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ABSTRACT

The College Success in Mathematics Project began in 1996 with the production of a framework for student success in university-level mathematics courses at the University of Guam. The project continued with a one-week workshop followed by the preparation of "Preparing for Mathematics" at the University of Guam (UOG), which outlined specific topics and described methods for testing the ability of students to succeed in the placement examinations for mathematics at UOG. In July 1999, a Curriculum Alignment and Teacher Training Workshop was held that made several recommendations at the administrative level, developed a list of topics to be emphasized in high school mathematics courses, and produced an outline of teaching methodologies to be used to enhance students' learning of mathematics. Building on the 1999 workshop recommendations, a Teacher Training Workshop was held, the first part of which was devoted to the discussion of Preparing for Mathematics at the University of Guam. The National Council of Teachers of Mathematics (NCTM) Standards were introduced and discussed, particularly as they apply to the Federated States of Micronesia situation, so a summary of relevant parts of the NCTM Standards have also been included. This manual contains suggestions, teaching tips, and ideas for making these changes in mathematics classrooms. (ASK)

College Success in Mathematics

Teacher's Manual

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A Joint Project of:

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&
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1 Introduction

The College Success in Mathematics Project began in 1996 with the production of a framework for student success in university-level mathematics courses, particularly at the University of Guam. The Project continued with a one-week workshop in Chuuk in May, 1998, followed by the preparation of *Preparing for Mathematics at the University of Guam*, which outlines specific topics and gives methods for testing the ability of students to succeed in the placement examinations for mathematics at UOG.

In July, 1999, a Curriculum Alignment and Teacher Training Workshop was held in Pohnpei with participants from Chuuk, Kosrae, and Pohnpei. This workshop made several recommendations at the administrative level, developed a list of topics to be emphasized in high school mathematics courses, and produced an outline of teaching methodologies to be used to enhance students' learning of mathematics.

Building on the 1999 Workshop recommendations, a Teacher Training Workshop was held from January 17 through January 21, 2000, in Pohnpei with participants from Chuuk, Kosrae, and Pohnpei. The first part of this workshop was devoted to discussion of *Preparing for Mathematics at the University of Guam* and the recommendations of the July, 1999, Workshop, so both these documents are summarized below.

The National Council of Teachers of Mathematics *Standards* were introduced and discussed, particularly as they apply to the FSM situation, so a summary of relevant parts of the *NCTM Standards* have been included below. The teaching methodologies from the July, 1999, Workshop were expanded and illustrated both for the participants and in College of Micronesia classrooms, and participants gave a presentation of their own, which was then critiqued by everyone involved on the basis of the techniques which had been discussed during the week.

It is difficult to convey on paper the flavor, context, and "mathematical culture" of the workshop due to its highly interactive nature, the group work involved, and the work done on the blackboard. There is no substitute in mathematics for actually being there and actively participating! However, perhaps the teaching tips included in this manual will in some small way substitute for actually attending the workshop.

Finally, the recommendations and teaching tips included here were generated by FSM mathematics teachers themselves! Guidance from outside is useful and good, but each teacher can make changes in the classroom that the teacher is assigned, and each department in each high school can make changes in that department. This manual contains suggestions and ideas for making these changes.

2 The importance of mathematics

Mathematics is the single most critical filter for success, not only in university-level studies but also in post-baccalaureate careers. Studies show that every high school mathematics course taken beyond Algebra I *doubles* a student's chance of success in college. Inability to think quantitatively restricts a person's access not only to technically oriented professions such as computer science and engineering, but also to professions such as nursing, finance, management, and in these days of computers and statistics, even sociology, psychology, and history. Students must have a solid foundation in mathematics and must continue to build on that foundation throughout their academic careers in order to keep all vocational options open.

Nurses must deal with fractions and ratios and proportions and rates of drug elimination. Sociologists and psychologists and social workers must understand statistical measures of central tendencies and dispersion and must read and interpret graphs of one quantity versus another. Artists and art teachers must understand geometric perspective and scaling. Musicians must understand the ratios involved in musical scales. Elementary school teachers must understand not only *how* to add, subtract, multiply and divide numbers and fractions, but also *why* these operations are carried out as they are and how they are related to real-world phenomena. This is not to mention the mathematics involved in power generation and distribution, traffic planning, road construction, telephone networks, and the study of tuna fish populations.

Specifically of interest in our region is the fact that to complete a degree at the University of Guam every student must take a computer or statistics course AND at least one other university-level mathematics course. Some colleges and majors require more than this minimum level or specify which course their students must take. For example, the College of Business and Public Administration requires *Finite Mathematics* for all their students, plus further quantitative courses in business and economics. A degree in agriculture requires *College Algebra*. Majors in biology are required to take *Basic Calculus* and majors in chemistry must take two calculus courses beyond *Basic Calculus*. This is just a sample of mathematics requirements.

3 Mathematics courses at UOG

Instructors cannot prepare students for success in college without having some knowledge of exactly what is in store for the students when they do enter college. In this section we outline what awaits a student who is entering the University of Guam. A similar experience awaits someone entering any institution of higher learning; details may differ but the general structure will be the same. The University of Guam's procedures are given only because the author is most familiar with those.

A mathematics placement examination is given to determine which mathematics course is appropriate for an incoming student. This examination is given in conjunction with the English language placement examination. The Division of Mathematical Sciences has chosen the *Basic Algebra Test* provided by the Mathematical Association of America, with cut-off scores based on those used nationally by the colleges and universities using this method of assessment.

The *Basic Algebra Test* used for UOG placement in mathematics is exactly what its name implies. There are no "word problems," no geometry, and no statistics on the test. Rather, it concentrates on algebraic manipulations involving signed numbers, fractions, exponents, polynomials, inequalities, absolute values, simplification of radicals, solution of simple linear equations and systems of equations, and graphing of straight lines. These skills seem to be those covered by the FSM curriculum standards numbered 48, 49, and 61.

The test itself is a machine-graded multiple-choice exam, each question having five possible answers. There are twenty-five questions and the time limit is thirty minutes. With ten or fewer correct, a student in placed into *Fundamentals of Mathematics*, a developmental course which counts toward a full academic load but does not count toward any degree. Scores between eleven and fourteen inclusive place a student into courses which do carry degree credit, but are not quite at the level of college algebra. A score of fifteen or above qualifies a student to take *College Algebra*; with a very high score the student may take a calculus preparedness test and thus may be able to enroll in calculus immediately. Finally, a score of 500 or more on the mathematics portion of the Scholastic Aptitude Test also places a student in college algebra.

MA085 FUNDAMENTALS OF MATHEMATICS. This course carries no credit toward graduation but does count as three credit hours toward the student's academic load. Upon entry into MA085, the student takes a Diagnostic Examination which determines the student's starting place in the textbook. This is a self-paced course in which the student studies a chapter in the text until satisfied that the material has been absorbed (some instructors require them to take the end-of-chapter test in the book), then takes an examination on that chapter. Passing is 80%, upon which the student proceeds to the next chapter. A score of less than 80% necessitates a return to that chapter, and then a re-take of the examination — a different version, of course. A faculty member and usually a student assistant are available for individual tutoring during the class period as well as for administering the chapter tests. The course consists of eighteen chapters. About 10% of the MA085 students are able to complete the course in one semester. Most take at least two and usually three or more semesters to finish. Since this course does not carry degree credit, this is a substantial waste of the student's time, effort, and money.

BEGINNING COLLEGE-LEVEL COURSES. The structure and conduct of courses MA110 *Finite Mathematics* through MA203 *Basic Calculus* are essentially the same, though the details vary from course to course and instructor to instructor. Each of these classes normally has about twenty-five students and is a mix of lecture, discussion, question-and-answer, and individual or team-work. The pace is set by the instructor. Though attendance may or may not be taken, it is expected. Homework is assigned, how it is checked and used for evaluation depends on the instructor. There are normally two to four in-class midterm examinations, plus a final examination, which is usually comprehensive. In addition, some instructors make writing assignments or assign longer-term projects to carry out. The expectation is that a student will put in at least two hours of work outside class for each hour in class.

4 Academic Expectations Framework

To be successful in entry-level mathematics at UOG, a student must

1. have elementary algebra skills,
2. have general knowledge of the world,
3. be able to read carefully and precisely,
4. be able to write carefully and precisely,
5. have some knowledge of simple geometric ideas and formulas such as those for perimeter, area, and volume,
6. have some understanding of the mathematical abstraction involved in the ideas of variables and functions,
7. have some facility for translating written descriptions of situations into mathematical descriptions,
8. have some understanding of the correspondence between numbers and functions and their graphical representations, and
9. have some familiarity with calculators and their operation.

Elementary algebra skills

These are precisely the skills which would lead to a high score on the mathematics placement examination described above. Such skills always play an important role. It is extremely important in each course that a student avoid simple arithmetic and algebra mistakes and be able to apply these manipulative skills with reasonable speed and with great accuracy. It is not enough to know that negative times negative is positive, it must be done quickly and without mistakes.

General knowledge

Students must know about airplanes and speed and boats and time and water and evaporation and car payments and interest and temperatures of ovens and baked cakes and boxes of frozen chicken. The list could go on and on. Such things commonly arise in word problems and problems in which the student must create a mathematical model of such things. Mathematical modeling is a very important part of every college-level mathematics course and considerable general knowledge is needed to successfully build mathematical models of real-world activities.

Reading skills

Such skills are essential in the translation of real-world situations into mathematical models. The particular skill involved here might be called *mathematical reading,* to distinguish it from the skill needed to read a novel, say. This means that a student must be able to read *precisely*, to distinguish the important facts from the inessential ones, and to know the technical meaning of the words involved.

Writing skills

Mathematical pedagogy in general is moving away from purely computational and manipulative skills and toward explanatory and exploratory skills. Students are often asked not only to work problems, but *to explain to a fellow student how to work such problems.* In statistics, students are asked not only to compute sample means, but to *explain in writing what a sample mean actually is and how to compute it.*

Being able to carry out computations is no longer enough. Students must be able to write about and explain the computations and their purposes.

Knowledge of geometry

Simple geometric ideas and terms arise in every 100-level mathematics course. A student should be familiar with triangles, rectangles, squares, and circles, as well as with how to compute their perimeters and areas.

Mathematical abstraction, variables, functions

Variables and functions are the focus of all the 100-level mathematics courses. The idea of the *meaning of x* is particularly important, as it gives the basis on which algebraic manipulation is done. Students should be comfortable with working with x's and y's and a's and b's. Much of this is covered in the elementary algebra section, but the fundamental ideas are not, only the calculations.

Students should know

1. that variables represent numbers,
2. that sometimes the variable represents a number which we can freely choose,
3. that other times the variable represents a number which we want to find,
4. that a variable can be manipulated as if it were a number,
5. that a variable can be used in a problem to represent a quantity on which other quantities depend,
6. that dependence of one quantity on another can often be expressed by using a variable and algebraic expressions.

Mathematical descriptions of situations

Students should have familiarity with word problems. In college level mathematics, building the mathematical description of a real-life situation is fundamental. The focus may be on constructing a mathematical model by means of variables and functions, or on a probability and/or statistical model of a situation like political polling or free throw shooting, or on using a variable and constraints given in the problem to find the optimum value of the variable. These situations can be rather complex, and familiarity with the usual types of word problems in elementary algebra will make a student more successful at the college level.

Functions and their graphs

Fundamental to all mathematics at any level is the relationship between functions and their graphs. Statistics has graphs of the normal curve, of frequency distributions, and of the t-statistic. Mathematics courses have graphs of linear functions, quadratic functions, cubic functions, rational functions, the exponential functions, and logarithm functions.

Students should have some familiarity with the real line, the coordinate plane, and graphs of simple functions. Each course will expand on this theme by treating graphs of linear functions and quadratic functions in detail. Each course will also cover exponential functions. Students must be able to graph such functions and interpret graphs in general.

Calculator skills

Since the fall of 1996 students in *College Algebra* and above have been required to have a graphing calculator. Time will necessarily be spent on its operation, but familiarity with scientific calculators, their operation, their advantages, and their disadvantages will enhance the chances of success. Though the intensity of calculator use will vary from instructor to instructor, students will always be allowed to use calculators on examinations. Students should know how to carry out reasonably complicated calculations efficiently on a calculator and how the order of operations affects the buttons they'll push. They should also be aware of the fact that calculators rarely give *exact* answers and of techniques for making these answers as accurate as possible.

5 Inside and outside the classroom

Classroom activities, course pacing, homework activities, study habits and attitude, and instructor expectations are probably more important than factual knowledge or isolated skills for success in college. This section outlines some ideas and suggestions in each of these areas.

Classroom Activities

Students must be actively involved in their learning. They must feel responsible for it. A mathematics instructor should avoid lecturing and instead should serve as a questioner and recorder. Students should be asked what to do next and why, then gently guided toward the proper approach to a given problem. The instructor's role here is in the guidance and the recording on the chalkboard of the results of the students' suggestions.

This can be facilitated by using small group activities. Teams of two or three students should be set to work on given problems; the instructor's job is to circulate, oversee, answer questions, and make suggestions to each team. If time allows, teams can present their results to the entire class.

Another contributor to enjoyable activity in the classroom is making the mathematics relevant in some way. It is difficult to *always* make it relevant, sometimes pure mathematical skills need to be developed. But a real-world problem or situation which can be modeled using the mathematics should never be more than two steps away. Topics should be introduced by means of some physical or *real* activity. Here *real* means something that could happen or could be observed outside a mathematics classroom. If the instructor cannot think of any such activity, or if the text does not provide one, then serious consideration should be given to dropping the topic from the curriculum completely.

Mathematics classrooms should no longer be places where only the instructor talks. They should be full of activity!

Course pacing

A striking difference between high school and college classrooms is the pace of activity. While in a high school class one or two problems may be done, then students set to work on one or two more, in a typical college classroom five problems will be done and students will set to work on five more. Then there will be homework!

Efforts should be made to quicken the pace of mathematics classes. Certainly the pace in a freshman algebra class should not be as fast as that in a senior trigonometry class, but there should be a gradual increase in the tempo of activity in the class as the student progresses from ninth through twelfth grade.

The mathematics faculty should come to a consensus about the topics to be treated in each course and then should make every effort to cover those topics. Here is where the pace of the course should be decided. Knowledge of what goes on in other high schools as well as consultation with other high school and college faculty should lead to a satisfactory curriculum plan that is neither too fast nor too slow. An emerging principle in mathematics learning is that it is better to learn a few topics well rather than many topics poorly.

Homework activities

It is essential that students be given regular homework and that it be evaluated regularly and without delay. This means that students must have textbooks, or at the very least copies of problem lists. The college rule of thumb is that a student should spend two hours outside class for every hour in class. High school need not be this rigorous, but the habit of daily work in mathematics is extremely important and must be ingrained in each student.

Homework should be a mixture of problems similar to those done in class and problems which extend the thinking and techniques presented in the classroom. Both practice and thought process are important here. And, part of some homework should be active recognition of mathematics in the student's daily life.

Collaboration should not be discouraged; working in teams is a dominant mode of problem-solving outside the academic arena. However, each student must be responsible for the knowledge and skills developed in homework assignments.

Homework should be collected the next day and there should be no allowance for being late. It should be evaluated, either by the instructor or by grading the homework during class. It should be part of the grade in order to give the student sufficient incentive.

Study habits and attitude

Good study habits and a good intellectual attitude are the most important ingredients in a student's success in UOG mathematics courses. Students cannot and will not succeed without being able and willing to work efficiently outside the classroom. Students must realize that it is they who learn, not the instructor who teaches.

Classroom activities, course pacing, and homework can all contribute to the development of a good academic attitude. Students must expect to work and they must expect to struggle to understand mathematics; it is the instructor's job to motivate this work and help with the struggle. But mathematical skill and knowledge is not delivered to the student, it is actively acquired by that student. No one should come to a mathematics classroom without pencil and paper! No one should read a mathematics textbook without pencil and paper at hand!

Filling the mathematics classroom with student activity, setting a brisk pace, and judiciously assigning homework will go far toward encouraging the development of good study skills and good intellectual attitudes on the part of the student.

Instructor expectations

It is a tenet of current mathematical pedagogy that all students are capable of doing good, relevant mathematics, and that all students should be given the opportunity to learn good, relevant mathematics. This should form the basis of the instructor's expectations for the students. All of them should be expected to learn, all of them should be expected to attend class without fail, all of them should be expected to turn in homework on time, all of them should be expected to make public contributions to classroom activities. When you expect it, you get it.

6 July, 1999, Workshop Recommendations

The preceding sections have outlined the requirements for success in college-level mathematics. The goal now becomes finding a way to help high school students meet these requirements and thus succeed in their first university-level mathematics courses. With this objective in mind, under the direction of Aier Willyander a workshop was held in July, 1999, with four participants from Pohnpei, one from Kosrae, one from Chuuk, and consultant Dr. Arlo Schurle from the Division of Mathematics, University of Guam.

With the framework for success summarized thus far in their minds, the workshop participants examined high school curricula and their alignment with the requirements for college success, chose topics for increased emphasis, and began to develop teacher training materials which would assist high school instructors in the presentation of the emphasized topics.

After studying these documents and after some discussion, the participants concluded that the Pohnpeian **Strand I: Understand Number, Operation and Computation Concepts** and **Strand III: Understand Patterns, Functions and Algebra Concepts** should receive substantially increased emphasis. Only elementary topics from **Strand II: Geometry, Measurement and Trigonometric Concepts** were seen to be necessary to first-year college success, while **Strand IV: Understanding Statistics and Probability Concepts** was seen to be very important for students not going on to college, but less important for those who are college-bound. In summary, students must have a very solid foundation in Strands I and III, which should be taught by means of **Strand V: Problem Solving and Mathematical Reasoning, Strand VI: Mathematical Skills and Tools, Strand VII: Communicate Mathematically**, and **Strand VIII: Put Mathematics to Work.** Further details will be provided later in the recommendations and suggestions for teaching activities.

The participants then moved to a detailed study of the courses most directly dealing with Strands I and III, namely Prealgebra, Algebra I and Algebra II. The entire list of topics treated in each of these courses was put on view and then those which are most important for first year college success were highlighted. These topics were presented to the entire workshop and are listed in the recommendations section of this report.

Several different approaches to teacher training were then discussed, varying from bringing one or two teachers from each FSM high school to UOG all the way to providing workshops for all the teachers at each individual school. Recommendations follow.

Finally, the participants carefully went through the topics in Prealgebra which should receive increased emphasis and discussed teaching techniques and attitudes which could lead to improved student performance. Prealgebra was chosen since these topics are revisited at increasing levels of difficulty through Algebra I and Algebra II. Guidelines and details are provided in a later section of this report.

Recommendations

We present in summary form the recommendations of the mathematics workshop participants. Recommendations 1 to 5 require action at the administrative or system level, while the remaining recommendations can be implemented by teachers themselves.

1. There should be clear, frequent, and regular communication among teachers, principals, mathematics specialists, and directors' offices regarding the implementation of these recommendations.

2. All possible resources and support should be provided to implement these recommendations.

3. Workshops should be provided at each public high school in the FSM for all mathematics teachers to become acquainted with the topics needing increased emphasis and to learn and practice the teaching methodologies listed in this report.

4. Every student must pass Algebra I with a grade of C or better in order to graduate from high school.

5. Every student should take a mathematics course each year while in high school.

6. Mathematics instructors should do everything in their power to insure that students do homework every day in their classes.
7. Mathematics instructors should require students to *write* explanations of mathematical problem-solving methods and to *write* careful definitions of terms used in discussing mathematical ideas.
8. Mathematics instructors in Prealgebra, Algebra I and Algebra II should structure their lesson plans so as to stress the items listed in the section **Topics to be emphasized.**
9. Mathematics instructors should as much as possible incorporate the ideas and techniques listed in the section **Teaching methodologies** into their classroom activities.

Discussion

Recommendations 1–3 arise from the brutal fact that without additional training and support teachers cannot be expected to learn, understand, or incorporate new pedagogical ideas and consequently student performance cannot be expected to improve. The workshop participants felt that the most effective way to do this was on a school-by-school basis in such a way that *all* mathematics instructors at a given school would be exposed to the ideas in this report.

During the workshop it was pointed out that many of the students who must take developmental or remedial mathematics in college have never been exposed to algebra in any form. It was also felt that the learning of mathematics at the Algebra I level was a vital part of the high school education of every student — regardless of future plans — in these days of increasing use of technology and mathematical ideas. Finally, the consultant pointed out that one of the reasons for poor performance on placement examinations and in the first year of college was that students had not been grappling with mathematical ideas for a year or more, and consequently anything they may once have known had been forgotten. Recommendations 4 and 5 are intended to remedy these situations.

A recurring theme in the workshop on both the mathematics and English sides was the lack of good study habits and attitudes on the part of the students. The transition to college involves increased freedom on the part of the student to do or not do required work in a timely manner. Good study habits, in particular, regular and unfailing attention to work outside class, go far toward success in college. This prompts Recommendation 6.

Current practice shows that students acquire and retain mathematical ideas better when they are required not only to use them, but also to explain and describe them in writing. Also, writing itself is a necessary skill for college success. Recommendation 7 is a consequence.

Recommendations 8 and 9 and the following two sections in which these recommendations are given substance came from the extended discussions during the workshops.

7 Topics to be emphasized

The summary of workshop activities includes a description of how the following topics were chosen. In essence, they arose from the collaboration of the teachers themselves, who provided the list of topics treated in the courses, with the consultant, who pointed out which of these topics were most important in first year college mathematics.

PREALGEBRA EMPHASIS FOR SUCCESS IN COLLEGE YEAR ONE

1. Operations on numbers, order of operations
2. Variables and simple equations
3. Arithmetic of integers, number line, absolute value
4. Factors and factoring
5. Monomials, powers, and exponents
6. Simplifying, interpreting and comparing fractions
7. Adding, subtracting, multiplying and dividing fractions and decimals
8. Solving equations
9. Graphing simple equations in the coordinate plane
10. Introduction to polynomial algebra
11. Perimeter, area, volume, angle measure
12. ALWAYS APPLICATIONS

ALGEBRA I EMPHASIS FOR SUCCESS IN COLLEGE YEAR ONE

1. Introduction to algebra, expressions, grouping, exponents, combining like terms
2. Operations with real numbers, number line, removing parentheses
3. Solving equations
4. Powers, multiplying and dividing monomials, positive and negative exponents
5. Simplifying, adding, subtracting, multiplying polynomials
6. Factoring polynomials, common factors, factoring trinomials, factoring $a^2 - b^2$, solving equations by factoring
7. Adding, subtracting, multiplying, dividing rational expressions, complex fractions
8. Coordinate plane, lines and linear equations
9. ALWAYS APPLICATIONS

ALGEBRA II EMPHASIS FOR SUCCESS IN COLLEGE YEAR ONE

1. Graphing linear equations, slope, direct variation, parallel and perpendicular lines
2. Operations with exponents
3. Operations with rational expressions, complex fractions, equations involving rational expressions
4. Adding, subtracting, multiplying, dividing, simplifying radicals, indices greater than 2, rational exponents
5. Quadratic equations: graphing, factoring, quadratic formula, completing the square
6. Multiply and Add Method for solving systems of linear equations
7. Exponential functions
8. ALWAYS APPLICATIONS

8 Teaching methodologies

The following teaching tips and techniques were generated by a free-flowing discussion of ways to approach the Prealgebra emphasized topics. The underlying assumption is that students learn and remember mathematical skills and ideas much better when they construct and discover them on their own, or with the instructor as a guide. Mathematical knowledge cannot be delivered into a student's head, rather, the student must construct it there on her own.

We first give a list of examples, suggestions, ideas, and techniques which are keyed to the Prealgebra emphasis topics, that is, the tips under Section 1 below are aimed at improving the teaching of emphasized topic 1 in the list above. Some of this may be repetitive, some of it may not work for a particular instructor, but all of it should be understood and tried!

1. In order to introduce the order of operations, ask the students to each calculate something like $4 \times 2 + 1 - 3 \div 3$. When they give several different answers, point out that we'd like to get just one correct answer in such a problem, so we need to specify in which order the operations should be done. Then give the accepted order, and have the students do more examples.

   It's always good to get students to work in groups, perhaps combining a good student with a medium student and a poor student. In order to prevent the good student from doing all the work, you can say that you'll choose one of the group to present the work when finished.

   As a short quiz, ask the students to *write down* in proper English the correct order of operations.

   Note that whether we like it or not, calculators have a certain order of operations built in, so we must know what that order is and use it correctly.

2. It's good to begin with $\square$ as a symbol for a variable, since then it's easy to talk about how a variable is like a box into which you will later put a number.

   Magic Tricks are a good way to introduce variables and their use. For example, tell the students to choose a number, any number. Then add 3. Multiply the sum by 4. Add 6 to the product. Subtract the original number from the sum. Divide the difference by 3. Now, if the student tells you the final result, you'll be able to tell the student what the original number was, just by subtracting 6. The reason you can do this is that you will start with $\square$ rather than a specific number and when you then follow the instructions, simplifying as you go, you will end with $\square + 6$. A variable is necessary here since you do not know what number any given student will start with.

   An equation like $\square + 4 = 9$ expresses a *desire* on our part. That is, we *want* to find a number to put into the box which will make the equation true.

   Always try to get students to experiment and do many examples before giving general rules. Then ask them to write descriptions of their experiments and results.

3. Locations above and below sea level are a good way to introduce negative numbers, which would represent positions below the sea's surface. Temperature works as well, but negative temperatures aren't very familiar to FSM students! Working with money also works, so that we would interpret negative numbers as debts, but the idea of a debt loses some concreteness — it's difficult to hold a debt in your hand.

   Point out that there is a difference between a negative number and the operation of subtraction. They are of course very closely related, but they are different. This will help students sort out the problems involved in subtracting negative numbers.

   Use the number line extensively here to illustrate and work with negative numbers. For example, ask the students how to make a number line, and get them to give you a first point other than 0. Then ask them for another point to the right of the first. Finally, by moving enough steps to the left, you'll be forced to introduce negative numbers.

   The number line also illustrates the difference between negative numbers and subtractions. A negative number is just a number to the left of 0, while subtracting b from a means that we start from a on the number line and move in a direction *opposite* the direction from 0 to b.

4. To introduce factors and factoring, ask the students how many ways there are to write the number 6, for example, as $3 + 3$, 2×3, etc. Then list these on the board and circle the ones which give a factoring of 6. Now do the same for, say, 8. Finally, ask the students if they can tell you what factoring means. If so, good, but if not, then do more examples until they grasp the concept on their own. Introduce the idea of prime numbers by starting with, say, 48, and factoring until you can't factor any more. Do more examples. THEN, and only then, give a definition of prime number. This also leads naturally to the idea of exponents, since you'll have expressions like $2 \times 2 \times 2 \times 2 \times 3$, and you can then say that exponents are just a short way to write the product of the four 2's as 2^4.

5. Introduce the laws of exponents by examples first and always by referring to the *meaning* of the exponent. So, for example, $2^4 \times 2^6$ means the product of four 2's multiplied by the product of six 2's, which certainly must just be the product of ten 2's. Hence $2^4 \times 2^6 = 2^{10}$. Do more examples until the law is understood, then have the students state it.

6. This is an extremely important topic, since students who can interpret and compare fractions will find it much easier to add and multiply them. If your classroom has a window with five rows of four panes each (That's what our workshop room had!), then use it to discuss fractions with denominators 4, 5 and 20. Use whatever concrete object is available to begin the discussion of fractions. Have the students draw pictures and write descriptions of simple fractions. Write some fractions on pieces of paper, hand them out to the students, and ask them to line up in order of size of their fractions.

7. Use the meaning of fractions developed in the previous section to help the students discover how to add fractions with the same denominators. Use the window panes or whatever objects again. Fold a paper in half to show that $1/2 + 1/2 = 1$. Then fold the folded paper in half again to deal with 1/4's. Fold yet again for eighths. Use objects or pictures or pizzas to show why you can't add fractions by just adding their numerators and adding their denominators. Be concrete. Use materials like window panes or floor tiles whenever possible. Do not quote rules, but explain why. Work from examples to summarizing rules. Have the students write, write, and write.

8. Motivate solving equations by first doing applications or word problems. Start with simple equations and move to more complex ones. Always involve the students actively either individually or in groups.

9. Look around the classroom for a concrete example of a coordinate plane. Use window panes, or the pattern created by floor tiles, or even arrange the students' desks so that, for example, the coordinate pair (1, 2) refers to an actual location in the room. Have students move around to coordinates that you give them. Ask other students what coordinates they are currently occupying.
   Note that to graph a straight line, you just need two points that are on it. How you get those two points is irrelevant. Try to use this single general method for all the different forms of the equation of a straight line.

10. Explore the meaning of *polynomial* and *term* by writing some expressions on the board and circling the terms and the polynomials. Always give some examples that are NOT polynomials.
    Re-emphasize the meaning of variable and always have the students substitute values and calculate. This reinforces arithmetic skills and never lets them lose sight of the fact that polynomials are a way of prescribing calculations.
    To introduce the idea of like terms, you could start with adding 3 bananas and 2 papayas and 5 bananas and 8 papayas, then move from these concrete examples to abstract ones using variables. You could also use D for *dollar bill* and F for *five dollar bill* and then explore expressions like $(6D + 9F) + (4D + 3F)$ by asking how much money they have in total.

11. To teach the ideas of perimeter, area, volume and angle measures, have the students measure actual objects in the classroom. Or go outside in the parking lot. Try to avoid memorization of formulas; instead concentrate on understanding the meaning of the words. Have the students write those meanings.

12. Always try to begin and end sections with applications.

After giving these topic-by-topic teaching suggestions, the workshop participants considered the above list and formed the following general rules for improving student performance in mathematics.

1. The classroom should be student-centered. The instructor should act as a facilitator or guide rather than as a teacher.
2. Teach the students, not the lesson.
3. Use probing questions.
4. Ask the students, don't tell the students.
5. Be concrete.
6. Do examples and applications first.
7. Do not emphasize rules and properties. These rules and properties should be internalized before they are stated.
8. Reinforce arithmetic when moving into algebra.
9. Maintain standards.
10. Encourage students to experiment.
11. Have students explain their solutions orally and in writing.

9 NCTM Standards

The topics to be emphasized, the teaching techniques, and the general rules listed above have been generated by people currently teaching in FSM schools! On the other hand, it's important to see how the FSM ideas match with those of other school systems and national organizations. In this section we discuss the *NCTM Curriculum Standards for Grades 9–12*, prepared in 1989 by the National Council of Teachers of Mathematics (NCTM).

The NCTM makes four assumptions about teaching mathematics.

1. The goal of teaching mathematics is to help all students develop mathematical power.
2. WHAT students learn is fundamentally connected with HOW they learn it.
3. All students can learn to think mathematically.
4. Teaching is a complex practice and hence not reducible to recipes or prescriptions.

There are two ideas in Assumption 1. First, *all* students should develop mathematical power. This fully justifies the recommendation of the July, 1999, Workshop to require Algebra I for all high school graduates. *Every single student should be exposed to the ideas and power of algebra!* Second, *mathematical power* means the ability to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods to effectively solve nonroutine problems. Notice the emphasis here on exploring, reasoning, and nonroutine. It is not enough to provide students with a template for solving a type of problem and then expect them to plug numbers into that template. They must be able to use mathematics in innovative ways to solve problems for which they do not have a pattern of solution.

These ideas are fundamentally connected with Assumption 2. A reader of the first part of this manual might think that an instructor only has to drill students on routine algebra problems in order for the students to succeed in placement examinations and university mathematics. But this would ignore Assumption 2. You cannot expect students to REMEMBER techniques of algebra if they don't connect those techniques with real-world problems or don't learn those techniques by exploring and conjecturing on their own.

KNOWLEDGE IS NOT POURED INTO A STUDENT'S HEAD. IT IS CONSTRUCTED THERE BY THE STUDENT.

Much of the remainder of this manual is devoted to techniques which can be used to encourage and guide students in their own construction of mathematical techniques.

Assumption 3 again uses the word *ALL*. The NCTM goals for students are that

1. they learn to value mathematics,
2. they become confident in their ability to do mathematics,
3. they become mathematical problem solvers,
4. they learn to communicate mathematically, and
5. they learn to reason mathematically.

It should be pointed out once again how well these goals correlate with the framework for success in college mathematics and the teaching techniques devised by the July, 1999, Workshop participants.

Assumption 4 says that teaching is complex. This certainly applies to this manual. Not everyone will or can or even should use all the methods and techniques described above and below every day and in every class, but everyone SHOULD give each of them full consideration, try them when and if possible, use those that work for a particular teacher and a particular class, and modify those that don't work well to find the personal style and assortment of tricks that work for each particular instructor.

♦ ♦ ♦ ♦ ♦ ♦ ♦ ♦

SUMMARY OF CHANGES IN CONTENT

TOPICS TO RECEIVE INCREASED ATTENTION

ALGEBRA

- The use of real-world problems to motivate and apply theory
- The use of computer utilities to develop conceptual understanding
- Computer-based methods such as successive approximations and graphing utilities for solving equations and inequalities
- The structure of number systems
- Matrices and their applications

GEOMETRY

- Integration across topics at all grade levels
- Coordinate and transformation approaches
- The development of short sequences of theorems
- Deductive arguments expressed orally and in sentence or paragraph form
- Computer-based explorations of 2-D and 3-D figures
- Three-dimensional geometry
- Real-world applications and modeling

TRIGONOMETRY

- The use of appropriate scientific calculators
- Realistic applications and modeling
- Connections among the right triangle ratios, trigonometric functions, and circular functions
- The use of graphing utilities for solving equations and inequalities

FUNCTIONS

- Integration across topics at all grade levels
- The connections among a problem situation, its model as a function in symbolic form, and the graph of that function
- Function equations expressed in standardized form as checks on the reasonableness of graphs produced by graphing utilities
- Functions that are constructed as models of real-world problems

STATISTICS

PROBABILITY

DISCRETE MATHEMATICS

TOPICS TO RECEIVE DECREASED ATTENTION

ALGEBRA
- Word problems by type, such as coin, digit, and work
- The simplification of radical expressions
- The use of factoring to solve equations and to simplify rational expressions
- Operations with rational expressions
- Paper-and-pencil graphing of equations by point plotting
- Logarithm calculations using tables and interpolation
- The solution of systems of equations using determinants
- Conic sections

GEOMETRY
- Euclidean geometry as a complete axiomatic system
- Proofs of incidence and betweenness theorems
- Geometry from a synthetic viewpoint
- Two-column proofs
- Inscribed and circumscribed polygons
- Theorems for circles involving segment ratios
- Analytic geometry as a separate course

TRIGONOMETRY
- The verification of complex identities
- Numerical applications of sum, difference, double-angle, and half-angle identities
- Calculations using tables and interpolation
- Paper-and-pencil solutions of trigonometric equations

FUNCTIONS
- Paper-and-pencil evaluation
- The graphing of functions by hand using tables of values
- Formulas given as models of real-world problems
- The expression of function equations in standardized form in order to graph them
- Treatment as a separate course

SUMMARY OF CHANGES IN INSTRUCTIONAL PRACTICES IN 9–12 MATHEMATICS

INCREASED ATTENTION to—

- The active involvement of students in constructing and applying mathematical ideas
- Problem solving as a means as well as a goal of instruction
- Effective questioning techniques that promote student interaction
- The use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole-class discussions, project work)
- The use of calculators and computers as tools for learning and doing mathematics
- Student communication of mathematical ideas orally and in writing
- The establishment and application of the interrelatedness of mathematical topics
- The systematic maintenance of student learnings and embedding review in the context of new topics and problem situations
- The assessment of learning as an integral part of instruction

DECREASED ATTENTION to—

- Teacher and text as exclusive sources of knowledge
- Rote memorization of facts and procedures
- Extended periods of individual seatwork practicing routine tasks
- Instruction by teacher exposition
- Paper-and-pencil manipulative skill work
- The relegation of testing to an adjunct role with the sole purpose of assigning grades

10 Teaching tips

In this section we report on and compile problems, solutions, teaching tips, different approaches to common ideas, questions to ask individuals or groups, cooperative work, etc., which arose in the January, 2000, Workshop. Some of these were contributed by the consultant, others by the participants. It is difficult at best to describe many of these approaches and techniques on paper; nothing replaces actually being part of the workshop. However, every reader ought to find something of interest and something which can be used in the classroom. We break the material into subsections according to topics.

10.1 Building a new arithmetic

Students have difficulty sometimes in algebra because they don't know arithmetic very well, or are slow in doing calculations in their heads. It's important that instructors keep this in mind. It's also important for instructors to be thoroughly familiar with ordinary positive integer arithmetic, place value, subtraction, and division. The workshop participants spent some time creating and using a new arithmetic.

PROBLEM: Suppose we only had three fingers on each hand. Then how would we count?

SOLUTION: We counted around the room, starting with one. However, when we came to what we'd usually call "six," we pointed out that if we had only that many fingers, then we'd call this number "one-zero." We used this term rather than "ten," because "ten" already carries too much meaning for us. The counting quickly progressed as follows: one, two, three, four, five, one-zero, one-one, one-two, one-three, one-four, one-five, two-zero, ...

This is a good place to emphasize careful use of language, for example, to distinguish between the number itself and the name we use for that number. For example, $4+4$ is the same number whether we use our base 10 system in which its name is "eight," or the base 6 system introduced above in which its name is "one-two."

ONE OF THE MOST IMPORTANT THINGS TO EMPHASIZE IN MATHEMATICS COURSES IS THE PRECISE USE OF LANGUAGE. THIS IS ONE OF THE SKILLS THAT STUDENTS SHOULD CARRY WITH THEM FROM ANY MATHEMATICS COURSE ANYWHERE THEY GO.

GROUP WORK: Pairs of participants worked the following problems in the new arithmetic, then presented their answers on the board.

1. $312 + 435$

2. 4×33

3. 35×531

4. $5341 - 2432$

Participants were asked not only to get the correct answers, but also to explain the process used. Students should also be asked to do this. Often two different correct methods can be used. For example, Morgan Stanley obtained $2 + 5$ in the new base six arithmetic by taking 1 from 2, putting that with the 5 to make one six, with one left over. Thus $2+5 = 11_6$.

The participants themselves brought up the question of division. Can it be done? How hard is it? So, together on the board we divided 220_6 by 4 to get 33_6. (Note: the subscript 6's indicate we are writing the numeral in base six.)

Participants asked about checking the answer. In these base six problems, there are often two ways to check an answer. As usual, we can add to check a subtraction and multiply to check a division. We can also change everything to base ten, do the problem there, and change it back again.

The consultant pointed out that this going from base six to base ten, carrying out an operation, then returning from base ten to base six, and finally realizing that this gives the same result as carrying out the operation completely in base six, is an example of the very important mathematical idea of *isomorphism*, that is, of two different systems behaving fundamentally in exactly the same way.

These connections to powerful mathematical ideas should always be pointed out. Knowing such connections is an important reason that mathematics instructors should know much more mathematics than directly appears in the courses they teach. Bringing such connections into the classroom makes the mathematical experience immeasurably richer and consequently more meaningful and interesting to the students.

The participants noted the importance of mental arithmetic. Students should be able to reasonably estimate numbers such as 278×14. The difficulty of testing such mental arithmetic is clear: instead of estimating students will just do the problem. Consequently this should be an important part of in-class work. Students should continually be asked to estimate answers, then to explain how they arrived at the estimate.

Finally, it's important that students themselves make up problems, either for the class to do with the instructor's guidance, or for other groups of students to do. This reinforces the idea that mathematics is not handed out by the instructor, but in fact is created by the students themselves.

10.2 Order of operations

The consultant began by asking each participant to pretend that there was no commonly accepted order for the algebraic operations, and then to give the result of carrying out the following:

$$2 + 3 \times 5 + 4 - 12 \times 3.$$

Here are some of the methods and the answers obtained:

1. left to right: result 51,

2. right to left: result -79,

3. split into the first three numbers and the last three, then do it left to right: result 1,

4. group or split differently: various results,

5. do additions first, then multiplications, then subtraction: result 9,

6. do multiplications first, then addition and subtraction left to right: result -15.

It's important for the students to realize several things about this situation.

1. This is an unacceptable situation in mathematics. Some subjects allow for differences of opinion, for example, two readers may interpret a poem in distinctly different ways, and as long as each can support the interpretation, this is quite acceptable. However, in mathematics we insist that a well-posed problem have a unique correct answer. There may be several methods to obtain that answer, but if the methods are all correct, the final answers they give should agree.

2. Because we cannot have several different correct answers to such a computation problem, we must agree on a fixed order in which to perform the operations. Before such agreement, all the answers above are equally right and wrong.

3. The universally accepted order of operations has been fixed and we must now abide by it. Indeed, it is this order of operations that is followed by scientific calculators and computers.

4. Once again, mathematics is not something presented to us as a finished product. We must often make decisions ourselves about what should be considered correct.

Students should, after some practice carrying out operations in the prescribed order, be asked to write down in detail the rules that they are to follow in such calculations.

The participants noted that the order of operations is important even when shopping for groceries. The clerk does not add up all the prices and then multiply by the number of items, instead, the number of items at a given price is multiplied by that price, then the number of items at another price is multiplied by *that* price, and so on, until only at the end are all the subtotals combined into a single sum.

10.3 Integer arithmetic

Participants were asked why negative numbers were needed. Responses included

1. specifying heights above or depths below sea level,
2. debits and credits in your financial statement, and
3. temperatures, though not anywhere in the FSM.

Participants noted that debits and credits were not quite as good as the other examples, since it is hard to think about things like taking a debt away, but it is easy to think about moving up or down in altitude, or having the temperature go up or down.

Some of the difficulties of negative numbers can be explained nicely using the number line. Participants were given 3×5 cards on each of which was a number, and then they were asked to line up along the wall in order of their numbers. A number line was thus created, but it depended on choosing the location of 0, choosing the unit length, and choosing which direction was to be positive. The standard number line has positive to the right, but often in physics problems it is more convenient to choose other directions.

Another set of number cards was distributed, this time including not only numbers such as 3 and 5, but also numbers such as 3,678,412 or $-2,897,561$. The participants had to decide how to display all such numbers. The choice is between distinguishing 3 from 5, which causes the very large or very small numbers to disappear outside the building, or clumping numbers such as 3 and 5 very closely together in order to keep the very large numbers in sight. Again, there is no right or wrong choice in general; whether a given choice is good in a given situation depends on what you really want to see along your number line.

The language problem involved in discussing numbers such as 3,678,412 or $-2,897,561$ was discussed. There is a strong psychological tendency to say each of these numbers is *very large*, but in fact only 3,678,412 is large, while the other, being negative, is very small. A distinction must be made between very small numbers and numbers which are close to zero. Again, the need for precise language is evident. This can also lead to a good discussion of absolute value, which is precisely what measures how far a number is from zero.

Some of the difficulties involved in the handling of plus and minus signs might be eased by carefully distinguishing between a negative number and the operation of subtraction. When you write -7, the $-$ sign is part of the number itself, but when you write $3 - 7$, the $-$ sign stands for the operation of subtraction.

Addition and subtraction of integers can be physically carried out using the number line. Addition means you go in the direction of the next number, while subtraction means you go in the direction opposite that of the next number. Here positive numbers have the direction *to the right*, while negative numbers have the direction *to the left*. Thus $3 + 2$ means to start on the number line at 3, then move 2 steps to the right. On the other hand, $3 + (-2)$ means to start at 3, then move 2 steps to the left, because the negative number -2 has the direction to the left. As for subtraction, $3 - 5$ means to start at 3 on the number line and move 5 units *to the left*, because subtraction means to go in the direction opposite the number being subtracted. Finally, $3 - (-5)$ means to start at 3, then go 5 units in the direction opposite the direction of -5, ie., go 5 units to the right.

A number line was constructed by taping numbered cards to the wall. Participants were then asked to act out numerical expressions like $3+2-4-3+6$, which does not involve any negative numbers, and then to do the same with expressions like $5-8+(-4)-(-2)+7+(-4)-(-8)$. Finally, one of a pair of participants was asked to go to the position given by $3-(-4)$ while the other was asked to go to $3+4$. Similarly, one person was asked to go to $2-3$ while another was asked to go to $2+(-3)$. In a classroom, after several such physical examples, students should be asked to write down a general rule for such calculations.

Rules should always be extracted from lots of examples, not the other way around. Also, whenever possible, physical manipulatives such as the number line taped to the wall and/or students carrying number cards, should be used to introduce algebraic concepts and rules. After students are very familiar with these physical representations, they can move to pictures. And, once they are very familiar with pictures, they can move to symbol manipulation. It should be pointed out that symbol manipulation is actually needed, as are statements of rules for these manipulations, so that calculations involving very large or very small numbers can be done. Numbers such as $-389,745,900,211,678,432$ are difficult to represent physically!

The number line can be used to represent addition and subtraction very nicely, but does not serve so well to illustrate multiplication and division. One way to approach multiplication of integers is to build from the fact that 3×5 can be interpreted as the sum of three 5's. Then 3×-2 must be the sum of three -2's, which from the number line considerations is -6. To do -2×3, notice that this is the same as 3×-2.

Another approach is to build on the idea that mathematics is the study of patterns. The pattern in question here can be found from these facts.

$$\begin{aligned} 3 \times 3 &= 9 \\ 3 \times 2 &= 6 \\ 3 \times 1 &= 3 \\ 3 \times 0 &= 0 \\ 3 \times -1 &= ? \end{aligned}$$

Notice that the right-hand-sides display a pattern of decreasing by 3 at each step. Thus, to go from 0 to ?, we should decrease 0 by 3. The number line says this is -3. So, 3×-1 should be -3. Similarly, 3×-2 should be -6, and so on. Once this is established, we then have the following.

$$\begin{aligned} -2 \times 3 &= -6 \\ -2 \times 2 &= -4 \\ -2 \times 1 &= -2 \\ -2 \times 0 &= 0 \\ -2 \times -1 &= ? \end{aligned}$$

In this case, the pattern of the right-hand-sides is to increase by 2 at each step (the number line shows this). So, to go from 0 to ? we should increase 0 by 2, so that -2×-1 should be 2. Continuing, we'll get $-2 \times -2 = 4$, and so on.

Investigating such patterns give students reasons for the rules of signs other than "the book or the teacher says so." Such reasons are crucial to both understanding and retention!

10.4 College of Micronesia classroom laboratories

The third day of the workshop was spent at the national campus of the College of Micronesia. Yen-ti and Ray Verg-in volunteered their classes to serve as "laboratories" in which Dr. Schurle conducted the class while the workshop participants observed and then recorded their impressions in journal-type entries.

The topics discussed in the classes were fractions, factoring, and variables and algebraic expressions. During lunch the participants had a lively discussion of $\sqrt{x^2}$. Each of these subjects will be treated in one of the following sections, in which at times several COM classroom experiences will be combined.

10.5 Fractions

Two COM classes were heavily involved in calculations with fractions. When asked whether they were good at this, the students generally said they were "pretty good." The consultant then handed out sheets of paper on each of which was printed a large rectangle subdivided into six rows of four rectangles each. (This sheet is on the next page.) This was called a "windowpane," since often windows are divided into panes in just this way. Students were then asked to shade 2/3 of the window. Only one or two of the entire class did this correctly.

The class then discussed the meaning of the fraction 2/3. One interpretation is that it says to divide the whole, in this case the entire large window or rectangle, into three equal parts, and then to shade two of them. Several more such examples were discussed. Then, students were asked to stand up along the front of the room, and then to sit down in 3/4 of the chairs in the room. Again, several more examples were acted out in this way.

In another class, students were asked what their favorite fraction was. Eventually, they agreed on the fraction 1/2. When students were asked what 1/2 means, answers were generally either .5 or 50. Then, when specifically asked whether 1/2 was the same as 50, everyone said yes.

Students should be encouraged to give answers! Sometimes it's even true that wrong answers are better for learning purposes than correct ones, and any answer is always better than no answer. Students may be reluctant to participate in class discussions; making them aware of this attitude on the part of the instructor will encourage active participation.

Students were asked to go to the board and draw pictures of 50 and/or 1/2. For the former, one student drew a picture frame with a 50 inside, and for the latter, a student drew a semicircle. Finally, someone drew a picture of five rows of ten baseballs in each row. The instructor completed the semicircle to a full circle, explaining that a picture of 1/2 must show the whole that is being considered. Finally, the class was at the stage at which the three parts of the meaning of a fraction could be discussed: the whole, the bottom or denominator which gives the number of equal pieces into which the whole should be divided, and the top or numerator which gives the number of these pieces to take or shade or use.

The windowpanes were then distributed and students were asked to shade a picture of the fraction 2/3, just as in the first class. Several did this correctly but all were collected and discussed. A nice chorus of "ooohs" indicated that many students were understanding the meaning of a fraction, perhaps for the first time.

Once again, physical representations of mathematical ideas lead to better understanding. Instructors should always look around the classroom to find ways to illustrate the mathematics. Sterile mindless manipulation of symbols must be avoided; rather, underlying meanings of the symbols must be understood. The algebra and arithmetic of fractions are a particularly difficult area for many students. The COM experiments show that many students can carry out the calculations without understanding the underlying meaning of what they are doing. This has important implications for the scope and sequence of the curriculum.

Much, much more could be said about teaching the algebra of fractions, but there was not enough time in this workshop. Perhaps at least the ideas above can contribute to better understanding.

10.6 Factoring

Students were asked to write 12 in as many different ways as possible, using $+, \times, -, \div$. At first most students used only addition, for example, $6+6$ and $3+3+3+3$ and so on. Then someone used multiplication, getting 2×6 and 3×4 and so on. The instructor introduced $2 \times 3 + 6$, which then unleashed a flood of new ways to write 12. All these names for 12 were written on the board, and students were asked to categorize them. For example, some of them used only $+$, while others used only $\times$. The instructor then said that the ones using only $\times$ were called "factorings" of 12 and the numbers involved are called "factors" of 12.

Students were then asked to factor larger numbers in such a way that no factor could be factored further. This led to exponents as a way to efficiently write long lists of the same factor, that is, 2^4 is just a short way to write $2 \times 2 \times 2 \times 2$. The instructor mentioned the number 10^{100}, which is called a googol. As homework, the students were asked to factor 72, 96, and 144 completely.

10.7 Lunchtime discussion of square roots

During lunch on the COM campus the workshop participants had a spirited discussion of square roots, particularly whether the equation $\sqrt{x^2} = x$ was always true for any x. It is not, as the example $x = -3$ shows. However, some argued that in fact this is still OK, because, for example, $\sqrt{16} = \pm 4$. Dr. Schurle said in his opinion this is bad, because at any level of mathematics it is poor practice for a mathematical notation to allow a choice, and that convention says that the symbol $\sqrt{a}$ represents the *positive* number whose square is a. If we want or need to allow either the positive or negative number whose square is a, then we write $\pm\sqrt{a}$, as is done in the Quadratic Formula for the roots of a quadratic equation.

10.8 Variables, algebraic expressions, and functions

One of the COM classes was already doing algebra, so Dr. Schurle introduced the idea of algebraic Magic Tricks. Students were asked to carry out the following instructions.

> Choose any number whatsoever. Add 3. Multiply the sum by 2. Add the original number to the product. Divide the sum by 3. Add 4 to the quotient.

First, this give practice in the correct use of language, for example, you "add to" but you "multiply by." Students also get practice using the names of the results of arithmetic operations, ie., sum and product and quotient. It gives good arithmetic practice and in fact, if the arithmetic is not done correctly, then the Magic Trick to follow will of course not work.

It turns out that no matter what number a student starts with, the student will end with that number plus six, if the arithmetic is done correctly. So, Dr. Schurle asks each student what the final number was, subtracts six from that, and is able to tell the student what the original number was.

How and why does this work? The first idea is that Dr. Schurle must use something that can represent *any number whatsoever,* because he does not know in advance which number a student will choose. This is the meaning of a variable; the variable x represents a number but does not specify exactly *which* number. Then, Dr. Schurle carries out the operations using the variable x and gets successively x, $x+3$, $2(x+3)$, $2(x+3)+x$, $\frac{2(x+3)+x}{3}$, and finally $\frac{2(x+3)+x}{3}+4$.

Notice that each of these algebraic expressions comes from a list of instructions, and conversely, an algebraic expression precisely defines a list of instructions that tells the reader what arithmetic operations to carry out on an arbitrarily chosen number. This is precisely the idea of a function: a process that takes an input and provides an output.

Next, notice that the algebraic expressions can be simplified, which means that they can be written in other forms *without changing the final result of the operations.* In fact, careful algebra shows that $\frac{2(x+3)+x}{3}+4$ simplifies to just $x+6$. So, while the student carries out a fairly complicated list of instructions, Dr. Schurle realizes that it all simplifies to just adding 6 to the chosen number. This justifies the Magic!

The class was asked to make up a new Magic Trick. One student suggested "multiply by 0" as an operation, which is a brilliant idea, because then every person doing the Magic Trick will end with the same result, regardless of which number that person chose originally. This is a second type of Magic Trick which can be made to work by arranging for the variable to vanish from the algebraic expression.

Note all the ideas involved in Magic Tricks: variables, algebraic expressions, order of operations, algebraic simplification, functions, arithmetic, multiplication by 0, and perhaps even others.

As homework the students were asked to make up a Magic Trick of their own, with Ms. Verg-in adding the requirement that it include at least five steps.

10.9 Connections: functions and the real world

It is important to build connections between different mathematics courses, e.g., between prealgebra and geometry, and between Algebra I and Algebra II, and between Geometry and Algebra I, and between Algebra I and Statistics. It is even more important to build connections between all these mathematical subjects and the world outside the classroom. Word problems are one of the ways to build these connections, but instructors must always keep their eyes open to see other ways to reinforce such connections.

One of the most overriding and fundamental connections between all these topics is the general notion of a function. The idea of functions not only connects algebra to geometry, but connects all the purely mathematical ideas in algebra and geometry with real world situations.

The participants were asked to give their notion of what a function really is, or to give an example of a function. Here are some of them.

1. If you eat too much, you'll get fat.

2. A function is doing something to produce another thing.

3. A function is a set of instructions that takes an input and produces an output.

4. A function is set of guidelines to accomplish something.

All these ideas have the flavor of the function idea, but Number 3 above captures it best.

Functions are used to model real-world situations in which one quantity depends on another. For example, we could say that the weight of a tuna depends on its age, so that a newly-born tuna weighs very little, while a mature tuna weighs more. In terms of input-process-output, this can be modeled as follows.

INPUT: The age of the tuna.
PROCESS: Catch a tuna that is the input age and weigh it.
OUTPUT: Weight of the tuna.

Asking students to describe function models of such dependencies reinforces connections between mathematics and real life. It also asks students to use language in mathematics, which not only helps them understand the mathematics but also improves their English skills, both oral and written.

10.10 Connections: functions and algebra

We've already discussed the elements of this connection in Section 10.8 above. The point is that an algebraic expression precisely describes a procedure to apply to a chosen number. The variable, say x, itself gives the instruction to choose a number. The rest of the algebraic expression then gives arithmetic operations to carry out on that number. For example, the algebraic expression

$$\left[\frac{(3x-4)+x}{4}+7\right]-x$$

translates into these verbal instructions:

> Choose any number. Multiply it by 3. Subtract 4 from the product. Add the original number to the difference. Divide the sum by 4. Add 7 to the quotient. Subtract the original number from the sum.

Conversely, a list of verbal instructions can be translated into an algebraic expression. Participants were asked to carry out some of these translations.

How does algebraic simplification and all its rules enter this picture? One list of instructions could say to choose a number, add it to itself, add it again to the sum, and add it yet again to the sum. Another instruction could say to choose a number and multiply it by 4. The expression $x + x + x + x$ algebraically simplifies to $4x$, which means that though these lists of instructions are different and even involve different operations, their outputs always agree. The point of algebraic simplification is precisely this: correct algebra does not change outputs.

This point of view brings life to algebra. Complicated algebraic expressions are not just magical things waiting to be simplified, they are dynamic processes waiting for someone to USE them. Similarly, algebraic simplification is not just something teachers make students do, but it has a purpose: to make our work easier when calculating the result of various lists of instructions.

At this point the participants brought up the idea of composition of functions. In terms of our input-process-output idea, this is easy. To compose the function f with the function g to obtain the new function $f \circ g$, one just takes the output of g and uses it as the input for f. Pictures of boxes containing the list of instructions, with an input funnel on top and an output funnel on the bottom, are good ways to represent functions in general, and the composition of functions in particular.

An excellent reference for these ideas, as well as others to come involving functions and geometry, is *Functions and Mathematical Modeling*, 1999 edition, by Arlo W. Schurle. It's available from the UOG Bookstore and perhaps even from workshop participants.

10.11 Connections: functions and geometry

There are two important connections between functions and geometry. The first comes from the input-output pairs of the function. Each function naturally provides lots and lots of input-output pairs. When these are pairs of numbers, then we can use the coordinate plane to plot them. We then have the graph of the function. The participants drew several graphs, then the consultant drew a graph and asked the participants to give the outputs for various inputs. The latter is an important idea: functions can be given directly by graphs without formulas. Much more could be said about graphs of functions, but time was short.

The second connection between functions and geometry is that many of the ideas in geometry are in fact themselves functions. For example, perimeter is a function which takes as input certain geometric shapes, measures the distance around the shape, and gives the perimeter of the shape as the output. Similarly, area and volume can also be viewed as functions.

Participants were asked to explain what it meant to say that an irregularly shaped kitchen floor has an area of 120 square feet. Such explanations reach directly to the meaning of area and avoid mindless memorization of formulas. Meanings of terms such as perimeter and area and volume are more important than the formulas for computing such things. In fact, a good understanding of the meaning sometimes makes the formulas completely unnecessary. In any case, many examples and concrete physical representations should be explored before any formulas are given.

Writing explanations is an excellent assessment tool. Students must understand the concept if they are to write something sensible about it, whereas they can plug numbers into formulas without any real grasp of the idea. The participants wrote their explanations and then turned them in anonymously, and then these explanations were discussed by the entire group. Sometimes this works very well; other times the instructor may want to have the work identified.

10.12 Connections: similar triangles and palm trees

Participants were each given a quadrant (the last page of the manual shows four of these quadrants), a plastic straw, and some string. They were asked to go out and pick up a small rock. They then tied the string to the rock and tied the other end through a hole they made at the square corner of the quadrant. Finally, they taped the straw along the TOP of the quadrant so that it could serve as a sighting device. The resulting instrument can be used to measure angles between horizontal and, for example, the top of a palm tree.

Participants decided what measurements would be needed to determine the height of a palm tree in the parking lot, and then went out and took these measurements. They returned to the classroom and agreed that when they were 40 feet from the tree, the angle was 40°. We used these data to construct on paper a small triangle that would be similar to the one made by the ground and the palm tree. Using measurements from this paper triangle and the property of similarity, the height of the palm tree was found to be about 43 feet.

It is an easy step from this exercise involving similar triangles to a discussion of the trigonometric functions, which is exactly what the workshop participants did. One can then compare the results obtained from constructing the triangle and measuring with the results obtained from using values of the trigonometric functions, in this case tangent. Once again a connection is established between mathematics and trees in parking lots.

10.13 Exponential functions

Participants were asked how they would introduce exponents and exponential functions to an algebra class. Most said they would define what exponents meant and go from there, giving the algebraic manipulations and rules of exponents.

It is better to start with a real-life example. The participants discussed which of the following would be best.

1. The decay of radioactive thorium

2. Termite populations in the jungle

3. Savings account at the Bank of the Federated States of Micronesia

The problem with the first and third option is that they require considerable terminology and prior knowledge, either of radioactivity and chemistry, or of compound interest and finance. The example chosen was the following.

Example. The termite population in the jungle doubles every year. On January 1, 2000, there were 10 termites. Make a table of years from January 1, 2000, the corresponding dates, and the number of termites there will be on those dates.

Participants quickly made the following table.

YEARS	DATE	NUMBER OF TERMITES
0	January 1, 2000	10
1	January 1, 2001	20
2	January 1, 2002	40
3	January 1, 2003	80
4	January 1, 2004	160

Patterns (notice the appearance of patterns yet again) are not obvious from this table. However, participants noted that 10 was a special number in this problem, because that's how many termites were there at the beginning. They also noted that perhaps 2 would be a special number, because of the *doubling* of termite populations. Then we remade the table to exhibit more clearly these two special numbers.

YEARS	DATE	NUMBER OF TERMITES
0	January 1, 2000	10
1	January 1, 2001	$20 = 10 \cdot 2$
2	January 1, 2002	$40 = 10 \cdot 2 \cdot 2$
3	January 1, 2003	$80 = 10 \cdot 2 \cdot 2 \cdot 2$
4	January 1, 2004	$160 = 10 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Already a pattern is evident: the number in the YEARS column tells us exactly how many 2's to put in the TERMITES column. Also, if we want to consider more years, like 18 years or 24 years, we'd like some shorthand way to write all these 2's. This leads directly to whole number exponents. By 2^8 we merely mean the product of 8 2's. Using this shortcut notation, the table can be changed and extended as follows.

YEARS	DATE	NUMBER OF TERMITES
0	January 1, 2000	10
1	January 1, 2001	$20 = 10 \cdot 2^1$
2	January 1, 2002	$40 = 10 \cdot 2^2$
3	January 1, 2003	$80 = 10 \cdot 2^3$
4	January 1, 2004	$160 = 10 \cdot 2^4$
8	January 1, 2008	$10 \cdot 2^8$
17	January 1, 2017	$10 \cdot 2^{17}$
43	January 1, 2043	$10 \cdot 2^{43}$

Students should be asked to actually write out expressions like $10 \cdot 2^{17}$ and even $10 \cdot 2^{43}$. They should also be asked to actually compute some of the values! Most importantly, they should be asked to write an expression for the number of termites in the jungle after t years. The extended table's pattern certainly gives the answer as $10 \cdot 2^t$. Finally, with the exponential function 2^t now firmly in hand, a connection can be made with graphs by asking the students to draw a graph of this function.

This example makes quite clear that expressions like $2^{1/2}$ and $2^{2/3}$ should be meaningful, because they are precisely what is involved in determining the number of termites in the jungle after six months or eight months, that is, $1/2$ a year and $2/3$ of a year. One can discover the meanings of such expressions by using the laws of exponents, and one can discover the meaning of expressions like 2^{-3} by a study of patterns, as we did above for integer arithmetic. The workshop did not have time to deal with these topics. For more information, see the book *Functions and Mathematical Modeling* mentioned earlier.

11 Participant presentations

On the second day of the workshop the participants were asked to choose a topic from a course which they had taught or were teaching and present it to the rest of us as though we were the students. The following sections describe these presentations and give comments on the teaching techniques involved. Workshop participants are not identified by name, but are referred to as P1, P2, P3, P4, P5, and P6.

11.1 Finding the area of a triangle in the coordinate plane

P1 gave a demonstration on how to find the area of a triangle in a coordinate plane. He drew a right triangle in a coordinate plane and then asked the "students," that is, the other workshop members, to find its area by counting squares. They were asked to justify their answers. He then "doubled" the triangle to get a rectangle, used known facts about the area of the rectangle, then halved the result.

Comments on this presentation included the following.

1. Get students to explain more of their counting.
2. Do more such examples before writing any formulas.
3. Connect with the meaning of area. Don't TELL them to get the area by counting squares, rather ASK them what to do to get the area.
4. Put some triangles in strange positions to solidify the meanings of base and height.

11.2 The area of a right cylinder

P2 discussed the area of a right cylinder. He began by discussing figures in space, then specialized to cones, spheres, cylinders, and pyramids. He then defined a cylinder as follows.

> A cylinder consists of two congruent and parallel circular bases whose lateral surface is the infinite number of segments connecting the circles.

He then proposed a situation in which you are in charge of manufacturing cans. You must order sheet metal from which the cans will be made. What do you need to know? Answers are volume and surface area. He then discussed the meanings of the words diameter, circumference, height, and radius, and illustrated them by rolling up a sheet of paper.

The problem is then how we will use these measurements to determine the amount of material needed. P2 drew a picture of a cylinder cut along its edges and down its side and unrolled/unfolded. He then wrote formulas: $A_B = \pi r^2$, $A_L = lw$, $l = 2\pi rh$, and $A_B = \pi r^2$ again. He then pointed out that $A_L = 2\pi rh$ and $A_B = 2\pi r^2$ and that the total area would be

$$A_T = A_L + A_B = 2\pi rh + 2\pi r^2,$$

or $A_T = 2\pi r(h + r)$.

The class then worked together on this problem.

> A factory wants to make 5,000 cans each with radius 3.5 centimeters and height 8 centimeters. How many 4×8 sheets of metal will be needed?

First, the class computed the surface area of each can, and then multiplied this by 5,000 to see that 1,265,000 square centimeters of metal would be needed. They then found that each sheet of metal contained 29,768 square centimeters, divided 1,265,000 by 29,768 to get 42.495, and concluded that 43 sheets of metal would be required.

Comments on this presentation from the participants and the consultant included the following.

1. You should always give examples first, and only then, if at all, should you give formulas.

2. You should come prepared with visual aids when possible. For example, in this case toilet paper rolls would be excellent examples of cylinders. In fact, a little advance planning would suffice to have each student in possession of an actual example of a cylinder that they could each then cut apart.

3. You should not overwhelm students with definitions full of long words, as in the definition given of a cylinder. Students do not need this. It is enough that they understand what a cylinder is and be able to recognize one and distinguish it from other shapes like pyramids and spheres. Precise definitions will come later in their mathematical studies.

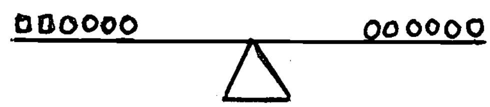
4. Don't use letters or variables when you can use words. Write out things like the formula for the area of a circle as $\pi(\text{radius})^2$, rather than as πr^2.

5. Avoid formulas if possible. In this case, you don't need the formula $A_T = 2\pi r(h + r)$ if you just remember how to find areas of circles and rectangles and point out how to slice the cylinder to get these pieces.

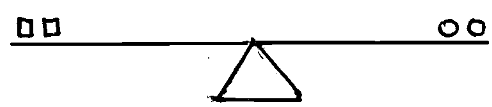
6. Be careful to use consistent notation in those few cases when you need such notation. For example, in this presentation the notation A_B was used for two different things.

11.3 Solving two-step equations

Participant P3 gave a presentation on solving two-step equations by using a balance scale to represent the equation. The example was the equation $2x + 4 = 6$, which he represented by the following diagram, in which the variable was pictured as $\Box$ and the constant unit by $\bigcirc$.



The idea is that the scale must remain balanced, so what we remove from one side we must also remove from the other. Thus, we remove four $\bigcirc$'s from each side to get this picture:



From this it's clear that $\Box = \bigcirc$, that is, $x = 1$. One can check this answer by replacing each $\Box$ by a $\bigcirc$ in the original picture.

Comments included the following.

1. This is an excellent way to introduce the method of solving such equations. In fact, if you could actually find a balance beam scale, it would be even better.

2. The choice of ◯ is unfortunate, since it is so close to a zero. Be very careful in choice of notation!

3. It would have been better to use an equation like $2x + 4 = 12$ so that □ doesn't end up being identical with ◯.

4. When a student asks a question, throw it back to another student instead of automatically answering it yourself.

11.4 The substitution method for solving linear systems

Participant P4 stated the topic, then began to discuss the example

$$\begin{aligned} x + y &= 4 \\ x - y &= 2 \end{aligned}$$

This led to the following slightly paraphrased discussion. S1 through S5 will be used to designate the participants who are acting as Students in this presentation.

P4:	How can we solve this system?
S5:	Guess and check!
S3:	Try 3 for x.
P4:	You need two values, one for x and one for y. How do we solve by substitution?
S6:	Can't we finish by guess and check?
Group:	Sure, try $y = 1$ and it works.
P4:	I'm going to solve by solving one of the equations for one variable. Which one?
S5:	Why? We've already gotten the solution.
P4:	Choose the second equation. Solve for what?
S5:	Solve for x.
P4:	No, I want to solve for y. We have $x = y - 2$ and we throw x to the other side to get $-y = -x + 2$. Now what? We need to get rid of the $-$.
S5:	Just erase the negative.

P4: No.
S6: Why not?
P4: Divide both sides by -1. [Some algebra went on here.] We get $y = x - 2$. Which shall we substitute this into?
S2: Use the second one.
P4: Well, then we get $x - (x - 2) = 2$, and then $x - x + 2 = 2$ and then $0 + 2 = 2$ and finally $2 = 2$.
S2: Does this mean the system is dependent and we get many solutions?
P4: Ah, OK. We cannot substitute into the second equation. So, we use the first equation and we get $x + (x - 2) = 4$, then $x + x - 2 = 4$, then $2x - 2 = 4$, then $2x = 4 + 2$, and $2x = 6$ and finally $x = 3$. Are there any questions?
S7: Could I multiply by -1 instead of dividing by -1?
P4: Yes.

Some comments on this presentation follow.

1. Though these presentations are out of context, it's good to remember that topics should be introduced by real-world examples. In this case, you need to think of a real-world situation which would lead to two equations in two unknowns.

2. The student interaction was great. But you need to be aware that students may lead you away from what you want to teach. It's hard to balance encouraging student comments with control of the classroom.

3. Beginning examples should be simple, but not so simple that students can immediately see the answer without using the technique you want to teach. In this case, probably slightly more complicated equations should have been used, so that guess and check wouldn't work so well.

4. The first method to use to solve a pair of simultaneous equations should be graphing, since this builds another connection!

11.5 Manipulatives and equations

Participant P6 used glasses and paperwads to solve the equation $2x - 3 = 1$. He asked how many paperwads should go into each glass, then moved three wads to the other side, then split the resulting four wads between two glasses. In this case the empty glass was used for the variable, which is a good idea. It's hard to represent negatives with manipulatives such as paperwads; perhaps different colors would be good. Be sure to read the next presentation for more on this.

11.6 Adding and subtracting integers

Participant P5 started by asking if there was anything any of the "students" would like to share. He then said that the lesson for today would be adding and subtracting integers, and asked what we mean by integers? The class responded with "whole numbers and opposites." P5 then asked for examples, and the class responded with 1, -1, 4, and 0. This prompted a student to ask whether zero is an integer. P5 said yes and asked whether 2/3 was an integer. A student said no, and P5 asked why not? This prompted a good discussion of names for numbers, and they then made a number line indicating the integers. There was very good interaction, just what you want in classroom, and there was very good debate about what an integer actually was, again something that should be part of every classroom.

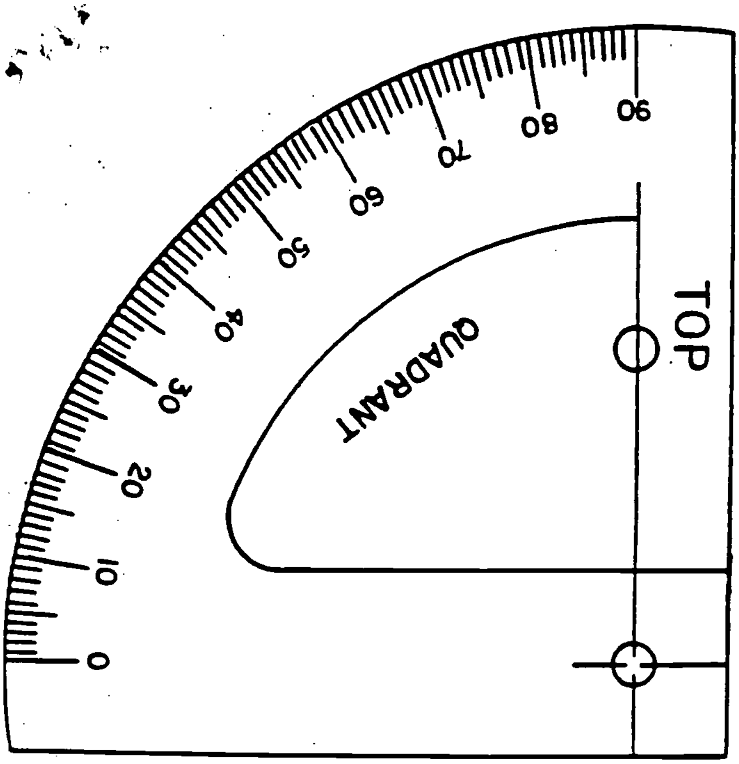
P5 returned the discussion to the addition of integers. To illustrate problems such as $9 + 2$, $-9 + (-2)$, $-5 + 3$, and $5 + (-3)$, he used both the number line and algebra tiles. After several examples, he asked if the students saw any patterns and if they could state a rule for adding integers. Then he moved on to subtraction, again doing several examples using algebra tiles before asking for and stating a rule.

Algebra tiles are small squares colored differently on the two sides, for example, red and yellow. Then the red side represents a positive unit, and the yellow side represents a negative unit. The basic fact is that a red and yellow pair cancels out, so you can either take away such a pair or add such a pair to your collection without changing the collection's value.

The problem $-9 + (-2)$ would be done by starting with 9 yellow tiles, then adding two more yellow tiles, and noting that you end with 11 yellow tiles, so that $-9 + (-2) = -11$. The problem $-5 + 3$ would be handled by starting with 5 yellow tiles, adding 3 red tiles, then noticing that red-yellow pairs can be taken away. There are three such pairs, and you are left with 2 yellow tiles, so that $-5 + 3 = -2$. Finally, to do $9 - (-2)$, you would try to take away 2 yellow tiles from a pile of 9 red tiles. But your pile has no yellow tiles, so you add two red-yellow pairs to your pile, so you'll have some yellow tiles to take away. When you do, you end with a pile of 11 red tiles, so that $9 - (-2) = 11$.

Comments included the following.

1. Multiple representations of the same problems are always a good idea, as in this case using both algebra tiles and the number line.
2. Students generally need lots of examples before they are ready to see the pattern and state a rule. We already know the rule, so it doesn't take so many for us!
3. Manipulatives such as algebra tiles are wonderful and can usually be made out of locally and cheaply available materials. In this case, all you need is cardboard that can be colored red on one side and, say, yellow on the other, and then can be cut into tiles.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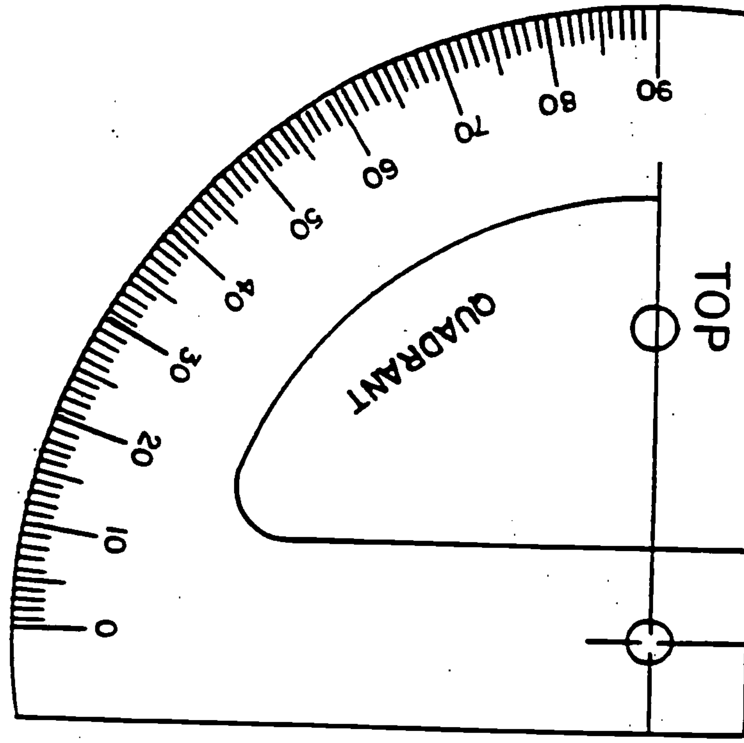
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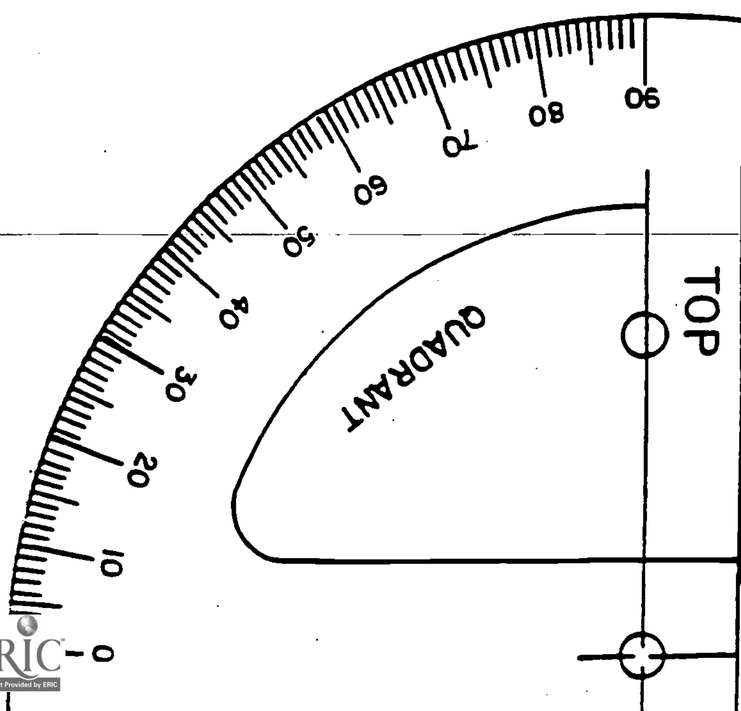
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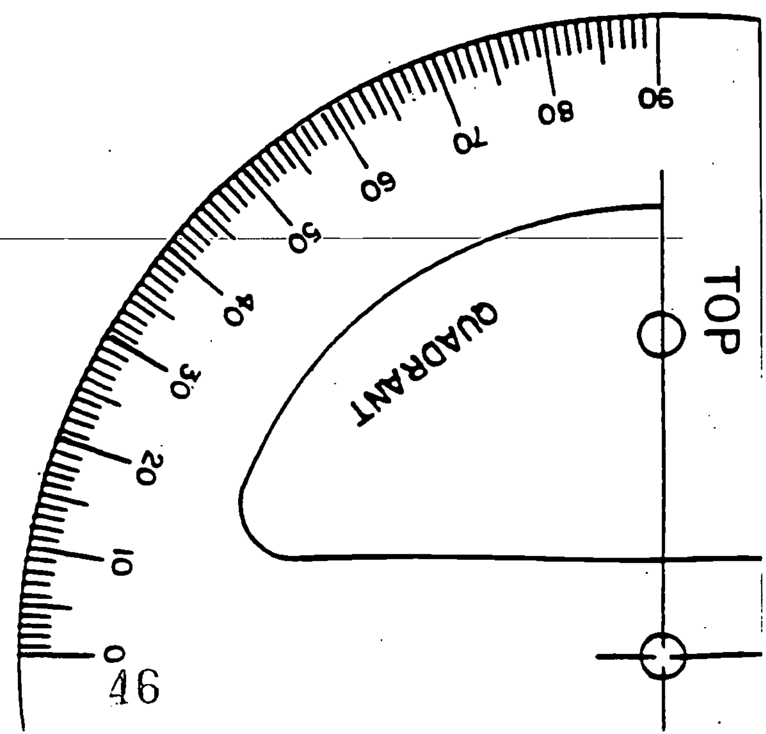
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