINAL with TOTAL. This was done in this example by using the MULTINOR program (Appendix B) by requesting scatterplots and noting elliptical patterns for the three possible combinations of variables. In Figure 10, the scatterplot for each possible pair reveals a clear ellipitical pattern between FINAL and TOTAL, but the scores in the scatterplots for PREVGPA with FINAL and PREVGPA with TOTAL are widely scattered and

are thus not bivariate normal. When the pattern of the scores in a bivariate plot are less clear, researchers can examine the percentage of scores that converge around the centroid (e.g., 80%, 60%, 40%, 20%, 10%) as a guide to deciding whether or not an elliptical pattern is displayed.

INSERT FIGURE 10 ABOUT HERE

At this stage of the analysis, a prudent researcher might stop and consider replacing PREVGPA with another dependent variable or go back and transform the scores in each of the univariate distributions to make them more normal. As noted earlier, Tabachnick and Fidell (1996) recommended that researchers start by taking the square root of the scores, but the scores can also be squared, or the natural log or log-ten (LG10) can be used:

> ... transformations may improve the analysis, and may have the further advantage of reducing the impact of outliers. Our recommendation, then, is to consider transformation of variables in all situations unless there is some reason not to. If you decide to transform, it is important to check that the variable is normally or near-normally distributed after transformation. Often you need to try one transformation and then another until you find the transformation that produces the skewness and kurtosis values nearest zero, the prettiest picture, and/or the fewest outliers. (p. 82)

After transforming the univariate distributions, the bivariate distributions could be examined again to determine if the three pairs of variables have become bivariate normal due to the univariate transformation of scores. For this set of scores, four data transformations were conducted: (a) square root of scores (Figure 11), (b) squared scores (Figure 12) (c) natural log (Figure 13), and (d) log-10 (Figure 14).

INSERT FIGURES 11, 12, 13 AND 14 ABOUT HERE

In none of these transformations did the bivariate relationships between PREVGPA and TOTAL or PREVGPA and FINAL become bivariate normal. Because PREVGPA appeared to be the problematic DV, a decision was made to create a new DV that was comprised of the sum of the quiz grades in the course. This new DV was named QUIZTOT and a new evaluation of unvariate, bivariate, and multivariate normality was conducted as before. The syntax commands for the new variable are shown in Appendix D. Figure 15 shows that the variable QUIZTOT has a bivariate normal relationship with both FINAL and TOTAL and is a big improvement over the variable PREVGPA.

INSERT FIGURE 15 ABOUT HERE

Multivariate Normality

Assuming that both univariate and bivariate normality are attained after transforming the univariate scores or replacing a dependent variable (as done in this example), the third level of assessment is to examine the Mahalanobis Distance by chi-square scatterplot to assess multivariate normality. As noted earlier, the Mahalabonis distance is accepted by researchers as the measure of distance between two multivariate populations and it is independent of sample size (Krzanowski, 1988; Stevens, 1996). If we examine the scatterplot of Mahalanobis Distance (D^2) values with chi-squares (Thompson, 1990) for this data set in Figure 16 we can see that we have a fairly straight line, which suggests multivariate normality. The second issue is the presence of outliers. This scatterplot has one extreme score in the upper right hand corner that is well off the line with an approximate D^2 score of 62 and a chi-square score of 12. If we look at the listing of Mahalanobis distances which are

ranked from lowest to highest in Figure 16, we can determine that the outlier is case 61 and the D^2 is more than four times larger than the next largest D^2 (case #36). Because case #36 in turn is twice as large as the next largest D^2 (case #45), both case #61 and #36 can be considered outliers.

INSERT FIGURE 16 ABOUT HERE

Again, assuming univariate and bivariate normality has been demonstrated, because we have multivariate normality except for two outliers, we can remove or transform the outliers and then look at the univariate and bivariate relationships again because removal of the extreme scores will change the means for both variable X and variable Y, which means that the Mahalanobis distance for each variable will change. If after examining the raw data for cases #61 and #36 we discover that they both had very high quiz scores (QUIZTOT) and very low scores on the FINAL, we might call these two students and ask why they did so poorly on the final exam. If we learn that they both had the flu the day of the exam, the scores for these two individuals can be deleted from the data set because this event produced "flukey" or abnormal scores (i.e. high quiz scores and low final exam scores). Figure 17 shows the Mahalanobis distance and chi square values for this data set after the outliers are removed. Note that while the line appears to become less straight, in actuality the scale for the Mahalanobis distance is being reduced from 70 units to 12 units, thus showing more precisely the linear relationships between the two variables

INSERT FIGURE 17 ABOUT HERE

An alternative to the stair-step approach of examining the univariate, bivariate, and multivariate normality of the proposed dependent variables in sequence for the multivariate analysis is to plot the Mahalanobis distance against the chi-square values straight away--if you get a straight

line, you can stop there because multivariate normality subsumes univariate and bivariate normality. However, plotting Mahalanobis distance against chi-square is only useful with samples greater than 25. If you fail to obtain a straight line, you can remove scores when you can justify doing so, or transform an individual's scores or a set of scores.

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Appendix A

SPSS Commands for Female Group (n=64)

```
SET BLANKS=SYSMIS UNDEFINED=WARN printback=list.
TITLE 'MULTINOR.SPS   tests multivar normality graphically****'.
COMMENT ******************************************************.
COMMENT The original MULTINOR computer program was presented,
COMMENT with examples, in:
COMMENT    Thompson, B. (1990). MULTINOR: A FORTRAN program that
COMMENT    assists in evaluating multivariate normality.
COMMENT    _Educational and Psychological Measurement_, 50,
COMMENT    845-848.
COMMENT
COMMENT The data source for the example are from:
COMMENT    George, D. J., and Mallery, P. (1999). _SPSS
COMMENT    for Windows step by step_.
COMMENT    Boston: Allyn & Bacon.
COMMENT
**********************************************************.
COMMENT  Here there are 3 variables for which multivariate
COMMENT  normality is being confirmed.
DATA LIST
  FILE='a:normgrad.dat' FIXED RECORDS=1 TABLE
  /1 gender 1 ethnicit 3 year 5 lowup 7 section 9 prevgpa 11-14
(1) final 16-17 (1)  total 19-21 (1) .
list variables=all/cases=9999/format=numbered .
COMMENT  'y' is a variable automatically created by the program,
COMMENT  and does not have to modified for different data sets.
select if (gender eq 1) .
compute y=$casenum .
print formats y(F5) .
regression variables=y prevgpa to total/
  descriptive=mean stddev corr/
  dependent=y/enter prevgpa to total/
  save=mahal(mahal) .
sort cases by mahal(a) .
execute .
list variables=y prevgpa to total mahal/cases=9999/format=numbered .
COMMENT  In the next TWO lines, for a given data set put the
COMMENT  actual n in place of the number '64' used for the
COMMENT  example data set.
loop #i=1 to 64 .
compute p=($casenum - .5) / 64.
COMMENT  In the next line, change '3' to whatever is the number
COMMENT  of variables.
COMMENT  The p critical value of chi square for a given case
COMMENT  is set as [the case number (after sorting) - .5] / the
COMMENT  sample size].
if (gender eq 1) chisq=idf.chisq(p,3) .
end loop .
print formats p chisq (F8.5) .
list variables=y p mahal chisq/cases=9999/format=numbered .
plot
  vertical='chi square'/
  horizontal='Mahalanobis distance'/
  plot=chisq with mahal
```

Appendix B

SPSS Commands for Male Group

```
SET BLANKS=SYSMIS UNDEFINED=WARN printback=list.
TITLE 'MULTINOR.SPS  tests multivar normality graphically****'.
COMMENT *****************************************************.
COMMENT The original MULTINOR computer program was presented,
COMMENT with examples, in:
COMMENT    Thompson, B. (1990). MULTINOR: A FORTRAN program that
COMMENT    assists in evaluating multivariate normality.
COMMENT    _Educational and Psychological Measurement_, 50,
COMMENT    845-848.
COMMENT
COMMENT The data source for the example are from:
COMMENT    George, D. J., and Mallery, P. (1999). _SPSS
COMMENT    for Windows step by step_.
COMMENT    Boston: Allyn & Bacon.
COMMENT
*********************************************************.
COMMENT  Here there are 3 variables for which multivariate
COMMENT  normality is being confirmed.
DATA LIST
 FILE='a:normgrad.dat' FIXED RECORDS=1 TABLE
 /1 gender 1 ethnicit 3 year 5 lowup 7 section 9 prevgpa 11-14 (1)   final 16-17 (1)
 total 19-21 (1) .
list variables=all/cases=9999/format=numbered .
COMMENT  'y' is a variable automatically created by the program,
COMMENT  and does not have to modified for different data sets.
select if (gender eq 2) .
compute y=$casenum .
print formats y(F5) .
regression variables=y prevgpa to total/
 descriptive=mean stddev corr/
 dependent=y/enter prevgpa to total/
 save=mahal(mahal) .
sort cases by mahal(a) .
execute .
list variables=y prevgpa to total mahal/cases=9999/format=numbered .
COMMENT  In the next TWO lines, for a given data set put the
COMMENT  actual n in place of the number '41' used for the
COMMENT  example data set.
loop #i=1 to 41 .
compute p=($casenum - .5) / 41.
COMMENT  In the next line, change '3' to whatever is the number
COMMENT  of variables.
COMMENT     The p critical value of chi square for a given case
COMMENT  is set as [the case number (after sorting) - .5] / the
COMMENT  sample size].
if (gender eq 2) chisq=idf.chisq(p,3) .
end loop .
print formats p chisq (F8.5) .
list variables=y p mahal chisq/cases=9999/format=numbered .
plot
 vertical='chi square'/
 horizontal='Mahalanobis distance'/
 plot=chisq with mahal .
```

Appendix C

SPSS Syntax for Evaluating Univariate and Bivariate Normality

```
PPLOT
 /VARIABLES=prevgpa
 /NOLOG
 /NOSTANDARDIZE
 /TYPE=Q-Q
 /TIES=MEAN
 /DIST=NORMAL .
GRAPH
 /HISTOGRAM=prevgpa .
EXAMINE
 VARIABLES=prevgpa final total
 /PLOT BOXPLOT STEMLEAF HISTOGRAM NPPLOT
 /COMPARE GROUP
 /STATISTICS DESCRIPTIVES
 /CINTERVAL 95
 /MISSING LISTWISE
 /NOTOTAL .
GRAPH
 /SCATTERPLOT (BIVAR)=prevgpa WITH total
 /MISSING=LISTWISE .
PLOT
 /VERTICAL='prevgpa' REFERENCE (6,4)
 /HORIZONTAL='total' REFERENCE (6,7)
 /PLOT=prevgpa WITH total .
GRAPH
 /SCATTERPLOT (BIVAR)=prevgpa with final
 /MISSING=LISTWISE .
PLOT
 /VERTICAL='prevgpa' REFERENCE (6,4)
 /HORIZONTAL='final' REFERENCE (6,9)
 /PLOT=prevgpa WITH final .
GRAPH
 /SCATTERPLOT (BIVAR)=final with total
 /MISSING=LISTWISE .
PLOT
 /VERTICAL='final' REFERENCE (6,9)
 /HORIZONTAL='total' REFERENCE (6,7)
 /PLOT=final WITH total .
```

Appendix D

SPSS Commands for New Dependent Variable

```
SET BLANKS=SYSMIS UNDEFINED=WARN printback=list.
TITLE 'MULTINOR.SPS   tests multivar normality graphically****'.
COMMENT ******************************************************.
COMMENT The original MULTINOR computer program was presented,
COMMENT with examples, in:
COMMENT    Thompson, B. (1990). MULTINOR: A FORTRAN program that
COMMENT    assists in evaluating multivariate normality.
COMMENT    _Educational and Psychological Measurement_, 50,
COMMENT    845-848.
COMMENT
COMMENT The data source for the example are from:
COMMENT    George, D. J., and Mallery, P. (1999). _SPSS
COMMENT    for Windows step by step_.
COMMENT    Boston: Allyn & Bacon.
COMMENT
**********************************************************.
COMMENT  Here there are 3 variables for which multivariate
COMMENT  normality is being confirmed.
DATA LIST
 FILE='a:norgrades.txt' FIXED RECORDS=1 TABLE
 /1 quiztot 1-2 (1) final 4-5 (1) total 7-9 (1) .
list variables=all/cases=9999/format=numbered .
COMMENT  'y' is a variable automatically created by the program,
COMMENT  and does not have to modified for different data sets .
compute y=$casenum .
execute .
print formats y(F5) .
regression variables=y quiztot to total/
 descriptive=mean stddev corr/
 dependent=y/enter quiztot to total/
 save=mahal (mahal) .
sort cases by mahal(a) .
execute .
list variables=y quiztot to total mahal/cases=9999/format=numbered .
COMMENT  In the next TWO lines, for a given data set put the
COMMENT  actual n in place of the number '62' used for the
COMMENT  example data set.
loop #i=1 to 62 .
COMMENT  In the next line, change '3' to whatever is the number
COMMENT  of variables.
COMMENT      The p critical value of chi square for a given case
COMMENT  is set as [the case number (after sorting) - .5] / the
COMMENT  sample size].
compute p=($casenum - .5)/62 .
compute chisq=idf.chisq(p,3) .
end loop .
print formats p chisq (F8.5) .
list variables=y p mahal chisq/cases=9999/format=numbered .
plot
 vertical='chi square'/
 horizontal='Mahalanobis distance'/
 plot=chisq with mahal .
```

Table 1

Skewness and Kurtosis for Variables PREVGPA, FINAL, and TOTAL

			Statistic	Std. Error
PREVGPA	Mean		2.898	9.350E-02
	95% Confidence Interval for Mean	Lower Bound	2.711	
		Upper Bound	3.085	
	5% Trimmed Mean		2.922	
	Median		2.910	
	Variance		.559	
	Std. Deviation		.748	
	Minimum		1.2	
	Maximum		4.0	
	Range		2.8	
	Interquartile Range		1.235	
	Skewness		-.155	.299
	Kurtosis		-.926	.590
FINAL	Mean		6.233	9.439E-02
	95% Confidence Interval for Mean	Lower Bound	6.044	
		Upper Bound	6.421	
	5% Trimmed Mean		6.253	
	Median		6.250	
	Variance		.570	
	Std. Deviation		.755	
	Minimum		4.2	
	Maximum		7.5	
	Range		3.3	
	Interquartile Range		1.100	
	Skewness		-.272	.299
	Kurtosis		-.499	.590
TOTAL	Mean		10.203	.174
	95% Confidence Interval for Mean	Lower Bound	9.856	
		Upper Bound	10.550	
	5% Trimmed Mean		10.287	
	Median		10.300	
	Variance		1.931	
	Std. Deviation		1.390	
	Minimum		5.1	
	Maximum		12.4	
	Range		7.3	
	Interquartile Range		1.625	
	Skewness		-.925	.299
	Kurtosis		1.958	.590

Table 2

Kolmogorov-Smirnov Test with Variables PREVGPA, FINAL, and TOTAL

Tests of Normality

	Kolmogorov-Smirnov[a]		
	Statistic	df	Sig.
PREVGPA	.098	64	.200*
FINAL	.093	64	.200*
TOTAL	.081	64	.200*

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Figure Captions

Figure 1. Contour diagram for a bivariate normal surface

Figure 2. Bivariate frequency distribution for a large group of persons measured on total GRE scores (X) with self-esteem scores (Y)

Figure 3. Example of a bivariate normal distribution

Figure 4. Example of an elliptical bivariate normal distribution for two variables with dissimilar standard deviations and means

Figure 5. Scatterplot of chi-square with Mahalanobis distance for 64 females without transforming or deleting scores

Figure 6. Histogram of female scores on PREVGPA, FINAL, and TOTAL

Figure 7. Box-and-whisker plots of female scores on PREVGPA, FINAL, and TOTAL

Figure 8. Stem-and-leaf plots of female scores on PREVGPA, FINAL, and TOTAL

Figure 9. Quantile vs. Quantile plots for variables PREVGPA, FINAL, and TOTAL

Figure 10.Bivariate scatterplot of FINAL with TOTAL, PREVGPA with TOTAL, and PREVGPA with FINAL

Figure 11. Bivariate scatterplot of PREVGPA with TOTAL and PREVGPA with FINAL using a square root transformation

Figure 12. Bivariate scatterplot of PREVGPA with TOTAL and PREVGPA with FINAL using a squared score transformation

Figure 13. Bivariate scatterplot of PREVGPA with TOTAL and PREVGPA with FINAL using a natural log transformation

Figure 14. Bivariate scatterplot of PREVGPA with TOTAL and PREVGPA with FINAL using a log-10 transformation

Figure 15. Bivariate scatterplot of QUIZTOT with TOTAL and QUIZTOT with FINAL and FINAL with TOTAL demonstrating bivariate normal distributions

Figure 16. Scatterplot of chi-square with Mahalanobis distance for 64 females after replacing PREVGPA with QUIZTOT

Figure 17. Scatterplot of chi-square with Mahalanobis distance for 62 females after replacing PREVGPA with QUIZTOT and deleting two outliers (case # 61 and case #36)

Figure 1.

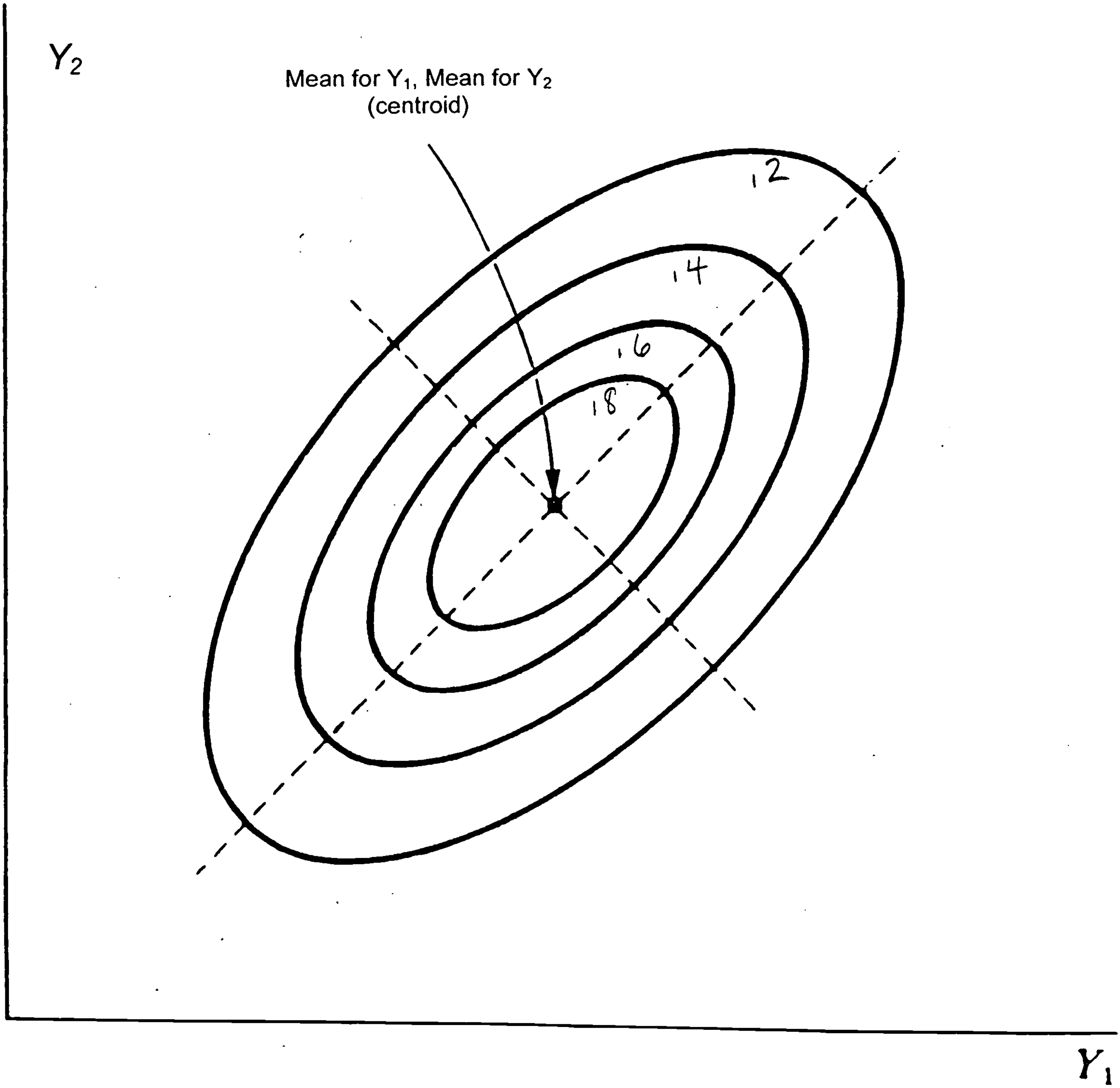
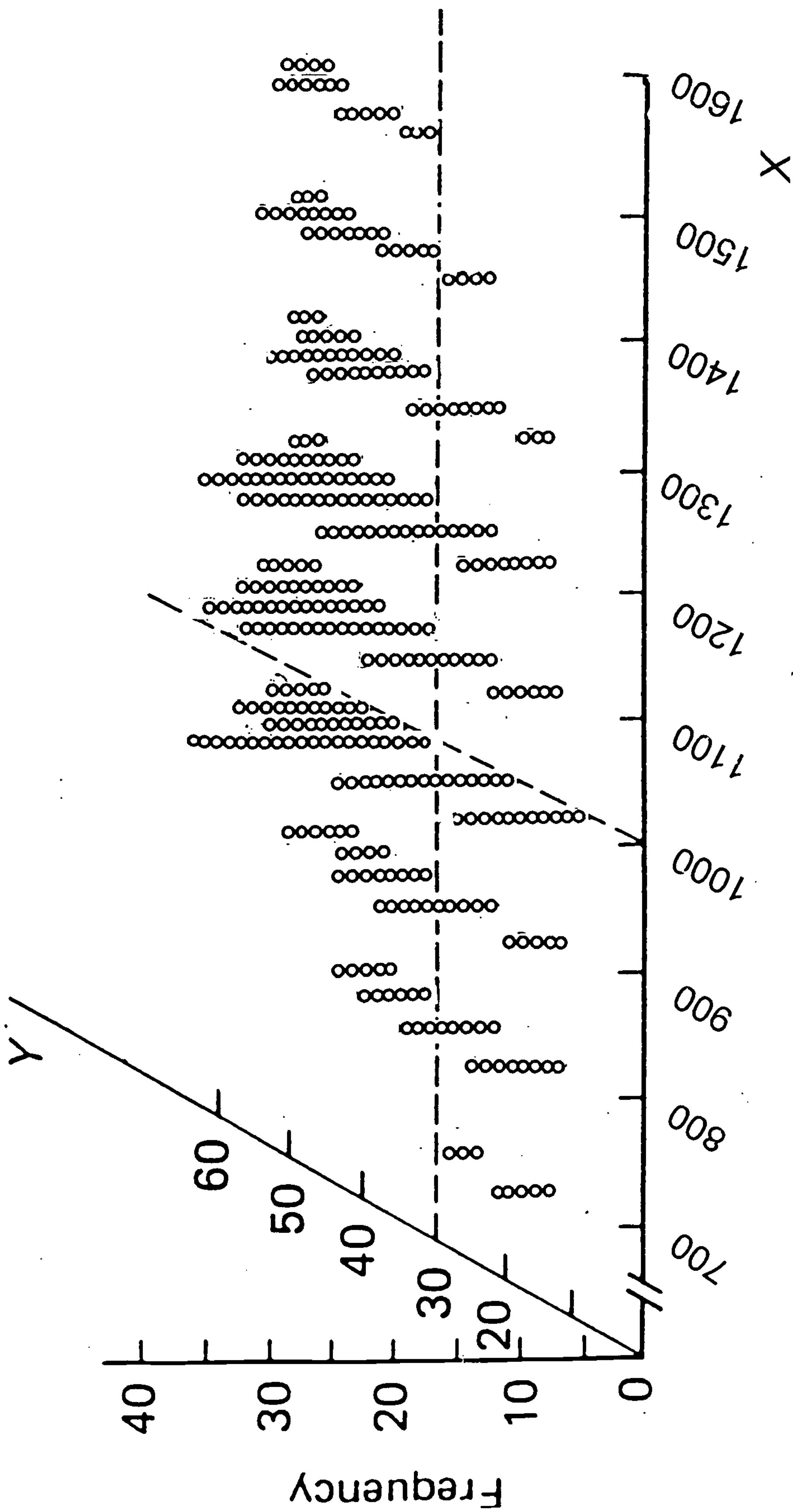


Figure 2.


Frequency
0
10
20
30
40
20
30
40
50
60
Y
700
800
900
1000
1100
1200
1300
1400
1500
1600
X

<u>Figure 3</u>.

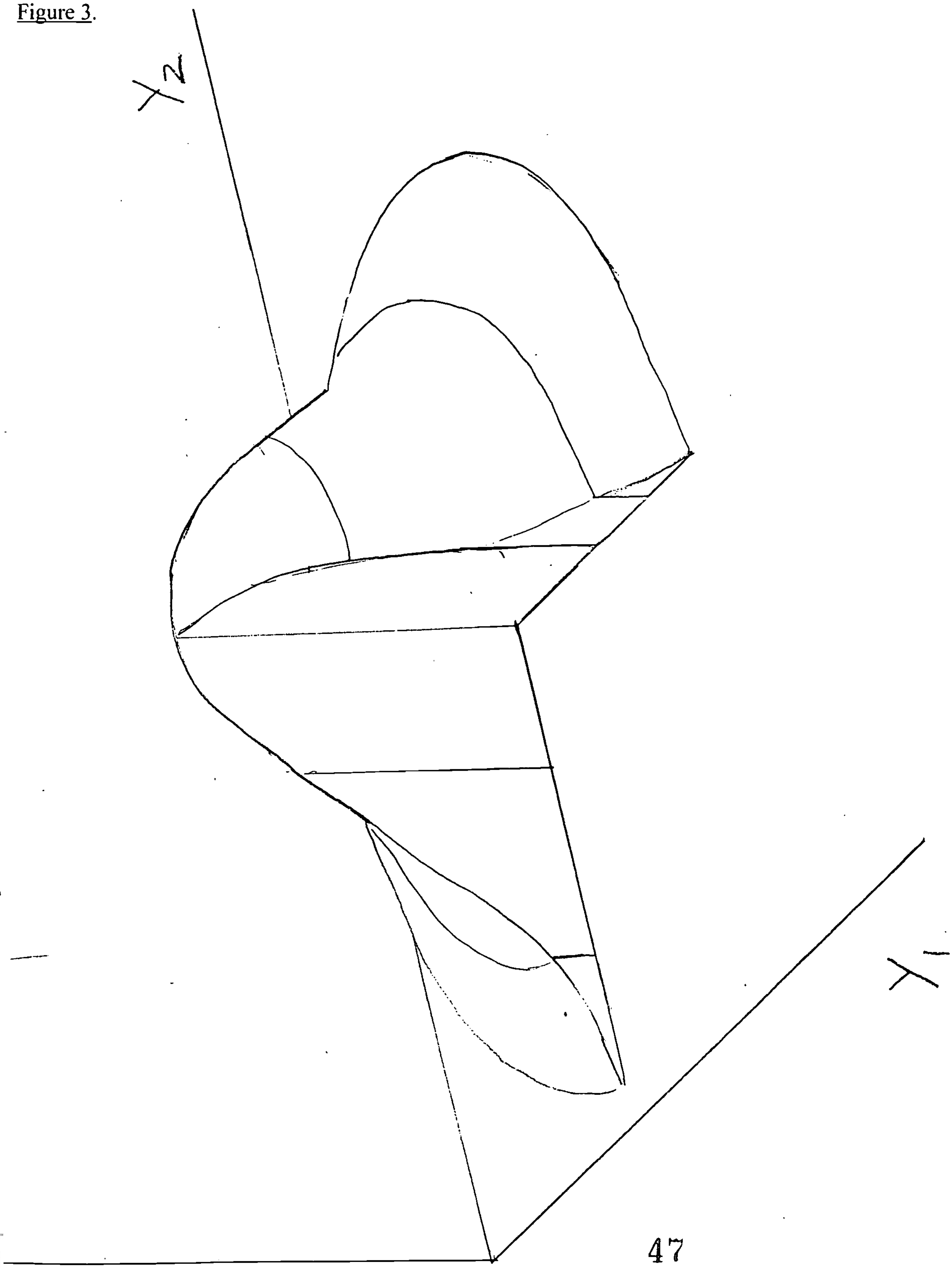
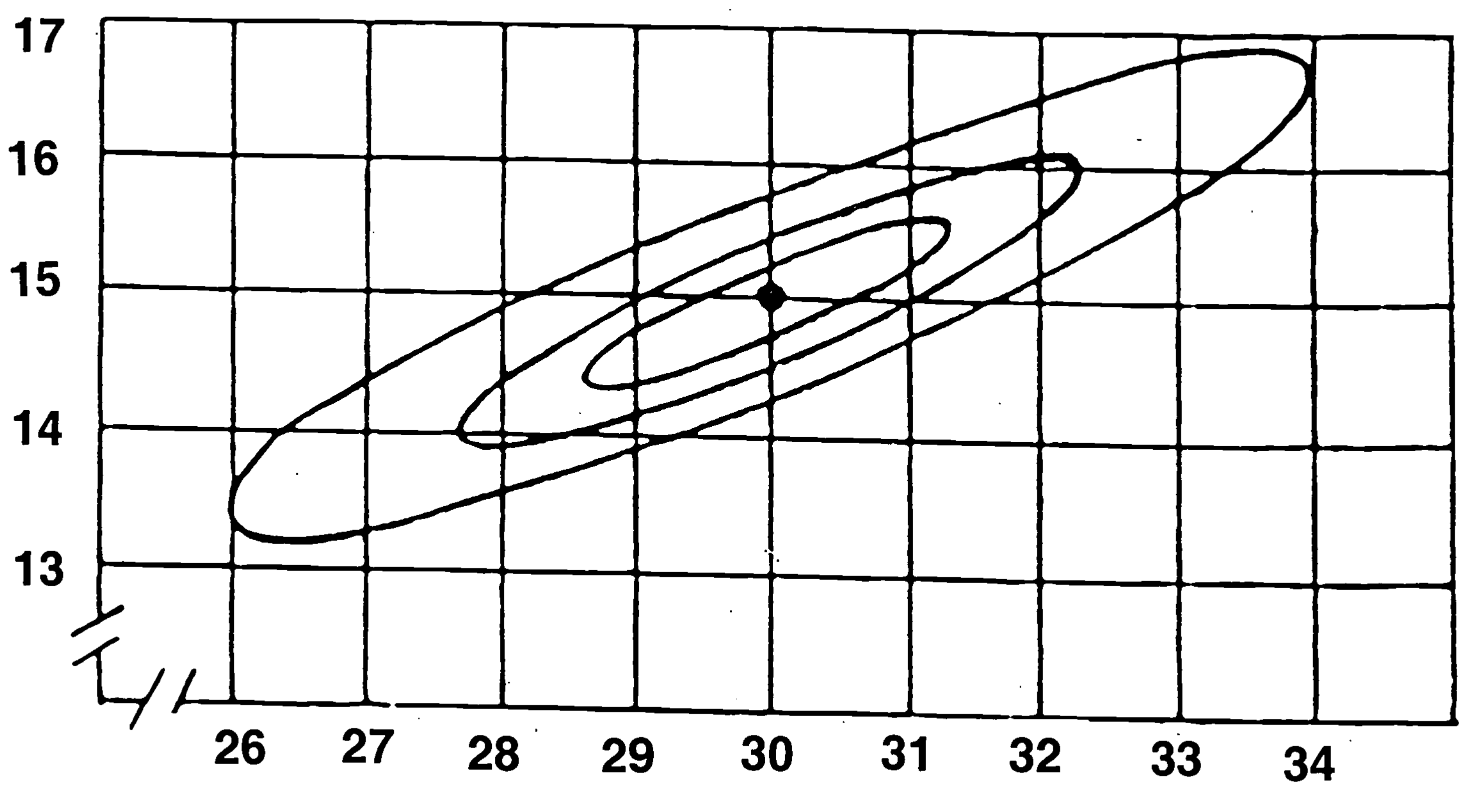


Figure 4.



Mean for X_1 = 30, SD = 2

Mean for X_2 = 15, SD = 1

$$r = .8$$

Figure 5.

	Y	PREVGPA	FINAL	TOTAL	MAHAL
1	32	3.0	6.1	9.8	.12894
2	37	3.1	6.5	10.8	.19202
3	62	3.2	6.5	10.7	.20924
4	23	2.7	6.0	9.6	.27020
5	43	3.1	6.6	10.6	.31206
6	24	2.8	6.3	9.9	.47757
7	50	2.6	5.7	9.5	.55113
8	7	2.3	5.9	9.7	.60748
9	26	2.3	5.9	9.6	.60807
10	28	2.7	6.0	10.4	.76627
11	35	3.5	6.8	10.9	.80149
12	1	2.5	5.7	9.8	.94261
13	51	2.9	6.8	10.8	.97197
14	64	3.1	6.8	10.7	.98038
15	18	2.3	6.1	9.7	1.06917
16	27	3.2	7.0	11.3	1.09685
17	9	2.5	6.1	9.4	1.11502
18	63	2.4	6.3	10.9	1.12942
19	15	2.3	6.3	10.8	1.13850
20	34	3.4	6.9	10.7	1.45279
21	55	3.5	7.1	11.8	1.52051
22	60	2.3	6.0	10.6	1.64273
23	56	3.3	7.2	12.0	1.77941
24	58	3.5	7.2	11.5	1.80680
25	40	2.6	5.8	8.7	1.86005
26	47	2.2	6.2	10.9	1.88054
27	5	3.6	7.1	12.0	1.92325
28	8	2.9	5.5	8.4	1.93248
29	13	2.5	6.8	10.9	1.97093
30	11	3.9	6.8	11.4	2.18579
31	61	2.4	5.3	9.3	2.23487
32	49	3.9	6.6	11.1	2.27174
33	25	2.4	5.6	8.4	2.28717
34	42	3.9	6.7	11.3	2.30406
35	39	3.6	7.2	12.2	2.36403
36	30	3.9	6.3	9.8	2.44809
37	54	1.9	5.7	9.0	2.48116
38	52	3.9	7.3	11.8	2.56152
39	46	4.0	7.0	11.7	2.63689
40	31	3.9	7.4	12.1	2.87585
41	59	3.9	6.7	10.1	2.87749
42	6	3.5	7.5	12.4	2.88134
43	2	4.0	6.8	10.3	3.04228
44	4	4.0	7.4	12.2	3.13920
45	33	3.9	7.5	12.3	3.22701
46	20	1.9	6.6	10.9	3.40986
47	17	2.8	5.7	8.1	3.76446
48	3	1.8	6.6	10.8	3.87187
49	41	1.5	5.7	9.0	4.19712
50	48	2.0	5.3	9.8	4.23244
51	21	3.5	5.4	9.2	4.23439
52	57	2.0	5.2	9.7	4.68805
53	38	1.2	5.3	8.8	4.98346
54	14	2.1	6.2	9.0	5.01279
55	12	2.2	5.2	9.8	5.08212
56	16	2.0	7.1	12.0	5.23050
57	19	3.0	5.5	10.3	5.33996
58	44	3.9	5.7	9.9	5.43204
59	10	3.7	5.5	7.8	6.31631
60	29	1.3	5.8	8.8	6.57535
61	22	2.6	5.3	10.3	7.12962
62	45	3.0	4.9	6.6	8.29773
63	53	2.3	4.7	9.3	9.40548
64	36	3.6	4.2	5.1	18.84009

Number of cases read: 64 Number of cases listed: 64

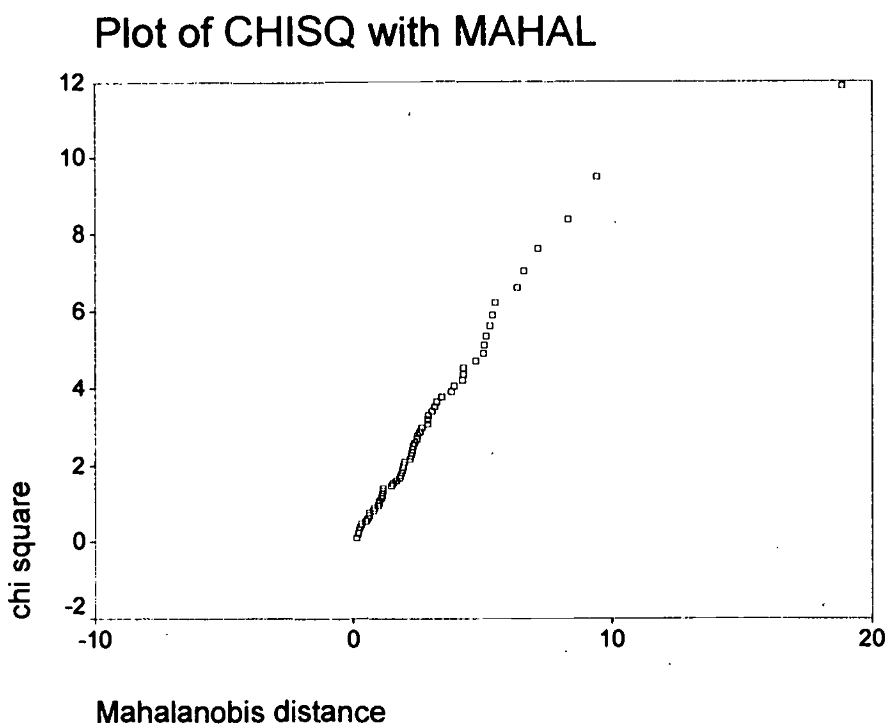


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Figure 6.

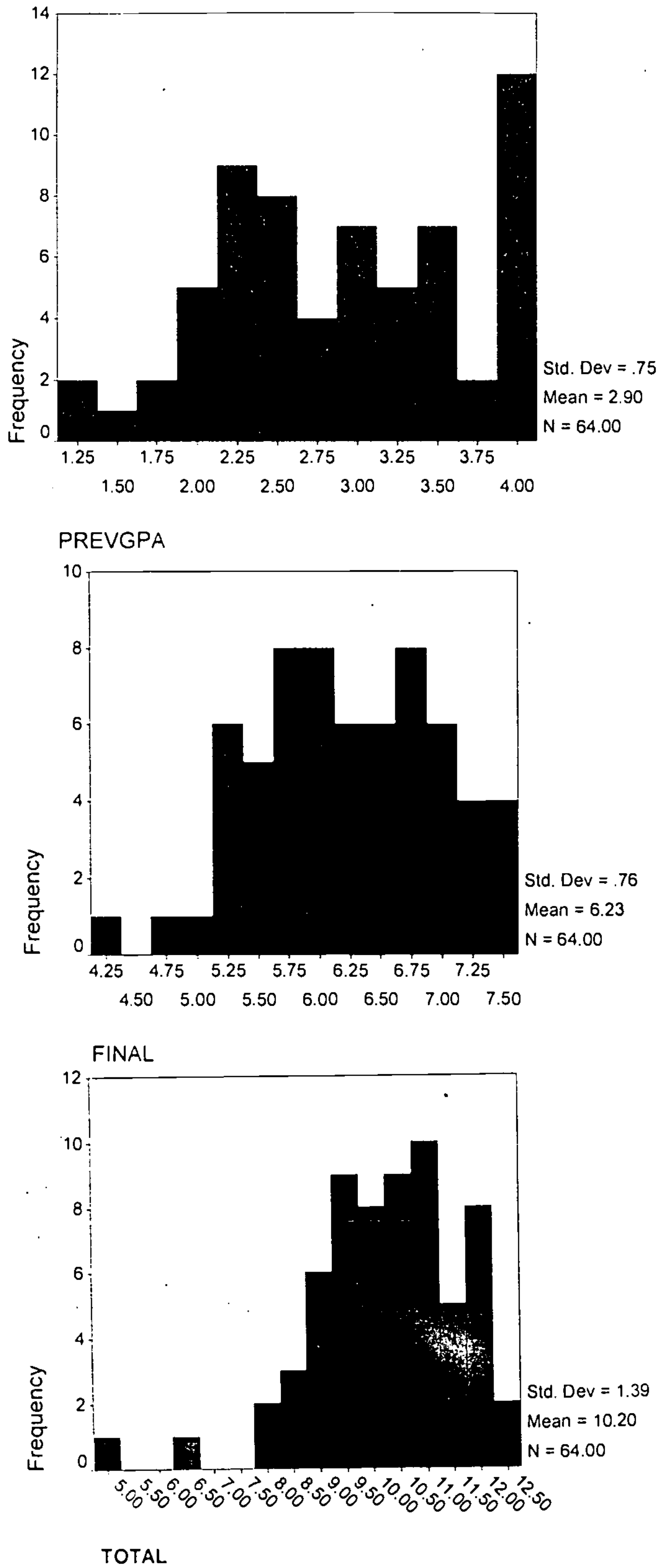

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Figure 7.

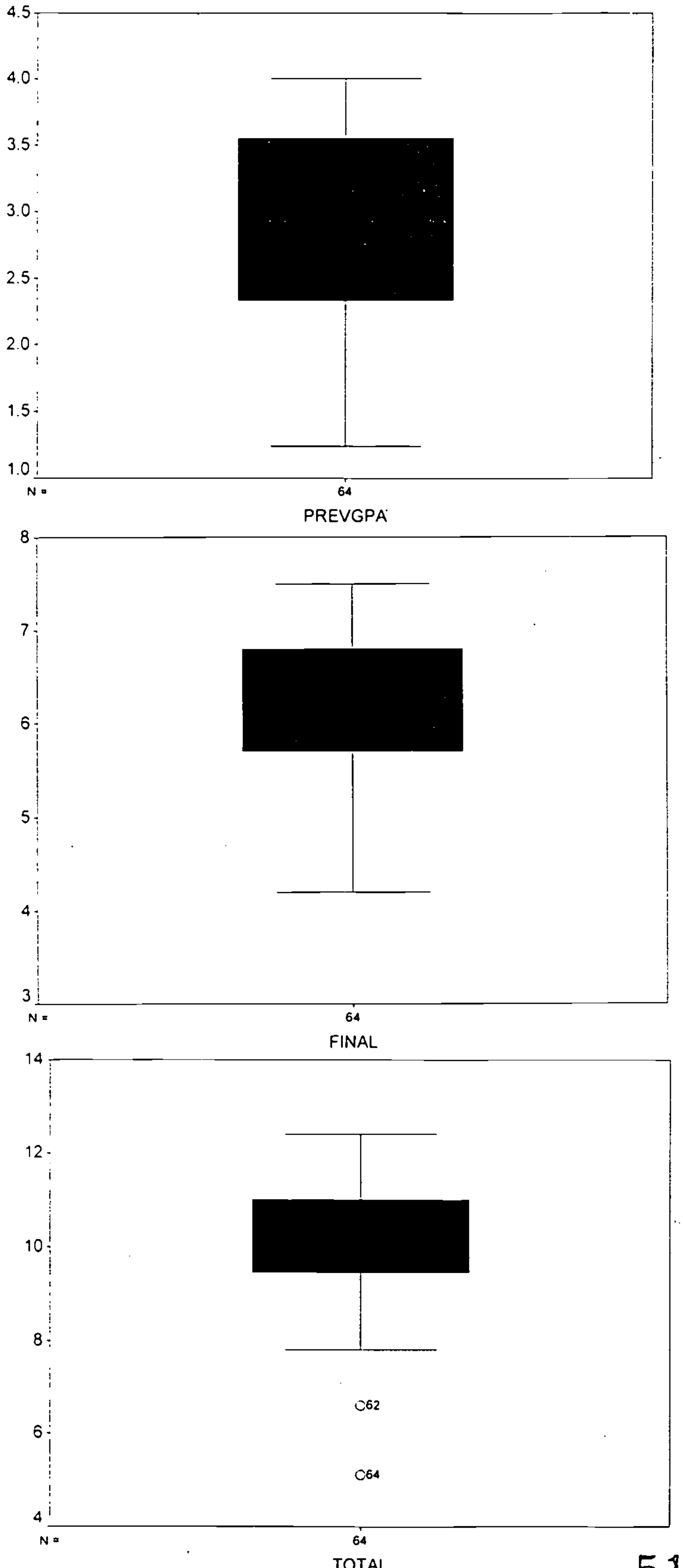
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
N =
64
PREVGPA
8
7
6
5
4
3
N =
64
FINAL
14
12
10
8
6
4
62
64
N =
64
TOTAL

Figure 8.

PREVGPA

```
PREVGPA Stem-and-Leaf Plot

 Frequency    Stem &  Leaf

     2.00        1 .  23
     5.00        1 .  58899
    16.00        2 .  0002223333333444
    11.00        2 .  55566778999
    12.00        3 .  000011123444
    17.00        3 .  55556799999999999
     1.00        4 .  0

 Stem width:    1.0
 Each leaf:       1 case(s)
```

FINAL

```
FINAL Stem-and-Leaf Plot

 Frequency    Stem &  Leaf

     1.00        4 .  2
     2.00        4 .  79
     7.00        5 .  2233334
    14.00        5 .  55567777778899
    12.00        6 .  000111223333
    15.00        6 .  556666778888889
    11.00        7 .  00111222344
     2.00        7 .  55

 Stem width:  1.0
 Each leaf:       1 case(s)
```

TOTAL

```
TOTAL Stem-and-Leaf Plot

 Frequency    Stem &  Leaf

     2.00 Extremes    (=<6.6)
     1.00        7 .  8
     6.00        8 .  144788
    20.00        9 .  00023345667778888899
    19.00       10 .  1333466777888899999
     8.00       11 .  13345788
     8.00       12 .  00012234

 Stem width:   1.0
 Each leaf:       1 case(s)
```

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Figure 9.

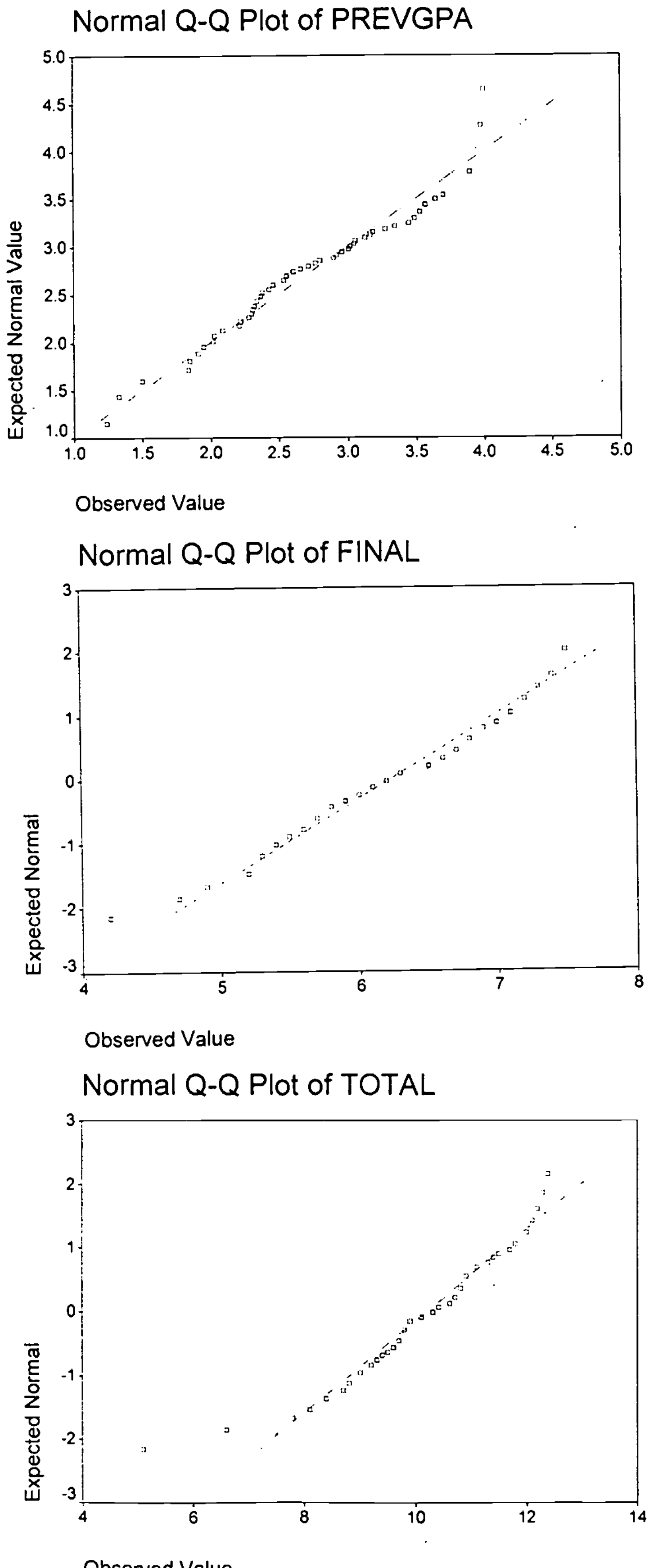

Normal Q-Q Plot of PREVGPA
Expected Normal Value
Observed Value
Normal Q-Q Plot of FINAL
Expected Normal
Observed Value
Normal Q-Q Plot of TOTAL
Expected Normal
Observed Value

Figure 10.

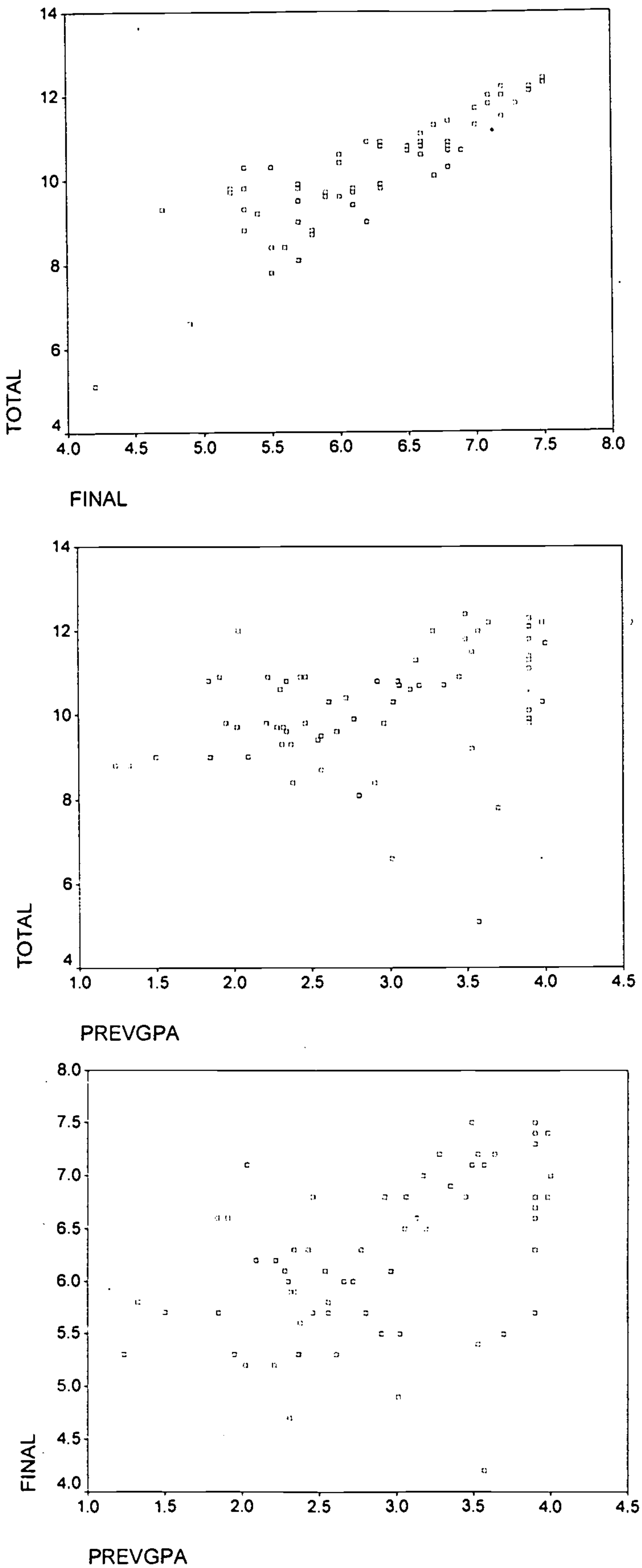
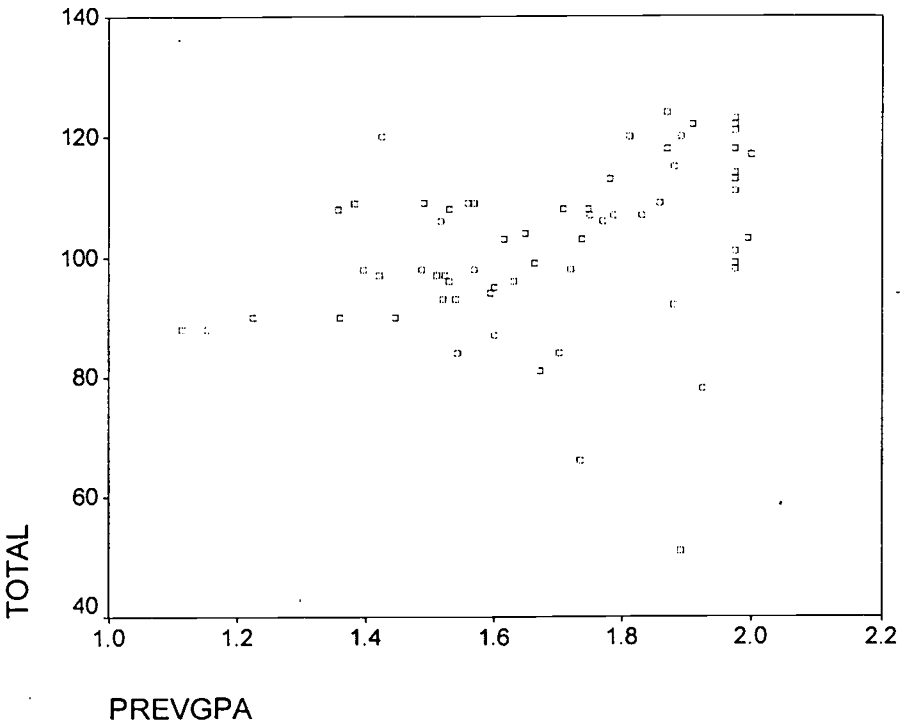

TOTAL
14
12
10
8
6
4
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
FINAL
TOTAL
14
12
10
8
6
4
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
PREVGPA
FINAL
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
PREVGPA

Figure 11.


140
120
100
80
60
40
TOTAL
1.0
1.2
1.4
1.6
1.8
2.0
2.2
PREVGPA

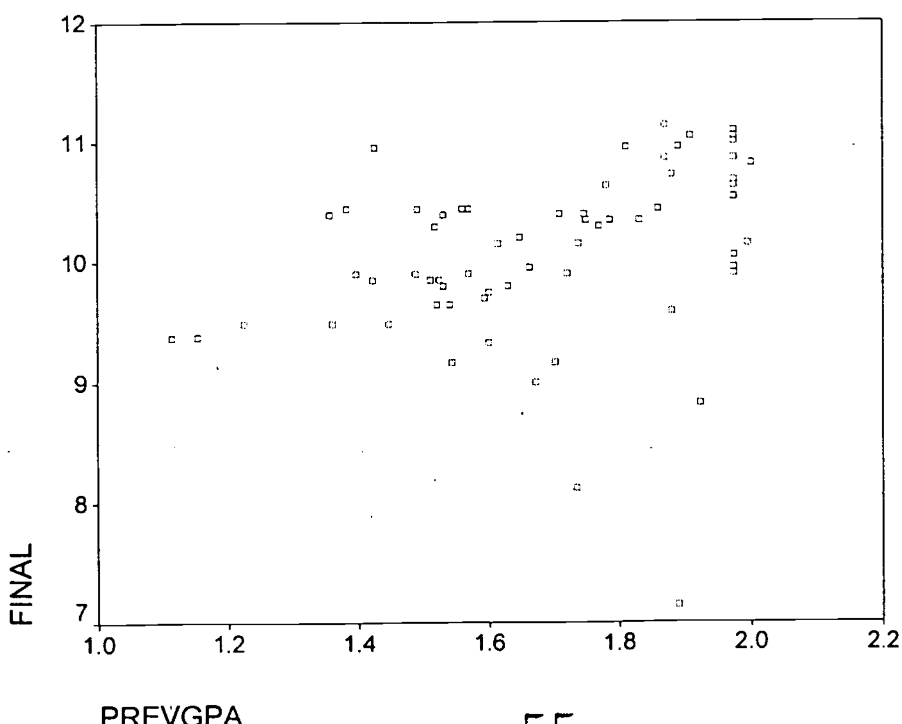
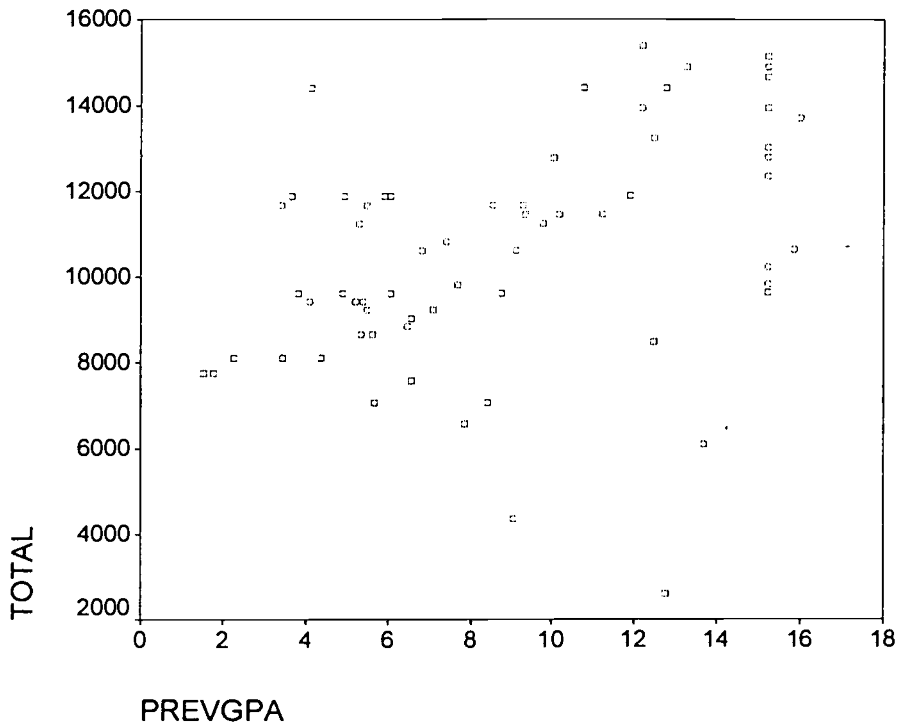

12
11
10
9
8
7
FINAL
1.0
1.2
1.4
1.6
1.8
2.0
2.2
PREVGPA

Figure 12.


16000
14000
12000
10000
8000
6000
4000
2000
TOTAL
0
2
4
6
8
10
12
14
16
18
PREVGPA

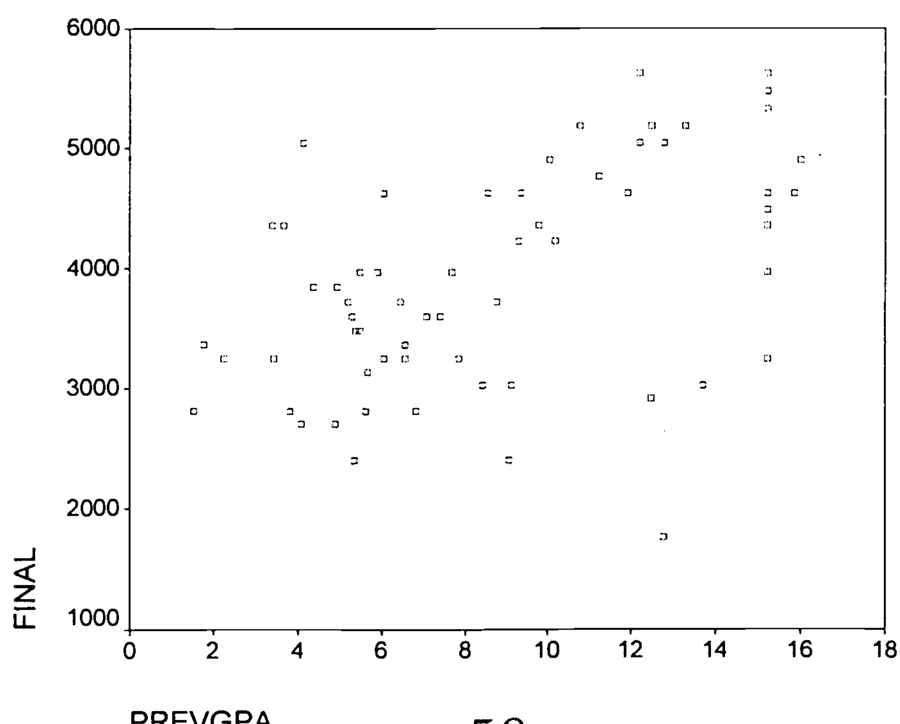
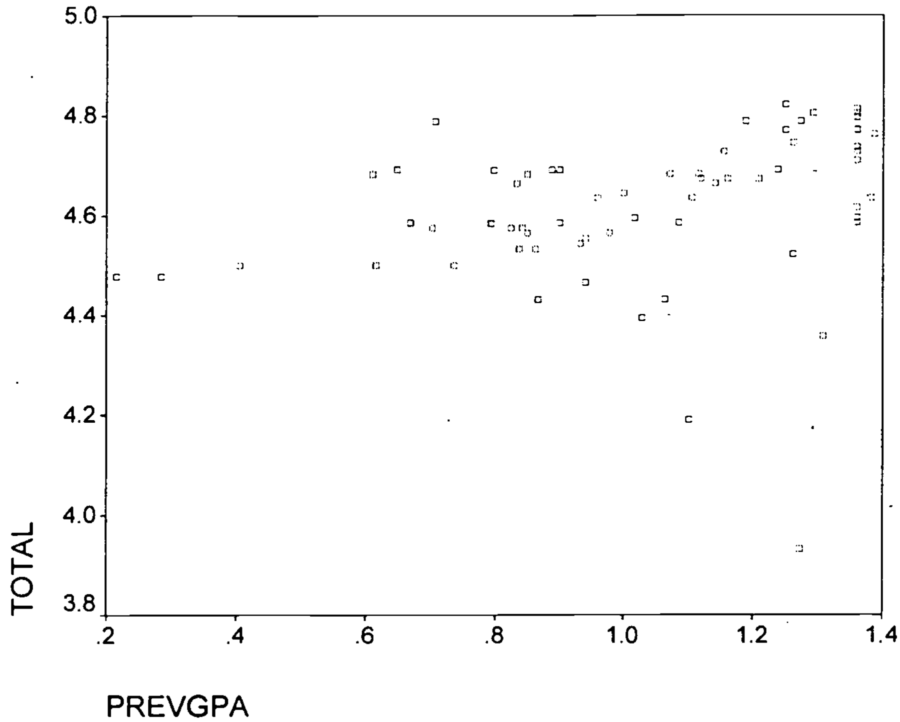

6000
5000
4000
3000
2000
1000
FINAL
0
2
4
6
8
10
12
14
16
18
PREVGPA

Figure 13.


5.0
4.8
4.6
4.4
4.2
4.0
3.8
TOTAL
.2
.4
.6
.8
1.0
1.2
1.4
PREVGPA

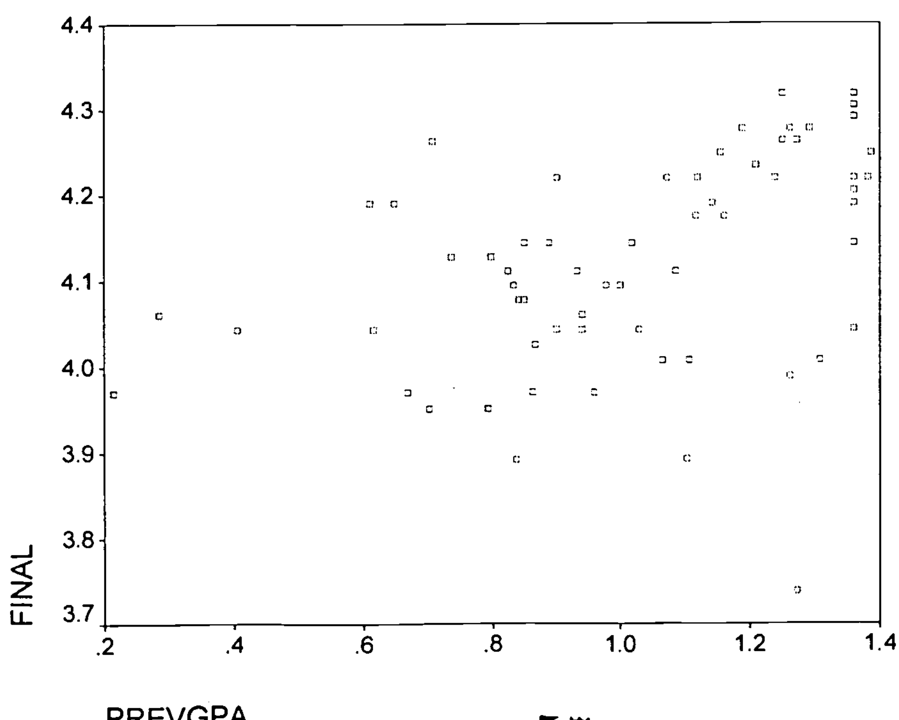
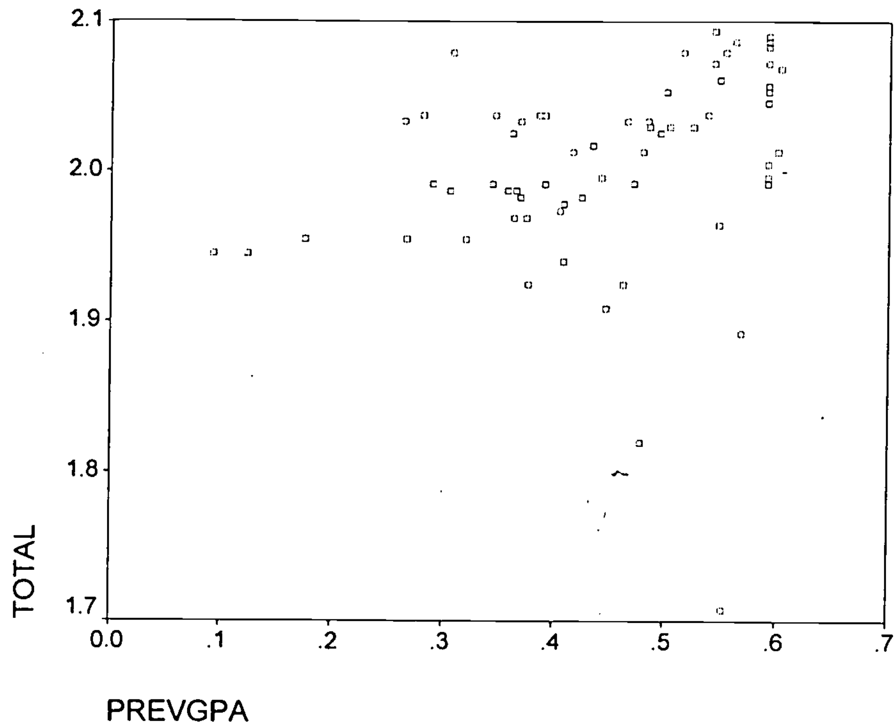

4.4
4.3
4.2
4.1
4.0
3.9
3.8
3.7
FINAL
.2
.4
.6
.8
1.0
1.2
1.4
PREVGPA

Figure 14.


2.1
2.0
1.9
1.8
1.7
TOTAL
0.0
.1
.2
.3
.4
.5
.6
.7
PREVGPA

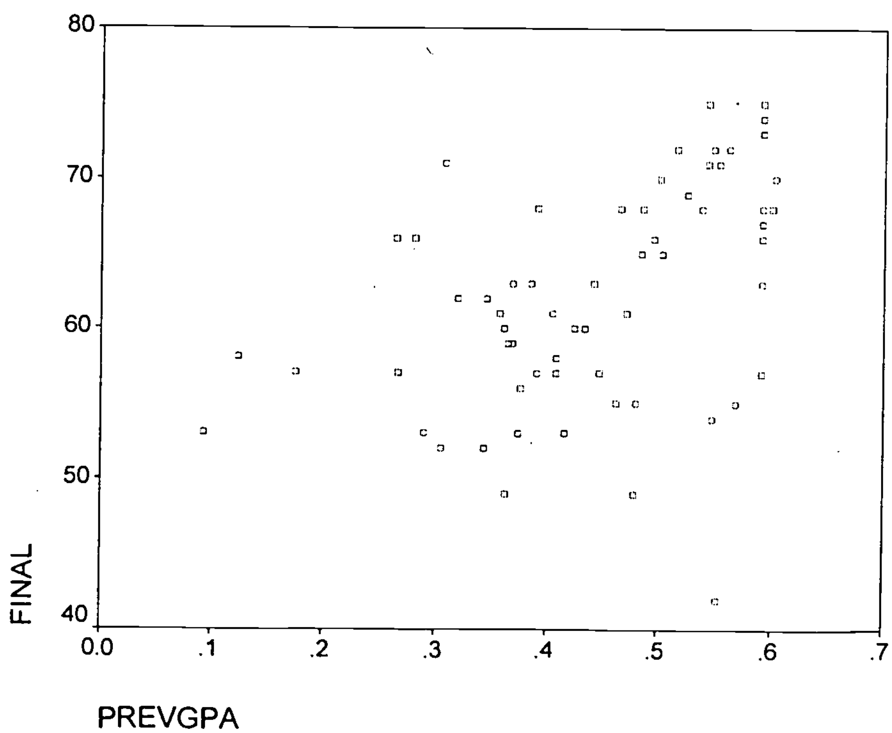

80
70
60
50
40
FINAL
0.0
.1
.2
.3
.4
.5
.6
.7
PREVGPA

Figure 15.

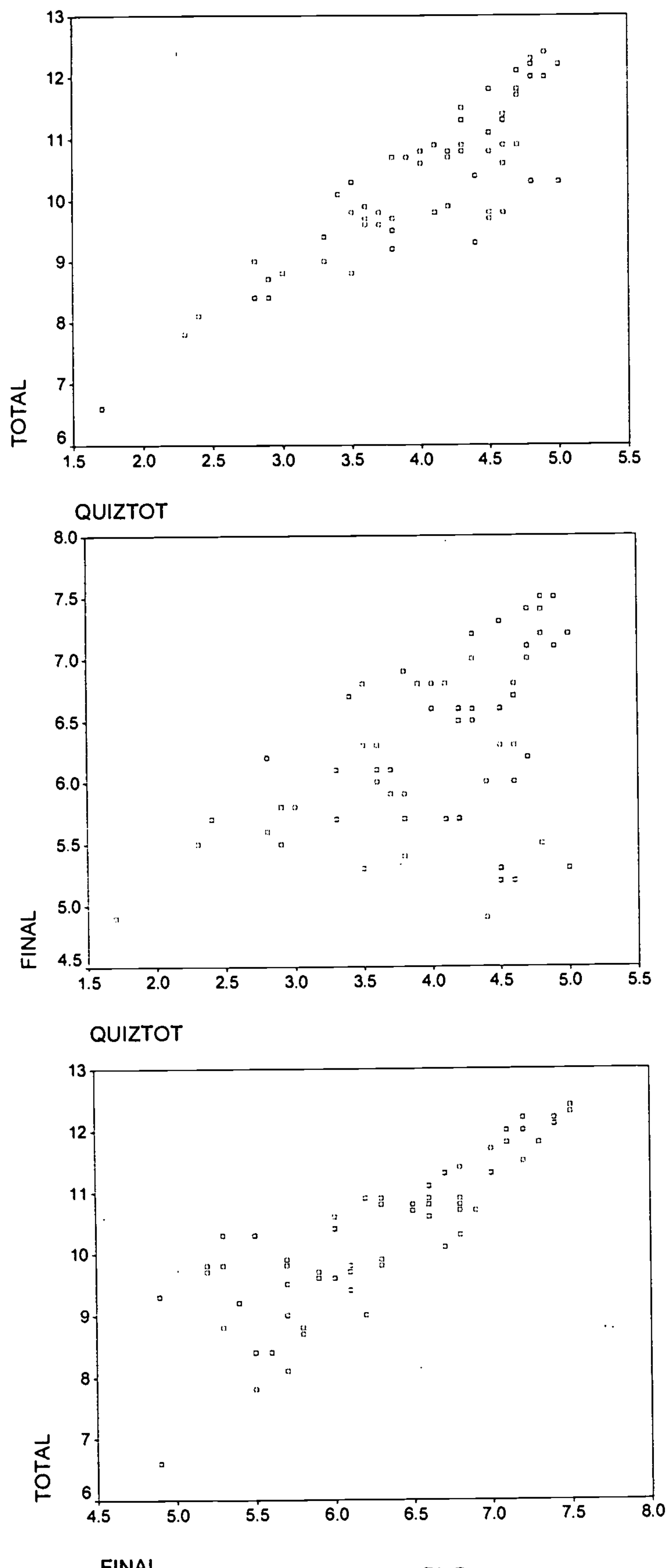

TOTAL
13 12 11 10 9 8 7 6
1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5
QUIZTOT
FINAL
8.0 7.5 7.0 6.5 6.0 5.5 5.0 4.5
1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5
QUIZTOT
TOTAL
13 12 11 10 9 8 7 6
4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0
FINAL

Figure 16.

	Y	P	MAHAL	CHISQ
1	60	.00806	.10744	.09918
2	36	.02419	.16680	.21093
3	32	.04032	.18702	.30186
4	3	.05645	.20321	.38390
5	20	.07258	.23811	.46078
6	42	.08871	.27738	.53432
7	7	.10484	.29880	.60555
8	18	.12097	.31775	.67515
9	26	.13710	.33262	.74360
10	23	.15323	.33964	.81125
11	24	.16935	.43263	.87837
12	48	.18548	.45189	.94517
13	15	.20161	.54342	1.01184
14	13	.21774	.61544	1.07854
15	35	.23387	.61544	1.14539
16	30	.25000	.66028	1.21253
17	41	.26613	.68158	1.28007
18	49	.28226	.70489	1.34811
19	50	.29839	.72545	1.41675
20	11	.31452	.78527	1.48609
21	61	.33065	.79569	1.55623
22	28	.34677	.83328	1.62726
23	62	.36290	.88345	1.69927
24	9	.37903	.99789	1.77236
25	1	.39516	1.03819	1.84664
26	27	.41129	1.05224	1.92220
27	40	.42742	1.10923	1.99916
28	53	.44355	1.10923	2.07763
29	43	.45968	1.24528	2.15773
30	45	.47581	1.28034	2.23958
31	46	.49194	1.28117	2.32334
32	59	.50806	1.43661	2.40914
33	34	.52419	1.44174	2.49715
34	54	.54032	1.51824	2.58754
35	21	.55645	1.71069	2.68052
36	57	.57258	1.77663	2.77628
37	5	.58871	1.87132	2.87507
38	16	.60484	1.87132	2.97715
39	29	.62097	1.89396	3.08281
40	55	.63710	1.93711	3.19239
41	37	.65323	1.95679	3.30625
42	2	.66935	1.99538	3.42481
43	58	.68548	1.99608	3.54858
44	51	.70161	2.08945	3.67810
45	39	.71774	2.28725	3.81403
46	38	.73387	2.33725	3.95715
47	8	.75000	2.47385	4.10834
48	31	.76613	2.54928	4.26872
49	4	.78226	2.62843	4.43960
50	25	.79839	2.75111	4.62259
51	14	.78906	2.81123	4.51521
52	33	.80469	2.87295	4.69778
53	6	.82031	2.91365	4.89450
54	17	.83594	3.78978	5.10793
55	10	.85156	4.12775	5.34141
56	48	.86719	4.45766	5.59941
57	19	.88281	4.75461	5.88801
58	57	.89844	5.14311	6.21598
59	12	.91406	5.68402	6.59639
60	53	.92969	6.94300	7.05027
61	22	.94531	7.45882	7.61453
62	45	.96094	7.68039	8.36377
63	36	.97656	14.50223	9.49005
64	61	.99219	62.01562	11.87813

er of cases read: 64 Number of cases listed: 64

Plot of CHISQ with MAHAL

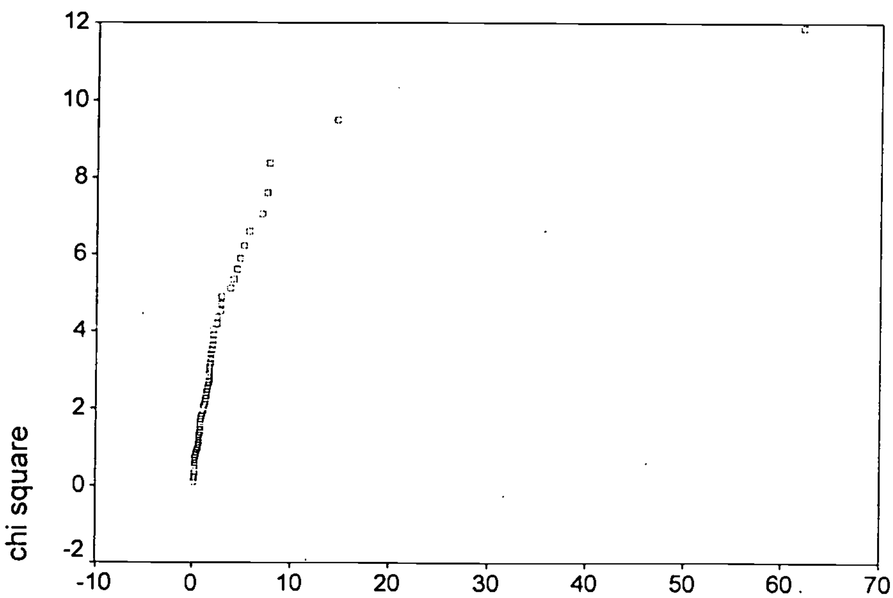


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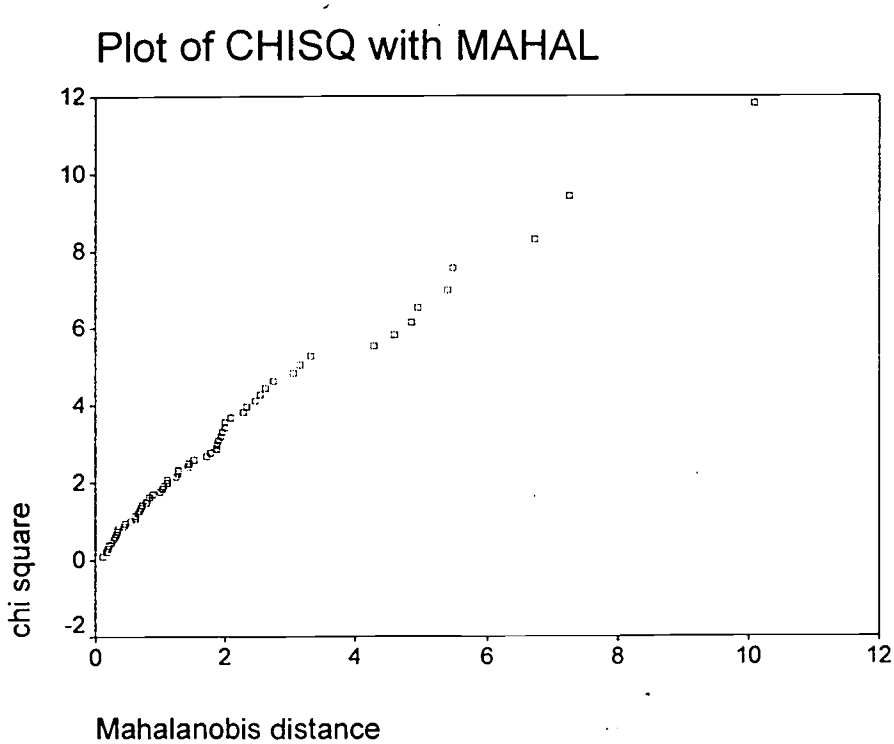
Figure 17.

	Y	P	MAHAL	CHISQ
1	60	.00806	.10744	.09918
2	36	.02419	.16680	.21093
3	32	.04032	.18702	.30186
4	3	.05645	.20321	.38390
5	20	.07258	.23811	.46078
6	42	.08871	.27738	.53432
7	7	.10484	.29880	.60555
8	18	.12097	.31775	.67515
9	26	.13710	.33262	.74360
10	23	.15323	.33964	.81125
11	24	.16935	.43263	.87837
12	48	.18548	.45189	.94517
13	15	.20161	.54342	1.01184
14	13	.21774	.61544	1.07854
15	35	.23387	.61544	1.14539
16	30	.25000	.66028	1.21253
17	41	.26613	.68158	1.28007
18	49	.28226	.70489	1.34811
19	50	.29839	.72545	1.41675
20	11	.31452	.78527	1.48609
21	61	.33065	.79569	1.55623
22	28	.34677	.83328	1.62726
23	62	.36290	.88345	1.69927
24	9	.37903	.99789	1.77236
25	1	.39516	1.03819	1.84664
26	27	.41129	1.05224	1.92220
27	40	.42742	1.10923	1.99916
28	53	.44355	1.10923	2.07763
29	43	.45968	1.24528	2.15773
30	45	.47581	1.28034	2.23958
31	46	.49194	1.28117	2.32334
32	59	.50806	1.43661	2.40914
33	34	.52419	1.44174	2.49715
34	54	.54032	1.51824	2.58754
35	21	.55645	1.71069	2.68052
36	57	.57258	1.77663	2.77628
37	5	.58871	1.87132	2.87507
38	16	.60484	1.87132	2.97715
39	29	.62097	1.89396	3.08281
40	55	.63710	1.93711	3.19239
41	37	.65323	1.95679	3.30625
42	2	.66935	1.99538	3.42481
43	58	.68548	1.99608	3.54858
44	51	.70161	2.08945	3.67810
45	39	.71774	2.28725	3.81403
46	38	.73387	2.33725	3.95715
47	8	.75000	2.47385	4.10834
48	31	.76613	2.54928	4.26872
49	4	.78226	2.62843	4.43960
50	25	.79839	2.75111	4.62259
51	33	.81452	3.05342	4.81972
52	6	.83065	3.15610	5.03358
53	14	.84677	3.31802	5.26749
54	47	.86290	4.28631	5.52592
55	19	.87903	4.60254	5.81497
56	17	.89516	4.86054	6.14339
57	56	.91129	4.95149	6.52428
58	10	.92742	5.40105	6.97867
59	12	.94355	5.47286	7.54348
60	52	.95968	6.71760	8.29335
61	22	.97581	7.25077	9.42039
62	44	.99194	10.09578	11.80964



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