This literature review asserts that although findings indicate that children from a wide array of cultures and circumstances acquire similar mathematical understandings with little formal instruction, there is also substantial evidence that mathematical knowledge varies across social classes and cultural groups. The paper explores the literature devoted to explaining cultural differences in children's mathematics knowledge, including: (1) cultural variations in young children's mathematical activities (for example, Brazilian street children's competence with currency but not written numbers); and (2) linguistic variations in number systems. Educational implications are also considered. The paper concludes by noting that from the perspective of culturally-relevant instruction, racial and ethnic group differences in young children's mathematics knowledge reflect variation in the opportunities children have to engage in mathematical activities, rather than reflecting children's inherent ability. Teachers will need to move beyond a view of mathematics as a decontextualized and sequenced set of skills, and toward asking questions about, and giving value to how children use mathematics in their everyday lives. (Contains 47 references.) (EV)
Cultural Aspects of Young Children's Mathematics Knowledge

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"There are those who suggest that mathematics is culture-free and that it does not matter who is doing mathematics; the tasks remain the same. But these are people who do not understand the nature of culture and its profound impact on cognition." (Ladson-Billings, 1997, p. 700)

As the chapters in this volume document, there is abundant evidence that very young children perhaps even infants have some understanding of enumeration and arithmetic. Before they receive formal instruction in mathematics, most children are capable of counting and comparing small sets of objects, and most demonstrate some understanding of basic arithmetical operations, such as addition and subtraction. The procedures children use often differ from those taught in school, although they may share underlying mathematical principles that form the basis of school instruction (Carraher, Schliemann, & Carraher, 1988; Klein & Starkey, 1988; Nunes, 1995; Resnick, 1986). Findings that children from a wide array of cultures and circumstances acquire similar mathematical understandings with little formal instruction, have led some researchers to highlight universals in children's mathematical thinking and to suggest that "number is a natural domain of human knowledge" (Klein & Starkey, 1988, p. 6).

Perhaps it should be no surprise that children growing up in remarkably different settings develop similar mathematical knowledge. As Ginsburg and Baron (1993, p. 5) ask, "In what culture, however impoverished, does the child lack things to count? In what culture cannot one add to what one had before? Mathematical events and phenomena appear to be universal in the physical world." Support for universals in children's developing mathematical thought comes from research documenting substantial
and similar mathematical understanding in American subgroups that vary by race and social class (Ginsburg & Baron, 1993; Ginsburg & Russell, 1981; Saxe, Guberman, & Gearhart, 1987), and across groups of children from diverse cultures (Ginsburg, Posner, & Russell, 1981; Guberman, 1996; Petitto & Ginsburg, 1982; Saxe, 1991; Song & Ginsburg, 1987).

At the same time, there is substantial evidence that mathematical knowledge varies across social classes and cultural groups. National and social class differences in children's mathematics achievement in school, which have received considerable attention from the public and from researchers (Secada, 1992; Stevenson & Stigler, 1992; Tate, 1997), are mirrored to some extent by differences in children's performance before they begin school. Before receiving any formal instruction in mathematics, Asian children tend to perform on many mathematical tasks at higher levels than do American children (Geary, Bow-Thomas, Fan, & Siegler, 1993; Miura, 1987; Song & Ginsburg, 1987). In the U.S., Ginsburg & Russell (1981) found that social class was related to how children performed on several assessment tasks, although they emphasized the strength of preschool children's mathematical thought regardless of social class and race. Similarly, Saxe et al. (1987) reported that the preschoolers they studied, children from white middle- and working-class families, showed considerable mathematical knowledge, although middle-class children performed at more advanced levels than working-class children on tasks involving cardinality, numerical reproduction, and arithmetic. After reviewing research on social class disparities in preschool children's mathematics, Secada (1992, p. 633) concluded that "there is evidence to suggest that many poor children enter school at an academic disadvantage to their middle class peers."

Researchers have offered a variety of explanations for differences in the school achievement of children from different nations and backgrounds. Common sources of group variation in the mathematics achievement of school-aged children include the rigor and structure of the mathematics curriculum; the expectations and attributions for success that children, parents, and teachers possess; discontinuities between home and school; and cultural
values about mathematics and schooling (Geary et al., 1993; Gutstein, Lipman, Hernandez, & de los Reyes, 1997; Ladson-Billings, 1997; Pellegrini & Stanic, 1993; Stevenson & Stigler, 1992). We know much less about the reasons for group differences in the mathematical achievements of children prior to beginning their formal education.

Explaining Cultural Differences in Children's Mathematics Knowledge

The most common explanation for group differences in young children's mathematics knowledge proposes that differences are largely superficial. For instance, Klein and Starkey (1988) point out that although Oksapmin children of Papua New Guinea count by using a number system that refers to 27-body parts and has no base structure (Saxe, 1981), Oksapmin and Western children's counting are alike in that both adhere to core principles, such as one-to-one correspondence between number words and objects. Similarly, Nunes (1995) notes that when the street mathematics used by child vendors in Brazil and the mathematics taught in school were compared "in terms of the mathematical properties they implicitly used, the properties turned out to be the same" (p. 94). And, although Song and Ginsburg (1987) made an important distinction between children's formal and informal mathematics knowledge, they assessed Korean children's informal mathematics using tasks they had designed for American children, thereby revealing an unstated assumption that even children's informal mathematics knowledge may vary in rate but not in kind. From this perspective, the surface properties of children's mathematics and the rate of its development may vary across groups, but the acquisition of mathematical principles proceeds in a sequence that is both invariant and universal (Klein & Starkey, 1988). Similar explanations have been offered from a Piagetian perspective to account for cultural differences in cognitive development (Dasen, 1980; Piaget, 1972).

An alternative approach to understanding the development of children's mathematics knowledge, derived from the work of Vygotsky (1978) and his followers (Forman, Minick, & Stone, 1993; Wertsch, 1985), is especially well-suited for examining social and
cultural aspects of children's mathematical thinking. Terezinha Carraher (1989, p. 320) described this position well: "I think mathematical knowledge is not the result of the unfolding of cognitive development but a cultural practice in which people become more proficient as they learn and understand particular ways or representing numbers and quantity and operating on them." From this "cultural practices" perspective (Goodnow, Miller, & Kessel, 1995; Scribner & Cole, 1981), children's developing mathematics knowledge reflects the activities in which they participate and the cultural tools (e.g., number systems, algorithms) used in them (Saxe, 1991). Two bodies of research—one on children's mathematical practices outside of school and one on linguistic aspects of number systems illustrate this approach.

Cultural Variation in Young Children's Mathematical Activities

An obvious but little researched source of cultural differences in young children's mathematics knowledge is that communities vary in the opportunities they provide for children to engage in mathematical activities. For instance, Song and Ginsburg (1987) suggest that Korean preschool children displayed less competence than U.S. children on tasks designed to tap their understanding of informal mathematics, because there are few opportunities for Korean children to engage in mathematical activities before entering school. Cultural values that discourage many Korean parents from instructing their children in counting and money, and thereby, influence children's mathematical activities and achievements.

Children's everyday activities, and the mathematics that children learn by participating in them, are shaped by a multitude of factors. For many poor children, engaging in commercial transactions is an economic necessity (Nunes, Schliemann, & Carraher, 1993; Oloko, 1994; Saxe, 1991). In order to do so, they develop mathematical systems that are well suited to their purposes. Studies of young children engaged in candy selling in Brazil (Saxe, 1991) and street trading in Nigeria (Oloko, 1993) show that children with little formal instruction in mathematics develop particular mathematical skills in their everyday commercial transactions, including the ability to compute sales.
quickly and give correct change without recording numbers on paper. Only rarely do children use written numbers and calculation used in street mathematics (Nunes et al., 1993). Indeed, Brazilian candy sellers who were quite competent using currency had considerable difficulty identifying written numbers (Saxe, 1991). Similar to other forms of everyday cognition (Guberman & Greenfield, 1991; Scribner, 1984), the mathematics knowledge that children acquire in everyday activities can be used flexibly to solve a wide range of problems that arise in their practice, although it may be of limited utility in other settings (Lave, 1988; Schliemann, Araujo, Cassundi, Macedo, & Nicias, 1998). For instance, Oloko (1993) reported that Nigerian children who work in street trading performed much worse than nonworking children on a timed assessment of arithmetic skills, perhaps because the informal procedures used in street trading take more time than do conventional algorithms.

Children's mathematical activities also vary across American ethnic groups. In a study comparing Latino and Korean American children in first, second, and third grades, cultural values about teaching children to use money and parents' expectations for their children's school achievement were associated with differences in children's out-of-school uses of mathematics (Guberman, 1994). Although both Latino and Korean American children frequently engaged in mathematical activities, Korean American children more often engaged in activities intended to support the mathematics they were learning in school, such as being quizzed by parents on multiplication facts; in contrast, Latino children more often engage in activities that employed informal mathematics to accomplish a nonmathematical goal, such as adding coins to accomplish a commercial transaction. Differences in children's activities were associated with distinct strengths demonstrated on an assessment of children's formal and informal mathematics knowledge.

Only a few studies have examined the mathematical practices of children before they enter school, although it appears that most American preschool children engage frequently in a wide array of informal mathematical activities. Interviews with the mothers of American preschoolers indicated that children from both middle-
and working-class families engaged in many types of counting and calculation activities (Saxe et al., 1987). Some of the activities cited by mothers included nursery rhymes with number words in them, counting fingers and toes, reading number books together, board games that employ dice to determine how far to move ones token, card games that require comparing written values, and adding small sets of coins. Additionally, almost all mothers reported that their children regularly watched educational TV (Sesame Street) that included segments on counting and basic calculation.

Studying how children use mathematics in their everyday activities outside of school is important for educational practice. In order to build on the informal understandings and attitudes about mathematics that children bring with them to school, teachers need to understand and value children's everyday mathematical activities and the informal mathematics knowledge acquired in them (Fuson, Zecker, Lo Cicero, & Pilar, 1995; Sleeter, 1997; Tate, 1997). Everyday activities may serve as models for classroom-based instruction that builds on children's natural motivation and helps students to see the real-world application of the mathematics taught in school (Fuson et al., 1995).

Cultural Tools: Linguistic Variations in Number Systems

Participating in cultural practices typically entails the use of cultural artifacts or tools developed over the course of social history. The mathematical knowledge acquired through participation in cultural practices is interwoven with the mathematical tools used in them. Number systems, one example of a cultural tool, are human inventions that vary across time and location (Ifrah, 1985). Miura and her colleagues (Miura, 1987; Miura, Okamoto, Kim, Chang, Steere, & Fayol, 1994; see also Fuson & Kwon, 1991; 1992) conducted a series of studies indicating that properties of number systems may facilitate or impede the development of children's mathematical understanding. Miura (1987) notes that "Asian languages that have their roots in ancient Chinese (among them, Chinese, Japanese, and Korean) are organized so that numerical names are congruent with the traditional Base 10 numeration
system" (p. 79). In these Asian languages, spoken numbers correspond exactly to their written form: 14 is spoken as "ten four" and 57 as "five ten seven." In contrast, most European systems of number words are considerably irregular through 100.

Linguistic variations in numeration systems impose distinct demands on children learning to count. Children who speak Chinese, Japanese, or Korean need to memorize the first nine number words, the words for powers of ten (ten, hundred, thousand), and the order in which words are said (from the largest value to the smallest) (Fuson & Kwon, 1991). English-speaking children must memorize, in addition, the number words from 11 through 19, and the decade names (twenty, thirty, etc.) through one hundred. Apparently as a consequence of the differences, Chinese children make many fewer errors in saying number words to 19 than do English-speaking children in the U.S. (Miller & Stigler, 1987), and Korean children demonstrate mastery of counting much earlier than do American children (Song & Ginsburg, 1987).

Linguistic aspects of numeration systems have an impact on children's developing mathematics that extends beyond the rate at which they master the number sequence. By making apparent the values of each power of ten, and their strict correspondence between spoken and written numbers, Asian numeration systems facilitate children's understanding of base structure, place value, and associated arithmetical computations (Fuson & Kwon, 1991, 1992; Miura, 1987; Miura et al., 1994). Asian children demonstrated understanding the base-10 structure of two- and three-digit written numbers earlier than American first graders and before being introduced to tens and ones in school. When asked to represent two-digit numbers using base-10 blocks, Chinese, Japanese, and Korean children were more likely than children in France, Sweden, and the U.S. to create canonical representations that employ ten-unit blocks (e.g., four tens and two units for 42); in contrast, the U.S. and European children were more likely to use only single-unit blocks (42 units) in their constructions (Miura et al., 1994). Similarly, when asked the value of "carry" marks placed above the tens column in written addition problems, Korean children in second and third grades were more likely than American children to correctly identify it as a
value of ten, an indication of understanding place value (Fuson & Kwon, 1992).

Miura and Fuson suggest that differences in Asian and American children's mathematics performance reflect distinct cognitive representations of number: "for speakers of Asian languages, numbers are organized as structures of tens and ones; place value seems to be an integral part of the cognitive representation" (Miura, 1987, p. 82). English-speaking children are slower to construct these "ten-structured conceptions of number" (Fuson et al., 1997), are more likely to have conceptions based on single units, and are less likely to understand the meaning of individual digits in written numbers.

Other aspects of the languages used for numeration systems influence young children's mathematical competence. Stigler, Lee, and Stevenson (1986) found that the speed with which number words can be pronounced varies across languages and is associated with national differences in children's memory span for numbers. In Chinese, for instance, number words can be said more quickly than in English, and Chinese kindergarten children have a numerical span that exceeds that of English-speaking children by 2.6 digits (Geary et al., 1993). The ability to keep more Chinese than English number words in short term memory appears to influence early mathematical skills that require counting, such as simple addition problems, which young children typically solve by counting. Chinese kindergarten children solved three times as many addition problems (with addends less than five) than did American children, and Chinese children were more likely than American children to use verbal counting in their solutions (Geary et al., 1993).

The finding that linguistic characteristics of numeration systems are associated with young children's mastery of counting, understanding of place value and base-10 structure, and calculation indicates that language may be an important source of national differences in young childrens mathematics. Although the long-term impact of these early differences is unknown, comparing children's mathematics across languages serves to highlight some of the difficulties American children have in mastering the
elementary school mathematics curriculum, much of which is concerned with helping children master the concepts that are the focus of the comparative studies. Fuson and Kwon (1991) suggest that teachers should provide supports to children to compensate for irregularities in English number words.

**Educational Implications: Toward a Culturally-Relevant Pedagogy**

The knowledge that children bring with them to school has a powerful influence on how they interpret and learn the mathematics taught in school. Evidence indicates that programs such as Cognitively Guided Instruction (See Warfield, this volume; Fennema, Carpenter, & Lamon, 1991) that assist teachers to build on children's informal knowledge, help "children use their intellect well, make meaning out of mathematical situations, learn mathematics with understanding, and connect their informal knowledge to school mathematics" (Gutstein et al., 1997, p. 711). Valuing and building on children's informal mathematics knowledge is stressed in recent calls for the reform of mathematics instruction (National Council of Teachers of Mathematics, 1991). Often neglected, though, are cultural aspects of children's everyday experiences with and attitudes toward mathematics. What is needed is a "culturally-relevant" mathematics instruction (Gutstein et al., 1997), a pedagogy that is embedded in children's everyday contexts and connects with student's cultural ways of knowing. Ladson-Billings (1997) notes that mathematics is taught in schools in ways that may give an advantage to children from middle-class backgrounds. She writes that "middle-class culture demands efficiency, consensus, abstraction, and rationality" (p.699), whereas "features of African American cultural expression include rhythm, orality, communalism, spirituality, expressive individualism, social time perspective, verve, and movement" (p. 700). The curriculum, assessment, and pedagogy of school mathematics, she argues, are more congruent with the cultural experiences of middle-class children than they are for children from other backgrounds.

From the perspective of culturally-relevant instruction, racial and ethnic group differences in young children's mathematics knowledge reflect variation in the opportunities
children have to engage in mathematical activities. Group differences do not reflect children's inherent ability to learn mathematics (Secada, 1992). Nor is the fact that some children begin school with informal knowledge that does little to prepare them for school learning an indication of a deficiency in their home culture. Rather, similar to the Brazilian candy sellers (Saxe, 1991) described above, all children acquire mathematical understandings that are adapted to their circumstances (Pellegrini & Stanic, 1993), understandings that may be continuous or discontinuous with school mathematics. As Sleeter (1997, pp. 683-684) points out, "school mathematics is a very narrow subset of the range of mathematical thinking in which people have engaged."

Culturally-relevant mathematics instruction presents new challenges for teachers and teacher educators. Building on children's informal mathematics knowledge will require going beyond a view of mathematics as a decontextualized and sequenced set of skills that students need to memorize, and toward asking questions about and valuing how children use mathematics in their everyday lives (Nunes & Bryant, 1996; Sleeter, 1997). It will necessitate, as Ladson-Billings (1997) suggests, that teachers study their students and their backgrounds, becoming students of their students.

References


Cultural variation in a basic cognitive skill. Cognitive Development, 2, 279-305.


of research on mathematics teaching and learning (pp. 623-660). New York: Macmillan.


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